Supplemental Material

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I. CMB AND HALO CONSTRAINTS

In this section, we discuss the CMB and halo constraints for the light-quark model introduced in the main text. For $m_K \sim 100$ MeV, the relevant CMB constraint on DM annihilations is approximately $\langle \sigma v \rangle \lesssim 10^{-29}\text{cm}^3/\text{s}$ [1–3]. Normally, this is a problem because the annihilation cross section at freeze-out required to produce the relic density is larger: $\langle \sigma v \rangle < 5 \times 10^{-26}\text{cm}^3\,\text{sec}^{-1}$. But in our case, processes like $K^+K^- \to A_D \to e^+e^-$ are P-wave and is suppressed by the ratio of DM veloctities at recombination and freeze-out, $(v_{\text{rec}}/v_{\text{fo}})^2 \approx 10^{-5}$.

On the other hand, we do need to worry about the P-wave process $K^+K^- \to \phi \to e^+e^-$ (and other similar channels with different SM final states) because it can go on resonance during recombination and in halos today. To be relevant for small-scale structure problems, we need $K^+K^- \to \phi \to K^+K^-$ to have $\langle \sigma v \rangle/m \sim 10^2\ \text{cm}^2/\text{g} \times \text{km/s}$, and thus $\langle \sigma v \rangle \sim 1.7 \times 10^{-18} \left(\frac{m}{100\ \text{MeV}}\right) \text{cm}^3/\text{s}$ [4]. $K^+K^- \to \phi \to e^+e^-$ and $K^+K^- \to A_D \to e^+e^-$ are related by the branching fraction $\phi \to A_D \to e^+e^-$. In QCD, $\phi \to e^+e^-$ branching fraction is a small value, 3×10^{-4}, due to the OZI rule. Since s has no charge, it couples to A_D only through a departure from the ideal mixing. In SM QCD, this is only at the 1% level, leading to a suppression by 10^{-4}. On top of both, it is suppressed by ϵ^2 as well as $\left(m_\phi^2/m_{A_D}^2\right)^2$. The total suppression is roughly

$$\text{BR}\left(\phi \to A_D \to e^+e^-\right) \tag{1}$$

$$\approx 3 \times 10^{-16} \frac{\alpha_D}{\left(\frac{1}{3}\right)^2 \alpha} \left(m_\phi/m_{A_D}\right)^4 \left(\frac{\epsilon}{10^{-4}}\right)^2 \tag{2}$$

Thus, this process is safe from CMB and galactic halo constraints, even if s is not neutral under $U(1)_D$. For simplicity, one may still take s neutral and assume ideal mixing so that $K^+K^- \to \phi \to e^+e^-$ is forbidden. The argument is true for the other similar relevant channels with different SM final states.

II. SM-DARK QCD MESON SUMMARY

Here, we first discuss the resonance structure in the theory of QCD and meson bound states. This is directly shown in Fig. 1 of the main text and the details are described in the caption. In Table I, we consider the vector meson to pseudoscalar meson couplings for four meson systems as examples. We consider the matrix element

$$< 0|j_\mu(x)|Y >= g_V \epsilon_\mu e^{-ipx} \tag{3}$$

g_V quantifies the strength of the process, $|Y>$ is the state of two pseudoscalar mesons, $j_\mu = \bar{b}\gamma_\mu b$ is the vector current, and ϵ is the polarization vector. This matrix element includes all hadronic corrections already and thus g_V can have values larger than the usual coupling constants in a perturbation theory.

