

THE UNIVERSITY OF CHICAGO

OCEAN CIRCULATION ON ICY MOONS

A DISSERTATION SUBMITTED TO
THE FACULTY OF THE DIVISION OF THE PHYSICAL SCIENCES
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

DEPARTMENT OF THE GEOPHYSICAL SCIENCES

BY
YAOXUAN ZENG

CHICAGO, ILLINOIS

AUGUST 2026

© 2026 by Yaoxuan Zeng

ORCID iD: <https://orcid.org/0000-0002-2624-8579>

To my parents

For my part I know nothing with any certainty, but the sight of the stars makes me dream

TABLE OF CONTENTS

LIST OF FIGURES	viii
LIST OF TABLES	x
ACKNOWLEDGMENTS	xi
ABSTRACT	xiv
1 INTRODUCTION	1
2 THE EFFECT OF SALINITY ON OCEAN CIRCULATION AND ICE-OCEAN INTERACTION ON ENCELADUS	8
2.1 Introduction	8
2.2 Experimental design	12
2.3 Simulation results	14
2.4 Discussion and conclusion	25
Appendix 2.A: Model description	30
2.A.1 Parameterization of sub-grid scale processes	30
2.A.2 Boundary conditions	31
2.A.3 Integration of the simulations	34
2.A.4 Sensitivity to domain width	36
Appendix 2.B: Heat budget	38
2.B.1 Meridional heat transport	38
2.B.2 Ocean heat budget of the surface layer	39
Appendix 2.C: Multiple equilibria in high salinity simulations	41
3 SYMMETRIC INSTABILITY IN A BOUSSINESQ FLUID ON A ROTATING PLANET	42
3.1 Introduction	42
3.2 Linear instability analysis	45
3.2.1 Instability criteria	45
3.2.2 Most unstable mode and growth rate	50
3.2.3 Instability diagram	51
3.2.4 Gravitational instability, inertial instability, and mixed symmetric in- stability	53
3.2.5 Low Rossby number limit	56
3.3 Numerical simulations	58
3.3.1 Simulation set up	58
3.3.2 Simulation results	60
3.4 Discussions	63
3.5 Conclusions	65
Appendix 3.A: Derivation of the dispersion relation	67
Appendix 3.B: Heat and zonal momentum transport by the most unstable mode .	72

	Appendix 3.C: Instability analysis for $b_r \sin \theta_0 = 0$	74
	Appendix 3.D: Instability analysis for $N^2 < 0$ and $\eta/f < 0$	75
	Appendix 3.E: Details of the simulation setup	78
4	SLANTWISE CONVECTION AND HEAT TRANSPORT IN ICY MOON OCEANS	81
4.1	Introduction	81
4.2	Numerical method	83
4.3	Flux-gradient relation of the heat transport	88
4.4	Implications for the global ocean heat transport and ice shell thickness on icy moons	97
4.5	Discussion	98
	Appendix 4.A: Numerical details	101
	Appendix 4.B: Eddy scale of the heat transport	106
	Appendix 4.C: Effects of zonal jets	109
5	PARAMETERIZING SLANTWISE CONVECTION IN ICY MOON OCEANS .	111
5.1	Introduction	111
5.2	Formulation of the scheme	112
5.2.1	Governing equations and coordinates	112
5.2.2	From traditional upright convection to slantwise convection	114
5.2.3	Tensor transformation	116
5.3	Benchmark tests	117
5.3.1	Model configuration	117
5.3.2	Simulation results	119
5.3.3	Sensitivity tests	124
5.4	Discussion	127
	Appendix 5.A: Rayleigh number and plume scale	130
	Appendix 5.B: An alternative choice of H_z outside the tangent cylinder	132
6	HEAT TRANSPORT IN ICY MOON OCEANS: INTERPLAY BETWEEN SLANTWISE CONVECTION AND BAROCLINIC EDDIES	133
6.1	Introduction	133
6.2	Theoretical prediction	134
6.3	Simulation design	138
6.4	Results	142
6.4.1	Europa-like simulations	142
6.4.2	Enceladus-like simulations	144
6.5	Application to different icy moons	146
6.6	Discussion	148
6.7	Conclusion	150
	Appendix 6.A: Baroclinic eddy length scale	153
	Appendix 6.B: Decomposition of the meridional heat transport	154
7	CONCLUSIONS	155

REFERENCES	159
----------------------	-----

LIST OF FIGURES

2.1	Simulation setup.	13
2.2	Temperature, salinity, and density fields.	15
2.3	Thermal wind balance in the quasi-equilibrium state of all simulations.	16
2.4	Flow fields.	17
2.5	Heat transports.	19
2.6	Meridional temperature gradients.	20
2.7	Temporal variability in the high salinity simulation with low horizontal diffusivity (HS_l).	21
2.8	Eddy fields.	22
2.9	Sensitivity test on conductive heat loss.	33
2.10	Sensitivity test on zonal extent.	36
2.11	Vertical heat fluxes near the surface of the ocean.	39
2.12	Sensitivity test on symmetry breaking.	40
3.1	Sketch of the coordinate system.	47
3.2	Instability diagrams for $\theta_0=60^\circ$	52
3.3	Growth rate and orientation of the most unstable mode with positive b_r here shown for latitude $\theta_0=60^\circ$	55
3.4	Background states in the simulations.	59
3.5	Time series for domain-averaged eddy kinetic energy (EKE).	60
3.6	Snapshots of the normalized buoyancy anomaly fields from simulations with eddy diffusivity and viscosity.	61
3.7	Instability diagram for $\theta_0 = 60^\circ$	76
3.8	Simulations with no diffusivity and viscosity.	80
4.1	Numerical simulation set up.	85
4.2	Snapshots of the flow and temperature fields.	89
4.3	Buoyancy flux–gradient relationships.	91
4.4	The CIA scaling compared with simulation results.	92
4.5	Time series of the temperature profile and buoyancy flux in simulation $L15_{0.625\Omega}$	93
4.6	Scaling of the radial (e_s) flux-gradient relation with best-fit exponents.	96
4.7	Interior and boundary layer.	102
4.8	Horizontal cross-spectra of the eddy heat transport.	107
4.9	Horizontal snapshots of the horizontal velocity fields at mid-depth ($r = 600$ m).	110
5.1	Coordinate systems.	113
5.2	Scalings of diffusivities.	115
5.3	Snapshots of convective plumes in convection-resolving (CR) simulations.	120
5.4	Benchmark test results.	121
5.5	Heat transport and temperature fields in the sensitivity tests with $Ro^* = 5.6 \times 10^{-3}$	123

5.6	Flow fields in the sensitivity tests with $\text{Ro}^* = 5.6 \times 10^{-3}$	124
5.7	Heat transport and temperature fields in the sensitivity tests with $\text{Ro}^* = 5.6 \times 10^{-3}$ and varying H_z outside the tangent cylinder.	132
6.1	Heat flux angle for slantwise convection.	135
6.2	Europa simulation setup.	139
6.3	Simulations for Europa-like oceans.	141
6.4	Simulations for Enceladus-like oceans.	145
6.5	Critical equator-to-pole ice thickness contrast for different icy moons.	147

LIST OF TABLES

1.1	Icy moon ocean parameters.	2
2.1	Parameters for Enceladus simulations.	12
2.2	Experimental Design.	12
3.1	Numerical simulation setups.	60
4.1	Numerical simulation parameters.	87
4.2	Time-averaged simulation results.	94
4.3	Ekman and Rayleigh numbers for numerical simulations.	104
5.1	Simulation setup.	118
5.2	Plume scales and diagnosed Rayleigh numbers for the convection-resolving simulations.	130
6.1	Comparison of the critical ice thickness contrast in different studies. .	150

ACKNOWLEDGMENTS

First of all, I would like to express my deepest gratitude to my brilliant advisor, Malte Jansen, for his guidance throughout this journey. Our weekly meetings have always inspired me, giving me confidence whenever I was stuck on a problem and helping me recognize important questions that I have overlooked. He always gave me the freedom to explore my own ideas while being there whenever I needed help. Talking with him and attending his lectures taught me that even ideas that seem well established in textbooks or research papers can still be questioned and understood more deeply. I could not have been more fortunate to be his student, and I only wish I had the opportunity to work with him for many more years.

I would also like to thank my committee members, Dorian Abbot, Edwin Kite, and Wanying Kang. I thank Dorian for continually reminding me of the significance of my research in a broader scientific community and for the thoughtful philosophical questions he raised during committee meetings, which have continually encouraged me to think about how and why I do research. I thank Edwin for his generous support in introducing me to the field of icy moons and helping me build a solid background in the field. He has also continually shared the latest developments in icy moon research with me and broadened my perspective on the field. I would especially like to thank Wanying, who has been not only an outstanding committee member and scientist, but also a wonderful friend. Besides offering invaluable advice in my research, she also provided tremendous help during conferences and throughout the postdoctoral application process. I would not have had the opportunity to meet so many scientists in the icy moon community or receive my postdoctoral fellowship without her generous support.

I would like to thank Wanying Kang, Geoff Vallis, and Glenn Flierl for their mentorship during the 2023 Geophysical Fluid Dynamics (GFD) Summer Program. I am also grateful to Tiffany Shaw for encouraging me to apply and recommending me to the program. I am

also thankful to Hao Fu, who was my roommate during the program. His friendship and encouragement helped me through the intense yet rewarding ten weeks. The GFD Summer Program was a transformative experience that showed me the beauty of scientific research and reinforced my determination to pursue an academic career.

I am also grateful to the other faculty members at the University of Chicago, including Noboru Nakamura, Tiffany Shaw, Da Yang (now at Stanford), Mary Silber, Elisabeth Moyer, Clara Blättler, Douglas MacAyeal, Meghana Ranganathan, and Pedram Hassanzadeh. I greatly appreciate their teaching and insightful suggestions in classes, Dynamics Group Meetings, and Climate Journal Clubs.

I would also like to thank the former Ph.D. students in Malte's group, Hailu and Tatsu, for helping me settle into the University of Chicago. I am grateful to my fellow students and postdocs for countless discussions and suggestions that have strengthened both my own research and my understanding of scientific research in general, especially Aidi, Allison, Bowen, Giorgio, Hao, Hung-I, Hsing-Hung, Jiacheng, Joonsuk, Katharina, Lichuan, Lin, Nuanliang, Osamu, Pengcheng, PJ, Pumeng, Rajat, Siming, Xuan, and Ziwei. I would also like to thank my friends outside the University of Chicago: Heng, Huazhi, Qiyu, Shuang, Sihe, Yingtian, and Yixiao, for their friendship and for the many valuable discussions that have shaped both my research and my academic career. I would especially like to thank Pengcheng, who has been my roommate at almost every conference, for the many discussions we shared during our travels.

Finally, I would like to thank the people who made everyday life in Chicago enjoyable. I am especially grateful to my roommates, Ziqi (along with his lovely cats, Rachel and Ross), Yixu, and Lichuan, as well as my neighbors Jiayu, Junyang, Kaixin, Lin, and Siming, and my own cat, Yuumi, for making my apartment at Windermere House, where I spent the past five years, feel like home. I would like to thank Lin and Xuan for walking through the final stage of PhD together. Our mutual support throughout postdoctoral applications and

graduation made the journey far less lonely. I would also like to thank my friends, especially Aoxuan, Bowen, Carly, Changyu, Chenxi, Hao, Kaixin, Kailong, Katy, Lai, Lichuan, Lin, Lingrui, Minxuan, Pengcheng, Pumeng, Qiao, Sandy, Sifang, Siming, Taige, Weidong, Wenyu, Xiaolin, Xuan, Yuke, Zherui, along with many other friends who have made my life in Chicago so joyful. Their friendship made my years in Chicago truly unforgettable. Although Chicago is 7,500 miles from my hometown, it never felt far from home because of them.

ABSTRACT

Icy moons are satellites orbiting the giant planets in the outer solar system. Some of them are believed to harbor global liquid oceans beneath ice shells, and are therefore prime targets in the search for life beyond Earth. Direct observations of these oceans are nearly impossible because the oceans are hidden beneath ice shells tens of kilometers thick. One of the few observational constraints is the ice shell thickness distribution inferred from shape and gravity measurements, which is closely linked to ocean circulation and heat transport. Understanding the dynamical processes that govern ocean heat transport is therefore essential for interpreting observations and assessing the habitability of icy moon oceans.

Ocean heat transport in icy moon oceans is primarily controlled by two processes: baroclinic eddies driven by buoyancy forcing at the ice-ocean interface associated with ice thickness variations, and slantwise convection driven by heating from the underlying silicate core. In this dissertation, we first study how salinity influences ocean stratification and circulation, as well as the meridional heat transport by baroclinic eddies on Enceladus. Then, we turn to slantwise convection in rapidly rotating icy moon oceans. Unlike the upright convection commonly assumed in terrestrial ocean models, slantwise convection in icy moon oceans is strongly influenced by planetary rotation and develops preferentially along the rotation axis. Traditional convection parameterizations cannot adequately represent convection in rapidly rotating icy moon oceans. We therefore use theoretical analysis and numerical simulations to investigate the instability criterion and heat transport scaling laws for slantwise convection in icy moon oceans. Based on these results, we develop a new slantwise convection parameterization for global ocean simulations that captures the poleward heat transport neglected by traditional convection schemes. We apply this parameterization to study the interaction between baroclinic eddies, which transport heat equatorward on Enceladus, and slantwise convection, which transports heat poleward.

Scaling analysis suggests that on Europa, Titan, and Ganymede, baroclinic eddies dom-

inate the meridional heat transport when the equator-to-pole ice thickness contrast exceeds about 10 m, whereas slantwise convection dominates when the contrast is smaller. On Enceladus, the entire ocean can convect when the salinity is sufficiently high, whereas a global stably stratified layer forms near the ocean surface when the salinity is sufficiently low. Regardless of salinity, baroclinic eddies are expected to dominate the meridional heat transport on Enceladus due to its large ice shell thickness gradient inferred from shape and gravity measurements. The resulting equatorward heat transport acts to smooth the poleward-thinning ice shell geometry, indicating that ocean heat transport alone is unlikely to maintain the present-day ice shell topography on Enceladus.

CHAPTER 1

INTRODUCTION

Icy moons are satellites orbiting giant planets in the solar system, mainly around Jupiter and Saturn (Table 1.1). Our curiosity about the icy moons can be traced back to their first discovery by Galileo in the 1600s. Centuries later, the *Galileo* and *Cassini* space missions have turned this curiosity into scientific discovery, uncovering subsurface oceans beneath the ice through gravitational and magnetic field measurements (Khurana et al., 1998; Neubauer, 1998; Zimmer et al., 2000; Kivelson et al., 2002; Iess et al., 2012; McKinnon, 2015; Thomas et al., 2016). Some of these hidden oceans are in contact with rocky cores and may hold the keys to life’s potential beyond Earth, making icy moons prime targets in the search for extraterrestrial life in the solar system (McKay et al., 2008; Hsu et al., 2015; Sekine et al., 2015; Waite et al., 2017).

Hidden beneath ice shells tens of kilometers thick, direct measurements of the subsurface oceans are infeasible. The only exception so far is Enceladus, where tidal stresses open cracks at the south pole, allowing material from the ocean interior to be ejected into space as geysers, some of which forms Saturn’s E-ring (Showalter et al., 1991; Nicholson et al., 1996; Spahn et al., 2006; Schmidt et al., 2008; Kempf et al., 2008), providing a unique opportunity for direct sampling. In the ejecta, mass spectrometry data from *Cassini* detected salts, CO₂, CH₄, H₂, and silica nanoparticles (Waite et al., 2006; Hillier et al., 2007; Postberg et al., 2009a,b; Hsu et al., 2015; Waite et al., 2017). The coexistence of H₂ and silica points to hydrothermal activity at the seafloor (Hsu et al., 2015; Sekine et al., 2015; Waite et al., 2017), processes that sustain chemosynthetic ecosystems and may have fostered life’s origin on Earth (Holm and Andersson, 2005; Shock, 2007; McKay et al., 2008). More recent analyses have identified six primary bioessential elements (C, H, O, N, P, and S; Postberg et al., 2023) and complex macromolecular organics (Postberg et al., 2018; Khawaja et al., 2025), further suggesting that Enceladus’s ocean could be chemically habitable. To better connect these

Table 1.1: **Icy moon ocean parameters.** The first section is adapted from Table 1 in Soderlund (2019). The heat flux Q , planetary radius R , and ocean depth H are based on Vance et al. (2018); the thermal expansivity α_T is taken from Vance (2017). Note that these parameters are representative values derived from selected interior models rather than direct observations. Estimates involving negative thermal expansivity are excluded, as such values suppress convection driven by bottom heating (Zeng and Jansen, 2021). Ω is the planetary rotation rate, and g is the surface gravity. The second section presents derived quantities based on these parameters. The natural Rossby number is defined as $\text{Ro}^* \equiv B^{1/2}(2\Omega)^{-3/2}H^{-1}$, where $B = Qg\alpha_T/(\rho c_p)$ is the bottom buoyancy flux, $\rho \approx 1000 \text{ kg m}^{-3}$ is the density, and $c_p = 4000 \text{ J kg}^{-1} \text{ K}^{-1}$ is the heat capacity of water. The plume scale is estimated as $l_{\text{plume}} = 5B^{1/4}(2\Omega)^{-3/4}H^{1/2}$ (see Chapter 4 for details). The existence of a global ocean on Titan remains under debate (Petricca et al., 2025), but we still include its parameters here for reference.

Parameter	Enceladus	Titan	Europa	Ganymede
$Q \text{ (mW m}^{-2}\text{)}$	16–83	14–20	23–123	15–107
$R \text{ (km)}$	252	2575	1561	2631
$g \text{ (m s}^{-2}\text{)}$	0.1	1.4	1.3	1.4
$\alpha_T \text{ (10}^{-4} \text{ K}^{-1}\text{)}$	0.1–1.3	0.4–4.2	1.9–2.5	1.9–4.4
$2\Omega \text{ (s}^{-1}\text{)}$	1.1×10^{-4}	9.2×10^{-6}	4.2×10^{-5}	2.0×10^{-5}
$H \text{ (km)}$	11–63	91–420	97–131	24–518
$B \text{ (10}^{-14} \text{ m}^2 \text{ s}^{-3}\text{)}$	0.4–27.0	19.6–294	148–938	120–1650
$l_{\text{plume}} \text{ (}\times 0.01^\circ\text{)}$	0.6–3.9	2.7–11.3	2.4–4.4	2.6–10.6
$\text{Ro}^* \text{ (}\times 10^{-5}\text{)}$	0.1–4	4–50	4–10	2–40

measurements and future observations to the ocean habitability, it is important to trace the observed surface materials back to their sources and formation processes within the ocean, which in turn requires a better understanding of ocean circulation.

Because Enceladus provides the most direct observational access to a subsurface ocean through its geyser activity, we will use it as the primary example throughout this introduction. Nevertheless, many of the fundamental processes governing ocean circulation and heat transport are shared among icy moons, and the concepts discussed here are broadly applicable to other ocean worlds, although their detailed planetary parameters may differ (Table 1.1). Enceladus has a solid core of about 190 km in radius, an ocean of about 40 km in depth, and is covered by an ice shell with an average thickness of 20 km, according to gravity (Iess et al., 2014) and topography (Nimmo et al., 2011; Tajeddine et al., 2017) observations and models. To maintain a global ocean beneath the ice shell and to account for the observed heat loss of approximately 10 GW from the south polar region (Spencer et al., 2006; Howett et al., 2011; Spencer et al., 2013), Enceladus must possess an internal energy source, most likely associated with tidal dissipation. Solar heating is insufficient because Enceladus receives little solar radiation due to its large distance from the Sun, the high surface albedo, and the inability of the remaining solar energy to penetrate the ice shell over tens of kilometers. Instead, the observed $O(10 \text{ GW})$ heat loss may be supplied by tidal dissipation within the porous rocky core and/or the ice shell, although the partitioning between them remains poorly constrained. Tidal dissipation within the ice shell, as well as turbulent dissipation of water in conduits at the south pole (Kite and Rubin, 2016), can potentially generate up to $O(10 \text{ GW})$ of heat (Beuthe, 2019; Souček et al., 2019). However, the rheology of Enceladus’s ice shell remains poorly understood, leading to substantial uncertainty in estimates of dissipation in the ice shell. The low bulk density of Enceladus suggests that the core may be porous, potentially allowing tidal dissipation rates of $O(10 \text{ GW})$ (Iess et al., 2014; Roberts, 2015; Travis and Schubert, 2015; Čadež et al., 2016; Choblet et al., 2017), although

the core porosity is also poorly constrained. By contrast, tidal dissipation within the ocean itself is generally considered negligible relative to dissipation in the ice shell and rocky core (Nimmo et al., 2018), owing to the small obliquity of Enceladus (Chen and Nimmo, 2011), the large ocean depth (Tyler, 2011; Hay and Matsuyama, 2017, 2019), and the presence of an overlying ice shell (Beuthe, 2016). In addition, libration has been proposed as a source of dissipative heating within the ocean, contributing up to $O(0.1 \text{ GW})$ (Lemasquerier et al., 2017; Wilson and Kerswell, 2018; Rekier et al., 2019).

Even for Enceladus, direct measurements of ocean circulation are difficult because of the surface ice shell. We therefore have to rely on indirect measurements to infer the ocean state, and one of them is the ice thickness distribution. On Enceladus, the ice shell thickness is not uniform. Based on shape, gravity, and libration measurements, Hemingway and Mittal (2019) inferred an equator-to-pole ice shell thickness contrast of approximately 10–30 km¹, assuming Airy isostatic compensation. The significant gradient in the ice shell thickness suggests an equator-to-pole ice flow. If the ice shell on Enceladus is in an equilibrium state, the equator-to-pole mass transport of the ice flow must be balanced by ice freezing at the equator and melting at the pole. The freezing and melting rate required to balance the ice flow is estimated to be a few millimeters per year², or equivalently, $O(0.01 \text{ W m}^{-2})$ of latent heat caused by freezing and melting (Čadek et al., 2019). This spatially varying latent heating could possibly be a result of the tidal dissipation in the ice shell, which is expected to peak at the poles and be amplified where the ice shell is thin (Kang and Flierl, 2020). Meanwhile, the tidal heating in the bottom solid core also peaks at the pole, and thus could also help to maintain the ice geometry. However, the ocean transports heat meridionally, which modulates the bottom heating pattern before reaching the surface. Understanding ocean

1. These values are based on the preferred model. For discussions of alternative assumptions regarding isostatic compensation and elastic flexural support, see Hemingway and Mittal (2019).

2. Throughout this dissertation we will use years (and similarly months and days) as referring to Earth-years.

heat transport is therefore important in interpreting the ocean circulation and stratification from surface observations of the ice shell thickness distribution.

We focus on two key dynamical processes that govern ocean circulation in icy moon oceans. First, heating from the silicate core (Choblet et al., 2017) drives convection. Under the strong rotational constraints expected on icy moons, convection tends to organize into columnar structures aligned with the planetary rotation axis rather than purely vertical plumes, namely slantwise convection. Second, horizontal density gradients at the ice-ocean interface drive overturning circulations and baroclinic eddies (Kang et al., 2022c; Zhang et al., 2024, 2025b). These gradients originate from freezing point variations caused by ice thickness gradients, as well as salt fluxes from spatial variations in freezing and melting rates.

These processes have very different spatial scales, making numerical modeling difficult. Specifically, convective plumes under icy moon conditions are extremely small, about $0.01\text{--}0.1^\circ$ in latitude (Table 1.1), and thus not resolved in current global ocean models. Most models, therefore, rely on convective adjustment schemes developed for Earth’s oceans (Marotzke, 1991), which assume purely vertical mixing (i.e., parallel to gravity). This assumption is incompatible with the rotationally aligned convection on icy moons, raising concerns about the accuracy of current parameterizations. In this dissertation, we use a hierarchy of models to study different heat transport mechanisms, first in isolation and then in combination.

In Chapter 2, we use a global ocean General Circulation Model (GCM) to investigate ocean circulation and heat transport on Enceladus. We focus on the role of mean ocean salinity. The results show that a stably stratified layer forms if the salinity is low enough, which suppresses convection in the upper ocean. Regardless of salinity, the meridional heat transport is dominated by baroclinic eddies and is equatorward, acting to smooth the observed poleward-thinning ice shell geometry. However, the convection scheme employed in this chapter is adapted from Earth’s oceans and does not represent the rotation-aligned slantwise convection expected on icy moons. This motivates the development of a new slantwise

convection scheme in Chapters 3 to 5.

In Chapter 3, we investigate the instability criteria for slantwise convection. We derive the general instability criterion for symmetric instability in a rotating Boussinesq fluid. For icy moons, the Rossby number is much smaller than unity (Table 1.1), so rotation plays a dominant role in the dynamics. In this regime, the instability criterion reduces to unstable stratification along the rotation axis, rather than along gravity as assumed in traditional convection schemes. This result demonstrates the need for a new convection parameterization that accounts for the rotation-aligned nature of convection in icy moon oceans.

In Chapter 4, we investigate heat transport by slantwise convection using local convection-resolving simulations in the icy moon parameter regime. We find that the heat transport along convective plumes follows the classical CIA (Coriolis–Inertial–Archimedean) scaling (Aurnou et al., 2020). In addition, slantwise convection produces significant heat transport across the plumes, for which we derive a new scaling law. The total heat transport is both upward and poleward, opposite to the equatorward transport associated with baroclinic eddies. This raises the question of how heat transport by slantwise convection compares with that by baroclinic eddies in global icy moon oceans. To address this question, the next chapter develops a parameterization for slantwise convection that represents the effects of unresolved convective plumes in global ocean models.

In Chapter 5, we develop a parameterization for slantwise convection in global ocean simulations. We benchmark the scheme against convection-resolving simulations and find that it successfully reproduces the meridional heat transport, although it does not fully capture the temperature structure, likely due to the lack of a proper representation of the boundary layers. By contrast, the traditional convection scheme fails to capture the meridional heat transport and produces even larger deviations in the temperature field. These results demonstrate that the scheme can represent the heat transport associated with unresolved slantwise convection in global ocean models, enabling direct comparisons between heat transport by

slantwise convection and baroclinic eddies.

In Chapter 6, we apply the scaling laws for heat transport by slantwise convection and baroclinic eddies to predict their relative importance in icy moon oceans, and test these predictions using global simulations with the slantwise convection parameterization developed in Chapter 5. We find that baroclinic eddies dominate the meridional heat transport on Enceladus, and the conclusions of Chapter 2 remain largely unchanged. On Europa, Ganymede, and Titan, the ice thickness contrast is poorly constrained. We show that the dominant transport mechanism depends on the amplitude of ice shell thickness variations: slantwise convection dominates when ice shell thickness variations are small, whereas baroclinic eddies dominate when thickness variations are sufficiently large.

In Chapter 7, we provide a summary and conclusion of the dissertation, as well as directions for future work.

CHAPTER 2

THE EFFECT OF SALINITY ON OCEAN CIRCULATION AND ICE-OCEAN INTERACTION ON ENCELADUS

An earlier version of this chapter was published as: Zeng, Yaoxuan, and Malte F. Jansen. “The effect of salinity on ocean circulation and ice–ocean interaction on Enceladus.” *The Planetary Science Journal* 5, no. 1 (2024): 13.

2.1 Introduction

Enceladus is believed to have a global ocean of about 40 km in depth under a global ice shell (e.g., Postberg et al., 2011; Patthoff and Kattenhorn, 2011; Thomas et al., 2016). The energy source that maintains the global liquid ocean and a heat loss of about 10 GW at the south polar region (Spencer et al., 2006; Howett et al., 2011; Spencer et al., 2013) is believed to primarily come from tidal dissipation (Iess et al., 2014; Roberts, 2015; Čadek et al., 2016; Choblet et al., 2017; Lemasquerier et al., 2017; Wilson and Kerswell, 2018; Beuthe, 2019; Souček et al., 2019; Rekier et al., 2019; Soderlund et al., 2020). Although the fraction of tidal heating coming from the solid core and the ice shell remains highly uncertain, some heating from the bottom solid core is expected, which fundamentally shapes the ocean circulation on Enceladus.

The ice shell on Enceladus is about 20 km thick on average, and is not flat: it is thickest at the equator (over 30 km) and thinnest at the south polar region (less than 10 km). There is also an interhemispheric asymmetry in the ice shell with ice being thinner at the south pole than at the north pole (Beuthe et al., 2016; Čadek et al., 2019; Hemingway and Mittal, 2019). The significant gradient in the ice shell thickness suggests an equator-to-pole ice flow. If the ice shell on Enceladus is in an equilibrium state, the equator-to-pole mass transport of the ice flow must be balanced by ice freezing at the equator and melting at the pole in

the order of 0.01 W m^{-2} (Čadek et al., 2019).

Freezing and melting of the ice is determined by the heat budget. The associated latent heat (Q_{LH} , with melting defined as positive) has to match the imbalance between the heat flux from the ocean to the ice (Q_{oi}) and the upward heat flux within the ice shell. Since the ice shell on Enceladus is unlikely to be convecting (Nimmo et al., 2018), we assume that the upward heat flux in the ice is purely due to conduction (Q_{cond}). The energy balance at the ice-ocean interface is then: $Q_{cond} + Q_{LH} = Q_{oi}$. The conductive heat flux at the ice-ocean interface in turn is modulated by the ice shell thickness, as well as the tidal heating in the ice shell (Q_i). Without strongly localized tidal heating in the ice shell, conductive heat loss is strongest at the pole where the ice is thinnest, which would favor ice freezing at the pole - the opposite of what is needed to maintain the ice geometry. Tidal dissipation in the ice shell could be a possible answer to the problem, since it is strongest at the pole and is amplified where the ice is thin (e.g., Beuthe, 2018, 2019), and thus could potentially allow for melting at the pole. Alternatively, the heat flux from the ocean, Q_{oi} , could be amplified at the pole. The global ocean heat flux to the ice comes from the tidal heating in the bottom solid core that peaks at the pole (Choblet et al., 2017), but the spatial pattern is modulated by the ocean circulation, which is the focus of this chapter.

Previous studies have explored the global ocean circulation on Enceladus and other icy moons (Soderlund et al., 2014; Travis and Schubert, 2015; Soderlund, 2019; Amit et al., 2020; Lobo et al., 2021; Kang et al., 2020; Zeng and Jansen, 2021; Ashkenazy and Tziperman, 2021; Bire et al., 2022; Kvorka and Čadek, 2022; Kang and Jansen, 2022; Kang, 2022; Kang et al., 2022a,c; Jansen et al., 2023). However, there have not yet been simulations with fully coupled ice-ocean thermodynamics in a 3-D global domain. The published simulation results vary substantially in their ocean circulation and ocean-ice heat fluxes, likely as a result of the large uncertainties in the external parameters and the difficulties in modeling the global ocean in a realistic parameter regime. The real regime of ocean circulation and heat transport on

Enceladus therefore remains uncertain.

One important factor determining the ocean circulation and heat transport is salinity (S). Salinity determines the equation of state of water, thus affecting ocean stratification and circulation. Most importantly, if the salinity is lower than a critical value, the thermal expansivity ($\alpha \equiv -(1/\rho)(\partial\rho/\partial T)$ where ρ is density and T is temperature) is negative near the freezing point. As a result, in such a low salinity ocean, a stably stratified layer forms in the upper ocean with bottom heating (Melosh et al., 2004; Zeng and Jansen, 2021). The threshold for the negative thermal expansivity to exist under the pressure found in Enceladus’s ocean is around 20 g kg^{-1} (Zeng and Jansen, 2021; Kang et al., 2022c). There are different methods to estimate the salinity of Enceladus’s ocean, based on the observations of salt-rich ice grains (Postberg et al., 2009b), the existence of silica nanoparticles (Hsu et al., 2015), using models of boiling water in ice conduits (Ingersoll and Nakajima, 2016), equilibrated water-rock interaction (Zolotov, 2007), ocean geochemistry (Glein et al., 2018), and ocean heat transport (Kang et al., 2022c). The estimates vary from study to study, giving a wide range of $2\text{--}40 \text{ g kg}^{-1}$. Therefore, both “high salinity” ($S > 20 \text{ g kg}^{-1}$) and “low salinity” ($S < 20 \text{ g kg}^{-1}$) scenarios should be carefully considered when studying the ocean circulation on Enceladus.

Salinity can also influence the overturning circulation driven by surface density contrasts, and modulate the ocean heat transport, which further affects the ice geometry. Kang et al. (2022c) model the ocean circulation with prescribed ice thickness and prescribe melting rates at the pole and freezing rates at the equator as required to balance the estimated ice flow induced by the ice thickness gradient. They find that the surface density contrast between the equator and the pole drives an overturning circulation in each hemisphere. The direction of the circulation depends on the mean salinity of the ocean, but the ocean heat transport is found to always be equatorward because the freezing point at the pole (with thinner ice and hence lower pressure) is higher than at the equator (with thicker ice and hence higher

pressure), and the heat transport is always from the warmer pole to the colder equator. The equatorward ocean heat transport favors freezing at the pole and melting at the equator and thereby acts to reduce the equator-to-pole ice shell thickness gradient, which is known as the ice pump effect (Lewis and Perkin, 1986).

The partitioning of tidal heating between the ice shell and the bottom solid core may also affect the ocean heat transport, further influencing the asymmetric ice shell geometry between the two hemispheres. Kang et al. (2022a) prescribe an ice geometry with an interhemispheric asymmetry where ice is slightly thinner in the southern hemisphere. They incorporate fully coupled ice-ocean thermodynamics in a 2-D domain with tidal dissipation in both the ice shell and the bottom solid core. They find that when the heating is dominated by tidal dissipation in the ice shell, there is stronger melting at the south pole than the north, and the overturning circulation transports heat to the south, thus enhancing the asymmetry; instead, when the heating is primarily in the core, there is stronger freezing at the south pole than the north, and the overturning transports heat to the north, thus suppressing the asymmetry.

Here we study the effect of salinity on the ocean circulation and ice-ocean interaction on Enceladus, focusing on the ocean heat transport. We carry out 3-D global ocean simulations using the Massachusetts Institute of Technology General Circulation Model (MITgcm) (Adcroft et al., 2018), with prescribed ice geometry and fully coupled ice-ocean thermodynamics. We exclude tidal heating in the ice to focus specifically on whether ocean heat transport from the solid core can maintain the observed ice topography and the associated pattern of freezing and melting. Section 2.2 describes the model configuration and experimental design. Section 2.3 introduces the numerical simulation results, focusing on ocean stratification, circulation, and heat transport. Section 2.4 compares the simulation results with previous studies and provides concluding remarks.

Table 2.1: **Parameters for Enceladus simulations.** The values are taken from Čadež et al. (2019) with some simplifications.

Parameters	Value
Surface gravity $g(r_s)$	0.113 m s^{-2}
Rotation rate Ω	$5.31 \times 10^{-5} \text{ s}^{-1}$
Ocean bottom radius r_c	192 km
Mean ice-ocean interface radius r_o	232 km
Mean ice surface radius r_i	252 km

Table 2.2: **Experimental Design.**

Case label	Case description	Mean ocean salinity S (g kg^{-1})	Horizontal turbulent diffusivity κ_h ($\text{m}^2 \text{ s}^{-1}$)
HS_h	High Salinity + high κ_h	35	0.5
HS_l	High Salinity + low κ_h	35	0.05
LS_h	Low Salinity + high κ_h	8.5	0.5
LS_l	Low Salinity + low κ_h	8.5	0.05

2.2 Experimental design

We perform numerical simulations using the MITgcm to solve the non-hydrostatic equations for a Boussinesq fluid in a rotating spherical shell in a 3-D domain, where all sphericity terms are preserved, including all components of the Coriolis force (Adcroft et al., 2018). Radius, rotation rate and gravity are set to be the same as Enceladus, and the vertical variation of gravity is also taken into consideration (see Table 2.1 and Zeng and Jansen, 2021). We use a non-linear equation of state (Jackett and McDougall, 1995) so that the thermal expansivity becomes negative near the freezing point in a low salinity ocean.

We simulate an ocean with a depth of 40 km on average over a zonal range of 15° with zonally periodic boundary conditions and a meridional range from 85.5°S to 85.5°N , with free-slip, no-normal flow conditions at the meridional boundaries. At the surface of the ocean, we prescribe the ice geometry and simulate the ice-ocean thermodynamics. In order to include the ice pump effect, we set the ice geometry to be an idealized (zonally-invariant

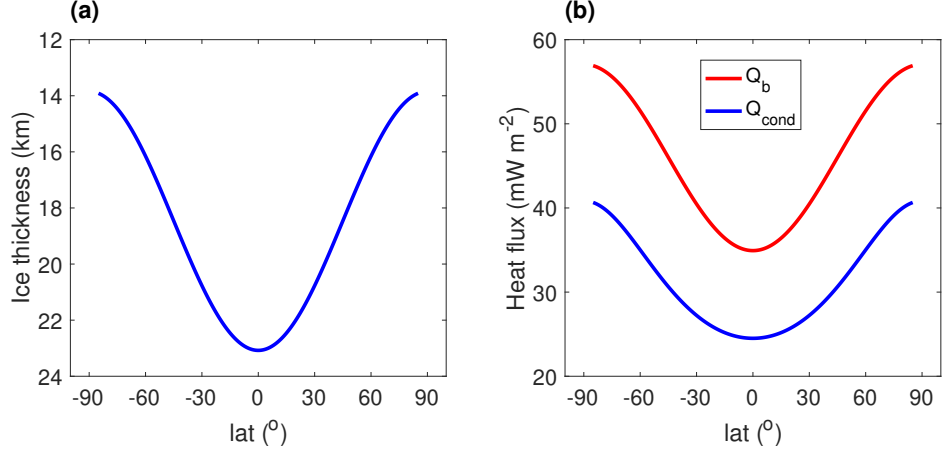


Figure 2.1: Prescribed ice topography (a), corresponding conductive heat loss (b, blue line), and bottom heating (b, red line) per unit area, as a function of latitude. The global integral of bottom heating balances the conductive heat loss. The apparent offset between the blue and red lines in panel (b) is due to the different surface areas at the bottom and ice-ocean interface.

and interhemispherically symmetric) approximation to Enceladus’s inferred ice topography (Tajeddine et al., 2017; Čadež et al., 2019), with ice thinning towards the pole. We do not allow the ice shell thickness to evolve (which would occur on a timescale that is much longer than the duration of the simulations) but compare the freezing and melting rate calculated in the simulations with the expected rate that is required to maintain the ice geometry against the ice flow. This approach is the same as used in Kang et al. (2022a) but different from Kang et al. (2022c) (where the freezing and melting rates are prescribed). At the bottom of the ocean we apply a prescribed heat flux based on Choblet et al. (2017) (Fig. 2.1). The detailed model description can be found in Appendix 2.A.

We carry out simulations with both high salinity (35 g kg^{-1} , same as Earth ocean) and low salinity (8.5 g kg^{-1} , suggested by Glein et al., 2018) to study the effect of salinity on ocean circulation and ice-ocean interaction on Enceladus. For each salinity, we use two different horizontal turbulent diffusivities (κ_h), $0.05 \text{ m}^2 \text{ s}^{-1}$ and $0.5 \text{ m}^2 \text{ s}^{-1}$, to test how sensitive the results are to this poorly constrained parameter, which is meant to mimic the effect of unresolved eddies and directly affects the meridional heat transport. The experimental

setups are summarized in Table 2.2. All simulations are integrated to a quasi-equilibrium state where the total energy imbalance is less than 3% of the seafloor heating rate (see Appendix 2.A.3).

2.3 Simulation results

Ocean stratification is highly dependent on the mean ocean salinity. The high salinity ocean is mostly unstratified or weakly stratified below the ice-ocean interface. Instead, as previously found in ocean-only simulations with a flat upper boundary and constant salinity, a stably stratified layer develops in the low salinity ocean (Fig. 2.2). In the stratified layer, the temperature decreases with height due to bottom heating, which, due to the negative thermal expansivity, amounts to a statically stable stratification (Zeng and Jansen, 2021). The stratified layer maintains a large vertical thermal contrast due to the difference between the freezing point and the critical temperature at which the thermal expansivity changes sign (around 2 °C difference with a salinity of 8.5 g kg⁻¹, Fig. 2.2a3 & a4). Despite the ice shell topography, isopycnals in the stratified layer are largely flat rather than following the ice shell topography, indicating that ocean circulation is efficient in removing any large horizontal density gradients that would be associated with an undulating stratified layer.

The temperature and salinity structure is strongly affected by the surface ice shell in both high and low salinity simulations. In the upper ocean, the ocean is in contact with the ice shell, and the temperature is near the freezing point at the ice-ocean interface, which decreases with increasing pressure and hence with increasing ice thickness. Therefore, the temperature increases poleward in the upper ocean (Fig. 2.2a). The salinity mostly increases poleward except for the equatorial region in the low salinity ocean (Fig. 2.2b). The density mostly increases poleward in all simulations, although the role that temperature and salinity play in setting this structure differs in the high versus low salinity oceans (Fig. 2.2c). In the high salinity ocean, the salinity and temperature gradients act in the opposite way,

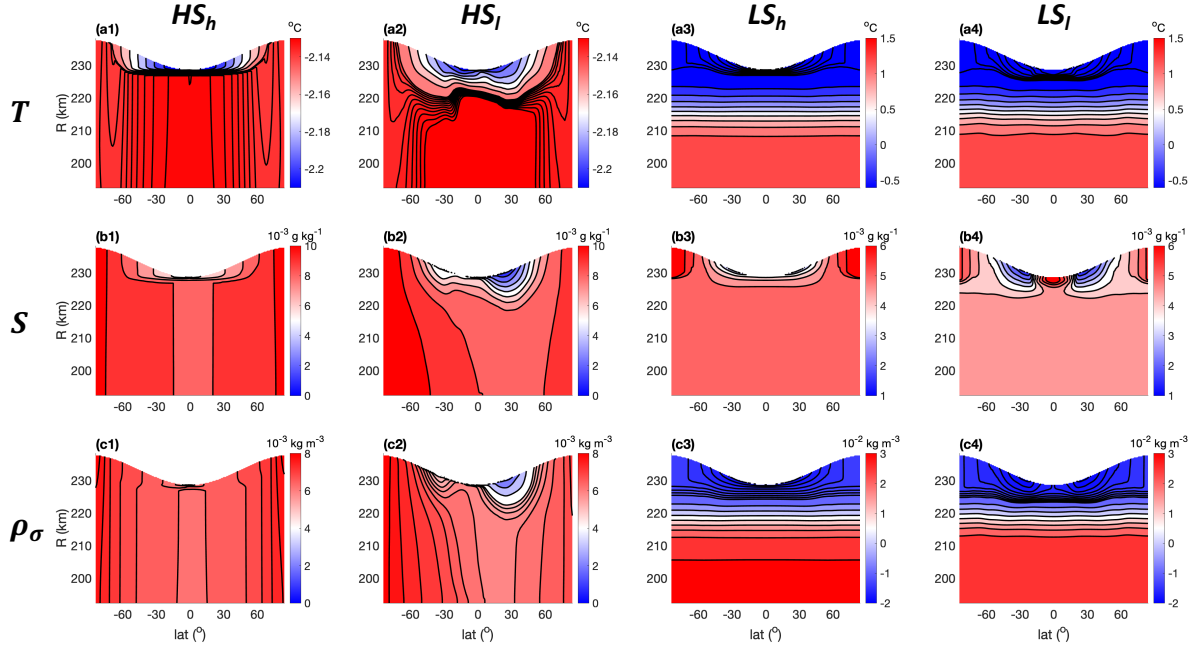


Figure 2.2: Zonal-mean temperature (T , a), salinity anomaly (S , b), and potential density anomaly (ρ_σ , with reference pressure near the ocean surface, c) in the quasi-equilibrium state of all simulations. The first to the fourth columns are results of the high salinity simulation with high horizontal diffusivity (HS_h), high salinity simulation with low horizontal diffusivity (HS_l), low salinity simulation with high horizontal diffusivity (LS_h), and low salinity simulation with low horizontal diffusivity (LS_l), respectively. In (a1) & (a2), the contour interval is 0.01°C below -2.14°C and 0.001°C above -2.14°C . In (a3) & (a4), the contour interval is 0.01°C below -0.6°C and 0.2°C above -0.6°C . In (b1) & (b2), a reference salinity of 34.97 g kg^{-1} is subtracted, and the contour interval is 10^{-3} g kg^{-1} . In (b3) & (b4), a reference salinity of 8.45 g kg^{-1} is subtracted, and the contour interval is $5 \times 10^{-4}\text{ g kg}^{-1}$. In (c1) & (c2), a reference density of 1029.13 kg m^{-3} is subtracted, and the contour interval is 10^{-3} kg m^{-3} below $5 \times 10^{-3}\text{ kg m}^{-3}$ and $3 \times 10^{-4}\text{ kg m}^{-3}$ above $5 \times 10^{-3}\text{ kg m}^{-3}$. In (c3) & (c4), a reference density of 1007.7 kg m^{-3} is subtracted, and the contour interval is 10^{-3} kg m^{-3} below $-10^{-2}\text{ kg m}^{-3}$ and $5 \times 10^{-3}\text{ kg m}^{-3}$ above $-10^{-2}\text{ kg m}^{-3}$.

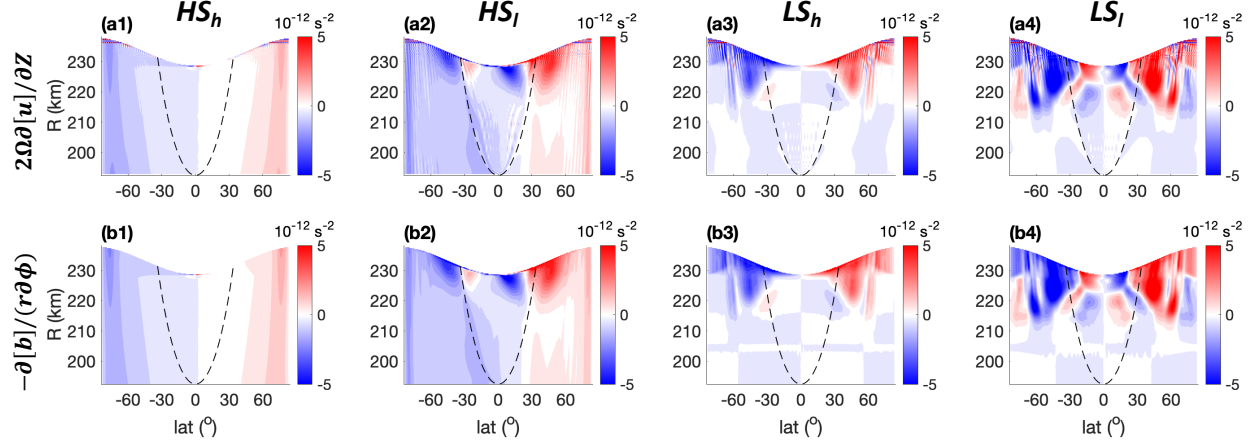


Figure 2.3: Thermal wind balance in the quasi-equilibrium state of all simulations. (a) shows $2\vec{\Omega} \cdot \nabla[u]$, and (b) shows $-\partial[b]/r\partial\phi$, respectively, where $[\cdot]$ denotes a zonal average. The first to the fourth columns are results of the high salinity simulation with high horizontal diffusivity (HS_h), high salinity simulation with low horizontal diffusivity (HS_l), low salinity simulation with high horizontal diffusivity (LS_h), and low salinity simulation with low horizontal diffusivity (LS_l), respectively.

but the contribution from the salinity anomaly is about 2-3 times larger than that of the temperature anomaly, so that the density increases poleward. In the low salinity ocean, the thermal expansivity is negative, so that both temperature and salinity anomalies contribute to the poleward-increasing density, with the contribution from the temperature anomaly 3-5 times larger than the salinity anomaly.

The density structure controls the zonal flow via the thermal wind relationship, which can be written as (see e.g. Kaspi et al., 2009; Bire et al., 2022):

$$2\vec{\Omega} \cdot \nabla u = 2\Omega \frac{\partial u}{\partial Z} = 2\Omega \left(\sin \phi \frac{\partial u}{\partial r} + \cos \phi \frac{\partial u}{r \partial \phi} \right) = -\frac{1}{r} \frac{\partial b}{\partial \phi}, \quad (2.1)$$

where $\vec{\Omega}$ is the planetary rotation rate, u is the zonal velocity, Z is the axis parallel to rotation, r is radius, ϕ is latitude, and $b \equiv -g\delta\rho/\rho_0$ is buoyancy, where g is gravity, $\delta\rho$ is the density anomaly and ρ_0 is a constant reference density. The thermal wind balance holds very well in all simulations (Fig. 2.3), except in regions where there is strong grid-scale convection (c.f. Fig. 2.4b & c). With density increasing poleward (i.e. $-\partial b/\partial\phi > 0$ in the

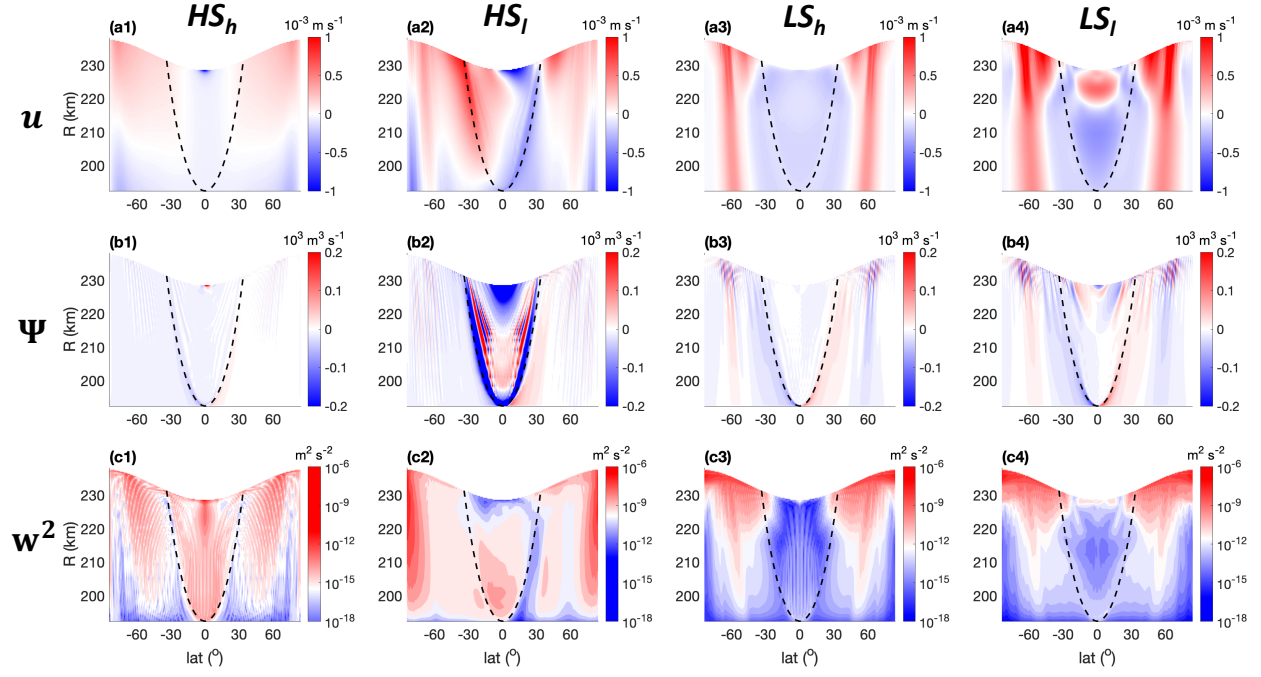


Figure 2.4: Zonal-mean zonal velocity (u , a), stream function (Ψ , with clockwise circulation defined as positive, b), and zonal-mean vertical kinetic energy (w^2 , in logarithmic scale, c) in the quasi-equilibrium state of all simulations. The first to the fourth columns are the results of the high salinity simulation with high horizontal diffusivity (HS_h), high salinity simulation with low horizontal diffusivity (HS_l), low salinity simulation with high horizontal diffusivity (LS_h), and low salinity simulation with low horizontal diffusivity (LS_l), respectively. The stream function is calculated as $\Psi = -\int_0^{\lambda_x} \int_{r_b}^r v r \cos \phi d\lambda dr$ where $\lambda_x = 15\pi/180$ is the longitudinal extent of the domain, v is the meridional velocity, λ is longitude, and r is radius. The black dashed lines denote the boundary of the tangent cylinder (a cylinder that is parallel to the rotational axis and is tangent to the bottom core).

northern hemisphere; and opposite in the southern hemisphere), at mid- and high-latitudes we therefore find $\partial u / \partial Z > 0$, and there are eastward jets in the upper ocean with the jet cores aligned with the axis of rotation. Near the equator, $\sin \phi$ is close to zero and therefore $\cos \phi \partial u / (r \partial \phi) > 0$, so that the zonal velocity decreases equatorward and there is a westward current near the equator (Fig. 2.4a1-a3). An exception is that in the low salinity simulation with small horizontal diffusivity (LS_l), an eastward flow exists near the equatorial surface (Fig. 2.4a4) as a result of the locally reversed density gradient (Fig. 2.2b4 & c4).

