

# No evidence for systematic voter fraud: A guide to statistical claims about the 2020 election

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After the 2020 US presidential election Donald Trump refused to concede, alleging widespread and unparalleled voter fraud. Trump's supporters deployed several statistical arguments in an attempt to cast doubt on the result. Reviewing the most prominent of these statistical claims, we conclude that none of them is even remotely convincing. The common logic behind these claims is that, if the election were fairly conducted, some feature of the observed 2020 election result would be unlikely or impossible. In each case, we find that the purportedly anomalous fact is either not a fact or not anomalous.

election security | fraud detection | science communication

Collowing the 2020 US elections, President Trump and other Republicans questioned Biden's victory in public statements and lawsuits. Although Trump's legal challenges were unsuccessful, many of his supporters were apparently convinced by his claims that the election was stolen: A survey in December 2020 found that over 75% of Republican voters found merit in claims that millions of fraudulent ballots were cast, voting machines were manipulated, and thousands of votes were recorded for dead people (1).

In this paper, we consider several widely disseminated claims purporting to call into question the 2020 US presidential election result. We focus on statistical claims, i.e., claims that are based on allegedly anomalous patterns in the official vote counts. The common logic of these claims is that some aspect of the 2020 result would be highly unlikely or even impossible if the election had been properly administered. We performed an extensive search to identify the most pervasive such claims appearing in social media posts, expert witness testimony, and research papers.<sup>\*</sup> Our purpose in this paper is to address several of the most pervasive statistical claims in one place and using a common conceptual framework.

We conclude that each of the statistical claims we consider fails in one of two ways. In some instances, accurate claims are made about the election results but they are not actually inconsistent with a free and fair election. In other instances, the supposedly anomalous fact about the 2020 election result turns out to be incorrect.

The 2020 election was remarkable in many ways (e.g., unusually high levels of mail-in voting and turnout), and election administration may well have been imperfect. But we see nothing in these statistical tests that supports Trump's claim of a stolen election.

This research builds on efforts to assess the prevalence of fraud in prior elections in the United States (2–4) and other democracies (5). We also work in parallel with a large number of legal briefs filed by political science experts after the 2020 election (for example, refs. 6 and 7).

### Claims Based on Facts That Are Not Actually Anomalous

Biden's Share of US Counties Is Not Anomalous. Conservative radio talk show host Charlie Kirk tweeted on 20 December 2020, "Does

anyone else have a hard time believing Joe Biden won a recordhigh number of votes despite winning a record-low number of counties?"<sup>†</sup> Later that day, he provided numbers to back up the claim, stating that Barack Obama won 69 million votes and 873 counties (in 2008) and Donald Trump won 74 million votes and 2,497 counties (in 2020), while Biden won 81 million votes and just 477 counties (also in 2020).<sup>‡</sup> While Kirk understated the number of counties Biden won (537, not 477), the basic fact is correct: Biden won far more votes than Trump or Obama while winning far fewer counties than Trump and somewhat fewer counties than Obama.<sup>§</sup> If Biden won so few counties, how could he have legitimately won so many votes?<sup>¶</sup>

Adding minimal context to Kirk's numbers reveals that there is nothing remotely suspicious or even anomalous about them. The reason Biden won a clear majority of votes while winning a minority of counties is that his support was concentrated in populous counties. This is typical of recent Democratic presidential candidates. Fig. 1 shows the proportion of votes and counties won by Democratic presidential candidates over the last several decades. As Democratic support has become more concentrated in cities, Democratic candidates have tended to win a smaller

#### **Significance**

President Donald Trump claimed that the 2020 US presidential election was stolen; millions of Americans apparently believed him. We assess the most prominent statistical claims offered by Trump and his allies as evidence of election fraud, including claims about Dominion voting machines switching votes from Trump to Biden, suspiciously high turnout in Democratic strongholds, and the supposedly inexplicable failure of Biden to win "bellwether counties." We use a combination of statistical reasoning and original data analysis to assess these claims. We hope our analysis contributes to public discussion about the integrity of the 2020 election and broader challenges of election security and election administration.