We also discuss the γ parameter considered in [4] for the purpose of fitting to data to determine the velocity dependence of the DM self-interaction, and determine the range of DM mass reading of the upper-right panel of Fig. 2 of [4].

systems	g_V	$\gamma = \frac{g_V^2}{(384\pi)}$	$\Delta \equiv 1 - \frac{2m_{\text{PS}}}{m_{\text{V}}}$	m_{DM} [GeV]
ϕ-K-K	4.5	0.02	0.02	*0.9 - 1.5
$\Upsilon(4S)$-B-B	25	0.50	2×10^{-3}	*2.8 - 4.7

Table I: In this table, we show two vector-to-pseudoscalar meson systems that inspired our models. g_V is the coupling between the vector mesons to pseudoscalar mesons defined in the text, and γ is considered in [4] and fitted to the small-scale structure data to determine the best range of DM masses that yield the desired velocity dependence of the self-interactions. Note that for the ϕ-K-K system here we use $K^\pm$ instead of K^0, to better match the discussions in Sec. II of the main text. A "*" indicates that, to get these m_{DM} ranges, we consider $\Delta \sim 10^{-7.8}$ instead of the SM values, to have the desired resonant self-interaction.

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III. MORE DETAILS ON THE LIGHT QUARK MODEL

In Fig. 1, the green band shows m_K versus f_K values which yield the correct self-interaction cross section in the low-velocity limit for the light-quark model in Sec. II of the main text. The dotted curve corresponds to $m_{\rm K}$ versus f_K values which equal the SM ratio with $m_K/f_K = m_{K,\rm SM}/f_{K,\rm SM}$.

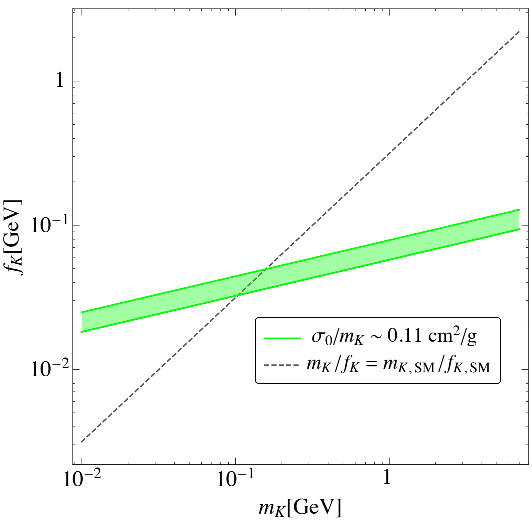

Figure 1: We show in green the dark-kaon masses and decay constants which yield the correct self-interaction cross-section in the low-velocity limit. The dotted curve follows the SM ratio, $m_K/f_K = m_{K,\rm SM}/f_{K,\rm SM}$.

Changing N_c for the light-quark model - Following the discussion in section II of the main text, we discuss the implications of changing the number of colors N_c for the $SU(N_c)$ gauge group. Note that the dependence of hadronic parameters on N_c can be discussed reliably only in the large N_c limit, and applying it to $N_c \sim 3$ is only qualitative and suggestive at best.

For the consideration of changing N_c, we fix the parameter Δ, and thus the $m_V/m_{\rm PS}$. Again, V is the vector resonance, and PS denotes the pseudoscalar DM, so that the resonant self-interaction is unaffected. We also fix several ratios of the parameters while conducting the N_c scaling. First, we fix m_K/Λ, and Λ is the scale of the dark QCD, and we also fix m_q/Λ. We set all the mass ratios to the SM ratios when $N_c = 3$, and then we change N_c to see the behavior of the change.

$$\frac{m_K}{f_K} = \left(\frac{m_{K,\rm SM}}{f_{K,\rm SM}}\right)\sqrt{\frac{3}{N_c}} \tag{4}$$

Now, for region I, we again apply the condition Eq. 11 of the main text, but now we have to modify $\sigma_0/m_{\rm DM}$ as

$$\begin{aligned}\frac{\sigma_0}{m_{\rm DM}} &= \frac{1}{32\pi}\left(\frac{m_K^4}{f_K^4}\right)\frac{1}{m_K^3} \\ &= \frac{1}{32\pi}\left(\frac{m_{K,\rm SM}}{f_{K,\rm SM}}\right)^4\left(\frac{3}{N_c}\right)^2\frac{1}{m_K^3}.\end{aligned} \tag{5}$$

Since we are fixing the value of $\sigma_0/m_{\rm DM}$ and $m_{K,\rm SM}/f_{K,\rm SM}$ is taken from data, $\left(\frac{3}{N_c}\right)^2\frac{1}{m_K^3}$ is a constant. Thus, $m_K = m_{K,N_c=3}\left(\frac{3}{N_c}\right)^{2/3}$ for the region I dark kaon mass range to scale with N_c.