In both high and low salinity oceans, there is a slantwise Hadley-like circulation that ascends from the seafloor at the equator, following the tangent cylinder (a surface of constant angular momentum), and descends at higher latitudes. In the low salinity ocean, there is another reversed circulation cell at mid-latitudes along with an eastward jet, which bears similarity to the Ferrel cell in Earth's atmosphere. We also see grid-scale slantwise convection near the surface at high latitudes (Fig. 2.4b). Although the radial stratification (parallel to the gravity vector) is near neutral or stable as maintained by the convective adjustment scheme in the model, the stratification along surfaces of constant angular momentum (mostly parallel to the rotational axis) can be unstable, which can cause slantwise convection.

A strong small-scale overturning cell exists near the equatorial surface in the high salinity simulation with high horizontal diffusivity (HS_h , Fig. 2.4b1), which is associated with a positive feedback loop. There is a strong but narrow (only several degrees in latitude) melting region at and just south of the equator (Fig. 2.5a1). As a result, a local density minimum and sharp density gradient exists at the equatorial surface (Fig. 2.2c1). Following the thermal wind relationship, this density gradient is associated with a strong westward jet near the equatorial surface (Fig. 2.4a1), and the corresponding frictional drag is associated with an Ekman transport that drives a localized overturning circulation. The local circulation cell then provides heat to maintain strong melting, forming a positive feedback loop. Note that this overturning is associated with a slight asymmetry around the equator where the peak

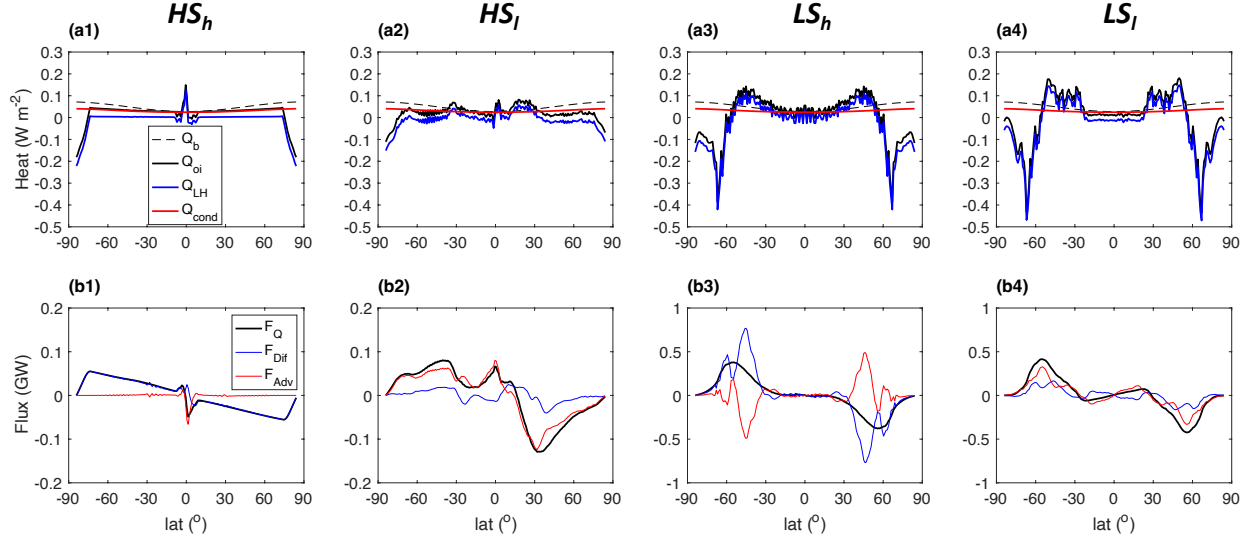


Figure 2.5: Zonal-mean surface heat flux (a) and zonally- and vertically-integrated meridional ocean heat transport (b, see Appendix 2.B.1 for details) in the quasi-equilibrium state of all simulations. The first to the fourth columns are results of the high salinity simulation with high horizontal diffusivity (HS_h), high salinity simulation with low horizontal diffusivity (HS_l), low salinity simulation with high horizontal diffusivity (LS_h), and low salinity simulation with low horizontal diffusivity (LS_l), respectively. In (a), the black dashed line is the bottom heating (Q_b), the blue line is the latent heating (Q_{LH}), the red line is the conductive heat loss (Q_{cond}), and the black solid line is the surface ocean-to-ice heat flux (Q_{oi}). In (b), the blue line is the diffusive heat flux (F_{Dif}), the red line is the advective heat flux (F_{Adv}), and the black line is the total heat flux (F_Q).

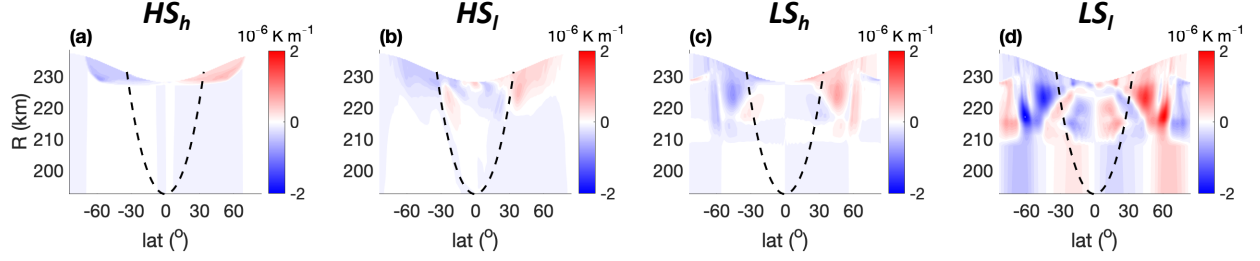


Figure 2.6: Meridional temperature gradient $\partial T/(r\partial\phi)$ in the quasi-equilibrium state of all simulations. The first to the fourth columns are results of the high salinity simulation with high horizontal diffusivity (HS_h), high salinity simulation with low horizontal diffusivity (HS_l), low salinity simulation with high horizontal diffusivity (LS_h), and low salinity simulation with low horizontal diffusivity (LS_l), respectively. The black dashed lines indicate the tangent cylinder.

of the melting and hence the salinity minimum is slightly displaced towards the southern hemisphere.

An even stronger asymmetry exists in the high salinity simulation with low horizontal diffusivity (HS_l). There are overturning cells across the equator (Fig. 2.4b2), and the salinity profile is asymmetric between the two hemispheres as a result of asymmetric freezing and melting patterns (Fig. 2.2b2). The advection of heat and associated freezing and melting generates a positive feedback loop that can result in spontaneous symmetry breaking between the two hemispheres: once we get more melting in one hemisphere, the density becomes smaller, driving an overturning circulation across the equator, with sinking in the hemisphere with higher density and rising in the hemisphere with lower density. Due to the bottom heating, the temperature increases with depth at low latitudes, so that the overturning transports warmer water to the hemisphere with lower density, enhancing melting in that hemisphere. Note that in our simulation, this positive feedback seems to be confined to outside the tangent cylinder at low latitudes, although the details of the results are likely affected by poorly constrained model parameters and insufficient resolution. There is almost no asymmetry in the low salinity simulations, likely because the density gradient is dominated by the temperature gradient in the low salinity ocean. The temperature gradient

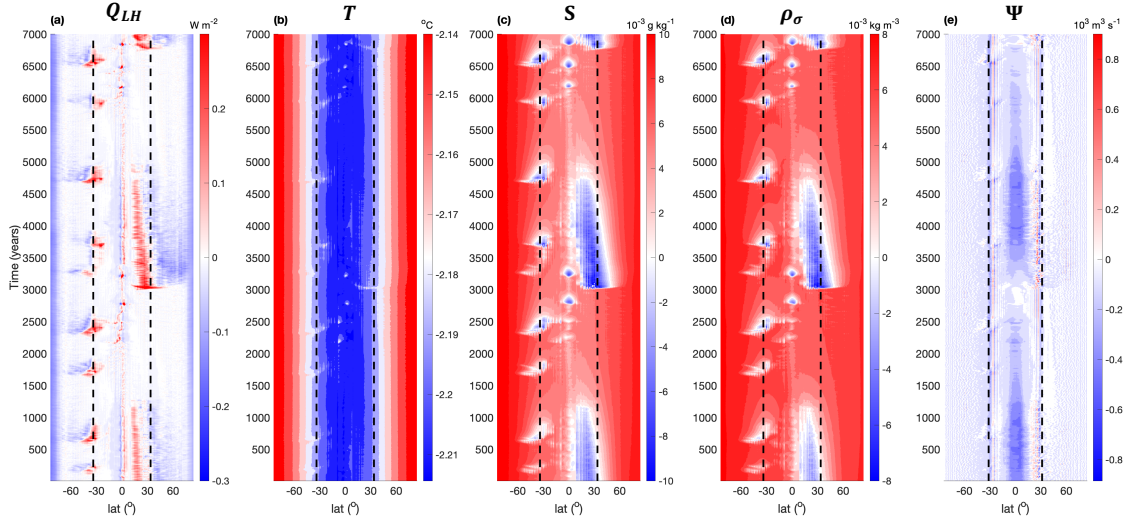


Figure 2.7: Temporal variability in the high salinity simulation with low horizontal diffusivity (HS_l). Zonal-mean latent heating (a), zonal-mean surface temperature (b), zonal-mean surface salinity (c, a reference salinity of 34.97 g kg^{-1} is subtracted), zonal-mean surface density (d, a reference density of $1029.13 \text{ kg m}^{-3}$ is subtracted), and stream function at $r = 225.375 \text{ km}$ (e) as a function of time and latitude. The black dashed lines are the latitudes where the tangent cylinder intersects the ocean surface in (a)–(d) and the latitudes where the tangent cylinder intersects $r = 225.375 \text{ km}$ in (e). The first 4000-year average is used as the quasi-equilibrium state of the simulation HS_l .

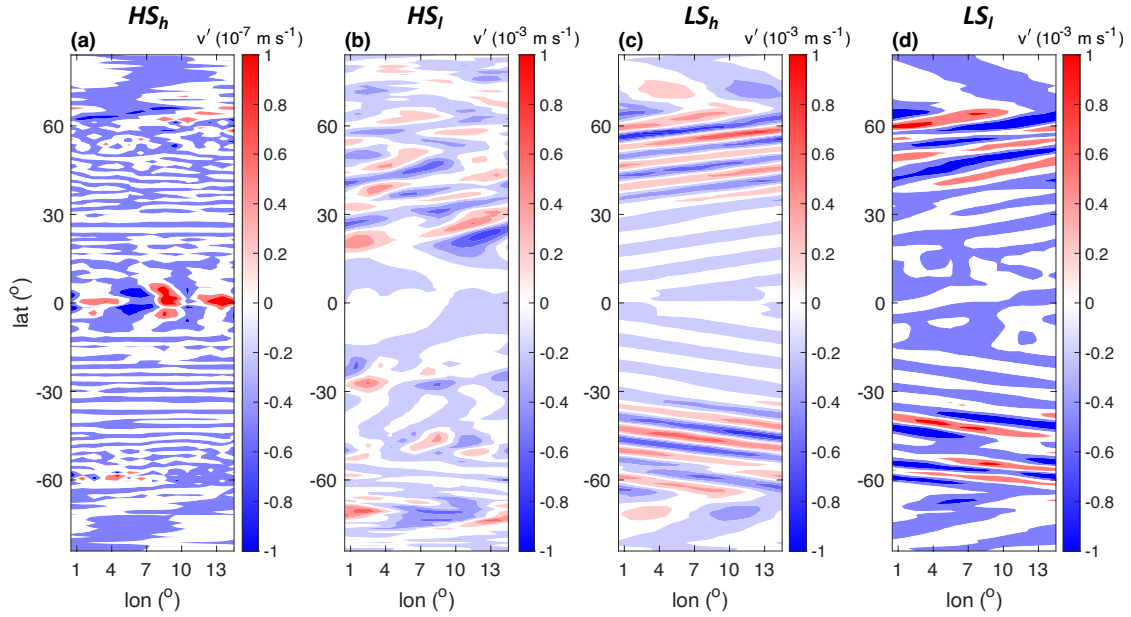


Figure 2.8: Snapshots of meridional eddy velocity v' (deviations from time- and zonal-mean) in all simulations at $r = 225.375$ km. The first to the fourth columns are results of the high salinity simulation with high horizontal diffusivity (HS_h), high salinity simulation with low horizontal diffusivity (HS_l), low salinity simulation with high horizontal diffusivity (LS_h), and low salinity simulation with low horizontal diffusivity (LS_l), respectively. Note that the magnitude of v' in (a) is 10^4 times smaller than in (b-d).

is largely determined by the surface freezing point, which in turn is determined by the ice shell topography. As long as the ice shell topography remains symmetric (as assumed in this study) the temperature field also stays approximately symmetric (Fig. 2.2a).

The ocean circulation drives ocean heat transport and modulates the surface density structure through the freezing and melting rate, and the density structure in turn modulates the ocean circulation. The heat budget at the ice-ocean interface is $Q_{cond} + Q_{LH} = Q_{oi}$, where the conductive heat loss (Q_{cond}) in the simulation is prescribed along with the ice geometry (see Appendix 2.A.2). If the ocean is in an equilibrium state, the ocean heat flux to the ice shell (Q_{oi}) is balanced by the bottom heating (Q_b) when globally integrated. However, locally, the ocean heat transport determines the latent heat (i.e., the freezing and melting rate) by modulating the bottom heating pattern and providing an ocean heat flux to the ice shell.

In all simulations, there is strong freezing at the pole and melting at lower latitudes (low latitudes in high salinity simulations and mid-latitudes in low salinity simulations; see Fig. 2.5a). Locally, the dominant balance is between the latent heat flux (Q_{LH}) and the sensible heat flux from the ocean to the ice (Q_{oi}), because the spatial variations in the meridional heat flux convergence are much larger than the variations in the bottom heating and the conductive heat loss (Fig. 2.5b). This latent heating profile, where freezing happens at higher latitudes than melting, is a result of the ocean heat transport mostly towards lower latitudes, induced by the ice pump effect. The meridional temperature gradient is strongest in the upper ocean where the ocean is in contact with the ice shell, so that the temperature gradient is largely determined by the freezing point at the ice-ocean interface (Fig. 2.6). With polar-thinning ice geometry, the freezing point increases poleward so that the temperature is generally warmer at the pole than at the equator. As a result, any down-gradient heat flux is transporting warm water towards lower latitudes, so that melting occurs at lower latitudes than freezing. The strong freezing at the pole is moreover associated with a

positive feedback: freezing at the pole leads to a density increase due to salt rejection, which destabilizes the stratification, thus promoting convection. Because the temperature decreases with depth under the ice shell at the poles (due to horizontal mixing with colder low-latitude water), the convection brings up colder water from below, which supports further freezing (see Appendix 2.B.2).

The meridional heat flux is larger in the low salinity ocean than in the high salinity ocean (Fig. 2.5b). This result can be understood by noting that the meridional heat flux has three components: diffusive heat flux, resolved eddy heat flux, and heat flux by the mean circulation (See Appendix 2.B.1). All three fluxes become larger when there is a stratified layer. The meridional heat transport by diffusion and resolved eddies both increase with the vertical integral of the meridional temperature gradient. The meridional temperature gradient near the surface is similar in both high and low salinity oceans, because the temperature here is controlled by the freezing point, and hence is determined by the ice geometry, which is the same in both simulations. The meridional temperature gradient in the unstratified convective layer is small in all simulations. However, in the low salinity ocean, the temperature gradient in the stratified layer is comparable to that near the surface, so that the vertical integral of the meridional temperature gradient (i.e. the vertical extent of the meridional temperature gradient) is larger in the low salinity ocean (Fig. 2.6), resulting in a higher meridional heat flux for a given eddy diffusivity. Additionally, the heat transport by the mean circulation is proportional to the strength of the overturning multiplied by the temperature contrast between the poleward and equatorward branches of the overturning circulation. Since the vertical temperature contrast is much larger in the low salinity ocean with a stratified layer, the heat transport by the mean circulation is also expected to be larger in the low salinity ocean.

Comparing simulations with different horizontal diffusivities indicates that, although freezing at the pole is found robustly across all simulations, the detailed pattern of melting

is sensitive to model parameters. In the high salinity simulations, there is persistent melting at the equator in the simulation with higher diffusivity (HS_h), while in the simulation with lower diffusivity (HS_l), the melting latitude varies temporally. In the northern hemisphere, there is an intermittent melting region outside the tangent cylinder (at low latitudes), associated with the spontaneous symmetry breaking mechanism (Fig. 2.7). The system switches between an asymmetric state and a more symmetric state with a period of about 1000-2000 years, but the cross-equatorial overturning never reverses in this simulation (although the sign of the circulation depends on the initial conditions – see Appendix 2.C). In the southern hemisphere, there is intermittent melting in a region inside the tangent cylinder (at mid-latitudes). The southern hemisphere melting only persists for several hundreds of years and is likely associated with an instability near the tangent cylinder (Fig. 2.7). The strong freezing at the pole and the pattern of melting in the low salinity simulations are less sensitive to horizontal diffusivities. In both high- and low-salinity ocean simulations, when the horizontal diffusivity κ_h is high, the meridional heat transport is dominated by diffusion; when κ_h is low, the advective heat transport becomes more important, while the overall heat flux changes relatively little (Fig. 2.5b). The results indicate that increased eddy heat transport, associated with increased eddy activity (Fig. 2.8), tends to compensate for the decrease in the prescribed parameterized diffusivity.

2.4 Discussion and conclusion

We find that ice topography and salinity fundamentally affect the ocean stratification, circulation, and heat transport on Enceladus. In both high and low salinity scenarios, the upper ocean temperature, salinity and density all increase poleward. The temperature is near the freezing point at the ice-ocean interface, which varies with pressure and is thus set by the ice shell thickness variation. The salinity pattern is strongly affected by the salinity flux through freezing and melting. The density anomalies are dominated by the salinity anomalies in the

high salinity simulations and are dominated by the temperature anomalies in the low salinity simulations, which is qualitatively consistent with Kang et al. (2022c).

In a low salinity ocean, with tidal heating at the bottom, a stably stratified layer forms in the upper ocean, as previously found by Zeng and Jansen (2021) in simulations with a flat upper boundary and no thermodynamic ice-ocean interface. This global thermally-driven stratified layer is caused by bottom heating and the negative thermal expansivity near the freezing point in a low salinity ocean, and is thus different from locally stable salt-stratified layers that form by surface melting, like the “freshwater lens” at the polar surface ocean discussed in Lobo et al. (2021), and the stratification in the “shell-heating” scenario in Kang et al. (2022a,c). Note that the depth of the stratified layer here is deeper than in the more idealized simulations of Zeng and Jansen (2021), but is thinner than in the “core-heating”, low salinity ocean in Kang et al. (2022a,c), where the entire ocean is stratified. The depth of the stratified layer H is proportional to the vertical temperature contrast $|\Delta T_r|$, vertical diffusivity κ_r , and the inverse of the vertical diffusive heat flux Q_{diff} : $H \propto \kappa_r |\Delta T_r| / Q_{diff}$ (Zeng and Jansen, 2021). The vertical turbulent diffusivity in Kang et al. (2022a,c) is set 100 times larger than in our simulations and in Zeng and Jansen (2021), such that the depth of the stratified layer exceeds the depth of the ocean in Kang et al. (2022a,c), leaving the entire ocean stratified. The smaller difference between our simulations and those in Zeng and Jansen (2021) can be understood by noting that all heat flux in the stratified layer is diffusive in the simulations in Zeng and Jansen (2021), while in the simulations presented in this chapter, some vertical heat transport is accomplished by advection. Therefore, Q_{diff} here is somewhat smaller and H is somewhat larger than in Zeng and Jansen (2021). This small discrepancy is likely due to the much larger meridional temperature gradient in this work, which in turn arises from the freezing point variations due to the ice shell topography.

In our simulations, the convergence of the meridional heat flux is much stronger than the variation of the bottom heating and surface conductive heat loss. As a result, the ocean heat

transport becomes the primary control on the pattern of ice freezing and melting, which is consistent with the core-heating scenario in Kang et al. (2022a) but is different from Zeng and Jansen (2021), which uses a flat ice shell. With ice thickness decreasing from equator to pole, the freezing point increases poleward, resulting in a much stronger meridional temperature gradient and hence a stronger meridional heat flux in this work and Kang et al. (2022a) compared to Zeng and Jansen (2021) where the ice shell is flat.

The meridional heat flux is mostly towards lower latitudes in both high- and low-salinity oceans, due to the ice pump effect. This heat transport can be understood qualitatively by noting that the freezing point increases poleward due to the thinning of the ice shell, and the ocean heat transport tends to be directed down the meridional temperature gradient. Consequently, we find freezing at the poles, which cannot maintain the ice geometry against the poleward ice flow, as also found in Kang et al. (2022a,c). The result that freezing happens at the poles indicates that either the ice shell on Enceladus is not in an equilibrium state, or that tidal heating in the ice shell is important in maintaining the ice geometry.

The stratified layer in the low salinity ocean tends to enhance the meridional heat transport by the mean overturning circulation through increasing the vertical temperature contrast, and enhance the meridional heat transport by diffusion and resolved eddies through increasing the vertical extent of the meridional temperature gradient. The first mechanism is also found in Kang et al. (2022c). In their core-heating scenario, the ocean heat transport is stronger in the low salinity ocean compared to the high salinity ocean, despite a weaker overturning.

In our high salinity simulation with small diffusivity, we find a positive feedback where enhanced melting in one hemisphere drives a cross-equatorial overturning circulation that further enhances the melting. This feedback can result in spontaneous symmetry breaking between the two hemispheres. Although the specific simulation results here cannot explain the observed ice thickness distribution, it is possible that a similar mechanism could play a

role in the interhemispheric asymmetry of Enceladus’s ice shell thickness. Indeed, a similar mechanism was previously identified by Kang et al. (2022a). In the core-heating scenario in Kang et al. (2022a), they however find that this ocean circulation acts to weaken the asymmetry of the ice shell, because melting occurs where the ice is thick in their simulations, which use an asymmetric ice shell topography. The combined results therefore indicate that the positive feedback may only hold as long as the asymmetry remains relatively small. With asymmetric tidal heating in the ice shell, Kang et al. (2022a) instead find enhanced melting in the hemisphere where the ice shell is thin, such that the ocean heat transport feedback can further enhance the asymmetry of the ice shell.

Besides Enceladus, previous studies have also investigated the heat transport in oceans on other icy moons (e.g., Soderlund, 2019; Amit et al., 2020; Ashkenazy and Tziperman, 2021; Bire et al., 2022; Kvorka and Čadek, 2022). In studies with direct numerical simulation (DNS) methods, simulations have been carried out without parameterizations of eddies and convection, and without including the effect of ice topography and ice-ocean thermodynamics (e.g., Soderlund, 2019; Amit et al., 2020; Bire et al., 2022; Kvorka and Čadek, 2022). These studies reveal how the ocean circulation and ocean heat transport vary over a wide non-dimensional parameter regime. However, care must be taken when interpreting the real icy moon ocean regime with these simulations because the non-dimensional parameters in the numerical simulations and real icy moon oceans differ by many orders of magnitudes due to numerical constraints (Jansen et al., 2023). More directly comparable to the present work are studies using General Circulation Models (GCMs), where the effect of sub-grid-scale eddies (and in some cases convection) are parameterized (e.g., Ashkenazy and Tziperman, 2021; Kang et al., 2022a). In Ashkenazy and Tziperman (2021), the authors use MITgcm to study the ocean circulation and heat transport on Europa considering the effect of salinity and ice-ocean thermodynamics with a flat ice shell. They find that the meridional ocean heat transport is sufficient to compensate for the local imbalance between bottom heating

and surface heat loss, thus allowing for a uniform ice thickness. This result is qualitatively consistent with our finding that ocean heat transport will tend to remove any ice thickness variations. However, the ice geometry is flat in the simulations of Ashkenazy and Tziperman (2021) so that there is no ice pump effect, and the planetary parameters of Europa and Enceladus are also significantly different, which makes it difficult to quantitatively compare the results of Ashkenazy and Tziperman (2021) to our work.

In global GCM ocean simulations of Enceladus and other icy moons, there are many processes that are difficult, or computationally impossible, to resolve. For example, slantwise convection cannot be adequately resolved in global ocean simulations but arises in the form of grid-scale convection in our simulations. Additionally, the effects of tides and librations are not explicitly included in our model - we only consider their effect in providing energy for eddy mixing in the choice for the vertical eddy diffusivity (see Section 2.2 in Zeng and Jansen, 2021). Therefore, we need parameterizations for these processes, which introduce external parameters (e.g., eddy diffusivities and viscosities) that are poorly constrained. Our results indicate that at least some of the results are sensitive to these parameterizations. The development of improved parameterizations for these unresolved processes in global icy moon ocean simulations therefore should be a high priority. In the following chapters, we focus on slantwise convection by examining its instability criterion, deriving heat transport scaling laws, and developing a parameterization for global ocean models.

Appendix 2.A: Model description

2.A.1 *Parameterization of sub-grid scale processes*

In our simulations, the vertical resolution is 300-400 m near the boundaries, about 1000 m in the upper ocean, where the stratified layer is formed in a low salinity ocean, and 5000 m in the deep ocean. The varying vertical resolution is designed to save computational resources and to better resolve ice-ocean boundaries and the stratified layer in the low salinity ocean. The horizontal resolution is 1° in the zonal and 0.95° in the meridional direction, which is around $4 \text{ km} \times 4 \text{ km}$ near the equatorial surface, decreasing to around $300 \text{ m} \times 3 \text{ km}$ near the seafloor at the highest latitudes.

Sub-grid scale processes are parameterized by turbulent viscosities and diffusivities. However, the turbulent viscosities and diffusivities are poorly constrained in simulations for ocean circulation on Enceladus. The vertical turbulent diffusivity in a stably stratified ocean can be constrained by the energy required to mix the water column (Wunsch and Ferrari, 2004). On Enceladus, this energy can come from tidal dissipation in the ocean and/or librations, based on which the vertical turbulent diffusivity in the stratified layer is estimated to be anywhere below $3 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$, although likely much smaller, according to Zeng and Jansen (2021). Here we choose a vertical turbulent diffusivity of $5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ so that the estimated depth of the stratified layer is around 14 km and can be explicitly resolved. We further set the vertical viscosity to $\nu_r = 5 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ to keep the vertical Prandtl number $Pr = 10$ (Soderlund, 2019). To ensure numerical stability, we apply a grid-dependent horizontal viscosity (ν_h), increasing linearly with the square of the zonal grid scale (L_x^2). The horizontal viscosity maximizes at about $6 \text{ m}^2 \text{ s}^{-1}$ near the equatorial surface and minimizes at about $0.5 \text{ m}^2 \text{ s}^{-1}$ near the seafloor at the highest latitudes. We test two different horizontal diffusivities, $0.05 \text{ m}^2 \text{ s}^{-1}$ and $0.5 \text{ m}^2 \text{ s}^{-1}$. These values are motivated by Kang et al. (2020) who estimate the horizontal eddy diffusivity to be $\kappa_h = 0.8 \text{ m}^2 \text{ s}^{-1}$ using a

scaling for geostrophically adjusted convection (Jones and Marshall, 1993).

The horizontal grid scale is not sufficient to resolve convective plumes. The length scale where rotation becomes important is estimated to be $l_r \approx 0.2$ m, many times smaller than our horizontal resolution, which means accurately resolving convective plumes is computationally impossible (Zeng and Jansen, 2021). We therefore apply a convective adjustment by increasing the vertical diffusivity from the background value $\kappa_r = 5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ to $\kappa_{conv} = 1 \text{ m}^2 \text{ s}^{-1}$ whenever the stratification becomes unstable to parameterize the effect of convection. The value of κ_{conv} is chosen such that the parameterized convective time scale ($\tau_{conv} = D_o^2 / \kappa_{conv} \approx 50$ years where D_o is the averaged depth of the ocean) is similar to the convective time scale estimated for the high salinity ocean (see Zeng and Jansen, 2021).

2.A.2 Boundary conditions

We set the ice geometry based on Enceladus’s topography and gravitational field as inferred by Čadek et al. (2019). We only apply the Y_{20} component of the spherical harmonic function, which means we exclude zonal variations and hemispheric asymmetry in the ice geometry for simplicity (Fig. 2.1a).

At the bottom of the ocean, we apply a fixed bottom heat flux pattern as thermal boundary condition, following the zonal-mean tidal forcing pattern in Choblet et al. (2017) with a total flux of 20 GW (Fig. 2.1b). We are applying a smoothly varying bottom heating pattern, although there could be strong local hot spots in the core heating (e.g., Choblet et al., 2017). While this may affect the details of the simulation results, it is not likely to affect the main conclusions. As recently shown by Kang et al. (2022b), warm water from a localized strong heat source will be quickly mixed with the surroundings due to baroclinic instability before it reaches the ice shell. As a result, the small-scale structure of the heating pattern is expected to have relatively little effect on the freezing and melting patterns at the ice-ocean interface. There is no salinity source or sink at the bottom of the ocean. We

apply a linear bottom drag with a drag coefficient $r_b \sim 10^{-4} \text{ m s}^{-1}$ as dynamic boundary condition (see Zeng and Jansen (2021) for a more detailed discussion of the bottom boundary condition). The same linear drag is also applied at the surface. The surface thermal and salinity boundary conditions are determined by the heat and salinity flux due to the ice-ocean interaction.

We apply the “shelfice” package in MITgcm to calculate the ice-ocean interaction (Adcroft et al., 2018). The heat flux balance at the ice-ocean interface is between the conductive heat loss, the latent heat, and the ocean heat flux to the ice-ocean boundary. The conductive heat loss in the shelfice model is assumed proportional to the inverse of the ice shell thickness and the temperature difference between the freezing point and the ice surface temperature, the latter of which is assumed to be spatially constant in our simulations. The magnitude of the conductive heat loss is chosen such as to exactly balance the total heat flux at the sea floor, as is necessary to obtain an equilibrated ocean state. The ocean heat flux to the ice shell is calculated by a turbulent exchange flux through the ice-ocean boundary layer, where the heat flux is assumed proportional to the temperature contrast between the local ocean temperature and the temperature at the bottom of the ice shell (assumed to be the freezing point). The freezing and melting rate is calculated based on the latent heat required to balance the heat budget at the ice-ocean interface. In the shelfice model, the freezing and melting rate is converted to a salinity flux and does not affect the freshwater volume. The volume flux associated with meltwater input is negligible in our simulations: the latent heat is $Q_{LH} \sim \mathcal{O}(0.1 \text{ W m}^{-2})$ in all simulations (Fig. 2.5a), so that the volume flux is $w_q = Q_{LH}/\rho L = 3 \times 10^{-10} \text{ m s}^{-1}$ where ρ is the density of the water and L is the latent heat of fusion. This flux is negligible compared with the typical vertical velocity in the ocean ($w_o \sim \mathcal{O}(10^{-6} \text{ m s}^{-1})$).

Our model does not account for the latitudinal variation of the ice surface temperature and the dependence of the heat diffusion coefficient of ice on the local ice temperature (both

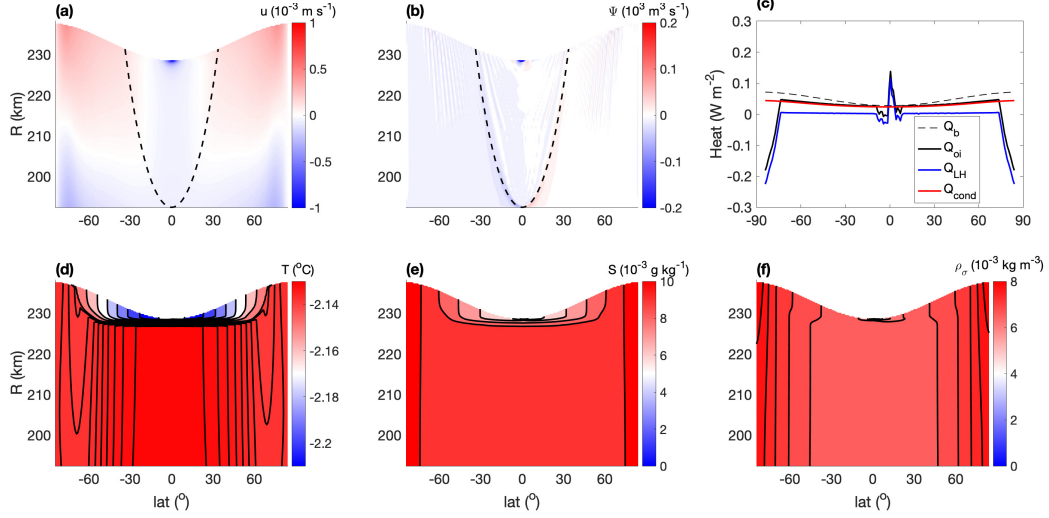


Figure 2.9: Simulation with parameters in HS_h but including surface temperature variation and temperature-dependent heat diffusion coefficient of the ice when prescribing the conductive heat loss (following Kang et al., 2022c). (a): zonal-mean zonal velocity u . (b): stream function Ψ , with clockwise circulation defined as positive. (c): surface heat budget, where the black dashed line is the bottom heating (Q_b), the blue line is the latent heating (Q_{LH}), the red line is the conductive heat loss (Q_{cond}), and the black solid line is the surface ocean-to-ice heat flux (Q_{oi}). (d): zonal-mean temperature T (the contour interval is 0.01°C below -2.14°C and 0.001°C above -2.14°C). (e): zonal-mean salinity S (a reference salinity of 34.97 g kg^{-1} is subtracted, and the contour interval is 10^{-3} g kg^{-1}). (f): zonal-mean potential density ρ_σ (a reference density of 1029.13 kg m^{-3} is subtracted, and the contour interval is 10^{-3} kg m^{-3} below $5 \times 10^{-3}\text{ kg m}^{-3}$ and $3 \times 10^{-4}\text{ kg m}^{-3}$ above $5 \times 10^{-3}\text{ kg m}^{-3}$).

parameters are assumed constant in our model). We carried out one sensitivity test using the parameters of the simulation HS_h but with the conductive heat loss through the ice shell computed using a latitudinally-varying surface temperature and a temperature-dependent heat diffusion coefficient, following Kang et al. (2022c). By comparing the results of this simulation with HS_h in the main text, we find that the results do not qualitatively change (Fig. 2.9). Therefore, we do not expect these variations to qualitatively affect our results, as long as the conductive heat loss is increasing towards the pole, as a result of the polar-thinning ice shell.

It is not known whether Enceladus's ocean and ice shell are in an equilibrium state or not. The adjustment time scale of the ice shell can be estimated as $\tau_i \sim D_i/w_q \approx 2 \times$

10^6 years where $D_i = 20$ km is the average ice thickness. For comparison, the (parameterized) convective time scale of the ocean is $\tau_{conv} \approx 50$ years, the overturning time scale of the ocean is $\tau_{adv} \sim V_o/\Psi \approx 3 \times 10^5$ years where V_o is the volume of the ocean and Ψ is the stream function, and the diffusive time scale of the stratified layer in the low salinity ocean is $\tau_{diff} \sim H^2/\kappa_r \approx 10^5$ years, where H is the thickness of the stratified layer and κ_r is the radial diffusivity. We find that the adjustment time scale of the ice shell is much longer than the adjustment time scale of the ocean. As a result, although we do not know if the ice shell on Enceladus is equilibrated, it is more likely that the ocean is near an equilibrium state.

2.A.3 Integration of the simulations

The ice-ocean coupled 3-D global ocean simulations are difficult to integrate to a fully equilibrated state. Therefore, we present quasi-equilibrium states where the total energy imbalance is less than 3% of the seafloor heating rate. Although there are still temperature and salinity trends in parts of the ocean, we expect that the remaining small imbalance does not qualitatively affect the general results discussed in this chapter. The presented results are 500-year averages for HS_h , LS_h , and LS_l , and 4000-year averages for HS_l which has larger low-frequency variability.

We start the integration of all simulations with a uniform salinity (35 g kg^{-1} for high salinity and 8.5 g kg^{-1} for low salinity). The initial temperature for the high salinity ocean is set to be near the freezing point (about -2°C) everywhere. For the low salinity ocean, we set the temperature in the upper ocean where the ocean is in contact with the ice shell to be near the freezing point (about -0.6°C). At the bottom of the ocean, we set the temperature to the critical temperature where the thermal expansivity changes sign and becomes positive (about 1.4°C). In between, we calculate the depth of the stratified layer using Eq. 1 in Zeng and Jansen (2021) by assuming that the diffusive flux is equal to the total bottom heating, and then prescribe a linear temperature profile to connect the upper and bottom ocean

layers, so that the initial conditions are reasonably close to the expected equilibrium state. Small-amplitude random noise is added to generate zonal variability.

This initial estimate still differs significantly from the final solution and the adjustment is very slow in the low salinity ocean. Therefore, we apply an acceleration method for the low salinity ocean simulations. For the temperature, we take the horizontally averaged profile for the temperature tendency ($\partial T/\partial t$), and accelerate the simulation by adding this tendency multiplied by a long time period (several thousands of years) to the original temperature profile. We only apply the tendency in layers where it has the same sign as the global temperature tendency: if the global ocean is net warming, we only apply positive temperature tendencies, and vice versa. After each acceleration step is applied, we integrate the model, initialized from the new profile, for about 200 years, and then repeat the acceleration step with the new tendencies. We repeat this procedure until the global temperature tendency changes sign, at which point we continue to integrate the model to a global thermal quasi-equilibrium state (defined as described above). It takes about 20 cycles (several tens of thousands of years) and a final integration of another several thousands of years for the low salinity ocean to reach a quasi-equilibrium state. During the acceleration steps, we did not accelerate the salinity, as the amplification of salinity tendencies from transient freezing and melting does not systematically nudge the simulated solution closer to its equilibrium state. Instead we apply a single adjustment step to the deep ocean salinity once a thermal quasi-equilibrium has been reached. In an equilibrium state, extrema of salinity can only exist at the surface of the ocean where there are salinity sources and sinks, so that every isoline of salinity should start or end at the surface. We hence find the lowermost isoline that intersects with the ice-ocean boundary and set the salinity below this isoline equal to this value. After this adjustment, we run the simulation for an additional 1000 years and use the last 500 years for analysis. Notice that the equilibrated solution is expected to be independent of the spin-up procedure and hence the acceleration method (unless multiple

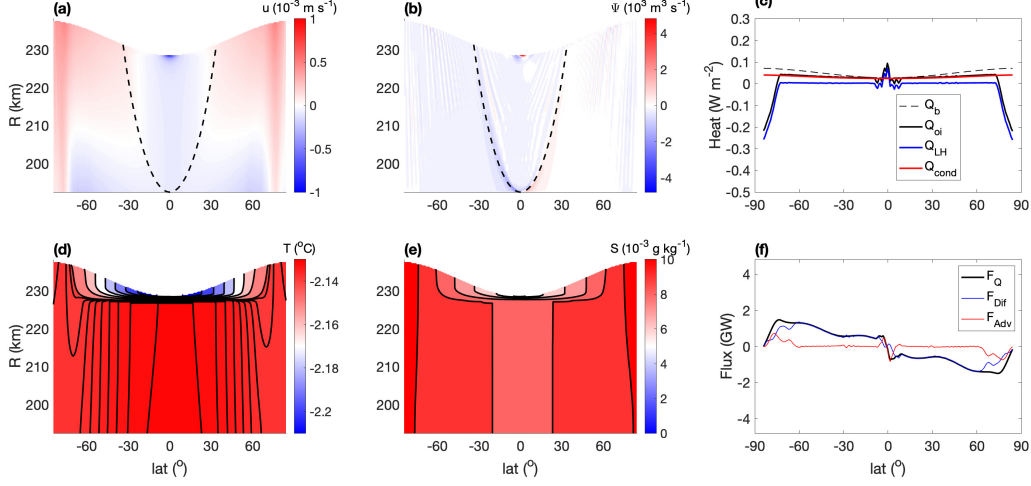


Figure 2.10: Simulation with the same parameters as in HS_h but with the zonal width of the domain extended from 15° to 360° . (a): zonal-mean zonal velocity u . (b): stream function Ψ , with clockwise circulation defined as positive. (c): surface heat budget, where the black dashed line is the bottom heating (Q_b), the blue line is the latent heating (Q_{LH}), the red line is the conductive heat loss (Q_{cond}), and the black solid line is the surface ocean-to-ice heat flux (Q_{oi}). (d): zonal-mean temperature T (the contour interval is 0.01°C below -2.14°C and 0.001°C above -2.14°C). (e): zonal-mean salinity S (a reference salinity of 34.97 g kg^{-1} is subtracted, and the contour interval is 10^{-3} g kg^{-1}). (f): zonally- and vertically-integrated meridional ocean heat transport. The blue line is the diffusive heat flux (F_{Dif}), the red line is the advective heat flux (F_{Adv}), and the black line is the total heat flux (F_Q). Note that the color bar range in (b) and the vertical axis range in (f) are multiplied by 24 compared to Fig. 2.4b1 and Fig. 2.5b1 to account for the larger zonal width of the domain and allow for easier comparison.

equilibria exist). While our low salinity simulation has not fully reached equilibrium, the remaining drifts are very small, suggesting that the results are not likely to differ qualitatively from the true equilibrium.

2.A.4 Sensitivity to domain width

In our simulations, we only simulate a zonal domain of 15° in longitude to save computational resources. However, the effect of horizontal eddies can be important in both ocean heat transport (see Fig. 2.5 and relevant discussions in the main text) and momentum transport (Ashkenazy and Tziperman, 2021). Therefore, we carried out one sensitivity test with the

same parameters as in HS_h , but with the zonal width of the domain extended from 15° to 360° (Fig. 2.10). To accelerate the simulation, we set the initial condition to be the zonal-mean field of the quasi-equilibrium state of HS_h with small random perturbations added to the temperature field. This simulation has been run for 120 years, at which point a quasi-equilibrium state is reached where the total heat imbalance is smaller than 7% of the seafloor heating rate. The results of this simulation are qualitatively similar to those in HS_h , in particular showing a very similar freezing and melting pattern.

Appendix 2.B: Heat budget

2.B.1 Meridional heat transport

The meridional heat flux F_Q has two components: advective heat flux F_{Adv} and diffusive heat flux F_{Dif} . Here we define F_Q as the zonally- and vertically-integrated, time-averaged heat flux, which is

$$F_Q = F_{Adv} + F_{Dif} = \rho c_p \int_0^{\lambda_x} \int_{r_b}^{r_{oi}} \overline{vT} r \cos \phi d\lambda dr + \rho c_p \int_0^{\lambda_x} \int_{r_b}^{r_{oi}} \kappa_h \left(-\frac{1}{r} \frac{\partial \overline{T}}{\partial \phi} \right) r \cos \phi d\lambda dr, \quad (2.2)$$

where r_b is the radius at the bottom of the ocean, r_{oi} is the radius at the ice-ocean interface, v is the meridional velocity, T is the temperature, λ is longitude, λ_x is the longitudinal extent of the domain, and the overbar indicates a time-average. The time- and zonally-averaged advection can be written as $[\overline{vT}] = [\overline{v}][\overline{T}] + [\overline{v'T'}]$, where the bracket denotes the zonal average and primes denote deviations from the zonal and temporal average - which we will refer to as the eddy component. Eq. 2.2 can then be written as

$$F_Q = \rho c_p \lambda_x \int_{r_b}^{r_{oi}} [\overline{v}][\overline{T}] r \cos \phi dr + \rho c_p \lambda_x \int_{r_b}^{r_{oi}} [\overline{v'T'}] r \cos \phi dr - \rho c_p \lambda_x \int_{r_b}^{r_{oi}} \kappa_h \frac{\partial [\overline{T}]}{\partial \phi} \cos \phi dr. \quad (2.3)$$

If we assume that the eddy heat transport is down-gradient and is proportional to the mean gradient of temperature, we can apply an equivalent “eddy diffusivity” κ_e to replace the eddy term: $[\overline{v'T'}] = -\kappa_e \partial [\overline{T}] / r \partial \phi$ (Vallis, 2017). Then Eq. 2.3 becomes

$$F_Q = \rho c_p \lambda_x \int_{r_b}^{r_{oi}} [\overline{v}][\overline{T}] r \cos \phi dr - \rho c_p \lambda_x \int_{r_b}^{r_{oi}} (\kappa_e + \kappa_h) \frac{\partial [\overline{T}]}{\partial \phi} \cos \phi dr, \quad (2.4)$$

where on the right hand side the first term indicates heat transport by the mean overturning

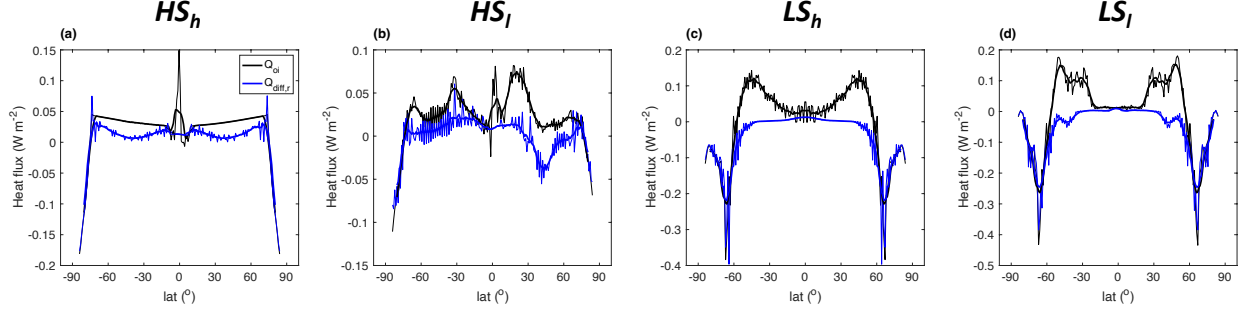


Figure 2.11: Vertical heat fluxes near the surface of the ocean (the uppermost ocean model layer in contact with the ice shell). The blue lines show the radial diffusive heat flux (including the parameterized convective heat flux) at the bottom of the surface ocean model layer, and the black line shows the ocean heat flux to the ice shell (Q_{oi}). The thinner lines are the raw data and the thicker lines are moving-averages of 9 grids to smooth grid-scale variabilities. The first to the fourth columns are results of the high salinity simulation with high horizontal diffusivity (HS_h), high salinity simulation with low horizontal diffusivity (HS_l), low salinity simulation with high horizontal diffusivity (LS_h), and low salinity simulation with low horizontal diffusivity (LS_l), respectively.

circulation, while the second term indicates heat transport by resolved eddies and diffusion (which may be thought of as representing the effect of unresolved eddies).

2.B.2 Ocean heat budget of the surface layer

To better understand how the parameterized upright convection is connected with the surface freezing and melting pattern, we compare the vertical diffusive heat flux near the ice-ocean interface (at the bottom of the model layer in contact with the ice shell) with the ocean heat flux to the ice shell Q_{oi} (Fig. 2.11). The parameterized vertical convective diffusivity ($\kappa_{conv} = 1 \text{ m}^2 \text{ s}^{-1}$) is much larger than the background vertical diffusivity ($\kappa_r = 5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$), so that the vertical diffusive heat flux primarily shows the effect of the parameterized upright convection. In all simulations, near the polar surface, Q_{oi} approximately matches the parameterized upright convective heat flux, indicating that the convection brings up colder water from below and supports freezing near the poles.

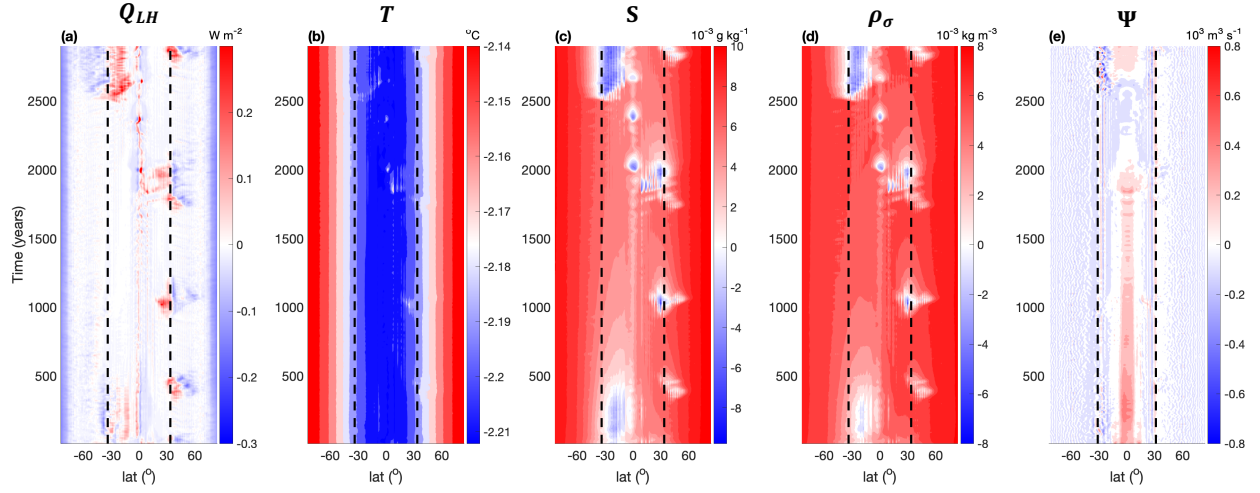


Figure 2.12: Simulation with the same parameters as in HS_l , but using the “mirrored” salinity field of the quasi-equilibrium state in HS_l as the initial condition. Zonal-mean latent heating (a), zonal-mean surface temperature (b), zonal-mean surface salinity (c, a reference salinity of 34.97 g kg^{-1} is subtracted), zonal-mean surface density (d, a reference density of $1029.13 \text{ kg m}^{-3}$ is subtracted), and stream function at $r = 225.375 \text{ km}$ (e) as a function of time and latitude. The black dashed lines are the latitudes where the tangent cylinder intersects the ocean surface in (a)-(d) and the latitudes where the tangent cylinder intersects $r = 225.375 \text{ km}$ in (e).

Appendix 2.C: Multiple equilibria in high salinity simulations

We have found a positive feedback which can result in spontaneous symmetry breaking in the high salinity simulations. This mechanism, however, should not have a preference between the northern and southern hemisphere. We hence carried out one sensitivity test with the same parameters as the HS_l simulation but using the “mirrored” salinity field of the quasi-equilibrium state in HS_l as the initial condition. In this sensitivity test, we observe that the cross-equatorial overturning circulation has the opposite sign. Consistently, we find stronger melting outside the tangent cylinder at low latitudes in the southern hemisphere (in contrast to the northern hemisphere in HS_l), and we still see the switching between an asymmetric state and a more symmetric state (Fig. 2.12). The nonlinear dynamics controlling the multiple quasi-equilibria could be an interesting topic for future studies.

CHAPTER 3

SYMMETRIC INSTABILITY IN A BOUSSINESQ FLUID ON A ROTATING PLANET

An earlier version of this chapter was published as Zeng, Yaoxuan, and Malte F. Jansen. “Symmetric instability in a Boussinesq fluid on a rotating planet.” *Journal of Geophysical Research: Planets* 130, no. 7 (2025): e2025JE009115.

3.1 Introduction

In this chapter, we focus on the instability criteria for slantwise convection, which is associated with symmetric instability. Symmetric instability describes the instability of axisymmetric flow. In an axisymmetric rotating fluid with background density stratification and angular momentum gradient, if a fluid parcel is perturbed from its origin, two restoring forces come into play: the buoyancy force and the inertial acceleration (Coriolis and centrifugal forces). These forces individually may result in gravitational instability or inertial instability. Moreover, even when the fluid is both gravitationally and inertially stable, the combined effects of the two force anomalies may result in symmetric instability (Solberg, 1936; Høiland, 1941; Hoskins, 1974; Haine and Marshall, 1998).

Symmetric instability has broad applications in geophysical fluid dynamics. On Earth, it is relevant to the formation of mesoscale rainbands in the midlatitude atmosphere (e.g., Emanuel, 1983, 1985) and instability in the ocean’s mixed layer (e.g., Straneo et al., 2002; Callies and Ferrari, 2018b). Beyond Earth, symmetric instability is closely tied to slantwise convection in the atmospheres of gas giants (e.g., Stone, 1967; Busse, 1970; Stone, 1971b; Walton, 1975; O’Neill and Kaspi, 2016) and in the oceans of their icy satellites (e.g., Soderlund, 2019; Ashkenazy and Tziperman, 2021; Kang et al., 2022c; Bire et al., 2022; Zeng and Jansen, 2024), where convection is tilted along angular momentum surfaces.

