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Online Appendix for

"Political Cycles and Stock Returns"

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This Appendix provides the proofs of all propositions in Pástor and Veronesi (2019) as well as additional theoretical and empirical results. The Appendix is organized as follows:

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  - Presents a simple example, with risk aversion taking two possible values
- **Section A2. Theory: Example 2**
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A1. Theory: Example 1

We now offer a simple example in which the function $\gamma(Y_t)$ can take only two values, high or low, depending on the state of the economy:

$$\gamma(Y_t) = \begin{cases} \gamma^H \,, \text{ where } \gamma^H > \overline{\gamma}\,, & \text{for } y_t < \overline{y} \\ \gamma^L \,, \text{ where } \gamma^L < \underline{\gamma}\,, & \text{for } y_t > \overline{y} \end{cases} , \tag{A1}$$

where $y_t = \log(Y_t)$ and $\overline{y} = \mathrm{E}[y_t] - \frac{1}{2}\sigma^2$.

Parameter values.

We pick risk aversion values of $\gamma^L = 1$ and $\gamma^H = 5$. These are plausible values commonly considered in the literature.

We select the tax rates $\tau^L = 32\%$ and $\tau^H = 34\%$. The two values are close to each other. We do not need a large difference between the tax rates imposed by the high-tax party and the low-tax party to generate a large difference between the average stock returns under the two parties.

We choose $g = -0.2$ in the function defining the government's contribution, $G_t = (1 - m_t)\, e^g$. Recall that the value of g can be interpreted as the average productivity of the public sector. We pick $g < 0$ so that the public sector is less productive than the private sector (recall that $\mathrm{E}(\mu_i) = 0$). We assume lower public sector productivity because our definition of government workers includes not only employees but also retirees and other non-workers living off taxes. The level of g affects the average growth rate in the economy but not the sign of the difference in growth rates under H and L.

For the remaining parameters, we choose $\sigma_\mu = 10\%$ per year, $\sigma = 20\%$ per year, $\sigma_1 = 50\%$ per year, and $\theta = 0.6$. Each electoral period lasts four years.

Results.

With the above parameter values, we obtain $\underline{\gamma} = 2.65$ and $\overline{\gamma} = 4.24$. Therefore, $\gamma^L < \underline{\gamma}$ and $\gamma^H > \overline{\gamma}$, so there is a unique equilibrium under each risk aversion. In the L equilibrium, the mass of entrepreneurs is $m_t^L = 55.5\%$; in the H equilibrium, it is $m_t^H = 47.2\%$. The transition probabilities are $\lambda^{H,L} = \lambda^{L,H} = 52.9\%$.

Both returns and growth rates are higher under party H. The expected returns are $\mathrm{E}(R_{t+1}|\tau^H) = 18\%$ and $\mathrm{E}(R_{t+1}|\tau^L) = 2\%$ per year, generating a presidential "puzzle" of 16% per year, even larger than in the data. The expected growth rates are $\mathrm{E}(Y_{t+1}|\tau^H) = 3.82\%$ and $\mathrm{E}(Y_{t+1}|\tau^L) = 3.61\%$ per year, generating a growth gap of 0.21%, somewhat smaller than in the data.

A2. Theory: Example 2

In this example, we keep the parameter values from Example 1, but we add one more value of γ_t to allow for the two-equilibrium scenario from Proposition 3. We let γ_t take three values:

$$\gamma(Y_t) = \begin{cases} \gamma^H = 5 & \text{for } y_t < \underline{y} \\ \gamma^M = 3 & \text{for } \underline{y} \leq y_t \leq \overline{y} \\ \gamma^L = 1 & \text{for } y_t > \overline{y} \end{cases}, \tag{A2}$$

where $\underline{y} < \overline{y}$. We choose $\underline{y}$ and $\overline{y}$ such that all three scenarios occur with equal probabilities. Since $\underline{\gamma} = 2.65$ and $\overline{\gamma} = 4.24$, we have $1 < \underline{\gamma} < 3 < \overline{\gamma} < 5$. Therefore, when $y_t < \underline{y}$, there is a unique H equilibrium, and when $y_t > \overline{y}$, there is a unique L equilibrium. When $\underline{y} \leq y_t \leq \overline{y}$, there are two possible equilibria, H and L, one of which is selected by a coin flip.

This setting features four regimes: $(\tau_t, \gamma_t) = (\tau^H, \gamma^H), (\tau^H, \gamma^M), (\tau^L, \gamma^M), (\tau^L, \gamma^L)$. We solve for the transition probabilities in closed form and present them later in Section A4. of this Online Appendix. From those, we compute the following quantities, all in annualized terms:

	τ_t	$\text{E}(R_{t+1})$	$\text{E}(Y_{t+1})$	m_t
Party H in power	34%	15.4%	3.8%	48.1%
Party L in power	32%	4.7%	3.6%	54.1%

The difference in average returns under parties H and L is 10.7% per year, which approximately matches the difference observed in the data. The model can thus match not only the sign but also the magnitude of the presidential puzzle. The difference in average growth rates is 0.2% per year, which is positive but smaller than the empirically observed difference.

Stock return volatility is equal to 21.4% per year under both parties. This volatility exceeds the instantaneous volatility of $\sigma = 20\%$ due to variation in the expected rate of return. Under each party, the expected return can take two different values, one in the unique equilibrium and one in the two-equilibrium scenario.

In this setting, political cycles arise naturally through the mechanism described earlier. To illustrate those cycles, we simulate the model over a 90-year-long period. Ninety years is approximately equal to the length of the sample used in our empirical work (1927-2015).

Figure A1 plots average stock market returns over the simulated sample, which features 12 H administrations and 10 L administrations. Market returns under H administrations tend to exceed those under L administrations. Moreover, the returns under L administrations are occasionally negative. Both results are also present in the data, as we show in the paper.

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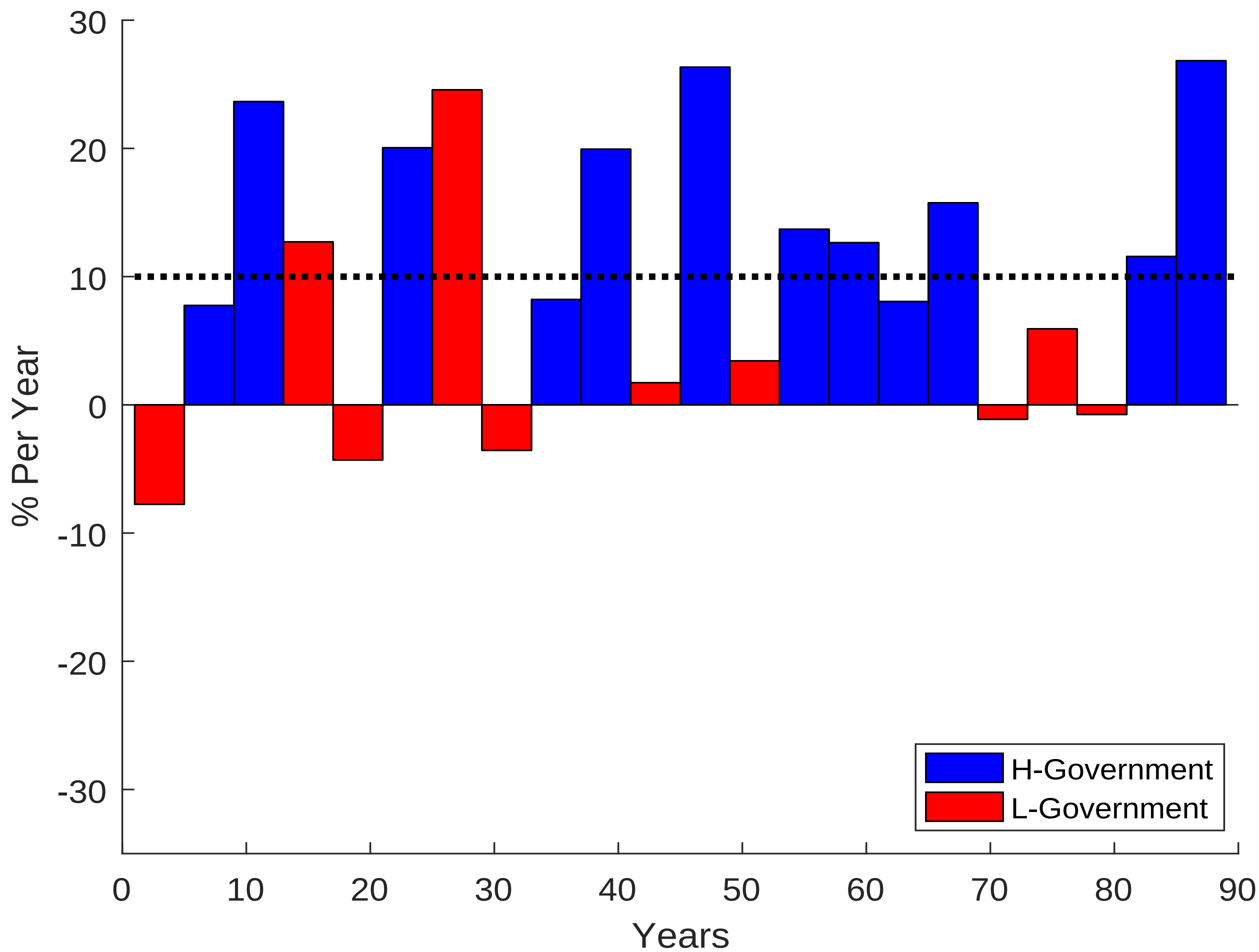

Figure A1. Average stock market returns simulated from the model. This figure plots average excess stock market returns for a 90-year-long illustrative segment of political cycles simulated from our model. The returns are plotted for periods over which party H is in power (blue bars) and periods over which party L is in power (red bars). The horizontal dotted line plots the unconditional mean return.

The examples presented in Sections A1. and A2. provide simple illustrations of the model's ability to generate political cycles. The model implies that stock returns and growth should both be higher under party H. While the return gap is about the same as in the data (and it can be larger, as Example 1 shows), the growth gap is smaller. We have experimented with other parameter values, finding similar results. There are many plausible parameter values generating a return gap of the right sign and magnitude. The growth gap also has the right sign, but its magnitude tends to be smaller than what we see in the data. The growth gap observed in the data is strongly influenced by the early years in the sample, especially the Great Depression era. But even its post-war value of 0.7% exceeds the 0.2% gap generated by the model in Examples 1 and 2.

To our knowledge, this is the first model that predicts a positive return gap. It is comforting that this simple model can match not only the sign but also the magnitude of the return gap, as well as the sign of the growth gap. Future work can aim to design a more sophisticated model that can match also the magnitude of the growth gap.

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A3. Theory: Announcement Returns

In this section, we fill in some details to accompany Proposition 8 in the paper. We provide closed-form formulae for the party-specific announcement returns and the electoral risk premium. We also provide a numerical example to illustrate the magnitudes of the announcement effects.

Part (a) of Proposition 8 states that the stock market reaction to the election outcome is positive if party L wins but negative if party H wins. That is,

$$AR_t^H \; < \; 0 \; < \; AR_t^L \; , \tag{A3}$$

where AR_t^k is the announcement return after the victory of party $k \in \{H, L\}$. To calculate the announcement returns, note that the value of the aggregate stock market portfolio immediately after the election is

$$M_{P,t}^L \;=\; \frac{1}{2} G_t \,\mathrm{E}\left[e^{\mu_i} | i \in \mathcal{I}_t\right] e^{-\gamma^M \sigma^2} \left(1 - \tau^L\right) \quad \text{if party } L \text{ wins} \tag{A4}$$

$$M_{P,t}^H \;=\; \frac{1}{2} G_t \,\mathrm{E}\left[e^{\mu_i} | i \in \mathcal{I}_t\right] e^{-\gamma^M \sigma^2} \left(1 - \tau^H\right) \quad \text{if party } H \text{ wins} \; . \tag{A5}$$

Immediately before the election, the market portfolio's value is between the above values:

$$M_{P,t} \;=\; \frac{1}{2} G_t \,\mathrm{E}\left[e^{\mu_i} | i \in \mathcal{I}_t\right] e^{-\gamma^M \sigma^2} \left[w \left(1 - \tau^L\right) + (1 - w) \left(1 - \tau^H\right)\right] \; , \tag{A6}$$

where

$$w \;=\; \frac{\left(1 - \tau^L\right)^{-\gamma^M}}{\left(1 - \tau^L\right)^{-\gamma^M} + \left(1 - \tau^H\right)^{-\gamma^M}} \; < \; \frac{1}{2} \; . \tag{A7}$$

The announcement return after the victory of party k is

$$AR_t^k = \frac{M_{P,t}^k}{M_{P,t}} - 1 \; , \tag{A8}$$

which is positive for $k = L$ and negative for $k = H$ because $M_{P,t}^H < M_{P,t} < M_{P,t}^L$.

Part (b) of Proposition 8 states that the risk premium for electoral uncertainty is positive. This risk premium is given by

$$\mathrm{E}\left(AR_t^k\right) \;=\; \frac{\left(\frac{1}{2} - w\right)\left(\tau^H - \tau^L\right)}{1 - \tau^H + w\left(\tau^H - \tau^L\right)} \; > \; 0 \; . \tag{A9}$$

This risk premium, which is equal to the expected value of the announcement return AR_t^k, compensates stockholders for the uncertainty about which of the two tax rates will be applied to their dividends at the end of period t. The derivations of all of the above formulae appear in Section A4. of this Online Appendix, along with the formula for γ^M that satisfies Proposition 8.

Finally, we illustrate the magnitudes of the announcement effects in the context of a numerical example. We take the parameter values from Example 2, except that we set $\gamma^M = 3.38$, which is the value for which Proposition 8 obtains. In that case, the announcement returns are $AR_t^H = -1.42\%$ and $AR_t^L = 1.57\%$. The risk premium for electoral uncertainty is the average of these two values, or 0.08%. The difference between AR_t^L and AR_t^H seems plausible given the evidence of Snowberg et al. (2007) that electing a Republican president raises equity valuations by two to three percent.

A4. Theory: Proofs

First, we define some notation that is useful in solving for the Nash equilibrium. Let V_i^E and V_i^G denote the expectations of the utility function in equation (1) in the paper conditional on agent i being an entrepreneur and government worker, respectively. In equilibrium, the set of agents who become entrepreneurs at the beginning of period t is given by

$$\mathcal{I}_t = \left\{ i : V_i^E(\gamma_t) \geq V_i^G(\gamma_t) \right\} \ . \tag{A10}$$

Both V_i^E and V_i^G depend on $\mathcal{I}_t$ itself: each agent's utility depends on the actions of other agents. Obtaining the equilibrium thus involves solving a complicated fixed-point problem. Agent i's decision whether to become an entrepreneur depends on V_i^E, which depends on the market value of firm i, which depends on the equilibrium state price density, which in general depends on who becomes an entrepreneur.

Proof of Proposition 1.

(a) Government workers. Consider an agent who decides at time t to be a government worker. From the government's budget equation, for a given tax rate τ, total tax receipts available at time $t+1$ are given by

$$\text{tax}_{t+1} = \tau \int_{j \in I_t} Y_{j,t+1} dj = \tau \left(\int_{j \in I_t} e^{\mu_j + \varepsilon_{j,t+1} + \varepsilon_{t+1}} dj \right) G_t = \tau G_t e^{\varepsilon_{t+1}} m_t E \left[e^{\mu_j} | j \in I_t \right] \ ,$$

where we used the law of large numbers

$$\int_{j \in I_t} e^{\mu_j + \varepsilon_{j,t+1}} dj = m_t E \left[e^{\mu_j + \varepsilon_{j,t+1}} | j \in I_t \right] = m_t E \left[e^{\mu_j} | j \in I_t \right] E_t \left[e^{\varepsilon_{j,t+1}} | j \in I_t \right] = m_t E \left[e^{\mu_j} | j \in I_t \right] \ .$$

Exploiting the balanced budget restriction, the consumption of a government worker is

$$C_{it+1}^{no} = \frac{\tau G_t e^{\varepsilon_{t+1}} m_t E \left[e^{\mu_j} | j \in I_t \right]}{1 - m_t} \ . \tag{A11}$$

When $\gamma_t \neq 1$, the expected one-period utility at the time of the voting decision is

$$E_t \left[U \left(C_{it+1}^{no} \right) | \tau \right] = \frac{\tau^{1-\gamma_t}}{1-\gamma_t} G_t^{1-\gamma_t} E_t \left[e^{(1-\gamma_t)\varepsilon_{t+1}} \right] E_t \left[e^{\mu_j} | j \in I_t \right]^{1-\gamma_t} \left(\frac{m_t}{1-m_t} \right)^{1-\gamma_t} \ . \tag{A12}$$

We immediately see that

$$E_t \left[U \left(C_{it+1}^{no} \right) | \tau^H \right] > E_t \left[U \left(C_{it+1}^{no} \right) | \tau^L \right]$$

if and only if

$$\tau^H > \tau^L \ .$$

Similarly, when $\gamma_t = 1$, then the utility function is log, and we obtain

$$E_t \left[U \left(C_{it+1}^{no} \right) | \tau \right] = \log(\tau) + E_t \left[\log \left[G_t e^{\varepsilon_{t+1}} m_t E \left[e^{\mu_j} | j \in I_t \right] \right] \right] - \log(1 - m_t) \ ,$$

so that the conclusion holds.

(b) Entrepreneurs. Consider now the consumption of an entrepreneur, under the assumption that m_t agents decide to be entrepreneurs in equilibrium. Each entrepreneur i sells $1-\theta$ shares and retains θ shares of his own company. All shares are one-period claims to the next-period dividend, net of taxes:

$$M_{i,t} = E_t \left[\pi_{t,t+1} Y_{i,t+1} \left(1-\tau_t\right) \right] \ ,$$

where $\pi_{t,t+1}$ is the equilibrium stochastic discount factor and τ_t is the tax rate decided at the election at time t. Each entrepreneur uses the shares sold at time t to purchase claims from other entrepreneurs. Let N_t^{ij} denote the fraction of firm j purchased by entrepreneur i at time t and let N_{it}^0 be the entrepreneur's (long or short) position in the bond. The entrepreneur's budget constraint is

$$\left(1-\theta\right) M_{it} = \int_{j\neq i} N_t^{ij} M_{jt} dj + N_{it}^0 \ ,$$

where we normalize the price of bonds to one. Since there is no intertemporal consumption/saving choice, the value of a bond at time t is indeterminate. We assume it is equal to one and acts as the numeraire. If agent i chooses to be an entrepreneur, his consumption at time $t+1$ (for given τ) is

$$C_{it+1} = \theta Y_{i,t+1} \left(1-\tau_t\right) + \int_{j\in I} N_t^{ij} Y_{j,t+1} \left(1-\tau_t\right) dj + N_{it}^0 \ .$$

From Proposition A1 below, $N_{it}^0 = 0$ and $N_t^{ij} = \left(1-\theta\right) \frac{e^{\mu_i}}{\int_{k\in I} e^{\mu_k} di}$, so that

$$C_{it+1} = \left(1-\tau\right) G_t e^{\mu_i} e^{\varepsilon_{t+1}} \left[\theta e^{\varepsilon_{i,t+1}} + \left(1-\theta\right)\right] \ . \tag{A13}$$

Then, for $\gamma_t \neq 1$, the expected utility of an entrepreneur is

$$\begin{aligned} E_t \left[U\left(C_{i,t+1}^{yes}\right) | \tau \right] &= \frac{\left(1-\tau\right)^{1-\gamma_t} G_t^{1-\gamma_t} e^{(1-\gamma_t)\mu_i}}{1-\gamma_t} E_t \left[e^{(1-\gamma_t)(\varepsilon_{t+1})} \left[\theta e^{\varepsilon_{i,t+1}} + \left(1-\theta\right)\right]^{1-\gamma_t} \right] \\ &= \frac{\left(1-\tau\right)^{1-\gamma_t} G_t^{1-\gamma_t} e^{(1-\gamma_t)\mu_i}}{1-\gamma_t} E_t \left[e^{(1-\gamma_t)(\varepsilon_{t+1})} \right] E\left[\left[\theta e^{\varepsilon_{i,t+1}} + \left(1-\theta\right)\right]^{1-\gamma_t} \right] \ . \end{aligned}$$

We clearly have

$$E_t \left[U\left(C_{i,t+1}^{yes}\right) | \tau^L \right] > E_t \left[U\left(C_{i,t+1}^{yes}\right) | \tau^H \right]$$

if and only if

$$\tau^L < \tau^H \ .$$

Similarly, if risk aversion $\gamma_t = 1$, then

$$E_t \left[U\left(C_{i,t+1}^{yes}\right) | \tau \right] = \log\left(1-\tau\right) + \log\left[G_t e^{\mu_i}\right] + E_t \left[\log\left[e^{\varepsilon_{t+1}} \left(\theta e^{\varepsilon_{i,t+1}} + \left(1-\theta\right)\right)\right]\right]$$

and the same conclusion holds. Q.E.D.

Proof of Proposition 2.

The argument is analogous to that in Pástor and Veronesi (2016). Consider $\gamma \neq 1$, tax rate τ^k, and let I^k be the equilibrium set of entrepreneurs and m^k be the equilibrium mass of entrepreneurs. For any agent i,

$$V_t^{i,yes} > V_t^{i,no}$$

if and only if

$$E_t\left[U\left(C_{it+1}^{yes}\right)|\tau^k, m^k\right] > E_t\left[U\left(C_{it+1}^{no}\right)|\tau^k, m^k\right] .$$

Using expressions (A12) and (A13), we obtain

$$\begin{aligned} &\frac{\left(1-\tau^k\right)^{1-\gamma_t} G_t^{1-\gamma_t} e^{(1-\gamma_t)\mu_i}}{1-\gamma_t} E_t\left[e^{(1-\gamma_t)\varepsilon_{t+1}}\right] E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma_t}\right] \\ > &\frac{1}{1-\gamma_t}\left(\tau^k\right)^{1-\gamma_t} G_t^{1-\gamma_t} E\left[e^{\mu_j}|j \in I^L\right]^{1-\gamma_t}\left(\frac{m^k}{1-m^k}\right)^{1-\gamma_t} E_t\left[e^{(1-\gamma_t)\varepsilon_{t+1}}\right] . \end{aligned}$$

Deleting common terms, taking logs, and re-arranging, we obtain

$$\begin{aligned} &\mu_i + \frac{1}{1-\gamma_t}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma_t}\right]\right) \\ > &\log\left(\frac{\tau^k}{1-\tau^k}\right) + \log\left(E\left[e^{\mu_j}|j \in I^L\right]\right) + \log\left(\frac{m^k}{1-m^k}\right) , \end{aligned}$$

or

$$\begin{aligned} \mu_i > \; &\underline{K}^k = \log\left(\frac{\tau^k}{1-\tau^k}\right) + \log\left(E\left[e^{\mu_j}|j \in I^L\right]\right) + \log\left(\frac{m^k}{1-m^k}\right) \\ &-\frac{1}{1-\gamma_t}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma_t}\right]\right) . \end{aligned}$$

We now derive m^k and $E\left[e^{\mu_j}|j \in I^L\right]$. From the definition of m^k and the distribution of skill $\mu_i \sim N\left(\overline{\mu}, \sigma_\mu^2\right)$, we obtain

$$m_t^k = \int_{\underline{K}^k}^\infty \phi\left(\mu_i; \overline{\mu}, \sigma_\mu^2\right) d\mu_i = 1 - \Phi\left(\underline{K}^k; \overline{\mu}, \sigma_\mu^2\right) .$$

In addition,

$$\begin{aligned} E\left[e^{\mu_j}|j \in I^k\right] &= \frac{1}{m_t^k}\int_{\underline{K}^k}^\infty e^{\mu_j}\phi\left(\mu_j; \overline{\mu}, \sigma_\mu^2\right) d\mu_j \\ &= \frac{e^{\overline{\mu}+\frac{1}{2}\sigma_\mu^2}\left(1-\Phi\left(\underline{K}^k; \overline{\mu}+\sigma_\mu^2, \sigma_\mu^2\right)\right)}{m_t^k} . \end{aligned} \tag{A14}$$

Therefore, substituting in the expression for $\underline{K}^k$, we obtain

$$\begin{aligned} \underline{K}^k = \; &\log\left(\frac{\tau^k}{1-\tau^k}\right) + \overline{\mu} + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{K}^k; \overline{\mu}+\sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^k; \overline{\mu}, \sigma_\mu^2\right)}\right) \\ &-\frac{1}{1-\gamma_t}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma_t}\right]\right) . \end{aligned}$$

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Define
$$\underline{\mu}^k = \underline{K}^k - \overline{\mu}$$
and exploit the properties of the normal distribution to obtain

$$\begin{aligned}\underline{K}^k - \overline{\mu} &= \log\left(\frac{\tau^k}{1-\tau^k}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{K}^k - \overline{\mu}; \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{K}^k - \overline{\mu}; 0, \sigma_\mu^2\right)}\right) \\ &\quad -\frac{1}{1-\gamma_t}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma_t}\right]\right) ,\end{aligned}$$

or

$$\begin{aligned}\underline{\mu}^k &= \log\left(\frac{\tau^k}{1-\tau^k}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{\mu}^k; \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^k; 0, \sigma_\mu^2\right)}\right) \\ &\quad -\frac{1}{1-\gamma_t}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma_t}\right]\right) ,\end{aligned}$$

which defines the equation to solve.

Finally,

$$m_t^k = \int_{\underline{K}^k}^{\infty} \phi\left(\mu_i; \overline{\mu}, \sigma_\mu^2\right) d\mu_i = 1 - \Phi\left(\underline{K}^k - \overline{\mu}; 0, \sigma_\mu^2\right) = 1 - \Phi\left(\underline{\mu}^k; 0, \sigma_\mu^2\right) .$$

When $\gamma_t = 1$, we instead have that

$$E_t\left[U\left(C_{it+1}^{yes}\right)|\tau^k, m^k\right] > E_t\left[U\left(C_{it+1}^{no}\right)|\tau^k, m^k\right]$$

holds if and only if

$$\begin{aligned}&\log\left(1-\tau^k\right) + E_t\left[\log\left[G_t e^{\mu_i} e^{\varepsilon_{t+1}}\left(\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right)\right]\right] \\ > \;&\log\left(\tau^k\right) + E_t\left[\log\left[\frac{G_t e^{\varepsilon_{t+1}} m_t^k E\left[e^{\mu_j}|j \in I_t\right]}{1-m_t^k}\right]\right] .\end{aligned}$$

Deleting common terms and re-arranging, we find

$$\begin{aligned}\mu_i \;>\; &\underline{K}^k = \log\left(\frac{\tau^k}{1-\tau^k}\right) + \log\left[\frac{m_t^k}{1-m_t^k}\right] + \log\left[E_t\left[e^{\mu_j}|j \in I_t\right]\right] \\ &-E_t\left[\log\left(\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right)\right] .\end{aligned}$$

The same argument as above establishes $m_t^k = 1 - \Phi\left(\underline{\mu}^k; 0, \sigma_\mu^2\right)$, where

$$\begin{aligned}\underline{\mu}^k &= \log\left(\frac{\tau^k}{1-\tau^k}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{\mu}^k; \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^k; 0, \sigma_\mu^2\right)}\right) \\ &\quad -E_t\left[\log\left(\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right)\right] .\end{aligned}$$

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Finally, we show that the equilibrium mass of entrepreneurs, m_t^k from Proposition 2, is always strictly between zero and one. If it were zero, there would be no output for agents to consume, and it would be worthwhile for some agents to become entrepreneurs. If it were one, there would be a large unallocated tax to be shared, and it would be worthwhile for some agents to become government workers and enjoy large tax-financed consumption. Therefore, our model guarantees an interior solution for the equilibrium mass of entrepreneurs.

Q.E.D.

Corollary 1. The equilibrium mass of entrepreneurs m_t^k is decreasing in (a) the tax rate τ^k, (b) risk aversion γ_t (for $\gamma_t > 1$), (c) the degree of market incompleteness θ, and (d) idiosyncratic volatility σ_1.

Proof of Corollary 1:

(a) The implicit function defined by $\underline{\mu}^k = F\left(\tau^k, \underline{\mu}^k\right)$ is clearly increasing in τ^k as we can see by taking the total derivative

$$d\underline{\mu}^k = \frac{\partial F\left(\tau^k, \underline{\mu}^k\right)}{\partial \tau^k} d\tau^k + \frac{\partial F\left(\tau^k, \underline{\mu}^k\right)}{\partial \underline{\mu}^k} d\underline{\mu}^k \, .$$

From the total derivative, one obtains the implicit function theorem

$$\frac{d\underline{\mu}^k}{d\tau^k} = \frac{\frac{\partial F\left(\tau^k, K^k\right)}{\partial \tau^k}}{1 - \frac{\partial F\left(\tau^k, K^k\right)}{\partial K^k}} > 0 \, ,$$

as $\partial F\left(\tau^k, \underline{\mu}^k\right)/\partial \tau^k > 0$ and $\partial F\left(\tau^k, \underline{\mu}^k\right)/\partial \underline{\mu}^k < 0$. That is, higher taxes increase the threshold and decrease the mass of entrepreneurs $m^k = 1 - \Phi\left(\underline{\mu}^k, 0, \sigma_\mu^2\right)$.

(b) First, consider the function

$$U\left(\gamma\right) = \frac{1}{1-\gamma} \log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + \left(1-\theta\right)\right]^{1-\gamma}\right]\right) \, .$$

The first derivative is

$$U'\left(\gamma\right) = \frac{1}{\left(1-\gamma\right)^2} \log\left(E\left[\left(\theta\left(e^{\varepsilon}-1\right)+1\right)^{1-\gamma}\right]\right) - \frac{1}{1-\gamma} \frac{E\left[\left(\theta\left(e^{\varepsilon}-1\right)+1\right)^{1-\gamma} \log\left(\left(\theta\left(e^{\varepsilon}-1\right)+1\right)\right)\right]}{E\left[\left(\theta\left(e^{\varepsilon}-1\right)+1\right)^{1-\gamma}\right]} \, .$$

Define $X = \left(\theta\left(e^{\varepsilon}-1\right)+1\right)^{1-\gamma}$ for convenience, and factor out $1/\left(1-\gamma\right)^2 > 0$ to obtain

$$U'\left(\gamma\right) = \frac{1}{\left(1-\gamma^2\right)} \left[\log\left(E\left[X\right]\right) - \frac{E\left[X \log\left(X\right)\right]}{E\left[X\right]}\right] \, .$$

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As $X > 0$ and the function $f(X) = Xlog(X)$ is convex ($f''(X) = 1/X > 0$), from Jensen's inequality we have $E[X\log(X)] > E[X]\log(E[X])$. Therefore,

$$U'(\gamma) = \frac{1}{(1-\gamma^2)}\left[\log(E[X]) - \frac{E[X\log(X)]}{E[X]}\right]$$
$$< \frac{1}{(1-\gamma^2)}\left[\log(E[X]) - \frac{E[X]\log(E[X])}{E[X]}\right] = 0\,.$$

Note that this proof holds for any $\gamma \neq 1$.

We now define now the implicit function $\underline{\mu}^k = F\left(\gamma, \underline{\mu}^k\right)$ where we emphasize γ rather than τ. We then have

$$d\underline{\mu}^k = \frac{\partial F\left(\gamma, \underline{\mu}^k\right)}{\partial \gamma} d\gamma + \frac{\partial F\left(\gamma, \underline{\mu}^k\right)}{\partial \underline{\mu}^k} d\underline{\mu}^k\,.$$

From the total derivative, one obtains the implicit function theorem

$$\frac{d\underline{\mu}^k}{d\gamma} = \frac{\frac{\partial F\left(\gamma, \underline{\mu}^k\right)}{\partial \gamma}}{1 - \frac{\partial F\left(\gamma, \underline{\mu}^k\right)}{\partial \underline{\mu}^k}} > 0\,,$$

as we have shown $\partial F\left(\gamma, \underline{\mu}^k\right)/\partial\gamma = -U'(\gamma) > 0$ and $\partial F\left(\gamma, \underline{\mu}^k\right)/\partial\underline{\mu}^k < 0$. That is, higher risk aversion increases the threshold $\underline{\mu}^k$. Hence

$$\frac{d\underline{\mu}(\tau_k, \gamma)}{d\gamma} > 0\,.$$

Thus, higher γ_t decreases the mass of entrepreneurs $m_t^k = 1 - \Phi\left(\underline{\mu}^k, 0, \sigma_\mu^2\right)$, that is,

$$\frac{dm^k}{d\gamma} < 0\,.$$

(c) Let

$$U(\theta) = \frac{1}{1-\gamma}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma}\right]\right)\,.$$

Then

$$U'(\theta) = \frac{1}{1-\gamma}\frac{1}{E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma}\right]} E\left[(1-\gamma)\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{-\gamma}\left(e^{\varepsilon_{i,t+1}} - 1\right)\right]$$
$$= \frac{E\left[\left[\theta\left(e^{\varepsilon_{i,t+1}} - 1\right) + 1\right]^{-\gamma}\left(e^{\varepsilon_{i,t+1}} - 1\right)\right]}{E\left[\left[\theta\left(e^{\varepsilon_{i,t+1}} - 1\right) + 1\right]^{1-\gamma}\right]} < 0\,,$$

which holds if and only if

$$E\left[\left[\theta\left(e^{\varepsilon_{i,t+1}} - 1\right) + 1\right]^{-\gamma}\left(e^{\varepsilon_{i,t+1}} - 1\right)\right] < 0\,,$$

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which holds if and only if

$$Cov\left[\left[\theta\left(e^{\varepsilon_{i,t+1}}-1\right)+1\right]^{-\gamma},\left(e^{\varepsilon_{i,t+1}}-1\right)\right]+E\left[\left[\theta\left(e^{\varepsilon_{i,t+1}}-1\right)+1\right]^{-\gamma}\right]E\left[e^{\varepsilon_{i,t+1}}-1\right]<0\,,$$

which holds if and only if

$$Cov\left[\left[\theta\left(e^{\varepsilon_{i,t+1}}-1\right)+1\right]^{-\gamma},e^{\varepsilon_{i,t+1}}-1\right]<0\,.$$

Because $\left[\theta\left(e^{\varepsilon_{i,t+1}}-1\right)+1\right]^{-\gamma}$ is decreasing in $\varepsilon_{i,t+1}$ and $\left(e^{\varepsilon_{i,t+1}}-1\right)$ is increasing in $\varepsilon_{i,t+1}$, the result follows.

