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To my grandfather

Seed of our legacy at the University of Chicago

Thank you for introducing me to economics

I will miss you deeply.

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ABSTRACT

I study the effect of international trade on aggregate efficiency to unpack the relationship between a country's openness to international trade and its technological capabilities. I study this question by looking at how trade policy can be used as a tool to correct domestic distortions either coming from firm's market power (Chapter 2) or due to the presence of scale economies (Chapter 3). I argue that trade is a powerful tool to achieve efficiency gains. See the abstract of each chapter for more details on the contributions of each paper.

CHAPTER 1

PREFACE

As an international trade economist, I'm interested in understanding the effect of international trade on aggregate efficiency to unpack the relationship between a country's openness to international trade and its technological capabilities. In my dissertation I study this question by looking at how trade policy can be used as a tool to correct domestic distortions.

Chapter 2, titled *Labor Market Power and the Pro-competitive Gains from Trade*, starts by investigating whether higher exposure to foreign competition is a useful tool to correct for the misallocation of resources caused by a firm's market power in product and labor markets. I approach this question by building into a standard trade model with oligopolistic competition labor market power, strategic complementarities in wage-setting, and a flexible structure of product and labor markets that accommodates the observed structure in the data. I combine the model with confidential firm tax return data from Australia to quantify the welfare and efficiency gains from trade liberalization.

I show in this chapter that closing the economy to trade, i.e., eliminating foreign competition, has significant and negative effects on welfare and aggregate labor productivity with firms' market power accounting for the vast majority of this effect.[1] As it is standard in the literature, the lack of foreign competition increases the amount of market power domestic producers can exert in product markets distorting relative prices. Low-productivity firms grow larger than they should be at the expense of their high-productivity competitor.

However, contrary to common belief, this is not followed by a decrease in a firm's labor market power that could mitigate the previous negative effect. Instead labor and product market power interacts with each other amplifying rather than dampening the welfare losses. Relative to an economy where labor markets are perfectly competitive, labor market power

1. The rest of the effect are accounted for love-for-variety effects. That is the loss in welfare due to the reduction in available good varieties for consumption.

adds curvature to the cost function of the firms changing the elasticity of its supply curve. Those firms that were originally most distorted are also the more elastic, and therefore in response to closing the economy to trade they increases their markups even more than the case with perfectly competive labor markets. This amplification effect is economically significant and accounts for almost half of the total welfare losses.

These results suggest that simply opening an economy to trade is a practical solution to obtain large productivity gains from an improved allocation of resources. Furthermore, they highlight the importance of market structure, firm competition, and strategic complementarities for the quantification of the gains from trade liberalizations.

In practice, trade between countries is not free but is subject to import tariffs and export taxes. One characteristic of import tariffs in the data is that they tend to be higher for final goods than for inputs, a phenomenon commonly referred to as tariff escalation. In Chapter 3, titled *Trade Policy and Global Sourcing: An Efficiency Rationale for Tariff Escalation*, I in joint work with Pol Antràs, Teresa Fort, and Felix Tintelnot, investigate whether this feature of the data can be rationalized as the outcome of an optimal policy, and what are the reasons that justifies such arrangement of import tariff.

While neoclassical trade theory cannot easily rationalize this pattern, we show that tariff escalation can be rationalized on efficiency grounds in the presence of scale economies. When both downstream and upstream sectors produce under increasing returns to scale, a unilateral tariff in either sector boosts the size and productivity of that sector, raising welfare. While these forces are reinforced up the chain for final-good tariffs, input tariffs may drive final-good producers to relocate abroad, mitigating their potential productivity benefits and therefore justifying setting them at a lower rate.

These projects constitue the beggining of a larger researcher agenda that, as I explained before, is concern with understanding how and how much can international trade shape a country's technological capabilities which I will continue to develop in the coming years.

