Supplemental Material for
"Entanglement and Bell nonlocality with bottom-quark pairs at hadron colliders"

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POLARIZATION RETENTION FACTORS

As analyzed in [1] and developed further in [2], in the heavy-quark limit, the polarization retention factors in $b \to \Lambda_b$ fragmentation can be expressed in terms of two nonperturbative QCD parameters, A and w_1, as

$$r_L \approx \frac{1 + A\,(0.23 + 0.38 w_1)}{1 + A}\,, \tag{1}$$

$$r_T \approx \frac{1 + A\,(0.62 - 0.19 w_1)}{1 + A}\,. \tag{2}$$

These quantities, r_L and r_T, are the fractions of the initial b-quark polarization (in the longitudinal and transverse directions, respectively, relative to the fragmentation axis, i.e., the b quark momentum direction) that are retained in the final Λ_b polarization. The above expressions describe the dominant polarization loss effect, due to the contribution to the Λ_b sample from $\Sigma_b^{(*)} \to \Lambda_b \pi$ decays. The parameter

$$A = \frac{\text{prob}\,(\Sigma_b^{(*)})}{\text{prob}\,(\Lambda_b)} = 9\,\frac{\text{prob}\,(T)}{\text{prob}\,(S)} \tag{3}$$

is the ratio of the $\Sigma_b^{(*)}$-decay and direct Λ_b production rates. It is related to the probability for the two light quarks in the baryon to form any of the nine spin-triplet, isospin-triplet diquark states T and the probability to form the spin-singlet, isospin-singlet diquark state S. The statistical hadronization model (for a brief overview, see [3]) predicts $A \approx 2.6$ [2], but it is unclear how accurate this number is. The parameter

$$w_1 = \frac{\text{prob}\,(T_{\pm 1})}{\text{prob}\,(T)} \tag{4}$$

accounts for the possibility that the fragmentation axis breaks the rotational symmetry in the spin-triplet diquark production. It describes the probability for the diquark to be produced with spin component $+1$ or -1 (but not 0) along the fragmentation axis. The isotropic case is obtained for $w_1 = 2/3$. The value of w_1 can be determined from the angular distributions of the pions in the $\Sigma_b^{(*)} \to \Lambda_b \pi$ decays [1, 2]. Measurements of both A and w_1 can certainly be done at LHCb [4] and perhaps even at ATLAS and CMS. The white dotted polygons in Fig. 2 of the main text correspond to the range

$$1 \le A \le 5\,, \qquad 0 \le w_1 \le 1\,, \tag{5}$$

where the chosen range for A reflects a large systematic uncertainty.

BACKGROUND DUE TO SEMILEPTONIC B-MESON DECAYS

An important background to the semileptonic Λ_b decays is due to semileptonic decays of B mesons. Reference [2] proposed three possible approaches to dealing with this background. The first approach ("Inclusive Selection") does not attempt to reduce it. This results in low sample purity, but keeps the signal efficiency high. The B-meson background can be reduced (with a corresponding cost in signal statistics) by requiring the jet to contain a reconstructed Λ_c^+ baryon (via one of its fully reconstructible decay modes, such as $\Lambda_c^+ \to pK^-\pi^+$) or a Λ baryon (reconstructed via its $\Lambda \to p\pi^-$ decay). These are referred to as "Exclusive Selection" and "Semi-Inclusive Selection", respectively [2]. Each of these requirements can be applied to either one or both sides of the event, leading to six possible analysis channels, all of which were analyzed in [5]. The statistical uncertainty was found to be the lowest for the inclusive/exclusive channel (although the other channels turned out rather comparable). The sample purity obtained in this channel is $\sim 4.9\%$. A higher sample purity of $\sim 44\%$ is possible in the exclusive/exclusive channel at the price of lowering the signal efficiency by a factor of ~ 22 and increasing the statistical uncertainty for the spin correlation components by a factor of ~ 1.6. We base our estimates in this Letter on the inclusive/exclusive channel, while noting that sensitivity can be improved by combining all six channels or by utilizing electrons in addition to muons, which we do not pursue here.

STATISTICAL UNCERTAINTY ESTIMATION

The expected statistical uncertainty in a measurement of the spin correlation matrix components C_{ij}, using a fit of the data to Eq. (3) from the main text and the relation in Eq. (4) therein, is approximately [5]

$$\sigma_{C_{ij}}^{\text{stat}} \simeq \frac{3}{r_i r_j \alpha^2 \sqrt{fN}}\,, \tag{6}$$

where N is the expected number of signal events after the full event selection, f is the sample purity $N/(N + N_B)$, with N_B being the number of background events, and we have approximated the angular distribution of the background to be similar to that for $C_{ij} = 0$.

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