(d) Consider

$$U\left(\sigma\right)=\frac{1}{1-\gamma}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+\left(1-\theta\right)\right]^{1-\gamma}\right]\right)$$

(now as a function of σ). We want to show that as σ increases, $U\left(\sigma\right)$ decreases. Define $X=\theta e^{\varepsilon_{i,t+1}}+\left(1-\theta\right)=\theta\left(e^{\varepsilon_{i,t+1}}-1\right)+1$, so that

$$U\left(\sigma\right)=\frac{1}{1-\gamma}\log\left(E\left[\left[X\right]^{1-\gamma}\right]\right)\,.$$

Because this is a concave function of X, the result is shown if the cdf $F_X\left(x;\sigma_1\right)$ is a mean-preserving spread of $F_X\left(x,\sigma_0\right)$ with $\sigma_1>\sigma_0$. First, note that $E\left[X\right]=1$. Consider now the cdf $F_X\left(x;\sigma\right)=\Pr\left(X<x\right)=\Pr\left(\theta\left(e^{\varepsilon_{i,t+1}}-1\right)+1<x\right)=\Pr\left(\varepsilon_{i,t+1}<\log\left(1+\frac{x-1}{\theta}\right)\right)$. Let $\eta_{it}\sim N(0,1)$ so that $\varepsilon_{it}=-\frac{1}{2}\sigma^2+\sigma\eta_{it}$. Thus

$$\begin{aligned}F_X\left(x;\sigma\right)&=\Pr\left(-\frac{1}{2}\sigma^2+\sigma\eta_{it}<\log\left(1+\frac{x-1}{\theta}\right)\right)\\&=\Pr\left(\eta_{it}<\frac{1}{2}\sigma+\frac{1}{\sigma}\log\left(\frac{x-\left(1-\theta\right)}{\theta}\right)\right)\\&=\Phi\left(\frac{1}{2}\sigma+\frac{1}{\sigma}\log\left(\frac{x-\left(1-\theta\right)}{\theta}\right)\right)\\&=\int_{-\infty}^{\frac{1}{2}\sigma+\frac{1}{\sigma}\log\left(\frac{x-(1-\theta)}{\theta}\right)}\frac{e^{-\frac{1}{2}\eta^2}}{\sqrt{2\pi}}d\eta\,.\end{aligned}$$

We now show that if $\sigma_1>\sigma_0$, then $F_X\left(x;\sigma_1,\theta\right)$ is a mean-preserving spread of $F_X\left(x;\sigma_0,\theta\right)$. We already know the two distributions have the same mean. Therefore, the claim is shown if for every x',

$$\int^{x'}F_X\left(x;\sigma_1\right)dx>\int^{x'}F_X\left(x;\sigma_0\right)dx\,,$$

that is, if for every x', the function

$$H\left(\sigma;x'\right)=\int^{x'}\Phi\left(\frac{1}{2}\sigma+\frac{1}{\sigma}\log\left(\frac{x-\left(1-\theta\right)}{\theta}\right)\right)dx$$

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is increasing in σ. We can write

$$\begin{aligned} H(\sigma; x') &= \int^{x'} \Phi\left(\frac{1}{2}\sigma + \frac{1}{\sigma}\log\left(\frac{x-(1-\theta)}{\theta}\right)\right) dx \\ &= \int^{x'} \int_{-\infty}^{\frac{1}{2}\sigma + \frac{1}{\sigma}\log\left(\frac{x-(1-\theta)}{\theta}\right)} \frac{e^{-\frac{1}{2}\eta^2}}{\sqrt{2\pi}} d\eta dx \,. \end{aligned}$$

Therefore,

$$\frac{\partial H(\sigma; x')}{\partial \sigma} = \int^{x'} \frac{e^{-\frac{1}{2}\left(\frac{1}{2}\sigma + \frac{1}{\sigma}\log\left(\frac{x-(1-\theta)}{\theta}\right)\right)^2}}{\sqrt{2\pi}} \left(\frac{1}{2} - \frac{1}{\sigma^2}\log\left(\frac{x-(1-\theta)}{\theta}\right)\right) dx \,.$$

Consider the change of variable

$$\varepsilon = \frac{1}{2}\sigma + \frac{1}{\sigma}\log\left(\frac{x-(1-\theta)}{\theta}\right) ,$$

so that

$$d\varepsilon = \frac{1}{\sigma}\frac{1}{x-(1-\theta)} dx \,,$$

or

$$(x-(1-\theta))\,\sigma d\varepsilon = dx \,.$$

Moreover,

$$\theta e^{\varepsilon\sigma - \frac{1}{2}\sigma^2} = (x-(1-\theta)) \,,$$

which implies

$$\theta e^{\varepsilon\sigma - \frac{1}{2}\sigma^2} \sigma d\varepsilon = dx$$

and

$$\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1-\theta) = x' \,.$$

Thus,

$$\begin{aligned} \frac{\partial H(\sigma; x')}{\partial \sigma} &= \int^{x'} \frac{e^{-\frac{1}{2}\left(\frac{1}{2}\sigma + \frac{1}{\sigma}\log\left(\frac{x-(1-\theta)}{\theta}\right)\right)^2}}{\sqrt{2\pi}} \left(\frac{1}{2} - \frac{1}{\sigma^2}\log\left(\frac{x-(1-\theta)}{\theta}\right)\right) dx \\ &= \int^{\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1-\theta)} \frac{e^{-\frac{1}{2}\varepsilon^2}}{\sqrt{2\pi}} \left(\frac{1}{2} - \frac{1}{\sigma^2}\log\left(e^{\sigma\varepsilon - \frac{1}{2}\sigma^2}\right)\right) \theta e^{\sigma\varepsilon - \frac{1}{2}\sigma^2} \sigma d\varepsilon \\ &= \int^{\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1-\theta)} \frac{e^{-\frac{1}{2}\varepsilon^2}}{\sqrt{2\pi}} \left(\frac{1}{2} - \frac{1}{\sigma}\left(\varepsilon - \frac{1}{2}\sigma\right)\right) \theta e^{\sigma\varepsilon - \frac{1}{2}\sigma^2} \sigma d\varepsilon \\ &= \theta \int^{\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1-\theta)} e^{\sigma\varepsilon - \frac{1}{2}\sigma^2} \frac{e^{-\frac{1}{2}\varepsilon^2}}{\sqrt{2\pi}} (\sigma - \varepsilon)\, d\varepsilon \\ &= \theta \int^{\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1-\theta)} e^{-\frac{1}{2}\sigma^2} \frac{e^{-\frac{1}{2}\varepsilon^2 + \sigma\varepsilon}}{\sqrt{2\pi}} (\sigma - \varepsilon)\, d\varepsilon \,. \end{aligned}$$

We have

$$e^{-\frac{1}{2}\varepsilon^2+\sigma\varepsilon} = e^{-\frac{1}{2}\left(\varepsilon^2-2\sigma\varepsilon\right)} = e^{-\frac{1}{2}\left(\varepsilon^2-2\sigma\varepsilon+\sigma^2-\sigma^2\right)} = e^{-\frac{1}{2}\left(\left(\varepsilon^2-2\sigma\varepsilon+\sigma^2\right)-\sigma^2\right)} = e^{-\frac{1}{2}(\varepsilon-\sigma)^2+\frac{1}{2}\sigma^2} .$$

Therefore,

$$\frac{\partial H(\sigma)}{\partial\sigma} = \theta\int^{\theta e^{\sigma\varepsilon'-\frac{1}{2}\sigma^2}+(1-\theta)} \frac{e^{-\frac{1}{2}(\varepsilon-\sigma)^2}}{\sqrt{2\pi}}(\sigma-\varepsilon)\, d\varepsilon .$$

Define

$$\begin{aligned} \eta &= \varepsilon-\sigma \\ d\eta &= d\varepsilon \\ \eta'+\sigma &= \varepsilon' \end{aligned}$$

to get

$$\frac{\partial H(\sigma;\eta')}{\partial\sigma} = -\theta\int^{\theta e^{\sigma\eta'+\frac{1}{2}\sigma^2}+(1-\theta)} \frac{e^{-\frac{1}{2}(\eta)^2}}{\sqrt{2\pi}}\eta d\eta .$$

If $\eta'\to\infty$ then $\theta e^{\sigma\eta'+\frac{1}{2}\sigma^2}+(1-\theta)\to\infty$ and hence

$$\frac{\partial H(\sigma;\infty)}{\partial\sigma} = -\theta\int^{\infty} \frac{e^{-\frac{1}{2}(\eta)^2}}{\sqrt{2\pi}}\eta d\eta = -E[\eta] = 0 .$$

Moreover, the function

$$L(\eta') = \frac{\partial H(\sigma;\eta')}{\partial\sigma} = -\theta\int^{\theta e^{\sigma\eta'+\frac{1}{2}\sigma^2}+(1-\theta)} \frac{e^{-\frac{1}{2}(\eta)^2}}{\sqrt{2\pi}}\eta d\eta$$

is monotonically decreasing in η' because

$$\frac{\partial L}{\partial\eta'} = -\theta\frac{e^{-\frac{1}{2}\left(\theta e^{\sigma\eta'+\frac{1}{2}\sigma^2}+(1-\theta)\right)^2}}{\sqrt{2\pi}}\left(\theta e^{\sigma\eta'+\frac{1}{2}\sigma^2}+(1-\theta)\right) < 0 .$$

Therefore, $\frac{\partial H(\sigma;\eta')}{\partial\sigma} > 0$ for every η'. It follows that the distribution under a higher σ is a mean-preserving spread of a distribution under a lower σ. Thus, every concave function is decreasing in σ, and so is $U(\sigma)$. Q.E.D.

Assumption A1: For a given value of σ_μ^2 and both $k\in\{H,L\}$, the tax rates τ^k satisfy

$$\tau^k < \overline{\tau} = \frac{1}{1+2e^{0.5\sigma_\mu^2}\left(1-\Phi\left(0;\sigma_\mu^2,\sigma_\mu^2\right)\right)} .$$

Assumption A1 guarantees that for $\gamma\to 0$ the thresholds $\underline{\mu}^k$ converge to negative values. This can be easily seen as $\overline{\tau}$ above is the value of τ for which $\underline{\mu}=0$ is a solution to the equation

$$\underline{\mu} = \log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{2}\sigma_\mu^2+\log\left(\frac{1-\Phi\left(\underline{\mu};\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu};0,\sigma_\mu^2\right)}\right)$$

We assume that Assumption A1 holds throughout. In addition,

Corollary A1. The threshold $\underline{\mu}^k \to \infty$ as $\gamma \to \infty$, for both $k \in \{H, L\}$.

Proof of Corollary A1: The proof of part (b) of Corollary 1 shows that the function

$$U(\gamma) = \frac{1}{1-\gamma}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma}\right]\right) = \log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma}\right]^{1-\gamma}\right)$$

is decreasing. Defining here $X = [\theta e^{\varepsilon_{i,t+1}} + (1-\theta)] > 0$ we see $E[X] = 1$ and thus the probability density of X gives positive mass to both $X < 1$ and $X > 1$. It follows that $E\left[\frac{1}{X^{\gamma-1}}\right] \to \infty$ as $\gamma \to \infty$. Therefore $\frac{1}{E\left[\frac{1}{X^{\gamma-1}}\right]^{\gamma-1}} \to 0$ and hence $U(\gamma) = \log([E[X^{1-\gamma}]^{1-\gamma}) \to -\infty$. Recall that the threshold $\underline{\mu}^k$ solves

$$\underline{\mu} \quad = \quad \log\left(\frac{\tau^k}{1-\tau^k}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{\mu}^k;\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^k;0,\sigma_\mu^2\right)}\right) - U(\gamma)\,.$$

As the right-hand side diverges to infinity, so must $\underline{\mu}$. Q.E.D.

Proof of Proposition 3:

First, we complete the expression of Proposition 3 by presenting the formulae for the two thresholds, $\underline{\gamma}$ and $\overline{\gamma}$. These thresholds represent solutions to the following equations:

$$\frac{1}{2} \quad = \quad 1 - \Phi\left(\underline{\mu}_t^H\left(\underline{\gamma}\right);0,\sigma_\mu^2\right) \tag{A15}$$

$$\frac{1}{2} \quad = \quad 1 - \Phi\left(\underline{\mu}_t^L\left(\overline{\gamma}\right);0,\sigma_\mu^2\right)\,. \tag{A16}$$

That these thresholds, $\underline{\gamma}$ and $\overline{\gamma}$, exist follows from Corollary A1 and Assumption A1.

We now begin the proof. We know from Corollary 1 that both $\underline{\mu}\left(\tau^L,\gamma_t\right)$ and $\underline{\mu}\left(\tau^H,\gamma_t\right)$ are increasing in γ, and we have $\underline{\mu}\left(\tau^L,\gamma_t\right) < \underline{\mu}\left(\tau^H,\gamma_t\right)$. Let $\overline{\gamma}$ be such that

$$0.5 = 1 - \Phi\left(\underline{\mu}^L\left(\overline{\gamma}\right),0,\sigma_\mu^2\right)\,.$$

Then, for $\gamma_t > \overline{\gamma}$, we have $m_t^L < 0.5$. Clearly, also $m_t^H < m_t^L$. So, regardless of the tax rate (H or L), the maximum mass of entrepreneus is below 0.5, and therefore τ^L cannot win. When $\gamma_t > \overline{\gamma}$, the unique equilibrium must be τ^H. As all agents expect this to be the case, the equilibrium mass of entrepreneus is $m_t = m_t^H = 1 - \Phi\left(\underline{\mu}^H\left(\gamma_t\right),0,\sigma_\mu^2\right)$.

Similarly, let $\underline{\gamma}$ such that

$$0.5 = 1 - \Phi\left(\underline{\mu}^H\left(\underline{\gamma}\right),0,\sigma_\mu^2\right)\,.$$

Then for $\gamma_t < \underline{\gamma}$, we have $m_t^H > 0.5$. Clearly, also $m_t^L > m_t^H$, that is, even under high taxes, the majority of agents are entrepreneurs. Therefore, τ^H cannot win, and the unique equilibrium has

τ^L. As all agents expect this to be the case, the equilibrium mass of entrepreneurs is $m_t = m_t^L = 1 - \Phi\left(\underline{\mu}^L\left(\gamma_t\right), 0, \sigma_\mu^2\right)$.

For $\underline{\gamma} < \gamma_t < \overline{\gamma}$, the above arguments imply that both equilibria can be supported. Q.E.D.

Proposition A1. In equilibrium,

1. The stochastic discount factor is
$$\pi_{t,t+1} \propto e^{-\gamma\varepsilon_{t+1}} .$$
2. Entrepreneurs invest
$$N_t^{ij} = (1-\theta)\frac{e^{\mu_i}}{\int_{k\in I} e^{\mu_k} di}; \quad N_{it}^0 = 0$$
in stocks and bonds, respectively.
3. Asset prices are
$$M_t^i = (1-\tau_t)\, e^{\mu_i - \gamma_t\sigma^2} G_t .$$
4. The aggregate market value is
$$\begin{aligned} M_t^P &= (1-\tau_t)\, e^{-\gamma_t\sigma^2} E\left[e^{\mu_i} | i \in I_t\right] G_t m_t \\ &= (1-\tau_t)\, e^{-\gamma_t\sigma^2} \frac{\left(1 - \Phi\left(\underline{\mu}_t^k; \sigma_1^2, \sigma_1^2\right)\right)}{\left(1 - \Phi\left(\underline{\mu}_t^k; 0, \sigma_1^2\right)\right)} G_t m_t . \end{aligned}$$
5. The expected rate of return on each stock i is given by
$$E\left(R^i\right) = e^{\gamma_t\sigma^2} - 1 .$$

Proof of Proposition A1.

The claims follow from Corollary C1 (a) - (e) in the technical appendix of Pástor and Veronesi (2016). The only difference is that $T = 1$ and total production is multiplied by G_t, which is known at time t and therefore does not change any calculations. The expression for $E\left[e^{\mu_j} | j \in I\right]$ is in equation (A14).

Q.E.D.

Proof of Proposition 4.

The proof follows from $E_t\left[R\right] = e^{\gamma_t\sigma^2} - 1$ being uniformly increasing γ_t, the fact that $\gamma_t > \overline{\gamma}$ selects a high-tax equilibrium, $\gamma_t < \underline{\gamma}$ selects a low-tax equilibrium, and for intermediate γ_t the high-tax equilibrium is selected with 50-50 chance. See the discussion following the proposition in the text.

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Q.E.D.

Proof of Proposition 5.

Consider the expected growth formula under tax regime k:

$$E\left[e^{\mu_i}|i \in I_t^k\right] = \frac{1-\Phi\left(\underline{\mu}_t^k;\sigma^2,\sigma^2\right)}{1-\Phi\left(\underline{\mu}_t^k;0,\sigma^2\right)} e^{\overline{\mu}+\frac{1}{2}\sigma_\mu^2}.$$

In any H equilibrium, we must have $m_t^H = 1-\Phi\left(\underline{\mu}_t^H;0,\sigma^2\right) < 0.5$ and in any L equilibrium, we must have $m_t^L = 1-\Phi\left(\underline{\mu}_t^L;0,\sigma^2\right) > 0.5$. It follows that for any H and L equilibria, $\underline{\mu}_t^H > \underline{\mu}_t^L$ (see Figure 3). Consider any pair of equilibrium thresholds $\underline{\mu}_t^H > \underline{\mu}_t^L$. The claim follows from showing that the function

$$F\left(\underline{\mu}\right) = \frac{1-\Phi\left(\underline{\mu};\sigma^2,\sigma^2\right)}{1-\Phi\left(\underline{\mu};0,\sigma^2\right)}$$

is increasing in $\underline{\mu}$. We have

$$F'\left(\underline{\mu}\right) = \frac{-\phi\left(\underline{\mu};\sigma^2,\sigma^2\right)\left[1-\Phi\left(\underline{\mu};0,\sigma^2\right)\right]+\left[1-\Phi\left(\underline{\mu};\sigma^2,\sigma^2\right)\right]\phi\left(\underline{\mu};0,\sigma^2\right)}{\left[1-\Phi\left(\underline{\mu};0,\sigma^2\right)\right]^2} > 0$$

if and only if

$$-\phi\left(\underline{\mu};\sigma^2,\sigma^2\right)\left[1-\Phi\left(\underline{\mu};0,\sigma^2\right)\right]+\left[1-\Phi\left(\underline{\mu};\sigma^2,\sigma^2\right)\right]\phi\left(\underline{\mu};0,\sigma^2\right) > 0$$

or

$$\frac{\phi\left(\underline{\mu};0,\sigma^2\right)}{1-\Phi\left(\underline{\mu};0,\sigma^2\right)} > \frac{\phi\left(\underline{\mu};\sigma^2,\sigma^2\right)}{1-\Phi\left(\underline{\mu};\sigma^2,\sigma^2\right)} = \frac{\phi\left(\underline{\mu}-\sigma^2;0,\sigma^2\right)}{1-\Phi\left(\underline{\mu}-\sigma^2;0,\sigma^2\right)},$$

where the last equality uses the properties of the normal distribution. The ratio $\phi\left(\underline{\mu};0,\sigma^2\right)/\left[1-\Phi\left(\underline{\mu};0,\sigma^2\right)\right]$ is the hazard function of the normal distribution, which is increasing in $\underline{\mu}$. Thus, this inequality is always satisfied, which confirms the claim. Q.E.D.

Proof of Proposition 6:

We consider the more general version in which $G_t = (1-m_t)^\alpha e^g$. In this case, output in tax regime k at time t is

$$Y_{t+1} = \left(1-\Phi\left(\underline{\mu}_t^k;\sigma_\mu^2,\sigma_\mu^2\right)\right) e^{\overline{\mu}+\frac{1}{2}\sigma_\mu^2}\Phi\left(\underline{\mu}_t^k;0,\sigma_\mu^2\right)^\alpha e^g e^{\varepsilon_{t+1}}.$$

Therefore,

$$\begin{aligned} E\left[Y_{t+1}|H\right] &= \left(1-\Phi\left(\underline{\mu}_t^H;\sigma_\mu^2,\sigma_\mu^2\right)\right) e^{\overline{\mu}+\frac{1}{2}\sigma_\mu^2}\Phi\left(\underline{\mu}_t^H;0,\sigma_\mu^2\right)^\alpha e^g \\ &> \left(1-\Phi\left(\underline{\mu}_t^L;\sigma_\mu^2,\sigma_\mu^2\right)\right) e^{\overline{\mu}+\frac{1}{2}\sigma_\mu^2}\Phi\left(\underline{\mu}_t^L;0,\sigma_\mu^2\right)^\alpha e^g = E\left[Y_{t+1}|L\right] \end{aligned}$$

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holds if and only if

$$\left(\frac{\Phi\left(\underline{\mu}_t^H;0,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}_t^L;0,\sigma_\mu^2\right)}\right)^\alpha > \frac{1-\Phi\left(\underline{\mu}_t^L;\sigma_\mu^2,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}_t^H;\sigma_\mu^2,\sigma_\mu^2\right)} . \tag{A17}$$

This condition is never satisfied for $\alpha = 0$, as $\Phi\left(\underline{\mu}_t;\sigma_\mu^2,\sigma_\mu^2\right)$ is increasing in $\underline{\mu}_t$ and $\underline{\mu}_t^H > \underline{\mu}_t^L$. We now show that it is always satisfied for $\alpha = 1$. Indeed, under the assumption of equilibrium symmetry, $\underline{\mu}_t^L = -\underline{\mu}_t^H$, that is, cutoffs are symmetric around 0, which implies $m_t^H < 0.5 < m_t^L$ are symmetric around 0.5. Hence, we can rewrite the term to the left as

$$\frac{1-\Phi\left(\underline{\mu}_t^L;0,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}_t^H;0,\sigma_\mu^2\right)} > \frac{1-\Phi\left(\underline{\mu}_t^L;\sigma_\mu^2,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}_t^H;\sigma_\mu^2,\sigma_\mu^2\right)}$$

or

$$F\left(\underline{\mu}_t^H\right) = \frac{1-\Phi\left(\underline{\mu}_t^H;\sigma_\mu^2,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}_t^H;0,\sigma_\mu^2\right)} > \frac{1-\Phi\left(\underline{\mu}_t^L;\sigma_\mu^2,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}_t^L;0,\sigma_\mu^2\right)} = F\left(\underline{\mu}_t^L\right) .$$

The claim follows from the proof of Proposition 5, which shows that $F\left(\underline{\mu}\right) = \frac{1-\Phi\left(\underline{\mu};\sigma_\mu^2,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu};0,\sigma_\mu^2\right)}$ is an increasing function of $\underline{\mu}$ and the fact that $\underline{\mu}_t^H > \underline{\mu}_t^L$.

Finally, the above arguments show that condition (A17) holds for $\alpha = 1$ and does not hold for $\alpha = 0$. By continuity, there exists a value $\underline{\alpha} < 1$ such that the condition always holds for $\alpha > \underline{\alpha}$. This is the value of α for which condition (A17) holds with equality, that is,

$$\underline{\alpha} = \frac{\log\left(\frac{1-\Phi\left(\underline{\mu}_t^L;\sigma_\mu^2,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}_t^H;\sigma_\mu^2,\sigma_\mu^2\right)}\right)}{\log\left(\frac{\Phi\left(\underline{\mu}_t^H;0,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}_t^L;0,\sigma_\mu^2\right)}\right)} .$$

Q.E.D.

Proposition A2: The welfare-maximizing allocation of human capital is

$$m_t = 1 - \Phi\left(\frac{1}{2}\sigma_\mu^2, 0; \sigma_\mu^2\right) < 0.5 ,$$

which corresponds to the threshold $\underline{\mu}^* = \frac{1}{2}\sigma_\mu^2$ determining which agents become entrepreneurs.

Proof of Proposition A2.

Let $\underline{\mu}^*$ be the threshold maximizing output. This is given by

$$E[Y] = \left(1 - \Phi\left(\underline{\mu}^*;\sigma_\mu^2,\sigma_\mu^2\right)\right) e^{\overline{\mu}+\sigma_\mu^2}\Phi\left(\underline{\mu}^*;0,\sigma_\mu^2\right) e^g .$$

The maximum over $\underline{\mu}^*$ can be obtained from the first order conditions:

$$\frac{\partial E[Y]}{\partial \underline{\mu}^*} = -\phi\left(\underline{\mu}^*;\sigma_\mu^2,\sigma_\mu^2\right)e^{\overline{\mu}+\sigma_\mu^2}\Phi\left(\underline{\mu}^*;0,\sigma_\mu^2\right)e^g + \left(1-\Phi\left(\underline{\mu}^*;\sigma_\mu^2,\sigma_\mu^2\right)\right)e^{\overline{\mu}+\sigma_\mu^2}\phi\left(\underline{\mu}^*;0,\sigma_\mu^2\right)e^g = 0\,,$$

or

$$\left(1-\Phi\left(\underline{\mu}^*;\sigma_\mu^2,\sigma_\mu^2\right)\right)\phi\left(\underline{\mu}^*;0,\sigma_\mu^2\right) = \phi\left(\underline{\mu}^*;\sigma_\mu^2,\sigma_\mu^2\right)\Phi\left(\underline{\mu}^*;0,\sigma_\mu^2\right)\ ,$$

or

$$\frac{\phi\left(\underline{\mu}^*;0,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^*;0,\sigma_\mu^2\right)} = \frac{\phi\left(\underline{\mu}^*;\sigma_\mu^2,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}^*;\sigma_\mu^2,\sigma_\mu^2\right)}\ .$$

The density $\phi\left(\underline{\mu}^*;0,\sigma_\mu^2\right)$ is symmetric around zero; therefore,

$$\frac{\phi\left(-\underline{\mu}^*;0,\sigma_\mu^2\right)}{1-\Phi\left(-\underline{\mu}^*;0,\sigma_\mu^2\right)} = \frac{\phi\left(\underline{\mu}^*;\sigma_\mu^2,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}^*;\sigma_\mu^2,\sigma_\mu^2\right)} = \frac{\phi\left(\underline{\mu}^*-\sigma_\mu^2;0,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}^*-\sigma_\mu^2;0,\sigma_\mu^2\right)}\ .$$

Note that these are hazard rates, which are always strictly increasing functions. Therefore, this equality can hold if and only if

$$-\underline{\mu}^* = \underline{\mu}^* - \sigma_\mu^2\ ,$$

or

$$\underline{\mu}^* = \frac{1}{2}\sigma_\mu^2\ .$$

Therefore, the socially optimal allocation has

$$m_t = 1-\Phi\left(\frac{1}{2}\sigma_\mu^2, 0; \sigma_\mu^2\right) < 0.5\ .$$

Q.E.D.

Proof of Proposition 7:

Because $\gamma^L < \underline{\gamma}$ and $\gamma^H > \overline{\gamma}$, there are only two equilibrium masses of agents, $m^L = 1 - \Phi\left(\underline{\mu}^L, 0, \sigma_\mu^2\right) > 0.5$ and $m^H = 1-\Phi\left(\underline{\mu}^H, 0, \sigma_\mu^2\right) < 0.5$. Denoting $y_t = \log(Y_t)$, we have

$$E\left[y_{t+1}|k\right] = g + \log\left(\Phi\left(\underline{\mu}^k;0,\sigma_\mu^2\right)\right) + \log\left(1-\Phi\left(\underline{\mu}^k;\sigma_\mu^2,\sigma_\mu^2\right)\right) + \overline{\mu} + \frac{1}{2}\sigma_\mu^2 - \frac{1}{2}\sigma^2\ ,$$

where we use

$$\varepsilon_{t+1} \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right)\ .$$

Recall that Proposition 6 shows

$$E\left[y_{t+1}|H\right] > E\left[y_{t+1}|L\right]$$

and denote the difference

$$\begin{aligned}
\overline{\Delta y} &= E\left[y_{t+1}|H\right] - E\left[y_{t+1}|L\right] \\
&= \log\left(\frac{1-\Phi\left(\underline{\mu}^H;\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^H;0,\sigma_\mu^2\right)}\right) - \log\left(\frac{1-\Phi\left(\underline{\mu}^L;\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^L;0,\sigma_\mu^2\right)}\right) \\
&> 0\ .
\end{aligned}$$

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Let f be the fraction of time spent in the L government, on average, in equilibrium and define the unconditional average as

$$\overline{y} = E[y] - \frac{1}{2}\sigma^2 = (1-f)\,E[y_{t+1}|H] + fE[y_{t+1}|L] - \frac{1}{2}\sigma^2 .$$

Suppose party H is in power. The probability of a regime change (from H to L) is

$$\begin{aligned}
\lambda^{HL} &= \Pr(y_{t+1} > \overline{y}) = \Pr(E[y_{t+1}|H] + \varepsilon_{t+1} > \overline{y}) \\
&= \Pr(\varepsilon_{t+1} > \overline{y} - E[y_{t+1}|H]) \\
&= \Pr\left(\varepsilon_{t+1} > -f[E[y_{t+1}|H] - E[y_{t+1}|L]] - \frac{1}{2}\sigma^2\right) \\
&= 1 - \Phi\left(-f\overline{\Delta y} - \frac{1}{2}\sigma^2, -\frac{1}{2}\sigma^2, \sigma^2\right) \\
&= 1 - \Phi\left(-f\overline{\Delta y}, 0, \sigma^2\right) \\
&> 0.5 .
\end{aligned}$$

Now suppose party L is in power. Note that

$$E[y_{t+1}|L] < \overline{y} .$$

Therefore, the probability of a regime change (from L to H) is

$$\begin{aligned}
\lambda^{LH} &= \Pr(y_{t+1} < \overline{y}) = \Pr(E[y_{t+1}|L] + \varepsilon_{t+1} < \overline{y}) \\
&= \Pr(\varepsilon_{t+1} < \overline{y} - E[y_{t+1}|L]) \\
&= \Pr\left(\varepsilon_{t+1} < (1-f)\,E[y_{t+1}|H] + fE[y_{t+1}|L] - \frac{1}{2}\sigma^2 - E[y_{t+1}|L]\right) \\
&= \Pr\left(\varepsilon_{t+1} < (1-f)\,[E[y_{t+1}|H] - E[y_{t+1}|L]] - \frac{1}{2}\sigma^2\right) \\
&= \Phi\left((1-f)\,\overline{\Delta y} - \frac{1}{2}\sigma^2; -\frac{1}{2}\sigma^2, \sigma^2\right) \\
&= \Phi\left((1-f)\,\overline{\Delta y}; 0, \sigma^2\right) > 0 .
\end{aligned}$$

The ergodic distribution of regime L implies

$$f = \frac{\lambda^{HL}}{\lambda^{HL} + \lambda^{LH}} = \frac{\Phi\left(f\overline{\Delta y}, 0, \sigma^2\right)}{\Phi\left(f\overline{\Delta y}, 0, \sigma^2\right) + \Phi\left((1-f)\,\overline{\Delta y}; 0, \sigma^2\right)} .$$

To show that $f = 0.5$ is the unique solution to this equation, rewrite the equation as

$$f\Phi\left((1-f)\,\overline{\Delta y}; 0, \sigma^2\right) = \Phi\left(f\overline{\Delta y}; 0, \sigma^2\right)(1-f)$$

The symmetry of the problem shows that there is only one solution, $f = 0.5$. If $f > 0.5$, then the left-hand side is greater than 0.25 while the right-hand side is smaller than 0.25, and vice versa. Substituting $f = 1/2$ in $\lambda^{H,L}$ and $\lambda^{L,H}$ yields the claim. Q.E.D.

Example 2: Three values of risk aversion.

Define the function

$$\gamma_t = \begin{cases} \gamma^H & \text{if} \quad y_t < \underline{y} \\ \gamma^M & \text{if} \quad \underline{y} < y_t < \overline{y} \\ \gamma^L & \text{if} \quad y_t > \overline{y} \end{cases}$$

with $\gamma^H > \overline{\gamma} > \gamma^M > \underline{\gamma} > \gamma^L$. That is, when log output $y_t = \log(Y_t)$ is very low, agents' risk aversion is sufficiently high to put the economy in the unique H-tax equilibrium. When output is very high, risk aversion is sufficiently low to put the economy in the unique L-tax equilibrium. For intermediate output, risk aversion is intermediate, leading to two possible equilibria. In this case, there is probability $p = 0.5$ that H-tax or L-tax equilibria will be selected. We can select the output thresholds $\underline{y}$ and $\overline{y}$ to give equal unconditional probabilities to the H and L equilibria, given that in the data the two types of administrations tend to have roughly the same presence in office.

We now compute the transition probabilities, for given $\underline{y}$ and $\overline{y}$. Recall that aggregate output under tax regime τ^k and risk aversion γ_t is

$$\begin{aligned} Y_{t+1} &= e^g \left(1 - m_t^k\right) \left(1 - \Phi\left(\underline{\mu}^k\left(\gamma_t\right), \sigma_\mu^2, \sigma_\mu^2\right)\right) e^{\frac{1}{2}\sigma_\mu^2 + \varepsilon_{t+1}} \\ &= e^g \, \Phi\left(\underline{\mu}^k\left(\gamma_t\right), 0, \sigma_\mu^2\right) \left(1 - \Phi\left(\underline{\mu}^k\left(\gamma_t\right), \sigma_\mu^2, \sigma_\mu^2 .\right)\right) e^{\frac{1}{2}\sigma_\mu^2 + \varepsilon_{t+1}} . \end{aligned}$$

Therefore, if $\gamma_t = \gamma^H > \overline{\gamma}$, then there is a unique equilibrium at time t and log final output is

$$\begin{aligned} y_{t+1} &= g + \log\left[\Phi\left(\underline{\mu}^H\left(\gamma^H\right), 0, \sigma_\mu^2\right)\right] + \log\left[1 - \Phi\left(\underline{\mu}^H\left(\gamma^H\right), \sigma_\mu^2, \sigma_\mu^2\right)\right] + \frac{1}{2}\sigma_\mu^2 + \varepsilon_{t+1} \\ &\sim N\left(\mu_y^{HH}, \sigma^2\right) , \end{aligned}$$

where

$$\mu_y^{HH} = g + \log\left[\Phi\left(\underline{\mu}^H\left(\gamma^H\right), 0, \sigma_\mu^2\right)\right] + \log\left[1 - \Phi\left(\underline{\mu}^H\left(\gamma^H\right), \sigma_\mu^2, \sigma_\mu^2\right)\right] + \frac{1}{2}\sigma_\mu^2 - \frac{1}{2}\sigma^2 .$$

Similarly, if $\gamma_t = \gamma^L < \underline{\gamma}$, then there is a unique equilibrium at t and log final output is

$$y_{t+1} \sim N\left(\mu_y^{LL}, \sigma^2\right) ,$$

where

$$\mu_y^{LL} = g + \log\left[\Phi\left(\underline{\mu}^L\left(\gamma^L\right), 0, \sigma_\mu^2\right)\right] + \log\left[1 - \Phi\left(\underline{\mu}^L\left(\gamma^L\right), \sigma_\mu^2, \sigma_\mu^2\right)\right] + \frac{1}{2}\sigma_\mu^2 - \frac{1}{2}\sigma^2 .$$

We know from previous results that $\mu_y^{LL} < \mu_y^{HH}$.

If instead $\gamma_t = \gamma^M \in (\underline{\gamma}, \overline{\gamma})$, then there are two equilibria, and

$$y_{t+1} \sim \begin{cases} N\left(\mu_y^{LM}, \sigma^2\right) & \text{with } p = 0.5 \\ N\left(\mu_y^{HM}, \sigma^2\right) & \text{with } p = 0.5 \end{cases}$$

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where

$$\begin{aligned}
\mu_y^{HM} &= g + \log\left[\Phi\left(\underline{\mu}^H\left(\gamma^M\right), 0, \sigma_\mu^2\right)\right] + \log\left[1 - \Phi\left(\underline{\mu}^H\left(\gamma^M\right), \sigma_\mu^2, \sigma_\mu^2\right)\right] + \frac{1}{2}\sigma_\mu^2 - \frac{1}{2}\sigma^2 \\
\mu_y^{LM} &= g + \log\left[\Phi\left(\underline{\mu}^L\left(\gamma^M\right), 0, \sigma_\mu^2\right)\right] + \log\left[1 - \Phi\left(\underline{\mu}^L\left(\gamma^M\right), \sigma_\mu^2, \sigma_\mu^2\right)\right] + \frac{1}{2}\sigma_\mu^2 - \frac{1}{2}\sigma^2 .
\end{aligned}$$

Therefore, conditional on each of the four possible events $(\tau, \gamma) = (LL, LM, HM, HH)$, the distribution of output is normal. The model is thus a four-state regime shift model whose transition probabilities depend on $\underline{y}$ and $\overline{y}$. In particular, we find the following transition probability matrix $\mathbf{\Lambda}$:

$$\text{Transition Matrix } \mathbf{\Lambda} = \left[\begin{array}{c|cccc}
(\tau,\gamma) & LL & LM & HM & HH \\
\hline
LL & 1-\Phi\left(\overline{y}, \mu_y^{LL}, \sigma^2\right) & 0.5\left[\Phi\left(\overline{y}, \mu_y^{LL}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{LL}, \sigma^2\right)\right] & 0.5\left[\Phi\left(\overline{y}, \mu_y^{LL}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{LL}, \sigma^2\right)\right] & \Phi\left(\underline{y}, \mu_y^{LL}, \sigma^2\right) \\
LM & 1-\Phi\left(\overline{y}, \mu_y^{LM}, \sigma^2\right) & 0.5\left[\Phi\left(\overline{y}, \mu_y^{LM}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{LM}, \sigma^2\right)\right] & 0.5\left[\Phi\left(\overline{y}, \mu_y^{LM}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{LM}, \sigma^2\right)\right] & \Phi\left(\underline{y}, \mu_y^{LM}, \sigma^2\right) \\
HM & 1-\Phi\left(\overline{y}, \mu_y^{HM}, \sigma^2\right) & 0.5\left[\Phi\left(\overline{y}, \mu_y^{HM}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{HM}, \sigma^2\right)\right] & 0.5\left[\Phi\left(\overline{y}, \mu_y^{HM}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{HM}, \sigma^2\right)\right] & \Phi\left(\underline{y}, \mu_y^{HM}, \sigma^2\right) \\
HH & 1-\Phi\left(\overline{y}, \mu_y^{HH}, \sigma^2\right) & 0.5\left[\Phi\left(\overline{y}, \mu_y^{HH}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{HH}, \sigma^2\right)\right] & 0.5\left[\Phi\left(\overline{y}, \mu_y^{HH}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{HH}, \sigma^2\right)\right] & \Phi\left(\underline{y}, \mu_y^{HH}, \sigma^2\right)
\end{array}\right].$$

There are four cases of interest:

1. If $(\tau, \gamma) = (H, H)$, then

$$\begin{aligned}
\Pr(LL) &= \Pr\left(y_{t+1} > \overline{y}\right) = 1 - \Phi\left(\overline{y}, \mu_y^{HH}, \sigma^2\right) \\
\Pr(HL) &= \Pr(LM) = 0.5\Pr\left(\underline{y} < y_{t+1} < \overline{y}\right) = 0.5\left[\Phi\left(\overline{y}, \mu_y^{HH}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{HH}, \sigma^2\right)\right] \\
\Pr(HH) &= \Pr\left(y_{t+1} < \underline{y}\right) = \Phi\left(\underline{y}, \mu_y^{HH}, \sigma^2\right) .
\end{aligned}$$

2. If $(\tau, \gamma) = (L, L)$, then

$$\begin{aligned}
\Pr(LL) &= \Pr\left(y_{t+1} > \overline{y}\right) = 1 - \Phi\left(\overline{y}, \mu_y^{LL}, \sigma^2\right) \\
\Pr(HL) &= \Pr(LM) = 0.5\Pr\left(\underline{y} < y_{t+1} < \overline{y}\right) = 0.5\left[\Phi\left(\overline{y}, \mu_y^{LL}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{LL}, \sigma^2\right)\right] \\
\Pr(HH) &= \Pr\left(y_{t+1} < \underline{y}\right) = \Phi\left(\underline{y}, \mu_y^{LL}, \sigma^2\right) .
\end{aligned}$$

3. If $(\tau, \gamma) = (H, M)$, then

$$\begin{aligned}
\Pr(LL) &= \Pr\left(y_{t+1} > \overline{y}\right) = 1 - \Phi\left(\overline{y}, \mu_y^{HM}, \sigma^2\right) \\
\Pr(HL) &= \Pr(LM) = 0.5\Pr\left(\underline{y} < y_{t+1} < \overline{y}\right) = 0.5\left[\Phi\left(\overline{y}, \mu_y^{HM}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{HM}, \sigma^2\right)\right] \\
\Pr(HH) &= \Pr\left(y_{t+1} < \underline{y}\right) = \Phi\left(\underline{y}, \mu_y^{HM}, \sigma^2\right) .
\end{aligned}$$

4. If $(\tau, \gamma) = (L, M)$, then

$$\begin{aligned}
\Pr(LL) &= \Pr\left(y_{t+1} > \overline{y}\right) = 1 - \Phi\left(\overline{y}, \mu_y^{LM}, \sigma^2\right) \\
\Pr(HL) &= \Pr(LM) = 0.5\Pr\left(\underline{y} < y_{t+1} < \overline{y}\right) = 0.5\left[\Phi\left(\overline{y}, \mu_y^{LM}, \sigma^2\right) - \Phi\left(\underline{y}, \mu_y^{LM}, \sigma^2\right)\right] \\
\Pr(HH) &= \Pr\left(y_{t+1} < \underline{y}\right) = \Phi\left(\underline{y}, \mu_y^{LM}, \sigma^2\right) .
\end{aligned}$$

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Given the matrix, we can compute the ergodic distribution satisfying stationarity:

$$\pi = \Lambda^{\infty} .$$

Given the ergodic distribution, we obtain

$$\begin{aligned}
E\left[r|\tau^{H}\right] &= \frac{\pi^{HH}}{\pi^{HH}+\pi^{HM}}E\left[r|\tau^{H},\gamma^{H}\right]+\frac{\pi^{HM}}{\pi^{HH}+\pi^{HM}}E\left[r|\tau^{H},\gamma^{M}\right] \\
&= \frac{\pi^{HH}}{\pi^{HH}+\pi^{HM}}\left(\gamma^{H}-\frac{1}{2}\right)\sigma^{2}+\frac{\pi^{HH}}{\pi^{HH}+\pi^{HM}}\left(\gamma^{M}-\frac{1}{2}\right)\sigma^{2} \\
E\left[r|\tau^{H}\right] &= \frac{\pi^{LM}}{\pi^{LM}+\pi^{LL}}E\left[r|\tau^{L},\gamma^{M}\right]+\frac{\pi^{LL}}{\pi^{LM}+\pi^{LL}}E\left[r|\tau^{L},\gamma^{L}\right] \\
&= \frac{\pi^{LM}}{\pi^{LM}+\pi^{LL}}\left(\gamma^{H}-\frac{1}{2}\right)\sigma^{2}+\frac{\pi^{LL}}{\pi^{LM}+\pi^{LL}}\left(\gamma^{M}-\frac{1}{2}\right)\sigma^{2} ,
\end{aligned}$$

and, similarly,

$$\begin{aligned}
E\left[Y|\tau^{H}\right] &= \frac{\pi^{HH}}{\pi^{HH}+\pi^{HM}}E\left[Y|\tau^{H},\gamma^{H}\right]+\frac{\pi^{HM}}{\pi^{HH}+\pi^{HM}}E\left[Y|\tau^{H},\gamma^{M}\right] \\
&= \frac{\pi^{HH}}{\pi^{HH}+\pi^{HM}}e^{\mu_{y}^{HH}+\frac{1}{2}\sigma^{2}}+\frac{\pi^{HH}}{\pi^{HH}+\pi^{HM}}e^{\mu_{y}^{HM}+\frac{1}{2}\sigma^{2}} \\
E\left[Y|\tau^{H}\right] &= \frac{\pi^{LM}}{\pi^{LM}+\pi^{LL}}E\left[Y|\tau^{L},\gamma^{M}\right]+\frac{\pi^{LL}}{\pi^{LM}+\pi^{LL}}E\left[Y|\tau^{L},\gamma^{L}\right] \\
&= \frac{\pi^{LM}}{\pi^{LM}+\pi^{LL}}e^{\mu_{y}^{LH}+\frac{1}{2}\sigma^{2}}+\frac{\pi^{LL}}{\pi^{LM}+\pi^{LL}}e^{\mu_{y}^{LL}+\frac{1}{2}\sigma^{2}} .
\end{aligned}$$

We choose the two thresholds $\underline{y}$ and $\overline{y}$ so as to obtain reasonable values for expected returns conditional on the two regimes, and some amount of symmetry so as to have $\Pr\left(\tau^{H}\right)\approx\Pr\left(\tau^{L}\right)\approx 0.5$. In particular, for the parameter values in the text, we choose $\underline{y}=-2.0979$ and $\overline{y}=-1.7539$. The transition matrix is

$$\mathbf{\Lambda}=\left[\begin{array}{c|cccc}
(\tau,\gamma) & LL & LM & HM & HH \\ \hline
LL & 0.3032 & 0.1658 & 0.1658 & 0.3651 \\
LM & 0.3340 & 0.1664 & 0.1664 & 0.3332 \\
HM & 0.3458 & 0.1663 & 0.1663 & 0.3216 \\
HH & 0.3556 & 0.1661 & 0.1661 & 0.3122
\end{array}\right] .$$

The unconditional probability to transit from H to L and from L to H are, respectively,

$$Prob(\tau_{t+1}=\tau^{H}|\tau_{t}=\tau^{L})=0.5205; \quad Prob(\tau_{t+1}=\tau^{L}|\tau_{t}=\tau^{H})=0.5185 .$$

Proof of Proposition 8:

The proof for a mixed equilibrium is complicated by the fact that tax uncertainty at time t affects the state price density and hence the equilibrium price of the stock when the entrepreneur issues shares. Luckily, the same arguments as in Pástor and Veronesi (2016) go through, as we now verify.