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The authors declare no competing interest.

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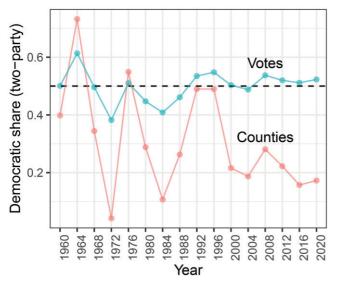
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## <sup>†</sup> https://twitter.com/charliekirk11/status/1340692425635979266. <sup>‡</sup> https://archive.vn/0phvm#selection-3045.143-3045.190.

 ${}^{\$}\textsc{By}$  "counties" we mean counties and county equivalents, e.g., parishes in Louisiana.

<sup>\*</sup>SI Appendix, section A describes our search process.

<sup>&</sup>lt;sup>¶</sup>Turning Kirk's question around, one could ask, If Trump won so few votes, how could he have legitimately won so many counties? The same point could be made for many of these claims.



**Fig. 1.** Biden's share of votes and counties won in 2020 is typical of that of recent Democratic presidential candidates.

share of counties even as their share of votes holds steady. Judging by both votes and counties, Biden did slightly better than Hillary Clinton in 2016 and worse than Obama in 2008. (Biden won many more votes than Obama, as Kirk pointed out, but a smaller share of votes; turnout in 2020 was extraordinarily high.) Thus, the supposedly incredible discrepancy Charlie Kirk highlighted is simply the continuation of a stable trend in US presidential elections.

**Biden's Share of Bellwether Counties Is Not Anomalous.** A related claim was made about Biden's performance in "bellwether" counties, which are counties where a majority of voters have supported the election winner in several consecutive elections (8, 9). Of the 19 counties that voted for the eventual winner in every presidential election from 1980 to 2016, Biden defeated Trump in only one. Several commentators viewed this fact as anomalous. As stated in *The Federalist*, "Amazingly, [Biden] managed to secure victory while also losing in almost every bellwether county across the country. No presidential candidate has been capable of such electoral jujitsu until now" (10). Trump recited this fact in a rally in Georgia (11).

Biden's poor performance in bellwether counties makes sense given two facts. First, at the county level there was remarkable continuity between 2016 and 2020.<sup>#</sup> Not only did Biden win roughly the same proportion of counties as Clinton in 2016 (as shown in Fig. 1), but also he won almost the same set of counties: As shown in Fig. 2A only 63 counties switched from Trump to Biden. (For each county, we show Democratic vote margin in 2016 on the horizontal axis and in 2020 on the vertical.) The 19 bellwether counties are highlighted in red. Visual inspection suggests that, like other counties, they voted in 2020 roughly as they did in 2016; given this (and given that many of these counties went solidly for Trump in 2016), it is unsurprising that Biden won only one of them. Indeed, if we model the probability of Biden winning a county as a function of the county's Democratic margin in 2016 (making no distinction between bellwethers and others), we find that Biden would be expected to win between one and two bellwethers. Fig. 2B shows the probability of Biden winning a county in 2020 given the 2016 Democratic margin in the county, with the conditional relationship calculated using a generalized additive model. The expected number of bellwethers won by Biden is just 1.65 under this model; with alternative models we get estimates between 1.2 and 1.8.

Fig. 2*A* suggests, and the analysis in Fig. 2*B* assumes, that bellwether counties have no special tendency to side with the winner, conditional on the prior election result. Further analysis indicates that this has long been the case (8). To assess whether bellwethers are more likely than other counties to side with the winner in the future, we analyzed each election since 1996. We modeled a county's probability of correctly choosing the winner in a given election as a function of the Democratic margin in the county in the previous election and an indicator for whether the county had sided with the winner in each past election since 1980. We find only one election since 1996 in which bellwethers were more likely to side with the winner than other counties conditional on the county's previous election result (*SI Appendix*, Fig. 1).