CHAPTER 2

LABOR MARKET POWER AND THE PRO-COMPETITIVE GAINS FROM TRADE

Abstract. We build a general equilibrium model of trade with market power in product and labor markets, strategic complementarities in price- and wage-setting, and a rich structure of product and labor markets to study the pro-competitive gains from trade. We combine our model with confidential firm-level data from Australia to quantify the welfare and efficiency gains of higher exposure to international trade. We find that closing the economy to trade has significant and negative effects on welfare and aggregate labor productivity. Firms' market power accounts for the vast majority of the effect: In a perfectly competitive economy, where there is no firm market power, welfare losses are 60 percent of the total effect. More important, contrary to common belief, we find that labor and product market power interacts with each other amplifying rather than dampening the welfare losses. This amplification effect is economically significant and accounts for 42 percent of the total welfare losses. Our results highlight the importance of market structure, firm competition, and strategic complementarities for the quantification of the gains from trade liberalizations.

2.1 Introduction

An old idea in international trade is that a more open economy can alleviate distortions caused by market power in product markets. The conventional view sustains that as an economy opens up to trade, local producers are more exposed to higher competition from foreign firms. Small and less productive firms shrink and exit the market, while larger and more productive firms concentrate local resources and expand to new markets. This pattern of reallocation has been documented, for example, in Pavcnik (2002), Trefler (2004), and

Mayer et al. (2014). The higher levels of competition alter the underlying distribution of markups in the economy, and the resulting reallocation of resources places inputs where their value is higher, increasing aggregate productivity (Holmes et al., 2014; Edmond et al., 2015; Arkolakis et al., 2018). Because these effects result from exposure to greater competition, the literature has referred to this mechanism as the pro-competitive gains from trade.

On the other hand, recent evidence from factor markets suggest firms may also exert market power over the local factors of production. Empirical research has estimated positive markdowns[1] on wages for the vast majority of manufacturing plants in the US (Hershbein et al., 2019; Kirov and Traina, 2021; Kroft et al., 2022; Lamadon et al., 2022).[2] In this context, the higher concentration of local resources that results from exposing the economy to trade has consequences for the prices of these factors of production due to firm market power. For example, Felix (2021) finds that after a trade liberalization, in labor markets where firms face a more substantial reduction in the import tariff of their product, there is a stronger increase in markdowns due to an increase in market concentration. However, it remains an open question how this effect on relative costs across firms feeds back into their product market power, and more important whether, and how, these forces affect the level and distribution of the welfare gains from trade.

Our goal is to use producer-level data to quantify the overall effects of trade on productivity and welfare when producers exert market power in both product and labor markets. We study these pro-competitive effects in a quantitative trade model with endogenously variable markups and markdowns. For this, we build a general equilibrium model of trade with market power in product and labor markets, strategic complementarities in price- and wage-setting,

1. The firm's markdown is defined as the gap between its marginal revenue product of labor and the wage it pays its workers. In perfectly competitive markets, this gap is zero. On the contrary, when the firm exerts market power, this gap is positive, reflecting the ability of the firm to pay wages below its marginal revenue product of labor.

2. Likewise, a negative relationship has been documented between the buyer share of a firm sales and the price it pays for inputs consistent with buyer power on input trade (Morlacco, 2020).

and a rich structure of product and labor markets.

The model combines features of Atkeson and Burstein (2008) and Berger et al. (2022a). In the model, competition in product markets is as in Atkeson and Burstein (2008), where in any given market, a small number of producers engage in oligopolistic competition. Firms are non-atomistic within product markets and compete strategically for consumers internalizing how they expect other firms to respond to their production and price policies. Different from this paper is our treatment of the labor market. First, we extend Atkeson and Burstein (2008) by incorporating labor supply decisions endogeneizing the total supply of factor of production in the economy. Second, we depart from the assumption of a perfectly competitive country-level labor market. Instead, within the country there are multiple local labor markets which are perceived as imperfect substitutes from the point of view of the household. Competition in labor markets is as in Berger et al. (2022a) where in any given labor market, there is a small number of producers who engage in oligopsonistic competition. Firms are non-atomistic within labor markets and compete strategically for workers internalizing how they expect other firms to respond to their hiring and wage policies. We accommodate this structure using a nested constant elasticity of substitution (CES) product-demand and labor-supply system that under imperfect competition generates both variable markups and markdowns.