Let $\mathcal{I}$ be the set of agents who choose to become entrepreneurs. Let M_t^i be the market value of firm i at time t. The net-of-tax dividend paid by firm i is $D_{t+1}^i = (1-\tau_{t+})\, G_t e^{\mu_i+\varepsilon_{t+1}+\varepsilon_{i,t+1}}$, where we use the subscript $t+$ to emphasize that this rate is not known at time t when agents make their occupation choice.

Proposition A4: In the mixed equilibrium, the state price density at the end of period t (beginning of $t+1$) is

$$\pi_{t+1} = h_t \left(1-\tau_{t+}\right)^{-\gamma} e^{-\gamma\varepsilon_{t+1}}$$

for a constant h_t known at time t. Denote the two stochastic discount factors at t and $t+$ as

$$\pi_{t,t+1} = \frac{\pi_{t+1}}{E_t\left[\pi_{t+1}\right]}; \quad \pi_{t+,t+1} = \frac{\pi_{t+1}}{E_{t+}\left[\pi_{t+1}\right]} \ , \tag{A18}$$

where $t+$ is the announcement of the party winning the election. Then the asset prices satisfy

$$M_t^i = E_t\left[\pi_{t,t+1} D_{t+1}^i\right] \tag{A19}$$

$$M_{t+}^i = E_{t+}\left[\pi_{t+,t+1} D_{t+1}^i\right] \ , \tag{A20}$$

In addition, entrepreneur i's consumption at time $t+1$ is

$$C_{i,t+1}^{yes} = (1-\tau_{t+})\, G_t e^{\mu_i+\varepsilon_{t+1}}\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right] \ .$$

Proof of Proposition A4:

We verify below that the state price density depends on only two shocks, ε_{t+1} and τ_{t+}:

$$\pi_{t+1} = \pi\left(\varepsilon_{t+1}, \tau_{t+}\right) \ ,$$

for some function $\pi\left(\varepsilon_{t+1}, \tau_{t+}\right)$.

Given the conjectured state price density and the definition of the stochastic discount factor $\pi_{t,t+1} = \pi_{t+1}/E_t\left[\pi_{t+1}\right]$, we can compute the price of each asset at time t as

$$\begin{aligned} M_t^i &= \frac{E_t\left[\pi_{t+1}\left(1-\tau_{t+}\right) G_t e^{\mu_i+\varepsilon_{t+1}+\varepsilon_{i,t+1}}\right]}{E_t\left[\pi_{t+1}\right]} = e^{\mu_i}\frac{E_t\left[\pi\left(\varepsilon_{t+1},\tau_{t+}\right)\left(1-\tau_{t+1}\right) e^{\varepsilon_{t+1}+\varepsilon_{it+1}}\right]}{E_t\left[\pi_{t+1}\right]} \quad \text{(A21)} \\ &= e^{\mu_i}\frac{E_t\left[\pi\left(\varepsilon_{t+1},\tau_{t+}\right)\left(1-\tau_{t+}\right) e^{\varepsilon_{t+1}}\right]}{E_t\left[\pi_{t+1}\right]} E_t\left[e^{\varepsilon_{i,t+1}}\right] \\ &= e^{\mu_i}\frac{E_t\left[\pi\left(\varepsilon_{t+1},\tau_{t+}\right)\left(1-\tau_{t+}\right) e^{\varepsilon_{t+1}}\right]}{E_t\left[\pi_{t+1}\right]} \\ &= e^{\mu_i} Z_t \ , \end{aligned} \tag{A22}$$

where we define Z_t as

$$Z_t = \frac{E_t\left[\pi\left(\varepsilon_{t+1}, \tau_{t+}\right)\left(1-\tau_{t+}\right)e^{\varepsilon_{t+1}}\right]}{E_t\left[\pi_{t+1}\right]} ,$$

which is the time-t price of a security with payoff $\left(1-\tau_{t+}\right)e^{\varepsilon_{t+1}}$ at time $t+1$. For later reference, note that the aggregate market value of the market portfolio is

$$M_t^P = \int_{\mathcal{I}} M_t^i di = Z_t \int_{\mathcal{I}} e^{\mu_i} di$$

and the total dividend

$$D_{t+1}^{Mkt} = \left(1-\tau_{t+}\right)e^{\varepsilon_{t+1}} \int_{\mathcal{I}} e^{\mu_i} di ,$$

so that the market return is

$$R^{Mkt} = \frac{D_{t+1}^{Mkt}}{M_t^P} - 1 = \frac{\left(1-\tau_{t+}\right)e^{\varepsilon_{t+1}}}{Z_t} - 1 .$$

In the arguments below, we will also make use of the fact that each individual stock is infinitesimal, that is, removing one stock from a continuum does not change the value of the market portfolio. In particular, we will use the following equality for $j \neq i$:

$$\int_{\mathcal{I}\backslash i} M_t^j dj = \int_{\mathcal{I}\backslash j} M_t^i di .$$

Consider the budget constraint of each entrepreneur i. At time t, entrepreneur i issues $1-\theta$ shares of his own firm i. From the proceeds, the entrepreneur purchases N_t^{ij} shares of firm j and N_t^{i0} bonds. As we show below, if unrestricted ($\theta = 0$), each entrepreneur would sell all of his firm and purchase the market portfolio, which would entail an infinitesimal position in his own firm. The θ constraint is always binding; for any given θ, each entrepreneur restricts his holdings of his own firm to exactly θ shares. All quantities are expressed in terms of our numeraire, which is the zero-coupon bond with maturity $t+1$ that is a claim to one unit of capital at $t+1$. The bond price is thus equal to one at both times t and $t+1$. Bonds are in zero net supply. The budget constraint is

$$(1-\theta) M_t^i = \int_{\mathcal{I}\backslash i} N_t^{ij} M_t^j dj + N_t^{i0} . \tag{A23}$$

Within each period, agents only trade once, at time t, and they hold their positions until time $t+1$. At time $t+1$, agent i's consumption is

$$C_{i,t+1} = \theta D_{t+1}^i + \int_{\mathcal{I}\backslash i} N_t^{ij} D_{t+1}^j dj + N_t^{i0} . \tag{A24}$$

As we shall see, in equilibrium $C_{i,t+1} > 0$ with probability one.

Before we analyze the optimal choice of each individual, we consider the market-clearing condition. Each entrepreneur j issues exactly $1-\theta$ shares. Therefore, we must have that in equilibrium all shares issued are bought by somebody. That is, the sum of all the j shares bought by agents i must equal $1-\theta$:

$$1-\theta = \int_{\mathcal{I}\backslash j} N_t^{ij} di \; .$$

Compared to the budget equation, the integral here is over i and not over j. The bond market must clear, too, and given that bonds are in zero net supply, we must have

$$\int_{\mathcal{I}} N_t^{i0} di = 0 \; .$$

The utility function of entrepreneur $i \in \mathcal{I}$ is:*

$$\begin{aligned} E\left[U\left(C_{i,t+1}\right)\right] &= \frac{1}{1-\gamma} E\left[\left(C_{i,t+1}\right)^{1-\gamma}\right] \\ &= \frac{1}{1-\gamma} E\left[\left(\theta D_{t+1}^i + \int_{I\backslash i} N_t^{ij} D_{t+1}^j dj + N_t^{i0}\right)^{1-\gamma}\right] . \end{aligned}$$

Consider again the budget equation of agent i, now rewritten as

$$\left(1-\theta\right) M_t^i - \int_{\mathcal{I}\backslash i} N_t^{ij} M_t^j dj = N_t^{i0} \; .$$

Substitute for N_t^{i0} in the utility function to find

$$E\left[U\left(C_{i,t+1}\right)\right] = \frac{1}{1-\gamma} E\left[\left(\theta\left(D_{t+1}^i - M_t^i\right) + \int_{\mathcal{I}\backslash i} N_t^{ij}\left(D_{t+1}^j - M_t^j\right) dj + M_t^i\right)^{1-\gamma}\right] .$$

The first-order conditions (FOC) with respect to N_t^{ij} are

$$E\left[\left(\theta\left(D_{t+1}^i - M_t^i\right) + \int_{\mathcal{I}\backslash i} N_t^{ij}\left(D_{t+1}^j - M_t^j\right) dj + M_t^i\right)^{-\gamma}\left(D_{t+1}^j - M_t^j\right)\right] = 0 \; .$$

We can rewrite this expression as

$$E\left[\left(\theta\left(\frac{D_{t+1}^i}{M_t^i} - 1\right) M_t^i + M_t^i \int_{\mathcal{I}\backslash i} \frac{N_t^{ij} M_t^j}{M_t^i}\left(\frac{D_{t+1}^j}{M_t^j} - 1\right) dj + M_t^i\right)^{-\gamma}\left(D_{t+1}^j - M_t^j\right)\right] = 0 \; .$$

Factoring M_t^i out of the expectation and simplifying, we can rewrite the FOC as

$$E\left[\left(\theta\left(\frac{D_{t+1}^i}{M_t^i} - 1\right) + \int_{\mathcal{I}\backslash i} \frac{N_t^{ij} M_t^j}{M_t^i}\left(\frac{D_{t+1}^j}{M_t^j} - 1\right) dj + 1\right)^{-\gamma}\left(\frac{D_{t+1}^j}{M_t^j} - 1\right)\right] = 0 \; .$$

*The argument below also applies to agents with $\gamma_i = 1$, that is, log utility investors, as the main equations only depend on marginal utility $C_{i,T}^{-\gamma_i}$, which are independent of whether $\gamma_i = 1$ or not.

Define ω_t^{ij} as

$$\omega_t^{ij} = \frac{N_t^{ij} M_t^j}{M_t^i} \; .$$

Note that for every j, the net-of-tax arithmetic return on investment is

$$R_{t+1}^j = \frac{D_{t+1}^j}{M_t^j} - 1 = \frac{(1-\tau_{t+})\, e^{\mu_j + \varepsilon_{j,t+1} + \varepsilon_{t+1}}}{e^{\mu_j} Z_t} - 1 = \frac{(1-\tau_{t+})\, e^{\varepsilon_{j,t+1} + \varepsilon_{t+1}}}{Z_t} - 1 \; .$$

That is, the return R^j is the same across firms, except for the realization of the idiosyncratic shock $\varepsilon_{j,t+1}$. All stocks have the same expected return equal to

$$E_t\left[R_{t+1}^j\right] = \left(1 - E_t\left[\tau_{t+}\right]\right) Z_t^{-1} - 1 \; .$$

We can rewrite the FOC of agent i as

$$E\left[\left(\theta R_{t,t+1}^i + \int_{\mathcal{I}\setminus i} \omega^{ij} R_{t,t+1}^j dj + 1\right)^{-\gamma} R_{t,t+1}^j\right] = 0 \; .$$

From the above discussion, all R_{t+1}^j have the same risk-return characteristics. Therefore, the properties of the expectation are the same, and hence the FOC for each agent i or i' are identical. It follows that $\omega^{ij} = \omega^{i'j} = \omega^j$ for all i:

$$\omega_t^{ij} = \omega_t^j \; .$$

That is, each agent i invests the same fraction ω_t^j of their wealth in each stock j.

Finally, by imposing market clearing in the stock and bond market, we obtain the state price density. Express first the number of shares bought N_t^{ij} as a function of ω_t^{ij} and thus ω_t^j:

$$\omega_t^{ij} = \omega_t^j = \frac{N_t^{ij} M_t^j}{M_t^i} \text{ for } j \neq i \; .$$

Solving for N_t^{ij}, we obtain the number of shares bought by each agent i:

$$N_t^{ij} = \omega_t^j \frac{M_t^i}{M_t^j} \text{ for } j \neq i \; . \tag{A25}$$

We now impose the market-clearing condition in the stock market. Recall that the total number of shares issued by firm j satisfies

$$1 - \theta = \int_{\mathcal{I}\setminus j} N_t^{ij} di \; .$$

Substitute for N_t^{ij}:

$$1 - \theta = \int_{\mathcal{I}\setminus j} \omega_t^j \frac{M_t^i}{M_t^j} di$$

or

$$(1-\theta)\, M_t^j = \omega_t^j \int_{\mathcal{I}\setminus j} M_t^i di \; .$$

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That is, for every agent i, their exposure to stock j, ω_t^j, must satisfy

$$\omega_t^j = (1-\theta)\,\frac{M_t^j}{\int_{\mathcal{I}\backslash j} M_t^i di}\,, \tag{A26}$$

which implies

$$N_t^{ij} = \omega_t^j \frac{M_t^i}{M_t^j} = (1-\theta)\,\frac{M_t^j}{\int_{\mathcal{I}\backslash j} M_t^k dk}\frac{M_t^i}{M_t^j} = (1-\theta)\,\frac{M_t^i}{\int_{\mathcal{I}\backslash j} M_t^k dk}\,. \tag{A27}$$

That is, each agent i purchases a number of shares N_t^{ij} proportional to his wealth M_t^i.

Consider now the budget equation of agent i:

$$(1-\theta)\,M_t^i = \int_{\mathcal{I}\backslash i} N_t^{ij} M_t^j dj + N_t^{i0}\,.$$

Substitute for N_t^{ij} from equation (A27):

$$(1-\theta)\,M_t^i = \int_{\mathcal{I}\backslash i} (1-\theta)\,\frac{M_t^i}{\int_{\mathcal{I}\backslash j} M_t^k dk} M_t^j dj + N_t^{i0}\,,$$

or

$$(1-\theta)\,M_t^i = (1-\theta)\,M_t^i \frac{\int_{\mathcal{I}\backslash i} M_t^j dj}{\int_{\mathcal{I}\backslash j} M_t^k dk} + N_t^{i0}\,,$$

or

$$(1-\theta)\,M_t^i = (1-\theta)\,M_t^i + N_t^{i0}\,. \tag{A28}$$

This implies that, for all i,

$$N_t^{i0} = 0\,.$$

That is, all agents have a zero position in bonds, which makes sense as all agents have the same risk aversion. Thus, the bond market clears.

We finally obtain the state price density that ensures that the FOC of all agents are satisfied by equation (A26). Consider again the FOC of agent i:

$$E\left[\left(\theta R_{t+1}^i + \int_{\mathcal{I}\backslash i} \omega_t^{ij} R_{t+1}^j dj + 1\right)^{-\gamma} R_{t+1}^j\right] = 0\,.$$

Substitute what we found earlier as the equilibrium weight of agent i into stock j :

$$\omega_t^{ij} = \omega_t^j = (1-\theta)\,\frac{M_t^j}{\int_{\mathcal{I}\backslash j} M_t^k dk}$$

to find that the FOC is

$$E_t\left[\left(\theta R_{t+1}^i + \int_{\mathcal{I}\backslash i} (1-\theta)\,\frac{M_t^j}{\int_{\mathcal{I}\backslash j} M_t^k dk} R_{t+1}^j dj + 1\right)^{-\gamma} R_{t+1}^j\right] = 0\,,$$

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or

$$E_t\left[\left(\theta R^i_{t+1} + (1-\theta)\int_{\mathcal{I}\backslash i}\frac{M^j_t}{\int_{\mathcal{I}\backslash j} M^k_t dk} R^j_{t+1} dj + 1\right)^{-\gamma} R^j_{t+1}\right] = 0\ ,$$

or

$$E\left[\left(\theta R^i_{t+1} + (1-\theta)\, R^{Mkt}_{t+1} + 1\right)^{-\gamma} R^j_{t+1}\right] = 0\ , \tag{A29}$$

where R^{Mkt}_{t+1} is the return on the market portfolio:

$$\begin{aligned} R^{Mkt}_{t+1} &= \int_{\mathcal{I}} \frac{M^j_t}{\int_{\mathcal{I}} M^k_t dk} R^j_{t+1} dj \\ &= \int_{\mathcal{I}} \frac{M^j_t}{\int_{\mathcal{I}} M^k_t dk}\left(\frac{D^j_{t+1}}{M^j_t} - 1\right) dj \\ &= \frac{\int_{\mathcal{I}} D^j_{t+1} dj}{\int_{\mathcal{I}} M^k_t dk} - 1\ . \end{aligned}$$

Ex ante, all R^i_{t+1}, R^j_{t+1} have the same characteristics, as we can write

$$R^i_{t+1} = \frac{(1-\tau_{t+})\, e^{\varepsilon_{i,t+1}+\varepsilon_{t+1}}}{Z_t} - 1 \tag{A30}$$

$$R^{Mkt}_{t+1} = \frac{(1-\tau_{t+})\, e^{\varepsilon_{t+1}}}{Z_t} - 1\ . \tag{A31}$$

Let us rewrite the FOC in terms of dividends again:

$$E\left[\left(\theta R^i_{t+1} + (1-\theta)\, R^{Mkt}_{t+1} + 1\right)^{-\gamma}\left(\frac{D^j_{t+1}}{M^j_t} - 1\right)\right] = 0\ .$$

For every i, we thus have

$$E\left[\left(\theta R^i_{t+1} + (1-\theta)\, R^{Mkt}_{t+1} + 1\right)^{-\gamma} D^j_{t+1}\right] = E\left[\left(\theta R^i_{t+1} + (1-\theta)\, R^{Mkt}_{t+1} + 1\right)^{-\gamma}\right] M^j_t\ .$$

Integrate across $i \in \mathcal{I}$ to obtain

$$E\left[\int_{\mathcal{I}}\left(\theta R^i_{t+1} + (1-\theta)\, R^{Mkt}_{t+1} + 1\right)^{-\gamma} di\ D^j_{t+1}\right] = E\left[\int_{\mathcal{I}}\left(\theta R^i_{t+1} + (1-\theta)\, R^{Mkt}_{t+1} + 1\right)^{-\gamma} di\right] M^j_t\ .$$

Define the state price density as

$$\pi_{t+1} = \int_{\mathcal{I}}\left(\theta R^i_{t+1} + (1-\theta)\, R^{Mkt}_{t+1} + 1\right)^{-\gamma} di\ , \tag{A32}$$

so that the above equation is

$$E_t\left[\pi_{t+1}\, D^j_T\right] = E_t\left[\pi_{t+1}\right] M^j_t\ ,$$

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which is the standard pricing equation. The stochastic discount factor is the $\pi_{t,t+1} = \pi_{t+1}/E_t[\pi_{t+1}]$. We now show that this state price density only depends on ε_{t+1} and τ_{t+} as initially conjectured. We have

$$\begin{aligned}
\pi_{t+1} &= \int_{\mathcal{I}} \left(\theta\left(\frac{D^i_{t+1}}{M^i_t}-1\right)+(1-\theta)\left(\frac{\int_{\mathcal{I}} D^j_{t+1}dj}{\int_{\mathcal{I}} M^k_t dk}-1\right)+1\right)^{-\gamma} di \\
&= \int_{\mathcal{I}} \left(\theta\left(\frac{(1-\tau_{t+})\,e^{\varepsilon_{i,t+1}+\varepsilon_{t+1}}}{Z_t}-1\right)+(1-\theta)\left(\frac{(1-\tau_{t+})\,e^{\varepsilon_{t+1}}}{Z_t}-1\right)+1\right)^{-\gamma} di \\
&= \int_{\mathcal{I}} \left(\theta\frac{(1-\tau_{t+1})\,e^{\varepsilon_{i,t+1}+\varepsilon_{t+1}}}{Z_t}+(1-\theta)\,\frac{(1-\tau_{t+1})\,e^{\varepsilon_{t+1}}}{Z_t}\right)^{-\gamma} di \\
&= \left((1-\tau_{t+})\,e^{\varepsilon_{t+1}}\right)^{-\gamma}\frac{1}{Z_t^{-\gamma}}\int_{\mathcal{I}}\left(\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right)^{-\gamma} di \\
&= h_t\,(1-\tau_{t+})^{-\gamma}\,e^{-\gamma\varepsilon_{t+1}}\ ,
\end{aligned}$$

where

$$h_t = \frac{1}{Z_t^{-\gamma}}\int_{\mathcal{I}}\left(\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right)^{-\gamma} di\ .$$

We can also solve for Z_t explicitly, from its definition

$$Z_t = \frac{E_t\left[\pi\left(\varepsilon_{t+1},\tau_{t+}\right)(1-\tau_{t+})\,e^{\varepsilon_{t+1}}\right]}{E_t\left[\pi_{t+1}\right]}\ ,$$

but it is not necessary to do so.

We now show that prices are well defined at both times t and $t+$ and that at $t+$ agents do not wish to rebalance their portfolios. In particular, we need to show that the state price density just obtained is well defined not only at t (before the announcement) but also at $t+$ (after the announcement), in the sense that it can still be derived from agents' first order conditions at $t+$, that it satisfies the martingale condition, and that deflated prices also satisfy the martingale conditions.

To see all this, recall the state price density at the end of period t, i.e., at $t+1$, is given by

$$\pi_{t+1} = h\,(1-\tau_{t+})^{-\gamma}\,e^{-\gamma\varepsilon_{t+1}}\ ,$$

where h is a constant. From the martingale condition, we have that the state price density values before time $t+1$, at times $t+$ and t, are given by

$$\begin{aligned}
E_{t+}\left[\pi_{t+1}\right] &= h\,(1-\tau_{t+})^{-\gamma}\,E_{t+}\left[e^{-\gamma\varepsilon_{t+1}}\right] \\
E_t\left[\pi_{t+1}\right] &= hE_t\left[(1-\tau_{t+})^{-\gamma}\,E_{t+}\left[e^{-\gamma\varepsilon_{t+1}}\right]\right]
\end{aligned}$$

The stock price must satisfy

$$\begin{aligned}
M_{t+}E_{t+}\left[\pi_{t+1}\right] &= E_{t+}\left[\pi_{t+1}M_{t+1}\right] \\
M_tE_t\left[\pi_{t+1}\right] &= E_t\left[\pi_{t+1}M_{t+1}\right]\ .
\end{aligned}$$

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The latter equation is clearly satisfied by the pricing formula, as this is how we obtained the state price density to begin with. We now show that the pricing equation at the intermediate time $t+$ determines the stock price obtained under the full information case:

$$
\begin{aligned}
M_{t+} &= \frac{E_{t+}\left[\pi_{t+1} M_{t+1}\right]}{E_{t+}\left[\pi_{t+1}\right]} \\
&= \frac{h\left(1-\tau_{t+}\right)^{-\gamma} E_{t+}\left[e^{-\gamma \varepsilon_{t+1}}\left(\left(1-\tau_{t+}\right) G_t e^{\mu_i+\varepsilon_{t+t}+\varepsilon_{i,t+1}}\right)\right]}{h\left(1-\tau_{t+}\right)^{-\gamma} E_{t+}\left[e^{-\gamma \varepsilon_{t+1}}\right]} \\
&= \frac{\left(1-\tau_{t+}\right) G_t e^{\mu_i} E_{t+}\left[e^{(1-\gamma) \varepsilon_{t+1}}\right]}{E_{t+}\left[e^{-\gamma \varepsilon_{t+1}}\right]},
\end{aligned}
$$

which is the same pricing formula we have for the case in which τ is known from the beginning (i.e. the pure strategy Nash equilibrium). Because also in that case the state price density is defined from agents' first order conditions, it follows that the state price density is well defined on both times. In particular, agents do not want to rebalance their portfolios after the revelation of the winning party at $t+$. To see this, we now show that each agent's wealth at $t+$ obtained from the initial investment at t also equals the wealth in the pure strategy equilibrium when the tax is announced at time t rather than $t+$. That is, their uncertainty at time t does not change the wealth position at the time of information about taxes, which in turn implies that their optimal choice conditional on taxes is unchanged compared to the pure strategy equilibrium:

$$
\begin{aligned}
W_{it} &= \theta M_{it} + (1-\theta) M_t^P \\
&= \theta G_t e^{\mu_i} \frac{\left[0.5\left(1-\tau^L\right)^{1-\gamma}+0.5\left(1-\tau^H\right)^{1-\gamma}\right]}{\left[0.5\left(1-\tau^L\right)^{-\gamma}+0.5\left(1-\tau^H\right)^{-\gamma}\right]} e^{-\gamma\sigma^2} \\
&\quad + (1-\theta) G_t m_t E\left[e^{\mu_i} | i \in I\right] \frac{\left[0.5\left(1-\tau^L\right)^{1-\gamma}+0.5\left(1-\tau^H\right)^{1-\gamma}\right]}{\left[0.5\left(1-\tau^L\right)^{-\gamma}+0.5\left(1-\tau^H\right)^{-\gamma}\right]} e^{-\gamma\sigma^2} \\
&= G_t \frac{\left[0.5\left(1-\tau^L\right)^{1-\gamma}+0.5\left(1-\tau^H\right)^{1-\gamma}\right]}{\left[0.5\left(1-\tau^L\right)^{-\gamma}+0.5\left(1-\tau^H\right)^{-\gamma}\right]} e^{-\gamma\sigma^2}\left[\theta e^{\mu_i} + (1-\theta) m_t E\left[e^{\mu_i} | i \in I\right]\right] .
\end{aligned}
$$

Agent i's wealth at $t+$ is

$$
\begin{aligned}
W_{it+} &= G_t \frac{\left[\left(1-\tau^+\right)^{1-\gamma}\right]}{\left[\left(1-\tau^+\right)^{-\gamma}\right]} e^{-\gamma\sigma^2}\left[\theta e^{\mu_i} + (1-\theta) m_t E\left[e^{\mu_i} | i \in I\right]\right] \\
&= \theta M_{it+} + (1-\theta) M_{t+}^P .
\end{aligned}
$$

Because the market portfolio and every individual stock price increase or decrease by the same percentage, no rebalancing takes place at time $\tau+$. In other words, the FOC are still the same for all agents even after the information release.

Finally, we note that even with tax uncertainty, consumption of entrepreneurs at $t+1$ is the same as in the case with tax certainty. From

$$
C_{i,t+1} = \theta D_{t+1}^i + \int_{\mathcal{I}\setminus i} N_t^{ij} D_{t+1}^j dj + N_t^{i0} \tag{A33}
$$

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and

$$\begin{aligned} N_t^{i0} &= 0 \\ N_t^{ij} &= (1-\theta)\,\frac{M_{it}}{M_t^P} = (1-\theta)\,\frac{e^{\mu_i}}{\int e^{\mu_k} dk}\ , \end{aligned}$$

we obtain

$$\begin{aligned} C_{i,t+1} &= \theta D_{t+1}^i + \int_{\mathcal{I}\backslash i} (1-\theta)\,\frac{e^{\mu_i} D_{t+1}^j}{\int e^{\mu_k} dk} dj \\ &= \theta\,(1-\tau_{t+})\,G_t e^{\mu_i+\varepsilon_{i,t+1}+\varepsilon_{t+1}} + e^{\mu_i} \int_{\mathcal{I}\backslash i} (1-\theta)\,\frac{(1-\tau_{t+})\,G_t e^{\mu_j+\varepsilon_{j,t+1}+\varepsilon_{t+1}}}{\int e^{\mu_k} dk} dj \\ &= (1-\tau_{t+})\,G_t e^{\mu_i+\varepsilon_{t+1}} \left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]\ . \qquad \text{(A34)} \end{aligned}$$

Thus, consumption $C_{i,t+1} > 0$ with probability one, as claimed earlier. Q.E.D.

Proof of Proposition 8 (cont'd).

We finally construct the mixed equilibrium. First, let the equilibrium mass be $m_t = 0.5$. We keep the general notation m_t as it will be useful later to prove uniqueness. As in previous cases, given m_t, truthful voting still implies entrepreneurs (E) vote for low taxes (L) and government workers (G) vote for high taxes (H). Let $p = 0.5$ be the probability that L wins. Agents take into account this uncertainty in deciding whether to be E (and vote L) or G (and vote H). In this case, agents take into account some consumption uncertainty at time $t+1$ which will depend on the voting outcome. In particular, if agent i chooses E, his consumption is (see equation (A34)):

$$C_{it+1}^{yes} = \begin{cases} \left(1-\tau^L\right) G_t e^{\mu_i} e^{\varepsilon_{t+1}} \left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right] & \text{with probability } 1/2 \\ \left(1-\tau^H\right) G_t e^{\mu_i} e^{\varepsilon_{t+1}} \left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right] & \text{with probability } 1/2 \end{cases}$$

If agent i chooses G, his/her consumption is

$$C_{it+1}^{no} = \begin{cases} \tau^L G_t e^{\varepsilon_{t+1}} E\left[e^{\mu_j} | j \in I_t\right] m_t / (1-m_t) & \text{with probability } 1/2 \\ \tau^H G_t e^{\varepsilon_{t+1}} E\left[e^{\mu_j} | j \in I_t\right] m_t / (1-m_t) & \text{with probability } 1/2 \end{cases}$$

Therefore,

$$V_t^{i,yes} > V_t^{i,no}$$

if and only if

$$\begin{aligned} &\frac{\left[0.5\left(1-\tau^L\right)^{1-\gamma^M} + 0.5\left(1-\tau^H\right)^{1-\gamma^M}\right] G_t^{1-\gamma^M} e^{\left(1-\gamma^M\right)\mu_i}}{1-\gamma^M} E_t\left[e^{\left(1-\gamma^M\right)\varepsilon_{t+1}}\right] E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma_t}\right] \\ > &\ \frac{1}{1-\gamma^M}\left(0.5\left(\tau^L\right)^{1-\gamma^M} + 0.5\left(\tau^H\right)^{1-\gamma^M}\right)\left(G_t E\left[e^{\mu_j} | j \in \mathcal{I}\right]\right)^{1-\gamma^M} E\left[e^{\left(1-\gamma^M\right)\varepsilon_{t+1}}\right]\left(\frac{m_t}{1-m_t}\right)^{1-\gamma^M} \end{aligned}$$

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if and only if

$$\begin{aligned} &\left[\left(1-\tau^{L}\right)^{1-\gamma^{M}}+\left(1-\tau^{H}\right)^{1-\gamma^{M}}\right] e^{\left(1-\gamma^{M}\right)\mu_i} E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma_t}\right] \\ < \ & \left(\left(\tau^{L}\right)^{1-\gamma^{M}}+\left(\tau^{H}\right)^{1-\gamma^{M}}\right) E\left[e^{\mu_j}|j\in I_t\right]^{1-\gamma^{M}}\left(\frac{m_t}{1-m_t}\right)^{1-\gamma^{M}} \end{aligned}$$

if and only if

$$\begin{aligned} & \log\left(\left(1-\tau^{L}\right)^{1-\gamma^{M}}+\left(1-\tau^{H}\right)^{1-\gamma^{M}}\right)+\left(1-\gamma^{M}\right)\mu_i+\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma}\right]\right) \\ < \ & \log\left(\left(\tau^{L}\right)^{1-\gamma^{M}}+\left(\tau^{H}\right)^{1-\gamma^{M}}\right)+\left(1-\gamma^{M}\right)\log\left(E\left[e^{\mu_j}|j\in I_t\right]\right)+\left(1-\gamma^{M}\right)\log\left(\frac{m_t}{1-m_t}\right) \end{aligned}$$

if and only if

$$\begin{aligned} \mu_i \ \geq \ K\left(\gamma^{M}\right) &= \frac{1}{\left(\gamma^{M}-1\right)}\log\left(\frac{\left(1-\tau^{L}\right)^{1-\gamma^{M}}+\left(1-\tau^{H}\right)^{1-\gamma^{M}}}{\left(\tau^{L}\right)^{1-\gamma^{M}}+\left(\tau^{H}\right)^{1-\gamma^{M}}}\right)+\log\left(E\left[e^{\mu_j}|j\in I_t\right]\right) \\ &+\frac{1}{\left(\gamma^{M}-1\right)}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma}\right]\right)+\log\left(\frac{m_t}{1-m_t}\right) . \end{aligned}$$

Therefore, the mass of agents who decide to become entrepreneurs is

$$m_t\left(\gamma^{M}\right)=\int_{i:\mu_i\geq K\left(\gamma^{M}\right)} di .$$

Let γ^M be such that $m_t\left(\gamma^{M}\right)=0.5$ (we show below that such $\gamma^M\in\left[\underline{\gamma},\overline{\gamma}\right]$ exists). By construction, the median voter i^* is such that $\mu_{i^*}=K\left(\gamma^{M}\right)$ and hence he is indifferent between E and G. Such a median voter i^* flips a coin and decides to become E with probability 0.5, supporting the equilibrium.