Considering that bellwether counties appear to have no special prognostic value in general, and that county-level results were very similar in 2020 and 2016, it is neither surprising nor suspicious that Biden won just one of 19 bellwethers in 2020.

**Differences between 2016 and 2020 Are Not Anomalous.** Trump advocates argued on the basis of a statistical analysis that there was a "one-in-a-quadrillion" chance that Joe Biden legitimately won the election. This claim comes from an expert report submitted as part of Texas Attorney General Ken Paxton's lawsuit against the Commonwealth of Pennsylvania. In that report (12), Paxton claims that the expert, Charles Cicchetti, calculated a one-in-a-quadrillion chance of Biden winning; Cicchetti concludes his report by arguing that "In my opinion, the outcome of Biden winning . . . is so statistically improbable, that it is not possible to dismiss fraud and biased changes in the ways ballots were processed, validated, and tabulated" (p. 9a).

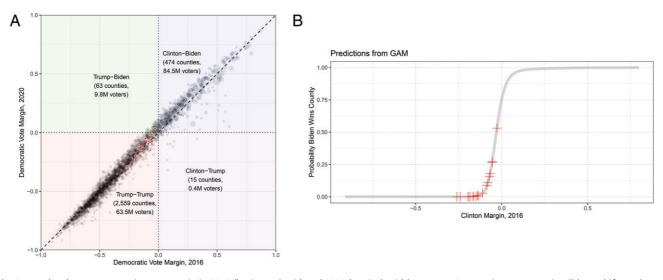
Cicchetti's assertion that Biden's victory was "statistically improbable" is based on a deeply misguided application of null hypothesis significance testing. Cicchetti never actually computes the probability of Biden winning. Instead, he tests the null hypothesis that Joe Biden in 2020 and Hillary Clinton in 2016 had the same expected number of votes in particular states.<sup>||</sup> But if the objective is to assess whether Biden won legitimately, then it is beside the point whether Biden and Clinton enjoyed the same expected support. Support can differ across candidates for any number of reasons, and it is absurd to think that any such difference constitutes evidence of election fraud.

More specifically, Cicchetti treats the number of Democratic votes in an election as a binomially distributed random variable and tests the hypothesis that the expected number of Democratic votes (e.g., in Arizona) was the same for Joe Biden in 2020 as it was for Hillary Clinton in 2016. Let SupportShare<sub>t</sub> denote the true probability that each voter votes Democratic in an election at time t, let Voters<sub>t</sub> denote the total number of voters in that election, and let VoteShare<sub>t</sub> denote the observed share of votes for the Democrat in that election. Then Cicchetti tests the null hypothesis that SupportShare<sub>t</sub> × Voters<sub>t</sub> = SupportShare<sub>t-1</sub> × Voters<sub>t-1</sub> using the test statistic

 $= \frac{\text{VoteShare}_t \text{Voters}_t - \text{VoteShare}_{t-1} \text{Voters}_{t-1}}{\sqrt{\frac{\text{VoteShare}_t (1 - \text{VoteShare}_t) \text{Voters}_t}{\sqrt{+\text{VoteShare}_{t-1} (1 - \text{VoteShare}_{t-1}) \text{Voters}_{t-1}}}}}$ 

<sup>#</sup>SI Appendix, Fig. 2 shows that the serial correlation in county-level election results has increased steadily to a new high in 2020.

He also tests the hypothesis that Biden's early and late vote counts were the same in specific states. This test is subject to the same critique, which we show in *SI Appendix*, section D.



**Fig. 2.** *A* plot shows Democratic vote margin in 2016 (horizontal axis) and 2020 (vertical axis) by county: Support in most counties did not shift much, and bellwethers (colored red) were no exception. *B* plot calculates the expected share of counties Biden won given the 2016 Democratic margin. Trump's margin in bellwether counties (red plus) was large and Biden won only a small share of those. We use several flexible models to calculate Biden's expected number of bellwether county wins if they behave like other counties and we find that Biden would be expected win between 1.24 and 1.75 bellwethers.