We express the model in relative changes and show we can identify the effect on welfare and aggregate productivity of closing the economy to trade using data only from the domestic country in the trade equilibrium. Our counterfactuals are performed with knowledge of a few parameters but are intensive in data requirements. In particular, it requires knowledge about: (i) the boundaries of each product and labor market, (ii) the equilibrium distribution within each market of firm's domestic sales, total payroll and employment, (iii) total import expending in each product market, and (iv) the elasticities of substitutions within and across product and labor markets. The first three elements allow us to compute firm-level market shares, the key input in our equilibrium conditions, and when combined with information on the elasticities of substitutions they determine the equilibrium distribution of firm's market power. This type of information is hardly available for a set of countries. My identification result circumvents the challenge of obtaining such granular data for several countries by establishing that, when we focus on the counterfactual in which the economy does not participate in international trade, one only needs information for the domestic economy.[3]

We quantify the model using a new data set containing detailed confidential firm-level data from Australia. The data comes from the Business Longitudinal Analysis Data Environment (BLADE), an economic data tool that combines tax, trade, and intellectual property data on all active businesses in Australia sourced from surveys conducted by the Australian Bureau of Statistics Business Register and the Australian Taxation Office. We use the information in BLADE on firm's domestic and export sales, total payroll, employment, and the firm's industry classification and location for all manufacturing plants actively producing in the fiscal year of 2018/2019 to discipline the model along the four key elements described before.

We partition the economy into a finite number of product and labor markets and use the information on a firm's location and industry classification to define the boundaries of each market. Several studies have shown that labor markets are local because workers find it costly to search for jobs in locations far from where they live. For example, Manning and Petrongolo (2017) estimates that the attractiveness of jobs decays sharply with distance. At the same time, workers require specific skills to produce a particular good, and therefore workers cannot move easily across different industries either (Neal, 1995; Parent, 2000). To capture these features, we define labor markets based on sector-region pairs using a unique characteristic of our data. BLADE matches its information on firm-level tax data with the Australian Statistical Geography Standard, a classification of Australia into statistical areas designed to represent labor markets.[4] We augment this definition of labor markets by interacting each

3. To study a larger set of counterfactuals with this approach, for example cases with more or less degrees of openness to trade, we would require to observe this information not only for the domestic economy but also for each of its trade partners.

4. Each hierarchy is designed to measure specific economic outcomes with the Statistical Areas - Level 4

statistical area with a 3-digit manufacturing industry of the Australian and New Zealand Standard Industry classification from 2006 (ANZSIC06), giving us approximately 3500 local labor markets. Firm's output, on the other hand, is considerably more mobile than labor, so we define product markets based only on different manufacturing industries as in Edmond et al. (2015) and Gaubert and Itskhoki (2021).[5] In total, we divide the economy into 143 distinct product markets, each a 4-digit manufacturing industry of ANZSIC06. Finally, we calibrate the elasticities of substitution within and across these markets using numbers from the literature and information about the sales to payroll ratio in BLADE. We show that our results are robust to a range of parameters in the literature that relies on the nested CES structure for product-demand and labor-supply.

We find that closing the economy to international trade generates a 2% decrease in welfare. These losses result from of a lower amount of aggregate consumption and a higher number of hours worked, which tries to compensate for the fall in output due to the absence of trade. Although the aggregate effect is small, the imperfectly competitive nature of product and labor markets accounts for the majority of these losses. In a perfectly competitive economy, where the allocation of resources is efficient, and there is no firm market power, the welfare losses of moving the economy to autarky account for 60% of the total effect.

To understand the forces at play that drive the losses in welfare and efficiency, we quantify the effects of trade across different classes of models nested in our specification. We start by looking at versions of the model where firms behave strategically in only one market while in the other we impose perfect competition. In these cases, the model reduces to either a variant of Edmond et al. (2015) with many local labor markets or an open economy version of Berger et al. (2022a). We find in these cases that the estimated welfare and efficiency

(SA4) classification designed to represent labor markets. Statistical Areas - Level 4 (SA4) were designed by taking into account information on where people live (labor demand) and where people work (labor supply).