For region II, the argument is much simpler. Since $g_V \propto \sqrt{\frac{1}{N_c}}$, we find $\gamma \propto \frac{1}{N_c}$. Based on [4], we have $m_K \propto \gamma^{1/3}$, so $m_K = m_{K,N_c=3}\left(\frac{3}{N_c}\right)^{1/3}$ when one scales the region II parameter according to different N_c.

One can see that, based on this simple naive argument, regions I and II move closer to each other for small N_c. However, as we know the large N_c expression breaks down for small N_c, so the above scaling is just a suggestive consideration. Nonetheless, it may indicate that the two regions could coincide in small N_c, and motivate the future lattice study.

Note that other gauge groups could work better for this purpose. For instance, an $Sp(2N_c)$ gauge theory with $N_f = 2$ (four Weyl fermions in the fundamental representation) would lead to chiral Lagrangian with the coset space $SU(4)/Sp(4) = SO(6)/SO(5) = S^5$. Vector mesons are in the adjoint representation of $SO(5)$. Upon gauging a $U(1)$ subgroup, the symmetry breaks to $U(2)$ and the Nambu–Goldstone bosons split as $(K^+, K^-, K_1^0, K_2^0, K_3^0)$. When the neutrals are heavier, the low-lying spectrum is the same as the $SU(N_c)$ gauge theories we discussed, while one of the vector mesons can appear in the K^+K^- channel. Similarly, an $SO(N_c)$ gauge theory with $N_f = 2$ (two Weyl fermions in the vector representation) has the coset space $SU(2)/SO(2) = S^2$ which only has $K^\pm$ states. We expect a single vector meson in the K^+K^- channel in this case as well. Quantitative discussions on such possibilities are beyond the scope of this paper and will be discussed in [5].

IV. A BRIEF INTRODUCTION OF SIMP AND ELDER DM

The SIMP mechanism allows thermal DM which is lighter than typical WIMPs by altering the number-changing process. In typical WIMP scenarios, the DM relic abundance is set by processes such as $\bar{\chi}\chi \to \bar{f}f$, where f is a SM particle. In contrast, in SIMP scenarios, the DM relic abundance is primarily set by processes in the dark sector, usually taken to be $3 \to 2$ DM processes. This alternate thermal history allows SIMPs to evade CMB bounds which typically constrain such light

DM masses [2]. Since the annihilation of 3 DM to 2 leaves the latter with excess energy, SIMP scenarios also require some mediation of this extra entropy to the SM. How one connects the dark and SM sectors to ferry this entropy without spoiling the SIMP mechanism yields rich phenomenology with many interesting signatures (see, *e.g.* [6–20]).

The ELDER scenario is similar to the SIMP mechanism: the relevant, DM-number-changing process is assumed to be one in the dark sector such as a $3 \to 2$ process. Any annihilation processes such as $\bar{\chi}\chi \to \bar{f}f$ are taken to be subdominant and assumed to decouple early. Additionally, there is a mediation process that maintains kinetic equilibrium between the dark and SM sectors, as in the SIMP mechanism. The key difference between ELDERs and SIMPs is the order in which processes decouple. In the SIMP paradigm, the DM-number-changing process stops while kinetic equilibrium is still maintained between the dark and SM baths. In the ELDER scenario, DM elastically decouples first, *i.e.*, the kinetic equilibrium between the two baths stops before the DM-number-changing process. In this case, when the DM elastically decouples ends up being the primary factor that determines the relic abundance.

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