For example, Biden won 0.494 of 3.33 million votes in Arizona in 2020, while Clinton won 0.446 of 2.41 million votes in Arizona in 2016; this yields z = 477.09, for a *P* value very close to zero. Given that Biden won a substantially larger share of a much larger total, it should not be surprising that we soundly reject the null hypothesis that the two candidates had the same expected vote total. But it is preposterous to attribute that difference to fraud rather than the myriad innocuous differences between the two elections. It would be similarly preposterous to conclude that something was suspicious about TV ratings because fewer people watched the Super Bowl in 2020 than in 2016 (*z* statistic: 1,495) or to suspect foul play in COVID-19 vaccine trials because the number of infected participants differs between two trials using different vaccines on different numbers of participants.

To further highlight the absurdity of Cicchetti's test, we applied it to other years and states since 1960. Unsurprisingly, we nearly always reject the null hypothesis (1,488 state-year combinations of 1,498). By Cicchetti's logic, this suggests that fraud is commonplace across nearly all US states and elections. In fact, the test indicates simply that elections differ from each other, an unsurprising conclusion that tells us nothing about fraud.

Patterns of Straight-Ticket and Split-Ticket Voting in Michigan Not Anomalous. In a YouTube video with over 1 million views, Shiva Ayyadurai claimed to provide evidence that voting machines in Michigan decisively switched votes from Trump to Biden (13). The analysis compares Trump's share of straight-ticket votes and Trump's share of split-ticket votes across precincts in four Michigan counties. (Voters in Michigan can tick a single box to vote straight ticket for all candidates of one party or vote split ticket for individual candidates.) Ayyadurai argues that, if ballots were counted properly, the difference between those two proportions in a precinct should be unrelated to Trump's success among straight-ticket voters in that precinct. In the four counties he analyzes, Ayyadurai finds instead a negative linear relationship, which he interprets as evidence that Biden stole votes from Trump.

Ayyadurai's argument has been debunked by others, including two analysts who point out that the same logic would also imply that Trump stole votes from Biden in the same counties (14, 15). We show that the negative relationship Ayyadurai takes as evidence of fraud is an expected consequence of regression to the mean and that the same pattern should be found when fraud is absent.

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Let  $X_i$  and  $Y_i$  denote Trump's share of straight-ticket votes and split-ticket votes in precinct *i*, respectively. Ayyadurai's observation is then that  $Y_i - X_i$  is negatively related to  $X_i$ . Now, note that the slope coefficient from the regression of  $Y_i - X_i$  on  $X_i$  is

$$\frac{\operatorname{Cov}(Y_i - X_i, X_i)}{\operatorname{Var}(X_i)} = \frac{\operatorname{Cov}(Y_i, X_i)}{\operatorname{Var}(X_i)} - \frac{\operatorname{Cov}(X_i, X_i)}{\operatorname{Var}(X_i)}$$
$$= \frac{\operatorname{Cov}(Y_i, X_i)}{\operatorname{Var}(X_i)} - 1,$$

which is the slope coefficient from the regression of  $Y_i$  on  $X_i$ minus 1. Thus the relationship Ayyadurai investigates will be negative if the slope coefficient from regressing  $Y_i$  (Trump's split-ticket share) on  $X_i$  (Trump's straight-ticket share) is less than 1. But regression to the mean implies that this should be the case: If split-ticket support for Trump and straight-ticket support for Trump are noisy measures of the same thing (support for Trump), then regressing one on the other will yield a coefficient less than 1, and the relationship Ayyadurai investigates should be characterized by a negative slope.<sup>\*\*</sup> Thus Ayyadurai has it backward: The flat relationship he says would characterize a valid election would be highly surprising, and the relationship he observes is what we would expect if two measures of Trump support were imperfectly correlated, as they typically would be.