5. With our data we cannot incorporate regional components to our definition of product markets since we lack information on goods flow within Australia.

effects are significantly smaller. On average this models under predict the welfare gains by 42% and can miss by almost 90% the effect on aggregate productivity. While both models tend to miss most of the effect of trade their relative performance depends on the value of the elasticities of substitution within product and labor markets.

Furthermore, we find that the effects of imperfect competition and strategic complementarities in product and labor markets interact with each other. As we mentioned before, neither of these models can reproduce the total effect of trade and combined they still miss as much as 25%, indicating that the distortions caused by firms' markdowns tend to amplify those generated by their markups, and vice versa. These results confirm that studying product and labor market power in a unified framework is important to understand the overall effects of trade and that these features are quantitatively important.

Finally, we remove strategic complementarities in price- and wage-setting from the model but still allow firms to exert market power in product and labor markets. In this case, firms are monopolists (monopsonists) in product (labor) markets and charge a constant markup (markdown) as in Jha and Rodriguez-Lopez (2021). We find that, on average, welfare losses are only 14% lowe in this case. Arkolakis et al. (2018) shows that for a large class of demand functions the gains from trade predicted by models with variable markups are equal to those predicted by models with constant markups. While the demand system in this paper does not fall within the class of models studied by Arkolakis et al. (2018), our results suggest that this specification with constant levels of market power accounts for a significant share of the total effect and so the pro-competitive gains are in this sense *elusive* as defined by Arkolakis et al. (2018).[6] In several important economic models, the unconditional markup distribution is invariant to the level of trade costs. Therefore, using changes in the moments of the markup distribution to infer the effects of trade on welfare or aggregate productivity can lead to wrong

6. The losses in aggregate productivity are caused by the dispersion in firm's market power within product and labor markets. In the limit where firm market power is constant, there is no misallocation of resources. The equilibrium of this model, however, is not efficient since market power operates as an aggregate wedge inducing the household to work more than would be efficient.

conclusions. While there is not an equivalent treatment in the literature for the relationship between trade and the distribution of markdowns, we show that there are knife-edge cases in our model where the unconditional markdown distribution is also invariant to the level of trade costs. In fact, our quantitative results shows an almost invariant distribution for markdowns and welfare losses from closing the economy to international trade.

Quantifying potential welfare gains and costs from trade has become increasingly important over the years. Our paper relates to an extensive literature that evaluates trade policy and is mostly related to studies that quantify the pro-competitive gains from trade.

Since the seminal work by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), many papers have tried to understand what drives the misallocation of resources and what can be done about it. We focus on the role of endogenous firm market power variation as a source of misallocation and the effectiveness of international trade addressing it as in Epifani and Gancia (2011), Holmes et al. (2014), Edmond et al. (2015), Arkolakis et al. (2018), Jha and Rodriguez-Lopez (2021), Egger et al. (2021), Felix (2021), and MacKenzie (2021). Our modeling approach to output market power follows closely Atkeson and Burstein (2008), and therefore our work is closely related to Edmond et al. (2015). Nonetheless, our paper still differs from these studies in several dimensions. First, we explicitly consider endogenous labor supply decisions, and so trade not only affects the composition of labor across markets but also the total amount of factor of production in the economy. Secondly, we depart from the assumption of a perfectly competitive labor market. Within the country, multiple local labor markets are populated by heterogeneous firms exerting market power over the labor input. We show how accounting for the interrelation between imperfect competition in product and labor markets amplifies the welfare and efficiency gains from trade. Thirdly, we allow for arbitrary boundaries for product and labor market. Hence, the model accommodates a salient feature of the data in which a product is produced by firms operating in different labor markets and, at the same time, within one labor market firms produced different products.

Fourthly, the way in which we take the model to the data is very different compared to these studies. We solve our two-country and multi-market model in changes relative to a base year, allowing us to perform counterfactuals without relying on estimates of unobserved structural parameters, like fundamental productivity, nor imposing assumptions about the rest of the world. Finally, our identification strategy for the gains from trade allows us to handle fixed cost of exporting without the complications of problems with discrete choice or multiplicity of equilibrium that is common in environments that rely on imperfect competition à la Atkeson and Burstein (2008).