Symmetric instability is often studied under certain assumptions about the background field. In most literature, it is examined in inertially stable ($\eta/f > 0$) fluids with stable gravitational stratification ($N^2 > 0$), where $N^2 = -(g/\rho)(\partial\rho/\partial R)$, $\eta = f - (1/R)(\partial u/\partial\theta)$ is the absolute vorticity, g is gravity, ρ is density, u is the zonal component of the velocity, $f = 2\Omega \sin\theta$ is the component of planetary rotation parallel to gravity, $\theta \in (-\pi/2, \pi/2]$ is the latitude, R is the planetary radius, and $\partial/\partial R$ denotes the derivative along the gravitational direction. Under these assumptions, instability occurs when the planetary vorticity has the opposite sign to the potential vorticity. In the Northern Hemisphere, symmetric instability arises when the potential vorticity, defined as $q = \nabla b \cdot (2\mathbf{\Omega} + \nabla \times \mathbf{v})$, is negative, where $\mathbf{\Omega}$ represents planetary rotation, $\mathbf{v}$ is the three-dimensional (3-D) relative velocity in the rotating frame, and b is the buoyancy (Eliassen, 1951; Ooyama, 1966; Hoskins, 1974; Stevens, 1983). Although assuming $N^2 > 0$ is reasonable for most regions of Earth’s atmosphere and ocean, this condition may not hold on other planetary bodies such as icy moon oceans and gas giant atmospheres. Studies on rotating convective instability have examined scenarios with $N^2 < 0$, and have found that rotating systems can remain stable even when $N^2 < 0$ (Flasar and Gierasch, 1978; Hathaway et al., 1979). In Flasar and Gierasch (1978) and Hathaway et al. (1979), the background zonal shear is assumed to be purely vertical. However, meridional shears resulting from eddy angular momentum transport (e.g., Busse, 1970; Aurnou et al., 2007; Soderlund, 2019; Zeng and Jansen, 2021) are likely significant in the oceans of icy moons and the atmospheres of gas giants, thereby influencing the instability criteria.

Beyond the background field, various approximations have been employed in studies of symmetric instability. In early studies, the traditional approximation (Gerkema et al., 2008) has been commonly applied, wherein the problem is examined on an f -plane that considers only the component of planetary rotation parallel to gravity. In this framework, the instability criteria for gravitationally and inertially stable fluids can be expressed as $\text{Ri} < \eta/f$, the same as negative PV in the Northern Hemisphere (c.f. Hoskins, 1974; Haine and Mar-

shall, 1998), where $Ri = N^2/(\partial u/\partial R)^2$ is the Richardson number. However, the traditional approximation is not valid for planets with deep fluid layers, such as the atmospheres of gas giants and the oceans of icy moons, for flows with strong vertical motion, or for near-equatorial flows where $f \rightarrow 0$ (Gerkema et al., 2008). Later studies have considered a “tilted f -plane” that takes into account both vertical and horizontal components of the planetary rotation, $\mathbf{f} = f\mathbf{e}_R + \tilde{f}\mathbf{e}_\theta$ where $\tilde{f} = 2\Omega \cos \theta$, $\mathbf{e}_R$ denotes the upwards direction parallel to gravity, and $\mathbf{e}_\theta$ denotes the latitudinal direction defined to point northward (see Figure 3.1; Sun, 1995; Straneo et al., 2002; Fruman and Shepherd, 2008; Itano and Maruyama, 2009; Jeffery and Wingate, 2009). Sun (1995) concluded that as latitude decreases, i.e., the angle between planetary rotation and gravity becomes larger, the effect of $\tilde{f}$ in modulating the maximum growth rate of symmetric instability increases. Itano and Maruyama (2009) found that the parameter regime for the occurrence of symmetric instability is less sensitive to $\tilde{f}$ when $Ri > 0.25$ and $\eta/f > 1$, but is considerably influenced in other regions.

The instabilities discussed above are generally difficult to resolve in global ocean simulations due to their small spatial scales. In studies of icy moon oceans, two main approaches have been used. The first involves direct numerical simulations in parameter regimes that differ from those of actual icy moon oceans, with asymptotic scaling laws developed to extrapolate the results (Soderlund et al., 2014; Gastine et al., 2016; Soderlund, 2019; Amit et al., 2020; Kvorka and Čadež, 2022; Cabanes et al., 2024; Bouffard et al., 2025). Parameters are chosen such that convective instabilities can be explicitly resolved, but results often need to be extrapolated by several orders of magnitude to infer the circulation on icy moons, raising concerns about their reliability in representing real ocean dynamics. The second approach uses sub-grid parameterizations to represent unresolved processes (including convection), enabling simulations under realistic icy moon conditions (Kang et al., 2022a,c; Kang, 2023; Zeng and Jansen, 2024). However, most parameterizations are designed for Earth-like flows, and whether they are applicable to other planets remains unclear. Here,

we discuss symmetric instability conditions and unstable mode properties over a wide planetary parameter regime with slowly varying, but otherwise arbitrary zonal velocity shear and density stratification. We apply a local linear instability analysis to the adiabatic, inviscid, non-hydrostatic Boussinesq equations formulated in a cylindrical coordinate system, which is the most natural choice for analyzing fast-rotating planets, such as icy moon oceans, where motions are largely aligned with the rotation axis (Ashkenazy and Tziperman, 2021; Bire et al., 2022). We retain full Coriolis force and metric terms that arise from the cylindrical geometry of the coordinate system. We linearize around a zonally symmetric background state in hydrostatic and gradient wind balance, without imposing additional assumptions on this state. In particular, we do not require the background stratification to be stable in the gravitational direction ($N^2 > 0$), which, as we show, is in general neither a necessary nor sufficient condition for stability. We consider zonally-symmetric perturbations, which allow for gravitational, inertial, and mixed symmetric instabilities, as discussed in this chapter, while excluding baroclinic instabilities associated with nonzero zonal wavenumbers (Stone, 1966). In Section 3.2, the theoretical analysis for linear instability criteria, the most unstable mode, and the maximum growth rate are discussed. With hindsight, the instability criteria derived here are essentially the Boussinesq analog of the Solberg-Høiland criteria (Solberg, 1936; Høiland, 1941) as formulated in Ogilvie (2019). Section 3.3 presents the numerical simulation results for comparison with the theoretical analysis. Section 3.4 provides discussions. Section 3.5 provides concluding remarks.

3.2 Linear instability analysis

3.2.1 *Instability criteria*

The adiabatic, inviscid, nonhydrostatic equations for a Boussinesq fluid in a cylindrical coordinate are the three components of the Navier-Stokes equations:

$$\left(\frac{\partial}{\partial t} + \frac{u}{r} \frac{\partial}{\partial \phi} + v \frac{\partial}{\partial r} + w \frac{\partial}{\partial z}\right) u + 2\Omega v + \frac{uv}{r} = -\frac{1}{r} \frac{\partial \Phi}{\partial \phi}, \quad (3.1)$$

$$\left(\frac{\partial}{\partial t} + \frac{u}{r} \frac{\partial}{\partial \phi} + v \frac{\partial}{\partial r} + w \frac{\partial}{\partial z}\right) v - 2\Omega u - \frac{u^2}{r} = -\frac{\partial \Phi}{\partial r} + b \cos \theta, \quad (3.2)$$

$$\left(\frac{\partial}{\partial t} + \frac{u}{r} \frac{\partial}{\partial \phi} + v \frac{\partial}{\partial r} + w \frac{\partial}{\partial z}\right) w = -\frac{\partial \Phi}{\partial z} + b \sin \theta, \quad (3.3)$$

the buoyancy equation:

$$\left(\frac{\partial}{\partial t} + \frac{u}{r} \frac{\partial}{\partial \phi} + v \frac{\partial}{\partial r} + w \frac{\partial}{\partial z}\right) b = 0, \quad (3.4)$$

and the mass continuity equation:

$$\frac{1}{r} \frac{\partial u}{\partial \phi} + \frac{\partial v}{\partial r} + \frac{\partial w}{\partial z} + \frac{v}{r} = 0, \quad (3.5)$$

where r is the radial cylindrical direction, increasing outwards perpendicular to the rotation axis of the planet, ϕ is the longitude, denoting the zonal (azimuthal) direction, and z is the vertical (rotational) direction (Figure 3.1). u , v , w are velocity components in the $\mathbf{e}_\phi$, $\mathbf{e}_r$, and $\mathbf{e}_z$ directions, respectively. $\Phi = p'/\rho_0$, where p' is the pressure anomaly and ρ_0 is the reference density. $b = -g\rho'/\rho_0$ is buoyancy, where ρ' is the density anomaly (c.f. Vallis, 2017).

We consider an axisymmetric state around the planetary rotation axis where all variables are invariant in the zonal direction ($\partial/\partial\phi = 0$, i.e., zonally symmetric), and assume a background state with an arbitrary background zonal flow $\bar{u}(r, z)$ and buoyancy field $\bar{b}(r, z)$ that are invariant in time: $b = \bar{b}(r, z) + b'(r, z, t)$, $\Phi = \bar{\Phi}(r, z) + \Phi'(r, z, t)$, $u = \bar{u}(r, z) + u'(r, z, t)$, $v = v'(r, z, t)$, $w = w'(r, z, t)$. Assuming the perturbations are small, the zeroth-order balance reveals the gradient wind balance and hydrostatic balance for the background

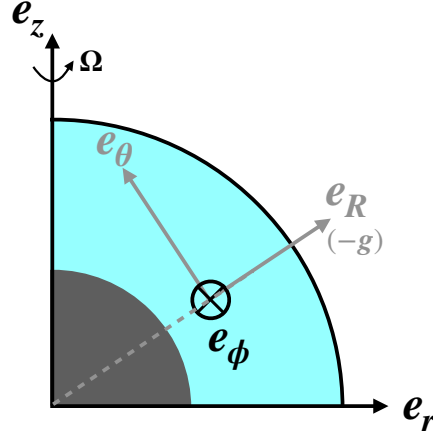


Figure 3.1: **Sketch of the coordinate system.** Black coordinates show the cylindrical coordinate system applied in this chapter ($\mathbf{e}_r$, $\mathbf{e}_\phi$, $\mathbf{e}_z$), where $\mathbf{e}_z$ is the planetary rotation axis and the origin locates at the center of the planet. Grey coordinates show the spherical coordinate system ($\mathbf{e}_\phi$, $\mathbf{e}_\theta$, $\mathbf{e}_R$), where $\mathbf{e}_R$ is opposite to gravity. Note that the zonal direction $\mathbf{e}_\phi$ is the same in both cylindrical and spherical coordinates.

state:

$$-2\Omega\bar{u} - \frac{\bar{u}^2}{r} = -\frac{\partial\bar{\Phi}}{\partial r} + \bar{b}\cos\theta, \quad (3.6)$$

$$0 = -\frac{\partial\bar{\Phi}}{\partial z} + \bar{b}\sin\theta. \quad (3.7)$$

With

$$\bar{f} = 2\Omega + 2\bar{\omega}, \quad (3.8)$$

being a modified Coriolis parameter where $\bar{\omega} \equiv \bar{u}/r$ is the angular velocity of the background zonal flow, Equations 3.6 & 3.7 yield

$$\bar{f}\frac{\partial\bar{u}}{\partial z} = \frac{\partial\bar{b}}{\partial r}\sin\theta - \frac{\partial\bar{b}}{\partial z}\cos\theta. \quad (3.9)$$

Equation 3.9 is similar to the thermal wind balance in a rapidly rotating (geostrophic) fluid

(Kaspi et al., 2009; Bire et al., 2022), but here $\bar{f}$ is modified by the background flow, with the effects of centrifugal force incorporated, thus giving the shear of a flow in gradient-wind balance rather than geostrophic balance.

In the first-order balance, we obtain linearized perturbation equations (Equations 3.49-3.53). In this chapter, we focus on local instability analysis, with the length scales of the perturbations small compared to the planetary radius. We therefore look for plane-wave solutions, where all variables are proportional to $\exp(ik_r r + ik_z z - i\xi t)$, where ξ is the angular frequency, and k_r and k_z are wavenumbers in the radial and vertical directions, respectively. We assume that variations in the background field are small across the perturbation length scale (Equation 3.41), allowing us to evaluate them locally at $(r = r_0, z = z_0)$, with k_r and k_z treated as constants. Under these assumptions, we have the dispersion relation (see Appendix 3.A for detailed derivation)

$$\xi^2 = \frac{b_z \sin \theta_0 k_r^2 + (f_0 M_r + b_r \cos \theta_0) k_z^2 - 2b_r \sin \theta_0 k_r k_z}{k_r^2 + k_z^2}, \quad (3.10)$$

where $b_z \equiv \partial \bar{b} / \partial z|_{(r_0, z_0)}$ and $b_r \equiv \partial \bar{b} / \partial r|_{(r_0, z_0)}$ are stratification, $f_0 \equiv 2\Omega + 2\bar{\omega}$ is the modified Coriolis parameter, and $M_r \equiv (1/r_0)(\partial \bar{m} / \partial r)|_{(r_0, z_0)} = f_0 + r_0(\partial \bar{\omega} / \partial r)|_{(r_0, z_0)}$ describes the radial gradient of the background angular momentum $\bar{m} = (\Omega + \bar{\omega})r^2$, all evaluated at (r_0, z_0) . $\theta_0 = \arctan(z_0/r_0)$ is the local latitude.

Stability requires that the frequency ξ has no imaginary part, which means the right-hand-side of Equation 3.10 is positive definite. Specifically, this requires the quadratic function $b_z \sin \theta_0 k_r^2 + (f_0 M_r + b_r \cos \theta_0) k_z^2 - 2b_r \sin \theta_0 k_r k_z > 0$ for all k_r and k_z . Consequently, the stability matrix

$$\begin{pmatrix} b_z \sin \theta_0 & -b_r \sin \theta_0 \\ -b_r \sin \theta_0 & f_0 M_r + b_r \cos \theta_0 \end{pmatrix}$$

must be positive definite. This stability matrix is essentially the same as the one derived

in previous studies on symmetric instability on a tilted f -plane (e.g., Itano and Maruyama, 2009) although here it is expressed in cylindrical coordinates instead of spherical coordinates to emphasize the symmetry in flows that are strongly rotationally constrained, and the Coriolis parameter f_0 is modified by the angular velocity of the background flow $\bar{\omega}$. It also closely resembles the corresponding expression for a compressible atmosphere in cylindrical coordinates (Ogilvie, 2019). The necessary and sufficient conditions for instability are that either the trace or determinant of this matrix be negative, i.e., either

$$b_z \sin \theta_0 + f_0 M_r + b_r \cos \theta_0 < 0, \quad (3.11)$$

or

$$b_z \sin \theta_0 (f_0 M_r + b_r \cos \theta_0) - b_r^2 \sin^2 \theta_0 < 0. \quad (3.12)$$

Equation 3.12 can also be expressed in terms of a condition for the background potential vorticity. The background potential vorticity is

$$q_0 = \nabla \bar{b} \cdot (2\mathbf{\Omega} + \nabla \times \bar{\mathbf{v}}) = M_r b_z - M_z b_r = M_r b_z - \frac{b_r}{f_0} (b_r \sin \theta_0 - b_z \cos \theta_0), \quad (3.13)$$

where $M_z \equiv (1/r_0)(\partial \bar{m}/\partial z)|_{(r_0, z_0)} = r_0(\partial \bar{\omega}/\partial z)|_{(r_0, z_0)}$ describes the vertical gradient of the background angular momentum, and the background gradient wind shear (Equation 3.9) is applied. We can hence rewrite Equation 3.12 as

$$q_0 f_0 \sin \theta_0 < 0, \quad (3.14)$$

i.e., instability occurs if the potential vorticity has the opposite sign to the generalized background planetary vorticity. This instability criterion is commonly referred to as the

symmetric instability criterion (e.g., Hoskins, 1974; Haine and Marshall, 1998), although it should be noted that in general, Equation 3.14 by itself is a sufficient but not necessary condition for instability.

3.2.2 *Most unstable mode and growth rate*

When ξ is imaginary, the perturbation fields will grow exponentially. Note that ξ^2 is a real number (Equation 3.10); therefore, ξ is either purely real (corresponding to neutral wave solutions) or purely imaginary (corresponding to exponentially growing or decaying solutions). By substituting $\xi = i\sigma$ in the dispersion relation (Equation 3.10), the e-folding growth rate σ of the unstable mode can be expressed as

$$\sigma = \left(-\frac{b_z \sin \theta_0 \tan^2 \delta + 2b_r \sin \theta_0 \tan \delta + f_0 M_r + b_r \cos \theta_0}{1 + \tan^2 \delta} \right)^{1/2}, \quad (3.15)$$

where $\delta \in (-\pi/2, \pi/2]$ is the angle between the unstable mode and $\mathbf{e}_r$ so that $\tan \delta = -k_r/k_z$. Equation 3.15 indicates that for a given background field and latitude, the growth rate is only a function of $\tan \delta$, i.e., the direction of the unstable mode, but not the magnitude of the wavenumber, i.e., the size of the mode. The result that the growth rate depends only on the direction of the mode is consistent with previous studies of symmetric instability on an f -plane (e.g., Ooyama, 1966) and a tilted f -plane (e.g., Sun, 1995). By calculating the derivatives of the function $\sigma(\tan \delta)$ and performing some algebra, we find that the maximum growth rate is obtained when

$$\tan \delta_m = \frac{b_z \sin \theta_0 - f_0 M_r - b_r \cos \theta_0 - [(f_0 M_r + b_r \cos \theta_0 - b_z \sin \theta_0)^2 + 4b_r^2 \sin^2 \theta_0]^{1/2}}{2b_r \sin \theta_0}. \quad (3.16)$$

It should be noted that δ_m also characterizes the net heat and zonal momentum transports in the r - z plane by the most unstable mode, and their directions align with the mode itself

(see Appendix 3.B). Substituting Equation 3.16 into Equation 3.15, we obtain the maximum growth rate as

$$\sigma_m = \left(-\frac{f_0 M_r + b_r \cos \theta_0 + b_z \sin \theta_0 - \sqrt{(f_0 M_r + b_r \cos \theta_0 - b_z \sin \theta_0)^2 + 4b_r^2 \sin^2 \theta_0}}{2} \right)^{1/2}. \quad (3.17)$$

3.2.3 Instability diagram

In this section, we discuss the instability criteria assuming $b_r \sin \theta_0 \neq 0$. The special case where $b_r \sin \theta_0 = 0$ is discussed in Appendix 3.C, where it is shown that the most unstable mode in this case always aligns with the radial (r) or vertical (z) directions. The instability criteria for fluid parcels perturbed purely along the $\mathbf{e}_r$ and $\mathbf{e}_z$ directions are $f_0 M_r + b_r \cos \theta_0 < 0$ and $b_z \sin \theta_0 < 0$, respectively, which are derived by solving $\xi^2 < 0$ (Equation 3.10) when $\delta = 0$ and $\delta = \pi/2$, respectively. It is convenient to define a 2-D nondimensionalized phase space based on the stability in these two orthogonal directions:

$$x = \frac{b_z \sin \theta_0}{|b_r \sin \theta_0|}, \quad y = \frac{f_0 M_r + b_r \cos \theta_0}{|b_r \sin \theta_0|}. \quad (3.18)$$

The stability matrix then becomes

$$\begin{pmatrix} x & -1 \\ -1 & y \end{pmatrix},$$

and positive definiteness requires that

$$x + y > 0, \text{ and } xy > 1, \quad (3.19)$$

which means only the region above the hyperbolic $xy = 1$ in the first quadrant ($x > 0, y > 0$)

is stable (Figure 3.2c & d).

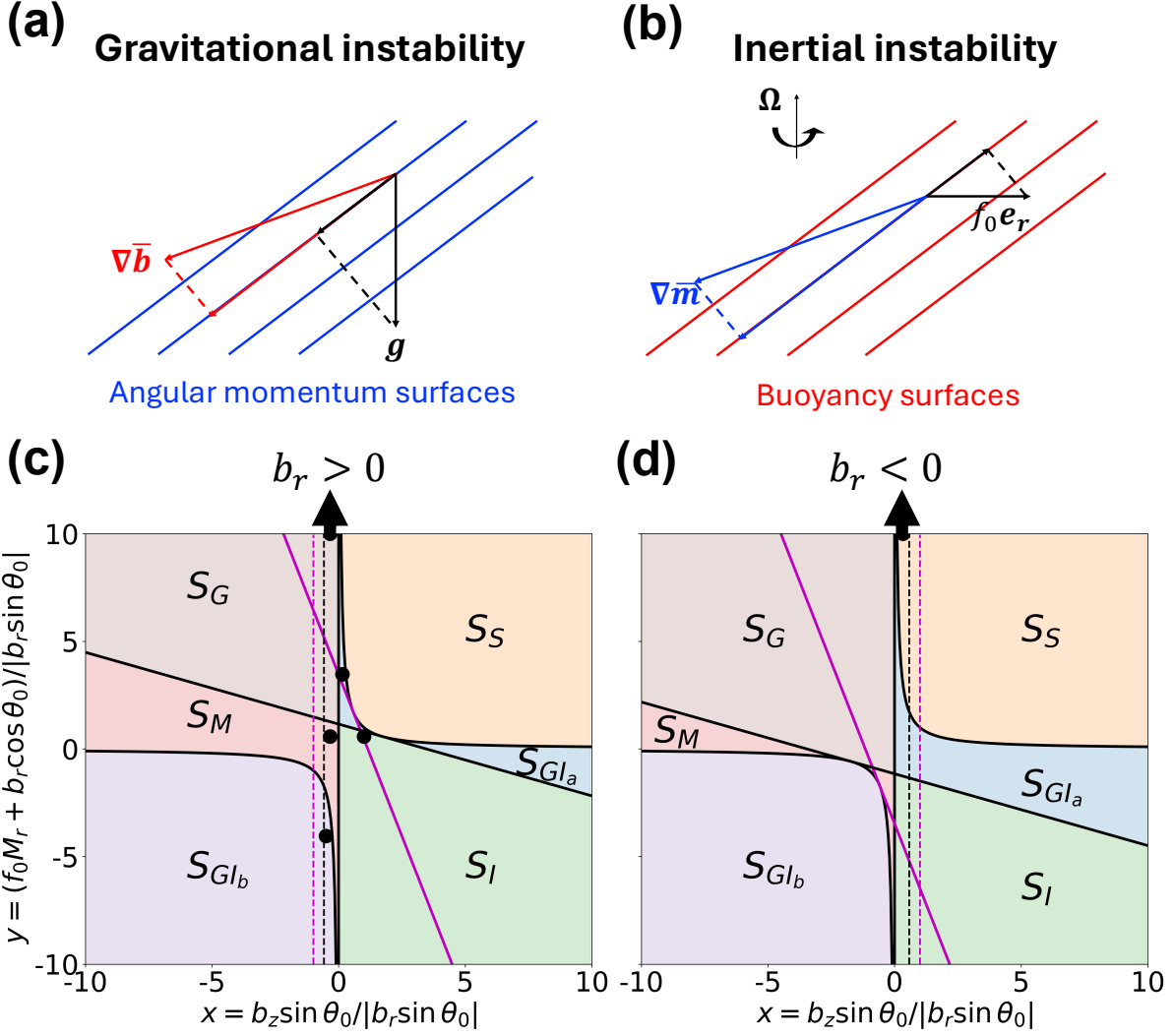


Figure 3.2: **Instability diagrams** for $\theta_0 = 60^\circ$. (a) and (b) sketch gravitational and inertial instabilities, respectively (see text for details). (c) and (d) show the instability regime diagram with $b_r > 0$ in (c) and $b_r < 0$ in (d). Subscripts in the labels indicate different types of instability: G for gravitational instability (Equation 3.20), I for inertial instability (Equation 3.21), M for mixed symmetric instability (Equation 3.14), and S for the stable regime. The GI regime is both gravitationally and inertially unstable, with GI_a corresponding to $q_0 f_0 \sin \theta_0 < 0$ and GI_b to $q_0 f_0 \sin \theta_0 > 0$. Dots in (c) and (d) mark simulation parameters listed in Table 3.1. In simulations s_G and s_S , the y -axis value is 1.15×10^4 , which lies outside the plotted range. Black solid lines in (c) and (d) indicate the criteria for gravitational, inertial, and mixed symmetric instabilities (Equations 3.22-3.24). The dashed lines indicate $N^2 = 0$, with regions to the left corresponding to $N^2 < 0$. The magenta lines indicate the criterion for $\theta_0 = 30^\circ$ for comparison.

3.2.4 *Gravitational instability, inertial instability, and mixed symmetric instability*

In a rotating fluid with background stratification and shear, two restoring forces, associated with gravity and rotation, act when a fluid parcel is displaced: buoyancy force and inertial acceleration. If a fluid parcel is displaced along a constant angular momentum surface, the inertial acceleration anomaly is zero and the only restoring force is the buoyancy force anomaly. As a result, pure gravitational instability can occur when the stratification is unstable along constant angular momentum surfaces. Similarly, if the displacement of the fluid parcel is along a constant buoyancy surface, the only restoring force is the inertial acceleration anomaly, hence pure inertial instability can occur when the angular momentum shear is unstable along constant buoyancy surfaces. Here, unstable stratification indicates buoyancy decreases in the opposite direction to gravity, and unstable angular momentum shear indicates the angular momentum gradient is opposite to the background planetary angular momentum gradient, $f_0 \mathbf{e}_r$ (Figure 3.2a & b). Therefore, gravitational instability can occur when

$$(\nabla \bar{b} \cdot \mathbf{e}_M)(-\mathbf{g} \cdot \mathbf{e}_M) = g(M_r b_z - M_z b_r)(M_r \sin \theta_0 - M_z \cos \theta_0) < 0, \quad (3.20)$$

where $\mathbf{e}_M = -M_z \mathbf{e}_r + M_r \mathbf{e}_z$ is the direction along a constant angular momentum surface. Inertial instability can occur when

$$(\nabla \bar{m} \cdot \mathbf{e}_b)(f_0 \mathbf{e}_r \cdot \mathbf{e}_b) = f_0 b_z r_0 (M_r b_z - M_z b_r) < 0, \quad (3.21)$$

where $\mathbf{e}_b = -b_z \mathbf{e}_r + b_r \mathbf{e}_z$ is the direction along a constant buoyancy surface. Using the gradient wind shear (Equation 3.9), and the definition of x and y (Equation 3.18), the gravitational instability criterion (Equation 3.20) becomes

$$[y + \cot^2 \theta_0 x - 2|\cot \theta_0| \text{sgn}(b_r)](xy - 1) < 0, \quad (3.22)$$

where sgn denotes the sign function, and the inertial instability criterion (Equation 3.21) becomes

$$x(xy - 1) < 0. \quad (3.23)$$

Comparison with the results derived in Section 3.2.1 shows that neither the gravitational nor inertial instability criteria are necessary for instability. Instability can still occur as a result of gravitational and inertial force anomalies when $q_0 f_0 \sin \theta_0 < 0$, or equivalently,

$$xy - 1 < 0. \quad (3.24)$$

We therefore refer to this instability (Equation 3.24) when the flow is both gravitationally stable and inertially stable as a mixed symmetric instability.

In the instability diagram, the phase plane is divided into several regions where gravitational instability (Equation 3.22), inertial instability (Equation 3.23), or mixed symmetric instability (Equation 3.24) occur. In all regions except for the stable region S_S located above the hyperbolic curve $xy = 1$ in the first quadrant, at least one type of instability is present (see Figure 3.2c & d and Figure 3.3a).

The alignment of the most unstable mode with the buoyancy and angular momentum surfaces is indicative of different types of instabilities. To illustrate how the most unstable mode aligns with the buoyancy and angular momentum surfaces, we define $\alpha \in (-\pi/2, \pi/2]$ as the angle between the direction of the most unstable mode and the buoyancy surface, and $\beta \in (-\pi/2, \pi/2]$ as the angle between the direction of the most unstable mode and the angular momentum surface, with the most unstable mode counterclockwise of the buoyancy/angular momentum surface defined as positive (Figure 3.3b). When $\alpha = 0$, the most

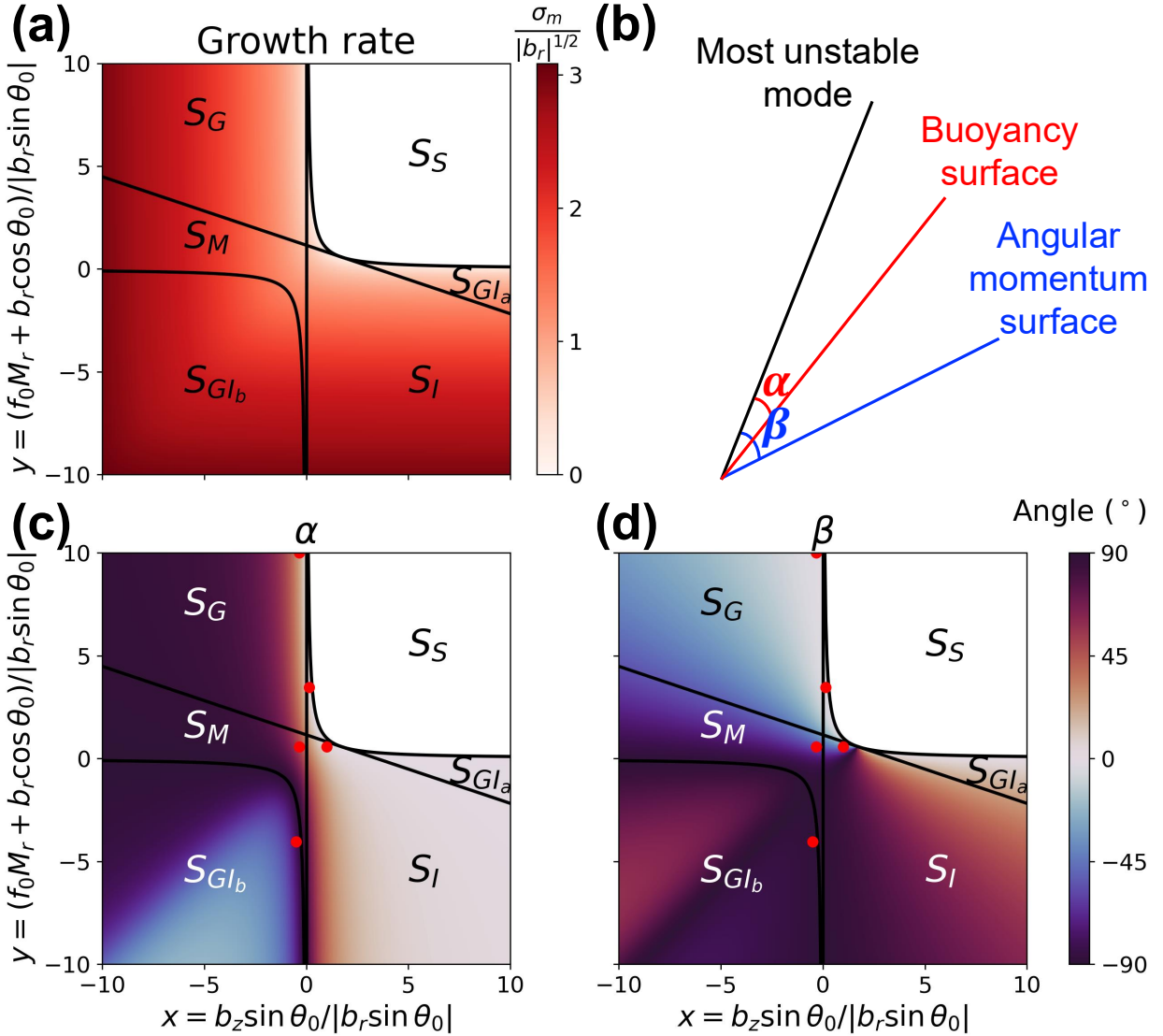


Figure 3.3: **Growth rate and orientation of the most unstable mode with positive b_r here shown for latitude $\theta_0 = 60^\circ$.** (a): growth rate of the most unstable mode, nondimensionalized with $|b_r|^{1/2}$: $\sigma_m/|b_r|^{1/2} = ((-x - y + ((x - y)^2 + 4)^{1/2})|\sin \theta_0|/2)^{1/2}$. (b) defines the angles of the most unstable mode relative to the buoyancy (α) and the angular momentum surface (β), shown in (c) and (d), respectively. In (a), (c), and (d), the dots, solid lines, and labels are the same as in Figure 3.2c. Note that $+90^\circ$ and -90° are equivalent as they correspond to a perturbation along the same physical surface.

unstable mode aligns with the buoyancy surface, and there is no buoyancy transport associated with the most unstable mode (Equation 3.60). As a result, the most unstable mode can only extract kinetic energy from the background angular momentum field (see Appendix 3.B

for detailed derivations), and regions where $\alpha \approx 0$ can be identified as inertially unstable (Figure 3.3c). Similarly, when the most unstable mode aligns with the angular momentum surface ($\beta = 0$), it can only extract potential energy from the background buoyancy field, and regions where $\beta \approx 0$ can be identified as gravitationally unstable (Figure 3.3d). However, the opposite is not always true, and in many parts of the parameter regime, the most unstable mode does not align with either surface. The special case where the Rossby number is small and the most unstable mode always aligns with the angular momentum surface will be discussed in Section 3.2.5.

Most previous studies on symmetric instability have focused on what we here call the “mixed symmetric instability”, thus neglecting regimes where $N^2 < 0$ or $\eta/f < 0$, as they typically assume stable stratification in the gravitational direction and stable zonal velocity shear in the latitudinal direction. However, we find that $N^2 < 0$ is neither sufficient nor necessary for instability. In Figure 3.2c & d, the dashed lines indicate $N^2 = 0$, with regions to the left of these lines corresponding to $N^2 < 0$. Therefore, when $b_r < 0$, the system can remain stable even if $N^2 < 0$. Meanwhile, $\eta/f < 0$ is a sufficient (but not necessary) condition for instability. When $\eta/f > 0$, the sufficient and necessary condition for instability becomes $q_0 f_0 \sin \theta_0 < 0$ (see detailed derivation in Appendix 3.D). This result is consistent with Flasar and Gierasch (1978) and Hathaway et al. (1979), who assumed that the background flow is in thermal wind balance with no meridional shear, such that $\eta/f = 1 > 0$. Under these assumptions, they found that symmetric instability would occur if and only if $q_0 f_0 \sin \theta_0 < 0$ (equivalent to Equation 38 in Hathaway et al. (1979) after substituting in the background thermal wind shear), consistent with our findings.

3.2.5 Low Rossby number limit

In the low Rossby number regime, $\text{Ro} = U/(f_0 L) \ll 1$ where U is the velocity scale and L is the length scale of motion, the rotational effect dominates over nonlinear advection (c.f.

Vallis, 2017). Many planetary flows in geophysical fluid dynamics likely fall into this regime, such as the deep convection in the bulk ocean away from the boundary layers on icy moons. For deep convection, the Rossby number can be estimated by the convective Rossby number, $\text{Ro}_C = B^{1/2} f_0^{-3/2} H^{-1}$ (Maxworthy and Narimousa, 1994), where B is the buoyancy flux and H is the depth of the ocean (Ro_C is equivalent to the square root of the modified flux Rayleigh number in Rayleigh–Bénard convection studies, c.f. Christensen and Aubert, 2006). In the bulk ocean interior, Jansen et al. (2023) suggest that the convective Rossby number is at most on the order 10^{-3} , indicating that the deep convection in the bulk icy moon ocean is strongly constrained by rotation.

In the low Rossby number limit, we have $\bar{\omega}$, $r\partial\bar{\omega}/\partial r$, $r\partial\bar{\omega}/\partial z \ll f_0$, which implies $M_r \approx f_0 \approx 2\Omega$ and $M_z \ll f_0$. In gradient wind balance, we moreover have the scaling that b_r , $b_z \sim f_0 M_z \ll f_0^2$. Consequently, the criterion for inertial instability, Equation 3.21, reduces to

$$f_0^2 b_z^2 < 0, \quad (3.25)$$

which cannot be satisfied, because small Rossby number flow is always inertially stable. Instability then occurs if, and only if,

$$b_z \sin \theta_0 < 0, \quad (3.26)$$

i.e., for an unstable stratification in the direction parallel to planetary rotation, which coincides with the angular momentum surface in the small Ro limit. The most unstable mode (Equation 3.16) is obtained when

$$|\tan \delta_m| \approx \left| \frac{-f_0^2 - [(f_0^2)^2]^{1/2}}{2b_r \sin \theta_0} \right| = \left| \frac{f_0^2}{b_r \sin \theta_0} \right| \gg 1, \quad \text{i.e., } \delta_m \approx \frac{\pi}{2}, \quad (3.27)$$

and the corresponding growth rate is

$$\sigma_m \approx (-b_z \sin \theta_0)^{1/2}. \quad (3.28)$$

Therefore, the most unstable mode is associated with slantwise convection aligned with the planetary angular momentum surface. This mode, parallel to the rotation axis, is likely to be important in the bulk oceans away from boundary layers on icy moons, which are thought to be characterized by low Rossby numbers. Such slantwise convection aligned with the rotation axis has been identified in numerical simulations of icy moon oceans (e.g., Ashkenazy and Tziperman, 2021; Kang et al., 2022c; Bire et al., 2022; Zeng and Jansen, 2024).

3.3 Numerical simulations

3.3.1 *Simulation set up*

To verify the theoretical results, we numerically integrate Equations 3.1–3.5 for a zonally symmetric flow ($\partial/\partial\phi = 0$) using Dedalus, which can solve initial-value partial differential equations using spectral methods (Burns et al., 2020). We nondimensionalize the equations using the rotational time scale ($T = f_0^{-1}$) and the domain length scale. We neglect variations of the background field and the metric term in the continuity equation (v/r), allowing the use of a local Cartesian coordinate system and double-periodic boundary conditions (c.f. Appendix 3.E). We prescribe a background state which satisfies the gradient wind balance (Equations 3.6 & 3.7), and solve for the evolution of the nondimensionalized perturbation fields ($\hat{u}, \hat{v}, \hat{w}, \hat{b}, \hat{\Phi}$). We apply a resolution with 256 grid points in both r and z directions. Simulations without viscosity and diffusivity develop unphysical grid-scale noise, once nonlinear effects become significant (Figure 3.8). In addition to inviscid simulations, we therefore perform simulations with a Leith sub-grid parameterization (Leith, 1996) to represent the effects of sub-grid-scale eddy mixing, and compare their results with the simulations without

viscosity and diffusivity. It takes some time for the most unstable mode to grow and become dominant, before which nonlinear effects may already induce turbulence in the system. To address this issue, we initialize the simulations with their most unstable modes (as inferred from an integration of the linearized equations). A detailed description of the simulation setup can be found in Appendix 3.E.

We carry out six simulations with the background field representing each region in the parameter space, where different types of instability (or no instability) occur (Figure 3.2c & d). The simulation parameters are summarized in Table 3.1 and the prescribed background fields are shown in Figure 3.4.

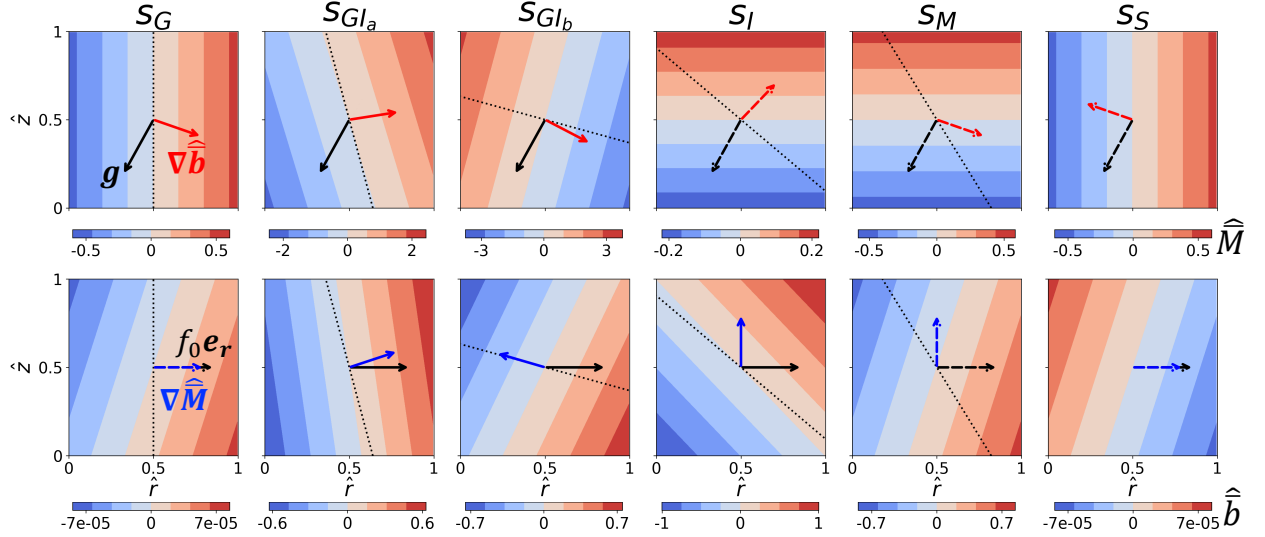


Figure 3.4: **Background states in the simulations.** The first row shows the background angular momentum surfaces ($\hat{M} = \hat{M}_r \hat{r} + \hat{M}_z \hat{z}$) and the second row shows the background buoyancy surfaces ($\hat{b} = \hat{b}_r \hat{r} + \hat{b}_z \hat{z}$), where the domain-averaged values are subtracted. Blue arrows indicate the direction of angular momentum gradients, red arrows indicate the direction of buoyancy gradients, and black arrows indicate gravity $\mathbf{g}$ in the upper row and the Coriolis vector $f_0 \mathbf{e}_r$ in the lower row, all consistent with sketches in Figure 3.2a & b. Solid arrows indicate gravitational (red) and/or inertial (blue) instability, while dashed arrows indicate stability. The black dotted line indicates the orientation of the most unstable mode predicted by Equation 3.16.

Simulation	$\hat{b}_r$	$\hat{b}_z$	$\hat{M}_r$	$\hat{q}_0$	$\hat{N}^2$	Instability	$\hat{\sigma}_m$
s_G	1e-4	-3.33e-5	1	-3.33e-5	2.11e-5	Grav	5.37e-3
s_{GI_a}	1	0.15	2.5	-0.416	0.630	Grav, Iner	0.333
s_{GI_b}	1	-0.5	-4	0.884	6.70e-2	Grav, Iner	1.93
s_I	1	1	0	-0.366	1.37	Iner	0.450
s_M	1	-0.333	0	-1.03	0.211	Mixed	0.920
s_S	-1e-4	3.33e-5	1	3.33e-5	-2.11e-5	Stable	/

Table 3.1: **Numerical simulation setups.** The latitudes for all simulations are $\theta_0 = 60^\circ$. The simulation parameters are nondimensionalized with the rotational time scale (f_0^{-1}) and the domain length scale, and hats are used to indicate nondimensionalized quantities (See Appendix 3.A). The types of instability in each simulation are indicated in the simulation name and the “Instability” column, where G and Grav indicate gravitational instability, I and Iner indicate inertial instability, M and Mixed indicate mixed symmetric instability, and S and Stable indicate the system is stable. Simulations s_G and s_S represent the low Rossby number limit.

3.3.2 Simulation results

For all simulations without diffusivity and viscosity, the eddy kinetic energy (EKE) grows exponentially, consistent with the maximum growth rate predicted by Equation 3.17. When the Leith closure is applied, the EKE growth rate decreases slightly due to viscous and

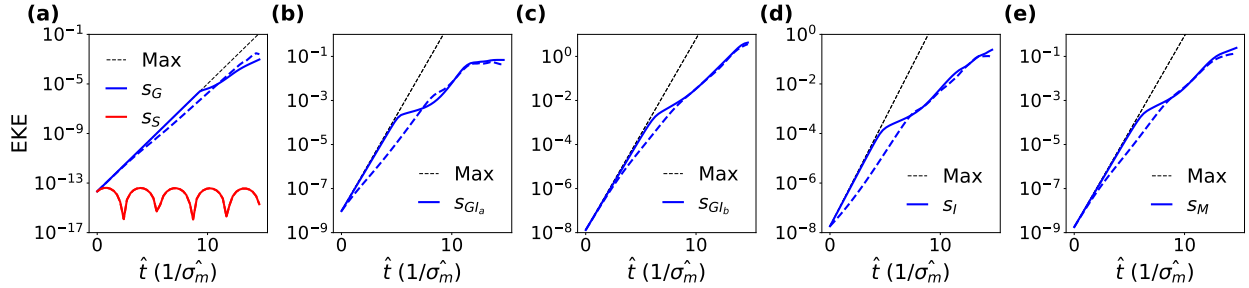


Figure 3.5: **Time series for domain-averaged eddy kinetic energy (EKE), $(\hat{u}^2 + \hat{v}^2 + \hat{w}^2)/2$.** The blue and red solid lines show the simulation results without diffusivity and viscosity, and the blue and red dashed lines show the simulation results with eddy diffusivity and viscosity. Note that the red dashed line overlaps the red solid line because the effect of eddy viscosity is weak. The black dashed line shows the predicted maximum growth rate of EKE ($2\hat{\sigma}_m$ due to the square, Equation 3.17). The time is normalized by the maximum growth rate in each simulation (for s_S , the normalization uses the maximum growth rate for simulation s_G).

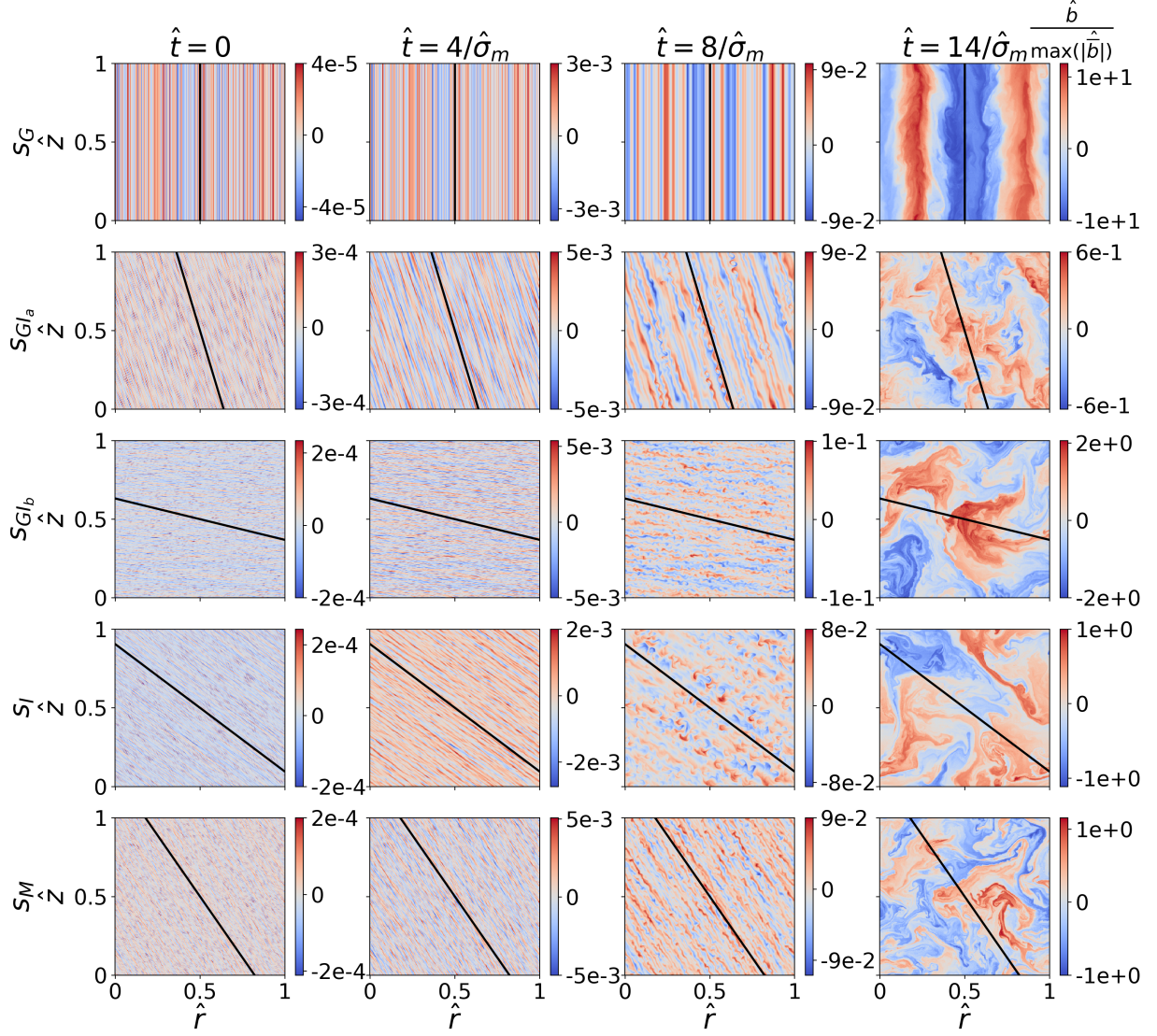


Figure 3.6: **Snapshots of the normalized buoyancy anomaly fields from simulations with eddy diffusivity and viscosity.** The buoyancy anomaly is normalized by the maximum value of the background buoyancy field ($\hat{b}/\max(|\hat{b}|)$) in each simulation. The time of each snapshot is indicated above the respective columns and are normalized by the maximum growth rate $\hat{\sigma}_m$ in each simulation. The black solid lines indicate the orientation of the most unstable mode predicted by Equation 3.16.

diffusive dissipation (Figure 3.5). During the exponential growth phase ($\hat{t} = 4/\hat{\sigma}_m$) of simulations with an unstable background state, the orientation of the perturbation fields aligns closely with the most unstable mode predicted by Equation 3.16 (Figure 3.6, second column). At $\hat{t} = 8/\hat{\sigma}_m$, nonlinear effects begin to become important, and the flow field becomes turbulent, while it is still dominated by small-scale eddies roughly aligned with the most unstable mode (Figure 3.6, third column). Eventually, at $\hat{t} = 14/\hat{\sigma}_m$, the nonlinear advection effects become dominant, the flow is characterized by domain-scale eddies, and the growth rates of EKE decrease significantly (Figure 3.6, last column). Notably, the EKE eventually converges for simulations with and without eddy viscosity and diffusivity (Figure 3.5).

The simulation with stable background state (s_S) does not show exponential growth but an oscillation behavior. The initial condition used in simulation s_S is the most unstable mode in simulation s_G , which has $\tan \delta = \pi/2$. In this case, we can estimate the oscillation period P according to the dispersion relation (Equation 3.10) as $P = 584.79$, or $3.14/\sigma_m$ with σ_m being the maximum growth rate of s_G , consistent with the oscillation period in the simulation (Figure 3.5a).

The numerical simulations also show that $N^2 < 0$ is neither a sufficient nor necessary condition for instability. In simulation s_S , $N^2 < 0$, but the system is stable. In simulations s_G and s_{GI_b} , $N^2 > 0$, but the systems are unstable, with either $q_0 f_0 \sin \theta_0 > 0$ (s_{GI_b}) or $q_0 f_0 \sin \theta_0 < 0$ (s_G).

In simulations with $\text{Ro} \ll 1$ (s_G and s_S), the instability criteria reduce to $b_z \sin \theta_0 < 0$. In the unstable case (simulation s_G), the most unstable mode is parallel to the rotation axis, i.e., slantwise convection parallel to planetary rotation.

3.4 Discussions

The analysis conducted in this chapter provides physical insight into symmetric instability within a generalized framework, applicable across a wide range of planetary parameter regimes. This framework offers useful guidance for developing more accurate convection schemes for icy moon oceans in Chapter 5.

The traditional convective adjustment schemes widely applied in ocean General Circulation Models (e.g., Marotzke, 1991) parameterize upright convection when $N^2 < 0$. However, our results show that a negative stratification in the gravitational direction ($N^2 < 0$) is neither a sufficient nor necessary condition for instability. This suggests that such schemes may not be appropriate for capturing slantwise convection in icy moon oceans. Given that $b_z \sin \theta_0 < 0$ is a sufficient condition for symmetric instability, and becomes a necessary and sufficient condition in the low Rossby number limit, we propose that stratification along the rotation axis should be treated as the criterion for static stability and used to trigger convective parameterizations in planetary ocean models.

In the linear regime, the angle of the most unstable mode controls both the direction and relative magnitude of the heat and momentum transports (see Section 3.2.4 and Appendix 3.B). However, our linear instability analysis does not capture nonlinear effects. As shown in Figure 3.6, once the flow becomes fully nonlinear and turbulent, it no longer follows the structure of the most unstable mode. A complete understanding of heat and momentum transport in fully nonlinear slantwise convection is left for future studies.

Although we consider Boussinesq fluids in this chapter, the results are also relevant for anelastic fluids, such as atmospheric convection on gas giants (e.g., O’Neill and Kaspi, 2016). In a compressible fluid, the Solberg–Høiland criteria in Equations 3.11 and 3.12 remain valid, but the buoyancy gradients must be replaced by the specific entropy gradient, as derived by Ogilvie (2019). In the low-Rossby number regime, where the angular momentum gradient is dominated by planetary rotation, the criteria reduce to unstable entropy gradients along

the rotation axis, similar to our findings under the Boussinesq approximation. Observations of the gravitational fields of Jupiter and Saturn suggest that the zonal jets on these planets are aligned with the rotation axis (Kaspi et al., 2018; Galanti et al., 2019; Kaspi et al., 2023), indicating that the prevailing modes are parallel to planetary rotation. The in-situ temperature profile measurement from the Galileo probe indicates a neutral stratification along constant angular momentum surfaces, which are parallel to the rotation axis outside the high-shear region in the upper weather layer (O’Neill et al., 2017). Numerical simulations of gas giant atmospheres also reveal slantwise convection aligned with planetary rotation (Christensen, 2002; Kaspi et al., 2009; Heimpel et al., 2016).