This suggests that we should find Ayyadurai's negative relationship in other elections in which voters may vote straight ticket or split ticket and fraud is not suspected. Conveniently, in a follow-up video Ayyadurai points out that the 2008 presidential election in Alabama was just such an election (16). We therefore check the 2008 Alabama election returns for patterns like the one Ayyadurai observes in Michigan in 2020. As expected, many Alabama counties exhibit precisely the negative relationship in 2008 that Ayyadurai considers evidence of fraud in Michigan counties in 2020, as shown in Fig. 3. (Each dot is a precinct, scaled

<sup>&</sup>lt;sup>\*\*</sup>To see this, suppose that underlying Trump support is given by  $T_i$  and that  $Y_i = T_i + \epsilon_i$ and  $X_i = T_i + \gamma_i$ , where  $\epsilon_i$  and  $\gamma_i$  are independent random draws from a distribution with mean zero and constant variance. Then  $\operatorname{cov}(Y_i, X_i) / \operatorname{var}(X_i) = \operatorname{var}(T_i) / (\operatorname{var}(T_i) + \operatorname{var}(\gamma))$  and  $0 < \operatorname{var}(T_i) / (\operatorname{var}(T_i) + \operatorname{var}(\gamma)) < 1$ .

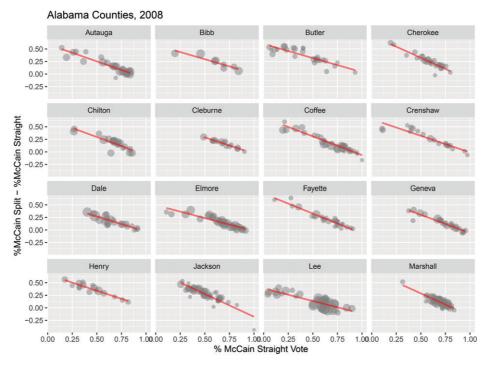


Fig. 3. Several counties in Alabama in 2008 show the same relationship between split-ticket voting and straight-ticket voting that Ayyadurai interprets as evidence of fraud in Michigan in 2020.

by the number of votes cast in the precinct; the red line is the linear prediction.) This confirms that the relationship Ayyadurai highlights is a feature of normal elections and not proof of fraud. In *SI Appendix*, Fig. 3 we show that in 32 of 35 Alabama counties the slope coefficient from a regression of McCain's split-ticket share on his straight-ticket share is less than 1, and in 29 of those counties we reject the null that the slope is 1.

#### **Claims Based on Facts That Are Not Actually Facts**

Dominion Voting Machines Do Not Decrease Trump Vote Share. Trump's legal team claimed after the election that voting machines run by Dominion Voting Systems switched votes from Trump to Biden. Trump lawyers Rudy Giuliani and Sidney Powell argued for a global conspiracy that undermined democracy everywhere Dominion was present. In late December, an anonymous analysis was widely circulated on social media claiming to show that Biden outperformed expectations in counties that used Dominion voting machines (17). The right-wing news outlet The Epoch Times reported that the analysis showed Biden outperformed expectations in 78% of the counties that use Dominion or Hart voting machines and that the analysis "also indicates that Biden consistently received 5.6 percent more votes in those counties than he should have" (18). Assessing whether a particular set of voting machines caused Biden to receive more votes is difficult, because machines are not randomly assigned to counties (19). Further, in *SI Appendix*, section E we present analyses indicating that the original study was the result of P hacking and careless data analysis.

Given these problems with the original analysis, we carry out our own analysis to check for evidence that Dominion machines switched votes from Trump to Biden. In Table 1, column 1 we show the results of a bivariate regression of Biden's share in 2020 on an indicator for whether the county used a Dominion machine, finding a very slight and statistically insignificant difference. In Table 1, column 2 we adjust for Clinton's share of the vote in 2016, which strongly predicts the 2020 outcome (note the  $R^2$  of 0.964); the Dominion coefficient becomes very slightly negative, although again it is not significant. In Table 1, column 3 we add a