We finally show that such $\gamma^M\in\left[\underline{\gamma},\overline{\gamma}\right]$ exists. Given the distribution of $\mu_i\sim N\left(\overline{\mu},\sigma_\mu^2\right)$, we have

$$m_t\left(\gamma^{M}\right)=\int_{K\left(\gamma^{M}\right)}^{\infty}\phi\left(\mu_i;\overline{\mu},\sigma_\mu^2\right)di=1-\Phi\left(K\left(\gamma^{M}\right);\overline{\mu},\sigma_\mu^2\right) .$$

In addition, recalling the conditional density

$$\begin{aligned} \phi\left(\mu_i|i\in I_t\right) &= \phi\left(\mu_i|\mu_i\geq K\left(\gamma^{M}\right)\right)=\frac{\phi\left(\mu_i;\overline{\mu},\sigma_\mu^2\right)1_{\{\mu_i>K(\gamma^M)\}}}{\int_{K(\gamma^M)}^{\infty}\phi\left(\mu_i;\overline{\mu},\sigma_\mu^2\right)d\mu_i} \\ &= \frac{\phi\left(\mu_i;\overline{\mu},\sigma_\mu^2\right)1_{\{\mu_i>K(\gamma^M)\}}}{1-\Phi\left(K\left(\gamma^{M}\right);\overline{\mu},\sigma_\mu^2\right)}=\frac{\phi\left(\mu_i;\overline{\mu},\sigma_\mu^2\right)1_{\{\mu_i>K(\gamma^M)\}}}{m_t} , \end{aligned}$$

we have

$$E\left[e^{\mu_j}|j\in I_t\right]=\frac{\int_{K(\gamma^M)}^{\infty}e^{\mu_i}\phi\left(\mu_i;\overline{\mu},\sigma_\mu^2\right)di}{m_t}=e^{\overline{\mu}+\frac{1}{2}\sigma_\mu^2}\frac{\left(1-\Phi\left(K\left(\gamma^{M}\right);\overline{\mu}+\sigma_\mu^2,\sigma_\mu^2\right)\right)}{m_t} .$$

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In fact,

$$\begin{aligned}
\int_{K(\gamma^M)}^{\infty} e^{\mu_i}\phi\left(\mu_i;\overline{\mu},\sigma_\mu^2\right)di &= \int_{K(\gamma^M)}^{\infty}\frac{e^{\mu_i-\frac{(\mu_i-\overline{\mu})^2}{2\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}}di = \int_{K(\gamma^M)}^{\infty}\frac{e^{-\frac{\mu_i^2+\overline{\mu}^2-2\mu_i\overline{\mu}-2\sigma_\mu^2\mu_i}{2\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}}di \\
&= \int_{K(\gamma^M)}^{\infty}\frac{e^{-\frac{\mu_i^2+\overline{\mu}^2-2\mu_i\left(\overline{\mu}+\sigma_\mu^2\right)+\left(\overline{\mu}+\sigma_\mu^2\right)^2-\left(\overline{\mu}+\sigma_\mu^2\right)^2}{2\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}}di \\
&= e^{\frac{1}{2}\sigma_\mu^2+\overline{\mu}}\int_{K(\gamma^M)}^{\infty}\frac{e^{-\frac{\left(\mu_i-\left(\overline{\mu}+\sigma_\mu^2\right)\right)^2}{2\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}}di \\
&= e^{\frac{1}{2}\sigma_\mu^2+\overline{\mu}}\left(1-\Phi\left(K\left(\gamma^M\right);\overline{\mu}+\sigma_\mu^2;\sigma_\mu^2\right)\right) .
\end{aligned}$$

Substituting everything inside the threshold, we find

$$\begin{aligned}
K\left(\gamma^M\right) &= \frac{1}{\left(\gamma^M-1\right)}\log\left(\frac{\left(1-\tau^L\right)^{1-\gamma^M}+\left(1-\tau^H\right)^{1-\gamma^M}}{\left(\tau^L\right)^{1-\gamma^M}+\left(\tau^H\right)^{1-\gamma^M}}\right) \\
&+\overline{\mu}+\frac{1}{2}\sigma_\mu^2+\log\left(\frac{\left(1-\Phi\left(K\left(\gamma^M\right);\overline{\mu}+\sigma_\mu^2,\sigma_\mu^2\right)\right)}{m_t}\right) \\
&+\frac{1}{\left(\gamma^M-1\right)}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma^M}\right]\right)
\end{aligned}$$

or, defining

$$\underline{\mu}\left(\gamma^M\right) = K\left(\gamma^M\right)-\overline{\mu} ,$$

we obtain

$$\begin{aligned}
\underline{\mu}\left(\gamma^M\right) &= \frac{1}{2}\sigma_\mu^2+\frac{1}{\left(\gamma^M-1\right)}\log\left(\frac{\left(1-\tau^L\right)^{1-\gamma^M}+\left(1-\tau^H\right)^{1-\gamma^M}}{\left(\tau^L\right)^{1-\gamma^M}+\left(\tau^H\right)^{1-\gamma^M}}\right) \\
&+\log\left(\frac{\left(1-\Phi\left(\underline{\mu}\left(\gamma^M\right);\sigma_\mu^2,\sigma_\mu^2\right)\right)}{\left(1-\Phi\left(\underline{\mu}\left(\gamma^M\right);0,\sigma_\mu^2\right)\right)}\right) \\
&+\frac{1}{\left(\gamma^M-1\right)}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma^M}\right]\right)+\log\left(\frac{\left(1-\Phi\left(\underline{\mu}\left(\gamma^M\right);0,\sigma_\mu^2\right)\right)}{\Phi\left(\underline{\mu}\left(\gamma^M\right);0,\sigma_\mu^2\right)}\right) ,
\end{aligned}$$

so that

$$\begin{aligned}
\underline{\mu}\left(\gamma^M\right) &= \frac{1}{2}\sigma_\mu^2+\frac{1}{\left(\gamma^M-1\right)}\log\left(\frac{\left(1-\tau^L\right)^{1-\gamma^M}+\left(1-\tau^H\right)^{1-\gamma^M}}{\left(\tau^L\right)^{1-\gamma^M}+\left(\tau^H\right)^{1-\gamma^M}}\right)+\log\left(\frac{1-\Phi\left(\underline{\mu}\left(\gamma^M\right);\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}\left(\gamma^M\right);0,\sigma_\mu^2\right)}\right) \\
&+\frac{1}{\left(\gamma^M-1\right)}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma^M}\right]\right) .
\end{aligned}$$

Finally, γ^M is chosen such that

$$\underline{\mu}\left(\gamma^M\right) = 0 ,$$

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so that

$$m_t = 1 - \Phi\left(\underline{\mu}\left(\gamma^M\right); 0, \sigma_\mu^2\right) = 1 - \Phi\left(0; 0, \sigma_\mu^2\right) = 0.5 \,,$$

which implies

$$\begin{aligned} 0 &= \frac{1}{2}\sigma_\mu^2 + \frac{1}{(\gamma^M - 1)} \log\left(\frac{\left(1-\tau^L\right)^{1-\gamma^M} + \left(1-\tau^H\right)^{1-\gamma^M}}{\left(\tau^L\right)^{1-\gamma^M} + \left(\tau^H\right)^{1-\gamma^M}}\right) + \log\left(\frac{1 - \Phi\left(0; \sigma_\mu^2, \sigma_\mu^2\right)}{0.5}\right) \\ &+ \frac{1}{(\gamma^M - 1)} \log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma^M}\right]\right) \,. \end{aligned}$$

The existence and uniqueness of a solution for $\gamma_M \in [\underline{\gamma}, \overline{\gamma}]$ can be obtained as follows. Define

$$\begin{aligned} \underline{\mu}(p,\gamma) &= \frac{1}{2}\sigma_\mu^2 + \frac{1}{(\gamma - 1)} \log\left(\frac{p\left(1-\tau^L\right)^{1-\gamma} + (1-p)\left(1-\tau^H\right)^{1-\gamma}}{p\left(\tau^L\right)^{1-\gamma} + (1-p)\left(\tau^H\right)^{1-\gamma}}\right) + \log\left(\frac{1 - \Phi\left(\underline{\mu}(p,\gamma); \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}(p,\gamma); 0, \sigma_\mu^2\right)}\right) \\ &+ \frac{1}{(\gamma - 1)} \log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma}\right]\right) \,. \end{aligned}$$

We know that for any $\gamma \in [\underline{\gamma}, \overline{\gamma}]$ and $p = 1$, the equation is

$$\begin{aligned} \underline{\mu}(1,\gamma) &= \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{\tau^L}{1-\tau^L}\right) + \log\left(\frac{1 - \Phi\left(\underline{\mu}(1,\gamma); \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}(1,\gamma); 0, \sigma_\mu^2\right)}\right) \\ &+ \frac{1}{(\gamma - 1)} \log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma}\right]\right) \\ &= \underline{\mu}^L < 0 \,, \end{aligned}$$

whereas for $p = 0$, the equation is

$$\begin{aligned} \underline{\mu}(0,\gamma) &= \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{\tau^H}{1-\tau^H}\right) + \log\left(\frac{1 - \Phi\left(\underline{\mu}(0,\gamma); \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}(0,\gamma); 0, \sigma_\mu^2\right)}\right) \\ &+ \frac{1}{(\gamma - 1)} \log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma}\right]\right) \\ &= \underline{\mu}^H > 0 \,. \end{aligned}$$

Thus, for any $\gamma \in [\underline{\gamma}, \overline{\gamma}]$ there exists a $p \in [0, 1]$ such that

$$\underline{\mu}(p,\gamma) = 0 \,. \tag{A35}$$

We also know that both $\underline{\mu}(1,\gamma)$ and $\underline{\mu}(0,\gamma)$ are increasing in γ. It then follows that there is unique value of γ for which equation (A35) is satisfied for $p = 0.5$. This concludes the proof of the existence of a mixed equilibrium.

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Announcement Returns.

We finally obtain the results about announcement returns. Given the state price density obtained in Proposition A4, we can finally compute the equilibrium under uncertainty. Let Prob $(L \text{ wins}) = 0.5$ and denote by γ^M the corresponding risk aversion. We have

$$\begin{aligned}
M_{it} &= \frac{E_t\left[\pi_{t+1} D_{it+1}\right]}{E_t\left[\pi_{t+1}\right]} = \frac{E_t\left[\left(1-\tau_{t+}\right)^{-\gamma^M} e^{-\gamma^M \varepsilon_{t+1}}\left(1-\tau_{t+}\right) G_t e^{\mu_i+\varepsilon_{it+1}+\varepsilon_{t+1}}\right]}{E_t\left[\left(1-\tau_{t+}\right)^{-\gamma^M} e^{-\gamma^M \varepsilon_{t+1}}\right]} \\
&= G_t e^{\mu_i} \frac{E_t\left[\left(1-\tau_{t+}\right)^{1-\gamma^M} e^{(1-\gamma)\varepsilon_{t+1}}\right]}{E_t\left[\left(1-\tau_{t+}\right)^{-\gamma^M} e^{-\gamma^M \varepsilon_{t+1}}\right]} \\
&= G_t e^{\mu_i} \frac{\left[0.5\left(1-\tau^L\right)^{1-\gamma^M}+0.5\left(1-\tau^H\right)^{1-\gamma^M}\right] e^{\left(1-\gamma^M\right)\left(-\frac{1}{2}\sigma^2\right)+\frac{1}{2}\left(1-\gamma^M\right)^2 \sigma^2}}{\left[0.5\left(1-\tau^L\right)^{-\gamma^M}+0.5\left(1-\tau^H\right)^{-\gamma^M}\right] e^{-\gamma^M\left(-\frac{1}{2}\sigma^2\right)+\frac{1}{2}\left(\gamma^M\right)^2 \sigma^2}} \\
&= G_t e^{\mu_i} \frac{\left[0.5\left(1-\tau^L\right)^{1-\gamma^M}+0.5\left(1-\tau^H\right)^{1-\gamma^M}\right] e^{\left(1-\gamma^M\right)\left(-\frac{1}{2}\sigma^2\right)+\frac{1}{2}\left(1+\left(\gamma^M\right)^2-2\gamma^M\right)\sigma^2+\gamma^M\left(-\frac{1}{2}\sigma^2\right)-\frac{1}{2}\left(\gamma^M\right)^2\sigma^2}}{\left[0.5\left(1-\tau^L\right)^{-\gamma^M}+0.5\left(1-\tau^H\right)^{-\gamma^M}\right]} \\
&= G_t e^{\mu_i} \frac{\left[0.5\left(1-\tau^L\right)^{1-\gamma^M}+0.5\left(1-\tau^H\right)^{1-\gamma^M}\right]}{\left[0.5\left(1-\tau^L\right)^{-\gamma^M}+0.5\left(1-\tau^H\right)^{-\gamma^M}\right]} e^{-\gamma^M \sigma^2} \\
&= G_t e^{\mu_i-\gamma^M \sigma^2}\left[\omega\left(1-\tau^L\right)+(1-\omega)\left(1-\tau^H\right)\right],
\end{aligned}$$

where

$$\omega = \frac{\left(1-\tau^L\right)^{-\gamma^M}}{\left(1-\tau^L\right)^{-\gamma^M}+\left(1-\tau^H\right)^{-\gamma^M}}.$$

The results follow from the fact that after the announcement, the price is $M_{it+} = G_t e^{\mu_i-\gamma^M\sigma^2}\left(1-\tau_{t+}\right)$, where τ_{t+} denotes the tax rate realized at time $t+$.

Q.E.D.

A5. Theory: Extension to Government Debt

In this section we extend our model to allow the government to run a budget deficit by borrowing from future generations. The government can use this deficit financing to mitigate risk aversion shocks. This mitigation, or smoothing, of risk aversion shocks happens endogenously through the voting process—when risk aversion is high, voters elect the party that favors deficit financing; when risk aversion is low, they elect the party that favors eliminating debt. As a preview, we show that the main results from our baseline model obtain also in this extended model.

We keep all of the modeling assumptions from our baseline model and, in addition, we allow the government to borrow a given amount δ of the consumption good from a future period. To keep the analysis tractable, we assume there are only two possible levels of government debt: either 0 or δ. If the current level of debt is 0 then the government can either preserve this zero level or raise it to δ by running a deficit. If the current debt level is δ then the government can either preserve it or reduce it to zero by paying off debt; in addition, it must pay interest on the existing debt. Any resources that the government raises through borrowing are used in the same way as tax revenue, that is, to increase the consumption of government workers.

The government can borrow across generations by using a borrowing technology whose cost per unit of consumption good borrowed is r. For example, one can imagine that the government has reserves of the consumption good that it can draw down every period, but then it needs to replenish the reserves by diverting tax revenue in the future, with the addition of a per-period cost r. If the government borrows δ at time t, then it must return $\delta(1 + r)$ at time $t + 1$, though it has the option of rolling its debt, δ, over. We assume that the interest rate r is exogenously given. In our model, agents live for one period so there is no intertemporal consumption-saving choice that would pin down r in equilibrium. Therefore, this model has no implications for bond returns over the presidential cycle.[†] Still, r affects the equilibrium, as we show below.

As in the baseline model, there are two parties, H and L, levying tax rates such that $\tau^H > \tau^L$. We now add the assumption that party H also favors "High debt" whereas party L favors "Low debt". As a result, party H runs a budget deficit whenever it can, whereas party L pays off debt whenever it can. Assuming that party H is not only high-tax but also high-debt (and that L is not only low-tax but also low-debt) seems plausible, for two reasons. First, thanks to this assumption, deficit financing is used to mitigate risk aversion shocks, as mentioned earlier. (This happens because party H, which favors deficits, is elected when risk aversion is high, as we show below.) Second, this assumption leads to the prediction that budget deficits tend to be higher under party H than under party L, on average. We find some empirical support for this prediction in Section A14. However, this evidence is not statistically significant: budget deficits are only insignificantly higher under Democrats than under Republicans (see Section A14. for details).

[†]In the data, excess bond returns are not significantly different between the Democratic and Republican presidencies, as we show in Section A15.

A5.1. Equilibrium

Proposition A5.1. *Entrepreneurs always vote L. Government workers always vote H.*

The consumption of entrepreneurs is identical to its value in the baseline model. Government borrowing has no effect on today's entrepreneurs because it is paid back by future entrepreneurs. Recall that, in our model, agents live for only one period, and their utility function does not include a concern for future generations. Therefore, an entrepreneur's consumption at the end of period t is the same as before,

$$C^E_{i,t+1} = (1-\tau_t)G_t e^{\mu_i+\varepsilon_{t+1}}[\theta e^{\varepsilon_{i,t+1}} + 1 - \theta] \tag{A36}$$

The consumption of government workers at time t depends on the deficit $\widetilde{\delta}_t$ announced by the government. Given the tax rate τ_t and total output $Y_{t+1} = G_t e^{\varepsilon_{t+1}} m_t \mathrm{E}\left[e^{\mu_i} | i \in \mathcal{I}_t\right]$, a government worker's consumption is

$$C^W_{i,t+1} = \frac{\tau_t Y_{t+1} + \widetilde{\delta}_t}{1-m_t} = \frac{\tau_t G_t e^{\varepsilon_{t+1}} m_t \mathrm{E}\left[e^{\mu_i} | i \in \mathcal{I}_t\right] + \widetilde{\delta}_t}{1-m_t} \tag{A37}$$

The total amount of consumption in the economy at time t is given by $Y_{t+1} + \widetilde{\delta}_t$. Compared to the baseline case presented in the paper, the consumption of government workers is either higher or lower, depending on whether the value of $\widetilde{\delta}_t$ is positive or negative. This value is positive ($\widetilde{\delta}_t = \delta$) if the government increases its debt by running a budget deficit. The same value is negative in two cases: if the governmant maintains a positive level of debt and simply makes interest payments on it ($\widetilde{\delta}_t = -\delta r$), and also if the government pays off its debt, with interest ($\widetilde{\delta}_t = -\delta(1+r)$).

Specifically, the variable $\widetilde{\delta}_t$ can take four different values, depending on the amount of government debt outstanding and which party is in power. Let B_t denote the current level of government debt (or Borrowing) at the beginning of period t. This variable, whose value is inherited from the past, is a new state variable compared to the baseline model in the paper. As mentioned earlier, debt can take two values, $B_t = 0$ or $B_t = \delta$, depending on the choice of the previous government. Our assumptions about deficit choices can be summarized as follows:

1. The H government announces

$$\widetilde{\delta}^H_t = \begin{cases} \delta & \text{if} \quad B_t = 0 \\ -\delta r & \text{if} \quad B_t = \delta \end{cases}$$

2. The L government announces

$$\widetilde{\delta}^L_t = \begin{cases} 0 & \text{if} \quad B_t = 0 \\ -\delta(1+r) & \text{if} \quad B_t = \delta \end{cases}$$

The dynamics of debt are therefore given by

$$B_{t+1} = B_t(1+r) + \widetilde{\delta}^k_t$$

Negative values of $\widetilde{\delta}_t$ reduce the consumption of government workers, $C^W_{i,t+1}$, in equation (A37). To ensure that $C^W_{i,t+1} > 0$, the aggregate shocks ε_{t+1} cannot have unbounded support because otherwise the combination of $\varepsilon_{t+1} \to -\infty$ and $\widetilde{\delta}_t < 0$ would result in negative consumption of government workers. We therefore assume that these shocks are bounded, $\varepsilon_{t+1} \in [\underline{\varepsilon}, \overline{\varepsilon}]$, with proper bounds such that $\mathrm{E}[e^{\varepsilon_{t+1}}] = 1$ and the equilibrium exists for the given choice of δ and r.

Proposition A5.2. *Suppose that government k is in power at time t, where $k \in \{H, L\}$. Agent i becomes an entrepreneur if and only if*

$$\mu_i > \underline{\mu}(\gamma_t, \widetilde{\delta}_t)$$

where the threshold solves the equation

$$\underline{\mu}\left(\gamma_t; \widetilde{\delta}_t\right) = \frac{1}{1-\gamma_t} \log \left[\frac{E\left[\left(\frac{\tau G_t e^{\varepsilon_{t+1}}\left[1-\Phi\left(\underline{\mu}\left(\gamma_t;\widetilde{\delta}_t\right)\right)\right] E\left[e^{\mu_i}|\mu_i > \underline{\mu}\left(\gamma_t;\widetilde{\delta}_t\right)\right] + \widetilde{\delta}_t}{\Phi\left(\underline{\mu}\left(\gamma_t;\widetilde{\delta}_t\right)\right)}\right)^{1-\gamma_t}\right]}{E\left[\left((1-\tau)\, G_t e^{\varepsilon_{t+1}} \left[\theta e^{\varepsilon_{i,t+1}} + 1 - \theta\right]\right)^{1-\gamma_t}\right]} \right] \tag{A38}$$

and we suppress the k superscripts on $\widetilde{\delta}_t$ and τ. For a given γ_t, the threshold is increasing in $\widetilde{\delta}_t$:

$$\frac{\partial \underline{\mu}(\gamma_t; \widetilde{\delta}_t)}{\partial \widetilde{\delta}_t} > 0$$

Proof: The proof follows immediately from comparing $\mathrm{E}[U(C^E_{i,t+1})]$ to $\mathrm{E}[U(C^W_{i,t+1})]$ and the fact that the right-hand side of equation (A38) is increasing in $\widetilde{\delta}_t$. Q.E.D.

Studying the equilibrium for a given value of γ_t is more challenging than in the baseline model in the paper. Due to the presence of an additional state variable, B_t, we have to consider several cases. To reduce the number of cases, we assume that the tax rates under parties H and L are (nearly) the same. That is, we assume

Assumption A2. The two tax rates are $\tau^L = \tau$ and $\tau^H = \tau + \nu$, where $\nu > 0$ and $\nu \to 0$.

That is, τ^H and τ^L are nearly the same and equal to τ. This modeling device allows us to break the indifference condition of entrepreneurs in the limit: entrepreneurs vote for party L as long as $\tau^H > \tau^L$ (although, in the limit, they are indifferent). The equilibrium has a discontinuity at $\tau^H = \tau^L = \tau$ but we avoid it by considering the limiting case in which $\tau^H > \tau^L$. Assumption A2 simplifies the exposition but is not necessary; we can obtain our main results also without it.

Assumption A2 allows us to limit our attention to five cases, which we summarize in Table 1. We now go over each case:

1. Case 1: The thresholds are all below 0, implying the mass of entrepreneurs above 0.5. As a result, party L is going to win no matter what deficit either party announces. If there

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Table 1: Equilibrium with Government Debt

		Case 1	*Case* 2	*Case* 3	*Case* 4	*Case* 5
		0 $\underline{\mu}(\gamma,\delta)$ $\underline{\mu}(\gamma,0)$ $\underline{\mu}(\gamma,-\delta r)$ $\underline{\mu}(\gamma,-\delta(1+r))$	$\underline{\mu}(\gamma,\delta)$ 0 $\underline{\mu}(\gamma,0)$ $\underline{\mu}(\gamma,-\delta r)$ $\underline{\mu}(\gamma,-\delta(1+r))$	$\underline{\mu}(\gamma,\delta)$ $\underline{\mu}(\gamma,0)$ 0 $\underline{\mu}(\gamma,-\delta r)$ $\underline{\mu}(\gamma,-\delta(1+r))$	$\underline{\mu}(\gamma,\delta)$ $\underline{\mu}(\gamma,0)$ $\underline{\mu}(\gamma,-\delta r)$ 0 $\underline{\mu}(\gamma,-\delta(1+r))$	$\underline{\mu}(\gamma,\delta)$ $\underline{\mu}(\gamma,0)$ $\underline{\mu}(\gamma,-\delta r)$ $\underline{\mu}(\gamma,-\delta(1+r))$ 0
$B_t = 0$	who wins?	L	Either	H	H	H
	new deficit $\widetilde{\delta}_t$	0	0 or δ	δ	δ	δ
$B_t = \delta$	who wins?	L	L	L	Either	H
	new deficit $\widetilde{\delta}_t$	$-\delta(1+r)$	$-\delta(1+r)$	$-\delta(1+r)$	$-\delta(1+r)$ or $-\delta r$	$-\delta r$

is some debt outstanding, party L will pay it off; if there is no existing debt, party L will maintain status quo. It follows that the relevant thresholds are $\underline{\mu}(\gamma,0)$ if debt $B_t = 0$ and $\underline{\mu}(\gamma,-\delta(1+r))$ if debt $B_t = \delta$.

2. Case 2: In this case, who wins the election depends on the amount of existing debt.

   (a) If $B_t = 0$, then H announces $\widetilde{\delta}_t^H = \delta$ and L announces $\widetilde{\delta}_t^L = 0$. The corresponding thresholds are $\underline{\mu}(\gamma,\delta) > 0 > \underline{\mu}(\gamma,0)$. Therefore, if for any reason agents believe that H will win, then $\underline{\mu}(\gamma,\delta) > 0$ is the equilibrium threshold, the mass of entrepreneurs $m_t = 1 - \Phi(\underline{\mu}(\gamma,\delta), 0, \sigma_\mu) < 0.5$, and H indeed wins. Vice versa, if for any reason agents believe that L will win, then $\underline{\mu}(\gamma,0) < 0$ is the equilibrium threshold, the mass of entrepreneurs is above 0.5, and L indeed wins. That is, either party can win if $B_t = 0$. This multiple-equilibrium solution echoes the sunspot equilibrium in the baseline model presented in the paper.

   (b) If $B_t = \delta$, then H announces $\widetilde{\delta}_t^H = -\delta r$ and L announces $\widetilde{\delta}_t^L = -\delta(1+r)$. The corresponding thresholds are $0 > \underline{\mu}(\gamma,-\delta r) > \underline{\mu}(\gamma,-\delta(1+r))$. Since both thresholds are below 0, all agents know that the mass of entrepreneurs $m_t > 0.5$. Thus L wins and the relevant threshold is $\underline{\mu}(\gamma,-\delta(1+r))$.

3. Case 3: Again, who wins depends on the amount of existing debt. However, unlike in the previous case, there is a unique equilibrium at each debt level.

   (a) If $B_t = 0$, then H announces $\widetilde{\delta}_t^H = \delta$ and L announces $\widetilde{\delta}_t^L = 0$. The corresponding thresholds are $\underline{\mu}(\gamma,\delta) > \underline{\mu}(\gamma,0) > 0$. Since both thresholds are above 0, all agents know that the mass of entrepreneurs is below 0.5. Therefore, H wins and the equilibrium threshold is $\underline{\mu}(\gamma,\delta)$.

   (b) If $B_t = \delta$, then H announces $\widetilde{\delta}_t^H = -\delta r$ and L announces $\widetilde{\delta}_t^L = -\delta(1+r)$. The corresponding thresholds are $0 > \underline{\mu}(\gamma,-\delta r) > \underline{\mu}(\gamma,-\delta(1+r))$. Since both thresholds are below 0, all agents know that the mass of entrepreneurs $m_t > 0.5$. Thus L wins and the relevant threshold is $\underline{\mu}\left(\gamma, -\delta(1+r)\right)$.

4. Case 4: This case is the reverse of Case 2. Again, who wins depends on the amount of existing debt.

   (a) If $B_t = 0$, then H announces $\widetilde{\delta}_t^H = \delta$ and L announces $\widetilde{\delta}_t^L = 0$. The corresponding thresholds are $\underline{\mu}\left(\gamma, \delta\right) > \underline{\mu}\left(\gamma, 0\right) > 0$. Since both thresholds are above 0, all agents know that the mass of entrepreneurs is below 0.5. Therefore, H wins and the equilibrium threshold is $\underline{\mu}\left(\gamma, \delta\right)$.

   (b) If $B_t = \delta$, then H announces $\widetilde{\delta}_t^H = -\delta r$ and L announces $\widetilde{\delta}_t^L = -\delta(1+r)$. The corresponding thresholds are $\underline{\mu}\left(\gamma, -\delta r\right) > 0 > \underline{\mu}\left(\gamma, -\delta(1+r)\right)$. In this case, if for any reason agents believe that H will win, then $\underline{\mu}\left(\gamma, -\delta r\right) > 0$ is the equilibrium threshold, the mass of entrepreneurs $m_t = 1 - \Phi(\underline{\mu}\left(\gamma, -\delta r\right), 0, \sigma_\mu) < 0.5$, and H indeed wins. Vice versa, if for any reason agents believe that L will win, then $\underline{\mu}\left(\gamma, -\delta(1+r)\right) < 0$ is the equilibrium threshold, the mass of entrepreneurs is above 0.5, and L indeed wins. That is, either party can win if $B_t = \delta$. This multiple-equilibrium solution echoes the sunspot equilibrium in the baseline model presented in the paper.

5. Case 5: This case is the reverse of Case 1. The thresholds are all above 0, implying the mass of entrepreneurs below 0.5. As a result, party H is going to win no matter what deficit either party announces. It follows that the relevant thresholds are $\underline{\mu}\left(\gamma, \delta\right)$ if debt $B_t = 0$ and $\underline{\mu}\left(\gamma, -\delta r\right)$ if debt $B_t = \delta$.

The final question is which of the cases in Table 1 is more likely to occur as a result of fluctuations in γ_t. Generally speaking, higher values of γ_t correspond to larger case numbers (that is, when γ_t is very high, the economy is in Case 5, whereas when it is very low, the economy is in Case 1). More formally, assume that $r = 0$. Under that assumption, Case 3 disappears and we only have four cases (1,2,4,5) that are symmetric: two of them have $\underline{\mu}(\gamma, 0) < 0$ and two have $\underline{\mu}(\gamma, 0) > 0$. We know from Corollary 1 that the threshold $\underline{\mu}(\gamma, 0)$ (which is relevant for $\widetilde{\delta} = 0$) is increasing in γ, and given Assumption A1, that $\underline{\mu}(\gamma, 0) < 0$ for sufficiently low γ and $\underline{\mu}(\gamma, 0) > 0$ for sufficiently high γ. From Table 1, we see that when $\underline{\mu}(\gamma, 0) < 0$, it is more likely that an L equilibrium arises, whereas when $\underline{\mu}(\gamma, 0) > 0$, an H equilibrium is more likely. We thus obtain the following proposition.

Proposition A5.3. *Let r be small and risk aversion γ_t sufficiently variable so that the threshold $\underline{\mu}(\gamma_t, 0) < 0$ for low γ_t and $\underline{\mu}(\gamma_t, 0) > 0$ for high γ_t are both attainable with positive probability. Then:*

(a) Party H is more likely to win when γ_t is high. Party L is more likely to win when γ_t is low.

(b) The budget deficit is higher under party H than under party L, on average.

As an example, suppose that γ_t can take only two values, γ^L and γ^H. These values are such that either Case 1 or Case 2 occurs for γ^L and either Case 4 or Case 5 occurs for γ^H. To keep things

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simple, assume that γ_t alternates between the two values deterministically, so that the probability of moving from γ^L to γ^H, and vice versa, is one. If $\gamma_t = \gamma^H$, then H wins and borrows δ. In the next period, we have $\gamma_{t+1} = \gamma^L$, so that L wins and repays the debt δ. We obtain a cycle in which periods with high risk aversion are accompanied by budget deficits, whereas periods with low risk aversion are marked by budget surpluses as prior deficits are repaid. Deficit financing thus helps the government mitigate the effect of risk aversion shocks.

With random fluctuations in risk aversion, we obtain the same conclusions. In this case, there is path dependence in the amount of deficit depending on whether the government already has any debt outstanding. Suppose that risk aversion is high ($\gamma_t = \gamma^H$). If there is no debt outstanding ($B_t = 0$), then party H wins and runs a deficit, as in the example above. If there already is debt outstanding ($B_t = \delta$), then party H still wins (certainly in Case 5, with 50% probability in Case 4) but it cannot borrow any more so it just pays interest δr. That is, party H keeps debt at δ but pays the per-period interest rate r. The opposite argument holds when $\gamma_t = \gamma^L$.

As a final point, note that the value of r affects the equilibrium. A large value of r implies high interest payments on existing debt. Since these payments come at the expense of the consumption of government workers, a large value of r makes government work less attractive, which in turn reduces the probability of party H getting elected. In Table 1, when r is large, Cases 4 and 5 may or may not exist, depending on the parameter values. It is possible that only Cases 1, 2, and 3 exist, in which case the equilibrium is asymmetric, with party L winning more often than party H. (That would be counterfactual because, in the data, Democrats and Republicans have spent about the same amount of time in office.) Still, if $B_t = 0$ and $\gamma_t = \gamma^H$, then Case 3 applies, so that party H wins and runs a deficit $\widetilde{\delta}_t = \delta$. In the next period, $B_{t+1} = \delta$, so that party L wins with probability one, regardless of the level of risk aversion. Party L then repays the debt by choosing $\widetilde{\delta}_t = -\delta(1 + r)$. Thus, $B_{t+2} = 0$ again, and the cycle continues, with similar dynamics as before. Our main results therefore continue to hold also when r is large.

A5.2. Prices and Returns

In the baseline model, our proofs of the results related to stock prices and returns made no assumptions about the distributions of shocks. Therefore, the same arguments hold here as well. The state price density is still given by

$$\pi_{t,t+1} \propto e^{-\gamma\varepsilon_{t+1}}$$

The fair market value of the aggregate stock market portfolio is

$$M_t = \frac{E\left[\pi_{t,t+1}Y_{t+1}\right]}{E\left[\pi_{t,t+1}\right]} = \frac{(1-\tau)\, GE\left[e^{(1-\gamma)\varepsilon_{t+1}}\right] E\left[e^{\mu_i}|i \in I^k\right]}{E\left[e^{-\gamma\varepsilon_{t+1}}\right]} = (1-\tau)\, GE\left[e^{\mu_i}|i \in I^k\right] Z$$

where

$$Z = \frac{E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right]}{E\left[e^{-\gamma\varepsilon_{t+1}}\right]}$$

The stock market return is

$$R^{mkt} = \frac{(1-\tau)\, Ge^{\varepsilon_{t+1}} E\left[e^{\mu}|i \in I^k\right]}{(1-\tau)\, GE\left[e^{\mu_i}|i \in I^k\right] Z} - 1 = \frac{e^{\varepsilon_{t+1}}}{Z} - 1$$

Finally

$$E\left[R^{mkt}\right]=\frac{1}{Z}-1$$

Proposition A5.4. *The expected stock market return is increasing in risk aversion γ:*

$$\frac{\partial E\left[R^{mkt}\right]}{\partial\gamma}>0$$

Proof.

$$\begin{aligned}\frac{\partial Z}{\partial\gamma} &= -\frac{E\left[e^{\varepsilon_{t+1}-\gamma\varepsilon_{t+1}}\varepsilon_{t+1}\right]}{E\left[e^{-\gamma\varepsilon_{t+1}}\right]}+\frac{E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right]E\left[e^{-\gamma\varepsilon_{t+1}}\varepsilon_{t+1}\right]}{E\left[e^{-\gamma\varepsilon_{t+1}}\right]^2}\\ &= Z\left[\frac{E\left[e^{-\gamma\varepsilon_{t+1}}\varepsilon_{t+1}\right]}{E\left[e^{-\gamma\varepsilon_{t+1}}\right]}-\frac{E\left[e^{(1-\gamma)\varepsilon_{t+1}}\varepsilon_{t+1}\right]}{E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right]}\right]\\ &= Z\left[E^*\left[\varepsilon_{t+1}\right]-E^{**}\left[\varepsilon_{t+1}\right]\right]\\ &< 0\end{aligned}$$

where the expectations are taken with respect to the following densities:

$$\begin{aligned}f^*(\varepsilon) &= \frac{e^{-\gamma\varepsilon}f(\varepsilon)}{E\left[e^{-\gamma\varepsilon}\right]}\\ f^{**}(\varepsilon) &= \frac{e^{(1-\gamma)\varepsilon}f(\varepsilon)}{E\left[e^{(1-\gamma)\varepsilon}\right]}\end{aligned}$$

The inequality in the last step of the proof follows from the fact that the expectation taken with respect to $e^{-\gamma\varepsilon}$ assigns a lower probability weight to high values of ε, which then implies that $E^*\left[\varepsilon_{t+1}\right]<E^{**}\left[\varepsilon_{t+1}\right]$. Q.E.D.

We conclude with the following proposition.

Proposition A5.5. *Under the conditions of Proposition A5.3, the expected stock market return is higher under party H than under party L.*

This proposition shows that the main result in the paper—the presidential puzzle—obtains also in this extended version of the model.

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A6. Theory: Extension to Optimal Tax Rates

In our baseline model, we take the tax rates τ^H and τ^L as given. In this section, we extend the model by endogenizing these tax rates. After adding some modeling assumptions, we solve for τ^H and τ^L that obtain as outcomes from the optimization problems of two electoral candidates who represent the interests of parties H and L. We show that this extended model is capable of delivering optimal tax rates $\tau^H > \tau^L$ for which the equilibrium obtains and, importantly, the expected stock market return under party H is higher than that under party L. Our mechanism is thus able to explain the presidential puzzle even with endogenous tax rates.

This model extension is far more complicated than the baseline model because it involves an additional fixed-point problem in solving the Nash equilibrium—each party chooses a tax rate that is optimal given the other party's tax rate. Moreover, in choosing their optimal tax rates, both parties take into consideration the impacts of those rates on agents' occupational choices (i.e., the equilibrium masses of entrepreneurs and government workers), voting decisions (i.e., which party gets elected in equilibrium), and equilibrium stock prices in our incomplete-market setting. While we are able to characterize some features of the equilibrium analytically, our solutions for the equilibrium tax rates and expected stock returns are numerical. We no longer obtain a general theorem characterizing expected stock returns, but we show that there exist plausible parameter values for which expected return is higher under the high-tax party, as it is in our baseline model. The presidential puzzle can therefore be explained also within this more general model.