Our study focuses exclusively on zonally symmetric instabilities. However, asymmetric modes could also play a significant role in fluid motions on a rotating planet. For instance, equatorial convective rolls driven by heating from the bottom of the fluid are believed to be important in driving the equatorial jets on gas giants (e.g., Busse and Hood, 1982; Busse and Or, 1986). Therefore, it is important to characterize the growth rate of different symmetric and asymmetric unstable modes to determine under which regime the symmetric mode dominates. Stone (1966) studied the growth rate of symmetric instability, Kelvin-Helmholtz instability, and baroclinic instability on an f -plane with hydrostatic balance. He concludes that on an f -plane, symmetric instability prevails when $0.25 < \text{Ri} < 0.95$; Kelvin-Helmholtz instability dominates for $\text{Ri} < 0.25$; and baroclinic instability is dominant when $\text{Ri} > 0.95$. When considering the horizontal component of planetary rotation, Jeffery and Wingate (2009) suggest that the transition point between symmetric and baroclinic instability dominance can exceed $\text{Ri} = 0.95$. Understanding how the symmetric mode, such as slantwise convection in the low Rossby number limit, interacts with baroclinic instability under arbitrary background stratification and shear, and conditions that separate symmetric and asymmetric mode dominance, are important future research directions. These questions will be revisited in Chapter 6. We applied a linear instability analysis method, which is valu-

able to understand stability criteria for small-amplitude perturbations but is not applicable to large-amplitude perturbations or non-linear instability problems. Bowman and Shepherd (1995) applied the energy-Casimir stability method (Holm et al., 1985; Shepherd, 1990; Cho et al., 1993) to study nonlinear symmetric instability where the hydrostatic approximation is made and the rotation is aligned with gravity. However, extending this approach to a more generalized setup (e.g., Fruman and Shepherd, 2008) presents challenges that remain to be addressed in future work. In our current study, solutions are sought on an infinitely large domain (local plane-wave solutions). Future work should also explore how boundary conditions might modify the characteristics of the most unstable modes.

3.5 Conclusions

We conduct local linear instability analysis with slowly-varying background shear and stratification, and find that the necessary and sufficient conditions for instability of zonally symmetric Boussinesq flow on a rotating planet are the background field is (1) gravitationally unstable, or (2) inertially unstable, or (3) the background potential vorticity has a different sign from the planetary vorticity, which is consistent with previous studies on symmetric instability (e.g., Hoskins, 1974). In our framework, we define the gravitational instability criterion as unstable buoyancy stratification along angular momentum surfaces (Equation 3.20 & Figure 3.2a) and the inertial instability criterion as unstable angular momentum shear along buoyancy surfaces (Equation 3.21 & Figure 3.2b), which ensures both to be sufficient conditions for instability. Mathematically, the criterion for instability can most compactly be expressed as either $b_z \sin \theta_0 < 0$ (i.e., unstable stratification along the planetary rotation axis) or $q_0 f_0 \sin \theta_0 < 0$ where q_0 is the background potential vorticity. These results establish the stability criterion for slantwise convection under planetary conditions relevant to icy moon oceans. When instability occurs, the growth rate is not sensitive to the magnitude of the radial and vertical wavenumbers, k_r and k_z , but is only a function of their

ratio, k_r/k_z , i.e., the tilting direction of the mode.

A negative stratification in the gravitational direction ($N^2 < 0$) is neither necessary nor sufficient for instability, indicating that traditional convective adjustment schemes used in ocean general circulation models (e.g., Marotzke, 1991) may not be appropriate for representing slantwise convection in icy moon oceans.

In the low Rossby number limit, instability occurs if and only if $b_z \sin \theta_0 < 0$, and the most unstable mode is slantwise convection parallel to the planetary rotation axis. This limit is particularly relevant for icy moon oceans and motivates the focus on the low Rossby number regime in the rest of the dissertation.

Our work provides criteria and physical insight into instabilities within a generalized framework applicable across diverse planetary parameter regimes. These results enhance our understanding of fluid motions on various planetary bodies and provide a foundation for developing parameterizations of subgrid-scale transport in large-scale models, such as the slantwise convection scheme for icy moon oceans presented in Chapter 5.

Appendix 3.A: Derivation of the dispersion relation

After subtracting the gradient wind balance and the hydrostatic balance for the background flow (Equations 3.6 & 3.7) from the original Equations 3.1-3.5, we have the perturbation equations:

$$\left(\frac{\partial}{\partial t} + v' \frac{\partial}{\partial r} + w' \frac{\partial}{\partial z} \right) u' + \left(2\Omega + \frac{\partial \bar{u}}{\partial r} + \frac{\bar{u}}{r} \right) v' + \frac{\partial \bar{u}}{\partial z} w' = 0, \quad (3.29)$$

$$\left(\frac{\partial}{\partial t} + v' \frac{\partial}{\partial r} + w' \frac{\partial}{\partial z} \right) v' - \left(2\Omega + \frac{2\bar{u}}{r} \right) u' - \cos \theta b' + \frac{\partial \Phi'}{\partial r} = 0, \quad (3.30)$$

$$\left(\frac{\partial}{\partial t} + v' \frac{\partial}{\partial r} + w' \frac{\partial}{\partial z} \right) w' - \sin \theta b' + \frac{\partial \Phi'}{\partial z} = 0, \quad (3.31)$$

$$\left(\frac{\partial}{\partial t} + v' \frac{\partial}{\partial r} + w' \frac{\partial}{\partial z} \right) b' + \frac{\partial \bar{b}}{\partial r} v' + \frac{\partial \bar{b}}{\partial z} w' = 0, \quad (3.32)$$

$$\frac{\partial v'}{\partial r} + \frac{\partial w'}{\partial z} + \frac{v'}{r} = 0. \quad (3.33)$$

We assume that the background fields vary slowly over the characteristic length scale of the perturbations, L , so that they can be treated as constants. This criterion can be formally obtained by performing the Taylor expansion of all background fields locally around $(r = r_0, z = z_0)$:

$$\begin{aligned} 2\Omega + \frac{2\bar{u}}{r} &= 2\Omega + 2\bar{\omega} \\ &= 2\Omega + 2\bar{\omega}(r_0, z_0) + 2 \left. \frac{\partial \bar{\omega}}{\partial r} \right|_{(r_0, z_0)} r' + 2 \left. \frac{\partial \bar{\omega}}{\partial z} \right|_{(r_0, z_0)} z' + \dots, \end{aligned} \quad (3.34)$$

$$\begin{aligned}
2\Omega + \frac{\partial \bar{u}}{\partial r} + \frac{\bar{u}}{r} &= 2\Omega + 2\bar{\omega} + r \frac{\partial \bar{\omega}}{\partial r} \\
&= 2\Omega + 2\bar{\omega}(r_0, z_0) + r_0 \left. \frac{\partial \bar{\omega}}{\partial r} \right|_{(r_0, z_0)} \\
&\quad + \left(3 \frac{\partial \bar{\omega}}{\partial r} + r_0 \frac{\partial^2 \bar{\omega}}{\partial r^2} \right)_{(r_0, z_0)} r' + \left(2 \frac{\partial \bar{\omega}}{\partial z} + r_0 \frac{\partial^2 \bar{\omega}}{\partial r \partial z} \right)_{(r_0, z_0)} z' + \dots,
\end{aligned} \tag{3.35}$$

$$\frac{\partial \bar{u}}{\partial z} = r \frac{\partial \bar{\omega}}{\partial z} = r_0 \left. \frac{\partial \bar{\omega}}{\partial z} \right|_{(r_0, z_0)} + \left(r_0 \frac{\partial^2 \bar{\omega}}{\partial r \partial z} + \frac{\partial \bar{\omega}}{\partial z} \right)_{(r_0, z_0)} r' + r_0 \left. \frac{\partial^2 \bar{\omega}}{\partial z^2} \right|_{(r_0, z_0)} z' + \dots, \tag{3.36}$$

$$\frac{\partial \bar{b}}{\partial r} = \left. \frac{\partial \bar{b}}{\partial r} \right|_{(r_0, z_0)} + \left. \frac{\partial^2 \bar{b}}{\partial r^2} \right|_{(r_0, z_0)} r' + \left. \frac{\partial^2 \bar{b}}{\partial r \partial z} \right|_{(r_0, z_0)} z' + \dots, \tag{3.37}$$

$$\frac{\partial \bar{b}}{\partial z} = \left. \frac{\partial \bar{b}}{\partial z} \right|_{(r_0, z_0)} + \left. \frac{\partial^2 \bar{b}}{\partial r \partial z} \right|_{(r_0, z_0)} r' + \left. \frac{\partial^2 \bar{b}}{\partial z^2} \right|_{(r_0, z_0)} z' + \dots, \tag{3.38}$$

$$\begin{aligned}
\sin \theta &= \sin \theta_0 + \left. \frac{\partial \sin \theta}{\partial r} \right|_{(r_0, z_0)} r' + \left. \frac{\partial \sin \theta}{\partial z} \right|_{(r_0, z_0)} z' + \dots \\
&= \sin \theta_0 - \sin \theta_0 \cos \theta_0 \frac{r'}{\sqrt{r_0^2 + z_0^2}} + \cos^2 \theta_0 \frac{z'}{\sqrt{r_0^2 + z_0^2}} + \dots,
\end{aligned} \tag{3.39}$$

$$\begin{aligned}
\cos \theta &= \cos \theta_0 + \left. \frac{\partial \cos \theta}{\partial r} \right|_{(r_0, z_0)} r' + \left. \frac{\partial \cos \theta}{\partial z} \right|_{(r_0, z_0)} z' + \dots \\
&= \cos \theta_0 + \sin^2 \theta_0 \frac{r'}{\sqrt{r_0^2 + z_0^2}} - \sin \theta_0 \cos \theta_0 \frac{z'}{\sqrt{r_0^2 + z_0^2}} + \dots,
\end{aligned} \tag{3.40}$$

where $\theta_0 = \arctan(z_0/r_0)$ is the local latitude, and $|r'| \sim |z'| \sim L$. We study the local instability of fluid in the outer region of the planet, where we assume $L \ll r_0$. The left-hand-sides of Equations 3.34–3.40 remain approximately constant over the scale L if

$$\begin{aligned}
\left(\frac{|\bar{\omega}|}{|d\bar{\omega}|}\right)_{(r_0, z_0)} &\gg L, & \left(\frac{|d\bar{\omega}|}{|d^2\bar{\omega}|}\right)_{(r_0, z_0)} &\gg L, \\
\left(\frac{|d\bar{b}|}{|d^2\bar{b}|}\right)_{(r_0, z_0)} &\gg L, & \frac{|\cos \theta_0|}{|\sin^2 \theta_0|}, \frac{|\sin \theta_0|}{|\cos^2 \theta_0|} &\gg \frac{L}{r_0},
\end{aligned} \tag{3.41}$$

where d and d^2 represent the first and second spatial derivatives, respectively. Equation 3.41 indicates that the approximation holds when $\bar{b}$ and $\bar{\omega}$ have a large radius of curvature relative to the characteristic perturbation length scale L , the variation in $\bar{\omega}$ is small compared to its average, and the region we consider is not too close to either the equator or the pole. Under this limit, we define

$$\begin{aligned}
f_0 &\equiv 2\Omega + 2\bar{\omega}(r_0, z_0), & M_r &\equiv f_0 + r_0 \left. \frac{\partial \bar{\omega}}{\partial r} \right|_{(r_0, z_0)}, & M_z &\equiv r_0 \left. \frac{\partial \bar{\omega}}{\partial z} \right|_{(r_0, z_0)}, \\
b_r &\equiv \left. \frac{\partial \bar{b}}{\partial r} \right|_{(r_0, z_0)}, & b_z &\equiv \left. \frac{\partial \bar{b}}{\partial z} \right|_{(r_0, z_0)}.
\end{aligned} \tag{3.42}$$

We nondimensionalize Equations 3.29-3.33 using the rotational time scale (f_0^{-1}) and the perturbation length scale (L), such that

$$\begin{aligned}
\hat{t} &= tf_0, & (\hat{u}, \hat{v}, \hat{w}) &= (u', v', w')/(f_0 L), & \hat{b} &= b'/(f_0^2 L), & \hat{\Phi} &= \Phi'/(f_0^2 L^2), \\
(\hat{r}, \hat{z}) &= (r', z')/L, & (\hat{M}_r, \hat{M}_z) &= (M_r, M_z)/f_0, & (\hat{b}_r, \hat{b}_z) &= (b_r, b_z)/f_0^2.
\end{aligned} \tag{3.43}$$

The nondimensionalized equations become

$$\left(\frac{\partial}{\partial \hat{t}} + \hat{v} \frac{\partial}{\partial \hat{r}} + \hat{w} \frac{\partial}{\partial \hat{z}} \right) \hat{u} + \hat{M}_r \hat{v} + \hat{M}_z \hat{w} = 0, \tag{3.44}$$

$$\left(\frac{\partial}{\partial \hat{t}} + \hat{v} \frac{\partial}{\partial \hat{r}} + \hat{w} \frac{\partial}{\partial \hat{z}} \right) \hat{v} - \hat{u} - \cos \theta_0 \hat{b} + \frac{\partial \hat{\Phi}}{\partial \hat{r}} = 0, \tag{3.45}$$

$$\left(\frac{\partial}{\partial \hat{t}} + \hat{v} \frac{\partial}{\partial \hat{r}} + \hat{w} \frac{\partial}{\partial \hat{z}}\right) \hat{w} - \sin \theta_0 \hat{b} + \frac{\partial \hat{\Phi}}{\partial \hat{z}} = 0, \quad (3.46)$$

$$\left(\frac{\partial}{\partial \hat{t}} + \hat{v} \frac{\partial}{\partial \hat{r}} + \hat{w} \frac{\partial}{\partial \hat{z}}\right) \hat{b} + \hat{b}_r \hat{v} + \hat{b}_z \hat{w} = 0, \quad (3.47)$$

$$\frac{\partial \hat{v}}{\partial \hat{r}} + \frac{\partial \hat{w}}{\partial \hat{z}} + \frac{\hat{v}}{r_0/L + \hat{r}} = 0. \quad (3.48)$$

We neglect the last term in Equation 3.48 because $r_0/L \gg 1$ for the local instability problem. In the linearized instability problem, we assume that the perturbation fields are much smaller than the background fields, and obtain the first-order linear perturbation equations:

$$\frac{\partial \hat{u}}{\partial \hat{t}} + \hat{M}_r \hat{v} + \hat{M}_z \hat{w} = 0, \quad (3.49)$$

$$\frac{\partial \hat{v}}{\partial \hat{t}} - \hat{u} - \cos \theta_0 \hat{b} + \frac{\partial \hat{\Phi}}{\partial \hat{r}} = 0, \quad (3.50)$$

$$\frac{\partial \hat{w}}{\partial \hat{t}} - \sin \theta_0 \hat{b} + \frac{\partial \hat{\Phi}}{\partial \hat{z}} = 0, \quad (3.51)$$

$$\frac{\partial \hat{b}}{\partial \hat{t}} + \hat{b}_r \hat{v} + \hat{b}_z \hat{w} = 0, \quad (3.52)$$

$$\frac{\partial \hat{v}}{\partial \hat{r}} + \frac{\partial \hat{w}}{\partial \hat{z}} = 0. \quad (3.53)$$

We look for plane-wave solutions of the linearized perturbation equations 3.49-3.53:

$$(\hat{u}, \hat{v}, \hat{w}, \hat{b}, \hat{\Phi}) = (A_u, A_v, A_w, A_b, A_\Phi) \exp(i\hat{k}_r \hat{r} + i\hat{k}_z \hat{z} - i\hat{\xi} \hat{t}), \quad (3.54)$$

where $A_u, A_v, A_w, A_b, A_\Phi$ are constant amplitudes, and $(\hat{k}_r, \hat{k}_z) = (k_r, k_z)L$.

Under these conditions and substituting Equation 3.54 into Equations 3.44–3.48, we get a linearized equation system. The linearized equations must have zero determinant to have non-trivial solutions, which requires

$$\begin{vmatrix} -i\hat{\xi} & \hat{M}_r & \hat{M}_z & 0 & 0 \\ -1 & -i\hat{\xi} & 0 & -\cos\theta_0 & i\hat{k}_r \\ 0 & 0 & -i\hat{\xi} & -\sin\theta_0 & i\hat{k}_z \\ 0 & \hat{b}_r & \hat{b}_z & -i\hat{\xi} & 0 \\ 0 & i\hat{k}_r & i\hat{k}_z & 0 & 0 \end{vmatrix} = 0. \quad (3.55)$$

This gives

$$\hat{\xi}[(\hat{k}_r^2 + \hat{k}_z^2)\hat{\xi}^2 - \hat{b}_z \sin\theta_0 \hat{k}_r^2 - (\hat{M}_r + \hat{b}_r \cos\theta_0)\hat{k}_z^2 + 2\hat{b}_r \sin\theta_0 \hat{k}_r \hat{k}_z] = 0. \quad (3.56)$$

Neglecting the trivial solutions $\hat{\xi} = 0$ and $\hat{k}_r = \hat{k}_z = 0$, we have the dispersion relation:

$$\hat{\xi}^2 = \frac{\hat{b}_z \sin\theta_0 \hat{k}_r^2 + (\hat{M}_r + \hat{b}_r \cos\theta_0)\hat{k}_z^2 - 2\hat{b}_r \sin\theta_0 \hat{k}_r \hat{k}_z}{\hat{k}_r^2 + \hat{k}_z^2}. \quad (3.57)$$

Restoring Equation 3.57 back to dimensionalized form, we have the dispersion relation:

$$\xi^2 = \frac{b_z \sin\theta_0 k_r^2 + (f_0 M_r + b_r \cos\theta_0)k_z^2 - 2b_r \sin\theta_0 k_r k_z}{k_r^2 + k_z^2}. \quad (3.58)$$

Appendix 3.B: Heat and zonal momentum transport by the most unstable mode

In this section, we analyze the heat transport ($\text{Re}(\hat{v}\hat{b}^*)$, $\text{Re}(\hat{w}\hat{b}^*)$) and zonal momentum transport ($\text{Re}(\hat{v}\hat{u}^*)$, $\text{Re}(\hat{w}\hat{u}^*)$) associated with the most unstable mode, where Re indicates the real part and the asterisk indicates the complex conjugation. By substituting Equation 3.54 into Equations 3.49, 3.52, and 3.53, we have

$$\begin{aligned}\hat{w}\hat{u}^* &= \frac{\hat{M}_r\hat{k}_z - \hat{M}_z\hat{k}_r}{-i\hat{\xi}\hat{k}_r}|\hat{w}|^2, \quad \hat{v}\hat{u}^* = \frac{\hat{k}_z(\hat{M}_r\hat{k}_z - \hat{M}_z\hat{k}_r)}{i\hat{\xi}\hat{k}_r^2}|\hat{w}|^2, \\ \hat{w}\hat{b}^* &= \frac{\hat{b}_r\hat{k}_z - \hat{b}_z\hat{k}_r}{-i\hat{\xi}\hat{k}_r}|\hat{w}|^2, \quad \hat{v}\hat{b}^* = \frac{\hat{k}_z(\hat{b}_r\hat{k}_z - \hat{b}_z\hat{k}_r)}{i\hat{\xi}\hat{k}_r^2}|\hat{w}|^2.\end{aligned}\tag{3.59}$$

For stable modes, the frequency $\hat{\xi}$ is real, so that $\hat{v}$ and $\hat{w}$ are $\pi/2$ out of phase with $\hat{u}$ and $\hat{b}$. As a result, we have $\text{Re}(\hat{w}\hat{u}^*) = \text{Re}(\hat{v}\hat{u}^*) = \text{Re}(\hat{w}\hat{b}^*) = \text{Re}(\hat{v}\hat{b}^*) = 0$, implying zero net heat and momentum transport.

For unstable modes, $-i\hat{\xi}$ is replaced by the positive growth rate $\hat{\sigma}$, which yields

$$\frac{\text{Re}(\hat{w}\hat{u}^*)}{\text{Re}(\hat{v}\hat{u}^*)} = \frac{\text{Re}(\hat{w}\hat{b}^*)}{\text{Re}(\hat{v}\hat{b}^*)} = -\frac{\hat{k}_r}{\hat{k}_z} = \tan \delta,\tag{3.60}$$

showing that the net heat and zonal momentum transports associated with the unstable mode are always aligned with the mode's orientation.

Equation 3.59 also provides the relative magnitude of the heat and zonal momentum transport:

$$\frac{|\mathbf{u}u|}{|\mathbf{u}b|} = \frac{\hat{M}_r\hat{k}_z - \hat{M}_z\hat{k}_r}{\hat{b}_r\hat{k}_z - \hat{b}_z\hat{k}_r} = \frac{|\nabla\hat{M}|\sin\beta}{|\nabla\hat{b}|\sin\alpha},\tag{3.61}$$

where $\mathbf{u} = v\mathbf{e}_r + w\mathbf{e}_z$, and α and β are the angles between the most unstable mode and the buoyancy and angular momentum surfaces, respectively (see Figure 3.3b). In addition to the overall magnitudes of the buoyancy and angular momentum gradients, the angles α

and β therefore determine the relative magnitude of the buoyancy and momentum fluxes. When the most unstable mode aligns with the angular momentum surface ($\beta = 0$), there is no zonal momentum transport, and the mode cannot extract kinetic energy from the background angular momentum field. Similarly, when $\alpha = 0$, the most unstable mode aligns with the buoyancy surface, and the mode cannot extract potential energy from the background buoyancy field.

Appendix 3.C: Instability analysis for $b_r \sin \theta_0 = 0$

In this section, we discuss the special case when $b_r \sin \theta_0 = 0$, which essentially suggests $b_r = 0$ since $\sin \theta_0 = 0$ violates Equation 3.41. In this case, the dispersion relation (Equation 3.10) becomes

$$\xi^2 = \frac{b_z \sin \theta_0 k_r^2 + f_0 M_r k_z^2}{k_r^2 + k_z^2}. \quad (3.62)$$

Therefore, the instability criteria reduce to $b_z \sin \theta_0 < 0$ or $f_0 M_r < 0$, i.e., the instability for perturbations in the vertical or radial directions (where the vertical is here defined as parallel to the rotation axis, while the radial is orthogonal to the rotation axis). When the system is unstable, the most unstable mode can be inferred by taking the limit $b_r \rightarrow 0$ of Equation 3.16, which gives

$$\lim_{b_r \rightarrow 0} \tan \delta_m \rightarrow \begin{cases} -\frac{b_r \sin \theta_0}{b_z \sin \theta_0 - f_0 M_r - b_r \cos \theta_0} \rightarrow 0 & \text{if } b_z \sin \theta_0 - f_0 M_r > 0, \\ \frac{b_z \sin \theta_0 - f_0 M_r - b_r \cos \theta_0}{b_r \sin \theta_0} \rightarrow \pm \infty & \text{if } b_z \sin \theta_0 - f_0 M_r < 0, \end{cases} \quad (3.63)$$

with the maximum growth rate being

$$\lim_{b_r \rightarrow 0} \sigma_m \rightarrow \begin{cases} (-f_0 M_r)^{1/2} & \text{if } b_z \sin \theta - f_0 M_r > 0, \\ (-b_z \sin \theta_0)^{1/2} & \text{if } b_z \sin \theta - f_0 M_r < 0. \end{cases} \quad (3.64)$$

This indicates that the most unstable mode always aligns with either the radial ($\delta_m = 0$) or vertical ($\delta_m = \pi/2$) directions, depending on the relative magnitude of the growth rate in these two directions, $(-f_0 M_r)^{1/2}$ and $(-b_z \sin \theta_0)^{1/2}$.

Appendix 3.D: Instability analysis for $N^2 < 0$ and $\eta/f < 0$

In this section, we show that $N^2 < 0$ is not a sufficient condition for instability in a zonally symmetric flow, while $\eta/f < 0$ is a sufficient condition for instability.

N^2 can be expressed as

$$\begin{aligned}
N^2 &= \frac{\partial \bar{b}}{\partial R} \\
&= b_z \sin \theta_0 + b_r \cos \theta_0 \\
&= |b_r \sin \theta_0| \left[\frac{b_z \sin \theta_0}{|b_r \sin \theta_0|} + \frac{b_r}{|b_r|} \frac{\cos \theta_0}{|\sin \theta_0|} \right] \\
&= |b_r \sin \theta_0| [x + |\cot \theta_0| \text{sgn}(b_r)],
\end{aligned} \tag{3.65}$$

where $x = b_z \sin \theta_0 / |b_r \sin \theta_0|$ is defined in Equation 3.18, and we used the definition that $\theta_0 \in (-\pi/2, \pi/2]$ so that $\cos \theta_0 > 0$. Therefore, $N^2 < 0$ is equivalent to

$$x < -|\cot \theta_0| \text{sgn}(b_r). \tag{3.66}$$

The blue lines mark Equation 3.66 in Figure 3.7 when $\theta_0 = 60^\circ$. As indicated in the plot, when $b_r < 0$, $N^2 < 0$ is not a sufficient condition for instability as the system can remain stable even when $N^2 < 0$, which is consistent with results shown in the numerical simulation *sS*.

We next consider the case $\eta/f < 0$, where, for easier comparison with previous work (e.g., Itano and Maruyama, 2009), we neglect the geometric terms so that $f_0 = 2\Omega$ and $M_r = f_0 + \partial \bar{u} / \partial r$. Note that

$$\frac{1}{R} \frac{\partial \bar{u}}{\partial \theta} = \frac{\partial \bar{u}}{\partial z} \cos \theta_0 - \frac{\partial \bar{u}}{\partial r} \sin \theta_0 = M_z \cos \theta_0 - (M_r - f_0) \sin \theta_0. \tag{3.67}$$

As a result,

$$\begin{aligned}
\frac{\eta}{f} &= \frac{f_0 \sin \theta_0 - \frac{1}{R} \frac{\partial \bar{u}}{\partial \theta_0}}{f_0 \sin \theta_0} \\
&= \frac{M_r \sin \theta_0 - M_z \cos \theta_0}{f_0 \sin \theta_0} \\
&= \frac{M_r \sin \theta_0 - \frac{b_r \sin \theta_0 - b_z \cos \theta_0}{f_0} \cos \theta_0}{f_0 \sin \theta_0} \\
&= \frac{f_0 M_r - b_r \cos \theta_0 + b_z \sin \theta_0 \cot^2 \theta_0}{f_0^2} \\
&= \frac{|b_r \sin \theta_0|}{f_0^2} \left[\frac{f_0 M_r + b_r \cos \theta_0}{|b_r \sin \theta_0|} + \frac{b_z \sin \theta_0 \cot^2 \theta_0}{|b_r \sin \theta_0|} - \frac{2b_r \cos \theta_0}{|b_r \sin \theta_0|} \right] \\
&= \frac{|b_r \sin \theta_0|}{f_0^2} \left[y + \cot^2 \theta_0 x - 2|\cot \theta_0| \text{sgn}(b_r) \right]. \tag{3.68}
\end{aligned}$$

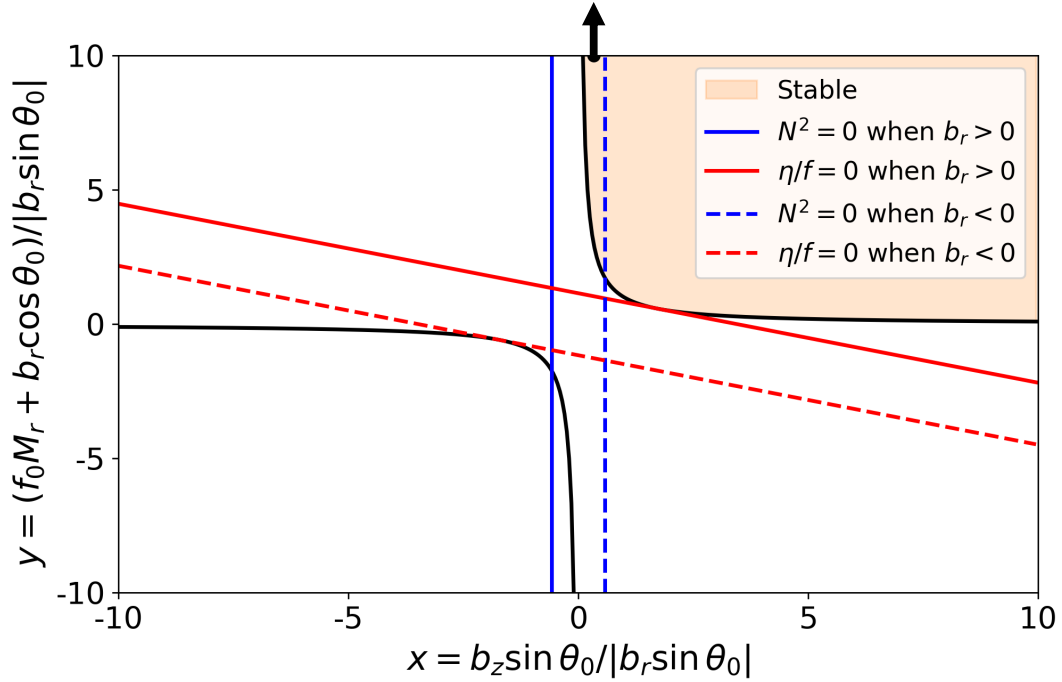


Figure 3.7: **Instability diagram for $\theta_0 = 60^\circ$ (other latitudes have similar results).** Blue lines indicate $N^2 = 0$, and red lines indicate $\eta/f = 0$, with solid lines indicating the results for $b_r > 0$ and dashed lines indicating the results for $b_r < 0$. The black lines indicate $xy = 1$. The shading indicates the stable region. The black dot indicates the parameters for simulation s_S . Note that for s_S , the y -axis value is 1.15×10^4 , outside the y -values shown in the plot.

where $y = (f_0 M_r + b_r \cos \theta_0)/|b_r \sin \theta_0|$ is defined in Equation 3.18. Therefore, $\eta/f < 0$ is equivalent to

$$y < -\cot^2 \theta_0 x + 2|\cot \theta_0| \operatorname{sgn}(b_r). \quad (3.69)$$

The separating line described by Equation 3.69 is always tangential to the hyperbolic curve, $xy = 1$ (see the red lines in Figure 3.7 for the case with $\theta_0 = 60^\circ$). $\eta/f < 0$ is therefore always a sufficient (although not necessary) condition for instability.

Appendix 3.E: Details of the simulation setup

The perturbation equations 3.44–3.48 form the system that we numerically integrate in time with Dedalus (Burns et al., 2020), where we choose the domain scale of the numerical simulations as the length scale L . We neglect the last term in Equation 3.48, consistent with the scaling analysis in 3.A. To search for plane-wave solutions in an infinitely large domain, we apply a Cartesian coordinate system with double-periodic boundary conditions. The equations are solved on a grid with 256×256 grid points.

The nonlinear effects may induce turbulence in the system before the most unstable mode becomes dominant when simulations are initialized with white noise. To address this issue and emphasize the initial exponential growth phase, we integrate the linearized equations (Equations 3.44–3.48 without the advection terms, $\hat{v}\partial/\partial\hat{r}$ and $\hat{w}\partial/\partial\hat{z}$) with white noise in the $\hat{b}$ field as the initial condition for at least $10/\hat{\sigma}_m$, where $\hat{\sigma}_m$ is the expected maximum growth rate, until the most unstable mode dominates. Subsequently, we use these modes, albeit with reduced amplitude, as the initial conditions for nonlinear simulations in each scenario (first column in Figure 3.6). Only the results of these nonlinear simulations are presented in this chapter. In s_S , we adopt the same initial conditions as in s_G for the nonlinear simulation, representing both stable and unstable scenarios with $\text{Ro} \ll 1$.

To address the problem of grid-scale noise in the non-linear simulations without viscosity and diffusivity (Figure 3.8), we employ sub-grid parameterizations to represent the effects of sub-grid-scale eddy mixing via a flow-dependent eddy diffusivity (κ_e) and viscosity (ν_e), computed following the parameterization of Leith (1996). The Leith sub-grid parameterization is based on the theory of 2-D turbulence, where enstrophy cascades toward smaller scales and is ultimately dissipated at the grid scale. As a result, the eddy diffusivity and viscosity are parameterized as follows:

$$\nu_e = \kappa_e = \left(\frac{K_{Leith}}{\pi} \right)^3 L_{grid}^3 \sqrt{\left[\frac{\partial}{\partial \hat{r}} \left(\frac{\partial \hat{v}}{\partial \hat{z}} - \frac{\partial \hat{w}}{\partial \hat{r}} \right) \right]^2 + \left[\frac{\partial}{\partial \hat{z}} \left(\frac{\partial \hat{v}}{\partial \hat{z}} - \frac{\partial \hat{w}}{\partial \hat{r}} \right) \right]^2}, \quad (3.70)$$

where L_{grid} is the grid scale and K_{Leith} is a proportionality parameter, chosen as $K_{Leith} = 1$ in our simulations. In addition to simulations with Leith viscosity and diffusivity, we also perform simulations without any explicit viscosity and diffusivity, and compare their results (see Figures 3.5, 3.6, and 3.8). During the initial, approximately exponential, growth stage, the two simulation sets behave similar albeit with a somewhat reduced growth rate in the simulations with eddy viscosity and diffusivity. Once the simulations become strongly nonlinear, the simulations without Leith viscosity and diffusivity develop excessive noise at small scales, although the large-scale results remain qualitatively similar over the time-scale considered in our simulations (Figure 3.8).

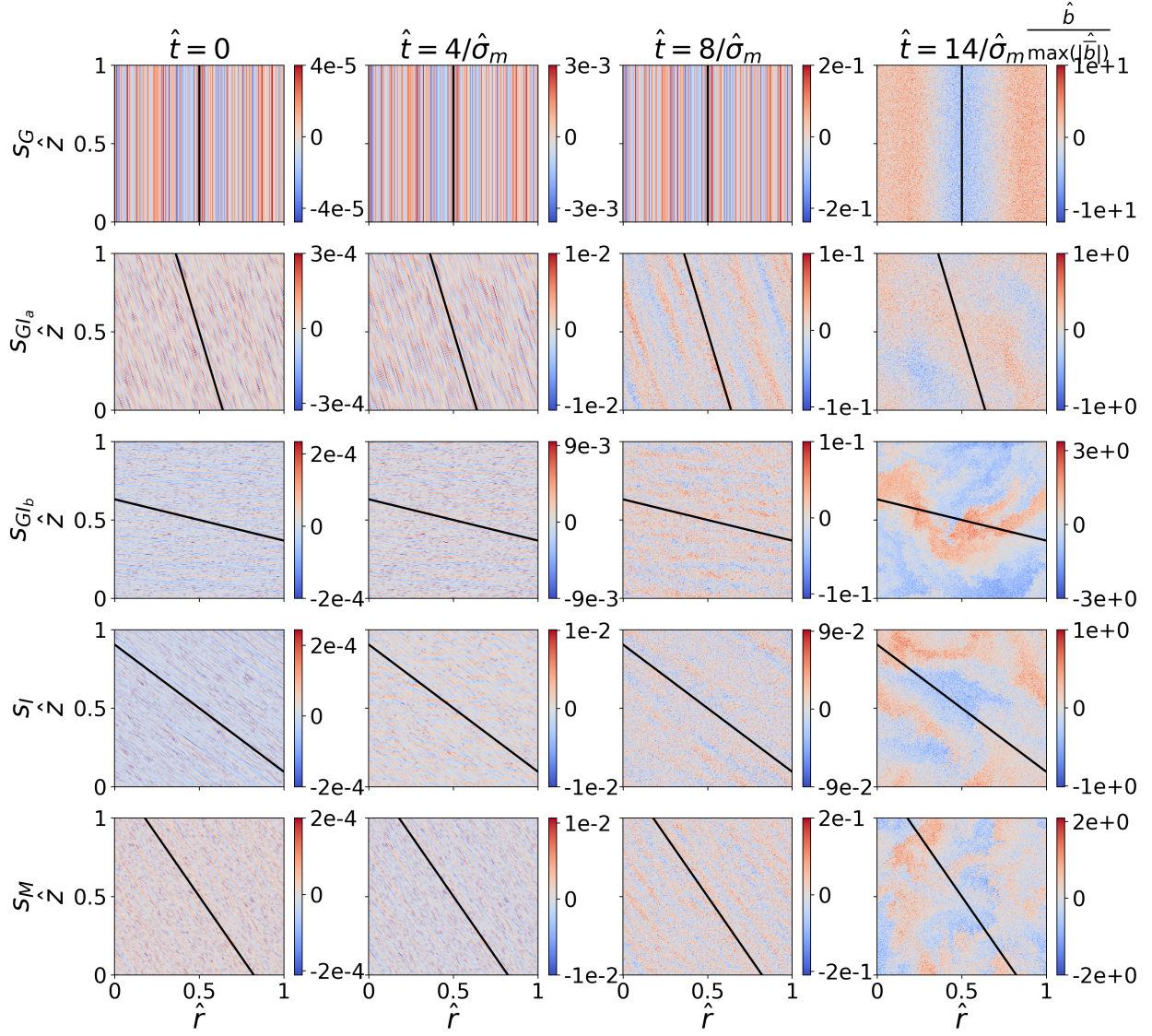


Figure 3.8: Same as Figure 3.6, but for simulations with no diffusivity and viscosity.

CHAPTER 4

SLANTWISE CONVECTION AND HEAT TRANSPORT IN ICY MOON OCEANS

An earlier version of this chapter was published as Zeng, Yaoxuan, and Malte F. Jansen. “Slantwise convection and heat transport in icy moon oceans.” *Geophysical Research Letters* 53, no. 3 (2026): e2025GL118761.

4.1 Introduction

In this chapter, we focus on heat transport by slantwise convection, another key process in icy moon oceans besides the baroclinic eddies studied in Chapter 2. As shown in Chapter 3, convection in icy moon oceans is strongly constrained by planetary rotation and organizes into columns aligned with the rotation axis, known as slantwise convection (Ashkenazy and Tziperman, 2021; Zeng and Jansen, 2021; Bire et al., 2022; Kang, 2023). Convective plumes under icy moon conditions are extremely small, about $0.01\text{--}0.1^\circ$ in latitude (Table 1.1), and thus not resolved in current GCMs. Most models, therefore, rely on convective adjustment schemes developed for Earth’s oceans (Marotzke, 1991), which assume purely vertical mixing (i.e., parallel to gravity). This assumption is incompatible with the rotationally aligned convection on icy moons (Chapter 3), raising concerns about the accuracy of current parameterizations. It is therefore essential to characterize heat transport by slantwise convection, which is the primary goal of this chapter and provides the basis for the parameterization developed in Chapter 5.

The simplest abstraction for ocean convection is Rayleigh–Bénard convection, which models buoyancy-driven flow between two parallel plates heated from below and cooled from above. Rotational effects are often studied using an f -plane, where the rotation axis is aligned with gravity and perpendicular to the plates (see Ecke and Shishkina (2023) and

references therein). In the rapidly rotating, turbulent regime, assuming a dominant balance among the Coriolis force, inertial acceleration, and buoyancy force leads to a CIA (Coriolis–Inertial–Archimedean) scaling (c.f. Aurnou et al. (2020) and references therein) for the flux-gradient relation:

$$|\nabla b| = \Delta b/H \propto B^{2/5}(2\Omega)^{4/5}H^{-4/5}, \quad (4.1)$$

where ∇b is the buoyancy gradient, $\Delta b = g\alpha_T\Delta T$ is the buoyancy contrast between the two plates, $B = (Qg\alpha_T)/(\rho_0c_p)$ is the buoyancy flux, Ω is the rotation rate, H is the depth scale, and g , α_T , ΔT , Q , ρ_0 , and c_p are gravity, thermal expansion coefficient, temperature contrast, heat flux, reference density, and heat capacity, respectively. This scaling is consistent with the scaling law in the diffusion-free limit, also known as the ultimate regime, for rapidly rotating fluids, where planetary rotation is sufficiently strong and the flow is sufficiently turbulent that the effects of molecular viscosity and diffusivity become negligible (c.f. Julien et al. (2012); Gastine et al. (2016)).

On planetary scales, the rotation axis is often tilted relative to gravity. To capture this, many studies adopt a tilted f -plane, where the rotation vector has both vertical and horizontal components (Flasar and Gierasch, 1978; Hathaway et al., 1979, 1980; Hathaway and Somerville, 1983; Von Hardenberg et al., 2015; Novi et al., 2019; Currie et al., 2020; Tro et al., 2024). The horizontal component of the rotation vector breaks horizontal symmetry and stretches convective structures along the rotation axis. At high rotation rates and large thermal contrasts, strong zonal flows can emerge, shearing the plumes and causing intermittent convection near the equator (Von Hardenberg et al., 2015).

Recent studies have also explored Rayleigh–Bénard convection in spherical shells (Soderlund et al., 2014; Gastine et al., 2016; Soderlund, 2019; Amit et al., 2020; Wang et al., 2021; Kverka and Čadek, 2022; Bire et al., 2022; Lemasquerier et al., 2023; Gastine and Aurnou, 2023; Hartmann et al., 2024; Fan et al., 2024). Several of these studies have examined

whether bottom heat is preferentially transported toward the poles or the equator (Soderlund, 2019; Amit et al., 2020; Kverka and Čadež, 2022; Bire et al., 2022; Lemasquerier et al., 2023). The parameters of these simulations remain far from those relevant to icy moons, so their results should be interpreted with caution when applied to real ocean conditions. For example, if the viscosity (ν) is set too high for numerical stability, leading to a high Ekman number ($\text{Ek} = \nu\Omega^{-1}H^{-2}$), high-latitude convective plumes can be artificially suppressed. This suppression may not occur in the low-viscosity (small Ek) regime that likely best represents icy moon ocean interiors. In this low-viscosity limit, Bire et al. (2022) found enhancement of vertical heat transport toward the poles under uniform bottom heating.

Slantwise convection drives heat transport both vertically and meridionally, and its meridional component may influence ice shell thickness variations by modifying the pattern of basal melting and freezing. Theoretical studies have shown that, in rapidly rotating regimes, the heat transport associated with the most unstable linear mode is aligned with the rotation axis, implying transport in both directions (Flasar and Gierasch, 1978; Hathaway et al., 1979). However, most studies of rotating Rayleigh–Bénard convection have focused on vertical transport, with the meridional component less explored. In this work, we conduct convection-resolving simulations to study the relationship between buoyancy flux and buoyancy gradient in icy moon oceans, considering heat transport in both directions. We target a parameter regime comparable to real planetary conditions on icy moons. Section 4.2 describes our numerical method. Section 4.3 presents simulation results and scaling laws. Section 4.4 discusses the implications for ocean heat transport and ice shell topography on icy moons. Section 4.5 provides discussion and concluding remarks.

4.2 Numerical method

We use the Massachusetts Institute of Technology General Circulation Model (MITgcm) (Adcroft et al., 2018) to perform convection-resolving simulations in a local Cartesian box,

with heating imposed at the bottom and cooling at the surface (Fig. 4.1a). The simulations use the Boussinesq approximation, and the governing equations are:

$$\frac{D\mathbf{v}}{Dt} + 2\boldsymbol{\Omega} \times \mathbf{v} = -\nabla\Phi + b\mathbf{e}_r + \nu\nabla^2\mathbf{v}, \quad (4.2)$$

$$\frac{Db}{Dt} = \kappa\nabla^2b, \quad (4.3)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (4.4)$$

where $\mathbf{v}$ is the velocity vector, $\nabla\Phi$ is the pressure gradient term, b is the buoyancy, $\mathbf{e}_r$ denotes the vertical direction (opposite to gravity), and ν and κ are the eddy viscosity and diffusivity, respectively. We assume a linear equation of state with constant salinity: $b = g\alpha_T T$, where T is the temperature anomaly from a constant reference, $\alpha_T = 4 \times 10^{-5} \text{ K}^{-1}$, and $g = 0.1 \text{ m s}^{-2}$.

Baroclinic eddies typically have much larger scales than slantwise convection, making it numerically infeasible to resolve both simultaneously. We therefore focus on the heat transport by slantwise convection in the absence of baroclinic instability. Equations 4.2–4.4 are solved in a local Cartesian domain with doubly periodic boundary conditions in the zonal ($\mathbf{e}_x$) and meridional ($\mathbf{e}_y$) directions, which removes any mean meridional temperature gradient and excludes the baroclinic instability associated with it. We apply free-slip boundary conditions, with a no-penetration bottom and a free surface top (a rigid lid at the top yields similar results; see Appendix 4.A). A constant buoyancy flux B_0 is prescribed at both vertical boundaries (Fig. 4.1b):

$$\kappa \frac{\partial b}{\partial R} = B_0 \quad \text{at } r = 0 \text{ and } L_r, \quad (4.5)$$

where r denotes the vertical direction and L_r is the vertical domain extent.

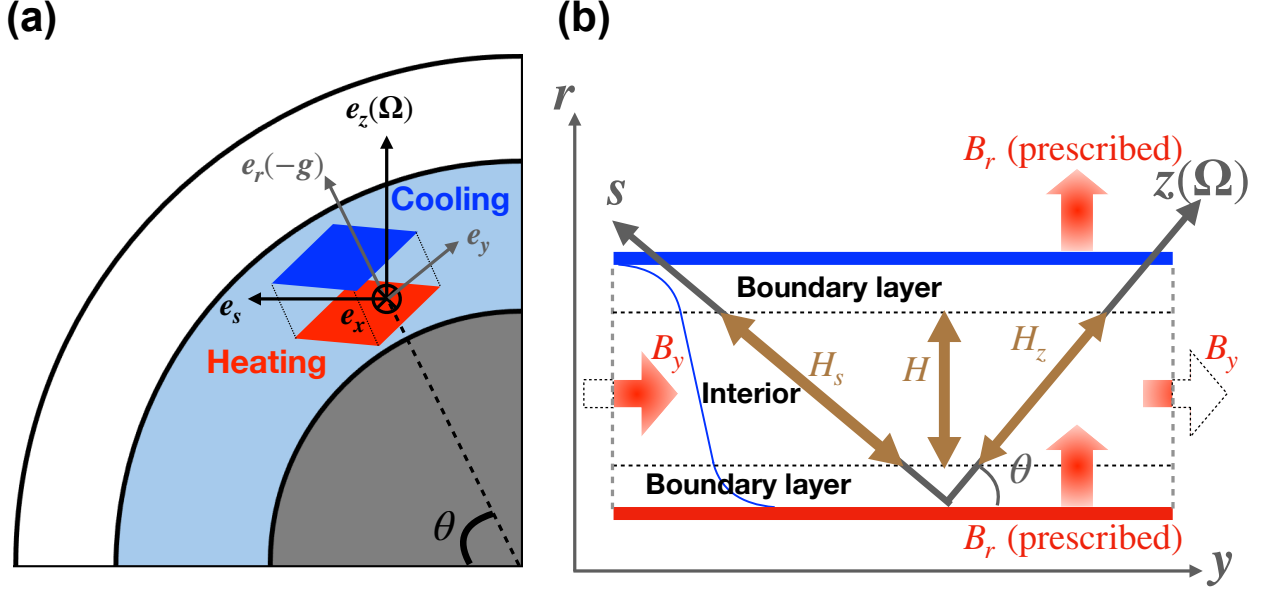


Figure 4.1: **Numerical simulation set up.** (a) Schematic of the coordinate system. $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_r$ are the zonal, meridional, and vertical (opposite to gravity) directions, respectively. $\mathbf{e}_z$ is the axial direction aligned with planetary rotation, and $\mathbf{e}_s$ is the radial direction perpendicular to it. (b) Sketch of the numerical domain in the meridional-vertical plane. The vertical buoyancy flux B_r is prescribed at the top and bottom boundaries. Horizontal boundaries are doubly periodic. The meridional buoyancy flux B_y is not prescribed and is free to adjust. The domain is divided into an interior region and two boundary layers, distinguished by whether the temperature profile (sketched as the blue line) is linear or not (see Appendix 4.A for details). H , $H_z = H/\sin \theta$, and $H_s = H/\cos \theta$ represent the interior depth measured along the vertical, axial, and radial directions, respectively.

We explore a parameter regime relevant to icy moon oceans (Table 4.1). These oceans lie in a transitional regime between rapidly rotating and non-rotating convection (Gastine et al., 2016), where rotation dominates the bulk interior dynamics but not the boundary layers and the smallest scales. To develop a parameterization for bulk heat transport by slantwise convection, we focus on the ocean interior, defined as the region where the horizontally and temporally averaged temperature gradient is approximately constant (Appendix 4.A). We hypothesize that the CIA scaling law remains applicable in the ocean interior. To test this, we run simulations for four buoyancy fluxes ($B_0 = 10^{-14}$, 4×10^{-14} , 1.6×10^{-13} ,

and $6.4 \times 10^{-13} \text{ m}^2 \text{ s}^{-3}$) and four rotation rates ($2\Omega = 3.9 \times 10^{-5}$, 6.25×10^{-5} , 10^{-4} , and $1.6 \times 10^{-4} \text{ s}^{-1}$), all relevant to the icy moon conditions (Table 1.1). Due to the high computational cost of simulating the full ocean depth on icy moons (typically 10^4 – 10^5 m), we test four reduced domain depths ($H_0 = 300, 600, 1200$, and 2400 m). We consider latitudes of $\theta = 15^\circ, 30^\circ, 60^\circ$, and 90° , excluding equatorial cases due to large temporal variability (Von Hardenberg et al., 2015). The natural Rossby number (Jones and Marshall, 1993; Maxworthy and Narimousa, 1994; Bire et al., 2022), $\text{Ro}^* \equiv B^{1/2}(2\Omega)^{-3/2}H^{-1}$, in our simulations is within the estimated range for Titan, Europa and Ganymede, and about 10-100 times larger than the estimate for Enceladus (comparing Tables 1.1 and 4.1). Importantly, the condition $\text{Ro}^* \ll 1$ in both real icy moon oceans and our simulations confirms that they lie within the same regime, where convection is strongly influenced by rotation. All simulations are initialized with small-amplitude white noise in temperature and integrated until reaching a statistically steady state.

The typical plume size is estimated as $l_{\text{plume}} = kB^{1/4}(2\Omega)^{-3/4}H^{1/2} \approx 100 \text{ m}$ using simulation parameters and an empirical pre-factor $k = 5$ (Fernando et al., 1989; Jones and Marshall, 1993; Bire et al., 2022). To resolve these plumes, we use a grid spacing of 4 m in most simulations. We use 300 grid points in the meridional and vertical directions to ensure scale separation between the domain size and the plume scale, and use 140 points in the zonal direction to save computational costs. The shorter zonal domain relative to the meridional extent also promotes the formation of zonal rather than meridional jets (Julien et al., 2018; Currie et al., 2020), more realistically representing icy moon conditions (Soderlund, 2019; Ashkenazy and Tziperman, 2021; Kvorka and Čadek, 2022; Bire et al., 2022; Kang, 2023; Cabanes et al., 2024). To test the effect of jets on heat transport, we also conduct sensitivity experiments with different horizontal aspect ratios ($L60_{0.5L_y}$ and $L60_{2L_x0.5L_y}$), as well as a simulation with linear bottom drag that suppresses jet formation ($L60_{\text{drag}}$). For simulations with varying depth, we adjust the resolution to maintain a constant total number of grid

Table 4.1: Numerical simulation parameters.