Table 1. Dominion voting systems did not cause an increase in Biden votes

Dependent	variable:	Biden vote	share, 2020	)
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	•			,
	1	2	3	4
Dominion machines	0.007 (0.010)	-0.002 (0.002)	-0.009 (0.002)	-0.006 (0.003)
Clinton share of vote, 2016		1.032 (0.004)	1.029 (0.004)	1.011 (0.004)
Observations R <sup>2</sup> Dummy for "Dominion state"	3,111 0.0002	3,111 0.964	3,111 0.965 √	3,111 0.975
State fixed effects				√

Data from all states and the coding of Dominion voting systems from the US Election Assistance Commission are used. SEs in parentheses.

dummy variable indicating whether the county is in a state where any Dominion machines were used and in Table 1, column 4 we add a fixed effect for each state; in both cases we find coefficients that are statistically significant in the negative (i.e., pro-Trump) direction, although very small in magnitude. In Table 1 we find the same null effect of Dominion voting machines persists regardless of how we classify a county as using Dominion machines, once we account for confounding at the state level and for county-level demographics. In short, using the most rigorous specifications we find no evidence that Biden outperformed expectations in counties where Dominion machines were used.

Absentee Ballot Counting Procedures Do Not Decrease Trump Vote Share. Another focus of the Trump team's accusations was the processing of absentee ballots in key states that Biden narrowly won. Among other claims, they alleged that Fulton County, GA, and Allegheny County, PA, were major centers of voter fraud in the 2020 election. Most of these allegations relied upon hearsay affidavits or debunked videos purportedly showing voters stuffing ballots. But in a paper posted in late December 2020, Lott (20) claims to provide statistical evidence that irregularities in the absentee vote counting procedure in Fulton County and Allegheny County suppressed votes for Trump and bolstered Biden's vote count. Lott examined precincts along the border of Fulton and Allegheny Counties and argued that he detected anomalous support for Biden in his absentee ballot share relative to his in-person share of ballots in Fulton and Allegheny Counties. Lott's paper received immediate and widespread attention. Peter Navarro, Assistant to the President and Director of the Office of Trade and Manufacturing Policy, touted the claim as solid evidence of fraud. President Trump tweeted out a link to the paper.

Lott's claims, however, do not withstand scrutiny. Using Lott's own data, we show in *SI Appendix*, section G that the specification he uses to analyze absentee voting patterns produces different conclusions depending on the entirely arbitrary order in which counties are entered in the dataset. Briefly, Lott posits that, if absentee ballots were correctly handled, the difference in Trump support across a boundary that separates a Democratic county from a Republican county should be similar to the difference in Trump support across a boundary that separates one Republican county from another. But Lott's conclusion depends entirely on the order in which the differences are computed for the Republican–Republican pairs. The conclusion is reversed when an alternative and equally justified order is used.

To achieve Lott's objective of comparing voting patterns across county boundaries, we reanalyze Lott's data using a more standard specification that does not suffer from these problems. We use the same pairs of precincts that Lott (20) used in his analysis to limit the confounding between precincts in different counties, but we now use a simple fixed-effects model that resolves the issue with Lott's (20) original specification. The regression equation for this model can be written as

Absentee<sub>i</sub> =  $\beta_1$ InPerson<sub>i</sub> +  $\delta$ SuspectCounty<sub>i</sub>

$$+\sum_{k=1}^{K} \alpha_k I(\text{pair}_i = k) + \epsilon_i, \qquad [1]$$

where  $Absentee_i$  and  $InPerson_i$  denote Trump's share of the absentee and in-person vote (respectively) in precinct *i*; SuspectCounty, indicates whether precinct i is located in a "suspect" county (Fulton or Allegheny, depending on the state being analyzed); and each precinct is identified with one of Kprecinct pairs indexed by k, with  $\alpha_k$  denoting the fixed effect for pair k. In the updated analysis, there is no significant difference in Trump's absentee support (conditional on his in-person support) across the key county boundaries, consistent with the null hypothesis that absentee ballots were handled correctly. We report the results of the fixed-effect analyses for Georgia and Pennsylvania in Table 2. In column 1, we regress Trump's share of the absentee vote on Trump's share of the in-person vote and a dummy for Fulton County; in column 2 we add precinct-pair fixed effects as in Eq. 1, essentially allowing the intercept to vary across Lott's precinct pairs. Neither specification shows a substantively or statistically significant difference between Trump's share of the absentee vote in Fulton County precincts and other precincts. The same is also true in Pennsylvania, as reported in Table 2.