We assume that running for an election are two candidates: a representative entrepreneur E and a representative government worker W. Agent E is a high-entrepreneurial-skill agent who always chooses to become an entrepreneur for any plausible tax rate. Agent W is a low-entrepreneurial-skill agent who always chooses to become a government worker for any plausible tax rate. Agent E therefore represents the "entrepreneur" party, which we interpret as Republicans, and agent W represents the "worker" party, which we interpret as Democrats. Both candidates run on single-dimensional policy platforms that consist of a flat tax rate levied on entrepreneurs' income. Each candidate chooses the tax rate that maximizes his own utility over the next period. Because E is always an entrepreneur, he chooses a tax rate, τ_t^E, that is beneficial to all entrepreneurs. Because W is always a government worker, he chooses a tax rate, τ_t^W, that is beneficial to all government workers. When making their tax policy choices, both agents consider their impact on the equilibrium mass of entrepreneurs, the election outcome, and stock prices.

Besides their different entrepreneurial skills, the two candidates are also endowed with personal characteristics, χ_t^E and χ_t^W, that are orthogonal to their tax policy choices. The values of χ_t^E and χ_t^W summarize various non-economic characteristics of the two candidates, such as their personal charisma and rapport with voters. These values affect voting decisions because they enter into agents' utility functions. In particular, agent i's utility function at time $t+1$ is

$$U\left(C_{i,t+1}, \chi_t\right) = \frac{C_{i,t+1}^{1-\gamma}}{1-\gamma} + \eta\chi_t \ , \tag{A39}$$

where $\chi_t \in \left\{\chi_t^E, \chi_t^W\right\}$ is the characteristic of the candidate who wins the election. It makes sense for agents' voting decisions to be affected by not only economic but also non-economic considerations. But the main purpose of introducing χ_t is to add electoral uncertainty, which ensures that the candidates' optimal tax choice is meaningful in all states of the world. In the absence of uncertain χ_t, there would be states of the world in which the candidates would know for certain in advance that they would lose the election, as a result of which their tax policy choice would be irrelevant. We assume that the values of χ_t^E and χ_t^W are unknown at the time E and W make their tax policy choices, but they are revealed by the time of the election.

What matters for the equilibrium is the difference $\Delta\chi_t = \chi_t^E - \chi_t^W$ (i.e., given $\Delta\chi_t$, the individual values χ_t^E and χ_t^W do not matter). To simplify the analysis, we assume the following probability distribution for $\Delta\chi_t$ before the characteristics are revealed:

$$\Delta\chi_t = \begin{cases} \infty & \text{with probability} \quad p \\ 0 & \text{with probability} \quad 1-2p \\ -\infty & \text{with probability} \quad p\,, \end{cases} \tag{A40}$$

where $p < 0.5$. That is, with probability $p + p = 2p$, the election outcome is dominated by candidates' personal characteristics, whereas with probability $1-2p$, the election outcome depends solely on candidates' tax policy choices.

The sequence of events in each period t is as follows:

1. Risk aversion γ_t is selected.
2. Candidates E and W simultaneously announce their tax policies, τ_t^E and τ_t^W.
3. Agents choose their professions, entrepreneur or government worker.
4. Characteristics χ_t^E and χ_t^W are revealed.
5. Agents vote for candidate E or W.
6. The candidate who wins the election implements his tax policy. Redistribution takes place.
7. Agents consume.

This is where the model's assumptions end. In the absence of steps 2 and 4, taking the values of τ_t^E and τ_t^W as given and setting $\Delta\chi_t = 0$, this extended model would collapse into the baseline model analyzed in the paper. Going forward, we simplify the notation by suppressing the t subscripts on γ_t, τ_t^E, τ_t^W, and all other related quantities.

We solve the model in two stages. Stage 1 involves the candidates' optimal choices of their tax policies (step 2 of the above timeline). Stage 2 involves agents' occupational and electoral choices (steps 3 and 5 of the above timeline). We solve the model backwards, first characterizing the second-stage equilibrium analytically and then solving for the first-stage equilibrium values of τ^E and τ^W numerically. We ensure that the first-stage tax policy choices internalize their effects on the second-stage occupational and electoral choices.

The following four-point proposition characterizes the second-stage equilibrium.

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Proposition A6.1: *Take the values of risk aversion γ and the policy tax rates τ^E and τ^W as given, with $\tau^E \leq \tau^W$. Let $\Gamma \equiv \{\gamma, \tau^E, \tau^W\}$ and define $\underline{\mu}(\Gamma)$ as the solution to the equation*

$$\underline{\mu}(\Gamma) = \frac{1}{1-\gamma}\log\left(\frac{\mathrm{E}[\tau^{1-\gamma}|\underline{\mu}(\Gamma)]}{\mathrm{E}[(1-\tau)^{1-\gamma}|\underline{\mu}(\Gamma)]}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{\mu}(\Gamma), \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}(\Gamma), 0, \sigma_\mu^2\right)}\right) - \frac{1}{1-\gamma}\log\left(\mathrm{E}\left[\left(\theta e^{\varepsilon_{i,t+1}} + 1 - \theta\right)^{1-\gamma}\right]\right) , \quad \text{(A41)}$$

where τ is a random variable that can take two values, τ^E and τ^W, and for any function $f(\tau)$, we define

$$\mathrm{E}[f(\tau)|\underline{\mu}(\Gamma)] = \{pf(\tau^E) + (1-p)f(\tau^W)\}I_{\{\Phi\left(\underline{\mu}(\Gamma),0,\sigma_\mu^2\right)>0.5\}} + \{pf(\tau^W) + (1-p)f(\tau^E)\}I_{\{\Phi\left(\underline{\mu}(\Gamma),0,\sigma_\mu^2\right)\leq 0.5\}} , \quad \text{(A42)}$$

where I is an indicator function equal to one if the statement attached to it is true and zero otherwise. Then we have the following results:

1. *Taking the mass of entrepreneurs as given, each entrepreneur prefers the lower tax rate and each government worker prefers the higher tax rate.*
2. *Equation (A41) has either one or two solutions.*
3. *If equation (A41) has one solution, denoted by $\underline{\mu}(\Gamma)$, then there is a unique equilibrium in which agent i becomes an entrepreneur if and only if his entrepreneurial skill μ_i satisfies*

   $$\mu_i > \underline{\mu}(\Gamma) .$$

   The mass of entrepreneurs is then

   $$m(\underline{\mu}(\Gamma)) = 1 - \Phi\left(\underline{\mu}(\Gamma), 0, \sigma_\mu^2\right) .$$

   If $m(\underline{\mu}(\Gamma)) > 0.5$ then E wins with probability $1-p$ and W wins with probability p. If $m(\underline{\mu}(\Gamma)) < 0.5$ then W wins with probability $1-p$ and E wins with probability p.
4. *If equation (A41) has two solutions, denoted by $\underline{\mu}^E(\Gamma)$ and $\underline{\mu}^W(\Gamma)$, then there is a sunspot equilibrium, in which two scenarios can occur:*

   (a) *Scenario 1: Agents believe that at least half of them will choose to become entrepreneurs (i.e., $m \geq 0.5$). Then agent i becomes an entrepreneur if and only if*

   $$\mu^i > \underline{\mu}^E(\Gamma) ,$$

   where $\underline{\mu}^E(\Gamma)$ solves equation (A41) with the function from equation (A42) redefined as

   $$E[f(\tau)|\underline{\mu}^E(\Gamma)] = pf(\tau^W) + (1-p)f(\tau^E) \quad \text{(A43)}$$

   and indeed $m(\underline{\mu}^E(\Gamma)) = 1 - \Phi\left(\underline{\mu}^E(\Gamma), 0, \sigma_\mu^2\right) \geq 0.5$.

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(b) *Scenario 2: Agents believe that fewer than half of them will become entrepreneurs (i.e., $m < 0.5$). Then agent i becomes an entrepreneur if and only if*

$$\mu^i > \underline{\mu}^W(\Gamma) \, ,$$

where $\underline{\mu}^W(\Gamma)$ solves equation (A41) with the function from equation (A42) redefined as

$$E[f(\tau)|\underline{\mu}^W(\Gamma)] \;=\; pf(\tau^E) + (1-p)f(\tau^W) \tag{A44}$$

and indeed $m(\underline{\mu}^W(\Gamma)) = 1 - \Phi\left(\underline{\mu}^W(\Gamma), 0, \sigma_\mu^2\right) < 0.5$.

To understand this proposition, it helps to consider the special case of $p = 0$. In this case, the equilibrium in Proposition A6.1 is the same as the equilibrium described in Propositions 1 through 3 in the paper. When $p = 0$, the skill threshold is

$$\begin{aligned}\underline{\mu}(\Gamma) \;=\;& \frac{1}{1-\gamma}\log\left(\frac{(\tau^W)^{(1-\gamma)} I_{\{\Phi(\underline{\mu}(\Gamma),0,\sigma_\mu^2)>0.5\}} + (\tau^E)^{(1-\gamma)} I_{\{\Phi(\underline{\mu}(\Gamma),0,\sigma_\mu^2)\le 0.5\}}}{(1-\tau^W)^{(1-\gamma)} I_{\{\Phi(\underline{\mu}(\Gamma),0,\sigma_\mu^2)>0.5\}} + (1-\tau^E)^{(1-\gamma)} I_{\{\Phi(\underline{\mu}(\Gamma),0,\sigma_\mu^2)\le 0.5\}}}\right) \\ &+ \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{\mu}(\Gamma), \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}(\Gamma), 0, \sigma_\mu^2\right)}\right) - \frac{1}{1-\gamma}\mathrm{E}\left[\left(\theta e^{\varepsilon_{i,t+1}} + 1 - \theta\right)^{1-\gamma}\right] \, , \end{aligned} \tag{A45}$$

which matches the threshold in Proposition 2 in the paper (because in both the numerator and the denominator of the first term, one of the indicators is equal to one and the other to zero). Equation (A45) can have one or two solutions. When the solution is unique, there is a unique equilibrium. Tracing out this unique solution for various γ values gives the bold line in the figure in the paper that plots the equilibrium outcomes. When equation (A45) has two solutions, we have a sunspot equilibrium, which is depicted between $\underline{\gamma}$ and $\overline{\gamma}$ in that figure in the paper. The two thresholds are then characterized in point 4 of Proposition A6.1:

$$\begin{aligned}\underline{\mu}^E(\Gamma) \;=\;& \frac{1}{1-\gamma}\log\left(\frac{(\tau^E)^{(1-\gamma)}}{(1-\tau^E)^{(1-\gamma)}}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{\mu}^E(\Gamma), \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^E(\Gamma), 0, \sigma_\mu^2\right)}\right) \\ &- \frac{1}{1-\gamma}\mathrm{E}\left[\left(\theta e^{\varepsilon_{i,t+1}} + 1 - \theta\right)^{1-\gamma}\right]\end{aligned}$$

if agents believe that E will win, and

$$\begin{aligned}\underline{\mu}^W(\Gamma) \;=\;& \frac{1}{1-\gamma}\log\left(\frac{(\tau^W)^{(1-\gamma)}}{(1-\tau^W)^{(1-\gamma)}}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{\mu}^W(\Gamma), \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^W(\Gamma), 0, \sigma_\mu^2\right)}\right) \\ &- \frac{1}{1-\gamma}\mathrm{E}\left[\left(\theta e^{\varepsilon_{i,t+1}} + 1 - \theta\right)^{1-\gamma}\right]\end{aligned}$$

if agents believe that W will win. These thresholds are the same as in Proposition 2 in the paper.

Proposition A6.1 generalizes this case to the case in which there is a probability $2p > 0$ that one of the two candidates will win not because of his tax policy but because of his personal characteristic (recall equation (A40)). In this more general case, the terms that involve the tax rates must be modified to account for the random aspect of the election outcome.

Now that we have characterized the second-stage equilibrium, we turn to the first-stage equilibrium and solve for the optimal tax policy choices of candidates E and W. Each candidate chooses a tax rate that maximizes his utility, taking as given the other candidate's equilibrium tax choice. Each candidate also takes into account the effect of his choice on the second-stage equilibrium. In particular, candidates choose τ^E and τ^W understanding that these choices determine whether the second-stage equilibrium will be unique or sunspot as well as what the equilibrium mass of entrepreneurs will be (see Proposition A6.1).

First, consider the choice of the W candidate. This candidate chooses τ^W, taking τ^E as given and also conditioning on the second-stage equilibrium. If that equilibrium is unique, W's expected utility is given by

$$E\left[U^W|\text{Unique}\right] = \frac{E[\tau^{1-\gamma}|\underline{\mu}(\Gamma)]G(\underline{\mu}(\Gamma))^{1-\gamma}}{1-\gamma}E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right]E\left[e^{\mu_j}|j\in I\right]^{1-\gamma}\left(\frac{m(\underline{\mu}(\Gamma))}{1-m(\underline{\mu}(\Gamma))}\right)^{1-\gamma}$$

where $\underline{\mu}(\Gamma)$ solves equation (A41) and $E[\tau^{1-\gamma}|\underline{\mu}(\Gamma)]$ is given by equation (A42). To obtain a sharper expression, we follow Section 4.3 in the paper in assuming that

$$G(\underline{\mu}(\Gamma)) = \left(1-m(\underline{\mu}(\Gamma))\right)^{\alpha} e^{g}$$

with $\alpha \le 1$. Recalling that

$$\begin{aligned} m(\underline{\mu}(\Gamma)) &= 1-\Phi\left(\underline{\mu}(\Gamma),0,\sigma_\mu^2\right) \\ E\left[e^{\mu_j}|j\in I\right] &= e^{\frac{1}{2}\sigma_\mu^2}\left(\frac{1-\Phi\left(\underline{\mu}(\Gamma),\sigma_\mu^2,\sigma_\mu^2\right)}{1-\Phi\left(\underline{\mu}(\Gamma),0,\sigma_\mu^2\right)}\right), \end{aligned}$$

we obtain

$$E\left[U^W|\text{Unique}\right] = \frac{E[\tau^{1-\gamma}|\underline{\mu}(\Gamma)]}{1-\gamma}\left(\frac{1-\Phi\left(\underline{\mu}(\Gamma),\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}(\Gamma),0,\sigma_\mu^2\right)^{1-\alpha}}\right)^{1-\gamma}H^W, \tag{A46}$$

where

$$H^W = e^{g(1-\gamma)}E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right]e^{\frac{1-\gamma}{2}\sigma_\mu^2}$$

does not depend on the tax rate.

If the second-stage equilibrium is sunspot instead, we need to recognize that the events $m \ge 0.5$ and $m < 0.5$ can both occur with 50% probability. We then have

$$\begin{aligned} E\left[U^W|\text{Sunspot}, m<0.5\right] &= \frac{E[\tau^{1-\gamma}|\underline{\mu}^W(\Gamma)]G(\underline{\mu}^W(\Gamma))^{1-\gamma}}{1-\gamma}E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right]E\left[e^{\mu_j}|j\in I\right]^{1-\gamma} \\ &\quad\times\left(\frac{m(\underline{\mu}^W(\Gamma))}{1-m(\underline{\mu}^W(\Gamma))}\right)^{1-\gamma} \\ &= \frac{E[\tau^{1-\gamma}|\underline{\mu}^W(\Gamma)]}{1-\gamma}\left(\frac{1-\Phi\left(\underline{\mu}^W(\Gamma),\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^W(\Gamma),0,\sigma_\mu^2\right)^{1-\alpha}}\right)^{1-\gamma}H^W \end{aligned}$$

$$\begin{aligned} E\left[U^W|\text{Sunspot}, m \geq 0.5\right] &= \frac{E[\tau^{1-\gamma}|\underline{\mu}^E(\Gamma)]G(\underline{\mu}^E(\Gamma))^{1-\gamma}}{1-\gamma} E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right] E\left[e^{\mu_j}|j \in I\right]^{1-\gamma} \\ &\quad \times \left(\frac{m(\underline{\mu}^E(\Gamma))}{1-m(\underline{\mu}^E(\Gamma))}\right)^{1-\gamma} \\ &= \frac{E[\tau^{1-\gamma}|\underline{\mu}^E(\Gamma)]}{1-\gamma} \left(\frac{1-\Phi\left(\underline{\mu}^E(\Gamma), \sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}^E(\Gamma), 0, \sigma_\mu^2\right)^{1-\alpha}}\right)^{1-\gamma} H^W \end{aligned}$$

and W's ex-ante expected utility in the sunspot equilibrium is

$$E\left[U^W|\text{Sunspot}\right] = \frac{1}{2} E\left[U^W|\text{Sunspot}, m < 0.5\right] + \frac{1}{2} E\left[U^W|\text{Sunspot}, m \geq 0.5\right] . \qquad \text{(A47)}$$

The final step in this case is to define

$$E\left[U^W\right] = \begin{cases} E\left[U^W|\text{Unique}\right] & \text{if} \quad \text{Unique Equilibrium} \\ E\left[U^W|\text{Sunspot}\right] & \text{if} \quad \text{Sunspot Equilibrium} . \end{cases}$$

Second, consider the choice of the E candidate. This candidate chooses τ^E, taking τ^W as given and also conditioning on the second-stage equilibrium. If that equilibrium is unique, E's expected utility is given by

$$\begin{aligned} E\left[U^E|\text{Unique}\right] &= \frac{E[(1-\tau)^{1-\gamma}|\underline{\mu}(\Gamma)]}{1-\gamma} G(\underline{\mu}(\Gamma))^{1-\gamma} e^{(1-\gamma)\mu_i} E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right] E\left[\left(\theta e^{\varepsilon_{i,t+1}} + 1 - \theta\right)^{1-\gamma}\right] \\ &= \frac{E[(1-\tau)^{1-\gamma}|\underline{\mu}(\Gamma)]\Phi\left(\underline{\mu}(\Gamma), 0, \sigma_\mu^2\right)^{\alpha(1-\gamma)}}{1-\gamma} H_i^E \end{aligned}$$

where

$$H_i^E = e^{(1-\gamma)(\mu_i+g)} E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right] E\left[\left(\theta e^{\varepsilon_{i,t+1}} + 1 - \theta\right)^{1-\gamma}\right]$$

does not depend on the tax rate. The value of H_i^E does depend on skill μ_i, making $E\left[U^E|\text{Unique}\right]$ dependent on μ_i as well. However, $E\left[U^E|\text{Unique}\right]$ is a product of two terms—one that depends on the tax rate but not on μ_i, and the other—H_i^E—that depends on μ_i but not on the tax rate. Therefore, E's optimal choice of the tax rate does not depend on μ_i. In other words, regardless of E's skill, his choice of the tax rate maximizes the expected utility of all entrepreneurs, not just his own. The same is true if the second-stage equilibrium is sunspot, in which case

$$\begin{aligned} E\left[U^E|\text{Sunspot}, m < 0.5\right] &= \frac{E[(1-\tau)^{1-\gamma}|\underline{\mu}^W(\Gamma)]\Phi\left(\underline{\mu}^W(\Gamma), 0, \sigma_\mu^2\right)^{\alpha(1-\gamma)}}{1-\gamma} H_i^E \\ E\left[U^E|\text{Sunspot}, m \geq 0.5\right] &= \frac{E[(1-\tau)^{1-\gamma}|\underline{\mu}^E(\Gamma)]\Phi\left(\underline{\mu}^E(\Gamma), 0, \sigma_\mu^2\right)^{\alpha(1-\gamma)}}{1-\gamma} H_i^E . \end{aligned}$$

The ex-ante expected utility of E in the sunspot equilibrium is thus

$$E\left[U^E|\text{Sunspot}\right] = \frac{1}{2} E\left[U^E|\text{Sunspot}, m < 0.5\right] + \frac{1}{2} E\left[U^E|\text{Sunspot}, m \geq 0.5\right] .$$

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The final step is to define

$$E\left[U^E\right] = \begin{cases} E\left[U^E|\text{Unique}\right] & \text{if} \quad \text{Unique Equilibrium} \\ E\left[U^E|\text{Sunspot}\right] & \text{if} \quad \text{Sunspot Equilibrium} \end{cases}.$$

While Proposition A6.1 proves that the second-stage equilibrium is either unique or sunspot, parametric conditions under which such equilibria occur are difficult to determine theoretically.

The first-stage equilibrium is a perfect Bayesian equilibrium. Each candidate chooses the tax rate while taking the other candidate's tax rate as given, and also taking into account whether the second-stage equilibrium is unique or sunspot. Whether the second-stage equilibrium is unique or sunspot depends on the tax rates announced in the first stage. We consider only tax policies such that $\tau^E < \tau^W$ to guarantee the existence of a second-stage equilibrium. Formally, we define

$$\tau^{*E}(\tau^W) = \arg\max_{\tau^E} \left\{E[U^E]\right\} \quad \text{(A48)}$$

$$\tau^{*W}(\tau^E) = \arg\max_{\tau^W} \left\{E[U^W]\right\}, \quad \text{(A49)}$$

and a perfect Bayesian equilibrium is the pair $\{\tau^{*E}(\tau^{*W}), \tau^{*W}(\tau^{*E})\}$.

Parametric illustration.

To illustrate the perfect Bayesian equilibrium, we assume that there are two possible risk aversion values, $\gamma \in \{\gamma^L, \gamma^H\}$, as in Example 1 in Section A1., and that the parameters are given in the table below. Both in Example 1 and here, $\gamma_L = 1$. In that case, we use logarithmic utility over consumption in equation (A39). Of course, log utility is the limit of CRRA utility when $\gamma \to 1$.

Table: Parameter values

σ_μ	σ_i	σ	θ	γ^L	γ^H	α	p
0.1	0.5	0.2	0.9	1	5	0.7	0.3

Figures A2 and A3 plot the optimal response functions $\tau^{E*}(\tau^W)$ and $\tau^{W*}(\tau^E)$ when $\gamma = 5$ and $\gamma = 1$, respectively. In both figures, the three colored areas show the parameter values of τ^E and τ^W for which we obtain the three possible second-stage equilibrium outcomes. First, the blue area shows a unique equilibrium in which $m \geq 0.5$—that is, at least half of all agents are entrepreneurs, and candidate E wins with probability $1 - p > 0.5$. Second, the green area shows a unique equilibrium in which $m < 0.5$—that is, fewer than half of all agents are entrepreneurs, and candidate W wins with probability $1-p > 0.5$. Third, the yellow area shows a sunspot equilibrium in which either $m \geq 0.5$ or $m < 0.5$ can occur with equal probabilities. For each value of τ^W on the x axis, the green circled line shows the optimal response $\tau^{E*}(\tau^W)$. For each value of τ^E on the y axis, the red star line shows the optimal response $\tau^{W*}(\tau^E)$. A perfect Bayesian equilibrium obtains at the point where the two optimal response functions intersect.

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Figure A2.

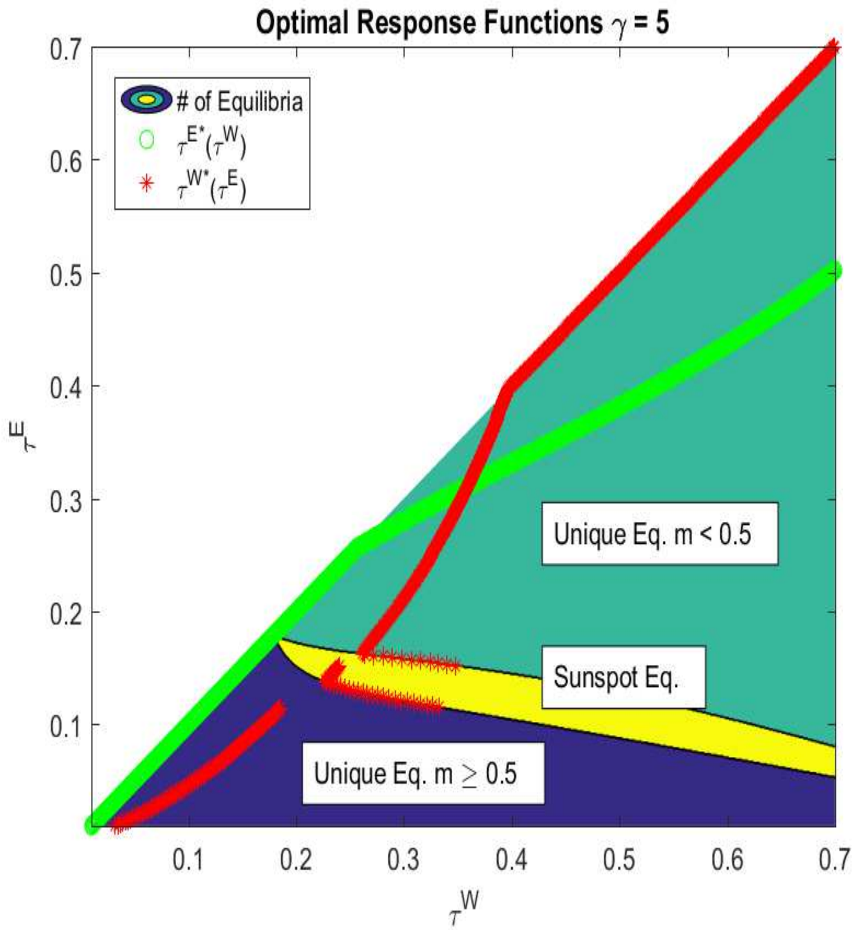
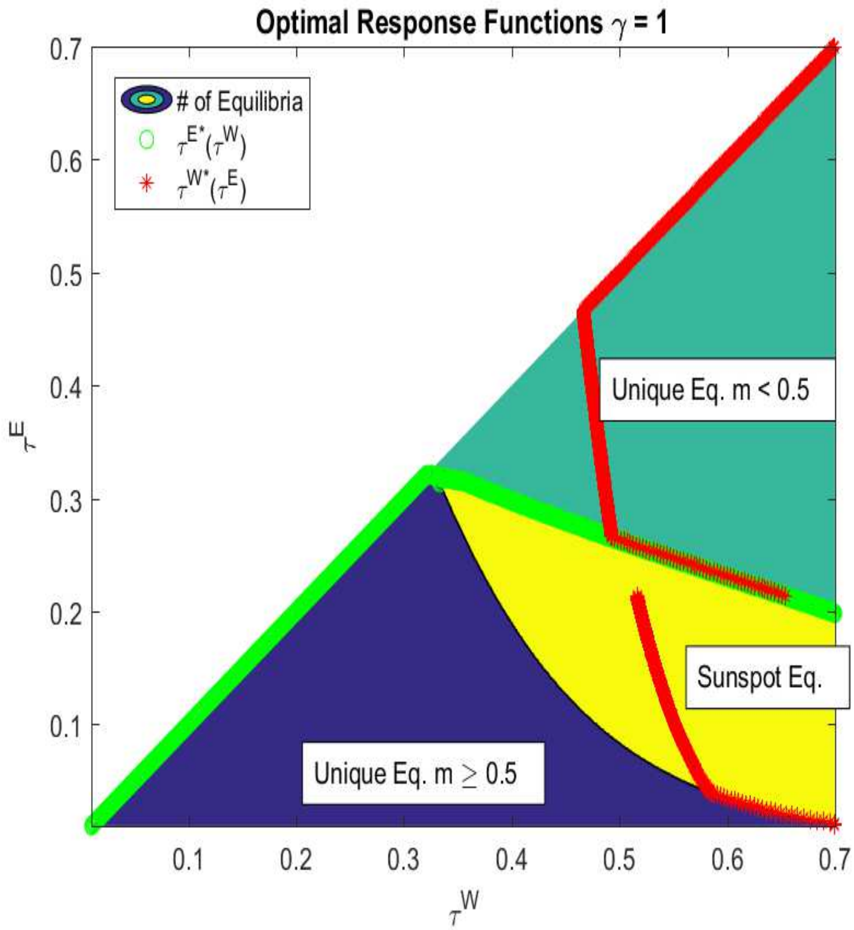


Figure A2 conditions on $\gamma = 5$. In that figure, the yellow sunspot-equilibrium area is relatively small. The two response functions intersect in the green area indicating a unique second-stage equilibrium in which candidate W wins with probability $1 - p$ and candidate E wins with probability p. The equilibrium tax policy rates are $\tau^{E*} = 31\%$ and $\tau^{W*} = 36\%$.

Figure A3 conditions on $\gamma = 1$. In that figure, the two optimal response functions intersect at the border of the yellow area. The candidates thus find it optimal to choose tax policies that induce a sunspot equilibrium. Interestingly, the intersection consists of more than one point—there is a continuum of pairs of tax rates, $\{\tau^{E*}, \tau^{W*}\}$, that correspond to a Bayesian perfect equilibrium, ranging from $\{27\%, 49\%\}$ to $\{21\%, 65\%\}$.

Figure A3.



Why are the first-stage equilibrium tax rates at the border of the second-stage sunspot equilibrium area? Conditional on being in a sunspot equilibrium, both candidates maximize their utility under the constraint that the sunspot equilibrium will occur. The result that the sunspot equilibrium is at the border of the yellow area simply indicates that this constraint is binding for at least one of the two candidates. For example, given candidate W's tax policy rate $\tau^W = 0.5$, the optimal tax policy rate for candidate E, conditional on winning, might be 0.4. However, if E were to choose $\tau^E = 0.4$, he would induce a unique equilibrium in which W wins with probability $1-p = 0.7$ and E wins with probability $p = 0.3$ (the green area in the figure). Because his 30% probability of winning in that unique equilibrium is much lower than his 50% probability of winning in the sunspot equilibrium, E prefers to push his tax policy rate lower, all the way to the yellow-green border, which yields the highest possible utility conditional on being in a sunspot equilibrium. Moving the rate further down would be suboptimal because it would not change E's 50% probability of winning but it would further reduce E's expected utility.

The equilibrium in this case features a continuum of pairs of tax rates, $\{\tau^{E*}, \tau^{W*}\}$, because the above argument is valid not only for the specific value of $\tau^W = 0.5$ but also for a range of other values of τ^W. For a number of different values of τ^W, the optimal response of candidate E

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is to induce a sunspot equilibrium by moving to the yellow-green border. The optimal response of candidate W is at the same border because even though a higher τ^W would raise W's probability of winning, it would reduce W's expected utility due to the lower total output induced by the high tax rate.

Expected stock returns.

The expected return depends on aggregate risk ε_{t+1} as well as on tax risk because agents at time t do not know which candidate will win the election at the end of the period. The following proposition shows how to compute expected returns in this setting.

Proposition A6.2: *The expected excess stock return is given by*

$$E\left[R_{t+1}|\gamma_t\right] = \frac{E\left[(1-\tau)\,|\gamma_t\right]E\left[(1-\tau)^{-\gamma_t}\,|\gamma_t\right]}{E\left[(1-\tau)^{1-\gamma_t}\,|\gamma_t\right]} \times e^{\gamma_t\sigma^2} - 1$$

where

$$E\left[f\left(\tau\right)|\gamma_t\right] = \begin{cases} pf\left(\tau^E\right) + (1-p)f\left(\tau^W\right) & \text{if Unique Equilibrium, } m_t < 0.5 \\ pf\left(\tau^W\right) + (1-p)f\left(\tau^E\right) & \text{if Unique Equilibrium, } m_t \geq 0.5 \\ 0.5f\left(\tau^W\right) + 0.5f\left(\tau^E\right) & \text{if Sunspot Equilibrium} \end{cases}$$

According to Figure A2, when $\gamma = 5$, the two candidates optimally announce the equilibrium pair of tax policies $\tau^{E*}(\tau^{W*}) = 0.31$ and $\tau^{W*}(\tau^{E*}) = 0.36$. Under those policies, candidate W wins with probability $1-p = 0.7$ and candidate E wins with probability $p = 0.3$. From the above proposition, the expected return is then given by

$$E\left[R_{t+1}|\gamma_t = 5\right] = \frac{E\left[(1-\tau)\,|\gamma_t = 5\right]E\left[(1-\tau)^{-\gamma_t}\,|\gamma_t = 5\right]}{E\left[(1-\tau)^{1-\gamma_t}\,|\gamma_t = 5\right]} \times e^{\gamma_t\sigma^2} - 1\,,$$

where

$$E\left[f\left(\tau\right)|\gamma_t = 5\right] = pf\left(\tau^E\right) + (1-p)f\left(\tau^W\right)\ .$$

Plugging in the values of $\gamma = 5$, $\sigma = 0.2$, $p = 0.3$, $\tau^{E*} = 0.31$ and $\tau^{W*} = 0.36$, we obtain the expected stock market return in this unique equilibrium:

$$E[R_{t+1}|\gamma_t = 5] = 22.8\%\ .$$

According to Figure A3, when $\gamma = 1$, the two candidates optimally announce the equilibrium tax rates on a previously described continuum. Those rates induce a sunspot equilibrium in which each candidate wins with 50% probability. The expected return depends on the choice of the optimal tax rates in the continuum. We compute the values of expected returns for the extreme cases $\{27\%, 49\%\}$ and $\{21\%, 65\%\}$. In particular,

$$E\left[R_{t+1}|\gamma_t = 1\right] = \frac{E\left[(1-\tau)\,|\gamma_t = 1\right]E\left[(1-\tau)^{-\gamma_t}\,|\gamma_t = 1\right]}{E\left[(1-\tau)^{1-\gamma_t}\,|\gamma_t = 1\right]} \times e^{\gamma_t\sigma^2} - 1\,,$$

where now

$$E\left[f(\tau)\,|\gamma_t = 1\right] = 0.5 f\left(\tau^E\right) + 0.5 f\left(\tau^W\right) \; .$$

Plugging in $\gamma = 1$, $\sigma = 0.2$, $\tau^E = 27\%$, and $\tau^W = 49\%$, the expected stock market return is

$$E[R_{t+1}|\gamma_t = 1] = 7.5\% \; .$$

For the opposite extreme case $\tau^E = 21\%$ and $\tau^W = 65\%$, we have

$$E[R_{t+1}|\gamma_t = 1] = 22.3\% \; .$$

The large increase in expected return in this case is due to tax risk, namely, the large difference in potential tax rates (21% and 65%) that can occur with 50% probability.

From these results, it follows that the expected return under "Republicans" is lower than the expected return under "Democrats"—that is, the expected return conditional on candidate E winning is lower than the expected return conditional on candidate W winning. Let q denote the unconditional probability of $\gamma_t = 5$. For this example, assume $q = 0.5$, and recall that $p = 0.3$. Therefore, for the case $\tau^E = 27\%$, and $\tau^W = 49\%$,

$$\begin{aligned}
E[R_{t+1}|W \text{ wins}] &= E[R_{t+1}|W \text{ wins}, \gamma_t = 5]Pr(\gamma_t = 5|W \text{ wins}) \\
&\quad + E[R_{t+1}|W \text{ wins}, \gamma_t = 1]Pr(\gamma_t = 1|W \text{ wins}) \\
&= 22.8\% \times \frac{(1-p)q}{(1-p)q + 0.5(1-q)} + 7.5\% \times \frac{0.5(1-q)}{(1-p)q + 0.5(1-q)} \\
&= 16.42\%
\end{aligned}$$

$$\begin{aligned}
E[R_{t+1}|E \text{ wins}] &= E[R_{t+1}|E \text{ wins}, \gamma_t = 5]Pr(\gamma_t = 5|E \text{ wins}) \\
&\quad + E[R_{t+1}|E \text{ wins}, \gamma_t = 1]Pr(\gamma_t = 1|E \text{ wins}) \\
&= 22.8\% \times \frac{pq}{pq + 0.5(1-q)} + 7.5\% \times \frac{0.5(1-q)}{pq + 0.5(1-q)} \\
&= 13.22\% \; ,
\end{aligned}$$

where we apply the Bayes formula as follows:

$$\begin{aligned}
Pr(\gamma_t = 5|W \text{ wins}) &= \frac{Pr(W \text{ wins}|\gamma_t = 5)Pr(\gamma_t = 5)}{Pr(W \text{ wins}|\gamma_t = 5)Pr(\gamma_t = 5) + Pr(W \text{ wins}|\gamma_t = 1)Pr(\gamma_t = 1)} \\
&= \frac{(1-p)q}{(1-p)q + 0.5(1-q)} \\
&= 58.33\% \\
Pr(\gamma_t = 1|W \text{ wins}) &= 1 - Pr(\gamma_t = 5|W \text{ wins}) = \frac{0.5(1-q)}{(1-p)q + 0.5(1-q)} = 41.67\% \\
Pr(\gamma_t = 5|E \text{ wins}) &= \frac{Pr(E \text{ wins}|\gamma_t = 5)Pr(\gamma_t = 5)}{Pr(E \text{ wins}|\gamma_t = 5)Pr(\gamma_t = 5) + Pr(E \text{ wins}|\gamma_t = 1)Pr(\gamma_t = 1)} \\
&= \frac{pq}{pq + 0.5(1-q)} \\
&= 37.50\% \\
Pr(\gamma_t = 1|E \text{ wins}) &= 1 - Pr(\gamma_t = 5|E \text{ wins}) = \frac{0.5(1-q)}{pq + 0.5(1-q)} = 62.50\%
\end{aligned}$$

In this example, we clearly have

$$E[R_{t+1}|W \text{ wins}] > E[R_{t+1}|E \text{ wins}] \ .$$

For the opposite extreme case with $\tau^E = 21\%$, and $\tau^W = 65\%$, we obtain

$$\begin{aligned} E[R_{t+1}|W \text{ wins}] &= 22.6\% \\ E[R_{t+1}|E \text{ wins}] &= 22.3\% \ . \end{aligned}$$

In this extreme case, the difference $E[R_{t+1}|W \text{ wins}] - E[R_{t+1}|E \text{ wins}]$ is at its lowest, but it is still positive. From Figure A3, all the other possible equilibrium tax rates when $\gamma_t = 1$ feature a smaller difference between τ^W and τ^E, which then imply a larger difference $E[R_{t+1}|W \text{ wins}] - E[R_{t+1}|E \text{ wins}]$. In general, under this parameterization, we have that the expected stock market return under candidate W is higher than under candidate E. In other words, the expected return is higher under Democrats than under Republicans.

Simulated Time Series.