Name	θ ($^\circ$)	B_0 ($\text{m}^2 \text{s}^{-3}$)	2Ω (s^{-1})	H_0 (m)	$N_x \times N_y \times N_r$	$A_{\nu, \text{smag}}$	Ro^*
$L60_{\text{Ctrl}}$	60	4×10^{-14}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	1.67×10^{-4}
$L60_{0.25B}$	60	1×10^{-14}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	8.33×10^{-5}
$L60_{4B}$	60	1.6×10^{-13}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	3.33×10^{-4}
$L60_{16B}$	60	6.4×10^{-13}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	6.67×10^{-4}
$L60_{0.39\Omega}$	60	4×10^{-14}	3.9×10^{-5}	1200	$140 \times 300 \times 300$	10^{-2}	6.84×10^{-4}
$L60_{0.625\Omega}$	60	4×10^{-14}	6.25×10^{-5}	1200	$140 \times 300 \times 300$	10^{-2}	3.37×10^{-4}
$L60_{1.6\Omega}$	60	4×10^{-14}	1.6×10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	8.24×10^{-5}
$L60_{0.25H}$	60	4×10^{-14}	10^{-4}	300	$140 \times 300 \times 300$	10^{-2}	6.67×10^{-4}
$L60_{0.5H}$	60	4×10^{-14}	10^{-4}	600	$140 \times 300 \times 300$	10^{-2}	3.33×10^{-4}
$L60_{2H}$	60	4×10^{-14}	10^{-4}	2400	$140 \times 300 \times 300$	10^{-2}	8.33×10^{-5}
$L15_{\text{Ctrl}}$	15	4×10^{-14}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	1.67×10^{-4}
$L15_{0.25B}$	15	1×10^{-14}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	8.33×10^{-5}
$L15_{0.625\Omega}$	15	4×10^{-14}	6.25×10^{-5}	1200	$140 \times 300 \times 300$	10^{-2}	3.37×10^{-4}
$L15_{2H}$	15	4×10^{-14}	10^{-4}	2400	$140 \times 300 \times 300$	10^{-2}	8.33×10^{-5}
$L30_{\text{Ctrl}}$	30	4×10^{-14}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	1.67×10^{-4}
$L30_{0.25B}$	30	1×10^{-14}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	8.33×10^{-5}
$L30_{0.625\Omega}$	30	4×10^{-14}	6.25×10^{-5}	1200	$140 \times 300 \times 300$	10^{-2}	3.37×10^{-4}
$L30_{2H}$	30	4×10^{-14}	10^{-4}	2400	$140 \times 300 \times 300$	10^{-2}	8.33×10^{-5}
$L90_{\text{Ctrl}}$	90	4×10^{-14}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	1.67×10^{-4}
$L90_{0.25B}$	90	1×10^{-14}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-2}	8.33×10^{-5}
$L90_{0.625\Omega}$	90	4×10^{-14}	6.25×10^{-5}	1200	$140 \times 300 \times 300$	10^{-2}	3.37×10^{-4}
$L90_{2H}$	90	4×10^{-14}	10^{-4}	2400	$140 \times 300 \times 300$	10^{-2}	8.33×10^{-5}
$L60_{\text{lowvisc}}$	60	4×10^{-14}	10^{-4}	1200	$140 \times 300 \times 300$	10^{-3}	1.67×10^{-4}
$L60_{\text{lowres}}$	60	4×10^{-14}	10^{-4}	1200	$70 \times 150 \times 150$	10^{-2}	1.67×10^{-4}
$L60_{\text{RigidLid}}$	60	4×10^{-14}	10^{-4}	1200	$140 \times 150 \times 300$	10^{-2}	1.67×10^{-4}
$L60_{\text{drag}}$	60	4×10^{-14}	10^{-4}	1200	$140 \times 150 \times 300$	10^{-2}	1.67×10^{-4}
$L60_{0.5L_y}$	60	4×10^{-14}	10^{-4}	1200	$140 \times 150 \times 300$	10^{-2}	1.67×10^{-4}
$L60_{2L_x 0.5L_y}$	60	4×10^{-14}	10^{-4}	1200	$280 \times 150 \times 300$	10^{-2}	1.67×10^{-4}

points.

Fully resolving turbulence down to viscous scales is computationally infeasible. We therefore apply a Smagorinsky closure to parameterize subgrid-scale mixing, which assumes an isotropic forward energy cascade at the grid scale (Smagorinsky, 1963; Smagorinsky et al., 1993). Although the “plume scale” is resolved in our simulations, the characteristic scale at which rotation becomes important, $l_\Omega = B^{1/2}(2\Omega)^{-3/2}$, is approximately 0.2 m in our simulations, smaller than the grid spacing. This means that rotational effects may remain important below the grid scale, but are not adequately accounted for by the Smagorinsky closure. To test the sensitivity to the Smagorinsky closure, we performed simulations $L60_{\text{lowvisc}}$ and $L60_{\text{lowres}}$ to assess the effects of viscosity and resolution, and found no significant differences (see Appendix 4.A). We set $\nu/\kappa = 10$ to maintain minimally diffusive stable simulations. Based on the posterior-diagnosed Smagorinsky eddy diffusivity and viscosity, the simulation parameters fall within the transitional regime, consistent with that expected for icy moon oceans (Appendix 4.A).

4.3 Flux-gradient relation of the heat transport

Strong zonal jets as well as slantwise convection plumes form in the simulations, all aligned with the planetary rotation axis (Fig. 4.2). Notably, the radial velocity component v_s is comparable in magnitude to the axial component v_z (Fig. 4.2, second and third rows), while both are roughly one order of magnitude smaller than the zonal jet velocities. This similarity ($v_s \sim v_z$) suggests that radial transport may be as important as axial transport. Moreover, the length scales of v_s and v_z are similar, indicating no clear scale separation between the two directions. This finding is supported by the heat transport spectra (Appendix 4.B), implying that both components should be treated concurrently in any parameterization of slantwise convection. Thermal boundary layers form near the top and bottom boundaries, maintaining a large temperature contrast between them (Appendix 4.A). Temperature anomalies are also

organized along the rotation axis, with horizontal temperature variations roughly an order of magnitude smaller than the full vertical temperature contrast (Fig. 4.2, last row).

We diagnose buoyancy gradients and fluxes from the simulations with the goal of inform-

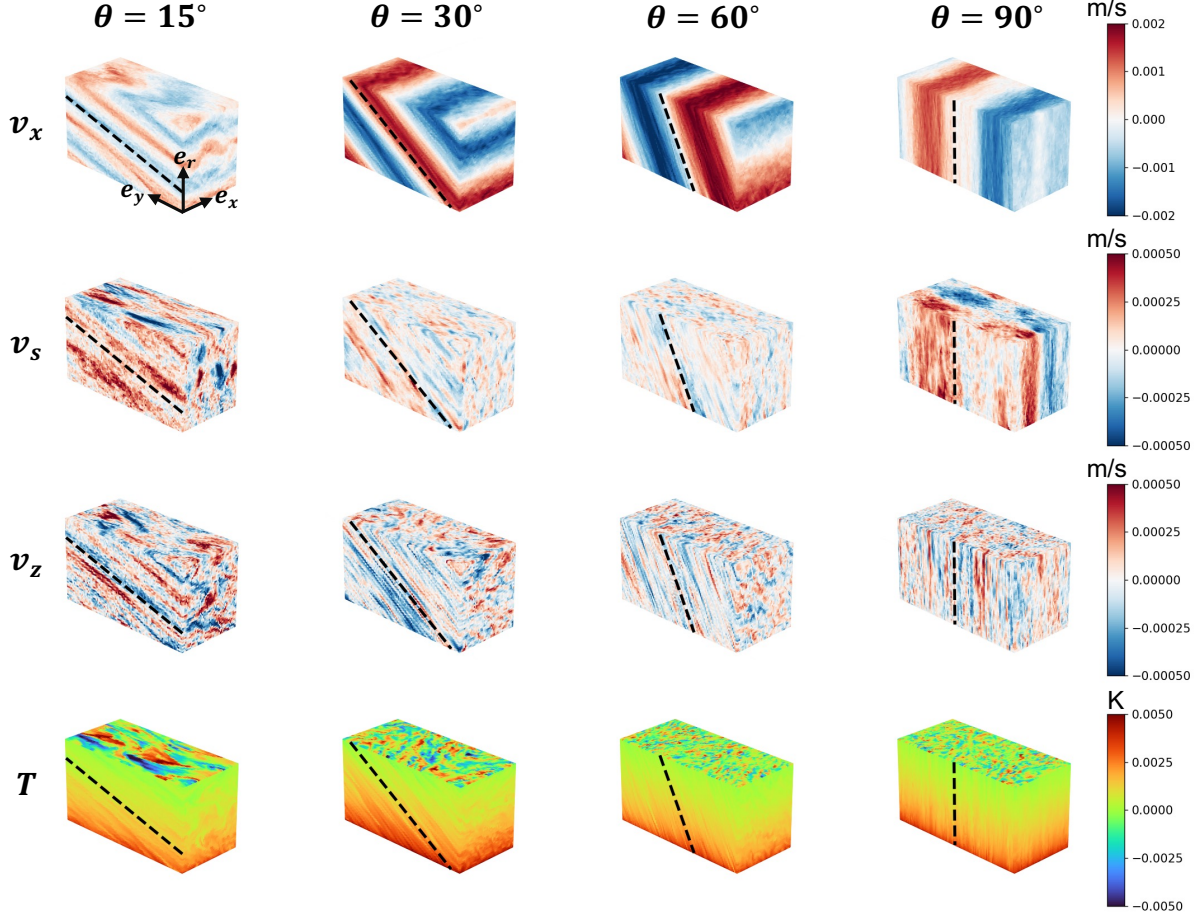


Figure 4.2: **Snapshots of the flow and temperature fields.** The first to fourth columns show results from the control simulations at different latitudes: $L15_{\text{Ctrl}}$, $L30_{\text{Ctrl}}$, $L60_{\text{Ctrl}}$, and $L90_{\text{Ctrl}}$, respectively. The first to fourth rows show the velocity components v_x , v_s , v_z (c.f. Fig. 4.1a), and the temperature anomaly T , respectively. In each panel, the front face shows a slice in the y - r plane at $x = 0$, the right face shows a slice in the x - r plane at $y = L_y$, and the top face shows a slice in the x - y plane at $r = L_r/2$, where L_x , L_y , and L_r are the domain extents in the zonal, meridional, and vertical directions, respectively. Only the lower half of the domain is shown due to symmetry in the vertical direction, and to illustrate the flow and temperature fields in the interior. In the top face of the last row, the temperature anomaly is scaled up by a factor of 10 to enhance visibility. Dashed lines in the y - r plane indicate the rotation axis ($\mathbf{e}_z$).

ing parameterizations of bulk ocean heat transport in global models (Fig. 4.3). Specifically, we focus on the ocean interior, away from boundary layers, where the horizontally and temporally averaged temperature profile is approximately linear (Fig. 4.1b). Within this region, we compute the buoyancy gradient $|\nabla b|$, the interior depth H , and the buoyancy flux B . The total buoyancy flux is decomposed into the axial component (B_z , parallel to the convective columns) and the radial component (B_s , perpendicular to the columns). The zonal heat flux is dynamically uninteresting because the averaged flux has to be approximately zero due to symmetry.

The CIA scaling reasonably captures the total heat transport magnitude (Fig. 4.4). Motivated by this, we assume this scaling holds for heat transport along the convective plumes (i.e., the rotation axis) at fixed latitude. When latitude varies, the plumes tilt along the rotation axis, and their height changes to $H_z = H/\sin \theta$ (Fig. 4.1b). We adjust H to H_z to account for the plume depth change, leading to the following flux–gradient scaling in the axial direction:

$$-\mathbf{e}_z \cdot \nabla b \equiv \frac{\Delta b}{H_z} = C_z B_z^{2/5} (2\Omega)^{4/5} H_z^{-4/5}, \quad H_z = \frac{H}{\sin \theta}, \quad (4.6)$$

where $C_z = 1.181$ is an empirical constant obtained from simulations at $\theta = 60^\circ$ and applied to all latitudes. This scaling performs well for all simulations with zonal jets (filled markers), with errors below 10% in most cases. The one notable exception is simulation $L15_{0.625\Omega}$ (red square), which exhibits large temporal variability in buoyancy flux and temperature profiles (Fig. 4.5). Even in this case, the scaling predicts the temperature gradient within a factor of 2 (Fig. 4.3a). One sensitivity test ($L60_{2L_x 0.5L_y}$, open upward triangle), which develops meridional rather than zonal jets, exhibits up-gradient axial heat transport and is not shown in Fig. 4.3a. However, because meridional jets are unlikely to occur in real icy moon oceans, we exclude this case from consideration here (see Section 4.5 and Appendix 4.C for further discussion).

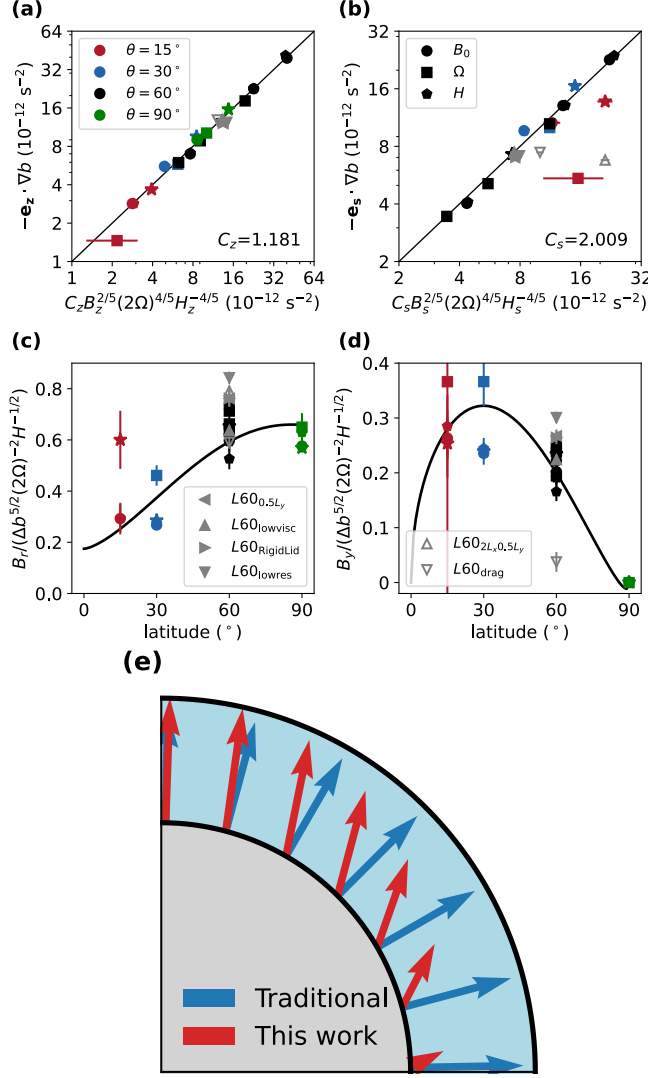


Figure 4.3: **Buoyancy flux–gradient relationships.** (a) and (b) show heat transport scaling in the axial and radial directions (Equations 4.6 and 4.7), respectively. (c) and (d) show the latitude dependence of normalized vertical (B_r) and meridional (B_y) buoyancy fluxes (Equations 4.8 and 4.9), respectively. Marker color denotes latitude (legend in a), shape denotes parameter variations (legend in b), and grey markers denote sensitivity tests (legend in c–d). Filled/open markers represent simulations with/without zonal jets. Error bars show one standard deviation of temporal variability, typically negligible. Black lines denote the one-to-one line in (a) and (b) and the theoretical prediction in (c) and (d). Simulation $L60_{2L_x 0.5L_y}$ is outside the range in (a) and (d); $L150_{0.625\Omega}$ is outside the range in (c). (e) illustrates the heat transport for a hypothetical icy moon ocean with uniform depth and vertical temperature contrast, with red and blue arrows showing the predicted heat flux from this study’s scaling laws versus a traditional convective adjustment scheme (see text for details). Arrow direction indicates the direction of heat flux, and arrow length is proportional to its magnitude.

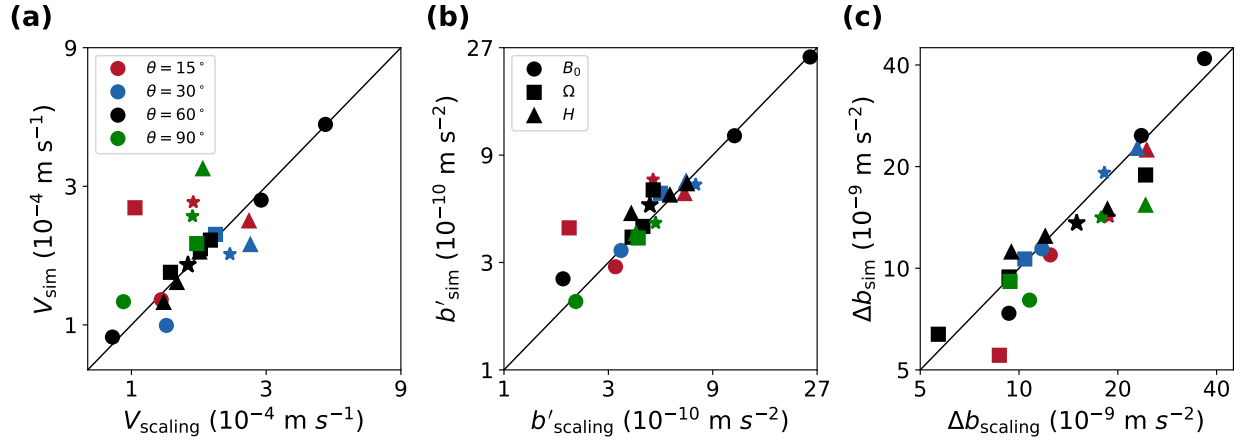


Figure 4.4: **The CIA scaling compared with simulation results.** (a)–(c) show the CIA scalings for the eddy velocity, $V = (v_s'^2 + v_z'^2)^{1/2}$, buoyancy perturbation b' , and buoyancy contrast Δb , respectively, where primes denote deviations from the horizontal average. The vertical axis shows simulation results, while the horizontal axis shows the corresponding theoretical CIA scalings: $V \sim \Delta b (2\Omega)^{-1}$, $b' \sim \Delta b^{3/2} (2\Omega)^{-1} H^{-1/2}$, and $\Delta b \sim B^{2/5} (2\Omega)^{4/5} H^{1/5}$ (c.f. Aurnou et al. (2020) and references therein). All scalings are evaluated from a single snapshot and averaged over the interior region. Note that B denotes the magnitude of the total buoyancy flux, $B = (B_s^2 + B_z^2)^{1/2}$, rather than the individual components B_s and B_z , which are discussed in Section 4.3. Pre-factors are fitted using simulations at $\theta = 60^\circ$ and applied uniformly across all cases: 1.16 in (a), 0.86 in (b), and 0.51 in (c). Symbols are consistent with those in Fig. 4.3.

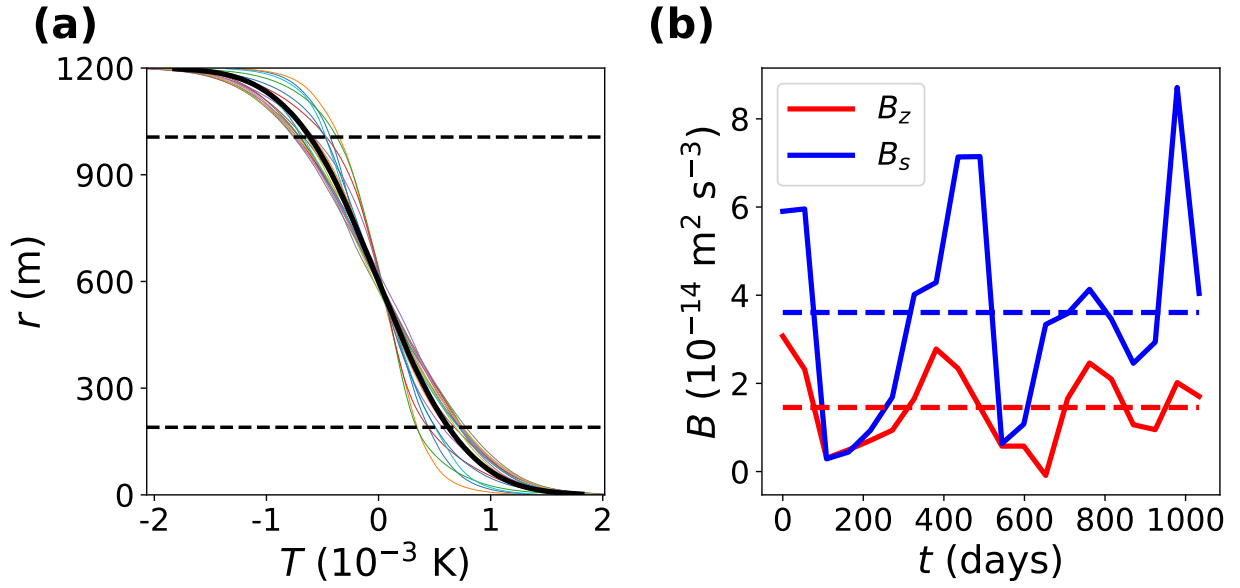


Figure 4.5: **Time series of the temperature profile and buoyancy flux in simulation $L15_{0.625\Omega}$.** (a) Horizontally averaged temperature profiles. Thin lines show individual snapshots, and the thick black line shows the time-averaged profile. Black dashed lines mark the boundaries between the interior and the boundary layers. (b) Time series of buoyancy flux in the axial (B_z , red) and radial (B_s , blue) directions. Solid lines show the time series, while dashed lines indicate the corresponding time averages.

Table 4.2: **Time-averaged simulation results.**

Name	$\Delta b/H$ (s^{-2})	B_s ($m^2 s^{-3}$)	B_z ($m^2 s^{-3}$)	H (m)
$L60_{Ctrl}$	1.46×10^{-11}	$7.64 \times 10^{-15} \pm 3.94 \times 10^{-16}$	$4.21 \times 10^{-14} \pm 7.31 \times 10^{-16}$	872
$L60_{0.25B}$	8.06×10^{-12}	$2.02 \times 10^{-15} \pm 1.48 \times 10^{-16}$	$1.04 \times 10^{-14} \pm 2.88 \times 10^{-16}$	856
$L60_{4B}$	2.61×10^{-11}	$3.35 \times 10^{-14} \pm 5.23 \times 10^{-15}$	$1.67 \times 10^{-13} \pm 1.03 \times 10^{-14}$	880
$L60_{16B}$	4.54×10^{-11}	$1.22 \times 10^{-13} \pm 1.56 \times 10^{-14}$	$6.72 \times 10^{-13} \pm 2.99 \times 10^{-14}$	864
$L60_{0.39\Omega}$	6.89×10^{-12}	$7.50 \times 10^{-15} \pm 1.15 \times 10^{-15}$	$4.15 \times 10^{-14} \pm 2.26 \times 10^{-15}$	856
$L60_{0.625\Omega}$	1.02×10^{-11}	$9.51 \times 10^{-15} \pm 6.67 \times 10^{-16}$	$4.08 \times 10^{-14} \pm 1.34 \times 10^{-15}$	856
$L60_{1.6\Omega}$	2.10×10^{-11}	$8.28 \times 10^{-15} \pm 3.44 \times 10^{-16}$	$4.19 \times 10^{-14} \pm 8.12 \times 10^{-16}$	848
$L60_{0.25H}$	4.77×10^{-11}	$9.09 \times 10^{-15} \pm 1.41 \times 10^{-15}$	$4.10 \times 10^{-14} \pm 2.71 \times 10^{-15}$	220
$L60_{0.5H}$	2.62×10^{-11}	$8.77 \times 10^{-15} \pm 6.99 \times 10^{-16}$	$4.07 \times 10^{-14} \pm 1.47 \times 10^{-15}$	436
$L60_{2H}$	8.19×10^{-12}	$8.35 \times 10^{-15} \pm 2.62 \times 10^{-16}$	$4.17 \times 10^{-14} \pm 5.25 \times 10^{-16}$	1712
$L15_{Ctrl}$	1.42×10^{-11}	$3.37 \times 10^{-14} \pm 6.94 \times 10^{-15}$	$2.62 \times 10^{-14} \pm 4.38 \times 10^{-15}$	928
$L15_{0.25B}$	1.10×10^{-11}	$7.54 \times 10^{-15} \pm 1.97 \times 10^{-15}$	$1.16 \times 10^{-14} \pm 1.41 \times 10^{-15}$	936
$L15_{0.625\Omega}$	5.36×10^{-12}	$3.61 \times 10^{-14} \pm 2.37 \times 10^{-14}$	$1.45 \times 10^{-14} \pm 8.70 \times 10^{-15}$	816
$L15_{2H}$	1.10×10^{-11}	$2.97 \times 10^{-14} \pm 6.85 \times 10^{-15}$	$4.96 \times 10^{-14} \pm 7.92 \times 10^{-15}$	1872
$L30_{Ctrl}$	1.91×10^{-11}	$1.75 \times 10^{-14} \pm 2.65 \times 10^{-15}$	$4.80 \times 10^{-14} \pm 3.47 \times 10^{-15}$	928
$L30_{0.25B}$	1.11×10^{-11}	$4.21 \times 10^{-15} \pm 2.89 \times 10^{-16}$	$1.25 \times 10^{-14} \pm 3.92 \times 10^{-16}$	944
$L30_{0.625\Omega}$	1.61×10^{-11}	$1.85 \times 10^{-14} \pm 2.69 \times 10^{-15}$	$4.69 \times 10^{-14} \pm 3.97 \times 10^{-15}$	856
$L30_{2H}$	1.11×10^{-11}	$1.67 \times 10^{-14} \pm 1.05 \times 10^{-15}$	$5.07 \times 10^{-14} \pm 1.43 \times 10^{-15}$	1872
$L90_{Ctrl}$	1.56×10^{-11}	$-3.31 \times 10^{-17} \pm 2.72 \times 10^{-18}$	$4.09 \times 10^{-14} \pm 1.11 \times 10^{-15}$	864
$L90_{0.25B}$	9.04×10^{-12}	$-1.18 \times 10^{-17} \pm 4.39 \times 10^{-19}$	$1.04 \times 10^{-14} \pm 1.60 \times 10^{-16}$	856
$L90_{0.625\Omega}$	1.02×10^{-11}	$6.36 \times 10^{-18} \pm 4.18 \times 10^{-18}$	$4.10 \times 10^{-14} \pm 3.24 \times 10^{-15}$	864
$L90_{2H}$	8.86×10^{-12}	$4.44 \times 10^{-17} \pm 1.83 \times 10^{-18}$	$4.16 \times 10^{-14} \pm 7.96 \times 10^{-16}$	1680
$L60_{lowvisc}$	1.49×10^{-11}	$7.84 \times 10^{-15} \pm 5.18 \times 10^{-16}$	$4.19 \times 10^{-14} \pm 9.90 \times 10^{-16}$	856
$L60_{lowres}$	1.43×10^{-11}	$7.64 \times 10^{-15} \pm 4.82 \times 10^{-16}$	$4.18 \times 10^{-14} \pm 9.35 \times 10^{-16}$	784
$L60_{RigidLid}$	1.40×10^{-11}	$7.91 \times 10^{-15} \pm 7.27 \times 10^{-16}$	$4.12 \times 10^{-14} \pm 1.37 \times 10^{-15}$	840
$L60_{drag}$	1.49×10^{-11}	$1.77 \times 10^{-14} \pm 6.44 \times 10^{-16}$	$3.57 \times 10^{-14} \pm 1.17 \times 10^{-15}$	888
$L60_{0.5L_y}$	1.40×10^{-11}	$7.85 \times 10^{-15} \pm 7.22 \times 10^{-16}$	$4.17 \times 10^{-14} \pm 1.36 \times 10^{-15}$	848
$L60_{2L_x 0.5L_y}$	1.36×10^{-11}	$1.08 \times 10^{-13} \pm 4.42 \times 10^{-16}$	$-1.63 \times 10^{-14} \pm 1.19 \times 10^{-14}$	864

To the best of our knowledge, no established theory exists for heat transport in the radial direction ($\mathbf{e}_s$, perpendicular to the convective plumes). We propose a semi-empirical scaling by analogy with the axial direction, projecting the depth along the radial direction as $H_s = H / \cos \theta$ (Fig. 4.1b). This yields:

$$-\mathbf{e}_s \cdot \nabla b \equiv \frac{\Delta b}{H_s} = C_s B_s^{2/5} (2\Omega)^{4/5} H_s^{-4/5}, \quad H_s = \frac{H}{\cos \theta}, \quad (4.7)$$

where $C_s = 2.009$ is fitted using simulations at $\theta = 60^\circ$. Alternative formulations using best-fit exponents in the natural Rossby number for heat transport in the radial direction ($\text{Ro}_s^* = B_s^{1/2} (2\Omega)^{-3/2} H_s^{-1}$) were tested and found to yield similar performance (Fig. 4.6). Notably, Equation 4.7 implies that $H_s \rightarrow \infty$ at the pole, leading to $B_s \rightarrow 0$, which is consistent with the expected horizontal isotropy and agrees with simulation results showing $B_s/B_z \approx 0.1\%$ in all cases with $\theta = 90^\circ$ (Table 4.2). At lower latitudes, the tilt between the rotation axis and gravity breaks isotropy, generating correlated radial velocity and temperature anomalies that drive net radial heat transport (Appendix 4.B). For most simulations, this scaling predicts radial heat flux with errors below 15%.

Zonal jets influence the radial heat transport but have little effect on the heat transport along the rotation axis. This likely occurs because zonal jets are primarily barotropic along the rotation axis but have strong radial shear (Fig. 4.2). The radial shear deflects the flow, reducing the efficiency of radial heat transport (Aurnou et al., 2008; Guervilly and Cardin, 2017). Consequently, simulations without steady zonal jets exhibit stronger radial heat transport, producing a weaker buoyancy gradient than predicted. In simulations without zonal jets ($L60_{2L_x 0.5L_y}$ and $L60_{\text{drag}}$) and those with strongly meandering zonal jets ($L15_{\text{Ctrl}}$ and $L15_{0.625\Omega}$; see Appendix 4.C), the scaling tends to overestimate the buoyancy contrast (or equivalently, underestimate the buoyancy flux) in the radial direction. Even in these cases, the predicted values remain within a factor of 3 (Fig. 4.3b).

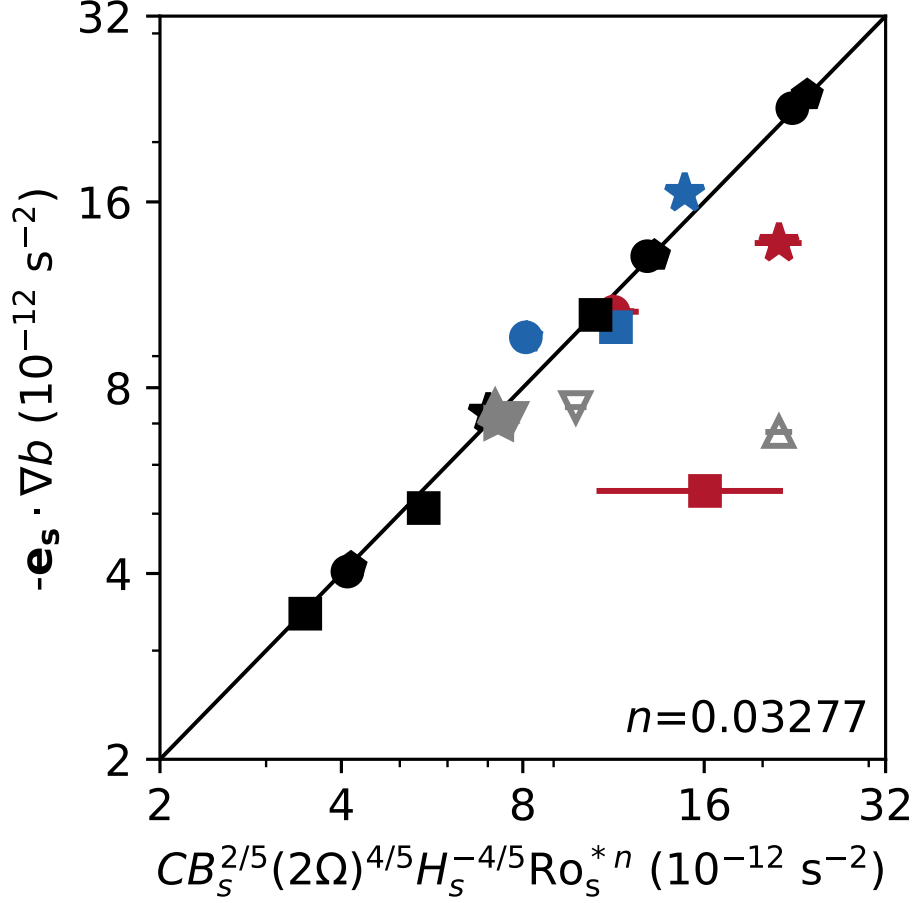


Figure 4.6: **Scaling of the radial ($\mathbf{e}_s$) flux-gradient relation with best-fit exponents.** According to the Buckingham π theorem, there is one non-dimensional parameter that governs the buoyancy flux-gradient relation. We define it as $\text{Ro}_s^* = |B_s|^{1/2} (2\Omega)^{-3/2} H_s^{-1}$, motivated by the natural Rossby number but using the interior depth projected in the radial direction, H_s , as the characteristic length scale. We assume a flux-gradient relation of the form $-\mathbf{e}_s \cdot \nabla b = CB_s^{2/5} (2\Omega)^{4/5} H_s^{-4/5} \text{Ro}_s^{*n}$, and fit the pre-factor C and exponent n using simulations at $\theta = 60^\circ$, then apply the fit across all latitudes. Symbols match those in Fig. 4.3. The fitted exponent n is close to zero, supporting the use of the semi-empirical scaling (Equation 4.7).

4.4 Implications for the global ocean heat transport and ice shell thickness on icy moons

The flux–gradient scaling laws (Equations 4.6 and 4.7) predict the dependence of heat transport on latitude. The normalized vertical and meridional buoyancy fluxes are given by

$$\tilde{B}_r \equiv \frac{B_r}{\Delta b^{5/2} (2\Omega)^{-2} H^{-1/2}} = C_z^{-5/2} \sin^{3/2} \theta + C_s^{-5/2} \cos^{3/2} \theta \quad (4.8)$$

and

$$\tilde{B}_y = \frac{B_y}{\Delta b^{5/2} (2\Omega)^{-2} H^{-1/2}} = C_z^{-5/2} \sin^{1/2} \theta \cos \theta - C_s^{-5/2} \cos^{1/2} \theta \sin \theta, \quad (4.9)$$

respectively.

The vertical heat transport increases with latitude for fixed buoyancy contrast, rotation rate, and depth, as predicted by Equation 4.8 (Fig. 4.3c). This trend aligns well with our simulation results within a factor of 1.5, except for the lowest latitude cases ($L15_{\text{Ctrl}}$ and $L15_{0.625\Omega}$). As discussed in Section 4.3, in these simulations, the jets are highly meandering, and the heat transport in the radial direction (close to vertical direction at low latitudes) is underestimated (Fig. 4.3b & c).

The meridional heat transport is poleward and exhibits a non-monotonic dependence on latitude, peaking near $\theta = 30^\circ$ and vanishing at both the equator and the pole, as predicted by Equation 4.9 (Fig. 4.3d). This pattern broadly agrees with the simulations within a factor of 1.5, although the scaling overestimates the meridional transport in simulation $L60_{\text{drag}}$, which does not develop jets. Only simulation $L60_{2L_x 0.5L_y}$, which develops meridional jets, exhibits equatorward heat transport. This case is not shown in this panel and is excluded from consideration, as the meridional jets are not representative of icy moon ocean conditions (see Section 4.5).

For comparison with traditional convective parameterizations, which assume heat trans-

port aligned with gravity, we evaluate the fluxes predicted by both the traditional scheme and the scaling laws developed in this study (Equations 4.8 and 4.9), assuming constant ocean depth and vertical buoyancy contrast, with latitude taken at the bottom boundary. For the traditional scheme, we adopt the scaling from Equation (3.10) of Gastine et al. (2016), which has previously been applied to icy moon ocean studies (e.g., Kang (2023)). Our scaling predicts weaker heat flux at lower latitudes and a heat flux vector that lies between the direction of gravity and the planetary rotation axis, resulting in a net poleward transport (Fig. 4.3e, red arrows). In contrast, traditional GCM convection schemes assume transport strictly aligned with gravity (Fig. 4.3e, blue arrows), thereby underestimating the poleward component. At the poles, our scaling converges with the traditional scheme, with a pre-factor difference of less than 10%.

The predicted latitude dependence, with increased vertical flux at high latitudes and poleward meridional flux, suggests that slantwise convection may play an important role in redistributing heat toward the poles. If it dominates the heat transport, this redistribution would help maintain poleward-thinning ice shells on icy moons, qualitatively consistent with observations of Enceladus (Beuthe et al., 2016; Čadek et al., 2019; Hemingway and Mittal, 2019).

4.5 Discussion

We investigate heat transport associated with slantwise convection in the ocean interior, away from boundary layers, under conditions relevant to icy moons. Using convection-resolving simulations of rotating Rayleigh-Bénard convection on a tilted f -plane, we examine heat fluxes in both the axial and radial directions. We find scaling laws consistent with the classical diffusion-free (CIA) scaling at fixed latitude, extended here to include latitude dependence (Equations 4.6 & 4.7). The heat transport in the axial and radial directions is of comparable magnitude, suggesting that both components should be included in parameterizations of

slantwise convection in icy moon oceans. These scalings show that the total convective heat flux vector lies between the directions of gravity and planetary rotation. The vertical heat transport increases with latitude, and the meridional transport is poleward, both of which support the formation of a poleward-thinning ice shell, qualitatively consistent with observations of Enceladus.

Zonal jets emerge in most simulations. Sensitivity tests show that the presence of zonal jets influences the radial heat transport, although the jet strength itself has little effect. We performed one simulation with meridional jets, where the meridional heat flux reverses sign and becomes equatorward (Table 4.2 and Appendix 4.C). However, in realistic icy moon oceans, the β -effect arising from planetary rotation and spherical geometry promotes zonal jets and suppresses meridional jets (Rhines, 1975; Vallis, 2017). We therefore focus on simulations with zonal jets only, where all simulations exhibit a poleward meridional heat transport.

Our simulations consider buoyancy driven solely by temperature. However, salinity can also contribute significantly to buoyancy stratification in icy moon oceans (Lobo et al., 2021; Zeng and Jansen, 2021; Kang et al., 2022c; Zeng and Jansen, 2024; Ames et al., 2025). In the turbulent regime studied here, mixing is dominated by turbulence, which mixes heat and salt similarly. Therefore, as long as the equation of state is approximately linear for both temperature and salinity, our conclusions should remain valid in the presence of both buoyancy sources.

We adopt a Rayleigh–Bénard setup with uniform heating. However, seafloor heating on icy moons may be heterogeneous or spatially localized, as has been argued specifically for Enceladus (Choblet et al., 2017; Bouffard et al., 2025). Such localized heating could generate isolated plumes rather than global convection (Goodman et al., 2004; Goodman and Lenferink, 2012; Kang et al., 2022b; Bire et al., 2023; Zhang et al., 2025a; Wang et al., 2025). On Enceladus, these localized plumes are likely to lose buoyancy through mixing

with the surrounding denser fluid, driven by baroclinic instability (Zhang et al., 2025a; Wang et al., 2025). Once buoyancy becomes horizontally homogenized, the system is expected to dynamically resemble the uniform heating case considered in this study. However, if highly localized heat sources exist on Europa, localized plumes there may or may not become well-mixed across the global ocean before reaching the surface (Wang et al., 2025). A quantitative analysis of the behavior of isolated slantwise convecting plumes under a tilted rotation axis remains an important topic for future work.

Our work is designed to investigate processes in the ocean interior, and the boundary layer is not well resolved due to numerical constraints. This may influence the vertical velocity and buoyancy anomalies at the outer edge of the boundary layer and therefore introduces some uncertainty in the heat transport. We note, however, that boundaries in real icy moons are rough, and a viscous boundary layer along a smooth boundary is in any case unlikely to be applicable to icy moons. The possible role of realistic boundary layer processes remains an important area of research.

Our study isolates the effects of slantwise convection in the absence of baroclinic instability, which arises from large-scale meridional temperature gradients. Chapter 2 has shown that baroclinic eddies tend to smooth out ice shell topography (Kang, 2022; Kang et al., 2022c), in contrast to the poleward-thinning tendency driven by slantwise convection. Kang (2023) examined the interaction between these competing processes using a traditional, gravity-aligned convective parameterization. Revisiting this problem using a more realistic representation of slantwise convection is therefore an important next step. In the next two chapters, we develop a parameterization scheme for heat transport by slantwise convection in icy moon oceans, and then apply it to investigate the competition between baroclinic eddies and slantwise convection.

Appendix 4.A: Numerical details

We set the parameters of our numerical simulations based on typical values for icy moon oceans. The buoyancy flux is calculated as $B = (Qg\alpha_T)/(\rho_0 c_p)$, where we use $\rho_0 = 10^3 \text{ kg m}^{-3}$ for reference density and $c_p = 4000 \text{ J kg}^{-1} \text{ K}^{-1}$ for the specific heat capacity of water. The bottom heat flux Q , gravitational acceleration g , and thermal expansivity α_T for various icy moons are listed in Table 1.1; in our simulations, we adopt $g = 0.1 \text{ m s}^{-2}$ and $\alpha_T = 4 \times 10^{-5} \text{ K}^{-1}$, and vary Q between 10 and 640 mW m^{-2} . A summary of the numerical simulation parameters is provided in Table 4.1. All simulations use uniform grid spacing in the zonal, meridional, and vertical directions.

We focus on the ocean interior, where the horizontally and temporally averaged temperature profile is approximately linear. To identify the boundaries between the interior and the thermal boundary layers, we perform a linear fit to the temperature profile, starting from the domain center and extending outward toward both the top and bottom. The fit is carried out incrementally over increasing depth and is terminated when the coefficient of determination falls below $r^2 = 0.998$. From this fit, we obtain the depth of the interior, H , and the corresponding buoyancy gradient, $|\nabla b| \equiv \Delta b/H$, as illustrated in Fig. 4.7.

In our simulations, subgrid-scale eddies are not explicitly resolved but are parameterized using the Smagorinsky closure, in which viscosity and diffusivity are determined by local velocity gradients, grid spacing, and prescribed coefficients $A_{\nu, \text{smag}}$ and $A_{\kappa, \text{smag}}$ (Smagorinsky, 1963; Smagorinsky et al., 1993):

$$\nu_{\text{smag}} = 2\sqrt{2}A_{\nu, \text{smag}}V_{\text{grid}}^{2/3}\sqrt{v_{xx}^2 + v_{yy}^2 + v_{zz}^2 + \frac{(v_{xy} + v_{yx})^2}{2} + \frac{(v_{xr} + v_{rx})^2}{2} + \frac{(v_{ry} + v_{yr})^2}{2}}, \quad (4.10)$$

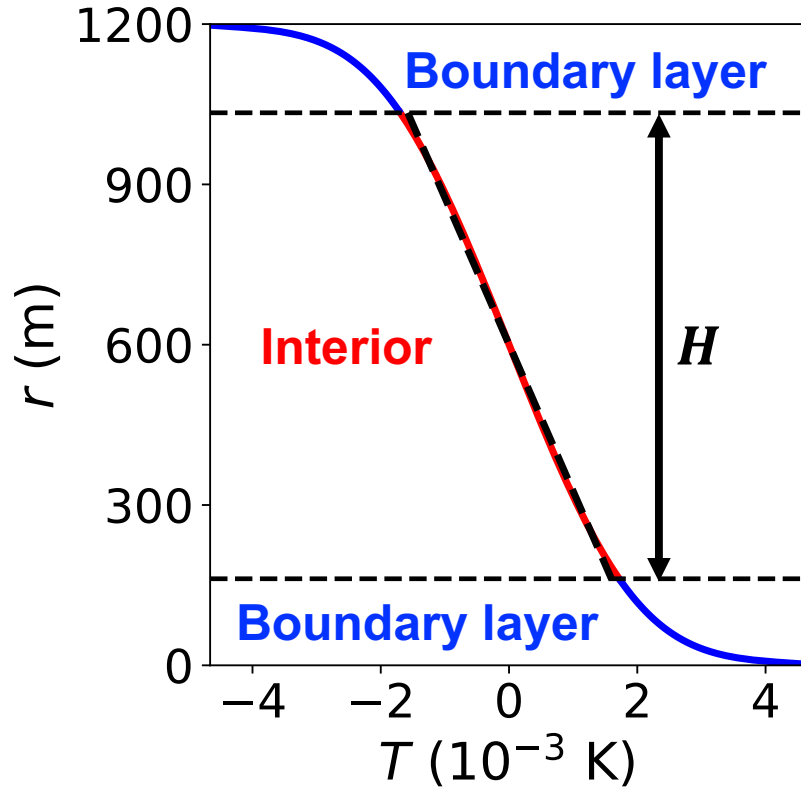


Figure 4.7: **Interior and boundary layer.** Example of the horizontally and temporally averaged temperature profile from simulation $L60_{\text{Ctrl}}$. Solid lines show the simulation result, and the dashed line indicates the best-fit slope for the interior temperature gradient.

$$\kappa_{smag} = 2\sqrt{2}A_{\kappa,smag}V_{grid}^{2/3}\sqrt{v_{xx}^2 + v_{yy}^2 + v_{rr}^2 + \frac{(v_{xy} + v_{yx})^2}{2} + \frac{(v_{xr} + v_{rx})^2}{2} + \frac{(v_{ry} + v_{yr})^2}{2}}, \quad (4.11)$$

where V_{grid} is the grid volume, and $v_{ij} \equiv \partial v_i / \partial j$, $i, j = x, y, r$ are the velocity gradients. Values of $A_{\nu,smag}$ are listed in Table 4.1 and we set $A_{\kappa,smag} = 0.1A_{\nu,smag}$ in our simulations.

For comparison with previous direct numerical simulations that characterize planetary parameters using the Rayleigh number $Ra = \Delta b_{domain} H_{domain}^3 / (\nu \kappa)$ and the Ekman number $Ek = \nu / (\Omega H_{domain}^2)$, we estimate Ra and Ek based on the diagnosed horizontally averaged buoyancy contrast between the two plates and the domain-averaged Smagorinsky eddy viscosity and diffusivity from a single snapshot (the eddy viscosity and diffusivity only vary within a factor of 2 vertically across the domain in our simulations). Here, both Δb_{domain} and H_{domain} are evaluated across the full domain, differing from the ocean interior definition used in the main text, to ensure consistency with previous studies. The transition to the non-rotating regime is given by $Ra_{T,NR} = 100Ek^{-12/7}$, and the transition to the rapidly rotating regime is given by $Ra_{T,RR} = 0.4Ek^{-8/5}$, following Gastine et al. (2016). The resulting values place our simulations in the transitional regime, $Ra_{T,RR} < Ra < Ra_{T,NR}$, consistent with that expected for icy moon oceans (Table 4.3).

To assess the sensitivity to the sub-grid eddy parameterization, we conducted one simulation with viscosity and diffusivity parameters, $A_{\nu,smag}$ and $A_{\kappa,smag}$, reduced by a factor of 10 ($L60_{lowvisc}$), and another low-resolution simulation with grid spacing doubled ($L60_{lowres}$), both compared to the control simulation $L60_{Ctrl}$. In the low-viscosity case, results changed by less than 3%, indicating that viscosity and diffusivity have minimal influence on interior ocean dynamics under our simulation parameters. In the low-resolution case, the buoyancy gradient and the meridional heat flux show very little change (less than 2%), while the interior depth decreased by about 10%. This suggests that the coarser resolution thickens the

Table 4.3: **Ekman and Rayleigh numbers for numerical simulations.** See Appendix 4.A for details.

Name	Ek	Ra	$Ra_{T,RR} = 0.4Ek^{-8/5}$	$Ra_{T,NR} = 100Ek^{-12/7}$
$L60_{Ctrl}$	7.91×10^{-8}	4.96×10^{12}	9.22×10^{10}	1.49×10^{14}
$L60_{0.25B}$	4.81×10^{-8}	8.42×10^{12}	2.05×10^{11}	3.51×10^{14}
$L60_{4B}$	1.21×10^{-7}	3.48×10^{12}	4.66×10^{10}	7.20×10^{13}
$L60_{16B}$	2.08×10^{-7}	2.00×10^{12}	1.96×10^{10}	2.84×10^{13}
$L60_{0.39\Omega}$	2.12×10^{-7}	1.95×10^{12}	1.90×10^{10}	2.75×10^{13}
$L60_{0.625\Omega}$	1.34×10^{-7}	2.85×10^{12}	3.97×10^{10}	6.06×10^{13}
$L60_{1.6\Omega}$	4.91×10^{-8}	8.01×10^{12}	1.98×10^{11}	3.38×10^{14}
$L60_{0.25H}$	2.13×10^{-7}	1.96×10^{12}	1.90×10^{10}	2.75×10^{13}
$L60_{0.5H}$	1.27×10^{-7}	3.18×10^{12}	4.34×10^{10}	6.66×10^{13}
$L60_{2H}$	4.71×10^{-8}	8.78×10^{12}	2.12×10^{11}	3.64×10^{14}
$L15_{Ctrl}$	1.48×10^{-7}	1.09×10^{12}	3.37×10^{10}	5.09×10^{13}
$L15_{0.25B}$	5.14×10^{-8}	6.86×10^{12}	1.84×10^{11}	3.13×10^{14}
$L15_{0.625\Omega}$	1.49×10^{-7}	1.39×10^{12}	3.34×10^{10}	5.03×10^{13}
$L15_{2H}$	5.22×10^{-8}	6.67×10^{12}	1.79×10^{11}	3.04×10^{14}
$L30_{Ctrl}$	8.19×10^{-8}	5.14×10^{12}	8.73×10^{10}	1.41×10^{14}
$L30_{0.25B}$	5.00×10^{-8}	8.76×10^{12}	1.92×10^{11}	3.28×10^{14}
$L30_{0.625\Omega}$	1.34×10^{-7}	3.06×10^{12}	3.96×10^{10}	6.04×10^{13}
$L30_{2H}$	4.93×10^{-8}	9.06×10^{12}	1.97×10^{11}	3.37×10^{14}
$L90_{Ctrl}$	8.18×10^{-8}	4.53×10^{12}	8.74×10^{10}	1.41×10^{14}
$L90_{0.25B}$	4.92×10^{-8}	7.41×10^{12}	1.98×10^{11}	3.38×10^{14}
$L90_{0.625\Omega}$	1.30×10^{-7}	3.01×10^{12}	4.17×10^{10}	6.39×10^{13}
$L90_{2H}$	1.90×10^{-7}	1.03×10^{12}	2.27×10^{10}	3.33×10^{13}
$L60_{lowvisc}$	1.04×10^{-8}	2.86×10^{14}	2.37×10^{12}	4.83×10^{15}
$L60_{lowres}$	2.14×10^{-7}	7.22×10^{11}	1.88×10^{10}	2.72×10^{13}
$L60_{RigidLid}$	8.90×10^{-8}	3.77×10^{12}	7.64×10^{10}	1.22×10^{14}
$L60_{drag}$	8.98×10^{-8}	3.12×10^{12}	7.54×10^{10}	1.20×10^{14}
$L60_{0.5L_y}$	8.24×10^{-8}	4.44×10^{12}	8.64×10^{10}	1.39×10^{14}
$L60_{2L_x 0.5L_y}$	8.50×10^{-8}	3.72×10^{12}	8.23×10^{10}	1.32×10^{14}

boundary layers, likely due to increased effective viscosity and diffusivity from the larger grid spacing, but does not significantly affect the interior dynamics.

We also use a free surface as the top boundary condition in our simulations. To assess its impact, we performed an additional simulation with a rigid lid at the top ($L60_{\text{RigidLid}}$), comparing it to the corresponding simulation with a free surface ($L60_{0.5L_y}$). The results only differed by 1%, indicating that the top boundary condition has little effect on heat transport. This insensitivity arises because the barotropic deformation radius, $L_D = \sqrt{gH}/(2\Omega)$, is much larger than the domain scale in all simulations, with L_D/L_{domain} ranging from 40 to 500. As a result, variations in surface elevation have negligible dynamical influence.

Appendix 4.B: Eddy scale of the heat transport

As shown in Section 4.3, the axial ($\mathbf{e}_z$) and the radial ($\mathbf{e}_s$) heat fluxes are of comparable magnitude. To guide the development of parameterizations, which are intended to represent the effects of unresolved subgrid-scale dynamics, it is important to assess whether the processes responsible for these transports occur at different spatial scales. If a clear scale separation exists, it may be possible to explicitly resolve the larger-scale process in global simulations while parameterizing the smaller-scale one.

We estimate the characteristic length scales of heat transport by computing the spectral centroids (k_x^c, k_y^c) of the axial and radial heat transport cross-spectra in the ocean interior. These spectra are defined by $\text{Re}(\hat{v}_z \hat{T}^*)$ for axial transport and $\text{Re}(\hat{v}_s \hat{T}^*)$ for radial transport, where the hat denotes the Fourier transform and the asterisk denotes the complex conjugate. The spectral centroids are given by:

$$k_x^c = \left(\frac{\iint k_x^2 |\hat{P}| dk_x dk_y}{\iint |\hat{P}| dk_x dk_y} \right)^{1/2}, \quad k_y^c = \left(\frac{\iint k_y^2 |\hat{P}| dk_x dk_y}{\iint |\hat{P}| dk_x dk_y} \right)^{1/2}, \quad (4.12)$$

where $\hat{P}$ represents the cross-spectrum of either $\text{Re}(\hat{v}_s \hat{T}^*)$ or $\text{Re}(\hat{v}_z \hat{T}^*)$.

We find that the eddy length scales associated with heat transport in the axial and radial directions are of comparable magnitude, indicating no clear scale separation between them (Fig. 4.8). In the polar simulation ($\theta = 90^\circ$), strong cancellation between positive and negative contributions to the radial heat transport results in a negligible net radial flux (Table 4.2).