**Turnout Was Not Unusually High in Counties Where Republicans Made Fraud Accusations.** Lott (20) also claims to show that 2020 turnout rates were higher than one would otherwise expect in a set of counties where Republicans have alleged that fraud took place. Lott argues that there was an "unexplained increase in voter turnout" ref. 20, p.13 in the key counties of between 1.26 and 2.42%, which Lott says is equivalent to 150,000 to 289,000 votes

## Table 2. Examining Lott's (20) claims about Allegheny and Fulton Counties

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	Geo	orgia	Pennsylvania			
	1	2	1	2		
Trump share, in person	0.760	0.606	0.511	0.307		
	(0.049)	(0.077)	(0.042)	(0.066)		
Suspect county	0.019	-0.003	0.003	0.003		
	(0.019)	(0.020)	(0.008)	(0.009)		
Observations	44	44	174	174		
Precinct-pair fixed effects		$\checkmark$		$\checkmark$		

A fixed-effects specification shows nothing suspicious in Fulton County, GA, and nothing suspicious in Allegheny County, PA. SEs in parentheses.

in those states. Lott concludes that this is evidence consistent with fraud.

To determine whether the "suspicious" counties had higher turnout, Lott checks whether turnout in the 2020 election was higher than would be expected (given previous turnout, political leaning, and local demographics) in counties where, according to Republican lawsuits filed after the election, fraud may have taken place. Lott identifies 19 counties across six swing states where Republicans made fraud allegations.<sup>††</sup> He then compares turnout in these counties to turnout in other counties in the same six states plus all counties in three other swing states (Florida, Ohio, and North Carolina). He argues that, if turnout is higher in these counties than would be expected given covariates, it would be evidence of fraud.

As we explain in *SI Appendix*, section H we dispute the premise of this analysis: Turnout varies across counties for many reasons, and it is unreasonable to ascribe a small unexplained difference to fraud. As it happens, Lott's finding is not robust to sensible departures from his chosen specification, so it is not necessary to dispute the premise.

Our analysis of county-level voting data for 2016 and 2020<sup>‡‡</sup> indicates that Lott's conclusions are driven by the inclusion of states that have lower turnout increases and no suspicious countiesnamely Florida, North Carolina, and Ohio. Fig. 4A shows that, conditional on turnout in 2016, turnout in these three states was lower than turnout in the six states that contain a suspicious county in Lott's analysis. This is relevant because Lott's analysis compares changes in turnout in suspicious counties with changes in turnout in all other counties, so these smaller increases in turnout rates across states will be conflated with the suspicious county indicator in his analysis. The smaller the turnout increase in these three "nonsuspect" states, the more turnout in the suspect counties will appear to be suspiciously high, even if the changes in turnout in these suspect counties are unremarkable relative to the changes in turnout in other counties in their own state.

Fig. 4*B* shows that, once we address the level differences across states, Lott's (20) estimates of the turnout differences in suspicious counties go to zero and become insignificant. We examine

<sup>&</sup>lt;sup>††</sup>Lott identifies the following suspicious counties—in Georgia, Fulton and DeKalb; in Pennsylvania, Allegheny, Centre, Chester, Delaware, Montgomery, Northampton, and Philadelphia; in Arizona, Apache, Coconino, Maricopa, and Navajo; in Michigan, Wayne; in Nevada, Clark and Washoe; and in Wisconsin, Dane.

<sup>&</sup>lt;sup>‡‡</sup> We use turnout rates for the county citizen voting-age population. For the number of voting-aged citizens we use the 5-y American Community Survey from 2019 and 2015. This follows best practice from McDonald (21). For total votes, we use Leip (22). We note that our estimates of turnout are lower than Lott's (20) average turnout rates, but closer to official statistics.