To further illustrate the implications of this model extension, we simulate its performance over multiple time periods. We use the same parameter values as above. For simplicity, we draw γ_t as either $\gamma^L = 1$ or $\gamma^H = 5$ with equal probabilities in each period. If $\gamma_t = 5$ then there is a unique equilibrium in which E wins the election with probability p, W wins with probability $1 - p$, and the optimal tax policy rates are $\tau^{E*} = 31\%$ and $\tau^{W*} = 36\%$ (see Figure A2). If $\gamma_t = 1$ then there is a sunspot equilibrium in which both E and W win with probability 0.5 and the optimal tax rates are drawn randomly from the continuum between $\{27\%, 49\%\}$ and $\{21\%, 65\%\}$ (see Figure A3). We simulate the model over 20 electoral periods and plot the results in Figure A4.

Panel A of Figure A4 plots the time series of the optimal tax policy rates τ^{E*} (solid line) and τ^{W*} (dashed line). The values of τ^{W*} are consistently larger than those of τ^{E*}, and the difference between them varies over time, ranging from 5% when $\gamma_t = 5$ to anywhere between 22% and 44% when $\gamma_t = 1$. We observe larger policy polarization when risk aversion is low, consistent with Figures A2 and A3.

Panel B of Figure A4 plots the time series of expected excess stock market returns. Expected excess return ranges from 8.6% (obtained for $\gamma_t = 1$) to 22.8% (obtained for $\gamma_t = 5$). Over this 20-period sample, expected return under E's ("Republican") presidency is 13.0% per year whereas that under W's ("Democratic") presidency is 20.6% per year.

Both panels also show the winner of the election in each of the 20 periods (E or W).

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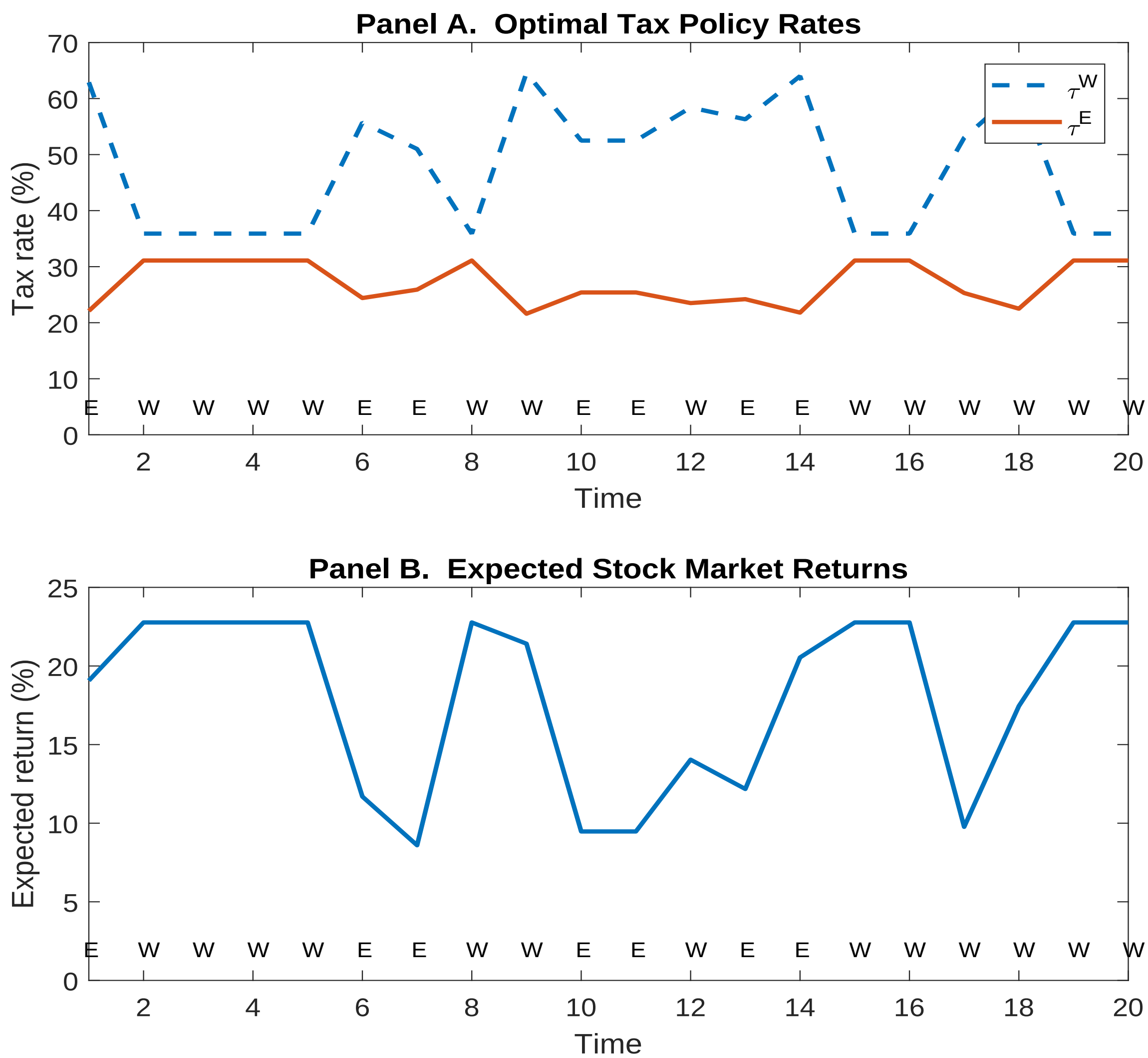
Figure A4.



Proof of Proposition A6.1.

In the baseline model's Nash equilibrium, agents make their occupational choices while taking their electoral choices as given, and vice versa. In this model extension, though, occupational choice takes place before the revelation of the personal characteristics χ^E and χ^W, whose difference co-determines the electoral outcome together with the mass of entrepreneurs, m_t. Therefore, in this extension, agents make their occupational choices while taking m_t, rather than the electoral outcome, as given. Since electoral choices also condition on m_t, just like they do in the baseline model, m_t is taken as given in the second-stage equilibrium. We consider two cases, depending on whether m_t is larger or smaller than half.

Case 1: $m_t \geq 0.5$.

In this case, agents know that candidate E will win the election with probability $1-p > 0.5$ and candidate W will win with probability $p < 0.5$. Specifically, candidate E wins if $\Delta\chi = 0$, which happens with probability $1-2p$, because entrepreneurs have a majority due to $m_t \geq 0.5$. Candidate E also wins if $\Delta\chi = \infty$, which happens with probability p. Candidate W wins only if $\Delta\chi = -\infty$, which happens with probability p.

Following the same steps as in Proposition A4, agent i's expected utility from becoming an entrepreneur is

$$E\left[U\left(C_{t+1}\right)|\text{choose job } E\right] = \frac{1}{1-\gamma}E\left[\left(1-\tau\right)^{1-\gamma}|m \geq 0.5\right]G_t^{1-\gamma}e^{\mu_i(1-\gamma)}E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right] \times \\ \times E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma}\right] + \eta E\left[\chi\right] , \tag{A50}$$

where

$$E\left[\left(1-\tau\right)^{1-\gamma}|m \geq 0.5\right] = p\left(1-\tau^W\right)^{1-\gamma} + (1-p)\left(1-\tau^E\right)^{1-\gamma} .$$

It follows immediately from equation (A50) that the expected utility of an entrepreneur is decreasing in both τ^E and τ^W. This proves that entrepreneurs prefer low tax rates, as stated in point 1 of Proposition A6.1.

Agent i's expected utility from becoming a government worker is

$$E\left[U\left(C_{t+1}\right)|\text{choose job } W\right] = \frac{1}{1-\gamma}E\left[\tau^{1-\gamma}|m \geq 0.5\right]G_t^{1-\gamma}E^{CS}\left[e^{\mu_j}|j \in I\right]^{1-\gamma}E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right] \times \\ \times \left(\frac{m}{1-m}\right)^{1-\gamma} + \eta E\left[\chi\right] , \tag{A51}$$

where

$$E\left[\tau^{1-\gamma}|m \geq 0.5\right] = p\left(\tau^W\right)^{1-\gamma} + (1-p)\left(\tau^E\right)^{1-\gamma} .$$

It follows immediately from equation (A51) that the expected utility of a government worker is increasing in both τ^E and τ^W. This proves that government workers prefer high tax rates, as stated in point 1 of Proposition A6.1.

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Comparing equations (A50) and (A51), agent i chooses to become an entrepreneur if and only if

$$E\left[U\left(C_{t+1}\right)|\text{choose job } E\right] \geq E\left[U\left(C_{t+1}\right)|\text{choose job } W\right] \ , \tag{A52}$$

which is true if and only if

$$\begin{aligned} & \frac{1}{1-\gamma}E\left[\left(1-\tau\right)^{1-\gamma}|m \geq 0.5\right]G_t^{1-\gamma}e^{\mu_i(1-\gamma)}E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right]E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma}\right]+\eta E\left[\chi\right] \\ \geq & \frac{1}{1-\gamma}E\left[\tau^{1-\gamma}|m \geq 0.5\right]G_t^{1-\gamma}E^{CS}\left[e^{\mu_j}|j \in I\right]^{1-\gamma}E\left[e^{(1-\gamma)\varepsilon_{t+1}}\right]\left(\frac{m}{1-m}\right)^{1-\gamma}+\eta E\left[\chi\right] \ , \end{aligned}$$

which holds if and only if (for $\gamma > 1$)

$$\begin{aligned} & E\left[\left(1-\tau\right)^{1-\gamma}|m \geq 0.5\right]e^{\mu_i(1-\gamma)}E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma}\right] \\ \leq & E\left[\tau^{1-\gamma}|m \geq 0.5\right]E^{CS}\left[e^{\mu_j}|j \in I\right]^{1-\gamma}\left(\frac{m}{1-m}\right)^{1-\gamma} \end{aligned}$$

if and only if

$$\begin{aligned} & \log\left(E\left[\left(1-\tau\right)^{1-\gamma}|m \geq 0.5\right]\right)+\mu_i\left(1-\gamma\right)+\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma}\right]\right) \\ \leq & \log\left(E\left[\tau^{1-\gamma}|m \geq 0.5\right]\right)+\left(1-\gamma\right)\log\left(E^{CS}\left[e^{\mu_j}|j \in I\right]\right)+\left(1-\gamma\right)\log\left(\frac{m}{1-m}\right) \end{aligned}$$

if and only if

$$\mu_i \geq \underline{\mu}\left(\Gamma\right) \ ,$$

where

$$\begin{aligned} \underline{\mu}\left(\Gamma\right) = & \frac{1}{1-\gamma}\log\left(\frac{E\left[\tau^{1-\gamma}|m \geq 0.5\right]}{E\left[\left(1-\tau\right)^{1-\gamma}|m \geq 0.5\right]}\right)+\log\left(E^{CS}\left[e^{\mu_j}|j \in I\right]\right) \\ & +\log\left(\frac{m}{1-m}\right)-\frac{1}{1-\gamma}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma}\right]\right) \ . \end{aligned} \tag{A53}$$

Case 2: $m_t < 0.5$.

In this case, agents know that candidate W will win the election with probability $1-p > 0.5$ and candidate E will win with probability $p < 0.5$. Specifically, candidate W wins if $\Delta\chi = 0$, which happens with probability $1-2p$, because government workers have a majority due to $m_t < 0.5$. Candidate W also wins if $\Delta\chi = -\infty$, which happens with probability p. Candidate E wins only if $\Delta\chi = \infty$, which happens with probability p.

Proceeding analogously to Case 1, we show that agent i becomes an entrepreneur if and only if

$$\mu_i > \underline{\mu}\left(\Gamma\right) \ ,$$

where

$$\begin{aligned} \underline{\mu}\left(\Gamma\right) = & \frac{1}{1-\gamma}\log\left(\frac{E\left[\tau^{1-\gamma}|m < 0.5\right]}{E\left[\left(1-\tau\right)^{1-\gamma}m < 0.5\right]}\right)+\log\left(E^{CS}\left[e^{\mu_j}|j \in I\right]\right) \\ & +\log\left(\frac{m}{1-m}\right)-\frac{1}{1-\gamma}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma}\right]\right) \ , \end{aligned} \tag{A54}$$

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and in this case,

$$\begin{aligned} E\left[\tau^{1-\gamma}|m<0.5\right] &= p\left(\tau^{E}\right)^{1-\gamma}+(1-p)\left(\tau^{W}\right)^{1-\gamma} \\ E\left[(1-\tau)^{1-\gamma}|m<0.5\right] &= p\left(1-\tau^{E}\right)^{1-\gamma}+(1-p)\left(1-\tau^{W}\right)^{1-\gamma} . \end{aligned}$$

Examining equations (A53) and (A54), we see that for any m, the skill threshold satisfies

$$\begin{aligned} \underline{\mu}(\Gamma) &= \frac{1}{1-\gamma}\log\left(\frac{E\left[\tau^{(1-\gamma}|m\right]}{E\left[(1-\tau)^{1-\gamma}|m\right]}\right)+\log\left(E^{CS}\left[e^{\mu_j}|j\in I\right]\right) \\ &\quad +\log\left(\frac{m}{1-m}\right)-\frac{1}{1-\gamma}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma}\right]\right) , \qquad \text{(A55)} \end{aligned}$$

where

$$\begin{aligned} E\left[\tau^{1-\gamma}|m\right] &= \begin{cases} p\left(\tau^{W}\right)^{1-\gamma}+(1-p)\left(\tau^{E}\right)^{1-\gamma} & \text{if} \quad m\geq 0.5 \\ p\left(\tau^{E}\right)^{1-\gamma}+(1-p)\left(\tau^{W}\right)^{1-\gamma} & \text{if} \quad m<0.5 \end{cases} \\ E\left[(1-\tau)^{1-\gamma}|m\right] &= \begin{cases} p\left(1-\tau^{W}\right)^{1-\gamma}+(1-p)\left(1-\tau^{E}\right)^{1-\gamma} & \text{if} \quad m\geq 0.5 \\ p\left(1-\tau^{E}\right)^{1-\gamma}+(1-p)\left(1-\tau^{W}\right)^{1-\gamma} & \text{if} \quad m<0.5 \end{cases} \end{aligned}$$

Recall that

$$\begin{aligned} m_t &= 1-\Phi\left(\underline{\mu}(\Gamma);0,\sigma_\mu^2\right) \\ E^{CS}\left[e^{\mu_j}|j\in I\right] &= e^{\frac{1}{2}\sigma_\mu^2}\frac{1-\Phi\left(\underline{\mu}(\Gamma);\sigma_\mu^2,\sigma_\mu^2\right)}{m_t} . \end{aligned}$$

Plugging these into equation (A55), we obtain the threshold from equation (A41):

$$\begin{aligned} \underline{\mu}(\Gamma) &= \frac{1}{1-\gamma}\log\left(F\left(\underline{\mu}(\Gamma)\right)\right)+\frac{1}{2}\sigma_\mu^2+\log\left(\frac{1-\Phi\left(\underline{\mu}(\Gamma);\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}(\Gamma);0,\sigma_\mu^2\right)}\right) \\ &\quad -\frac{1}{1-\gamma}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]^{1-\gamma}\right]\right) , \qquad \text{(A56)} \end{aligned}$$

where the function $F\left(\underline{\mu}(\Gamma)\right)$ is given by

$$F=\frac{\left\{p\left(\tau^{E}\right)^{1-\gamma}+(1-p)\left(\tau^{W}\right)^{1-\gamma}\right\}_{\Phi\left(\underline{\mu}(\Gamma);0,\sigma_\mu^2\right)>0.5}+\left\{p\left(\tau^{W}\right)^{1-\gamma}+(1-p)\left(\tau^{E}\right)^{1-\gamma}\right\}_{\Phi\left(\underline{\mu}(\Gamma);0,\sigma_\mu^2\right)\leq 0.5}}{\left\{p\left(1-\tau^{E}\right)^{1-\gamma}+(1-p)\left(1-\tau^{W}\right)^{1-\gamma}\right\}_{\Phi\left(\underline{\mu}(\Gamma);0,\sigma_\mu^2\right)>0.5}+\left\{p\left(1-\tau^{W}\right)^{1-\gamma}+(1-p)\left(1-\tau^{E}\right)^{1-\gamma}\right\}_{\Phi\left(\underline{\mu}(\Gamma);0,\sigma_\mu^2\right)\leq 0.5}}$$

The following lemma characterizes this function $F\left(\underline{\mu}(\Gamma)\right)$ more closely. For notational simplicity, we suppress the symbol "Γ" and refer to $F\left(\underline{\mu}(\Gamma)\right)$ simply as $F\left(\underline{\mu}\right)$.

Lemma A6.1. *The function $F\left(\underline{\mu}\right)$ jumps upward at $\underline{\mu}=0$ for $\tau^E>\tau^W$, it jumps downward at $\underline{\mu}=0$ for $\tau^E<\tau^W$, and it does not depend on $\underline{\mu}$ for $\tau^W=\tau^E$.*

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Proof of Lemma A6.1. From the basic properties of the cumulative density function of the normal distribution, we have $\Phi\left(\underline{\mu};0,\sigma_\mu^2\right) = 0.5$ for $\underline{\mu} = 0$, as well as $\Phi\left(\varepsilon;0,\sigma_\mu^2\right) > 0.5$ and $\Phi\left(-\varepsilon;0,\sigma_\mu^2\right) < 0.5$ for any $\varepsilon > 0$. Consider the values of the function $F\left(\underline{\mu}\right)$ around $\underline{\mu} = 0$:

$$\begin{aligned} F(\varepsilon) &= \frac{p\left(\tau^E\right)^{1-\gamma} + (1-p)\left(\tau^W\right)^{1-\gamma}}{p\left(1-\tau^E\right)^{1-\gamma} + (1-p)\left(1-\tau^W\right)^{1-\gamma}} \\ F(-\varepsilon) &= \frac{p\left(\tau^W\right)^{1-\gamma} + (1-p)\left(\tau^E\right)^{1-\gamma}}{p\left(1-\tau^W\right)^{1-\gamma} + (1-p)\left(1-\tau^E\right)^{1-\gamma}} \,. \end{aligned}$$

Therefore,

$$F(\varepsilon) > F(-\varepsilon)$$

if and only if

$$\frac{p\left(\tau^E\right)^{1-\gamma} + (1-p)\left(\tau^W\right)^{1-\gamma}}{p\left(1-\tau^E\right)^{1-\gamma} + (1-p)\left(1-\tau^W\right)^{1-\gamma}} > \frac{p\left(\tau^W\right)^{1-\gamma} + (1-p)\left(\tau^E\right)^{1-\gamma}}{p\left(1-\tau^W\right)^{1-\gamma} + (1-p)\left(1-\tau^E\right)^{1-\gamma}}$$

if and only if

$$\tau^W < \tau^E \,,$$

which follows from $p < 0.5$. The inequality follows because

$$\begin{aligned} p\left(\tau^E\right)^{1-\gamma} + (1-p)\left(\tau^W\right)^{1-\gamma} &> p\left(\tau^W\right)^{1-\gamma} + (1-p)\left(\tau^E\right)^{1-\gamma} \\ p\left(1-\tau^E\right)^{1-\gamma} + (1-p)\left(1-\tau^W\right)^{1-\gamma} &< p\left(1-\tau^W\right)^{1-\gamma} + (1-p)\left(1-\tau^E\right)^{1-\gamma} \,. \end{aligned}$$

Finally, note that $F(\varepsilon) = F(-\varepsilon)$ if and only if $\tau^W = \tau^E$. Q.E.D. (Lemma A6.1)

Define the following function of $\underline{\mu}$:

$$\begin{aligned} V\left(\underline{\mu}\right) &= -\underline{\mu} + \frac{1}{1-\gamma}\log\left(F\left(\underline{\mu}\right)\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi\left(\underline{\mu};\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu};0,\sigma_\mu^2\right)}\right) \\ &\quad - \frac{1}{1-\gamma}\log\left(E\left[\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]^{1-\gamma}\right]\right) \,. \end{aligned} \tag{A57}$$

Equation (A56), which characterizes the threshold $\underline{\mu}$, can then be rewritten as

$$V\left(\underline{\mu}\right) = 0 \,. \tag{A58}$$

The following lemma characterizes the function $V\left(\underline{\mu}\right)$ more closely.

Lemma A6.2: *The function $V\left(\underline{\mu}\right)$ has the following properties:*
(a) $V\left(\underline{\mu}\right)$ jumps discretely at $\underline{\mu} = 0$ from positive to negative if and only if $\tau^E > \tau^W$;
(b) $V\left(\underline{\mu}\right)$ is monotonically decreasing for $\underline{\mu} \neq 0$.

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Proof of Lemma A6.2, part (a): We first show that $V\left(\underline{\mu}\right)$ jumps from a positive to a negative value at $\underline{\mu} = 0$ if and only $\tau^E > \tau^W$. Because $\Phi\left(\underline{\mu}; a, b\right)$ is continuous in $\underline{\mu}$, as $\varepsilon \to 0$, we have

$$V(\varepsilon) - V(-\varepsilon) = -2\varepsilon + \frac{1}{1-\gamma}\log\left(\frac{F(\varepsilon)}{F(-\varepsilon)}\right) .$$

We know from Lemma A6.1 that $F(\varepsilon) > F(-\varepsilon)$ if and only if $\tau^E > \tau^W$. This implies that $\frac{1}{1-\gamma}\log\left(\frac{F\left(\varepsilon;\left\{\tau^W,\tau^E\right\}\right)}{F(-\varepsilon,\{\tau^W,\tau^E\})}\right) < 0$ as $\varepsilon \to 0$ if and only if $\tau^E > \tau^W$. That is, the function V jumps down discretely at 0 if and only if $\tau^E > \tau^W$, and by contrast, it jumps up discretely at 0 if and only if $\tau^E < \tau^W$. Q.E.D. (Lemma A6.2, part (a))

Proof of Lemma A6.2, part (b): When $\underline{\mu}$ is away from zero, the function $F\left(\underline{\mu}\right)$ does not depend on $\underline{\mu}$ because $\underline{\mu}$ enters $F\left(\underline{\mu}\right)$ only through the indicator function. Therefore, it follows from equation (A57) that

$$V'\left(\underline{\mu}\right) = -1 + \frac{\partial \log\left(\frac{1-\Phi\left(\underline{\mu};\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu};0,\sigma_\mu^2\right)}\right)}{\partial \underline{\mu}} < 0 ,$$

where the inequality stems from the fact that the fraction is negative because $\Phi\left(\underline{\mu}; a, b\right)$ is increasing in $\underline{\mu}$. Moreover, when $\underline{\mu} \to -\infty$, we have $V\left(\underline{\mu}\right) \to +\infty$ as both the first term and the last term in the first row of expression (A57) diverge to infinity. Similarly, when $\underline{\mu} \to \infty$, we have $V\left(\underline{\mu}\right) \to -\infty$ as both the first and the last term in the first row of (A57) converge to minus infinity. It follows that the function $V\left(\underline{\mu}\right)$ crosses zero either in a part in which $V\left(\underline{\mu}\right)$ is continuous ($\underline{\mu} \neq 0$), or when it jumps ($\underline{\mu} = 0$). Q.E.D. (Lemma A6.2, part (b))

Lemma A6.2 proves proves point 2 of Proposition A6.1 because it shows that the function $V\left(\underline{\mu}\right)$ has either one or two solutions for $\tau^E \leq \tau^W$. It has two solutions if and only if $V(-\varepsilon) < 0 < V(\varepsilon)$ for $\varepsilon \to 0$. This happens if the function discretely jumps from a negative value to a positive value as it crosses zero, because in that case it must intersect the $y = 0$ axis twice: once for $\underline{\mu} < 0$ and once for $\underline{\mu} > 0$. These two cases correspond to $m > 0.5$ and $m < 0.5$, respectively. In these two cases, we obtain the two sunspot equilibria described in point 4 of Proposition A6.1.

Similarly, Lemma A6.2 implies that the function $V\left(\underline{\mu}\right)$ has either one or no solution for $\tau^E > \tau^W$. It has one solution if the function $V\left(\underline{\mu}\right)$ intersects the $y = 0$ axis in a continuous part of $V\left(\underline{\mu}\right)$ and it has no solution if $V(-\varepsilon) > 0 > V(\varepsilon)$ for $\varepsilon \to 0$.

Q.E.D. (Proposition A6.1)

The Log Utility Case

The above derivations assume $\gamma > 1$, but similar arguments apply for $\gamma = 1$ (log utility). We now derive the condition characterizing the skill threshold above which a log-utility agent

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optimally chooses to become an entrepreneur. For any m, define

$$E\left[f(\tau)|m\right] = \begin{cases} pf\left(\tau^{W}\right)+(1-p)\,f\left(\tau^{E}\right) & \text{if} \quad m \geq 0.5 \\ pf\left(\tau^{E}\right)+(1-p)\,f\left(\tau^{W}\right) & \text{if} \quad m < 0.5 \end{cases}$$

Then the expected utility of agent i when he becomes an entrepreneur is

$$\begin{aligned} E\left[U\left(C_{t+1}\right)|\text{choose job } E\right] &= E\left[\log\left(1-\tau\right)|m\right]+\log\left(G_t\right)+\mu_i+E\left[\log\varepsilon_{t+1}\right] \\ &\quad +E\left[\log\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]\right]+\eta E\left[\chi\right] \end{aligned}$$

and the expected utility of agent i when he becomes a government worker is

$$\begin{aligned} E\left[U\left(C_{t+1}\right)|\text{choose job } W\right] &= E\left[\log\left(\tau\right)|m\right]+\log\left(G_t\right)+\log\left(E\left[e^{\mu_j}|j\in I\right]\right) \\ &\quad +E\left[\log\varepsilon_{t+1}\right]+\log\left(m\right)-\log\left(1-m\right)+\eta E\left[\chi\right] \ . \end{aligned}$$

The agent chooses to become an entrepreneur if

$$E\left[U\left(C_{t+1}\right)|\text{choose job } E\right] > E\left[U\left(C_{t+1}\right)|\text{choose job } W\right] \ ,$$

which holds if and only if

$$\begin{aligned} & E\left[\log\left(1-\tau\right)|m\right]+\mu_i+E\left[\log\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]\right] \\ > \ & E\left[\log\left(\tau\right)|m\right]+\log\left(E\left[e^{\mu_j}|j\in I\right]\right)+\log\left(m\right)-\log\left(1-m\right) \end{aligned}$$

if and only if

$$\begin{aligned} \mu_i \ > \ & E\left[\log\left(\frac{\tau}{1-\tau}\right)|m\right]+\log\left(E^{CS}\left[e^{\mu_j}|j\in I\right]\right)+\log\left(\frac{m}{1-m}\right) \\ & -E\left[\log\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]\right] \ . \end{aligned}$$

Substituting from

$$\begin{aligned} m_t &= 1-\Phi\left(\underline{\mu}\left(\Gamma\right);0,\sigma_\mu^2\right) \\ E^{CS}\left[e^{\mu_j}|j\in I\right] &= e^{\frac{1}{2}\sigma_\mu^2}\frac{1-\Phi\left(\underline{\mu}\left(\Gamma\right);\sigma_\mu^2,\sigma_\mu^2\right)}{m_t} \ , \end{aligned}$$

we obtain the condition for entrepreneurship as

$$\mu_i > \underline{\mu}\left(\Gamma\right) \ ,$$

where the skill threshold is given by

$$\underline{\mu}\left(\Gamma\right)=E\left[\log\left(\frac{\tau}{1-\tau}\right)|\underline{\mu}(\Gamma)\right]+\frac{1}{2}\sigma_\mu^2+\log\left(\frac{1-\Phi\left(\underline{\mu}\left(\Gamma\right);\sigma_\mu^2,\sigma_\mu^2\right)}{\Phi\left(\underline{\mu}\left(\Gamma\right);0,\sigma_\mu^2\right)}\right)-E\left[\log\left[\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right]\right]$$

and

$$E\left[f(\tau)|\underline{\mu}(\Gamma)\right] = \begin{cases} pf\left(\tau^{W}\right)+(1-p)\,f\left(\tau^{E}\right) & \text{if} \quad \Phi\left(\underline{\mu}\left(\Gamma\right);0,\sigma_\mu^2\right)\leq 0.5 \\ pf\left(\tau^{E}\right)+(1-p)\,f\left(\tau^{W}\right) & \text{if} \quad \Phi\left(\underline{\mu}\left(\Gamma\right);0,\sigma_\mu^2\right) > 0.5 \end{cases}$$

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Next, we derive the most relevant expressions for solving the first-stage equilibrium. Specifically, we compute the expected utilities of candidates E and W under log utility.

Candidate E's expected utility under the unique equilibrium is

$$\begin{aligned} E\left[U^E|\text{Unique}\right] &= E\left[\log\left(1-\tau\right)|\underline{\mu}(\Gamma)\right] + \log\left(G_t\right) + \mu_i + E\left[\log\varepsilon_{t+1}\right] \\ &\quad + E\left[\log\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]\right] \\ &= E\left[\log\left(1-\tau\right)|\underline{\mu}(\Gamma)\right] + \alpha\log\left(1-m\right) + \mu_i + E\left[\log\varepsilon_{t+1}\right] \\ &\quad + E\left[\log\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]\right] \\ &= E\left[\log\left(1-\tau\right)|\underline{\mu}(\Gamma)\right] + \alpha\log\left(\Phi\left(\underline{\mu}\left(\Gamma\right),0,\sigma_\mu^2\right)\right) + \mu_i \\ &\quad + E\left[\log\varepsilon_{t+1}\right] + E\left[\log\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]\right] \\ &= E\left[\log\left(1-\tau\right)|\underline{\mu}(\Gamma)\right] + \alpha\log\left(\Phi\left(\underline{\mu}\left(\Gamma\right),0,\sigma_\mu^2\right)\right) + H_i^E\ , \end{aligned}$$

where

$$H_i^E = \mu_i + E\left[\log\varepsilon_{t+1}\right] + E\left[\log\left[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right]\right]\ .$$

Similarly, in the sunspot equilibrium,

$$\begin{aligned} E\left[U^E|\text{Sunspot}, m\geq 0.5\right] &= E\left[\log\left(1-\tau\right)|\underline{\mu}^E(\Gamma)\right] + \alpha\log\left(\Phi\left(\underline{\mu}\left(\Gamma\right),0,\sigma_\mu^2\right)\right) + H_i^E \\ E\left[U^E|\text{Sunspot}, m< 0.5\right] &= E\left[\log\left(1-\tau\right)|\underline{\mu}^W(\Gamma)\right] + \alpha\log\left(\Phi\left(\underline{\mu}\left(\Gamma\right),0,\sigma_\mu^2\right)\right) + H_i^E \end{aligned}$$

and

$$E\left[U^E|\text{Sunspot}\right] = \frac{1}{2}E\left[U^E|\text{Sunspot}, m\geq 0.5\right] + \frac{1}{2}E\left[U^E|\text{Sunspot}, m< 0.5\right]\ .$$

Candidate W's expected utility under the unique equilibrium is

$$\begin{aligned} E\left[U^W|\text{Unique }\right] &= E\left[\log\left(\tau\right)\underline{\mu}(\Gamma)\right] + \log\left(G_t\right) + \log\left(E\left[e^{\mu_j}|j\in I\right]\right) \\ &\quad + E\left[\log\varepsilon_{t+1}\right] + \log\left(\frac{m}{1-m}\right) \\ &= E\left[\log\left(\tau\right)\underline{\mu}(\Gamma)\right] + \alpha\log\left(1-m\right) + \log\left(E\left[e^{\mu_j}|j\in I\right]\right) \\ &\quad + E\left[\log\varepsilon_{t+1}\right] + \log\left(\frac{m}{1-m}\right) \\ &= E\left[\log\left(\tau\right)|\underline{\mu}(\Gamma)\right] + (\alpha-1)\log\left(\Phi\left(\underline{\mu}\left(\Gamma\right),0,\sigma_\mu^2\right)\right) \\ &\quad + \log\left(1-\Phi\left(\underline{\mu}\left(\Gamma\right),\sigma_\mu^2,\sigma_\mu^2\right)\right) + E\left[\log\varepsilon_{t+1}\right]\ . \end{aligned}$$

Similarly, in the sunspot equilibrium,

$$\begin{aligned} E\left[U^W|\text{Sunspot }, m\geq 0.5\right] &= E\left[\log\left(\tau\right)|\underline{\mu}^E(\Gamma)\right] + (\alpha-1)\log\left(\Phi\left(\underline{\mu}^E\left(\Gamma\right),0,\sigma_\mu^2\right)\right) \\ &\quad + \log\left(1-\Phi\left(\underline{\mu}^E\left(\Gamma\right),\sigma_\mu^2,\sigma_\mu^2\right)\right) + E\left[\log\varepsilon_{t+1}\right] \\ E\left[U^W|\text{Sunspot }, m< 0.5\right] &= E\left[\log\left(\tau\right)|\underline{\mu}^W(\Gamma)\right] + (\alpha-1)\log\left(\Phi\left(\underline{\mu}^W\left(\Gamma\right),0,\sigma_\mu^2\right)\right) \\ &\quad + \log\left(1-\Phi\left(\underline{\mu}^W\left(\Gamma\right),\sigma_\mu^2,\sigma_\mu^2\right)\right) + E\left[\log\varepsilon_{t+1}\right] \end{aligned}$$

and

$$E\left[U^W|\text{Sunspot}\right] = \frac{1}{2}E\left[U^W|\text{Sunspot}, m \geq 0.5\right] + \frac{1}{2}E\left[U^W|\text{Sunspot}, m < 0.5\right] \ .$$

The rest of the argument is the same as in the CRRA case presented earlier.

Proof of Proposition A6.2. The proof is identical to the proof of Proposition A4. Q.E.D.

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A7. Theory: Extension to Retrospective Voting

In this section we extend our model to allow for persistent variation in government quality, which induces retrospective voting. We are able to solve this more complicated model in the special case from Example 1 (Section A1.) where risk aversion can take two possible values. As we show below, the presence of both retrospective voting and time-varying risk aversion makes it possible for this extended model to deliver both patterns in Table 5 in the paper: the strong state dependence of Republican-to-Democrat transitions and the weak state dependence of Democrat-to-Republican transitions. For the same parameter values, this extended model also predicts higher average stock market returns under Democrats, thereby preserving the main implication of the baseline model.

We model time-varying government quality as follows. In equation (18) in the paper, the value of g denotes the average productivity of the public sector. Here we allow g to be time-varying, denoting it by g_t and interpreting it as government quality. Equation (18) in the paper then becomes $G_t = (1 - m_t)e^{g_t}$. Combining it with equation (3) in the paper, firm i's output is

$$Y_{it+1} = e^{\mu_i + \varepsilon_{t+1} + \varepsilon_{it+1} + g_{t+1}} (1 - m_t) \ . \tag{A59}$$

We allow g_t to take two values, high and low, $g_t \in \{g^H, g^L\}$, where $g^H > g^L$. We refer to a government of high quality (g^H) as a "good government" and a government of low quality (g^L) as a "bad government." The value of g_t is observed by all agents.

As in the baseline model, there are two parties, H and L. These parties levy the tax rates τ^H and τ^L, respectively, where $\tau^H > \tau^L$. We refer to a government with party H in power as an "H government" and a government with party L in power as an "L government." Both parties can rule at high or low quality, so there are four possible states of the world regarding the government:

1. Good H government (g^H, τ^H)
2. Good L government (g^H, τ^L)
3. Bad H government (g^L, τ^H)
4. Bad L government (g^L, τ^L)

The transition matrix for g_t depends on whether the government changes (i.e., the party in power changes, either from H to L or the other way round) or stays the same after the election. We introduce persistence in government quality by assuming that if the government stays the same, it retains the same quality with probability greater than 50%:

$$g_{t+1}|_{g_t = g^H} = \begin{cases} g^H & \text{with probability} \quad p^H \\ g^L & \text{with probability} \quad 1 - p^H \end{cases} \tag{A60}$$

$$g_{t+1}|_{g_t = g^L} = \begin{cases} g^L & \text{with probability} \quad p^L \\ g^H & \text{with probability} \quad 1 - p^L \ , \end{cases} \tag{A61}$$

where $p^H > 0.5$ and $p^L > 0.5$. In contrast, if the government changes, the new government is equally likely to be good or bad (i.e., both g^H and g^L occur with 50% probability).

Under a bad government, the economy performs relatively poorly, on average, whereas under a good government, the economy performs relatively well. To see this formally, recall from equation (A59) that Y_t tends to be low when $g_t = g^L$, whereas Y_t tends to be high when $g_t = g^H$. Therefore, consistent with the approach we take in the baseline model in the paper, we assume that after a bad government, risk aversion is high, whereas after a good government, risk aversion is low:

$$\gamma_t = \begin{cases} \gamma^H & \text{if} \quad g_t = g^L \\ \gamma^L & \text{if} \quad g_t = g^H \end{cases}, \tag{A62}$$

where both γ^H and γ^L are greater than one. The timing is such that g_t is revealed at the end of period $t-1$. Based on this value of g_t, the value of γ_t is determined at the beginning of period t according to equation (A62).

At the beginning of period t, all agents observe which party is in power (i.e., $\tau_t \in \{\tau^H, \tau^L\}$) and what the government quality is (i.e., $g_t \in \{g^H, g^L\}$). By implication, they also observe γ_t (equation (A62)). Based on this information, agents make two decisions: which party to vote for (H or L) and which occupation to take (entrepreneur: E, or government worker: W).

To determine which party wins the election, we consider the four possible states of the government mentioned earlier: $(g_t, \tau_t) = \{(g^H, \tau^H), (g^H, \tau^L), (g^L, \tau^H), (g^L, \tau^L)\}$. We state the results below and provide the intuition behind them. The proofs are at the end of this section.