The radial heat transport spectrum (first row in Fig. 4.8) exhibits a dip in energy near $k_x = 0$, indicating that the zonally symmetric mode primarily transports heat along the rotation axis. This result is consistent with the linear instability analyses of Flasar and Gierasch (1978) and Hathaway et al. (1979), which showed that the most unstable symmetric modes correspond to slantwise convection and preferentially transport heat along the rotation

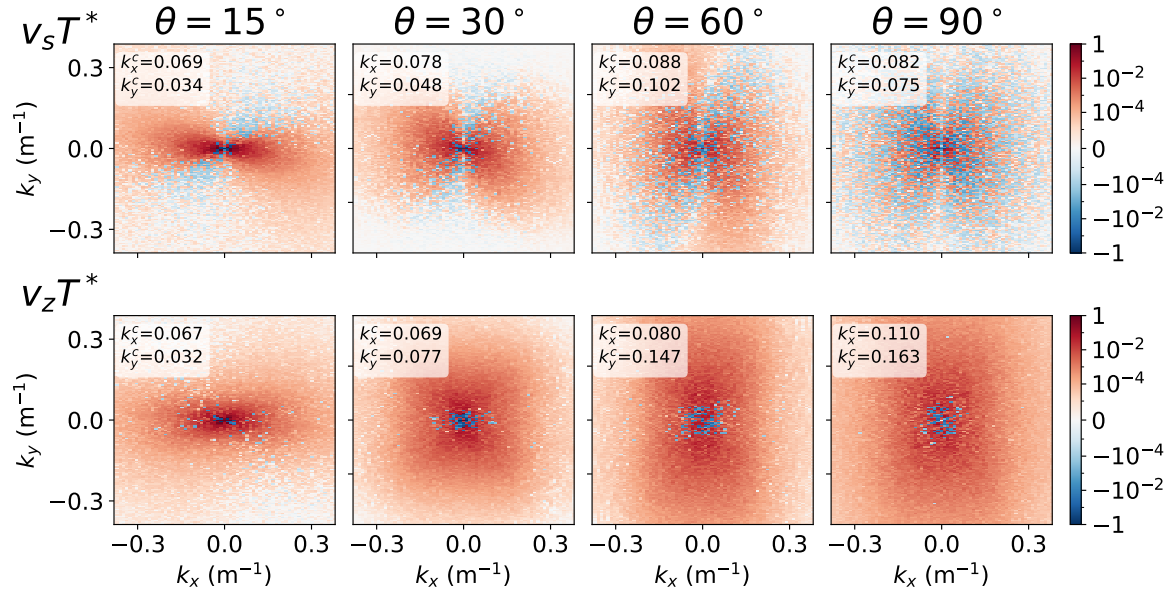


Figure 4.8: **Horizontal cross-spectra of the eddy heat transport.** From left to right, the columns show results from the control simulations at different latitudes: $L15_{\text{Ctrl}}$, $L30_{\text{Ctrl}}$, $L60_{\text{Ctrl}}$, and $L90_{\text{Ctrl}}$. The upper and lower rows show the cross-spectra for radial and axial heat transport, $\text{Re}(\hat{v}_s \hat{T}^*)$ and $\text{Re}(\hat{v}_z \hat{T}^*)$, respectively. Spectra are computed from averages of horizontal slices within the interior region using a single snapshot. Spectral centroids are indicated in the upper-left corner of each panel.

axis.

More generally, the heat transport spectrum is nearly isotropic near the pole and becomes increasingly anisotropic toward lower latitudes. There is some anisotropy present in the $\theta = 90^\circ$ simulation, which arises from the domain geometry, where the zonal extent is shorter than the meridional one. At lower latitudes, eddies become elongated in the meridional direction (i.e., shifted toward smaller k_y), as also evident in the snapshots (Fig. 4.2). This anisotropy is consistent with the findings of Currie et al. (2020) and likely results from the geometric projection of slantwise convection onto the horizontal plane: at the pole, where the rotation axis is vertical, the projection is isotropic; at lower latitudes, the rotation axis tilts in the meridional direction, introducing anisotropy and leading to meridional elongation.

Appendix 4.C: Effects of zonal jets

Previous studies have shown that zonal jets can influence convective heat transport. Aurnou et al. (2008) and Guervilly and Cardin (2017) found that the radial shear of the zonal jets decreases the radial heat transport. Novi et al. (2019) and Currie et al. (2020) found that vertical heat transport along gravity increases with latitude in the presence of zonal jets. Currie et al. (2020) further demonstrated that when the zonal domain is longer than the meridional domain, meridional jets form instead of zonal ones. In such cases, vertical heat transport decreases with latitude up to approximately 60° , and remains nearly constant higher than 60° (see their Fig. 4).

In our simulations, zonal jets are present, and the vertical heat transport (i.e., along the direction of gravity) increases with latitude, consistent with earlier findings. To investigate the influence of jets, we conducted sensitivity tests by modifying the horizontal domain aspect ratio and introducing linear bottom drag. At $\theta = 60^\circ$, we tested three aspect ratios: $L_x : L_y = 7 : 15$ ($L60_{\text{Ctrl}}$), $14 : 15$ ($L60_{0.5L_y}$), and $28 : 15$ ($L60_{2L_x0.5L_y}$). Additionally, we performed a simulation with linear bottom drag ($L60_{\text{drag}}$), in which bottom-layer velocities were relaxed to zero at a rate of $2.5 \times 10^{-4} \text{ s}^{-1}$, which suppresses jet formation.

In simulations $L60_{0.5L_y}$ and $L60_{\text{Ctrl}}$, zonal jets develop (Fig. 4.9a & b). Although their jet magnitudes differ by a factor of two, the resulting buoyancy gradient and flux vary by less than 4% (Table 4.2), suggesting that jet strength has a limited impact on heat transport. In $L60_{2L_x0.5L_y}$, where the zonal domain is wider than the meridional one, meridional jets form instead (Fig. 4.9c), consistent with Julien et al. (2018) and Currie et al. (2020). In this case, the meridional heat flux reverses sign ($B_y = B_z \cos \theta - B_s \sin \theta < 0$; Table 4.2). However, as discussed in Section 4.5, the β effect in real icy moon oceans is expected to inhibit the formation of meridional jets (Rhines, 1975; Vallis, 2017), making such a scenario unlikely to occur in real oceans. In $L60_{\text{drag}}$, linear drag suppresses jet formation, and domain-scale eddies dominate (Fig. 4.9d), reducing the meridional heat flux B_y by a factor of 5, although

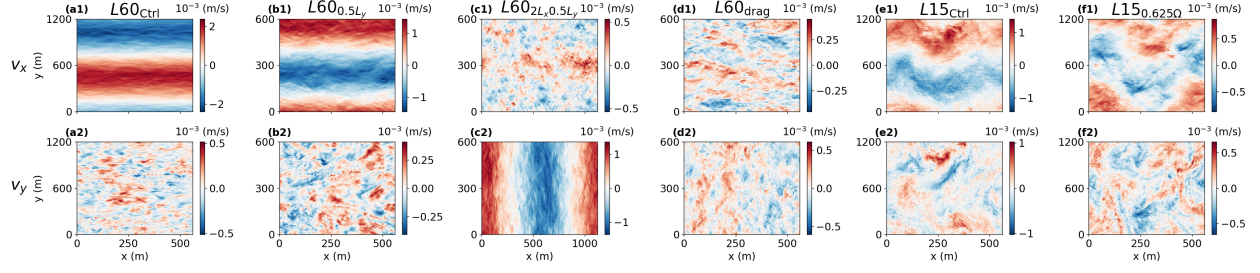


Figure 4.9: **Horizontal snapshots of the horizontal velocity fields at mid-depth ($r = 600$ m).** The first and second rows show the zonal velocity v_x and the meridional velocity v_y , respectively. (a)–(c) show simulations with varying horizontal aspect ratios: $L60_{\text{Ctrl}}$, $L60_{0.5L_y}$, and $L60_{2L_x 0.5L_y}$. (d) shows the simulation with linear bottom drag applied to suppress jet formation ($L60_{\text{drag}}$). (e) and (f) show simulations $L15_{\text{Ctrl}}$ and $L15_{0.625\Omega}$. In all other simulations, relatively strong and straight jets form, resembling those in $L60_{0.5L_y}$ and $L60_{\text{Ctrl}}$.

it remains poleward.

These results suggest that the presence and orientation of jets significantly affect meridional heat transport. Across all four simulations, however, the vertical buoyancy flux remains close to the imposed value, and buoyancy gradients vary by less than 10%, indicating that vertical transport at $\theta = 60^\circ$ is less sensitive to jet configuration. Notably, in the other two outlier cases in Fig. 4.3b ($L15_{\text{Ctrl}}$ and $L15_{0.625\Omega}$), jets exhibit strong meandering (Fig. 4.9e & f), in contrast to the relatively straight jets in other simulations. This also suggests that the structure of the zonal jets may influence convective heat transport. Quantifying this effect is left for future work.

CHAPTER 5

PARAMETERIZING SLANTWISE CONVECTION IN ICY MOON OCEANS

An earlier version of this chapter was posted on *arXiv* as Zeng, Yaoxuan, and Malte F. Jansen. "Parameterizing slantwise convection in icy moon oceans." arXiv:2606.22547 (2026).

5.1 Introduction

In this chapter, we apply the instability criterion developed in Chapter 3 and the heat transport scaling laws developed in Chapter 4 to build a parameterization scheme for slantwise convection that can be implemented in global ocean circulation models.

In icy moon oceans, the natural Rossby number is small, $Ro^* \equiv B^{1/2}(2\Omega)^{-3/2}H^{-1} \ll 1$, where B is the buoyancy flux (Jones and Marshall, 1993; Maxworthy and Narimousa, 1994; Bire et al., 2022). Under strong rotational constraints, convection in icy moon oceans organizes into columnar structures aligned with the planetary rotation axis. The ocean can then be divided into two regions separated by the tangent cylinder, a virtual cylinder parallel to the rotation axis and tangent to the solid core (Fig. 5.1). Inside the tangent cylinder, the bases of columns aligned with the rotation axis are in direct contact with the heated core, forming convective plumes known as slantwise convection. Outside the tangent cylinder, the columns are not directly connected to the core at their bases, and convection takes the form of equatorial rolls that circulate around the columns (Ashkenazy and Tziperman, 2021; Bire et al., 2022).

Most existing global simulations of icy moon oceans rely on convective adjustment schemes developed for Earth's oceans (Marotzke, 1991). However, these schemes cannot represent slantwise convection in icy moon oceans, as illustrated in Chapters 3 and 4. A new parameterization is therefore needed for global icy moon ocean models. The scaling laws for vertical

and meridional heat transport derived in Chapter 4 provide a framework for representing the effects of unresolved slantwise convection in global models. However, these scaling laws were derived from local Cartesian simulations, and their applicability in global models remains to be examined.

In this study, we develop a slantwise convection scheme based on the convection-resolving simulations in Chapter 4. This scheme captures the meridional transport by slantwise convection, which is neglected in the traditional convection scheme. The scheme is implemented in the Massachusetts Institute of Technology General Circulation Model (MITgcm, Adcroft et al., 2018), a numerical model widely used to simulate icy moon oceans (e.g., Zeng and Jansen, 2021; Kang et al., 2022c; Kang, 2023; Zeng and Jansen, 2024; Ames et al., 2025). We conduct benchmark tests in the low- Ro^* regime relevant to icy moon oceans. Section 5.2 describes the formulation of the slantwise convection scheme. Section 5.3 presents benchmark simulations in the low- Ro^* regime and compares the parameterized results with convection-resolving simulations in a global spherical shell. Section 5.4 provides discussion and concluding remarks.

5.2 Formulation of the scheme

5.2.1 Governing equations and coordinates

We solve the Navier-Stokes equations under the Boussinesq approximation for icy moon oceans:

$$\frac{D\mathbf{v}}{Dt} + 2\boldsymbol{\Omega} \times \mathbf{v} = -\nabla\Phi + b\mathbf{e}_r + \nabla \cdot (\boldsymbol{\nu}\nabla\mathbf{v}), \quad (5.1)$$

$$\frac{DT}{Dt} = \nabla \cdot (\boldsymbol{\kappa}\nabla T), \quad (5.2)$$

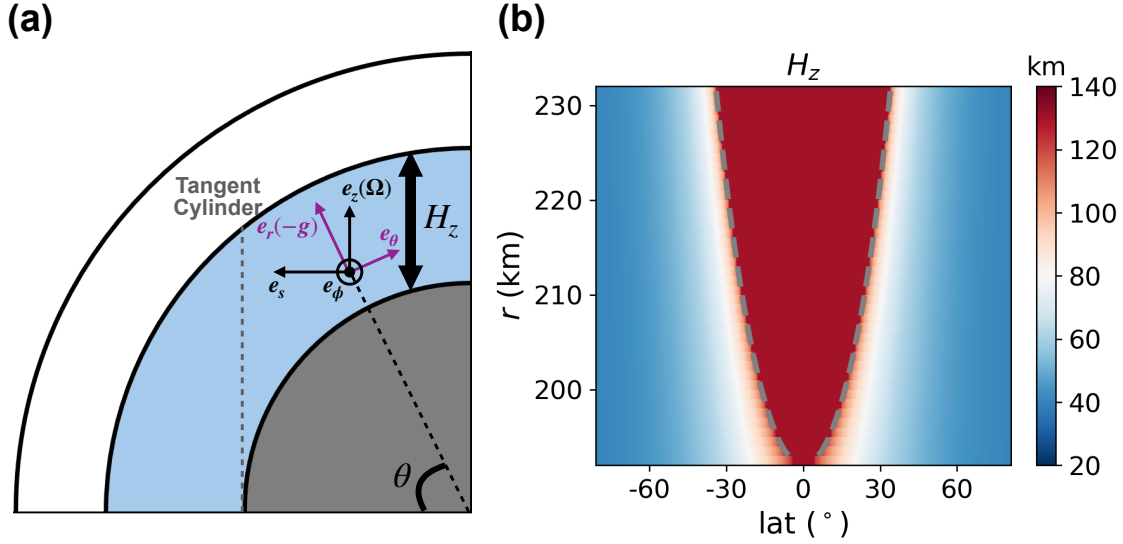


Figure 5.1: **Coordinate systems.** (a) Schematic of the coordinate systems. $\mathbf{e}_\theta$ and $\mathbf{e}_r$ are the meridional (latitudinal) and vertical (opposite to gravity) directions, respectively. $\mathbf{e}_z$ and $\mathbf{e}_s$ are the axial direction aligned with planetary rotation and the radial direction perpendicular to it, respectively. $\mathbf{e}_\phi$ represents both the zonal direction in spherical coordinates and the azimuthal direction in cylindrical coordinates. (b) Distribution of H_z in the simulation with Enceladus-like parameters: a core radius of 192 km and an ocean surface radius of 232 km. Outside the tangent cylinder, H_z is kept constant. In (a) and (b), gray dashed lines indicate the tangent cylinder.

$$\frac{DS}{Dt} = \nabla \cdot (\boldsymbol{\kappa} \nabla S), \quad (5.3)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (5.4)$$

where $\mathbf{v}$ is the velocity vector, $\nabla \Phi$ is the pressure gradient term, $b = b(S, T)$ is the buoyancy, S is the salinity, and T is the temperature. $\boldsymbol{\nu}$ and $\boldsymbol{\kappa}$ are the three-dimensional (3-D) eddy viscosity and diffusivity tensors representing unresolved subgrid-scale mixing processes. We use $(\mathbf{e}_\phi, \mathbf{e}_\theta, \mathbf{e}_r)$ to denote the zonal, meridional, and vertical directions in spherical coordinates, where ϕ is longitude and θ is latitude, and $(\mathbf{e}_\phi, \mathbf{e}_z, \mathbf{e}_s)$ to denote the azimuthal, axial,

and radial directions in cylindrical coordinates, respectively (Fig. 5.1a; note that the zonal direction in spherical coordinates and the azimuthal direction in cylindrical coordinates are the same).

5.2.2 *From traditional upright convection to slantwise convection*

To incorporate slantwise convection, two main aspects of the traditional convection scheme widely used for Earth’s ocean (Marotzke, 1991) must be modified. The first is the instability criterion. The traditional scheme is triggered when stratification is unstable along the gravitational direction, i.e., $\partial b / \partial r < 0$. This criterion is appropriate for upright convection on Earth. However, in icy moon oceans, the Rossby number is small, so angular momentum is dominated by planetary rotation, and surfaces of constant angular momentum are approximately parallel to the rotation axis. In this regime, the flow is always inertially stable, and the only possible buoyancy-driven instability is governed by stratification along the rotation axis. Instability therefore occurs if $(\partial b / \partial z) \sin \theta < 0$, where the factor $\sin \theta$ accounts for the projection of gravity relative to the rotation axis (Chapter 3).

The second aspect is the representation of convective buoyancy transports. The traditional scheme parameterizes convection using a large vertical diffusivity, $\kappa_{rr,c}$, typically prescribed as a constant, where the subscript c here denotes the convective contribution to the eddy diffusivity. This formulation represents only upright transport and adjustment of the buoyancy profile along the gravitational direction, while neglecting any meridional fluxes associated with the tilted structure of slantwise convection. Moreover, it does not depend on local buoyancy gradients, which control the strength of convection.

To address these limitations, we represent convective transport using an anisotropic diffusivity tensor $\kappa_{\mathbf{c}}$ derived from simulations in Chapter 4. The buoyancy flux along the rotation axis follows a CIA (Coriolis–Inertial–Archimedean) scaling (Aurnou et al., 2020):

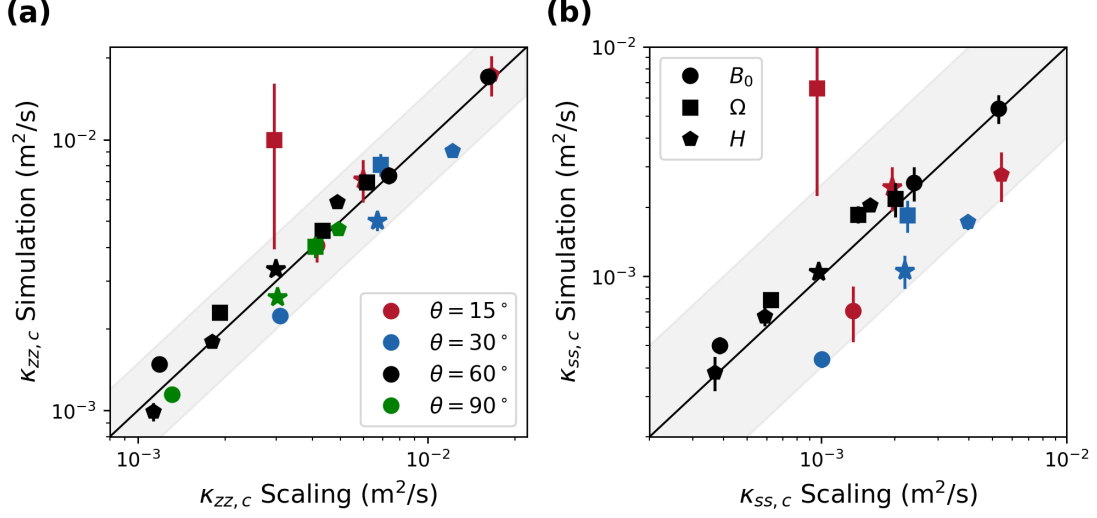


Figure 5.2: **Scalings of diffusivities.** Panels (a) and (b) compare the diffusivity scalings in the axial and radial directions (Equations 5.5 and 5.6) with convection-resolving simulations from Chapter 4, respectively. Marker color denotes latitude (legends in a), and shape denotes parameter variations (legends in b). Black lines denote the one-to-one line. Grey shading indicates the range between $0.67\times$ and $1.5\times$ the scaling in panel (a), and between $0.4\times$ and $2.5\times$ in panel (b). Most simulations lie within this range, except for one case exhibiting large temporal variability (see Chapter 4 for details).

$$B_z = -\kappa_{zz,c} \frac{\partial b}{\partial z}, \quad \kappa_{zz,c} = \begin{cases} K_z \left| \frac{\partial b}{\partial z} \right|^{3/2} (2\Omega)^{-2} H_z^2, & \text{if } \frac{\partial b}{\partial z} \sin \theta < 0, \\ 0, & \text{if } \frac{\partial b}{\partial z} \sin \theta \geq 0. \end{cases} \quad (5.5)$$

where H_z is the characteristic depth scale of convection plumes and $K_z = 0.660$ is an empirical constant. This scaling reproduces the simulation results in Chapter 4 well (Fig. 5.2a).

Inside the tangent cylinder of a spherical shell, the depth scale of slantwise convection plumes is constrained by the shell geometry and therefore varies with latitude, assuming that convection spans the full ocean depth inside the tangent cylinder. Outside the tangent cylinder, however, the dynamics are dominated by equatorial rolls rather than slantwise convection, making the appropriate choice of H_z less clear. In the present formulation, we keep H_z constant outside the tangent cylinder (Fig. 5.1b). We also tested alternative

formulations in which H_z is constrained by the spherical boundaries outside the tangent cylinder, but found no qualitative changes in the results (Appendix 5.B).

For the radial direction, we assume that the diffusivity is proportional to that in the axial direction but reduced by a constant factor:

$$B_s = -\kappa_{ss,c} \frac{\partial b}{\partial s}, \quad \kappa_{ss,c} = \gamma \kappa_{zz,c}, \quad (5.6)$$

where $\gamma = 0.326$ is an empirical constant obtained from the simulations in Chapter 4. With this factor, the scaling agrees with the simulations to within a factor of 2.5 (Fig. 5.2b). The diffusivity $\kappa_{zz,c}$ represents transport associated with slantwise convective motions along the rotation axis, while $\kappa_{ss,c}$ represents cross-column transport. Because these processes are dynamically distinct, we represent the diffusivity tensor as anisotropic but diagonal in cylindrical coordinates (i.e., $\kappa_{zs,c} = \kappa_{sz,c} = 0$).

5.2.3 Tensor transformation

General circulation models for global icy moon ocean simulations are formulated in spherical coordinates to accommodate realistic boundary conditions. We therefore transform the diffusivity tensor from cylindrical coordinates to spherical coordinates. We focus on the meridional heat transport and therefore only require the transformation between the $(\mathbf{e}_z, \mathbf{e}_s)$ and $(\mathbf{e}_\theta, \mathbf{e}_r)$ directions:

$$\boldsymbol{\kappa}_c = \begin{pmatrix} \kappa_{\theta\theta,c} & \kappa_{\theta r,c} \\ \kappa_{r\theta,c} & \kappa_{rr,c} \end{pmatrix} = \mathbf{R}^T \begin{pmatrix} \kappa_{zz,c} & 0 \\ 0 & \kappa_{ss,c} \end{pmatrix} \mathbf{R} = \kappa_{zz,c} \begin{pmatrix} \cos^2 \theta + \gamma \sin^2 \theta & (1 - \gamma) \sin \theta \cos \theta \\ (1 - \gamma) \sin \theta \cos \theta & \sin^2 \theta + \gamma \cos^2 \theta \end{pmatrix}, \quad (5.7)$$

where

$$\mathbf{R} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

is the rotation matrix. The slantwise convection diffusivity is added to the background eddy diffusivity to form the total eddy diffusivity, $\kappa = \kappa_b + \kappa_c$. The slantwise convection scheme does not modify the eddy viscosity, so that $\nu = \nu_b$. Here, the subscript b denotes the background contribution to the eddy diffusivity and viscosity. The scheme is implemented in MITgcm, which uses an Arakawa-C grid (Arakawa, 1977).

5.3 Benchmark tests

5.3.1 Model configuration

The scaling laws in Section 5.2.2 are derived from local Cartesian simulations. To assess their applicability in a more realistic geometry, we conduct convection-resolving simulations (CR simulations) in a global spherical shell using MITgcm. In addition, we perform simulations with the slantwise convection parameterization (Equations 5.5 & 5.6) under the same setup for comparison (SL simulations). These simulations serve as benchmark tests for the parameterization developed in this study. We also perform simulations with the traditional convection scheme (“convective adjustment scheme” in MITgcm, Adcroft et al., 2018) for comparison (TR simulations).

We impose a uniform bottom heat flux Q_{bot} and restore the surface temperature toward the freezing point (0°C) at a time scale τ_T to represent icy moon conditions. The uniform surface temperature suppresses lateral buoyancy gradients and therefore minimizes the influence of baroclinic eddies, allowing us to isolate the effect of slantwise convection. We fix the ocean salinity and adopt a linear equation of state with thermal expansion coefficient $\alpha_T = 2 \times 10^{-4} \text{ K}^{-1}$, such that $b = -g\alpha_T(T - T_{ref})$, where we choose $T_{ref} = 0^\circ\text{C}$. Other

Table 5.1: **Simulation setup.** CR indicates convection-resolving simulations, SL indicates simulations with the slantwise convection parameterization, and TR indicates the simulation with the traditional convection scheme. The restoring timescale τ_T is chosen to be approximately 1/50 of the vertical overturning timescale. In the TR simulations, we set $\kappa_{rr,c} = 3869, 972, 244$, and $61.3 \text{ m}^2 \text{ s}^{-1}$ for simulations from the highest to lowest heat fluxes, respectively. These values are chosen to be comparable to the diffusivities estimated from the corresponding SL simulations.

Scheme	Q_{bot}	Ro^*	$Ek = \nu\Omega^{-1}H^{-2}$	Resolution	τ_T	τ_U
CR	10^6 W m^{-2}	5.6×10^{-2}	5.8×10^{-4}	0.45°	10^4 s	10^5 s
	10^5 W m^{-2}	1.8×10^{-2}	2.7×10^{-4}	0.45°	$3 \times 10^4 \text{ s}$	10^5 s
	10^4 W m^{-2}	5.6×10^{-3}	1.3×10^{-4}	0.225°	$9 \times 10^4 \text{ s}$	10^5 s
	10^3 W m^{-2}	1.8×10^{-3}	5.8×10^{-5}	0.225°	$2.7 \times 10^5 \text{ s}$	10^6 s
SL	10^6 W m^{-2}	5.6×10^{-2}	3.8×10^{-3}	0.9°	10^4 s	10^5 s
	10^5 W m^{-2}	1.8×10^{-2}	1.7×10^{-3}	0.9°	$3 \times 10^4 \text{ s}$	10^5 s
	10^4 W m^{-2}	5.6×10^{-3}	8.1×10^{-4}	0.9°	$9 \times 10^4 \text{ s}$	10^5 s
	10^3 W m^{-2}	1.8×10^{-3}	3.8×10^{-4}	0.9°	$2.7 \times 10^5 \text{ s}$	10^6 s
TR	10^6 W m^{-2}	5.6×10^{-2}	3.8×10^{-3}	0.9°	10^4 s	10^5 s
	10^5 W m^{-2}	1.8×10^{-2}	1.7×10^{-3}	0.9°	$3 \times 10^4 \text{ s}$	10^5 s
	10^4 W m^{-2}	5.6×10^{-3}	8.1×10^{-4}	0.9°	$9 \times 10^4 \text{ s}$	10^5 s
	10^3 W m^{-2}	1.8×10^{-3}	3.8×10^{-4}	0.9°	$2.7 \times 10^5 \text{ s}$	10^6 s
SL	10^4 W m^{-2}	5.6×10^{-3}	3.2×10^{-4}	0.45°	$9 \times 10^4 \text{ s}$	10^5 s
	10^4 W m^{-2}	5.6×10^{-3}	1.4×10^{-3}	1.35°	$9 \times 10^4 \text{ s}$	10^5 s
	10^4 W m^{-2}	5.6×10^{-3}	2.4×10^{-3}	0.9°	$9 \times 10^4 \text{ s}$	10^5 s
	10^4 W m^{-2}	5.6×10^{-3}	2.7×10^{-4}	0.9°	$9 \times 10^4 \text{ s}$	10^5 s

parameters are chosen to be representative of Enceladus: rotation rate $\Omega = 5 \times 10^{-5} \text{ s}^{-1}$, gravity $g = 0.1 \text{ m s}^{-2}$, bottom radius $r_{bot} = 192 \text{ km}$, and surface radius $r_{surf} = 232 \text{ km}$. The resulting H_z profile is shown in Fig. 5.1b.

Resolving slantwise convection with realistic heat fluxes is computationally infeasible because the resulting convective plumes are too small (Table 1.1). We therefore apply enhanced heat fluxes to make the plumes resolvable, while keeping a small natural Rossby number: $Ro^* = 5.6 \times 10^{-2}$, 1.8×10^{-2} , 5.6×10^{-3} , and 1.8×10^{-3} . For reference, we also diagnose the horizontally averaged vertical temperature profile to estimate the Rayleigh number ($Ra = g\alpha_T\Delta TH^3/(\nu\kappa)$) for each CR simulation (see Table 5.2 in APPENDIX 5.A).

In all simulations, we apply isotropic background eddy diffusivity and viscosity and fix the Prandtl number $\text{Pr} = \nu_b/\kappa_b = 10$ to maintain numerical stability. The vertical resolution is 1000 m. For the CR simulations, the horizontal resolution is 0.45° for $\text{Ro}^* = 5.6 \times 10^{-2}$ and 1.8×10^{-2} , and 0.225° for $\text{Ro}^* = 5.6 \times 10^{-3}$ and 1.8×10^{-3} , in order to resolve the convective plumes (see APPENDIX 5.A for more details). The SL (slantwise convection parameterization) and TR (traditional convective parameterization) simulations use a coarser horizontal resolution of 0.9° with increased viscosity, since convection is parameterized rather than explicitly resolved.

To assess sensitivity to numerical parameters, we additionally perform SL simulations with different horizontal resolutions (0.45° and 1.35°)¹ and viscosities (3ν and $\nu/3$), while keeping $\text{Pr} = 10$. To reduce computational cost, the zonal domain is restricted to 13.5° with periodic boundary conditions, corresponding to an approximate 27-fold zonal symmetry. The meridional extent spans 81°S to 81°N to avoid numerical difficulties near the poles, where zonal grid spacing becomes small. We apply no-normal-flow and free-slip boundaries at the top and bottom. To prevent long-term drift of the zonal flow, we apply a linear bottom drag, which restores the velocity in the lowest model level toward zero with a timescale $\tau_U = 10^5$ – 10^6 s. All simulations are integrated to a statistically steady state, and time-averaged results are presented to reduce temporal variability, unless otherwise noted. Detailed simulation parameters are listed in Table 5.1.

5.3.2 *Simulation results*

In convection-resolving (CR) simulations, we observe distinct dynamical features in equatorial rolls (Fig. 5.3a) and mid-latitude slantwise convection (Fig. 5.3b) across different Ro^* . In the smaller- Ro^* simulations ($\text{Ro}^* = 5.6 \times 10^{-3}$ and 1.8×10^{-3}), mid-latitude convection

1. For simulations with varying horizontal resolution, the viscosity is also adjusted according to Kolmogorov scaling, $\nu \propto L_{\text{grid}}^{4/3}$ (cf. Appendix B in Zeng and Jansen, 2021).

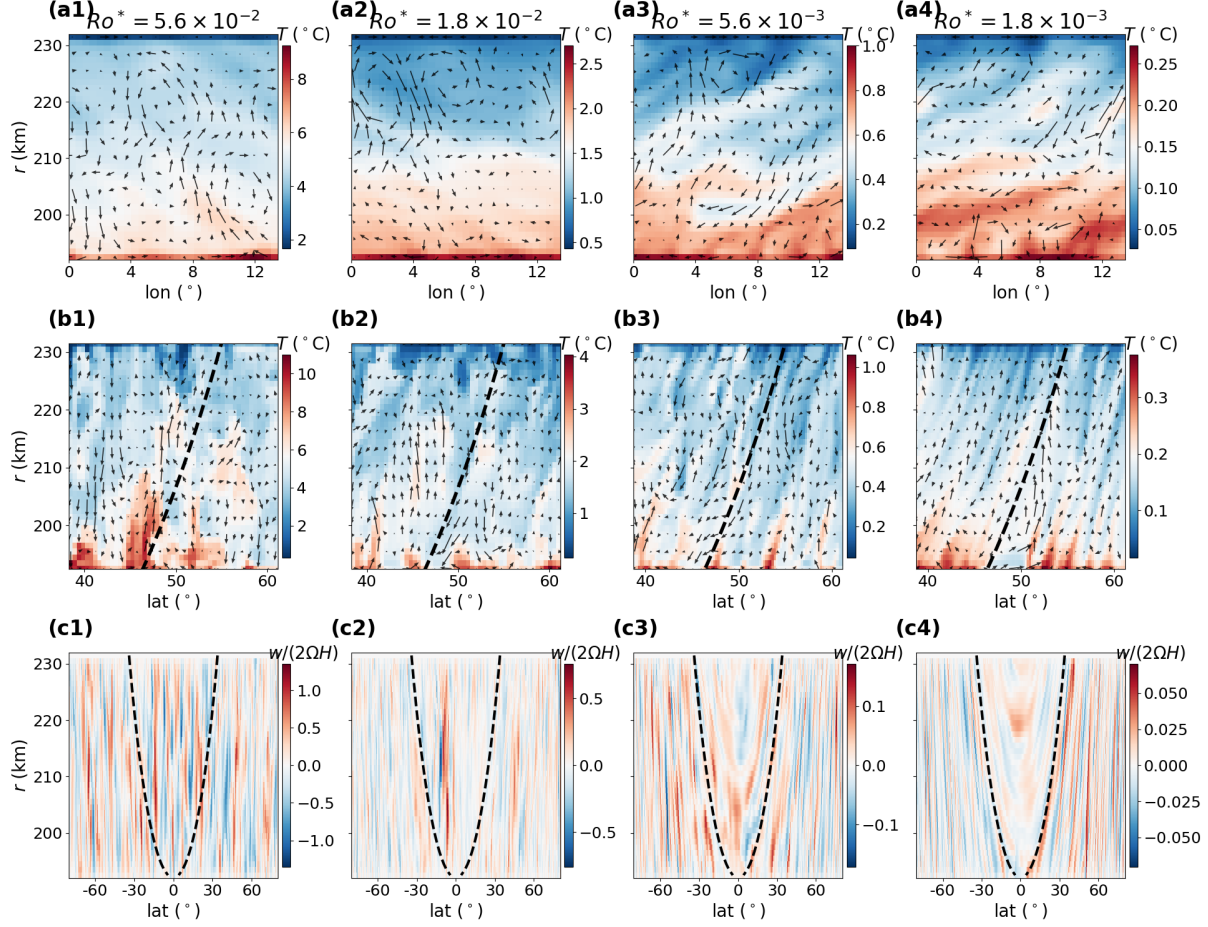


Figure 5.3: **Snapshots of convective plumes in convection-resolving (CR) simulations.** (a): Snapshots of equatorial rolls, longitude–depth sections averaged over a 10° band around the equator. (b): Snapshots of slantwise convection at mid-latitudes, latitude–depth sections for one longitude slice. From left to right, the columns correspond to convection-resolving simulations with $Ro^* = 5.6 \times 10^{-2}$, 1.8×10^{-2} , 5.6×10^{-3} , and 1.8×10^{-3} , respectively. In (a) and (b), color shadings indicate temperature, arrows indicate velocity anomalies with the zonal mean subtracted, and dashed lines indicate directions parallel to rotation ($s = r \cos \theta = \text{const}$). (c): Snapshots of vertical velocity w at one longitude slice. The velocity is non-dimensionalized by rotation rate Ω and ocean depth H , representing the Rossby number of the convective flow. In (c), black dashed lines indicate the tangent cylinder.

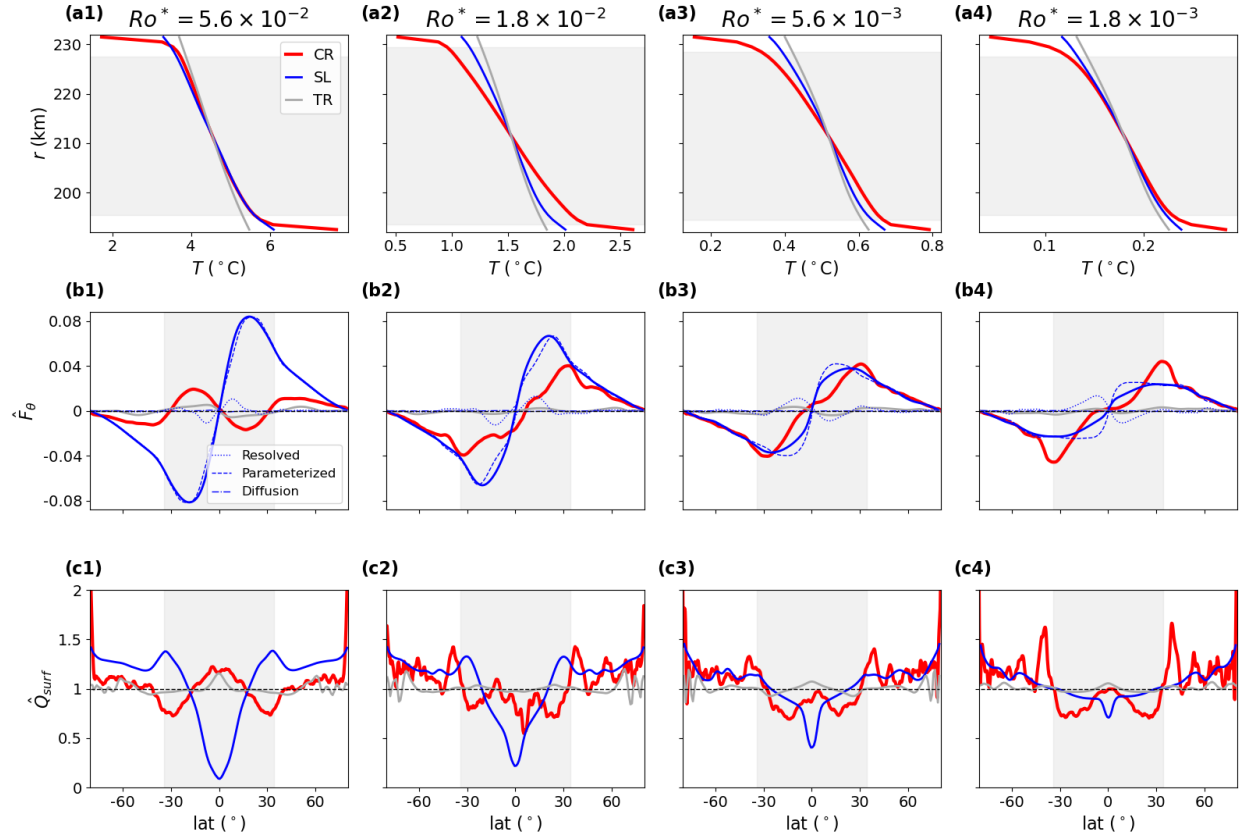


Figure 5.4: **Benchmark test results.** (a): Horizontally averaged temperature profiles. (b): Vertically integrated meridional heat flux F_θ , normalized by the total bottom heat flux as $\hat{F}_\theta = F_\theta / (Q_{bot} A_{bot})$, where A_{bot} is the total bottom area. (c) Surface heat flux per unit area Q_{surf} , normalized by the bottom heat flux rescaled to the surface as $\hat{Q}_{surf} = (Q_{surf} A_{surf}) / (Q_{bot} A_{bot})$, where A_{surf} is the total surface area. In (a)–(c), red lines indicate convection-resolving (CR) simulations, blue lines indicate simulations with the slantwise convection parameterization developed in this study (SL), and gray lines indicate simulations using the traditional convection scheme (TR). In (a), gray shading indicates the interior region, defined as the region where the vertical temperature profile is approximately linear (see Chapter 4 for details of the algorithm separating interior and boundary layers). Blue and gray lines are shifted by a constant offset for better comparison of the interior temperature gradient. In (b), blue dotted, dashed, and dash-dotted lines indicate contributions from resolved, parameterized, and background diffusive fluxes, respectively. In (b) and (c), gray shading indicates regions outside the tangent cylinder.

aligns with the rotation axis, consistent with the rapidly rotating regime (Aurnou et al., 2020). In the equatorial regions we see indications for eastward tilted rolls, resembling equatorial Busse modes associated with superrotation, i.e., an eastward jet at the equator (Busse and Hood, 1982). In contrast, in the larger- Ro^* simulations ($\text{Ro}^* = 5.6 \times 10^{-2}$ and 1.8×10^{-2}), the equatorial rolls tend to tilt westward and the mid-latitude convection no longer exhibits clear rotational alignment. These differences are also evident in the vertical velocity fields (Fig. 5.3c), indicating that the larger- Ro^* cases lie outside the rapidly rotating regime relevant to icy moon oceans. This transition is broadly consistent with Bire et al. (2022), who found that rotationally constrained convection occurs approximately for $\text{Ro}^* < 10^{-2}$. Realistic icy moon oceans typically have $\text{Ro}^* \sim 10^{-6}$ – 10^{-4} (Table 1.1), substantially smaller than the values explored here.

For the smaller- Ro^* cases, simulations with the slantwise convection parameterization (SL) reproduce the total meridional heat transport in the CR simulations well (Fig. 5.4b3 & b4). In particular, inside the tangent cylinder (i.e. at mid- to high-latitudes), the parameterized flux matches the total meridional heat transport in the CR simulations well (compare dashed blue and solid red lines), while the resolved flux (blue dotted lines) and background diffusive flux (blue dash-dotted lines) are negligible. Outside the tangent cylinder, the agreement between the parameterized and CR fluxes becomes weaker, consistent with the fact that the low latitudes are not dominated by slantwise convection. In the two simulations with larger Ro^* , the parameterized meridional heat transport deviates substantially from the CR results, as these cases are not within the rotation-constrained slantwise convection regime assumed in the parameterization.

The meridional heat transport significantly modifies the surface heat flux distribution. In all simulations, it modulates the surface heat flux by up to a factor of 2 relative to the uniform bottom heating (Fig. 5.4c). In contrast, simulations using the traditional convection scheme (grey lines in Fig. 5.4a3-c3) exhibit negligible meridional heat transport, resulting in a nearly

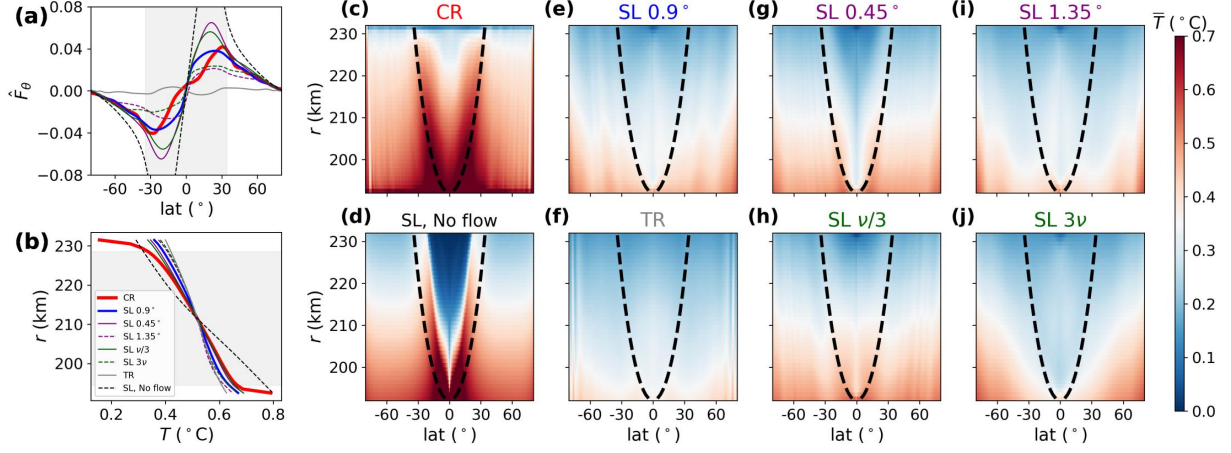


Figure 5.5: **Heat transport and temperature fields in the sensitivity tests with $Ro^* = 5.6 \times 10^{-3}$.** (a) and (b) are the same as Fig. 5.4b3 and a3, respectively, but include sensitivity tests. (c)-(j) show the zonal-mean temperature $\bar{T}$. Purple indicates sensitivity tests with varying horizontal resolution, and green indicates simulations with varying background viscosity. In (a), the gray shading indicates regions outside the tangent cylinder. In (b), the gray shading indicates the interior region. In (c)-(i), black dashed lines indicate the tangent cylinder.

uniform surface heat flux. This leads to an underestimation of polar heat accumulation for a uniform surface temperature, which in turn could affect predictions of ice shell thickness distribution.

Although the slantwise convection scheme reproduces the meridional heat transport well, the simulations with parameterized convection struggle to fully reproduce the temperature structure (Fig. 5.4a and Fig. 5.5). In the SL simulations, there are no thermal boundary layers seen in the CR simulations, where large vertical temperature gradients occur. This is because the parameterization is derived for the ocean interior and does not represent boundary layers (Chapter 4). The parameterized diffusivity is much larger than the background diffusivity, which smooths out sharp vertical gradients. Nevertheless, the interior temperature gradients in the SL simulations agree reasonably well with those in the CR simulations for the smaller- Ro^* cases (Fig. 5.4a3-a4), indicating that the scheme captures the bulk interior structure in the rotation-constrained regime.

Besides the missing boundary layers, differences also appear in the latitudinal distribu-

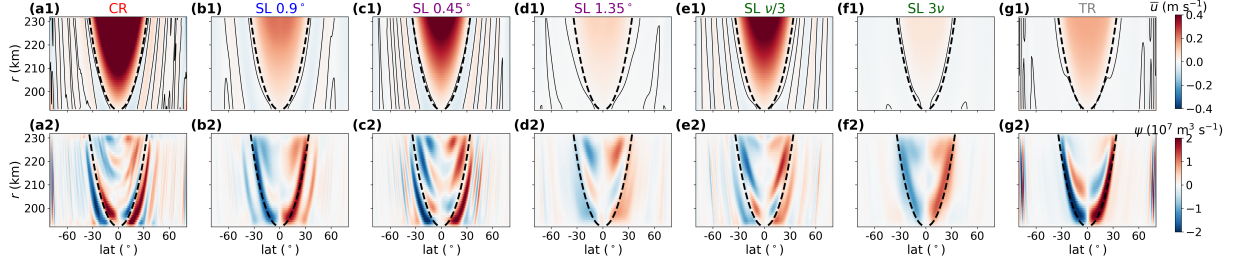


Figure 5.6: **Flow fields in the sensitivity tests with $\text{Ro}^* = 5.6 \times 10^{-3}$.** The first row shows the zonal-mean zonal velocity $\bar{u}$, and the second row shows the stream function ψ , calculated as $\psi = -2\pi \int_{r_{\text{core}}}^{r_{\text{surf}}} \bar{v} r \cos \phi dr$, where $\bar{v}$ is the zonal-mean meridional velocity, and r_{core} and r_{surf} are the radii of the core and ocean surface, respectively. Positive values of ψ indicate clockwise circulation. Black dashed lines indicate the tangent cylinder. In the first row, black solid lines indicate the $\bar{u} = 0$ contours.

tion of temperature fields. In the CR simulation, there is a temperature maximum near the tangent cylinder (Fig. 5.5c), associated with upwelling flow along the tangent cylinder (Fig. 5.6a2). This feature is not reproduced in the SL simulation, and the overall temperature contrast in the SL simulation is weaker compared to the CR simulation (Fig. 5.5e). This misfit is likely again attributable to the treatment of the boundary layers, since any mismatch in the meridional temperature structure at the outer edge of the boundary layer can project into the interior temperature field. However, it should also be noted that the boundary layers in our convection-resolving simulations are not expected to faithfully represent those in realistic icy moon oceans. Adequate parameterizations of the boundary layer dynamics are ultimately required, but developing such parameterizations is beyond the scope of this study.

5.3.3 Sensitivity tests

To better understand the discrepancy in the temperature fields, we perform a series of sensitivity tests for the $\text{Ro}^* = 5.6 \times 10^{-3}$ case. This case lies within the rapidly rotating regime while allowing the convective plumes to be better resolved than in the simulation with the smallest Ro^* . In the first experiment, the resolved ocean dynamics are turned off and all

heat redistribution is represented solely by the slantwise convection scheme, together with a very weak background diffusion. In this case, the simulation reproduces the temperature maximum near the tangent cylinder, although the maximum shifts slightly equatorward (Fig. 5.5d). This behavior is expected because heat transport is most efficient near the tangent cylinder. If all heat transport were aligned with the rotation axis, the temperature maximum would occur at the tangent cylinder, where the area projection from the bottom boundary onto columns parallel to the rotation axis is largest (cf. Appendix A in Zeng and Jansen 2021). In the parameterization, however, heat transport is not purely axial. The additional cross-column diffusivity shifts the effective transport direction slightly away from the rotation axis, causing the temperature maximum to move slightly equatorward. This simulation also overestimates the interior vertical temperature gradient and the meridional heat transport (Fig. 5.5a & b, black dashed lines). The likely explanation is that the scaling law underestimates the effective cross-column diffusivity outside the tangent cylinder, where the dynamics are dominated by equatorial rolls. A similar underestimation in the radial heat transport is also evident in the low-latitude simulations in Chapter 4. Near the equator, this cross-column diffusivity is approximately equivalent to the vertical diffusivity. Consequently, a larger temperature gradient is required to maintain the prescribed vertical heat flux. This enhanced temperature gradient then increases the heat transport along the rotation axis, which is approximately meridional near the equator. As a result, this simulation overestimates the meridional heat transport outside the tangent cylinder by approximately a factor of four (black dashed line in Fig. 5.5a).

The temperature field in the simulation with resolved dynamics turned off differs substantially from the simulation with active ocean dynamics, suggesting that the resolved dynamics play an important role in shaping the temperature structure and the associated heat transport even in the relatively coarse simulations with parameterized convection. To further examine this effect, we carry out sensitivity tests by varying the horizontal resolution

and background viscosity. In the simulation with finer horizontal resolution (SL 0.45°) and the simulation with reduced viscosity (SL $\nu/3$), the resolved dynamics become stronger, and the heat transport outside the tangent cylinder increases by about 50%. In contrast, in the simulation with coarser horizontal resolution (SL 1.35°) and the simulation with increased viscosity (SL 3ν), the resolved dynamics become weaker, and the meridional heat transport outside the tangent cylinder is reduced by about 50%. Despite these differences, all SL simulations reproduce the heat transport inside the tangent cylinder reasonably well and capture the bulk interior vertical temperature gradient, while none of them reproduces the exact latitudinal temperature structure in the CR simulation (Fig. 5.5). For comparison, the simulation with the traditional convection scheme (TR) exhibits even weaker temperature gradients and produces a very weak and largely wrong-signed meridional heat flux. This difference arises because the traditional parameterization applies strong diffusion only along the gravitational direction, without enhancing meridional diffusion.

Although the slantwise convection scheme does not include momentum transport, it is nevertheless interesting to examine its impact on the flow fields. In the CR simulation, a strong superrotating jet occupies the region outside the tangent cylinder, together with multiple alternating jets inside the tangent cylinder (Fig. 5.6a1). In all SL simulations and the TR simulation, there is a superrotating jet outside the tangent cylinder but it is much weaker. These simulations also fail to reproduce the multiple alternating jets inside the tangent cylinder. Only the SL 0.45° and SL $\nu_e/3$ simulations, which resolve more dynamical processes and hence can better represent the momentum fluxes, better reproduce the equatorial jet magnitude and the alternating jet structures (Fig. 5.6c1 & e1).

The meridional overturning circulation also differs substantially between the simulations. In the CR simulation, multiple small-scale slantwise overturning cells appear inside the tangent cylinder, together with larger overturning cells outside the tangent cylinder. Near the tangent cylinder, a pair of overturning cells forms, with upwelling at the tangent cylinder

and downwelling on its flanks (Fig. 5.6b1). This structure is not reproduced in the SL and TR simulations. Many of them instead exhibit a single overturning cell spanning across the tangent cylinder (second row in Fig. 5.6), which leads to equatorward convergence of heat by resolved advective transport (dotted blue line in Fig. 5.4b3).

5.4 Discussion

We developed a slantwise convection scheme to parameterize heat transport in the low- Ro^* regime relevant to icy moon oceans. The scheme captures the meridional heat transport associated with slantwise convection, which is absent in traditional convection schemes. This transport modulates the surface heat flux by up to a factor of 2 relative to the imposed bottom heat flux. Such redistribution affects the ice-ocean heat budget and may influence ice shell topography, one of the key observational constraints of icy moons.

The scheme performs best inside the tangent cylinder, where it accurately reproduces the meridional heat transport from convection-resolving simulations, and performs reasonably well outside the tangent cylinder where it captures the sign and general magnitude of the meridional heat transport. Our sensitivity tests show that heat transport outside the tangent cylinder remains sensitive to the resolved dynamics, while heat transport inside the tangent cylinder is reproduced robustly across a wide range of resolutions and viscosities. The reduced accuracy outside the tangent cylinder likely reflects the distinct dynamical regime dominated by equatorial-roll-like motions. Future work should therefore focus on understanding these dynamics and developing improved parameterizations for them.

Although the slantwise convection scheme performs well in representing the bulk vertical temperature gradient in the ocean interior, it does not fully reproduce the temperature field in the convection-resolving simulations. The likely explanation is that the scheme does not represent the boundary layers, which set the boundary conditions for temperature and heat transport in the interior. However, the viscous boundary layers along a smooth surface in

the convection-resolving simulations themselves are unrealistic, as even the CR simulations cannot adequately resolve the boundary layer dynamics, and the boundaries of real icy moon oceans are likely rough rather than smooth. Future studies should therefore investigate appropriate boundary-layer parameterizations for icy moon oceans and their influence on the temperature structure and heat transport.