Our paper is also related to the literature on imperfect competition in labor markets (Manning, 2011; Card et al., 2018; Manning, 2021; Berger et al., 2022a) and the subsequent papers that address the question of the optimal policy response (Berger et al., 2022b; Mousavi, 2021; Trottner, 2022). All of this papers have looked at the effects of labor market power within closed-economy models and therefore we contribute to this literature introducing international trade. Furthermore, the potential policy solutions that the literature has given to labor market power has focus on instruments that use prices, .e.g., minimum wage or taxes on labor. We show that opening to trade helps to address this problem by differently exposing markets to foreign competition.

The remainder of the paper proceeds as follows. Section 2.2 presents the model. Section 2.3 gives an overview of the data, and Section 2.4 explains how we use that data to quantify the model. In Section 2.5, we present our main results on the pro-competitive gains from trade and robustness to some of our parameter choices. Finally, in Section 2.6, we conclude.

2.2 Theoretical Framework

The world consist of two symmetric countries, Home and Foreign.[7] We assume a static environment with a single factor of production, labor, that is immobile between countries. We focus on describing the Home country in detail. We indicate Foreign variables with an asterisk.[8]

Each economy is partitioned into a finite (but large) number of product and labor markets. We use $\mathcal{K}$ and $\mathcal{L}$ to denote the set of product and labor markets, respectively.[9] At the same time, the economy is populated by a fixed and finite number of firms. These firms are denoted with i and j, and we use $\mathcal{M}$ to refer to the set of firms. Each firm $i \in \mathcal{M}$ hires workers from a specific labor market $l \in \mathcal{L}$ to produce a specific type of good $k \in \mathcal{K}$. We assume that firms have market power both in product and labor market.

We define two functions $\kappa : \mathcal{M} \cup \mathcal{M}^* \to \mathcal{K}$ and $\ell : \mathcal{M} \to \mathcal{L}$, that map firms into product and labor markets, respectively. In the model, these functions work as accounting devices that help to keep track of which firms operate in each market, and therefore determine the set of competitors for a firm. However, at their core they encode the definition of product and labor markets. At this point, these functions are arbitrary and flexible enough to accommodate different definitions of what constitutes a market.[10]

7. We impose this assumption here, but, in the quantitative section, we work with two asymmetric countries.

8. The description of the environment follows closely Edmond et al. (2015) with the distinction that labor markets are model as in Berger et al. (2022a).

9. In the Foreign country, the set of product and labor markets are $\mathcal{K}^*$ and $\mathcal{L}^*$

10. In Atkeson and Burstein (2008) and Edmond et al. (2015) firms produce for different sectors and hire workers from a unique labor market. This corresponds to $\mathcal{K}$ representing the different sectors in the economy, κ identifies the sector of the firm, and $|\mathcal{L}| = 1$. In Berger et al. (2022a), firms operate in different labor markets and produce varieties of a unique type of good. This correspond to $\mathcal{L}$ representing the different labor markets in the economy, ℓ identifies the labor market where the firm operates, and $|\mathcal{K}| = 1$. Finally, the literature that has worked with models of imperfect competition in product and labor markets impose the restrictive assumption that each product market is also a labor market. (Deb et al., 2022; Trottner, 2022). In this case, $\mathcal{K} = \mathcal{L}$. We want to think of cases outside these extremes where, for example, two firms from different industries are located in the same region, so although they do not compete for the same consumers they do compete for the same pool of workers.

Finally, there is unit mass of identical households that supply labor to all firms and consumes the final good which is an aggregate basket of all the goods produced in the Home and Foreign country.