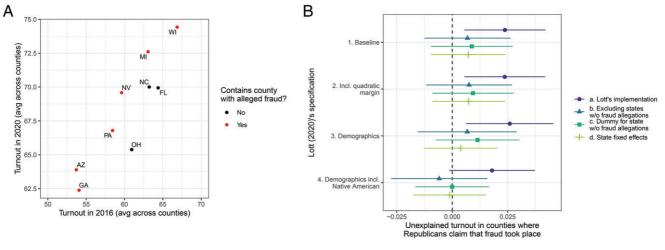


Fig. 4. No evidence suspicious counties had higher turnout. (A) Swing states without suspicious counties had smaller average turnout increases, which drives Lott's (20) results. (B) Lott's (20) estimates of suspicious county differences in turnout are zero and null once we address state-level differences.

all four of Lott's (20) models (organized on the vertical axis) and present the estimated coefficient on an indicator for "suspicious county" in a regression of 2020 turnout on that indicator plus 2016 turnout and covariates. The circle/purple estimates of suspicious county turnout depict the estimates using the four specifications for which Lott (20) presents results in his table 10. The triangle/dark-green estimates depict our estimates when we exclude Florida, Ohio, and North Carolina-three states in which no fraud was alleged. Across models, the difference in suspicious counties is close to zero and-in the case of model 4-the estimate is negative. The square/light-green estimates are from a model where we include all of Lott's states but add an indicator for a state that has suspicious counties. Again, this reduces the estimate to null. Finally, the last estimates (plus/lime green) include state-level fixed effects. Across models, this gives a close to zero and null difference for suspicious counties. Thus, simply by focusing only on states where at least one county had alleged fraud (i.e., swing states that Biden won) or allowing that state-wide turnout trends may differ across states or groups of states, we are able to explain what Lott (20) claimed was unexplained turnout in counties where Republicans had claimed fraud.

In short, there is no evidence that turnout was unusually high in the suspicious counties, let alone that turnout was inflated in these counties by fraud.

## Statistical Analyses of Elections, the Detection of Fraud, and the Spread of Misinformation

Even though the 2020 election is over and Donald Trump's attempt to overturn the results failed, the effects of the claims will reverberate for years. A large segment of the public remains skeptical that Biden won the election legitimately and Republican state lawmakers are taking steps to alter voting access in the name of preventing fraud. The Trump campaign delivered a blueprint for losing candidates to undermine support for the

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winner or even steal the election. It seems unlikely that he will be the last to try these tactics.

We have closely examined what we consider the most prominent statistical claims of fraud in the 2020 election. Although the claims are diverse, our conclusion is consistent: For each claim, we find that what is purported to be an anomalous fact about the election result is either not a fact or not anomalous. In many cases the alleged fact, if shown to withstand scrutiny, would hardly constitute convincing evidence that Biden was elected due to fraud: A modest advantage to Biden in counties that chose to use Dominion machines, for example, could be explained by chance, by factors not accounted for in statistical models, or indeed by pro-Trump fraud undertaken using other voting machines. As it happens, the allegedly anomalous features we consider appear mundane once properly measured or placed in the appropriate context.

In some cases, members of the public who are confronted with a statistical claim of election fraud can apply the approach we took in this paper: First, ask whether the allegedly anomalous fact is a fact; if so, ask whether it is anomalous. In many cases, assessing the validity and unexpectedness of an allegedly anomalous fact requires some statistical sophistication and even original data analysis. For these cases, we think academics (and data journalists and others with appropriate skills) have an important role to play. To safeguard future election results, it will be essential to have elections experts ready to evaluate claims made about whether an election is free and fair. We think that social media organizations can do more to broadcast these evidencebased claims rather than merely flagging questionable assertions as disputed or asserting that the election was free and fair.

Rebuilding trust in American elections requires that we fairly evaluate claims about their failures and communicate those claims to a skeptical public. This paper is an effort in that direction.

Data Availability. Election results data have been deposited in Code Ocean at https://codeocean.com/capsule/0007435/tree/v2.

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