In addition, the scheme does not account for momentum transport by slantwise convection, which may lead to nonsynchronous rotation of ice shells (Ashkenazy et al., 2023; Kang, 2024). Extending the scheme to include momentum redistribution by slantwise convection is therefore an important direction for future work.

In the benchmark tests, density is controlled solely by temperature. In realistic icy moon oceans, salinity can also play an important role (Zeng and Jansen, 2021; Kang et al., 2022c). The scheme can be directly extended to include salinity under a linear equation of state. However, the equation of state for seawater under icy moon conditions can be strongly nonlinear, and the impact of this nonlinearity on convective transport remains to be explored. The current formulation assumes that convection spans the full ocean depth. Previous studies suggest that for Enceladus, a stably stratified layer may exist above a convective layer when salinity is low (Zeng and Jansen, 2021, 2024; Ames et al., 2025). In such cases, the depth scale for slantwise convection H_z should be reduced to the depth of the convective layer, which can be estimated from the bottom heat flux and background vertical diffusivity (cf. Eq. 1 in Zeng and Jansen (2021)).

This study isolates the effects of slantwise convection and neglects baroclinic eddies. However, Chapter 2 has shown that baroclinic eddies can dominate heat transport when the ice shell is not flat. In the next chapter, we will use the slantwise convection scheme developed here to examine the interaction between slantwise convection and baroclinic eddies, and assess their relative importance for ocean heat transport.

Beyond icy moons, this scheme can also be applied to other oceans in the rapidly rotating

regime, i.e., $\text{Ro}^* \equiv B^{1/2}(2\Omega)^{-3/2}H^{-1} \ll 1$. This regime can be realized when the ocean depth H is large, and may therefore be relevant to deep ocean worlds on exoplanets (Kuchner, 2003; Léger et al., 2004; Sotin et al., 2007; Kite and Ford, 2018; Lai and Yang, 2022).

Appendix 5.A: Rayleigh number and plume scale

We estimate the convective plume scale in the simulations as $l_{\text{plume}} \sim B^{1/4}(2\Omega)^{-3/4}H^{1/2}$ (Fernando et al., 1989; Jones and Marshall, 1993). The actual plume size is larger than this scaling by a prefactor, reported to be approximately 20 in Bire et al. (2022) and 5 in Chapter 4. Taking this prefactor into account, to ensure that convection plumes are adequately resolved in the CR simulations, we choose horizontal resolutions smaller than $l_{\text{plume}}/10$.

Using the posteriori diagnosed horizontally-averaged vertical temperature contrast ΔT , we calculate the Rayleigh number for the convection-resolving simulations (Table 5.2). For comparison, we also compute the transitional Rayleigh numbers $\text{Ra}_{\text{T,RR}} = 0.4\text{Ek}^{-8/5}$ and $\text{Ra}_{\text{T,NR}} = 100\text{Ek}^{-12/7}$, which represent the transitions from the rapidly rotating regime to the transitional regime and from the transitional regime to the non-rotating regime, respectively (Gastine et al., 2016).

For the simulation with $\text{Ro}^* = 5.6 \times 10^{-2}$, we find $\text{Ra} \sim \text{Ra}_{\text{T,NR}}$, indicating that this case lies near the non-rotating regime. In this regime, convective flows are dominated more by buoyancy than by rotation, consistent with the absence of rotation-aligned structures and the tendency for plumes to align with gravity.

For the simulations with $\text{Ro}^* = 5.6 \times 10^{-3}$ and 1.8×10^{-3} , we find $\text{Ra}_{\text{T,NR}} < \text{Ra} <$

Table 5.2: **Plume scales and diagnosed Rayleigh numbers for the convection-resolving simulations.** The average ocean radius, $\bar{r} = (r_{\text{core}} + r_{\text{surf}})/2$, is used to convert the estimated plume size into an equivalent latitudinal scale ($\Delta\theta_{\text{plume}} \sim 5B^{1/4}(2\Omega)^{-3/4}H^{1/2}/\bar{r}$).

Ro^*	$\text{Ra} = g\alpha_T\Delta TH^3\nu^{-1}\kappa^{-1}$	$\text{Ra}_{\text{T,RR}}$	$\text{Ra}_{\text{T,NR}}$	$\Delta\theta_{\text{plume}}$
5.6×10^{-2}	2.5×10^7	6.0×10^4	3.5×10^7	12.8°
1.8×10^{-2}	4.1×10^7	2.1×10^5	1.3×10^8	7.2°
5.6×10^{-3}	5.7×10^7	7.0×10^5	4.9×10^8	4.1°
1.8×10^{-3}	9.9×10^7	2.4×10^6	1.8×10^9	2.3°

$Ra_{T,RR}$, with Ra approximately an order of magnitude away from both transition values, indicating that these cases lie within the transitional regime. In this transitional regime, the interior flow remains strongly influenced by rotation, while the boundary layers are not rotationally constrained. This is consistent with the presence of slantwise plumes aligned with the rotation axis in the interior. Icy moon oceans are expected to lie within this transitional regime. The good agreement in meridional heat transport for these low- Ro^* simulations therefore supports the applicability of the slantwise convection scheme developed in this study to realistic icy moon conditions.

Appendix 5.B: An alternative choice of H_z outside the tangent cylinder

The appropriate length scale H_z outside the tangent cylinder is not well constrained, as the dynamics there are dominated by equatorial rolls rather than slantwise convection. In the default setup, we prescribe a constant H_z outside the tangent cylinder. An obvious alternative choice is to define H_z as the distance between the ice shell and the equatorial plane, which yields an H_z profile that decreases to zero toward the equator (Fig. 5.7a).

To assess the sensitivity to this choice, we rerun $\text{Ro}^* = 5.6 \times 10^{-3}$ simulations using the alternative H_z profile while keeping all other parameters unchanged. The resulting meridional heat transport and temperature structure remain broadly unchanged (compare Fig. 5.5 and Fig. 5.7). The alternative formulation produces slightly better agreement with the convection-resolving simulation in the meridional heat transport outside the tangent cylinder. However, it yields a less realistic temperature structure in some simulations, with a temperature maximum outside the tangent cylinder rather than the temperature minimum seen in the convection-resolving simulation.

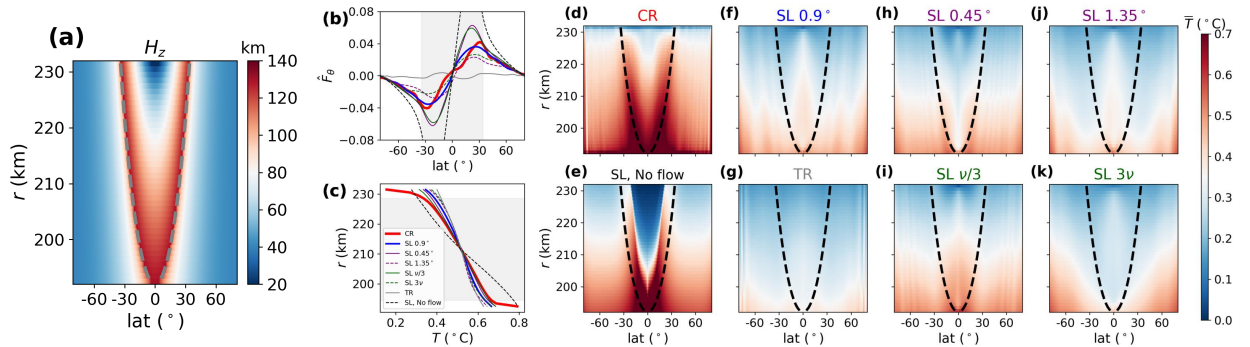


Figure 5.7: **Heat transport and temperature fields in the sensitivity tests with $\text{Ro}^* = 5.6 \times 10^{-3}$ and varying H_z outside the tangent cylinder.** This plot is the same as Fig. 5.5, but with varying H_z outside the tangent cylinder (see panel a).

CHAPTER 6

HEAT TRANSPORT IN ICY MOON OCEANS: INTERPLAY BETWEEN SLANTWISE CONVECTION AND BAROCLINIC EDDIES

6.1 Introduction

In icy moon oceans, two major processes control ocean heat transport: baroclinic eddies driven by meridional buoyancy gradients at the ice–ocean interface, and slantwise convection driven by heating from the silicate core. Previous studies have investigated the relative importance of baroclinic eddies and convection under a variety of theoretical frameworks and numerical model configurations (e.g., Stone, 1966, 1971a; Jeffery and Wingate, 2009; Callies and Ferrari, 2018b; Kang, 2023; Wang et al., 2026). For icy moon oceans in particular, Kang (2023) combined theoretical analysis and global ocean simulations to examine whether ocean heat transport driven by bottom heating can sustain the observed ice thickness variations on Enceladus and other icy moons. They showed that ice thickness variations on Enceladus generate strong meridional density gradients that drive baroclinic eddies, which dominate the meridional heat transport. The resulting equatorward heat transport efficiently redistributes heat from the poles toward lower latitudes, indicating that bottom heating alone cannot maintain the observed poleward-thinning ice shell geometry. This conclusion is consistent with the results presented in Chapter 2.

However, convection in icy moon oceans develops preferentially along the rotation axis rather than along the gravitational direction, as shown in Chapter 3. Previous studies either relied on under-resolved convection in global simulations, which likely underestimates the associated heat flux, or employed traditional convection parameterizations, which cannot represent the poleward heat transport by the slantwise convection in icy moon oceans, as shown in Chapter 4. This poleward heat transport is opposite to the equatorward heat

transport associated with baroclinic eddies. The relative importance of slantwise convection and baroclinic eddies in controlling heat transport therefore needs further examination.

In this chapter, we revisit the interaction between slantwise convection and baroclinic eddies using the slantwise convection parameterization developed in Chapter 5. For Enceladus, we assess how the conclusions of previous studies are modified when slantwise convection is more appropriately represented. For Europa, the equator-to-pole ice shell thickness variation remains poorly constrained. We therefore investigate the critical equator-to-pole ice thickness contrast that separates regimes where meridional heat transport is dominated by slantwise convection from those dominated by baroclinic eddies. Section 6.2 applies scaling laws to estimate this critical thickness contrast. Section 6.3 describes the numerical model configuration. Section 6.4 presents the simulation results. Section 6.5 discusses the implications for different icy moons. Section 6.6 provides discussions, and Section 6.7 provides concluding remarks.

6.2 Theoretical prediction

In this section, we apply scaling laws for heat transport by slantwise convection and baroclinic eddies to estimate the critical equator-to-pole ice thickness contrast at which the dominant heat transport mechanism transitions from slantwise convection to baroclinic eddies.

We start with the buoyancy transport by slantwise convection. In Chapter 4, we showed that the buoyancy flux by slantwise convection is directed between the gravitational direction and the rotation axis. Let ϕ_{SL} denote the angle between the heat flux direction and the local horizontal surface. Then

$$\frac{B_{y,SL}}{B_{r,SL}} = \cot \phi_{SL} \equiv k_{SL}, \quad (6.1)$$

where B is the buoyancy flux, subscripts y and r denote the meridional (latitudinal) and the

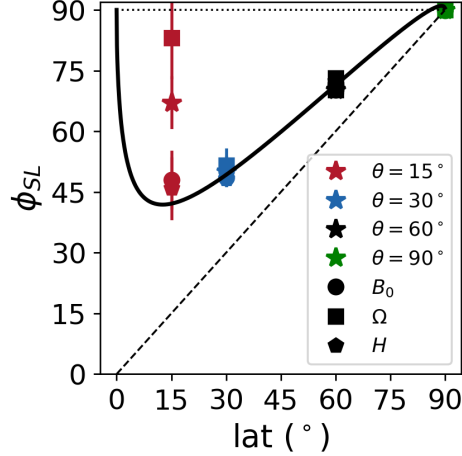


Figure 6.1: **Heat flux angle for slantwise convection.** Angle between the slantwise convective heat flux vector and the local horizontal surface, ϕ_{SL} , as a function of latitude. Markers show results from the convection-resolving simulations in Chapter 4: colors denote different latitudes, and marker shapes denote different sensitivity experiments. The black solid line shows the theoretical prediction from the scaling laws developed in Chapter 4. The black dashed line indicates the direction parallel to the rotation axis, and the black dotted line indicates the gravitational direction.

vertical (opposite to the gravitational) directions, respectively, and SL denotes heat transport by slantwise convection. In the convection-resolving simulations of Chapter 4, ϕ_{SL} ranges from 45° to 90° (Fig. 6.1), corresponding to $\cot \phi_{SL}$ values between 1 and 0. For scaling analysis presented here, we adopt $k_{SL} = 0.5$.

We then build a scaling law for heat transport by baroclinic eddies following previous work (Kang, 2023; Zhang et al., 2025b). The buoyancy transport by baroclinic eddies is largely adiabatic, which means the eddy buoyancy flux aligns with isopycnals (Vallis, 2017):

$$\frac{B_{r,BC}}{B_{y,BC}} = -\frac{\partial_y \bar{b}}{\partial_r \bar{b}} \equiv s, \quad (6.2)$$

where b is the buoyancy, s is the slope of the isopycnal, $\partial_y \equiv (1/r)(\partial/\partial\theta)$, $\partial_r \equiv \partial/\partial r$, r is radius, θ is latitude, the overline denotes zonal averages, and the subscript BC denotes contributions from baroclinic eddies. We assume that the meridional buoyancy flux can be expressed as an effective diffusive flux acting on the background buoyancy gradient:

$$B_{y,BC} = -\kappa_{BC}\partial_y\bar{b}, \quad (6.3)$$

where κ_{BC} is an effective baroclinic eddy diffusivity. Following derivations in Zhang et al. (2025b), based on Held and Larichev (1996), the effective diffusivity scales as (cf. Equations 16–21 in Zhang et al. (2025b))

$$\kappa_{BC} \sim k_{BC}(s|\partial_y\bar{b}|)^{3/2}\beta^{-2} \sim k_{BC}(s|\partial_y\bar{b}|)^{3/2}R^2(2\Omega)^{-2}, \quad (6.4)$$

where R is the planetary radius, Ω is the rotation rate, $\beta \sim 2\Omega/R$ is the meridional gradient of the planetary vorticity, and $k_{BC} = 0.25$ (Kang, 2022). With Equations 6.2–6.4, we have

$$B_{y,BC} \sim k_{BC}^{2/5}B_{r,BC}^{3/5}|\partial_y\bar{b}|R^{4/5}(2\Omega)^{-4/5} \sim k_{BC}^{2/5}B_{r,BC}^{3/5}\Delta bR^{-1/5}(2\Omega)^{-4/5}, \quad (6.5)$$

where Δb is the equator-to-pole buoyancy contrast so that $|\partial_y\bar{b}| \sim \Delta b/R$.

In the transition regime, we assume that the vertical heat transports by slantwise convection and baroclinic eddies are comparable, so that both are constrained by the buoyancy flux supplied from the core (B_{bot}),

$$B_{r,SL} \sim B_{r,BC} \sim B_{bot}. \quad (6.6)$$

As a result, the corresponding meridional buoyancy transports by slantwise convection and baroclinic eddies can be estimated as

$$B_{y,SL} \sim k_{SL}B_{bot}, \quad B_{y,BC} \sim k_{BC}^{2/5}B_{bot}^{3/5}\Delta bR^{-1/5}(2\Omega)^{-4/5}, \quad (6.7)$$

respectively. We further consider the condition under which the meridional buoyancy transports by slantwise convection and baroclinic eddies are equal, i.e., $B_{y,SL} = B_{y,BC}$. This defines a critical surface buoyancy contrast Δb_c at which the meridional buoyancy transport

transitions from a slantwise-convection-dominated regime to a baroclinic-eddy-dominated regime:

$$\Delta b_c \sim \frac{k_{SL}}{k_{BC}^{2/5}} B_{bot}^{2/5} (2\Omega)^{4/5} R^{1/5}. \quad (6.8)$$

The prefactor $k_{SL}/k_{BC}^{2/5} = 0.87$ is close to unity and is therefore neglected in the following analysis.

We now translate the buoyancy contrast to the equator-to-pole ice thickness contrast. For simplicity, we neglect the contribution of salinity to buoyancy, so that $B_{bot} = Q_{bot}g\alpha_T/(\rho c_p)$, and the surface buoyancy contrast is entirely due to the freezing point variation, $\Delta b = g\alpha_T\Delta T_f$, where Q_{bot} is the bottom heat flux, g is gravity, α_T is the thermal expansivity, $\rho = 1000 \text{ kg m}^{-3}$ is the density of the water, $c_p = 4000 \text{ J kg}^{-1} \text{ K}^{-1}$ is the heat capacity of the water, and ΔT_f is the equator-to-pole freezing point contrast at the ice-ocean interface. The freezing point contrast is associated with the ice thickness contrast:

$$|\Delta T_f| = k_p |\Delta p| = k_p \rho_i g |\Delta H_{ice}|, \quad (6.9)$$

where $k_p = 7.61 \times 10^{-8} \text{ K Pa}^{-1}$ and $\rho_i = 917 \text{ kg m}^{-3}$ is the density of the ice shell. As a result, the critical equator-to-pole ice thickness contrast is

$$\Delta H_{ice,c} \sim (k_p \rho_i)^{-1} \alpha_T^{-3/5} \left(\frac{Q_{bot}}{\rho c_p} \right)^{2/5} (2\Omega)^{4/5} R^{1/5} g^{-8/5}. \quad (6.10)$$

This scaling predicts that if the ice thickness contrast is larger than $\Delta H_{ice,c}$, baroclinic eddies dominate, while if the ice thickness contrast is smaller than $\Delta H_{ice,c}$, slantwise convection dominates the meridional heat transport. Note that if we fix the bulk density of the icy moon, we have approximately $g \propto R$, so that

$$\Delta H_{ice,c} \propto Q_{bot}^{2/5} \alpha_T^{-3/5} (2\Omega)^{4/5} R^{-7/5}, \quad (6.11)$$

suggesting that with the same ice thickness contrast, baroclinic eddies are more likely to dominate on moons with smaller bottom heat flux, larger thermal expansivity, slower planetary rotation, and larger planetary radius. This result is qualitatively consistent with the findings of Kang (2022), who showed that baroclinic eddies are more effective at smoothing ice thickness variations on larger icy moons.

6.3 Simulation design

We carry out numerical simulations using MITgcm with the slantwise convection parameterization developed in Chapter 5. Simulations are performed for two icy moons, Europa and Enceladus. At the ocean bottom, we apply a spatially varying heat flux that peaks at the poles, following the pattern of tidal dissipation (Beuthe, 2013, 2018, 2019), with different globally averaged heat fluxes for the two moons. We also account for the depth dependence of gravity because the ocean depth is not negligible compared with the planetary radius. The gravity profile is calculated assuming constant densities for both the ocean and the core and is scaled to match the prescribed surface gravity. Calculation details for the bottom heat flux and gravity profiles can be found in Appendix B of Zeng and Jansen (2021). At the ocean surface, temperature is restored toward the local freezing point using a piston velocity of $w_T = 9 \times 10^{-4} \text{ m s}^{-1}$ for Europa and $w_T = 6 \times 10^{-4} \text{ m s}^{-1}$ for Enceladus. No-normal-flow and free-slip boundary conditions are applied at both the upper and lower boundaries. To prevent long-term drift of the horizontal velocities, we include a linear bottom drag that restores the bottom horizontal velocity toward zero using a piston velocity of $w_u = 10^{-4} \text{ m s}^{-1}$.

For Europa, we adopt planetary parameters of $\alpha_T = 3 \times 10^{-4} \text{ K}^{-1}$ and $\Omega = 2.1 \times 10^{-5} \text{ s}^{-1}$. The ocean bottom and surface radii are $r_{bot} = 1431 \text{ km}$ and $r_{surf} = 1551 \text{ km}$, respectively, corresponding to an ocean depth of 120 km. The globally averaged bottom heat flux is 0.2 W m^{-2} , and the surface gravity is 1.3 m s^{-2} (Fig. 6.2a–b). Substituting these values

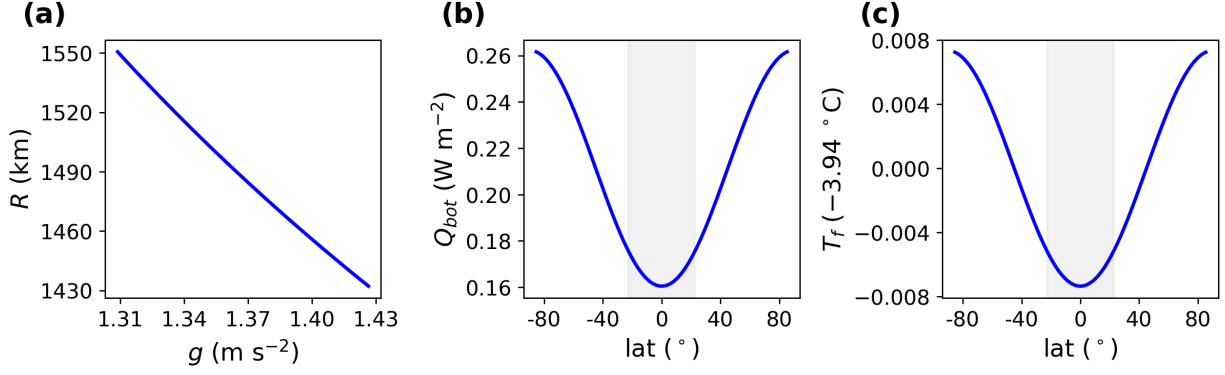


Figure 6.2: **Europa simulation setup.** Panel (a) shows the vertical gravity profile, panel (b) shows the bottom heat flux profile, and panel (c) shows the prescribed surface freezing point (relative to -3.94 °C) in the simulation with $\Delta H_{\text{ice}} = 150$ m. For simulations with $\Delta H_{\text{ice}} = 15$ m and 1.5 m, the amplitude of the freezing point variation decreases by factors of 10 and 100, respectively, while retaining the same spatial pattern. For the simulation with $\Delta H_{\text{ice}} = 0$ m, we set $T_f = -3.94$ °C everywhere. In panels (b) and (c), the gray shading indicates the region outside the tangent cylinder.

into Equation 6.10 yields a critical ice thickness contrast of $\Delta H_{\text{ice},c} \approx 8$ m. This corresponds to a very small ice shell slope, whose dynamical effect on the ocean circulation is expected to be negligible. In addition, such small surface height variations cannot be resolved with the vertical grid spacing that we can afford. We therefore employ a flat upper boundary in the ocean model and represent the effect of ice thickness variations solely through the prescribed surface freezing point distribution. To investigate the transition from slantwise-convection-dominated to baroclinic-eddy-dominated regimes, we perform simulations with $\Delta H_{\text{ice}} = 0, 1.5, 15,$ and 150 m, corresponding to freezing point contrasts of $\Delta T_f = 0, 1.47 \times 10^{-4}, 1.47 \times 10^{-3},$ and 1.47×10^{-2} K, respectively. We restore the surface temperature toward $T_s(\theta) = T_0 - \Delta T_f(\cos^2 \theta - 1/2)$, where $T_0 = -3.94$ °C is the mean freezing point (Fig. 6.2c).

For Enceladus, we adopt planetary parameters of $\alpha_T = 2 \times 10^{-4}$ K⁻¹ and $\Omega = 5.3 \times 10^{-5}$ s⁻¹. The ocean bottom and surface radii, gravity profile, and bottom heating profile are identical to those used in Chapter 2. Substituting these parameters into Equation 6.10 yields $\Delta H_{\text{ice},c} \approx 400$ m, which is substantially smaller than the observed equator-to-pole

ice thickness contrast of tens of kilometers. We therefore expect slantwise convection to play a negligible role compared to baroclinic eddies, and hence the choice of convection scheme has little influence on the simulated ocean circulation and heat transport. To test this prediction, we perform two simulations: one using the slantwise convection scheme and one using the traditional convection scheme. Both simulations use the same ice topography as in Chapter 2, with an equator-to-pole ice thickness contrast of approximately 9 km.

In all simulations, the vertical grid spacing is refined near the surface and bottom boundaries to better resolve boundary layers. For Enceladus-like simulations, the vertical resolution is 300 m near the ice-ocean interface, increases to about 2000 m in the ocean interior, and decreases to about 1000 m near the seafloor. For Europa-like simulations, the vertical grid spacing is three times larger than applied for Enceladus, reflecting its approximately three-fold greater ocean depth. The horizontal resolution is 0.25° for Europa-like simulations and is 0.5° for Enceladus-like simulations. The resolution cannot resolve all relevant dynamical process for Enceladus, and the impact of the resolution is discussed in Appendix 6.A. We apply background horizontal and vertical diffusivities of $\kappa_h = 4 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ and $\kappa_r = 5 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ for Europa, and $\kappa_h = 1 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ and $\kappa_r = 5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ for Enceladus. The corresponding viscosities are set to ten times these values to ensure numerical stability. To damp grid-scale numerical noise, we additionally apply a Smagorinsky viscosity in the horizontal direction. In the meridional direction, the model domain extends from 85.5°S to 85.5°N to avoid numerical singularities at the poles. In the zonal direction, we simulate only a 15° sector with periodic boundary conditions, corresponding to an assumed 24-fold symmetry, to reduce computational cost. All integrated quantities reported below are scaled to the full 360° domain. All simulations are integrated to a statistically steady state, in which the total horizontally integrated heat flux varies by less than 10%.

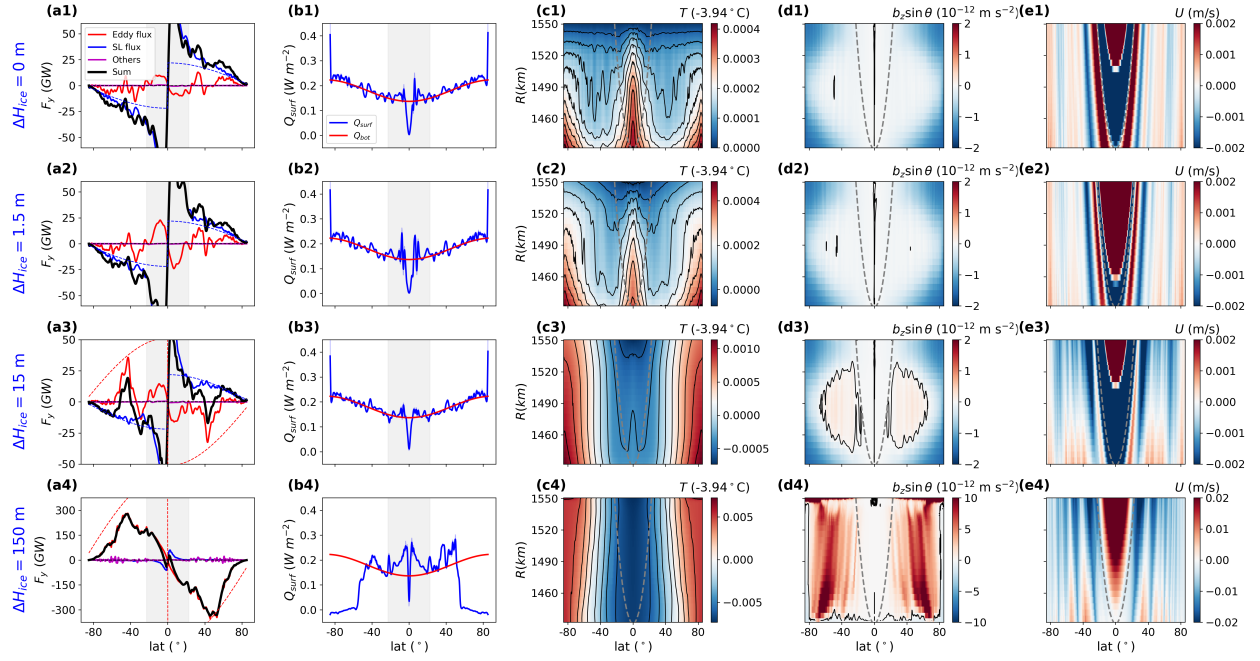


Figure 6.3: **Simulations for Europa-like oceans.** Panels (a) show the vertically integrated meridional heat transport. Solid lines indicate simulation results, where red lines indicate resolved eddy transport, blue lines indicate parameterized transport by slantwise convection, purple lines indicate transport by background diffusion and mean circulation, and black lines indicate the total transport (see Appendix 6.B for details). Dashed lines show the scaling estimates (Equation 6.7), with the red and blue lines representing baroclinic eddies and slantwise convection, respectively. Both scalings are divided by a factor of 5 to better match the simulation results. Panels (b) show the vertical heat flux at the ocean surface. Blue lines indicate the surface heat flux from model outputs, and red lines indicate the rescaled bottom heat flux, $Q_{bot}(r_{bot}/r_{surf})^2$, for comparison (when the two lines are the same, ocean heat flux is purely vertical). In panels (a) and (b), gray shadings indicate regions outside the tangent cylinder. In panels (a), the vertical axis range is chosen to better illustrate the meridional heat transport inside the tangent cylinder. The peak meridional heat transport occurs outside the tangent cylinder, although this may reflect an overestimation of the meridional heat transport by the slantwise convection scheme (see discussions in Section 6.4.1). In panels (b), darker lines indicate profiles smoothed using a 5-point running average, while fainter lines indicate the raw data. Panels (c) show the zonal-mean temperature $\bar{T}$. Panels (d) show the stability measure, $\partial\bar{b}/\partial z \sin \theta$, where positive and negative values indicate stable and unstable stratification along the rotation axis, respectively (Chapter 3). Panels (e) show the zonal-mean zonal velocity $\bar{U}$. The color scale is saturated outside the tangent cylinder to better illustrate the mid-latitude jets. The equatorial jet speeds are 0.023, 0.040, 0.020, and 0.065 m s^{-1} for the first through fourth rows, corresponding to simulations with $\Delta H_{ice} = 0, 1.5, 15, \text{ and } 150 \text{ m}$, respectively. Gray dashed lines in panels (c)-(e) indicate the tangent cylinder. In panels (d), black lines indicate the zero contour.

6.4 Results

6.4.1 Europa-like simulations

In simulations with Europa-like parameters, we observe a clear transition from a slantwise-convection-dominated regime to a baroclinic-eddy-dominated regime as we increase ΔH_{ice} (Fig. 6.3). In the simulations with $\Delta H_{ice} = 0$ m and 1.5 m, the results are largely similar. The meridional heat transport is dominated by slantwise convection (blue lines in Fig. 6.3a1 & a2), and its poleward heat transport results in a surface heat flux minimum near the equator and maxima near the poles (Fig. 6.3b1 & b2). The ocean stratification is unstable or close to neutral throughout most of the domain (Fig. 6.3d1 & d2), consistent with the dominance of slantwise convection. Both simulations exhibit a temperature maximum at the equator (Fig. 6.3c1 & c2), although it remains unclear whether this feature is physical or arises from limitations of the slantwise convection scheme in representing heat transport outside the tangent cylinder (see Chapter 5). The zonal flow exhibits a strong prograde jet at the equatorial surface, together with multiple alternating jets at mid-latitudes that are approximately barotropic along the rotation axis (Fig. 6.3e1 & e2). The only noticeable difference between these simulations is that the isopycnals are steeper near the surface in the $\Delta H_{ice} = 1.5$ m simulation than in the $\Delta H_{ice} = 0$ m simulation. This is because the imposed freezing point contrast in the $\Delta H_{ice} = 1.5$ m simulation (1.47×10^{-4} K) becomes comparable, though still smaller, to the temperature contrast generated by slantwise convection in the $\Delta H_{ice} = 0$ m case (4×10^{-4} K).

The simulation with $\Delta H_{ice} = 15$ m appears to represent the transitional regime. Inside the tangent cylinder, the meridional heat transport associated with slantwise convection and baroclinic eddies becomes comparable (compare the blue and red lines in Fig. 6.3a3), resulting in a weak net heat transport. The meridional heat transport within the tangent cylinder is equatorward and dominated by baroclinic eddies below about 60° , but poleward

and dominated by slantwise convection at higher latitudes. This transition occurs where the stability¹ changes sign: at lower latitudes, stable stratification suppresses slantwise convection, so that baroclinic eddies dominate; at higher latitudes, the fluid is unstable and slantwise convection dominates (Fig. 6.3d3). Outside the tangent cylinder, the meridional heat transport appears to remain dominated by the parameterized slantwise convection flux, although this may be due to the tendency of the scheme to overestimate meridional heat transport in this region (see Chapter 5). The isopycnals become substantially steeper as the meridional temperature gradient becomes larger than the vertical temperature gradient (Fig. 6.3c3). A strong prograde equatorial jet persists, but the mid-latitude jets exhibit stronger shear along the rotation axis, reflecting the increasing importance of thermal wind balance associated with the meridional buoyancy gradient (Vallis, 2017).

In the simulation with $\Delta H_{ice} = 150$ m, the meridional heat transport is dominated by baroclinic eddies and is substantially stronger than in the other three simulations (Fig. 6.3a4). As a result, the surface heat flux peaks near the equator rather than near the poles (Fig. 6.3b4). The isopycnals are nearly vertical throughout most of the domain, except for a strongly stratified layer near the polar surface (Fig. 6.3c4). Most of the ocean is stably stratified, with unstable regions confined to small areas near the poles (Fig. 6.3d4). The zonal jets exhibit clearer thermal wind shear, and both the temperature and velocity fields become smoother than in the other simulations because slantwise convection is largely suppressed throughout the domain, resulting in substantially less small-scale variability (Fig. 6.3e4).

Our scaling law predicts a transition at $\Delta H_{ice,c} \approx 8$ m, which is broadly consistent with the simulations, where the $\Delta H_{ice} = 15$ m case appears to represent the transition regime. It is nevertheless instructive to examine the scaling estimates derived in Section 6.2 in more detail. We compare the scaling estimates from Equation 6.7 with the simulation results. The

1. Here, stability is evaluated using the low-Rossby-number criterion derived in Chapter 3. In this limit, instability is determined by the buoyancy gradient along the rotation axis, $\partial b / \partial z \sin \theta < 0$, rather than by the gravitational stratification, $N^2 \equiv \partial b / \partial r < 0$. Thus, although N^2 is largely negative or close to zero everywhere (Fig. 6.3c3), regions with $\partial b / \partial z \sin \theta > 0$ can remain stable.

slantwise-convection scaling is not evaluated for the $\Delta H_{ice} = 150$ m simulation because the stratification is stable everywhere, and the baroclinic-eddy scaling is not evaluated for the $\Delta H_{ice} = 0$ m and 1.5 m simulations because the stratification is unstable everywhere. First, we note that both scalings overestimate the meridional heat transport. However, after dividing both scaling estimates by a factor of 5, the slantwise-convection scaling reproduces the meridional heat transport inside the tangent cylinder reasonably well, while the baroclinic-eddy scaling captures the peak meridional heat transport (dashed lines in Fig. 6.3a). The origin of this factor-of-five discrepancy remains unclear. For slantwise convection, it may arise because the scaling assumes a fixed heat flux angle, whereas the parameterization represents heat transport as anisotropic diffusion, making the direction of the heat flux sensitive to local buoyancy gradients. For baroclinic eddies, the adiabatic assumption used to relate vertical and meridional heat transport may not strictly apply. Despite this discrepancy, the fact that the required correction factors are similar for both scalings further suggests that the scaling framework successfully captures the leading-order transition between the slantwise-convection-dominated and baroclinic-eddy-dominated regimes.

6.4.2 *Enceladus-like simulations*

For Enceladus-like cases, the meridional heat transport is dominated by equatorward eddy heat transport, together with equatorward heat convergence associated with the mean circulation near the tangent cylinder, while the contributions from parameterized slantwise convection and background diffusion remain negligible (Fig. 6.4a). As a result, the surface heat flux peaks at lower latitudes and minimizes at the poles (Fig. 6.4b). The dense water formed near the equatorial surface sinks and spreads toward higher latitudes, producing a temperature structure in which warmer water generally overlies colder water (Fig. 6.4c). Consequently, most of the ocean remains stably stratified (Fig. 6.4d). Near the tangent cylinder, an overturning circulation develops, with dense equatorial surface water sinking

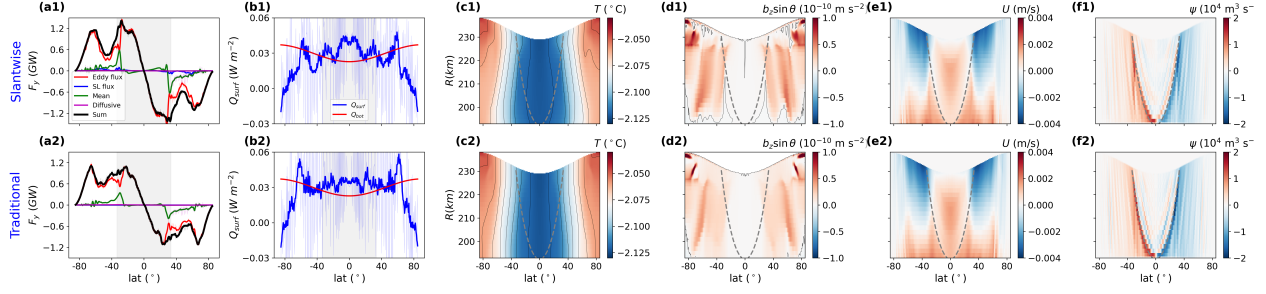


Figure 6.4: **Simulations for Enceladus-like oceans.** Panels (a)–(e) are the same as Fig. 6.3, except that panel (a) additionally separates the contributions from the mean circulation (green) and background diffusion (magenta), and the surface heat flux in (b) is shown after applying an 11-point running average. Panel (f) shows the stream function, calculated as $\psi = -2\pi \int_{r_{\text{bot}}}^{r_{\text{surf}}} \bar{v} r \cos \phi dr$, where $\bar{v}$ is the zonal-mean meridional velocity. Positive values indicate clockwise circulation, and negative values indicate counter-clockwise circulation. The streamfunction is smoothed in the meridional direction using a 5-point running average.

near the tangent cylinder and returning poleward at depth (Fig. 6.4f). Because the upper branch of this circulation is warmer than the lower branch and is equatorward, the resulting net heat transport is equatorward. The zonal flow exhibits prograde jets outside the tangent cylinder and clear thermal wind shear inside the tangent cylinder (Fig. 6.4e).

Simulations with the slantwise convection scheme and the traditional convection scheme produce very similar results, except that simulations with the slantwise convection scheme exhibit slightly stronger stratification and meridional heat transport than those with the traditional convection scheme. This similarity is consistent with the result that baroclinic eddies dominate heat transport on Enceladus, and our conclusions from Chapter 2 will not be substantially altered by replacing the traditional convection scheme with the slantwise convection scheme. Applying the baroclinic eddy scaling law together with the empirical prefactor estimated in Section 6.4.1 predicts a meridional heat transport of approximately 6.6 GW at 60° , which is about five times larger than that obtained in the simulations. This discrepancy likely reflects the limited resolution of the model, which underestimates the strength of baroclinic eddies and therefore their associated heat transport (see Appendix 6.A). Nevertheless, this quantitative underestimation does not affect the main conclusion. Both the

scaling analysis and the numerical simulations indicate that heat transport in Enceladus's ocean is equatorward and dominated by baroclinic eddies. Because the ocean remains nearly stably stratified throughout most of the domain, slantwise convection plays only a minor role in the overall heat budget. Consequently, the large-scale circulation and heat transport are largely insensitive to whether a traditional convection scheme or the slantwise convection scheme is employed.

6.5 Application to different icy moons

In this section, we apply the scaling law developed in Section 6.2 to estimate the critical equator-to-pole ice thickness contrast, $\Delta H_{ice,c}$, for different icy moons using the planetary parameters listed in Table 1.1. We rewrite Equation 6.10 as

$$\Delta H_{ice,c} = \frac{\Delta b_c}{\Delta b_{ice,1m}} = \frac{(Q_{bot} g \alpha_T \rho^{-1} c_p^{-1})^{2/5} (2\Omega)^{4/5} R^{1/5}}{k_p \rho_i \alpha_T g^2}, \quad (6.12)$$

where Δb_c is the critical equator-to-pole buoyancy contrast at which baroclinic eddies and slantwise convection contribute equally to the meridional heat transport, and $\Delta b_{ice,1m} = k_p \rho_i \alpha_T g^2$ is the buoyancy contrast created by a 1 m equator-to-pole ice thickness contrast through the pressure dependence of the freezing point. Figure 6.5 summarizes the predicted transition ice thickness contrast for different icy moons.

For Enceladus, we obtain $\Delta H_{ice,c} \approx 400\text{--}4000$ m. This value is substantially smaller than the inferred equator-to-pole ice thickness contrast of more than 20 km (Hemingway and Mittal, 2019). Enceladus therefore lies well within the baroclinic-eddy-dominated regime, consistent with both the numerical simulations presented in this chapter and previous studies (Kang, 2022, 2023; Zeng and Jansen, 2024). The strong equatorward heat convergence by baroclinic eddies therefore suggests that ocean heat transport alone is unlikely to maintain the observed poleward-thinning ice geometry on Enceladus.

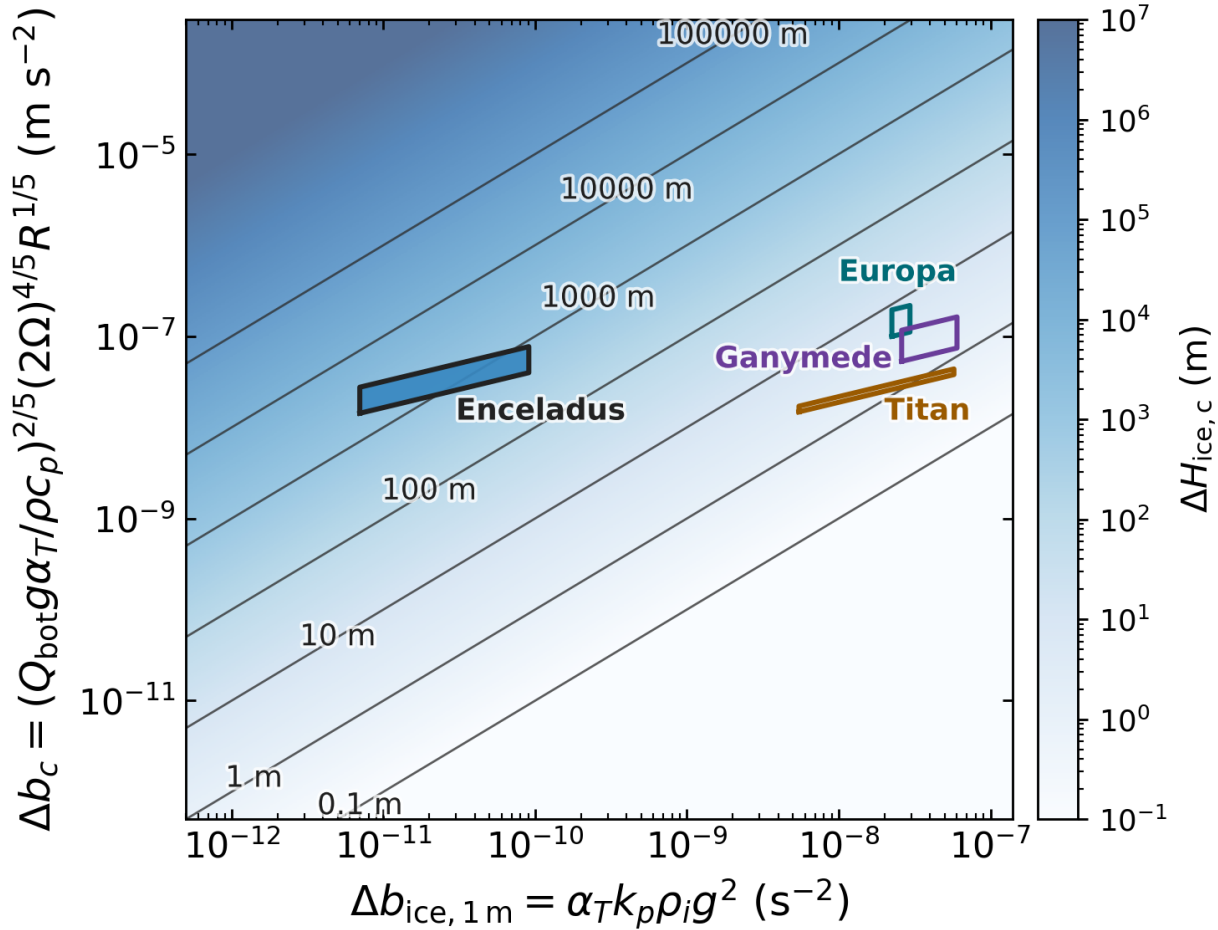


Figure 6.5: **Critical equator-to-pole ice thickness contrast for different icy moons.** The horizontal axis shows the buoyancy contrast produced by a 1 m ice thickness contrast, $\Delta b_{\text{ice},1\text{ m}}$, and the vertical axis shows the critical equator-to-pole buoyancy contrast, Δb_c , at which baroclinic eddies and slantwise convection are equally important for meridional heat transport (Equation 6.8). Background shading and contour lines indicate the corresponding critical ice thickness contrast, $\Delta H_{\text{ice},c} = \Delta b_c / \Delta b_{\text{ice},1\text{ m}}$. Boxes indicate the parameter ranges for different icy moons. The color shading within the Enceladus box indicates the inferred equator-to-pole ice thickness contrast from observations (Hemingway and Mittal, 2019).

For Europa, Titan, and Ganymede, the predicted transition ice thickness contrast is much smaller, typically of order 1–10 m. In the Europa simulations presented in Section 6.4.1, the transition occurs for ice thickness contrasts of the same order, providing additional support for the scaling analysis. Although the actual ice thickness contrasts on these moons remain poorly constrained, the results suggest that baroclinic eddies would dominate the meridional heat transport whenever equator-to-pole ice thickness variations exceed about 10 m.

6.6 Discussion

It is useful to compare the critical buoyancy contrast derived here with those obtained in previous studies. In Kang (2023) and Wang et al. (2026) (hereafter *KWL*), the transition between baroclinic-eddy-dominated and convection-dominated regimes is defined as the point at which convection can overcome the stable stratification established by baroclinic eddies. Specifically, the transition occurs when the isopycnal originating from the coldest surface water first reaches the ocean bottom. Under this definition, the critical buoyancy contrast is derived as

$$\Delta b_{c,KWL} \sim B_{bot}^{2/5} (2\Omega)^{4/5} R^{1/5} \frac{R}{H}, \quad (6.13)$$

where H is the ocean depth. We find that this scaling is similar to our result, differing by an additional factor of $R/H > 1$. This difference arises from the distinct transition criteria adopted. In *KWL*, the transition criterion is based on the onset of convection that overcomes the stable stratification established by baroclinic eddies. In contrast, our framework defines the transition as the point at which the meridional heat transport by slantwise convection becomes comparable to that by baroclinic eddies. Consequently, the transition in our study requires a weaker heat transport by baroclinic eddies, and therefore corresponds to a smaller critical buoyancy contrast than in *KWL*.

In the mixed layer of Earth’s ocean, Callies and Ferrari (2018b) considered the transition between baroclinic-eddy-dominated and convection-dominated regimes by comparing the destabilizing vertical buoyancy flux that drives convection with the vertical buoyancy flux associated with baroclinic eddies (Kempf et al., 2008; Callies and Ferrari, 2018a). They found a transition at approximately $B_{bot}/(2\Omega\Lambda^2H^2) \approx 0.1$, where $\Lambda = \partial u/\partial r$ is the vertical shear of the zonal flow. However, Callies and Ferrari (2018b) explored a parameter regime with $\Lambda/(2\Omega) = 1$, corresponding to order-unity shear relative to planetary rotation and therefore Rossby number (Ro) of order unity. This differs from the regime relevant to the interior of icy moon oceans, where $\text{Ro} \ll 1$. Therefore, the transition criterion derived in Callies and Ferrari (2018b) may not be directly applicable to the transition in icy moon oceans.

Besides scaling theories, previous studies have also examined the transition using linear instability analyses (e.g., Stone, 1966, 1971a; Jeffery and Wingate, 2009). In Stone (1966, 1971a), instability analyses on an f -plane showed that the dominance of baroclinic instability (relevant to baroclinic eddies) and symmetric instability (relevant to slantwise convection) depends on the Richardson number ($\text{Ri} \equiv N^2/\Lambda^2$, where $N^2 \equiv \partial b/\partial r$), with the transition occurring near $\text{Ri} \approx 1$. Jeffery and Wingate (2009) further extended this result to a tilted f -plane and showed that the transition Richardson number can be larger than unity with a positive shear ($\Lambda > 0$). These studies assume $N^2 > 0$ and examine symmetric instability associated with vertical shear. However, Chapter 3 showed that in the low-Rossby-number regime relevant to icy moon oceans, symmetric instability is controlled by the stratification along the rotation axis rather than by vertical shear. Therefore, the Richardson number criterion may not be directly applicable to the transition discussed in this study either.

We translate the critical buoyancy contrasts derived in *KWL* into the corresponding critical equator-to-pole ice thickness contrasts for different icy moons, as listed in Table 6.1. The values of $\Delta H_{ice,c}$ predicted by *KWL* are approximately an order of magnitude larger, reflecting the additional aspect-ratio factor R/H in the scaling. Nevertheless, the critical

Table 6.1: **Comparison of the critical ice thickness contrast in different studies.** Results are computed from the critical buoyancy contrasts given by Equation 6.8 for this study and Equation 6.13 for *KWL*, then converted to $\Delta H_{ice,c}$ using Equation 6.9. Planetary parameters are adopted from Table 1.1.

Moons	Enceladus	Titan	Europa	Ganymede
$\Delta H_{ice,c}$ (m)	439–3950	0.660–3.12	3.77–8.69	1.23–4.48
$\Delta H_{ice,c,KWL}$ (m)	1750–90500	4.04–88.2	44.9–140	6.26–491

ice thickness contrast remains small for Titan, Europa, and Ganymede. For Enceladus, the upper end of the *KWL* estimate exceeds the observed ice thickness contrast. However, this requires the lower-end estimates of both the thermal expansivity ($\alpha_T = 10^{-5} \text{ K}^{-1}$) and ocean depth ($H = 11 \text{ km}$). Despite these quantitative differences, both analyses lead to the same qualitative conclusion. Ocean heat transport alone is unlikely to maintain a large ice shell thickness variation on Titan, Europa, and Ganymede. For Enceladus, baroclinic eddies are likely to be the dominant contributor to meridional heat transport for most plausible ocean parameters, and therefore ocean heat transport is unlikely to be sufficient to maintain the observed poleward-thinning ice shell geometry.

6.7 Conclusion

In this chapter, we studied the transition between slantwise-convection-dominated and baroclinic-eddy-dominated regimes of meridional heat transport in icy moon oceans. Using scaling laws for heat transport by slantwise convection and baroclinic eddies, we derived a scaling for the critical equator-to-pole ice thickness contrast separating these two regimes (Equation 6.10 and Fig. 6.5). We then tested this scaling using global numerical simulations with the slantwise convection parameterization developed in Chapter 5.

For Enceladus, the inferred equator-to-pole ice thickness contrast exceeds 20 km (Hemingway and Mittal, 2019), substantially larger than the predicted transition thickness contrast of 400–4000 m. Both the scaling analysis and numerical simulations indicate that meridional

heat transport is dominated by baroclinic eddies and is directed equatorward. This result is consistent with previous studies (Kang, 2022, 2023; Zeng and Jansen, 2024) and further suggests that ocean heat transport alone is unlikely to maintain the observed ice shell topography on Enceladus. For Europa, Titan, and Ganymede, the predicted transition thickness contrast is only several meters. Slantwise convection can therefore dominate meridional heat transport only when ice thickness variations are very small. Once equator-to-pole ice thickness contrasts exceed about 10 m, baroclinic eddies are expected to dominate the meridional heat transport.

Throughout this chapter, we have assumed that the ice shell is thicker at lower latitudes than higher latitudes, so that the freezing point is lower at the equator and baroclinic eddies transport heat equatorward (Zhang et al., 2025b; Kang and Zhang, 2026). This configuration is consistent with the observed ice shell geometry of Enceladus and with theoretical expectations, because both tidal heating (Beuthe, 2013, 2018, 2019) and slantwise convection preferentially favor thinner polar ice shells, while ice flow and baroclinic eddies act to smooth out ice thickness gradients. Alternative scenarios involving reversed buoyancy gradients and the effect of ice-ocean interface slopes have been discussed in Zhang et al. (2025b). While these effects may modify the quantitative details of the circulation, we do not expect them to qualitatively alter the conclusions of this chapter.

In this study, we assume that buoyancy anomalies arise entirely from temperature variations and employ a linear equation of state. In reality, salinity variations may also contribute to the buoyancy contrast. In most cases, salinity effects are expected to reinforce the thermal buoyancy contrast because melting near regions of thin ice freshens the upper ocean and further reduces the density there. One exception may occur on Enceladus, where the thermal expansivity becomes negative at sufficiently low salinity (Zeng and Jansen, 2021). However, Chapter 2 suggests that even under the low-salinity scenario, the meridional heat transport on Enceladus remains dominated by baroclinic eddies and is directed equatorward, implying

that the qualitative conclusions of this chapter are unlikely to change because of salinity.