1. $(g_t, \tau_t) = (g^H, \tau^H)$: **Good government, High-tax party.**

   Results:

   - Government workers always vote for party H.
   - Entrepreneurs vote for party H if and only if

   $$\frac{1}{1-\gamma^L} \log\left(\frac{p^H e^{(1-\gamma^L)(g^H - g^L)} + (1-p^H)}{\frac{1}{2} e^{(1-\gamma^L)(g^H-g^L)} + \frac{1}{2}}\right) > \log\left(\frac{1-\tau^L}{1-\tau^H}\right) \tag{A63}$$

   Discussion:

   Government workers prefer party H, for two reasons. First, they prefer the higher tax rate, τ^H, because their consumption is increasing in the tax rate. Second, they like the fact that the incumbent H government is good, due to persistence in government quality. Government workers thus always vote to keep party H in power in this state of the world.

   Entrepreneurs also like the incumbent government's high quality, but they dislike its high-tax policy, so their voting decision depends on the parameter values. They are more likely to vote H if the two tax rates are close to each other, if $g^H - g^L$ is high, and if p^H is high. All of these relations follow from inequality (A63), both sides of which are positive. When

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$\tau^L \to \tau^H$, the right-hand side of equation (A63) converges to zero, and all entrepreneurs vote H. When either $g^H - g^L$ or p^H is high, the left-hand side of equation (A63) increases because the persistence effect is stronger, making party H more desirable.

2. $(g_t, \tau_t) = (g^L, \tau^H)$: **Bad government, High-tax party.**

   Results:

   - Entrepreneurs always vote for party L.
   - Government workers vote for party H if and only if

$$\frac{1}{1-\gamma^H} \log\left(\frac{\left(1-p^L\right) e^{\left(1-\gamma^H\right)\left(g^H-g^L\right)} + p^L}{\frac{1}{2} e^{\left(1-\gamma^H\right)\left(g^H-g^L\right)} + \frac{1}{2}}\right) > \log\left(\frac{\tau^L}{\tau^H}\right) \tag{A64}$$

   Discussion:

   Entrepreneurs dislike the incumbent H government's low quality as well as its high-tax policy, so they always vote against it.

   Government workers also dislike the government's low quality but they like its high-tax policy, so their voting decision depends on the parameter values. They are more likely to vote H if the two tax rates are further apart from each other, if $g^H - g^L$ is low, and if p^L is low. All of these relations follow from inequality (A64), both sides of which are negative.

3. $(g_t, \tau_t) = (g^H, \tau^L)$: **Good government, Low-tax party.**

   Results:

   - Entrepreneurs always vote for party L.
   - Government workers vote for party H if and only if

$$\frac{1}{1-\gamma^L} \log\left(\frac{\frac{1}{2} e^{\left(1-\gamma^L\right)\left(g^H-g^L\right)} + \frac{1}{2}}{p^H e^{\left(1-\gamma^L\right)\left(g^H-g^L\right)} + \left(1-p^H\right)}\right) > \log\left(\frac{\tau^L}{\tau^H}\right) \tag{A65}$$

   Discussion:

   Entrepreneurs like the incumbent L government's high quality as well as its low-tax policy, so they always vote for it.

   Government workers also like the government's high quality but they dislike its low-tax policy, so their voting decision depends on the parameter values. They are more likely to vote H if the two tax rates are further apart from each other, if $g^H - g^L$ is low, and if p^H is low. All of these relations follow from inequality (A65), both sides of which are negative.

4. $(g_t, \tau_t) = (g^L, \tau^L)$: **Bad government, Low-tax party.**

   Results:

- Government workers always vote for party H.
- Entrepreneurs vote for party H if and only if

$$\frac{1}{1-\gamma^H}\log\left(\frac{\frac{1}{2}e^{\left(1-\gamma^H\right)\left(g^H-g^L\right)}+\frac{1}{2}}{\left(1-p^L\right)e^{\left(1-\gamma^H\right)\left(g^H-g^L\right)}+p^L}\right)>\log\left(\frac{1-\tau^L}{1-\tau^H}\right) \tag{A66}$$

Discussion:

Government workers dislike the incumbent L government's low quality as well as its low-tax policy, so they always vote against it.

Entrepreneurs also dislike the government's low quality but they like its low-tax policy, so their voting decision depends on the parameter values. They are more likely to vote H if the two tax rates are closer to each other, if $g^H - g^L$ is high, and if p^L is high. All of these relations follow from inequality (A66), both sides of which are positive.

Equilibrium.

The equilibrium outcomes depend on the parameter values. We illustrate the equilibrium results by using the parameter values from Example 1 in Section A1. of this Online Appendix: $\gamma^H = 5$, $\gamma^L = 1$, $\tau^H = 34\%$, $\tau^L = 32\%$, $\sigma_\mu = 10\%$ per year, $\sigma = 20\%$ per year, $\sigma_1 = 50\%$ per year, and $\theta = 0.6$.[‡] In Example 1, government quality is $g = -0.2$. Here, we have two levels of government quality, and we pick them symmetrically around -0.2, namely, $g_L = -0.3$ and $g_H = -0.1$. For quality persistence, we pick the values $p^L = 0.9$ and $p^H = 0.6$. Given these values, poor government performance is more likely to persist than good performance. Voters are thus more willing to punish underperforming governments than to reward outperforming ones.

The following table summarizes the equilibrium outcomes in each of the four states for these parameter values:

	$\left(g^H,\tau^H\right)$	$\left(g^L,\tau^H\right)$	$\left(g^H,\tau^L\right)$	$\left(g^L,\tau^L\right)$
Risk aversion	γ^L	γ^H	γ^L	γ^H
Equilibrium winner	L	L	L	H

This parametric example illustrates both mechanisms—time-varying risk aversion (TVRA), the focus of our study, and retrospective voting (RV), the focus of this model extension. In columns 1 and 2, TVRA and RV pull in opposite directions. In column 1, TVRA favors party L because risk aversion is low, but RV favors party H because the incumbent H government is good. For these parameter values, TVRA wins. In column 2, TVRA favors party H because risk aversion is high, but RV favors party L because the incumbent H government is bad. For these parameter values, RV wins. In columns 3 and 4, TVRA and RV pull in the same directions. In column 3, TVRA favors party L because risk aversion is low, and RV favors party L because the incumbent

[‡]We set $\gamma^L = 1.0001$ to ensure that the condition $\gamma_t > 1$ is satisfied. It would be easy to pick risk aversion values more distant from one but we want to discipline this exercise by committing to parameter choices made earlier.

L government is good. In column 4, TVRA favors party H because risk aversion is high, and RV favors party H because the incumbent L government is bad.

Importantly, this example illustrates how the interaction of TVRA and RV can generate the transition patterns observed in Table 5 in the paper. First, a transition from party L to party H (Republicans to Democrats), which happens in column 4, is preceded by a weak economy (g^L). Therefore, a regression of such transitions on recent economic performance would produce a negative slope, just like the left column of Table 5. Second, transitions from party H to party L (Democrats to Republicans) happen in columns 1 and 2. One of them is preceded by a strong economy (g^H; column 1) and the other by a weak economy (g^L; column 2). Therefore, a regression of these transitions on recent performance would produce mixed results, just like the right column of Table 5. Its ability to explain the asymmetry in Table 5 is the raison d'être of this framework.

Just like our baseline model, this setting also delivers the "presidential puzzle"—a higher average excess stock market return under the H government (i.e., Democrats). We no longer obtain a general result akin to Proposition 4 in the paper, but the result obtains quite strongly for various parameter configurations, including this one. When party H is in power, risk aversion is high (i.e., $\gamma_t = \gamma^H$, see column 4), resulting in a high market risk premium. When party L is in power, risk aversion is either low (columns 1 and 3) or high (column 2), resulting in a lower market risk premium, on average. Given these parameter values, we obtain $\text{ER}(H) - \text{ER}(L) = 14\%$ per year, a value somewhat larger than, but similar to, the 11% difference observed in the data.

Proofs:

First, we establish some notation. For each of the four possible states of the world captured by (g_t, τ_t), we define

- $\text{E}[U|E, g_t, \tau_t]$ = expected utility from being an entrepreneur
- $\text{E}[U|W, g_t, \tau_t]$ = expected utility from being a government worker.

The above expected utility functions apply at times when the value of g_t for the upcoming government is already known. But the value of g_t is still unknown when agents make their electoral decision. Therefore, at the time of the election, we use the notation

- $\text{E}[U|E, p_t, \tau_t]$ = expected utility from being an entrepreneur
- $\text{E}[U|W, p_t, \tau_t]$ = expected utility from being a government worker,

where p_t is the probability that the incumbent government's quality will remain the same after the election, following equations (A60) and (A61). These expected utility functions are defined for each of the four combinations of $p_t \in \{p^H, p^L\}$ and $\tau_t \in \{\tau^H, \tau^L\}$. Specifically, when the current government's quality is g^j, where $j \in \{H, L\}$, and the current government's tax policy is τ^k, where $k \in \{H, L\}$, then

$$\text{E}\left[U|E, p^j, \tau^k\right] \quad = \quad p^j \text{E}\left[U|E, g^j, \tau^k\right] + \left(1 - p^j\right) \text{E}\left[U|E, g^{j'}, \tau^k\right] \tag{A67}$$

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$$\mathrm{E}\left[U|W,p^{j},\tau^{k}\right] = p^{j}\mathrm{E}\left[U|W,g^{j},\tau^{k}\right]+\left(1-p^{j}\right)\mathrm{E}\left[U|W,g^{j'},\tau^{k}\right], \quad \text{(A68)}$$

where $\mathrm{E}\left[U|E,g_t,\tau_t\right]$ and $\mathrm{E}\left[U|W,g_t,\tau_t\right]$ are defined earlier and j' is the complement of j in the two-element space $\{H,L\}$—that is, if $j=H$ then $j'=L$, and vice versa.

Given our model assumptions, the expected utility functions for given g_t are equal to[§]

$$\mathrm{E}\left[U|E,g^{j},\tau^{k}\right] = \left(1-\tau^{k}\right)^{1-\gamma_t}e^{(1-\gamma_t)\left(g^{j}+\mu_i\right)}\frac{\mathrm{E}\left[e^{(1-\gamma_t)\varepsilon_{t+1}}\right]\mathrm{E}\left[\left(\theta e^{\varepsilon_{i,t+1}}+(1-\theta)\right)^{1-\gamma_t}\right]}{1-\gamma_t} \quad \text{(A69)}$$

$$\mathrm{E}\left[U|W,g^{j},\tau^{k}\right] = \frac{e^{(1-\gamma_t)g^{j}}}{1-\gamma_t}\left(\frac{\tau^{k}e^{\varepsilon_{t+1}}m_t\mathrm{E}\left[e^{\mu_i}|i\in I_t\right]}{1-m_t}\right)^{1-\gamma_t} \quad \text{(A70)}$$

Proof for State 1: $(g_t,\tau_t)=\left(g^H,\tau^H\right)$.

- Government workers vote H if and only if

$$E\left[U|W,p^{H},\tau^{H}\right] > E\left[U|W,0.5,\tau^{L}\right]$$

$$p^{H}\mathrm{E}\left[U|W,g^{H},\tau^{H}\right]+\left(1-p^{H}\right)\mathrm{E}\left[U|W,g^{L},\tau^{H}\right] > \frac{1}{2}\mathrm{E}\left[U|W,g^{H},\tau^{L}\right]+\frac{1}{2}\mathrm{E}\left[U|W,g^{L},\tau^{L}\right]$$

The left-hand side (LHS) of this inequality, which is the expected utility from keeping party H in power, is given by

$$LHS = \frac{\left[p^{H}e^{\left(1-\gamma^{L}\right)g^{H}}+\left(1-p^{H}\right)e^{\left(1-\gamma^{L}\right)g^{L}}\right]}{1-\gamma^{L}}\left(\tau^{H}\right)^{1-\gamma^{L}}\frac{E\left[e^{\left(1-\gamma^{L}\right)\varepsilon_{t+1}}\right]m_t^{1-\gamma^{L}}E\left[e^{\mu_i}|i\in I_t\right]^{1-\gamma^{L}}}{\left(1-m_t\right)^{1-\gamma^{L}}},$$

recognizing from equation (A62) that $\gamma_t=\gamma^L$ in this case. The right-hand side (RHS) of the same inequality, which is the expected utility from electing party L, is given by

$$RHS = \frac{\left[\frac{1}{2}e^{\left(1-\gamma^{L}\right)g^{H}}+\frac{1}{2}e^{\left(1-\gamma^{L}\right)g^{L}}\right]}{1-\gamma^{L}}\left(\tau^{L}\right)^{1-\gamma^{L}}\frac{E\left[e^{\left(1-\gamma^{L}\right)\varepsilon_{t+1}}\right]m_t^{1-\gamma^{L}}E\left[e^{\mu_i}|i\in I_t\right]^{1-\gamma^{L}}}{\left(1-m_t\right)^{1-\gamma^{L}}}$$

Therefore, government workers vote H if and only if LHS > RHS, that is,

$$\frac{p^{H}e^{\left(1-\gamma^{L}\right)g^{H}}+\left(1-p^{H}\right)e^{\left(1-\gamma^{L}\right)g^{L}}}{1-\gamma^{L}}\left(\tau^{H}\right)^{1-\gamma^{L}} > \frac{\frac{1}{2}e^{\left(1-\gamma^{L}\right)g^{H}}+\frac{1}{2}e^{\left(1-\gamma^{L}\right)g^{L}}}{1-\gamma^{L}}\left(\tau^{L}\right)^{1-\gamma^{L}}$$

$$\frac{p^{H}e^{\left(1-\gamma^{L}\right)g^{H}}+\left(1-p^{H}\right)e^{\left(1-\gamma^{L}\right)g^{L}}}{\frac{1}{2}e^{\left(1-\gamma^{L}\right)g^{H}}+\frac{1}{2}e^{\left(1-\gamma^{L}\right)g^{L}}} < \left(\frac{\tau^{L}}{\tau^{H}}\right)^{1-\gamma^{L}}$$

$$\frac{1}{1-\gamma^{L}}\log\left(\frac{p^{H}e^{\left(1-\gamma^{L}\right)g^{H}}+\left(1-p^{H}\right)e^{\left(1-\gamma^{L}\right)g^{L}}}{\frac{1}{2}e^{\left(1-\gamma^{L}\right)g^{H}}+\frac{1}{2}e^{\left(1-\gamma^{L}\right)g^{L}}}\right) > \log\left(\frac{\tau^{L}}{\tau^{H}}\right)$$

$$\frac{1}{1-\gamma^{L}}\log\left(\frac{p^{H}e^{\left(1-\gamma^{L}\right)\left(g^{H}-g^{L}\right)}+\left(1-p^{H}\right)}{\frac{1}{2}e^{\left(1-\gamma^{L}\right)\left(g^{H}-g^{L}\right)}+\frac{1}{2}}\right) > \log\left(\frac{\tau^{L}}{\tau^{H}}\right).$$

[§]Entrepreneur i's expected utility depends on his skill μ_i. Therefore, a more precise notation for his expected utility would be $\mathrm{E}\left[U|E,g^{j},\tau^{k},i\right]$. We suppress the i argument to simplify the notation.

This condition is always satisfied because the left-hand side is always positive and the right-hand side is always negative (recall that $p^H > 0.5$, $\gamma^L > 1$, $g^H > g^L$, and $\tau^H > \tau^L$). Therefore, in the state of the world characterized by $(g_t, \tau_t) = \left(g^H, \tau^H\right)$, government workers always vote for party H, QED.

- Entrepreneurs vote for party H if and only if

$$E\left[U|E, p^H, \tau^H\right] \quad > \quad E\left[U|E, 0.5, \tau^L\right] \ .$$

From equations (A67) and (A69), we have, for $k \in \{H, L\}$ and any transition probability p,

$$\begin{aligned} E\left[U|E, p, \tau^k\right] \quad &= \quad \left(1-\tau^k\right)^{1-\gamma^L} \left[p e^{\left(1-\gamma^L\right)g^H} + (1-p)\, e^{\left(1-\gamma^L\right)g^L}\right] e^{\left(1-\gamma^L\right)\mu_i} \\ &\quad \times \frac{E\left[e^{\left(1-\gamma^L\right)\varepsilon_{t+1}}\right] E\left[\left(\theta e^{\varepsilon_{i,t+1}} + (1-\theta)\right)^{1-\gamma^L}\right]}{1-\gamma^L} \ . \end{aligned}$$

Plugging into the inequality above and canceling common terms, including $1-\gamma^L < 0$, entrepreneurs vote for party H if and only if

$$\begin{aligned} \left(1-\tau^L\right)^{1-\gamma^L} \left[\frac{1}{2} e^{\left(1-\gamma^L\right)g^H} + \frac{1}{2} e^{\left(1-\gamma^L\right)g^L}\right] \quad &> \quad \left(1-\tau^H\right)^{1-\gamma^L} \left[p^H e^{\left(1-\gamma^L\right)g^H} + \left(1-p^H\right) e^{\left(1-\gamma^L\right)g} \right. \\ \frac{1}{1-\gamma^L} \log\left(\frac{p^H e^{\left(1-\gamma^L\right)g^H} + \left(1-p^H\right) e^{\left(1-\gamma^L\right)g^L}}{\frac{1}{2} e^{\left(1-\gamma^L\right)g^H} + \frac{1}{2} e^{\left(1-\gamma^L\right)g^L}}\right) \quad &> \quad \log\left(\frac{1-\tau^L}{1-\tau^H}\right) \\ \frac{1}{1-\gamma^L} \log\left(\frac{p^H e^{\left(1-\gamma^L\right)\left(g^H-g^L\right)} + \left(1-p^H\right)}{\frac{1}{2} e^{\left(1-\gamma^L\right)\left(g^H-g^L\right)} + \frac{1}{2}}\right) \quad &> \quad \log\left(\frac{1-\tau^L}{1-\tau^H}\right) \ , \end{aligned}$$

which is equation (A63), QED.

Proof for State 2: $(g_t, \tau_t) = \left(g^L, \tau^H\right)$.

- Entrepreneurs vote for party H if and only if

$$\frac{1}{1-\gamma^H} \log\left(\frac{\left(1-p^L\right) e^{\left(1-\gamma^H\right)\left(g^H-g^L\right)} + p^L}{\frac{1}{2} e^{\left(1-\gamma^H\right)\left(g^H-g^L\right)} + \frac{1}{2}}\right) > \log\left(\frac{1-\tau^L}{1-\tau^H}\right) \ ,$$

which is never satisfied because the left-hand side is always negative while the right-hand side is always positive. Therefore, entrepreneurs always vote for party L. QED.

- Government workers vote for party H if and only if

$$\frac{1}{1-\gamma^H} \log\left(\frac{\left(1-p^L\right) e^{\left(1-\gamma^H\right)\left(g^H-g^L\right)} + p^L}{\frac{1}{2} e^{\left(1-\gamma^H\right)\left(g^H-g^L\right)} + \frac{1}{2}}\right) > \log\left(\frac{\tau^L}{\tau^H}\right) \ , \qquad \text{(A71)}$$

proceeding through the same steps as in state 1. QED.

Proof for State 3: $(g_t, \tau_t) = \left(g^H, \tau^L\right)$.

- Entrepreneurs vote for party H if and only if

$$\frac{1}{1-\gamma^L} \log \left(\frac{\frac{1}{2} e^{\left(1-\gamma^L\right) g^H} + \frac{1}{2} e^{\left(1-\gamma^L\right) g^L}}{p^H e^{(1-\gamma^L) g^H} + \left(1-p^H\right) e^{(1-\gamma^L) g^L}} \right) > \log \left(\frac{1-\tau^L}{1-\tau^H} \right)$$

This condition is never satisfied because the left-hand side is always negative while the right-hand side is always positive. Therefore, entrepreneurs always vote for party L, QED.

- Government workers vote for party H if and only if

$$\frac{1}{1-\gamma^L} \log \left(\frac{\frac{1}{2} e^{\left(1-\gamma^L\right)(g^H - g^L)} + \frac{1}{2}}{p^H e^{(1-\gamma^L)(g^H - g^L)} + \left(1-p^H\right)} \right) > \log \left(\frac{\tau^L}{\tau^H} \right) , \tag{A72}$$

QED.

Proof for State 4: $(g_t, \tau_t) = \left(g^L, \tau^L\right)$.

- Government workers vote for party H if and only if

$$\frac{1}{1-\gamma^H} \log \left(\frac{\frac{1}{2} e^{\left(1-\gamma^H\right) g^H} + \frac{1}{2} e^{\left(1-\gamma^H\right) g^L}}{\left(1-p^L\right) e^{(1-\gamma^H) g^H} + p^L e^{(1-\gamma^H) g^L}} \right) > \log \left(\frac{\tau^L}{\tau^H} \right) .$$

This condition is always satisfied because the left-hand side is always positive and the right-hand side is always negative. Therefore, government workers always vote for party H, QED.

- Entrepreneurs vote for party H if and only if

$$\frac{1}{1-\gamma^H} \log \left(\frac{\frac{1}{2} e^{\left(1-\gamma^H\right)(g^H - g^L)} + \frac{1}{2}}{\left(1-p^L\right) e^{(1-\gamma^H)(g^H - g^L)} + p^L} \right) > \log \left(\frac{1-\tau^L}{1-\tau^H} \right) ,$$

QED.

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A8. Evidence: International Stock Returns

This section presents international evidence on the presidential puzzle. We analyze stock returns in five large developed countries outside the U.S.: Australia, Canada, France, Germany, and the UK. These are the same five countries examined by Arnott, Cornell, and Kalesnik (2017).

For each of the five countries, we compare the average excess stock market returns computed over two different periods: when the U.S. president is a Democrat and when he is a Republican. Specifically, we examine the slope coefficient from the regression of each country's monthly excess stock market return on a dummy variable indicating whether the current U.S. president is a Democrat. A positive value of the slope coefficient indicates that the average excess stock market return is higher under a Democratic president. We report the estimated slope coefficients, along with their t-statistics, in Tables A1, A2, and A3 below.

Our perspective reflects the view that international stock markets are integrated in that stocks in each country are owned by a disperse set of global investors. We argue that the outcome of the U.S. presidential election—the largest democratic election in the world among developed countries—is a useful signal about the level of global risk aversion.

One could also relate each country's stock returns to the outcomes of the elections in that country rather than in the U.S., as Arnott, Cornell, and Kalesnik (2017) do. However, their perspective implicitly assumes that international stock markets are segmented—that French investors hold only French stocks, German investors hold only German stocks, etc., so that there are no cross-border equity holdings. While international markets do exhibit some degree of a home bias, the assumption of market segmentation is clearly counterfactual.

Another disadvantage of analyzing country-by-country election outcomes outside the U.S. is that in other countries it is more difficult to determine the vote shares of high-tax versus low-tax parties. No large country outside the U.S. has the same simple two-party system. Even in countries that come closest, such as the UK, there are additional parties (e.g., the Scottish National Party, Liberal Democrats, UKIP, etc.) that often enter into coalitions with one of the leading parties to form the government. Junior coalition partners often have significant bargaining power over the government's policy because their choice of the coalition partner can determine which of the two leading parties heads the government. The designation of a high-tax versus low-tax party is therefore more complicated than it might seem at first sight. One could in principle check whether the tax burden rises or falls under each governing coalition, similar to what we do when we compare tax changes under Democrats versus Republicans in the U.S., to obtain a noisy ex-post signal of the high- versus low-tax party designation. But Arnott, Cornell, and Kalesnik (2017) do not perform such an exercise, nor do they report the composition of each governing coalition, so it seems difficult to determine whether their designation of high-tax and low-tax parties is appropriate. This concern adds to the larger segmented-markets concern voiced in the previous paragraph.

We regress each country's monthly excess stock market return on a dummy variable that takes the value of one or zero depending on which party has the presidency at the beginning of the month. In any given month, the dummy variable takes the value of one (zero) when a Democrat

(Republican) is in the White House on the first day of the month. That is the right-hand-side variable in the regression. The left-hand-side variable is of the following form:

$$\log(1 + MKT) - \log(1 + Rf) \;, \tag{A73}$$

where MKT is the stock market return in the given month and Rf is the risk-free rate of return. We measure both MKT and Rf in two different ways, for robustness.

Measuring MKT.

We measure MKT in two different ways. In Tables A1 and A3, we use stock return data from the Global Financial Database (GFD), as do Arnott, Cornell, and Kalesnik (2017); in Table A2, we use data from Morgan Stanley Capital International (MSCI).

When using GFD, we use the following market proxies:

Country	GFD Stock Market Index
Australia	ASX All-Ordinaries (AORDD)
Canada	S&P/TSX 300 Composite (GSPTSED)
France	CAC All-Tradable (CACTD)
Germany	DAX 30 (IDDEUD)
United Kingdom	FTSE All-Share Capital Index (FTASD)
United States	S&P 500 (SPXD)

The GFD series are local-currency returns that include dividends. Some of these series have been extended by the GFD. For example, while the DAX 30 index was established in 1988, the series was extended by the GFD back to 1958. We choose these series to maximize coverage.

When using MSCI, we use the MSCI country-specific local-currency equity indices provided by Thomson Reuters Eikon (formerly Datastream):

Country	MSCI Index
Australia	MSAUSTL
Canada	MSCNDAL
France	MSFRNCL
Germany	MSGERML
United Kingdom	MSUTDKL
United States	MSUSAML

For both GFD and MSCI, we also convert local-currency stock market returns into U.S. dollar returns. We do that by using data on exchange rates that come from GFD. For France and Germany, we switch from the Dollar-to-Franc and Dollar-to-Mark exchange rates, respectively, to the Dollar-to-Euro exchange rate in January 2002 (when the French franc and the German mark were replaced by the euro). We report only dollar returns based on GFD (in Table A3) because GFD's data coverage is substantially longer than MSCI's for all countries.

Measuring Rf.

We use two sets of risk-free rates. For Tables A1 and A2, we use country-specific 90-day interbank rates from the FRED database. We use the following series:

Country	FRED series
Australia	IR3TIB01AUM156N.csv
Canada	IR3TIB01CAM156N.csv
France	IR3TIB01FRM156N.csv
Germany	IR3TIB01DEM156N.csv
United Kingdom	IR3TTS01GBM156N.csv
United States	IR3TIB01USM156N.csv

Coverage for these series typically extends back to the 1960s, which limits the lengths of the sample periods in Tables A1 and A2.

For Table A3, which reports U.S. dollar returns, we use the three-month U.S. Treasury bill rate, which is available from WRDS back to 1927.

As a final data-related note, data coverage differs across the datasets. The GFD stock index and exchange rate series have the longest coverage. They extend back to 1929 or longer in most instances, though there is variation across countries; for example, the stock market data for Germany are only available since the 1950s. MSCI data begin on December 31, 1969. The start dates of the FRED data are more variable; the earliest series begin in 1960. We always use the longest series possible in each of the tables below.

Regression results.

Tables A1, A2, and A3 report the slope coefficients and t-statistics from the regressions described above. To interpret the monthly slope coefficients as differences in average returns in percent per year, we multiply them by 1,200. The t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

All three tables tell the same story. The slope estimate is positive for each of the six countries in each of the three tables, indicating that international excess stock market returns tend to be higher when a Democrat is in the White House. Across Tables A1 through A3, the slope is statistically significant in 12 out of 18 cases. Even in the cases when the slope is not statistically significant, it is usually economically significant. In fact, the magnitude of the Democrat-minus-Republican difference in average returns for international markets is comparable to, and often larger than, the same difference for the U.S. market. For example, in Table A1, the difference ranges from 8.0% to 13.4% per year across the six countries, and for Australia, Canada, and France, it is larger than the

10.2% difference observed for the U.S.[¶] The U.S. evidence thus extends easily to an international setting. Our interpretation is that the outcome of the U.S. election is a useful signal of global risk aversion, which affects equity risk premia across the globe.

[¶]The numbers for the U.S. differ slightly from the numbers reported in the paper because the time period is different and because the U.S. index we use here is the S&P 500 index, not the CRSP total market index.

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Table A1
International Returns: GFD Stock Indices, Country-Specific Risk-Free Rate

This table reports the slope coefficient from the regression of the country's excess stock market return on the dummy variable that is equal to one when a Democratic president is in the White House and zero otherwise. Stock return data are from the Global Financial Database (GFD). Excess stock returns are computed monthly as the log return on the country's market index minus the log of the country-specific 90-day interbank rates from FRED. Returns are reported in percent per year. For each country, we extend the sample period as far back as data are available. Presidents are assumed to be in office until the end of the month during which they leave office. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Australia	Canada	France	Germany	UK	U.S.
Dem-Rep	13.35	12.54	13.40	8.30	8.01	10.17
	(2.60)	(2.94)	(2.33)	(1.57)	(1.61)	(2.45)
N	576	672	552	672	672	619
Start Year	1968	1960	1970	1960	1960	1964
End Year	2015	2015	2015	2015	2015	2015

Table A2
International Returns: MSCI Stock Indices, Country-Specific Risk-Free Rate

This table reports the slope coefficient from the regression of the country's excess stock market return on the dummy variable that is equal to one when a Democratic president is in the White House and zero otherwise. Stock return data are from Morgan Stanley Capital International (MSCI). Excess stock returns are computed monthly as the log return on the country's market index minus the log of the country-specific 90-day interbank rates from FRED. Returns are reported in percent per year. For each country, we extend the sample period as far back as data are available. Presidents are assumed to be in office until the end of the month during which they leave office. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Australia	Canada	France	Germany	UK	U.S.
Dem-Rep	11.31	13.62	13.78	11.63	7.33	10.62
	(2.05)	(2.78)	(2.33)	(2.02)	(1.38)	(2.39)
N	552	552	552	552	552	553
Start Year	1970	1970	1970	1970	1970	1969
End Year	2015	2015	2015	2015	2015	2015

Table A3
International Returns: GFD Stock Indices Converted to USD, U.S. Risk-Free Rate

This table reports the slope coefficient from the regression of the country's excess stock market return on the dummy variable that is equal to one when a Democratic president is in the White House and zero otherwise. Stock return data are from the Global Financial Database (GFD). Excess stock returns are computed monthly as the log return on the country's market index minus the log of the three-month U.S. Treasury bill from WRDS. Local-currency returns are converted to U.S. dollar returns at the exchange rates obtained from GFD. Returns are reported in percent per year. For each country, we extend the sample period as far back as data are available. Presidents are assumed to be in office until the end of the month during which they leave office. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Australia	Canada	France	Germany	UK	U.S.
Dem-Rep	9.39	10.97	1.36	2.92	7.33	9.90
	(2.11)	(2.50)	(0.24)	(0.51)	(1.72)	(2.35)
N	1067	1067	1067	675	1067	1067
Start Year	1927	1927	1927	1959	1927	1927
End Year	2015	2015	2015	2015	2015	2015

A9. Evidence: Economic Growth

This section plots the time series of U.S. real GDP growth under Democratic versus Republican presidents.

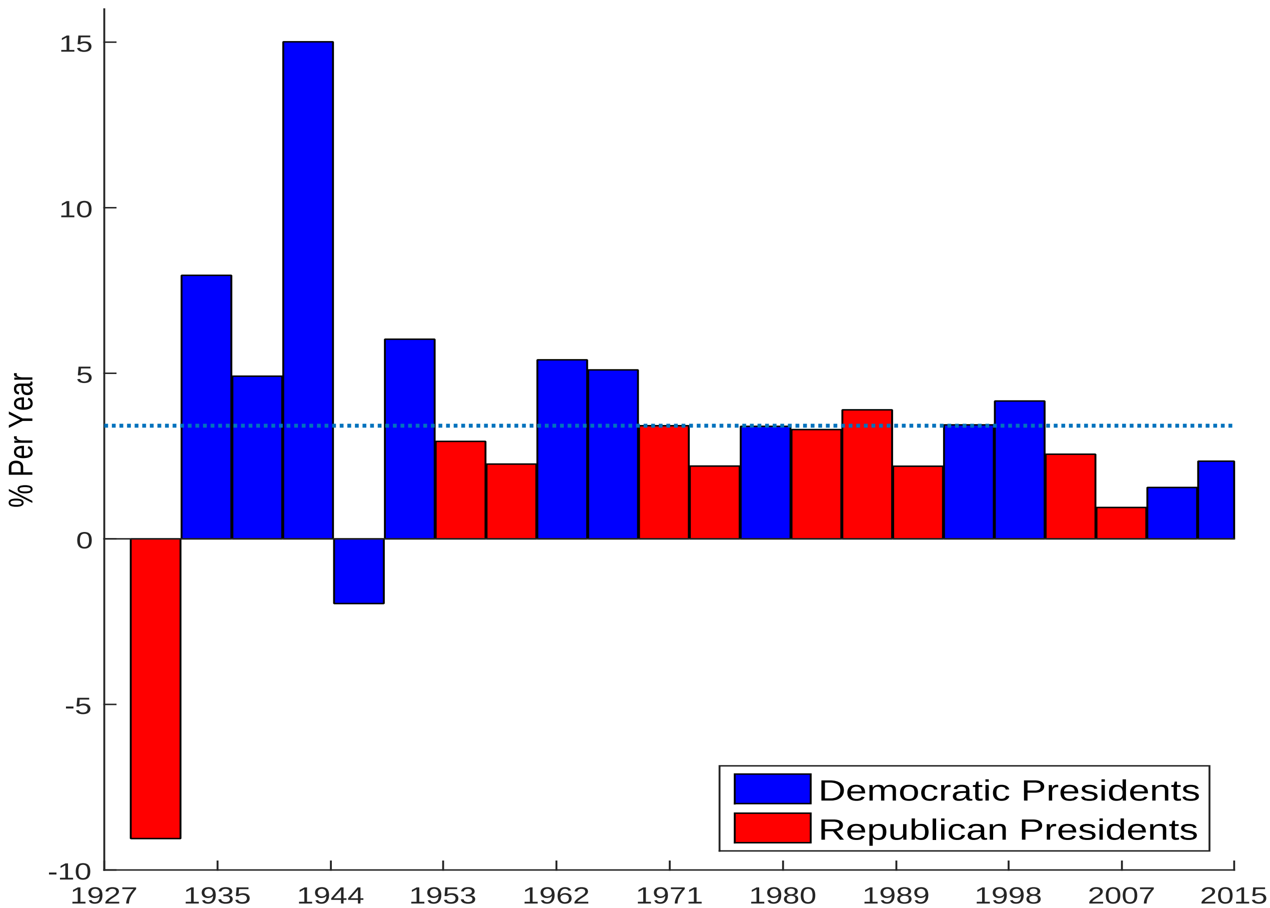

Figure A5. Average GDP growth under Democrat vs Republican presidents. This figure plots average U.S. real GDP growth under each of the 22 administrations between 1930 and 2015, from President F. D. Roosevelt through President Obama. We begin in 1930 because that is when GDP growth data from BEA begin. Presidents are assumed to be in office until the end of the month during which they leave office. The horizontal dotted line plots the unconditional mean growth rate.

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A10. Evidence: Time Series of Risk Aversion

This section describes the empirical proxies used in our analysis of the time series of risk aversion.

We use four proxies for risk aversion. The first proxy, which has the strongest theoretical justification, is the surplus consumption ratio of Campbell and Cochrane (1999). This ratio is defined in equation (2) of Campbell and Cochrane (1999) as $S_t = (C_t^a - X_t)/C_t^a$, where C_t^a denotes the average consumption across all agents and X_t is an external habit that depends on the history of aggregate consumption. In Campbell and Cochrane's model, the coefficient of relative risk aversion is given by γ/S_t, where γ denotes the curvature of the utility function. The variable S_t is thus perfectly negatively correlated with risk aversion in the time series. Our proxy for risk aversion is the negative of S_t, or $-S_t$. This proxy behaves better over time than $1/S_t$ because in the data, unlike in the model, S_t occasionally crosses zero. We construct the time series of S_t by following Cochrane (2017) and using his code.[‖] The data are available for the period of February 1959 through December 2015.

Our second proxy is the risk appetite measure of Pflueger et al. (2018), which the authors call the "price of volatile stocks," or PVS. The authors define PVS_t as the average book-to-market ratio of low-volatility stocks minus the average book-to-market ratio of high-volatility stocks. They contend that PVS_t measures the aggregate risk appetite in the economy. We download the data from Carolin Pflueger's website; specifically, from http://www.carolinpflueger.com/. The data are available quarterly, covering the period 1970Q2 through 2015Q4. Since PVS_t measures risk appetite, we use its negative, or $-\text{PVS}_t$, to proxy for risk aversion.

Our third proxy is the risk aversion measure of Miranda-Agrippino and Rey (2018). The authors first calculate a global factor in equity returns and then decompose this factor into a risk aversion component and a component due to other factors. They do not provide data on their measure of risk aversion, but they do provide data for their global factor and describe how to extract the risk aversion component. We follow their procedure. We first download the data for the global factor from Silvia Miranda-Agrippino's website; specifically, from http://silviamirandaagrippino.com/research/. The data are available from January 1990 to December 2010. We then follow their procedure to construct the risk aversion component, successfully replicating the risk aversion series plotted in Panel B of Figure 3 in the February 10, 2018 version of their paper.

Our fourth proxy for risk aversion is the unemployment rate. The idea behind this proxy is twofold. First, people without labor income are likely to be more reluctant to take risks. Second, and more important, unemployment tends to be higher during recessions, which tend to be preceded and accompanied by decreases in aggregate wealth. People who have suffered financial losses are likely to be more reluctant to take risks. The unemployment data, which are seasonally adjusted, come from FRED. They cover the period January 1948 through December 2015.