Although the scaling law developed here successfully predicts the transition between slantwise-convection-dominated and baroclinic-eddy-dominated regimes, it is less accurate in predicting the quantitative heat transport. Developing a more accurate quantitative theory for heat transport by both baroclinic eddies and slantwise convection is therefore an important direction for future work.

Appendix 6.A: Baroclinic eddy length scale

The characteristic length scale of the energy-containing baroclinic eddies is given by the β -scale (Vallis and Maltrud, 1993; Held and Larichev, 1996):

$$L_\beta \sim B_{r,BC}^{1/5} \beta^{-3/5} \sim B_{r,BC}^{1/5} (2\Omega)^{-3/5} R^{3/5}. \quad (6.14)$$

In our Enceladus-like simulations, the horizontal resolution is 0.5° , while $L_\beta \sim 0.28^\circ$. Consequently, baroclinic eddies, although present, are not properly resolved, suggesting that their contribution to meridional heat transport may be underestimated. This may also help explain the substantial overestimation of the meridional heat transport by the scaling law relative to the simulation results. For the Europa-like simulations, the horizontal resolution is 0.25° and $L_\beta \sim 0.58^\circ$, suggesting that the dominant baroclinic eddies are reasonably resolved.

Appendix 6.B: Decomposition of the meridional heat transport

By integrating the temperature equation (Vallis, 2017) in the zonal and vertical directions, the total meridional heat transport can be expressed as

$$F_y = \rho c_p \int_0^{2\pi} \int_{r_{bot}}^{r_{surf}} \overline{vT} r \cos \theta d\lambda dr - \rho c_p \int_0^{2\pi} \int_{r_{bot}}^{r_{surf}} \mathbf{e}_y \cdot (\boldsymbol{\kappa} \nabla \overline{T}) r \cos \theta d\lambda dr, \quad (6.15)$$

where λ is longitude, $\boldsymbol{\kappa} = \boldsymbol{\kappa}_b + \boldsymbol{\kappa}_{SL}$ is the total diffusivity tensor, consisting of the background diffusivity ($\boldsymbol{\kappa}_b$) and the parameterized diffusivity due to slantwise convection ($\boldsymbol{\kappa}_{SL}$), and the overline denotes a time and zonal average. The advection term can be decomposed as $\overline{vT} = \overline{v} \overline{T} + \overline{v'T'}$, where primes denote deviations from the average. Equation 6.15 can then be written as

$$\begin{aligned} F_y = & 2\pi \rho c_p \int_{r_{bot}}^{r_{surf}} \overline{v} \overline{T} r \cos \theta dr \\ & + 2\pi \rho c_p \int_{r_{bot}}^{r_{surf}} \overline{v'T'} r \cos \theta dr \\ & - 2\pi \rho c_p \int_{r_{bot}}^{r_{surf}} \mathbf{e}_y \cdot (\boldsymbol{\kappa}_{SL} \nabla \overline{T}) r \cos \theta dr \\ & - 2\pi \rho c_p \int_{r_{bot}}^{r_{surf}} \kappa_h \frac{\partial \overline{T}}{\partial \theta} \cos \theta dr, \end{aligned} \quad (6.16)$$

where κ_h is the background horizontal eddy diffusivity. The four terms on the right-hand side represent contributions from the mean circulation, resolved eddy heat flux, parameterized slantwise convection, and background horizontal diffusion, respectively.

CHAPTER 7

CONCLUSIONS

Oceans beneath the ice shells of icy moons are among the most promising environments in the search for life beyond Earth. However, observations of these subsurface oceans remain extremely limited because they are hidden beneath thick ice shells. One of the few observational constraints is the ice shell thickness distribution inferred from shape and gravity measurements, which is closely linked to ocean circulation and heat transport.

In this dissertation, we investigated ocean circulation and heat transport in icy moon oceans, with a particular focus on the role of slantwise convection and its interaction with baroclinic eddies. In Chapter 2, we examined the effects of salinity on ocean circulation and heat transport on Enceladus. Motivated by the limitations of traditional convection parameterizations used in Chapter 2 and previous work, we investigated the instability criterion and heat transport scaling laws of slantwise convection in Chapters 3 and 4. Using these theoretical results, we developed a slantwise convection parameterization for global ocean models in Chapter 5. Finally, in Chapter 6, we applied this parameterization to investigate the interaction between slantwise convection and baroclinic eddies and explored the implications for different icy moons.

Our main findings and takeaways are summarized below:

- Salinity strongly influences the stratification of Enceladus’s ocean. If the salinity is sufficiently low (below approximately 20 g kg^{-1}), a stably stratified layer forms near the surface. At higher salinity, bottom heating can drive convection throughout the ocean except in regions where stable stratification is maintained by baroclinic eddies. In either regime, baroclinic eddies transport heat equatorward, thereby acting to smooth the ice shell thickness gradient.
- Convection in icy moon oceans differs substantially from the upright convection com-

monly assumed in Earth ocean models. On icy moons, the buoyancy forcing is relatively weak and the ocean is deep, so the Rossby number is typically much smaller than unity. As a result, convection preferentially aligns with the planetary rotation axis, namely slantwise convection. In this regime, slantwise convection is triggered when the stratification becomes unstable along the rotation axis, corresponding to $\partial b / \partial z \sin \theta < 0$, where b is buoyancy, z is the direction parallel to the rotation axis, and θ is latitude.

- Slantwise convection transports heat both along and across convective plumes. Using convection-resolving simulations and scaling analysis, we developed scaling laws for these two components of heat transport. The resulting heat transport is both upward and poleward, opposite to the equatorward heat transport by baroclinic eddies on Enceladus.
- Traditional convection parameterizations cannot represent the meridional heat transport associated with slantwise convection. To address this limitation, we developed a slantwise convection parameterization for global icy moon ocean models. Benchmark simulations demonstrate that this scheme successfully reproduces the meridional heat transport in convection-resolving simulations. While the scheme reproduces the bulk temperature structure of the ocean interior, it does not accurately capture the detailed temperature distribution. This limitation is likely due to the absence of a proper boundary layer scheme. Despite this limitation, the scheme provides a practical representation of unresolved slantwise convection in global models and enables investigations of its interaction with other dynamical processes, including baroclinic eddies.
- Applying the parameterization and scaling laws to icy moon oceans reveals a transition between slantwise-convection-dominated and baroclinic-eddy-dominated regimes in meridional ocean heat transport. For Europa, Titan, and Ganymede, slantwise con-

vection dominates the meridional heat transport only when the equator-to-pole ice thickness contrast is less than $O(10 \text{ m})$. Larger ice thickness variations generate sufficiently strong baroclinic eddies to dominate the meridional heat transport, which tends to smooth the ice thickness gradient. These results suggest that ocean heat transport alone is unlikely to maintain large ice thickness contrasts on these moons. On Enceladus, the meridional heat transport is expected to be dominated by baroclinic eddies, and the resulting equatorward heat transport cannot maintain the observed ice shell geometry. If the present-day ice shell is in equilibrium, its thickness gradient is more likely to be maintained by spatial variations in tidal dissipation within the ice shell.

These results establish a framework for understanding convection and its interaction with other dynamical processes, such as baroclinic eddies. They also provide a pathway for connecting observations of ice shell structure to the underlying ocean circulation. As new measurements become available in the coming decades from missions such as *Europa Clipper* and *JUICE*, the theoretical and numerical tools developed in this dissertation can help interpret observed ice shell thickness distributions and constrain the role of ocean heat transport in shaping them. The slantwise convection parameterization developed here may also be extended to the transport of chemical tracers, providing a useful tool for assessing the habitability of icy moon oceans.

Several important directions remain for future work. First, the parameterization developed here primarily represents heat transport by slantwise convection inside the tangent cylinder, whereas heat transport outside the tangent cylinder is associated with equatorial rolls and remains less well understood. Future studies should investigate the dynamics of these equatorial rolls and develop corresponding scaling laws and parameterizations. Second, the current scheme does not explicitly represent boundary layer processes, which may play an important role in setting temperature and tracer distributions in the ocean. Developing a complementary boundary layer parameterization would therefore improve the representa-

tion of ocean heat and tracer transports. Third, the slantwise convection scheme developed in this dissertation accounts only for heat transport and neglects momentum transport. Convection-driven momentum transport may contribute to nonsynchronous rotation of the ice shell, which provides another potential observational constraint of ocean circulation on icy moons (Ashkenazy et al., 2023; Kang, 2024). Extending the parameterization to include momentum transport is therefore an important direction for future work. Lastly, the results presented here focus primarily on ocean heat transport. A complete understanding of icy moon evolution will require fully coupled models that incorporate ocean circulation, ice shell dynamics, and seafloor processes. Developing such coupled models is therefore an important next step toward understanding the long-term evolution and habitability of icy moon oceans.

REFERENCES

- Alistair Adcroft, Campin Jean-Michel, Dutkiewicz Stephanie, Evangelinos Constantinos, Ferreira David, Forget Gael, Fox-Kemper Baylor, Heimbach Patrick, Hill Chris, Hill Helen, et al. MITgcm user manual. Technical report, Online documentation: <https://mitgcm.readthedocs.io/en/latest/>, 2018.
- Flynn Ames, David Ferreira, Arnaud Czaja, and Adam Masters. Ocean stratification impedes particulate transport to the plumes of Enceladus. *Communications Earth & Environment*, 6(1):63, 2025. doi:10.1038/s43247-025-02036-3.
- Hagay Amit, Gaël Choblet, Gabriel Tobie, Filipe Terra-Nova, Ondřej Čadek, and Mathieu Bouffard. Cooling patterns in rotating thin spherical shells—application to Titan’s subsurface ocean. *Icarus*, 338:113509, 2020. doi:10.1016/j.icarus.2019.113509.
- A Arakawa. Computational design of the basic dynamical processes of the UCLA general circulation model. methods in computational physics. *Advances in Research and Application, Vol. 17: General circulation models of the atmosphere*, 337, 1977.
- Yosef Ashkenazy and Eli Tziperman. Dynamic Europa ocean shows transient taylor columns and convection driven by ice melting and salinity. *Nature communications*, 12(1):6376, 2021. doi:10.1038/s41467-021-26710-0.
- Yosef Ashkenazy, Eli Tziperman, and Francis Nimmo. Non-synchronous rotation on Europa driven by ocean currents. *AGU Advances*, 4(3):e2022AV000849, 2023.
- Jonathan Aurnou, Moritz Heimpel, and Johannes Wicht. The effects of vigorous mixing in a convective model of zonal flow on the ice giants. *Icarus*, 190(1):110–126, 2007.
- Jonathan Aurnou, Moritz Heimpel, Lorraine Allen, Eric King, and Johannes Wicht. Convective heat transfer and the pattern of thermal emission on the gas giants. *Geophysical Journal International*, 173(3):793–801, 2008. doi:10.1111/j.1365-246X.2008.03764.x.
- Jonathan M Aurnou, Susanne Horn, and Keith Julien. Connections between nonrotating, slowly rotating, and rapidly rotating turbulent convection transport scalings. *Physical Review Research*, 2(4):043115, 2020. doi:10.1103/PhysRevResearch.2.043115.
- Mikael Beuthe. Spatial patterns of tidal heating. *Icarus*, 223(1):308–329, 2013.
- Mikael Beuthe. Crustal control of dissipative ocean tides in Enceladus and other icy moons. *Icarus*, 280:278–299, 2016.
- Mikael Beuthe. Enceladus’s crust as a non-uniform thin shell: I tidal deformations. *Icarus*, 302:145–174, 2018.
- Mikael Beuthe. Enceladus’s crust as a non-uniform thin shell: II tidal dissipation. *Icarus*, 332:66–91, 2019.

- Mikael Beuthe, Attilio Rivoldini, and Antony Trinh. Enceladus’s and Dione’s floating ice shells supported by minimum stress isostasy. *Geophysical Research Letters*, 43(19):10–088, 2016. doi:10.1002/2016GL070650.
- Suyash Bire, Wanying Kang, Ali Ramadhan, Jean-Michel Campin, and John Marshall. Exploring ocean circulation on icy moons heated from below. *Journal of Geophysical Research: Planets*, 127(3):e2021JE007025, 2022. doi:10.1029/2021JE007025.
- Suyash Bire, Tushar Mittal, Wanying Kang, Ali Ramadhan, Philip J Tuckman, Christopher R German, Andreas M Thurnherr, and John Marshall. Divergent behavior of hydrothermal plumes in fresh versus salty icy ocean worlds. *Journal of Geophysical Research: Planets*, 128(11):e2023JE007740, 2023. doi:10.1029/2023JE007740.
- Mathieu Bouffard, Gaël Choblet, Hagay Amit, Gabriel Tobie, Ondřej Čadek, and Filipe Terra-Nova. Seafloor hydrothermal control over ocean dynamics in Enceladus. *Nature Astronomy*, pages 1–8, 2025. doi:10.1038/s41550-025-02490-1.
- John C Bowman and Theodore G Shepherd. Nonlinear symmetric stability of planetary atmospheres. *Journal of Fluid Mechanics*, 296:391–407, 1995.
- Keaton J Burns, Geoffrey M Vasil, Jeffrey S Oishi, Daniel Lecoanet, and Benjamin P Brown. Dedalus: A flexible framework for numerical simulations with spectral methods. *Physical Review Research*, 2(2):023068, 2020.
- F. H. Busse. Thermal instabilities in rapidly rotating systems. *Journal of Fluid Mechanics*, 44(3):441–460, November 1970. ISSN 0022-1120, 1469-7645. doi:10.1017/S0022112070001921. URL https://www.cambridge.org/core/product/identifier/S0022112070001921/type/journal_article.
- Friedrich H Busse and LL Hood. Differential rotation driven by convection in a rapidly rotating annulus. *Geophysical & Astrophysical Fluid Dynamics*, 21(1-2):59–74, 1982.
- Friedrich H Busse and AC Or. Convection in a rotating cylindrical annulus: thermal Rossby waves. *Journal of Fluid Mechanics*, 166:173–187, 1986.
- Simon Cabanes, Thomas Gastine, and Alexandre Fournier. Zonostrophic turbulence in the subsurface oceans of the Jovian and Saturnian moons. *Icarus*, 415:116047, 2024. doi:10.1016/j.icarus.2024.116047.
- Ondřej Čadek, Gabriel Tobie, Tim Van Hoolst, Marion Massé, Gaël Choblet, Axel Lefèvre, Giuseppe Mitri, Rose-Marie Baland, Marie Běhouňková, Olivier Bourgeois, et al. Enceladus’s internal ocean and ice shell constrained from Cassini gravity, shape, and libration data. *Geophysical Research Letters*, 43(11):5653–5660, 2016.
- Ondřej Čadek, Ondřej Souček, Marie Běhouňková, Gaël Choblet, Gabriel Tobie, and Jaroslav Hron. Long-term stability of Enceladus’ uneven ice shell. *Icarus*, 319:476–484, 2019. doi:10.1016/j.icarus.2018.10.003.

- Jörn Callies and Raffaele Ferrari. Note on the rate of restratification in the baroclinic spindown of fronts. *Journal of Physical Oceanography*, 48(7):1543–1553, 2018a.
- Jörn Callies and Raffaele Ferrari. Baroclinic Instability in the Presence of Convection. *Journal of Physical Oceanography*, 48(1):45–60, January 2018b. ISSN 0022-3670, 1520-0485. doi:10.1175/JPO-D-17-0028.1. URL <https://journals.ametsoc.org/view/journals/phoc/48/1/jpo-d-17-0028.1.xml>.
- EMA Chen and F Nimmo. Obliquity tides do not significantly heat Enceladus. *Icarus*, 214(2):779–781, 2011.
- HR Cho, TG Shepherd, and VA Vladimirov. Application of the direct liapunov method to the problem of symmetric stability in the atmosphere. *Journal of the atmospheric sciences*, 50(6):822–836, 1993.
- Gaël Choblet, Gabriel Tobie, Christophe Sotin, Marie Běhouňková, Ondřej Čadek, Frank Postberg, and Ondřej Souček. Powering prolonged hydrothermal activity inside Enceladus. *Nature Astronomy*, 1(12):841–847, 2017. doi:10.1038/s41550-017-0289-8.
- Ulrich R Christensen. Zonal flow driven by strongly supercritical convection in rotating spherical shells. *Journal of Fluid Mechanics*, 470:115–133, 2002.
- Ulrich R Christensen and Julien Aubert. Scaling properties of convection-driven dynamos in rotating spherical shells and application to planetary magnetic fields. *Geophysical Journal International*, 166(1):97–114, 2006.
- Laura K Currie, Adrian J Barker, Yoram Lithwick, and Matthew K Browning. Convection with misaligned gravity and rotation: simulations and rotating mixing length theory. *Monthly Notices of the Royal Astronomical Society*, 493(4):5233–5256, 2020. doi:10.1093/mnras/staa372.
- Robert E Ecke and Olga Shishkina. Turbulent rotating Rayleigh–Bénard convection. *Annual Review of Fluid Mechanics*, 55(1):603–638, 2023. doi:10.1146/annurev-fluid-120720-020446.
- Arnt Eliassen. Slow thermally or frictionally controlled meridional circulation in a circular vortex. *Astrofysica Norvegica*, v. 5, p. 19, 5:19, 1951.
- Kerry A Emanuel. The lagrangian parcel dynamics of moist symmetric instability. *Journal of Atmospheric Sciences*, 40(10):2368–2376, 1983.
- Kerry A Emanuel. Convective adjustment in baroclinic atmospheres. *The Jovian Atmospheres*, 2441:163–171, 1985.
- Wei Fan, Qi Wang, and Yufeng Lin. Scaling behaviour of rotating convection in a spherical shell with different Prandtl numbers. *Journal of Fluid Mechanics*, 998:A20, 2024. doi:10.1017/jfm.2024.893.

- Harindra JS Fernando, Don L Boyer, and Rui-Rong Chen. Turbulent thermal convection in rotating and stratified fluids. *Dynamics of atmospheres and oceans*, 13(1-2):95–121, 1989. doi:10.1016/0377-0265(89)90035-3.
- F. Michael Flasar and Peter J. Gierasch. Turbulent convection within rapidly rotating superadiabatic fluids with horizontal temperature gradients. *Geophysical & Astrophysical Fluid Dynamics*, 10(1):175–212, January 1978. ISSN 0309-1929, 1029-0419. doi:10.1080/03091927808242636. URL <http://www.tandfonline.com/doi/abs/10.1080/03091927808242636>.
- Mark D. Fruman and Theodore G. Shepherd. Symmetric Stability of Compressible Zonal Flows on a Generalized Equatorial β Plane. *Journal of the Atmospheric Sciences*, 65(6):1927–1940, June 2008. ISSN 1520-0469, 0022-4928. doi:10.1175/2007JAS2582.1. URL <https://journals.ametsoc.org/doi/10.1175/2007JAS2582.1>.
- Eli Galanti, Yohai Kaspi, Yamila Miguel, Tristan Guillot, Daniele Durante, Paolo Racioppa, and Luciano Iess. Saturn’s deep atmospheric flows revealed by the Cassini grand finale gravity measurements. *Geophysical Research Letters*, 46(2):616–624, 2019.
- Thomas Gastine and Jonathan M Aurnou. Latitudinal regionalization of rotating spherical shell convection. *Journal of Fluid Mechanics*, 954:R1, 2023. doi:10.1017/jfm.2022.1010.
- Thomas Gastine, Johannes Wicht, and Julien Aubert. Scaling regimes in spherical shell rotating convection. *Journal of Fluid Mechanics*, 808:690–732, 2016. doi:10.1017/jfm.2016.659.
- T Gerkema, JTF Zimmerman, LRM Maas, and H Van Haren. Geophysical and astrophysical fluid dynamics beyond the traditional approximation. *Reviews of Geophysics*, 46(2), 2008.
- CR Glein, F Postberg, and SD Vance. The geochemistry of Enceladus: Composition and controls. *Enceladus and the icy moons of Saturn*, 39, 2018.
- Jason C Goodman and Erik Lenferink. Numerical simulations of marine hydrothermal plumes for Europa and other icy worlds. *Icarus*, 221(2):970–983, 2012. doi:10.1016/j.icarus.2012.08.027.
- Jason C Goodman, Geoffrey C Collins, John Marshall, and Raymond T Pierrehumbert. Hydrothermal plume dynamics on Europa: Implications for chaos formation. *Journal of Geophysical Research: Planets*, 109(E3), 2004. doi:10.1029/2003JE002073.
- Céline Guervilly and Philippe Cardin. Multiple zonal jets and convective heat transport barriers in a quasi-geostrophic model of planetary cores. *Geophysical Journal International*, 211(1):455–471, 2017. doi:10.1093/gji/ggx315.
- Thomas W. N. Haine and John Marshall. Gravitational, Symmetric, and Baroclinic Instability of the Ocean Mixed Layer. *Journal of Physical Oceanography*, 28(4):634–658, April 1998. ISSN 0022-3670, 1520-0485. doi:10.1175/1520-0485(1998)028<0634:GSABIO>2.0.CO;2. URL [http://journals.ametsoc.org/doi/10.1175/1520-0485\(1998\)028<0634:GSABIO>2.0.CO;2](http://journals.ametsoc.org/doi/10.1175/1520-0485(1998)028<0634:GSABIO>2.0.CO;2).

- Robert Hartmann, Richard JAM Stevens, Detlef Lohse, and Roberto Verzicco. Toward understanding polar heat transport enhancement in subglacial oceans on icy moons. *Geophysical Research Letters*, 51(3):e2023GL105401, 2024. doi:10.1029/2023GL105401.
- David H Hathaway and Richard CJ Somerville. Three-dimensional simulations of convection in layers with tilted rotation vectors. *Journal of Fluid Mechanics*, 126:75–89, 1983. doi:10.1017/S0022112083000051.
- David H. Hathaway, Peter A. Gilman, and Juri Toomre. Convective instability when the temperature gradient and rotation vector are oblique to gravity. I. Fluids without diffusion. *Geophysical & Astrophysical Fluid Dynamics*, 13(1):289–316, January 1979. ISSN 0309-1929, 1029-0419. doi:10.1080/03091927908243778. URL <http://www.tandfonline.com/doi/abs/10.1080/03091927908243778>.
- David H Hathaway, Juri Toomre, and Peter A Gilman. Convective instability when the temperature gradient and rotation vector are oblique to gravity. II. Real fluids with effects of diffusion. *Geophysical & Astrophysical Fluid Dynamics*, 15(1):7–37, 1980. doi:10.1080/03091928008241168.
- Hamish CFC Hay and Isamu Matsuyama. Numerically modelling tidal dissipation with bottom drag in the oceans of Titan and Enceladus. *Icarus*, 281:342–356, 2017.
- Hamish CFC Hay and Isamu Matsuyama. Nonlinear tidal dissipation in the subsurface oceans of Enceladus and other icy satellites. *Icarus*, 319:68–85, 2019.
- Moritz Heimpel, Thomas Gastine, and Johannes Wicht. Simulation of deep-seated zonal jets and shallow vortices in gas giant atmospheres. *Nature Geoscience*, 9(1):19–23, 2016.
- Isaac M Held and Vitaly D Larichev. A scaling theory for horizontally homogeneous, baroclinically unstable flow on a beta plane. *Journal of Atmospheric Sciences*, 53(7):946–952, 1996.
- Douglas J Hemingway and Tushar Mittal. Enceladus’s ice shell structure as a window on internal heat production. *Icarus*, 332:111–131, 2019. doi:10.1016/j.icarus.2019.03.011.
- Jon K Hillier, SF Green, N McBride, JP Schwanethal, F Postberg, R Srama, S Kempf, G Moragas-Klostermeyer, JAM McDonnell, and E Grün. The composition of Saturn’s E ring. *Monthly notices of the royal astronomical society*, 377(4):1588–1596, 2007.
- Darryl D Holm, Jerrold E Marsden, Tudor Ratiu, and Alan Weinstein. Nonlinear stability of fluid and plasma equilibria. *Physics reports*, 123(1-2):1–116, 1985.
- Nils G Holm and Eva Andersson. Hydrothermal simulation experiments as a tool for studies of the origin of life on earth and other terrestrial planets: a review. *Astrobiology*, 5(4):444–460, 2005.

- B. J. Hoskins. The role of potential vorticity in symmetric stability and instability. *Quarterly Journal of the Royal Meteorological Society*, 100(425):480–482, July 1974. ISSN 0035-9009, 1477-870X. doi:10.1002/qj.49710042520. URL <https://rmets.onlinelibrary.wiley.com/doi/10.1002/qj.49710042520>.
- CJA Howett, JR Spencer, J Pearl, and M Segura. High heat flow from Enceladus’ south polar region measured using 10–600 cm⁻¹ Cassini/cirs data. *Journal of Geophysical Research: Planets*, 116(E3), 2011.
- Hsiang-Wen Hsu, Frank Postberg, Yasuhito Sekine, Takazo Shibuya, Sascha Kempf, Mihály Horányi, Antal Juhász, Nicolas Altobelli, Katsuhiko Suzuki, Yuka Masaki, et al. Ongoing hydrothermal activities within Enceladus. *Nature*, 519(7542):207–210, 2015. doi:10.1038/nature14262.
- Einar Høiland. On the stability of the circular vortex. *Avhandlingar Norske Videnskaps-Akademi i Oslo, I. Matematisk-Naturvitenskapelig Klasse*, 1(11):1–24, 1941.
- Luciano Iess, Robert A Jacobson, Marco Ducci, David J Stevenson, Jonathan I Lunine, John W Armstrong, Sami W Asmar, Paolo Racioppa, Nicole J Rappaport, and Paolo Tortora. The tides of Titan. *Science*, 337(6093):457–459, 2012. doi:10.1126/science.1219631.
- Luciano Iess, DJ Stevenson, M Parisi, D Hemingway, RA Jacobson, JI Lunine, F Nimmo, JW Armstrong, SW Asmar, M Ducci, et al. The gravity field and interior structure of Enceladus. *Science*, 344(6179):78–80, 2014.
- Andrew P Ingersoll and Miki Nakajima. Controlled boiling on Enceladus. 2. model of the liquid-filled cracks. *Icarus*, 272:319–326, 2016.
- Toshihisa Itano and Kiyoshi Maruyama. Symmetric Stability of Zonal Flow under Full-Component Coriolis Force -Effect of the Horizontal Component of the Planetary Vorticity-. *Journal of the Meteorological Society of Japan. Ser. II*, 87(4):747–753, 2009. ISSN 0026-1165, 2186-9057. doi:10.2151/jmsj.87.747. URL http://www.jstage.jst.go.jp/article/jmsj/87/4/87_4_747/_article.
- David R Jackett and Trevor J McDougall. Minimal adjustment of hydrographic profiles to achieve static stability. *Journal of Atmospheric and Oceanic Technology*, 12(2):381–389, 1995. doi:10.1175/1520-0426(1995)012<0381:MAOHPT>2.0.CO;2.
- Malte F Jansen, Wanying Kang, Edwin S Kite, and Yaoxuan Zeng. Energetic constraints on ocean circulations of icy ocean worlds. *The Planetary Science Journal*, 4(6):117, 2023.
- Nicole Jeffery and Beth Wingate. The Effect of Tilted Rotation on Shear Instabilities at Low Stratifications. *Journal of Physical Oceanography*, 39(12):3147–3161, December 2009. ISSN 1520-0485, 0022-3670. doi:10.1175/2009JPO4138.1. URL <http://journals.ametsoc.org/doi/10.1175/2009JP04138.1>.

- Helen Jones and John Marshall. Convection with rotation in a neutral ocean: A study of open-ocean deep convection. *Journal of Physical Oceanography*, 23(6):1009–1039, 1993. doi:10.1175/1520-0485(1993)023<1009:CWRIAN>2.0.CO;2.
- Keith Julien, Edgar Knobloch, Antonio M Rubio, and Geoffrey M Vasil. Heat transport in low-Rossby-number Rayleigh-Bénard convection. *Physical review letters*, 109(25):254503, 2012. doi:10.1103/PhysRevLett.109.254503.
- Keith Julien, Edgar Knobloch, and Meredith Plumley. Impact of domain anisotropy on the inverse cascade in geostrophic turbulent convection. *Journal of Fluid Mechanics*, 837:R4, 2018. doi:10.1017/jfm.2017.894.
- Wanying Kang. Different ice-shell geometries on Europa and Enceladus due to their different sizes: Impacts of ocean heat transport. *The Astrophysical Journal*, 934(2):116, 2022. doi:10.3847/1538-4357/ac779c.
- Wanying Kang. The modulation effect of ice thickness variations on convection in icy ocean worlds. *Monthly Notices of the Royal Astronomical Society*, 525(4):5251–5261, 2023. doi:10.1093/mnras/stad2638.
- Wanying Kang. Nonsynchronous rotation of icy moon ice shells: the thermal wind perspective. *Science Advances*, 10(4):eadk2277, 2024.
- Wanying Kang and Glenn Flierl. Spontaneous formation of geysers at only one pole on Enceladus’s ice shell. *Proceedings of the National Academy of Sciences*, 117(26):14764–14768, 2020.
- Wanying Kang and Malte Jansen. On icy ocean worlds, size controls ice shell geometry. *The Astrophysical Journal*, 935(2):103, 2022. doi:10.3847/1538-4357/ac7a32.
- Wanying Kang and Yixiao Zhang. Subsurface ocean salinity and dissipation rate inferred from Enceladus ice shell morphology. *arXiv preprint arXiv:2603.22602*, 2026.
- Wanying Kang, Suyash Bire, Jean-Michel Campin, Christophe Sotin, Christopher German, Andreas Thurnherr, and John Marshall. Differing enceladean ocean circulation and ice shell geometries driven by tidal heating in the ice versus the core. *arXiv preprint arXiv:2008.03764*, 2020.
- Wanying Kang, Suyash Bire, and John Marshall. The role of ocean circulation in driving hemispheric symmetry breaking of the ice shell of Enceladus. *Earth and Planetary Science Letters*, 599:117845, 2022a. ISSN 0012-821X. doi:https://doi.org/10.1016/j.epsl.2022.117845. URL <https://www.sciencedirect.com/science/article/pii/S0012821X22004812>.
- Wanying Kang, John Marshall, Tushar Mittal, and Suyash Bire. Ocean dynamics and tracer transport over the south pole geysers of Enceladus. *Monthly Notices of the Royal Astronomical Society*, 517(3):3485–3494, 2022b. doi:10.1093/mnras/stac2882.

- Wanying Kang, Tushar Mittal, Suyash Bire, Jean-Michel Campin, and John Marshall. How does salinity shape ocean circulation and ice geometry on Enceladus and other icy satellites? *Science advances*, 8(29):eabm4665, 2022c. doi:10.1126/sciadv.abm4665.
- Yohai Kaspi, Glenn R Flierl, and Adam P Showman. The deep wind structure of the giant planets: Results from an anelastic general circulation model. *Icarus*, 202(2):525–542, 2009.
- Yohai Kaspi, E Galanti, William B Hubbard, DJ Stevenson, SJ Bolton, L Iess, T Guillot, J Bloxham, JEP Connerney, H Cao, et al. Jupiter’s atmospheric jet streams extend thousands of kilometres deep. *Nature*, 555(7695):223–226, 2018.
- Yohai Kaspi, Eli Galanti, RS Park, K Duer, N Gavriel, D Durante, L Iess, M Parisi, DR Bucino, T Guillot, et al. Observational evidence for cylindrically oriented zonal flows on Jupiter. *Nature Astronomy*, 7(12):1463–1472, 2023.
- Sascha Kempf, Uwe Beckmann, Georg Moragas-Klostermeyer, Frank Postberg, Ralf Srama, Thanasis Economou, Jürgen Schmidt, Frank Spahn, and Eberhard Grün. The E ring in the vicinity of Enceladus: I. spatial distribution and properties of the ring particles. *Icarus*, 193(2):420–437, 2008.
- Nozair Khawaja, Frank Postberg, Thomas R O’Sullivan, Maryse Napoleoni, Sascha Kempf, Fabian Klenner, Yasuhito Sekine, Maxwell Craddock, Jon Hillier, Jonas Simolka, et al. Detection of organic compounds in freshly ejected ice grains from Enceladus’s ocean. *Nature Astronomy*, pages 1–10, 2025.
- KK Khurana, MG Kivelson, DJ Stevenson, G Schubert, CT Russell, RJ Walker, and C Polanskey. Induced magnetic fields as evidence for subsurface oceans in Europa and Callisto. *Nature*, 395(6704):777–780, 1998. doi:10.1038/27394.
- Edwin S Kite and Eric B Ford. Habitability of exoplanet waterworlds. *The Astrophysical Journal*, 864(1):75, 2018.
- Edwin S Kite and Allan M Rubin. Sustained eruptions on Enceladus explained by turbulent dissipation in tiger stripes. *Proceedings of the National Academy of Sciences*, 113(15):3972–3975, 2016.
- MG Kivelson, KK Khurana, and M Volwerk. The permanent and inductive magnetic moments of Ganymede. *Icarus*, 157(2):507–522, 2002. doi:10.1006/icar.2002.6834.
- Marc J Kuchner. Volatile-rich earth-mass planets in the habitable zone. *The Astrophysical Journal Letters*, 596(1):L105–L108, 2003.
- Jakub Kvorka and Ondřej Čadež. A numerical model of convective heat transfer in Titan’s subsurface ocean. *Icarus*, 376:114853, 2022. doi:10.1016/j.icarus.2021.114853.
- Yanhong Lai and Jun Yang. Thermocline depth on water-rich exoplanets. *The Astrophysical Journal*, 933(2):152, 2022.

- Alain Léger, Franck Selsis, Christophe Sotin, Tristan Guillot, Didier Despois, D Mawet, Marc Ollivier, A Labèque, C Valette, F Brachet, et al. A new family of planets? “ocean-planets”. *Icarus*, 169(2):499–504, 2004.
- CE Leith. Stochastic models of chaotic systems. *Physica D: Nonlinear Phenomena*, 98(2-4): 481–491, 1996.
- D Lemasquerier, AM Grannan, J Vidal, D Cébron, B Favier, M Le Bars, and JM Aurnou. Libration-driven flows in ellipsoidal shells. *Journal of Geophysical Research: Planets*, 122(9):1926–1950, 2017.
- Daphné G Lemasquerier, Carver J Bierson, and Krista M Soderlund. Europa’s ocean translates interior tidal heating patterns to the ice-ocean boundary. *AGU Advances*, 4(6): e2023AV000994, 2023. doi:10.1029/2023AV000994.
- EL Lewis and RG Perkin. Ice pumps and their rates. *Journal of Geophysical Research: Oceans*, 91(C10):11756–11762, 1986.
- Ana H Lobo, Andrew F Thompson, Steven D Vance, and Saikiran Tharimena. A pole-to-equator ocean overturning circulation on Enceladus. *Nature Geoscience*, 14(4):185–189, 2021. doi:10.1038/s41561-021-00706-3.
- Jochem Marotzke. Influence of convective adjustment on the stability of the thermohaline circulation. *Journal of Physical Oceanography*, 21(6):903–907, 1991. doi:10.1175/1520-0485(1991)021<0903:IOCAOT>2.0.CO;2.
- T Maxworthy and S Narimousa. Unsteady, turbulent convection into a homogeneous, rotating fluid, with oceanographic applications. *Journal of Physical Oceanography*, 24(5): 865–887, 1994. doi:10.1175/1520-0485(1994)024<0865:UTCIAH>2.0.CO;2.
- Christopher P McKay, Carolyn C Porco, Travis Altheide, Wanda L Davis, and Timothy A Kral. The possible origin and persistence of life on Enceladus and detection of biomarkers in the plume. *Astrobiology*, 8(5):909–919, 2008. doi:10.1089/ast.2008.0265.
- William B McKinnon. Effect of Enceladus’s rapid synchronous spin on interpretation of Cassini gravity. *Geophysical Research Letters*, 42(7):2137–2143, 2015. doi:10.1002/2015GL063384.
- H.J Melosh, A.G Ekholm, A.P Showman, and R.D Lorenz. The temperature of Europa’s subsurface water ocean. *Icarus*, 168(2):498–502, April 2004. ISSN 00191035. doi:10.1016/j.icarus.2003.11.026. URL <https://linkinghub.elsevier.com/retrieve/pii/S0019103503004251>.
- Fritz M Neubauer. The sub-Alfvénic interaction of the Galilean satellites with the Jovian magnetosphere. *Journal of Geophysical Research: Planets*, 103(E9):19843–19866, 1998. doi:10.1029/97JE03370.

- Philip D Nicholson, Mark R Showalter, Luke Dones, Richard G French, Stephen M Larson, Jack J Lissauer, Colleen A McGhee, Patrick Seitzer, Bruno Sicardy, and G Edward Danielson. Observations of Saturn’s ring-plane crossings in August and November 1995. *Science*, 272(5261):509–515, 1996.
- F Nimmo, BG Bills, and PC Thomas. Geophysical implications of the long-wavelength topography of the Saturnian satellites. *Journal of Geophysical Research: Planets*, 116(E11), 2011.
- Francis Nimmo, Amy C Barr, Marie Behouňková, and William B McKinnon. The thermal and orbital evolution of Enceladus: observational constraints and models. *Enceladus and the icy moons of Saturn*, 475:79–94, 2018.
- Lyuba Novi, Jost von Hardenberg, David W Hughes, Antonello Provenzale, and Edward A Spiegel. Rapidly rotating Rayleigh-Bénard convection with a tilted axis. *Physical Review E*, 99(5):053116, 2019. doi:10.1103/PhysRevE.99.053116.
- Gordon I Ogilvie. Internal waves and tides in stars and giant planets. In *Fluid Mechanics of Planets and Stars*, pages 1–30. Springer, 2019.
- Morgan E O’Neill and Yohai Kaspi. Slantwise convection on fluid planets. *Geophysical Research Letters*, 43(20), October 2016. ISSN 0094-8276, 1944-8007. doi:10.1002/2016GL071188. URL <https://agupubs.onlinelibrary.wiley.com/doi/10.1002/2016GL071188>.
- Morgan E O’Neill, Yohai Kaspi, and Leigh N Fletcher. Galileo probe interpretation indicating a neutrally stable layer in the Jovian troposphere. *Geophysical Research Letters*, 44(9):4008–4017, 2017.
- Katsuyuki Ooyama. On the stability of the baroclinic circular vortex: A sufficient criterion for instability. *Journal of Atmospheric Sciences*, 23(1):43–53, 1966.
- D Alex Patthoff and Simon A Kattenhorn. A fracture history on Enceladus provides evidence for a global ocean. *Geophysical Research Letters*, 38(18), 2011.
- Flavio Petricca, Steven D Vance, Marzia Parisi, Dustin Buccino, Gael Cascioli, Julie Castillo-Rogez, Brynna G Downey, Francis Nimmo, Gabriel Tobie, Baptiste Journaux, et al. Titan’s strong tidal dissipation precludes a subsurface ocean. *Nature*, 648(8094):556–561, 2025.
- F Postberg, S Kempf, D Rost, T Stephan, R Srama, M Tieloff, A Mocker, and M Goerlich. Discriminating contamination from particle components in spectra of Cassini’s dust detector cda. *Planetary and Space Science*, 57(12):1359–1374, 2009a.
- F Postberg, J Schmidt, J Hillier, Sascha Kempf, and Ralf Srama. A salt-water reservoir as the source of a compositionally stratified plume on Enceladus. *Nature*, 474(7353):620–622, 2011.

- Frank Postberg, Sascha Kempf, Jürgen Schmidt, N Brilliantov, Alexander Beinsen, Bernd Abel, Udo Buck, and Ralf Srama. Sodium salts in e-ring ice grains from an ocean below the surface of Enceladus. *Nature*, 459(7250):1098–1101, 2009b.
- Frank Postberg, Nozair Khawaja, Bernd Abel, Gael Choblet, Christopher R Glein, Murthy S Gudipati, Bryana L Henderson, Hsiang-Wen Hsu, Sascha Kempf, Fabian Klenner, et al. Macromolecular organic compounds from the depths of Enceladus. *Nature*, 558(7711):564–568, 2018.
- Frank Postberg, Yasuhito Sekine, Fabian Klenner, Christopher R. Glein, Zenghui Zou, Bernd Abel, Kento Furuya, et al. Detection of phosphate in Enceladus’ ocean with implications for geochemistry and habitability in the outer solar system. *Nature*, 618(7965):489–493, 2023.
- Jeremy Rekier, Antony Trinh, Santiago Andres Triana, and Véronique Dehant. Internal energy dissipation in Enceladus’s subsurface ocean from tides and libration and the role of inertial waves. *Journal of Geophysical Research: Planets*, 124(8):2198–2212, 2019.
- Peter B Rhines. Waves and turbulence on a beta-plane. *Journal of Fluid Mechanics*, 69(3):417–443, 1975. doi:10.1017/S0022112075001504.
- James H Roberts. The fluffy core of Enceladus. *Icarus*, 258:54–66, 2015.
- Jürgen Schmidt, Nikolai Brilliantov, Frank Spahn, and Sascha Kempf. Slow dust in Enceladus’ plume from condensation and wall collisions in tiger stripe fractures. *Nature*, 451(7179):685–688, 2008.
- Yasuhito Sekine, Takazo Shibuya, Frank Postberg, Hsiang-Wen Hsu, Katsuhiko Suzuki, Yuka Masaki, Tatsu Kuwatani, Megumi Mori, Peng K Hong, Motoko Yoshizaki, et al. High-temperature water–rock interactions and hydrothermal environments in the chondrite-like core of Enceladus. *Nature Communications*, 6(1):8604, 2015. doi:10.1038/ncomms9604.
- Theodore G Shepherd. Symmetries, conservation laws, and hamiltonian structure in geophysical fluid dynamics. In *Advances in Geophysics*, volume 32, pages 287–338. Elsevier, 1990.
- Everett L Shock. Hydrothermal systems as environments for the emergence of life. In *Ciba Foundation Symposium 202-Evolution of Hydrothermal Ecosystems on Earth (And Mars?) Evolution of Hydrothermal Ecosystems on Earth (And Mars?): Ciba Foundation Symposium 202*, pages 40–60. Wiley Online Library, 2007.
- Mark R Showalter, Jeffrey N Cuzzi, and Stephen M Larson. Structure and particle properties of saturn’s e ring. *Icarus*, 94(2):451–473, 1991.
- Joseph Smagorinsky. General circulation experiments with the primitive equations: I. The basic experiment. *Monthly weather review*, 91(3):99–164, 1963. doi:10.1175/1520-0493(1963)091<0099:GCEWTP>2.3.CO;2.

- Joseph Smagorinsky, B Galperin, and S Orszag. Large eddy simulation of complex engineering and geophysical flows. *Evolution of physical oceanography*, pages 3–36, 1993. doi:10.1111/j.1365-246X.1994.tb04036.x.
- KM Soderlund, BE Schmidt, J Wicht, and DD Blankenship. Ocean-driven heating of Europa’s icy shell at low latitudes. *Nature Geoscience*, 7(1):16–19, 2014. doi:10.1038/ngeo2021.
- Krista M Soderlund. Ocean dynamics of outer solar system satellites. *Geophysical Research Letters*, 46(15):8700–8710, 2019. doi:10.1029/2018GL081880.
- Krista M Soderlund, Klára Kalousová, Jacob J Buffo, Christopher R Glein, Jason C Goodman, Giuseppe Mitri, G Wesley Patterson, Frank Postberg, Marc Rovira-Navarro, Tina Rückriemen, et al. Ice-ocean exchange processes in the Jovian and Saturnian satellites. *Space Science Reviews*, 216(5):1–57, 2020.
- M Solberg. Le mouvement d’inertie de l’atmosphere stable et son role dans le theorie des cyclones. *Union Geodesique et Geophysique internationale VIeme assemblee, Edinburg*, pages 66–82, 1936.
- Ch Sotin, O Grasset, and A Mocquet. Mass–radius curve for extrasolar earth-like planets and ocean planets. *Icarus*, 191(1):337–351, 2007.
- Ondřej Souček, Marie Běhouňková, Ondřej Čadek, Jaroslav Hron, Gabriel Tobie, and Gaël Choblet. Tidal dissipation in Enceladus’ uneven, fractured ice shell. *Icarus*, 328:218–231, 2019.
- Frank Spahn, Jürgen Schmidt, Nicole Albers, Marcel Hörning, Martin Makuch, Martin Seiß, Sascha Kempf, Ralf Srama, Valeri Dikarev, Stefan Helfert, et al. Cassini dust measurements at Enceladus and implications for the origin of the E ring. *Science*, 311(5766):1416–1418, 2006.
- JR Spencer, JC Pearl, M Segura, FM Flasar, A Mamoutkine, P Romani, BJ Buratti, AR Hendrix, LJ Spilker, and RMC Lopes. Cassini encounters Enceladus: Background and the discovery of a south polar hot spot. *science*, 311(5766):1401–1405, 2006.
- JR Spencer, CJA Howett, A Verbiscer, TA Hurford, M Segura, and DC Spencer. Enceladus heat flow from high spatial resolution thermal emission observations. *EPSC Abstracts*, 8: 840–1, 2013.
- Duane E Stevens. On symmetric stability and instability of zonal mean flows near the equator. *Journal of the Atmospheric Sciences*, 40(4):882–893, 1983.
- Peter H Stone. On non-geostrophic baroclinic stability. *Journal of the Atmospheric Sciences*, 23(4):390–400, 1966.
- Peter H Stone. An application of baroclinic stability theory to the dynamics of the Jovian atmosphere. *Journal of Atmospheric Sciences*, 24(6):642–652, 1967.

- Peter H. Stone. Baroclinic stability under non-hydrostatic conditions. *Journal of Fluid Mechanics*, 45(4):659–671, February 1971a. ISSN 0022-1120, 1469-7645. doi:10.1017/S0022112071000260. URL https://www.cambridge.org/core/product/identifier/S0022112071000260/type/journal_article.
- Peter H. Stone. The symmetric baroclinic instability of an equatorial current. *Geophysical Fluid Dynamics*, 2(1):147–164, January 1971b. ISSN 0016-7991. doi:10.1080/03091927108236055. URL <https://www.tandfonline.com/doi/full/10.1080/03091927108236055>.
- Fiammetta Straneo, Mitsuhiro Kawase, and Stephen C. Riser. Idealized Models of Slantwise Convection in a Baroclinic Flow. *Journal of Physical Oceanography*, 32(2):558–572, February 2002. ISSN 0022-3670, 1520-0485. doi:10.1175/1520-0485(2002)032<0558:IMOSCI>2.0.CO;2. URL [http://journals.ametsoc.org/doi/10.1175/1520-0485\(2002\)032<0558:IMOSCI>2.0.CO;2](http://journals.ametsoc.org/doi/10.1175/1520-0485(2002)032<0558:IMOSCI>2.0.CO;2).
- Wen-Yih Sun. Unsymmetrical symmetric instability. *Quarterly Journal of the Royal Meteorological Society*, 121(522):419–431, January 1995. ISSN 0035-9009, 1477-870X. doi:10.1002/qj.49712152209. URL <https://rmets.onlinelibrary.wiley.com/doi/10.1002/qj.49712152209>.
- Radwan Tajeddine, Krista M Soderlund, Peter C Thomas, Paul Helfenstein, Matthew M Hedman, Joseph A Burns, and Paul M Schenk. True polar wander of Enceladus from topographic data. *Icarus*, 295:46–60, 2017.
- PC Thomas, R Tajeddine, MS Tiscareno, JA Burns, J Joseph, TJ Lored, P Helfenstein, and C Porco. Enceladus’s measured physical libration requires a global subsurface ocean. *Icarus*, 264:37–47, 2016. doi:10.1016/j.icarus.2015.08.037.
- BJ Travis and G Schubert. Keeping Enceladus warm. *Icarus*, 250:32–42, 2015.
- Sara Tro, Ian Grooms, and Keith Julien. Parameterized Ekman boundary layers on the tilted f-plane. *Journal of Fluid Mechanics*, 1000:A61, 2024. doi:10.1017/jfm.2024.561.
- R Tyler. Tidal dynamical considerations constrain the state of an ocean on Enceladus. *Icarus*, 211(1):770–779, 2011.
- Geoffrey K Vallis. *Atmospheric and oceanic fluid dynamics*. Cambridge University Press, 2017. doi:10.1017/9781107588417.
- Geoffrey K Vallis and Matthew E Maltrud. Generation of mean flows and jets on a beta plane and over topography. *Journal of physical oceanography*, 23(7):1346–1362, 1993.
- Steve Vance. vancesteven/planetprofile: Release for use in reproducing results submitted to journal of geophysical research-planets. *Zenodo*, 2017.

- Steven D Vance, Mark P Panning, Simon Stähler, Fabio Cammarano, Bruce G Bills, Gabriel Tobie, Shunichi Kamata, Sharon Kedar, Christophe Sotin, William T Pike, et al. Geophysical investigations of habitability in ice-covered ocean worlds. *Journal of Geophysical Research: Planets*, 123(1):180–205, 2018.
- J Von Hardenberg, D Goluskin, A Provenzale, and EA Spiegel. Generation of large-scale winds in horizontally anisotropic convection. *Physical review letters*, 115(13):134501, 2015. doi:10.1103/PhysRevLett.115.134501.
- J Hunter Waite, Michael R Combi, Wing-Huen Ip, Thomas E Cravens, Ralph L McNutt Jr, Wayne Kasprzak, Roger Yelle, Janet Luhmann, Hasso Niemann, David Gell, et al. Cassini ion and neutral mass spectrometer: Enceladus plume composition and structure. *Science*, 311(5766):1419–1422, 2006.
- J Hunter Waite, Christopher R Glein, Rebecca S Perryman, Ben D Teolis, Brian A Magee, Greg Miller, Jacob Grimes, Mark E Perry, Kelly E Miller, Alexis Bouquet, et al. Cassini finds molecular hydrogen in the Enceladus plume: evidence for hydrothermal processes. *Science*, 356(6334):155–159, 2017. doi:10.1126/science.aai8703.
- I. C. Walton. The viscous nonlinear symmetric baroclinic instability of a zonal shear flow. *Journal of Fluid Mechanics*, 68(4):757–768, April 1975. ISSN 0022-1120, 1469-7645. doi:10.1017/S0022112075001218. URL https://www.cambridge.org/core/product/identifier/S0022112075001218/type/journal_article.
- Guiquan Wang, Luca Santelli, Detlef Lohse, Roberto Verzicco, and Richard J. A. M. Stevens. Diffusion-free scaling in rotating spherical Rayleigh-Bénard convection. *Geophysical Research Letters*, 48(20):e2021GL095017, October 2021. ISSN 0094-8276, 1944-8007. doi:10.1029/2021GL095017.
- Shuang Wang, Wanying Kang, Yixiao Zhang, and John Marshall. The fate of rotating point plumes in an unstratified environment: from free growth to boundary interactions. *Journal of Fluid Mechanics*, 1018:A19, 2025. doi:10.1017/jfm.2025.10533.
- Shuang Wang, Wanying Kang, and Cheng Li. A tug-of-war between baroclinic eddies and convection: Implications for icy moon oceans. *arXiv preprint arXiv:2603.17185*, 2026.
- Alec Wilson and Rich R Kerswell. Can libration maintain Enceladus’s ocean? *Earth and Planetary Science Letters*, 500:41–46, 2018.
- Carl Wunsch and Raffaele Ferrari. Vertical mixing, energy, and the general circulation of the oceans. *Annu. Rev. Fluid Mech.*, 36:281–314, 2004.
- Yaoxuan Zeng and Malte F Jansen. Ocean circulation on Enceladus with a high-versus low-salinity ocean. *The Planetary Science Journal*, 2(4):151, 2021. doi:10.3847/PSJ/ac1114.
- Yaoxuan Zeng and Malte F Jansen. The effect of salinity on ocean circulation and ice–ocean interaction on Enceladus. *The Planetary Science Journal*, 5(1):13, 2024. doi:10.3847/PSJ/ad0cba.

- Yixiao Zhang, Wanying Kang, and John Marshall. Ocean weather systems on icy moons, with application to Enceladus. *Science Advances*, 10(45):eadn6857, 2024. doi:10.1126/sciadv.adn6857.
- Yixiao Zhang, Suyash Bire, Shuang Wang, Anika Nath, Ali Ramadhan, Wanying Kang, and John Marshall. Long transit time from the seafloor to the ice shell on Enceladus. *Monthly Notices of the Royal Astronomical Society*, page staf1008, 2025a. doi:10.1093/mnras/staf1008.
- Yixiao Zhang, Wanying Kang, and John Marshall. How does ice shell geometry shape ocean dynamics on icy moons? *arXiv preprint arXiv:2510.25988*, 2025b.
- Christophe Zimmer, Krishan K Khurana, and Margaret G Kivelson. Subsurface oceans on Europa and Callisto: Constraints from Galileo magnetometer observations. *Icarus*, 147(2):329–347, 2000. doi:10.1006/icar.2000.6456.
- Mikhail Y Zolotov. An oceanic composition on early and today’s Enceladus. *Geophysical Research Letters*, 34(23), 2007.