In addition to the four risk aversion proxies, we also use four proxies for the equity risk premium. These proxies are related to risk aversion to the extent that time variation in the risk premium is driven by time variation in risk aversion. Therefore, the connection of these proxies to

[‖]https://faculty.chicagobooth.edu/john.cochrane/research/Data_and_Programs/habit_habit/macro_finance_data_and_programs.zip.

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risk aversion is substantially less direct than the connection of the four proxies described earlier.

Our first proxy for the equity risk premium is the *cay* variable of Lettau and Ludvigson (2001). The authors show that this variable, which is based on the aggregate consumption-wealth ratio, has strong power to predict excess stock market returns (stronger than the dividend-price ratio, for example). We download the data from Martin Lettau's website; specifically, from https://sites.google.com/view/martinlettau/data. The data are quarterly, covering the period 1951Q1 through 2015Q4.

Our second proxy is the dividend-price ratio, the veteran of the literature on the predictability of stock market returns. The dividend-price ratio is equal to total dividends paid over the previous 12 months divided by the current total stock market capitalization. We compute this ratio from the with-dividend and without-dividend monthly returns on the value-weighted portfolio of all NYSE, Amex, and NASDAQ stocks, which we obtain from CRSP. The data are available from January 1927 through December 2015.

Our third proxy comes from Martin (2017). Martin uses index option prices to construct a volatility index, from which he derives a lower bound on the equity premium. We download the data from Ian Martin's website; specifically, from http://personal.lse.ac.uk/martiniw/epbounds.xls. The data cover the period January 1996 through January 2012. We use the one-year premium but the results for other horizons, such as 1, 2, 3, or 6 months, are virtually identical.

Our fourth and final proxy for the equity risk premium is IPO volume, or the number of initial public offerings. In the model of Pástor and Veronesi (2005), time variation in the equity risk premium leads to time variation in IPO volume. Low risk premia lead to high IPO volume and vice versa, so we take the negative of IPO volume to proxy for the risk premium. We download the data from Jay Ritter's website; specifically, from https://site.warrington.ufl.edu/ritter/files/2018/01/IPOALL_2017.xls. The data are available from January 1960 through December 2015. We divide the number of IPOs by 100 to obtain convenient magnitudes for the slope coefficients (this scaling obviously does not affect the statistical significance of the estimates).

A11. Evidence: Voter Characteristics

This section describes the datasets that we use in our cross-sectional analysis of U.S. and UK voters. It also provides additional empirical results complementing those presented in the paper.

A11.1. United States

We now describe the variables we use from the 2010-2014 panel of the Cooperative Congressional Election Study (CCES) survey. The 2014 survey that we use is a stratified sample survey that was administered by YouGov in the fall of 2014, with about 10,000 respondents.

Our left-hand-side variable is a dummy variable indicating support for the Democratic Party. The variable, which we derive from the CCES variable *CC12_410a*, is equal to one if the respondent voted for the Democratic candidate (Obama) in the 2012 presidential election and zero if they voted for the Republican candidate (Romney) or some other candidate. We exclude respondents who state that they did not vote or that they are not sure how they voted.

Our first right-hand-side variable is risk aversion. We measure it on a 1-to-4 scale based on the respondents' answers to whether they would accept risky gambles. We rely on the CCES variables *CC14_RISK1*, *CC14_RISK2a*, and *CC14_RISK2b*. The *CC14_RISK1* variable contains a yes-or-no answer to the following question: "Suppose you are the only income earner in the family, and you have a good job guaranteed to give you income every year for life. You are given the opportunity to take a new and equally good job, with a 50-50 chance it will double your income and a 50-50 chance that it will cut your income by a third. Would you take the new job?" If the respondent answers "yes," they are asked a follow-up question, *CC14_RISK2a*: "Suppose the chances were 50-50 that it would double your income, and 50-50 that it would cut it in half. Would you still take the new job?" If the respondent answers "no" to *CC14_RISK1*, they are asked a different follow-up question, *CC14_RISK2b*: "Suppose the chances were 50-50 that it would double your income and 50-50 that it would cut it by 20 percent. Would you then take the new job?" Based on the respondents' answers to both questions, we assign risk aversion scores as follows:

- 'Yes' to *CC14_RISK1*, 'Yes' to *CC14_RISK2a* $\rightarrow$ 1
- 'Yes' to *CC14_RISK1*, 'No' to *CC14_RISK2a* $\rightarrow$ 2
- 'No' to *CC14_RISK1*, 'Yes' to *CC14_RISK2b* $\rightarrow$ 3
- 'No' to *CC14_RISK1*, 'No' to *CC14_RISK2b*' $\rightarrow$ 4

That is, the lowest risk aversion is assigned to respondents who would accept both risky gambles and the highest risk aversion to those who would decline both gambles.

To measure entrepreneurship, we use the CCES variable *occupationcat_14*, which captures the respondent's occupational category. We classify the respondent as an entrepreneur if they indicate that they are independent contractors, business owners, owner-operators, or managers.

To identify government workers, we use the CCES variable *employercat_14*, which captures the respondent's employer category. We classify the respondent as a government worker if they indicate that they work for the government. In robustness analysis at the end of this section, we also consider more expansive definitions of government workers.

To measure income, we use the CCES variable *faminc_14*, which reports the response to the question "Thinking back over the last year, what was your family's annual income?" in multiple income ranges. We construct *Income* by assigning the values 1 through 16 as follows:

- Less than \$10,000 → 1
- \$10,000 to \$19,999 → 2
- \$20,000 to \$29,999 → 3
- \$30,000 to \$39,999 → 4
- \$40,000 to \$49,999 → 5
- \$50,000 to \$59,999 → 6
- \$60,000 to \$69,999 → 7
- \$70,000 to \$79,999 → 8
- \$80,000 to \$99,999 → 9
- \$100,000 to \$119,999 → 10
- \$120,000 to \$149,999 → 11
- \$150,000 to \$199,999 → 12
- \$150,000 or more → 12
- \$200,000 to \$249,999 → 13
- \$250,000 to \$349,999 → 14
- \$250,000 or more → 14
- \$350,000 to \$499,999 → 15
- \$500,000 or more → 16

To measure *Education*, we use the CCES variable *educ_14*, which contains the response to the question "What is the highest level of education you have completed?" We convert the six possible responses to integer values between 1 and 6 as follows:

- 'No high school' → 1
- 'High school graduate' → 2
- 'Some college' → 3
- '2-year' → 4
- '4-year' → 5
- 'Post-grad' → 6

We back out the respondent's age from the variable *birthyr_14*. Gender comes from *gender_14*.

Next, we present some additional evidence. First, we examine the voting preferences of stock owners. Second, we conduct two robustness exercises.

Additional evidence on stock ownership:

We classify a respondent as a stock owner if they respond "Yes" to the question "Do you personally (or jointly with a spouse), have any money invested in the stock market right now, either in an individual stock or in a mutual fund?". The responses are contained in the CCES variable *investor_14*. We add a stock owner dummy on the right-hand side of our baseline regression, which corresponds to Table 7 in the paper.[**]

Table A4 shows that stock owners are significantly less likely to vote Democrat. This evidence is consistent with our model, in which stock owners are entrepreneurs while agents who do not own stock are government workers. It is also comforting to see that all other variables—risk aversion, entrepreneurship, and government work—enter the regression in the same way whether or not stock ownership is excluded.

Robustness exercise:

In the first robustness exercise, we report the equivalent of the U.S. part of Table 7 in the paper when the estimation is conducted via probit instead of logit. The results are very similar, as shown in Table A5.

Robustness exercise:

In the second robustness exercise, we expand the definition of government workers to include not only those who work for the government but also those who are unemployed, temporarily laid off, or permanently disabled, as indicated by the CCES variable *employ_14*. The idea is to include not only government employees but also other agents who receive net transfers from fiscal redistribution. The results are similar, as shown in Table A6. The coefficient on government workers is positive and significant when risk aversion and entrepreneurship are included in the regression. The same coefficient loses its significance when other controls are added. But even with those other controls, risk aversion and entrepreneurship retain their model-predicted signs with significant t-statistics.

[**]The stock ownership variable is available for U.S. voters (in the CCES database) but not for UK voters (in BES).

Table A4
Stock Owners Vote Republican

This table is the counterpart of the U.S. part of Table 7 in the paper, except that stock ownership is added to the right-hand-side variables.

	(1)	(2)	(3)	(4)	(5)
Stock Owner	-0.25	-0.22	-0.18	-0.18	-0.16
	(-5.66)	(-4.92)	(-3.61)	(-3.73)	(-2.81)
Risk Aversion		0.13	0.11	0.11	0.12
		(6.82)	(5.78)	(5.61)	(5.27)
Entrepreneur			-0.26	-0.23	-0.15
			(-5.40)	(-4.72)	(-2.67)
Government Worker				0.20	0.12
				(3.50)	(1.87)
Income					-0.02
					(-2.52)
Education					0.27
					(13.53)
Age					-0.01
					(-3.97)
Gender (Male)					-0.61
					(-11.46)
Observations	8982	8852	7809	7771	6784

Table A5
U.S. Part of Table 7: Probit instead of Logit

This table is the counterpart of the U.S. part of Table 7 in the paper, except that the estimation is conducted by the probit rather than the logit model.

	(1)	(2)	(3)	(4)
Risk Aversion	0.08	0.07	0.07	0.07
	(7.30)	(6.07)	(5.91)	(5.39)
Entrepreneur		-0.17	-0.16	-0.09
		(-5.69)	(-5.04)	(-2.69)
Government Worker			0.12	0.08
			(3.39)	(1.94)
Income				-0.02
				(-3.46)
Education				0.16
				(13.39)
Age				-0.01
				(-4.53)
Gender (Male)				-0.38
				(-11.65)
Observations	8855	7809	7771	6784

Table A6
U.S. Part of Table 7: Broader Definition of Government Workers

This table is the counterpart of the U.S. part of Table 7 in the paper, except that government workers include not only those who work for the government but also those who are unemployed, temporarily laid off, or permanently disabled.

	(1)	(2)	(3)	(4)
Risk Aversion	0.13	0.12	0.12	0.12
	(7.28)	(6.04)	(5.95)	(5.48)
Entrepreneur		-0.28	-0.26	-0.16
		(-5.68)	(-5.25)	(-2.91)
Government Worker			0.14	0.05
			(2.59)	(0.86)
Income				-0.03
				(-3.25)
Education				0.26
				(13.41)
Age				-0.01
				(-4.44)
Gender (Male)				-0.61
				(-11.56)
Observations	8855	7809	7809	6811

A11.2. United Kingdom

Next, we describe the variables we use from the British Election Study (BES). This survey of British voters asks questions about political preferences, values, and demographic characteristics. The study is funded by the Economic and Social Research Council and is run by academics at the Universities of Manchester Oxford, and Nottingham (https://esrc.ukri.org/research/our-research/british-election-study/). We use the BES panel study dataset, which consists of responses to an online survey conducted between 2014 and 2018.

Our left-hand-side variable is a dummy variable indicating support for the Labour Party. The variable is equal to one if the respondent supports the Labour Party and zero if the respondent supports the Conservative Party. Some respondents support neither of these two parties; we drop them from the sample. We do this to maintain approximate symmetry with the U.S. setting in which there are essentially just two parties, Democrats and Republicans. In the UK, there are meaningful "third" parties, such as the Scottish National Party, Liberal Democrats, Greens, and UKIP. To measure party support, we use the BES variable *partyId*, which contains the response to the question "Generally speaking, do you think of yourself as Labour, Conservative, Liberal Democrat or what?" As the baseline value of *partyId*, we take the value from wave eight of the survey, which was conducted in 2016 on 33,502 respondents. If the wave-eight value of *partyId* is available, we use it; otherwise we use the value from the most recent wave in which it is available.

Our first right-hand-side variable is risk aversion. We measure it as the negative of the willingness to take risk. To construct this willingness, we use the BES variable *riskTaking*, which contains the response to the question "Generally speaking, how willing are you to take risks?" We convert the four possible responses to integer values between 0 and 3 as follows:

- 'Very unwilling to take risks' → 0
- 'Somewhat unwilling to take risks' → 1
- 'Somewhat willing to take risks' → 2
- 'Very willing to take risks' → 3

We transform the resulting willingness-to-take-risk variable to risk aversion by multiplying it by minus one. To maintain symmetry with our definition of party support, we use the wave-eight value of *riskTaking* if available; otherwise we use the value from the most recent wave in which it is available.

To measure entrepreneurship, we use the BES variable *selfOccStatusW6_W12*, which contains the response to the question "Are you an employee or self-employed/an independent contractor?" We convert this variable to the indicator variable entrepreneur as follows:

- Self-employed/independent contractor → 1
- Any other non-missing value → 0

Identifying government workers is difficult because unlike the CCES survey that we use for the U.S., the BES survey does not ask whether the respondent works for the government. We do the best we can under the circumstances. In the spirit of our model, we identify as government workers those respondents who are likely to be net recipients of fiscal redistribution. Specifically, we aggregate the BES variables *profile_work_statW7* and *disability* to classify the respondent as a government worker if they are either not working or disabled or both. The variable *profile_work_statW7* asks about employment status. We determine the respondent is not working if their reply is "Unemployed" or "Retired". The variable *disability* contains the response to the question "Are your day-to-day activities limited because of a health problem or disability?" There are three possible responses: "No", "Yes, limited a little", and "Yes, limited a lot". We determine the respondent is disabled if their response is either "Yes, limited a little" or "Yes, limited a lot".

To measure income, we use the BES variable *profile_gross_household*, which reports each household's annual gross income in one of 15 income ranges. We construct *Income* by assigning the values 1 through 15 as follows:

- under £5,000 per year → 1
- £5,000 to £9,999 per year → 2
- £10,000 to £14,999 per year → 3
- £15,000 to £19,999 per year → 4
- £20,000 to £24,999 per year → 5
- £25,000 to £29,999 per year → 6
- £30,000 to £34,999 per year → 7
- £35,000 to £39,999 per year → 8
- £40,000 to £44,999 per year → 9
- £45,000 to £49,999 per year → 10
- £50,000 to £59,999 per year → 11
- £60,000 to £69,999 per year → 12
- £70,000 to £99,999 per year → 13
- £100,000 to £149,999 per year → 14
- £150,000 and over → 15

We measure the respondent's education by using the BES variable *profile_education*, which contains the responses to the question "At what age did you finish full-time education?" We create a dummy variable *Education*, which is equal to zero if the response is 18 years or less and one otherwise. The value of one thus suggests some college education.

Finally, to measure the respondent's age, we use the BES variable *Age* ("What is your age?"), and to measure the respondent's gender, we use the BES variable *gender* ("Are you male or female?").

Robustness exercise:

As a robustness check, we report the equivalent of the UK part of Table 7 in the paper when the estimation is conducted via probit instead of logit. The results are very similar, as shown in Table A7.

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Table A7
UK Part of Table 7: Probit instead of Logit

This table is the counterpart of the UK part of Table 7 in the paper, except that the estimation is conducted by the probit rather than the logit model.

	(1)	(2)	(3)	(4)
Risk Aversion	0.08	0.09	0.10	0.09
	(8.24)	(5.24)	(4.77)	(3.69)
Entrepreneur		-0.25	-0.26	-0.24
		(-7.85)	(-6.40)	(-4.94)
Government Worker			0.14	0.16
			(4.08)	(3.97)
Income				-0.06
				(-12.96)
Education				0.27
				(7.95)
Age				-0.01
				(-4.86)
Gender (Male)				-0.10
				(-3.04)
Observations	30301	12626	7949	6279

A12. Evidence: Electoral Transitions

This section presents the evidence on occupational changes around electoral transitions. Our model predicts that Democrats are elected when the median voter is a government worker, while Republicans are elected when the median voter is an entrepreneur. Testing this prediction is challenging, in part given the difficulty in classifying real-world voters as government workers or entrepreneurs. Since the model features only two types of agents, the types must be interpreted broadly, as explained in the paper. In a realistic system of fiscal redistribution, we think of entrepreneurs as net contributors, or net tax payers, and government workers as net beneficiaries, or net tax recipients. Finding data on who is a net tax payer or contributor, when all costs and benefits of government are aggregated at the individual level, seems impossible. Moreover, an agent can migrate between the two groups without changing jobs. For example, a private sector worker can become a net beneficiary if her firm obtains a government contract. Our model only requires the median voter to shift between being a net contributor and a net beneficiary of fiscal redistribution.

To construct a crude measure of government workers, we add up two series: the fraction of the U.S. population working for the federal government and the unemployment rate. We thus assume that the unemployed are net tax recipients, through their collection of unemployment benefits. The data on government employees come from FRED, starting January 1939. The population data, which come from the U.S. Census Bureau, are available monthly from 1952 and annually before that. We interpolate the pre-1952 annual data to extend our monthly time series before 1952. The unemployment rate data, which come from FRED and are seasonally adjusted, begin in January 1948. Our government worker series thus covers the period January 1948 through December 2015.[††]

We measure entrepreneurship by the number of new firms entering the economy, which we obtain from the U.S. Census Bureau, Business Dynamics Statistics. The data are annual, covering the period 1977 through 2014. We convert it to monthly by dividing each annual value by 12.

Both time series, government workers and entrepreneurs, are highly persistent over time. We therefore focus on changes in these series around electoral transitions. Our model predicts that the number of government workers should increase around transitions from Republican to Democratic presidents and decrease around transitions from Democratic to Republican presidents. Entrepreneurship should exhibit the opposite pattern: it should increase around transitions from Democratic to Republican presidents and decrease around reverse transitions. The pattern need not be strong—in the model, we need just a single agent, the median voter, to change their occupation for an electoral transition to occur. Nonetheless, we look to see whether the pattern obtains in the data.

We run logistic regressions in which the dependent variable is a dummy variable indicating transitions from one party to the other. The variable is equal to one for months in which one party wins the presidential election while the incumbent president is from the other party. Given the small numbers of electoral transitions, we include only one independent variable at a time. We consider

[††]We also have data on the fraction of the population on disability. We do not include it in the measure of government workers, however, because the data start only in 2008.

two independent variables: government workers and entrepreneurs. Each independent variable is the difference between two averages of the corresponding quantity: the average computed over the window of m months after the election month minus the average computed over the m months prior to the election month, where $m \in \{3, 6, 12\}$.

Table A8 reports the estimated slopes and their t-statistics from a logistic regression model. The table shows that transitions from a Republican president to a Democratic president tend to be accompanied by increases in the number of government workers and decreases in entrepreneurship, as the model predicts. The statistical significance of these results is borderline, which is not surprising given the low number of electoral transitions. The evidence for reverse transitions, from Democrats to Republicans, is never statistically significant, though the coefficient estimates always have the model-predicted sign. With these caveats, we conclude that the evidence in Table A8 is consistent with our model.

Table A8
Occupational Changes Around Electoral Transitions

This table reports the estimated slopes and their t-statistics from a logistic regression model. The left-hand side variables, given in column headings, are dummy variables that are equal to one if the given electoral transition occurs in the current month and zero otherwise. The left (right) column reports results for elections resulting in transitions from a Republican president to a Democratic president (and vice versa). Each regression has a single right-hand side variable. The right-hand side variables are government workers, measured as the fraction of the population working for the federal government plus the unemployment rate, and entrepreneurs, measured by the number of new businesses entering the economy. Each right-hand side variable is the difference between two averages of the corresponding quantity: the average computed over the first m months after the election month minus the average computed over the m months immediately preceding the election month, where $m \in \{3, 6, 12\}$ varies across the three panels.

	Transition from Republicans to Democrats	Transition from Democrats to Republicans
	Panel A. 3-month window	
Government workers	135.73	-70.35
	(2.07)	(-0.59)
Entrepreneurs	-0.46	0.08
	(-2.37)	(0.24)
	Panel B. 6-month window	
Government workers	90.07	-39.24
	(1.92)	(-0.52)
Entrepreneurs	-0.51	0.06
	(-2.26)	(0.24)
	Panel C. 12-month window	
Government workers	60.03	7.88
	(1.63)	(0.18)
Entrepreneurs	-0.48	0.01
	(-2.06)	(0.03)

A13. Evidence: Tax Burden

In the context of our model, we interpret the high-tax party as Democrats and the low-tax party as Republicans. It is often argued that Democrats tend to favor bigger government than do Republicans. To take a closer look at the evidence, we compare changes in the tax burden under Democrat versus Republican presidents. We measure the tax burden by the ratio of total federal tax to GDP, which we obtain from the Bureau of Economic Analysis (BEA). Specifically, we use current tax receipts from Table 3.2. Federal Government Current Receipts and Expenditures. BEA provides quarterly data back to 1947Q2 and annual data back to 1929.

The tax/GDP series exhibits trends and high persistence. For example, it trends up from 3.3% in 1929 to 17.2% in 1951, before drifting down to 7.9% in 2009 and finishing at 12.0% in 2015. To account for this persistence, we focus on first differences in the tax/GDP ratio.

Table A9 shows that the tax burden tends to rise under Democratic presidents and fall under Republican presidents. Under Democrats, the tax/GDP ratio rises by 0.44% per year, on average, whereas under Republicans it falls by 0.30% per year. The Democrat-Republican difference of 0.74% per year is highly significant ($t = 3.15$). Subperiod results are very similar to the full-sample results. While the individual Democrat and Republican averages are sometimes insignificant, their difference is significant in both equally long subperiods ($t = 2.07$ in 1929–1972 and $t = 3.04$ in 1972–2015). The results look similar even in all three equally long subperiods, with lower significance due to shorter samples. In short, it seems reasonable to interpret Democratic presidents as favoring more tax-based redistribution and Republican presidents as favoring less.

In our simple model, the tax rate changes as soon as the new administration is elected. In reality, it takes time for tax changes to be implemented. Our assumption has some empirical support in that tax changes tend to happen early during presidential terms. When we isolate the presidents' first year in office, the Democrat-Republican difference is 2.19% per year ($t = 2.77$), three times higher than the full-term difference. When we isolate the first two years in office, the difference is 1.61% ($t = 3.85$), and when we look at the first three years, the difference is 1.08% ($t = 3.40$). All of these values exceed the full-term difference of 0.74% mentioned above.

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Table A9
Taxes under Democratic and Republican Presidents

This table reports average changes in the federal tax/GDP ratio under Democratic presidents, Republican presidents, and the Democrat-Republican difference. Changes in tax/GDP are in percent per year, for the full sample period as well as for equally long subperiods. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Democrat	Republican	Difference
1929:01–2015:12	0.44	-0.30	0.74
	(2.48)	(-1.94)	(3.15)
1929:01–1972:06	0.47	-0.26	0.73
	(1.60)	(-1.33)	(2.07)
1972:07–2015:12	0.41	-0.32	0.73
	(3.92)	(-1.47)	(3.04)
1929:01–1957:12	0.61	-0.17	0.78
	(1.51)	(-0.61)	(1.59)
1958:01–1986:12	0.17	-0.27	0.44
	(1.11)	(-1.11)	(1.52)
1987:01–2015:12	0.44	-0.36	0.81
	(3.64)	(-1.35)	(2.76)

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A14. Evidence: Government Budget Deficits

This section presents the evidence on the government's budget deficits under the two parties. We obtain the annual series of the federal government's budget surplus/deficit as percent of GDP from the Federal Reserve Bank of St. Louis. The data cover the period of 1929 through 2015.

Table A10 shows that the deficits have been larger under Democratic presidents, but insignificantly so (by 1.65%, $t = -1.24$). The larger deficits under Democrats are understandable because Democrats tend to get elected during crises—that is our main point!—and those crises force them to run larger deficits. Two prominent examples of Democrats elected during crises are Barack Obama and Franklin Delano Roosevelt (FDR). Under both of them, the federal budget deficit far exceeded its historical average of 3.0% GDP. Under Obama (200902-201512), the average deficit was 6.1% GDP, which is understandable as the period covers a good part of the Great Recession. Under FDR (193212-194504), the average deficit was even larger, 7.9% GDP, which makes sense as the period covers a part of the Great Depression and most of World War II. This is not about wars, though; even after controlling for a war dummy, Democrats run insignificantly larger deficits than Republicans. It is also not just about the Great Depression—in fact, Republican Hoover presided over some of the worst years of the Great Depression, yet the average deficit on his watch (192903-193302) was only 1.1% GDP.

In short, there is no significant difference between the deficits under Democrats and Republicans. The deficits have been somewhat larger under Democrats, indicating that Democrats engage in even more fiscal redistribution than the comparison of tax revenue would suggest. But the difference is far from being statistically significant, suggesting that our modeling approach of assuming budget deficits away is not unreasonable.

Table A10
Federal Budget Surpluses/Deficits under Democratic and Republican Presidents

This table reports average ratios of the federal government budget surplus or deficit (-) to GDP, in percent per year, under Democratic presidents, Republican presidents, and the Democrat-Republican difference. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Democrat	Republican	Difference
1929:01–2015:12	-3.76	-2.12	-1.65
			(-1.24)
1929:01–1972:06	-4.29	-0.74	-3.55
			(-1.88)
1972:07–2015:12	-2.99	-2.99	-0.005
			(-0.003)
1929:01–1957:12	-5.63	-0.63	-5.01
			(-2.10)
1958:01–1986:12	-1.40	-2.45	1.05
			(1.49)
1987:01–2015:12	-3.18	-2.68	-0.50
			(-0.28)

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A15. Evidence: Government Bond Returns

This section presents the evidence on government bond returns under the two parties. We obtain monthly returns on U.S. Treasury bonds at various maturities from the CRSP Treasuries file. We use Fixed Term Indexes with maturities 5, 10, 20, and 30 years as well as Fama bond portfolios with maturities between 5 and 10 years and greater than 10 years. For symmetry with our treatment of stocks (and Santa-Clara and Valkanov, 2003), we work with excess log returns $\log(1 + r_{\text{bond}}) - \log(1 + r_f)$, where r_f is the return on a three-month Treasury bill. Data coverage varies across maturities, with starting dates between June 1941 and January 1952 and the end date of December 2015.[‡‡]

Tables A11 through A16 show that excess bond returns are not significantly different between the Democratic and Republican presidencies. This conclusion follows for all bond maturities, both in the full sample period and in subperiods.

[‡‡]For the 10+ year Fama bond portfolio return series used in Table A16, the data from September 1962 through November 1971 are missing in the CRSP database.

Table A11
Treasury Bond Returns under Democratic and Republican Presidents: 5-Year Maturity

This table reports average excess returns on Treasury bond returns, in percent per year, under Democratic presidents, Republican presidents, and the Democrat-Republican difference. The bond returns are in excess of a three-month Treasury bill. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Democrat	Republican	Difference
1941:06–2015:12	0.31	1.94	-1.62
	(0.51)	(2.20)	(-1.52)
1941:06–1978:12	-0.14	0.34	-0.48
	(-0.34)	(0.30)	(-0.40)
1979:01–2015:12	0.89	3.22	-2.33
	(0.68)	(2.50)	(-1.28)

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Table A12
Treasury Bond Returns under Democratic and Republican Presidents: 10-Year Maturity

This table reports average excess returns on Treasury bond returns, in percent per year, under Democratic presidents, Republican presidents, and the Democrat-Republican difference. The bond returns are in excess of a three-month Treasury bill. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Democrat	Republican	Difference
1941:06–2015:12	0.24	1.95	-1.71
	(0.28)	(1.51)	(-1.09)
1941:06–1978:12	-0.75	-0.27	-0.47
	(-1.09)	(-0.16)	(-0.26)
1979:01–2015:12	1.50	3.72	-2.23
	(0.88)	(1.96)	(-0.85)

Table A13
Treasury Bond Returns under Democratic and Republican Presidents: 20-Year Maturity

This table reports average excess returns on Treasury bond returns, in percent per year, under Democratic presidents, Republican presidents, and the Democrat-Republican difference. The bond returns are in excess of a three-month Treasury bill. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Democrat	Republican	Difference
1942:01–2015:12	0.42	2.42	-2.00
	(0.34)	(1.58)	(-0.99)
1942:01–1978:12	-1.14	-0.47	-0.67
	(-1.26)	(-0.26)	(-0.34)
1979:01–2015:12	2.34	4.72	-2.38
	(0.94)	(1.95)	(-0.66)

Table A14
Treasury Bond Returns under Democratic and Republican Presidents: 30-Year Maturity

This table reports average excess returns on Treasury bond returns, in percent per year, under Democratic presidents, Republican presidents, and the Democrat-Republican difference. The bond returns are in excess of a three-month Treasury bill. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Democrat	Republican	Difference
1941:11–2015:12	0.01	2.23	-2.22
	(0.01)	(1.31)	(-0.96)
1941:11–1978:12	-1.00	-0.74	-0.26
	(-1.07)	(-0.41)	(-0.13)
1979:01–2015:12	1.27	4.60	-3.33
	(0.42)	(1.68)	(-0.78)

Table A15
Treasury Bond Returns under Democratic and Republican Presidents: 5 to 10 Year Maturity

This table reports average excess returns on Treasury bond returns, in percent per year, under Democratic presidents, Republican presidents, and the Democrat-Republican difference. The bond returns are in excess of a three-month Treasury bill. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Democrat	Republican	Difference
1952:01–2015:12	0.23	2.10	-1.87
	(0.22)	(2.09)	(-1.33)
1952:01–1983:12	-2.54	0.38	-2.92
	(-1.69)	(0.28)	(-1.49)
1984:01–2015:12	2.65	4.00	-1.35
	(2.09)	(2.71)	(-0.69)

Table A16
Treasury Bond Returns under Democratic and Republican Presidents: 10+ Year Maturity

This table reports average excess returns on Treasury bond returns, in percent per year, under Democratic presidents, Republican presidents, and the Democrat-Republican difference. The bond returns are in excess of a three-month Treasury bill. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation. The data from September 1962 through November 1971 are missing in the CRSP files for this series.

	Democrat	Republican	Difference
1952:01–2015:12	1.32	2.60	-1.29
	(0.69)	(1.71)	(-0.50)
1952:01–1983:12	-5.46	-0.31	-5.15
	(-1.64)	(-0.16)	(-1.35)
1984:01–2015:12	4.34	5.35	-1.00
	(1.90)	(2.24)	(-0.30)

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A16. Evidence: Economic Policy Uncertainty

This section presents the evidence on economic policy uncertainty under the two parties.

Our explanation for the Democrat-Republican return gap is based on risk aversion. We now examine a possible alternative explanation that is based on policy uncertainty. The role of policy uncertainty in generating risk premia in asset markets is analyzed, for example, by Pástor and Veronesi (2012, 2013) and Kelly et al. (2016). There are two ways in which policy uncertainty could in principle explain the presidential puzzle. The two hypotheses are not mutually exclusive.

First, policy uncertainty could decline over the course of a typical Democratic president's tenure, more so than during a typical Republican president's tenure. It seems plausible for policy uncertainty to be high at the beginning of a president's term and to resolve over time. The gradual resolution of uncertainty could lead to a reduction in the risk premium, resulting in high realized stock returns. This hypothesis could also potentially explain why the return gap is the highest during the first year in office. The key ingredient of this hypothesis is that the gradual decline in uncertainty must be larger under Democratic presidents.

Second, the average level of policy uncertainty could be higher under Democratic presidents than under Republican presidents. The higher average level of uncertainty would then translate into a higher level of the equity premium.

We test both hypotheses empirically. To proxy for policy uncertainty, we use the U.S. historical index of economic policy uncertainty (EPU) from Baker, Bloom, and Davis (2006), downloadable from http://policyuncertainty.com. This data is available from 1900 to October 2014, which allows us to use a long sample period 1927 through 2014.

To test the first hypothesis, we estimate the regression

$$\text{EPU}_t = \alpha + \beta_1 \text{Dem}_t + \beta_2 \text{TimeInOffice}_t + \beta_3 \text{Dem}_t \times \text{TimeInOffice}_t + \epsilon_t \ ,$$

where Dem_t is a dummy variable equal to 1 if the current president is a Democrat and 0 if he is a Republican and TimeInOffice_t is the number of months since the current president assumed office. The hypothesis predicts $\beta_3 < 0$.

Table A17 presents the regression results. We present the results for the full sample period as well as for two subperiods. In the full sample period, none of the slope coefficients are significantly different from zero. Moreover, the point estimate of β_3 has the "wrong" sign: $\beta_3 > 0$, indicating that the EPU in fact rises under Democratic presidents, albeit insignificantly. In the first subperiod, β_3 is significantly positive, clearly rejecting the policy uncertainty hypothesis. In the second subperiod, β_3 is insignificantly negative. We thus do not find evidence in favor of the first hypothesis.

To test the second hypothesis, we regress EPU_t on Dem_t. While the point estimate of the slope coefficient is positive, it is not statistically significant ($t = 1.20$). In addition, if policy uncertainty were indeed higher under Democrats, then return volatility should also be higher under Democrats (because the models of Pástor and Veronesi (2012, 2013) predict a positive relation between policy uncertainty and return volatility). However, that is not the case, as we report in the paper.

We conclude that we do not find evidence to support the policy uncertainty hypothesis.

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Table A17
The Role of Economic Policy Uncertainty

This table the slope coefficients from the regression of the Baker-Bloom-Davis historical index of economic policy uncertainty on the Democrat dummy, time in office, and their interaction. The Democrat dummy is equal to 1 if the current president is a Democrat and 0 if he is a Republican. Time in office is the number of months since the current president assumed office. The intercept is included in the regression. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Full sample 192701-201410	First half 192701-197011	Second half 197012-201410
Democrat	4.68	11.16	12.55
	(0.21)	(0.52)	(0.80)
Time in office	-0.04	-0.35	0.07
	(-0.13)	(-2.53)	(0.38)
(Democrat) × (Time in office)	0.23	0.71	-0.14
	(0.55)	(2.44)	(-0.48)
Obsevations	1054	527	527

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A17. Evidence: International GDP Growth

This section presents international evidence on GDP growth. Our analysis is analogous to that in Section A8., except that we replace international stock returns by international real GDP growth. We examine the same five large developed countries as in Section A8.: Australia, Canada, France, Germany, and the UK.

For each of the five countries, we compare the average real GDP growth rates computed over two different periods: when the U.S. president is a Democrat and when he is a Republican. Specifically, we examine the slope coefficient from the regression of each country's monthly growth on a dummy variable indicating whether the current U.S. president is a Democrat. In any given month, the dummy variable takes the value of one (zero) when a Democrat (Republican) is in the White House on the first day of the month. It is the same dummy variable used in Section A8. and in the paper.

We obtain GDP growth data for the five countries from the International Monetary Fund (IMF). On the IMF's website `http://data.imf.org`, we choose "International Financial Statistics," then "Data Tables," then "Data by Country," and then "GDP and Components." For each of the five countries, we download two seasonally adjusted annual series: nominal GDP (NGDP_SA_XDC) and GDP deflator (NGDP_D_SA_IX). To obtain real GDP, we divide nominal GDP by the deflator. From these real GDP numbers, we construct the annual growth series for each country. These series begin in different years for different countries, depending on data availability: in 1961 for Australia, 1948 for Canada, 1950 for France, 1992 for Germany, and 1949 for the UK. We do not choose these beginning dates; for each country, we use all available data, going as far back as possible in the IMF database.

To map annual growth data to monthly dummy data, for each country and each year, we assign the annual growth rate to each month of the same calendar year. For example, given Canada's 1948 growth rate of 1.52%, we assign the 1.52% value to each month in 1948 for Canada. Using data at the annual frequency is convenient because post-war presidential transitions always take place in January, which aligns the presidential terms almost perfectly with the annual frequency of GDP growth. Using quarterly growth rates, which are also available from the IMF, would not represent any improvement in this regard. On the contrary, it would result in a shorter time series for one of the countries (UK) due to more limited availability of its quarterly data. Nonetheless, the results based on quarterly data are very similar to those presented here.

Regression results.

Table A18 reports the slope coefficients and t-statistics from the country-level time-series regressions described above. The t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

The slope estimate is positive for each of the five countries, indicating that international growth rates tend to be higher when a Democrat is in the White House. However, the international Democrat-Republican growth gaps are smaller than in the U.S. (cf. Table 4 in the paper), and none of them are statistically significant. Both of these facts seem related to the shorter sample period. Whereas our U.S. GDP growth data go back to 1930, our international data series are shorter, as noted above. Table 4 in the paper shows that even in the U.S., the growth gap is smaller and insignificant when measured over the recent subperiods 1973–2015, 1958–1987, and 1987–2015. In other words, the U.S. growth gap has gradually shrunk over time, and Table A18 measures the international growth gap over post-war periods during which the U.S. growth gap is substantially smaller than in earlier periods.

There are also reasons to believe that our theory's implications for the international growth gap are weaker than those for the U.S. growth gap. Risk aversion of U.S. agents, which we argue affects the outcome of the U.S. presidential election, is unlikely to be perfectly correlated with risk aversion of non-U.S. agents, which helps shape employment choices of non-U.S. agents. In addition, the model's prediction of a positive growth gap obtains under the assumption of sufficient complementarity between the public and private sectors. That assumption may or may not be satisfied outside the U.S., where the public sector comprises a larger fraction of the total economy than in the U.S.

Table A18
International GDP Growth Gap

This table reports the slope coefficient from the regression of the country's real GDP growth on the dummy variable that is equal to one when a Democratic president is in the White House and zero otherwise. GDP growth data come from the IMF. The slope coefficients, which represent growth rate differences, are reported in percent per year. For each country, we extend the sample period as far back as data are available: 1961 for Australia, 1948 for Canada, 1950 for France, 1992 for Germany, and 1949 for the UK. All series end at the end of 2015. Presidents are assumed to be in office until the end of the month during which they leave office. t-statistics, reported in parentheses, are computed based on standard errors robust to heteroscedasticity and autocorrelation.

	Australia	Canada	France	Germany	UK
Dem-Rep	0.40	0.78	0.05	0.09	0.06
	(0.86)	(1.26)	(0.07)	(0.15)	(0.16)

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