2.2.1 Household

Preferences. The household chooses the number of workers to supply to each firm n_i and consumption of the final good C to maximize their value of utility. Preferences are described by the utility function

$$\mathcal{U} = \log C - \bar{\varphi}^{\frac{1}{\varphi}} \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}, \tag{2.1}$$

where N is the aggregate labor supply index and $\bar{\varphi} > 0$ and $\varphi > 0$ are two parameters capturing household disutility from labor and the Frisch elasticity of labor supply, respectively. We assume that the aggregate labor supply index follows the nested CES structure of Berger et al. (2022a) and is given by:

$$N = \left[\sum_{\ell \in \mathcal{L}} \upsilon_{n,l}^{-\frac{1}{\theta}} N_\ell^{\frac{\theta+1}{\theta}}\right]^{\frac{\theta}{\theta+1}}, \qquad N_\ell = \left[\sum_{\{i \in \mathcal{M}: \ell(i) = l\}} \xi_{n,i}^{-\frac{1}{\eta}} n_i^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}, \qquad \eta \geq \theta > 0, \tag{2.2}$$

where the inner nest aggregates employment across firms within a particular labor market $l \in \mathcal{L}$ while the outer nest aggregates this market level employment indexes across different labor markets. The two key parameters in equation (2.2) are η and θ which represent the elasticity of substitution across different employers within a market, and the elasticity of substitution across different labor markets, respectively. Finally, the parameters $\{\xi_{n,i}\}_{i \in \mathcal{M}}$ ($\{\upsilon_{n,l}\}_{l \in \mathcal{L}}$) are non-negative labor supply shifters that reflect the preferences of the household to work in one over another firm (labor market), such that,

$$\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} \xi_{n,i} = 1, \quad \text{and} \quad \sum_{l\in\mathcal{L}} \upsilon_{n,l} = 1.$$

The preferences over labor supply in equation (2.2) imply that jobs are differentiated from the perspective of the workers, this constitutes the source of labor market power in the model. Berger et al. (2022a) show that this type of preferences can be micro-funded in an environment with heterogeneous workers making independent decisions on where to work providing an exact map between the parameters η and θ and the distribution of relative net costs to individuals of moving between and across markets.

Utility Maximization. Consumers maximize utility $\mathcal{U}$ subject to the budget constraint,

$$C = \sum_{i\in\mathcal{M}} w_i n_i + \Pi,$$

where w_i is the wage offered by employer i and Π are aggregate profits. The expression for the budget constraint makes explicit that we use final consumption in Home as the numeraire.

Solving the household's problem, we obtain that the labor supply face by employer i in market l is given by:

$$n_i = \xi_{n,i}\upsilon_{n,\ell(i)} \left(\frac{w_i}{W_{\ell(i)}}\right)^{\eta} \left(\frac{W_{\ell(i)}}{W}\right)^{\theta} N, \tag{2.3}$$

with $W_{\ell(i)}$ and W the market and aggregate wage index, where $\ell(i) \in \mathcal{L}$ indicates the labor market where firm i operates. These indexes are defined as the numbers that satisfy

$$W_l N_l = \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} w_i n_i, \qquad WN = \sum_{l\in\mathcal{L}} W_l N_l.$$

Together with optimality condition (2.3) these definitions imply,

$$W_l^{1+\eta} = \sum_{\{i \in \mathcal{M} : \ell(i) = l\}} \xi_{n,i} w_i^{1+\eta}, \qquad W^{1+\theta} = \sum_{l \in \mathcal{L}} \upsilon_{n,l} W_l^{1+\theta}. \tag{2.4}$$

Finally, consumption of final good C (the numeraire) and the aggregate labor supply index N satisfy the standard consumption-labor trade-off equation,

$$(\bar{\varphi} N)^{\frac{1}{\varphi}} C = W. \tag{2.5}$$

2.2.2 Final Good Producers

Technology. Perfectly competitive firms in each country produce an homogeneous final consumption good Y using a bundle of inputs Y_k from each market $k \in \mathcal{K}$,

$$Y = \left[\sum_{k \in \mathcal{K}} \upsilon_{y,k}^{\frac{1}{\epsilon}} Y_k^{\frac{\epsilon - 1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon - 1}}, \tag{2.6}$$

where $\epsilon > 1$ is the elasticity of substitution across product markets $k \in \mathcal{K}$. Each product market consists of a finite number of domestic and foreign intermediate producers. In market k, output is produced using $M_k \in \mathbb{N}$ domestic intermediate inputs and potentially an additional $M_k^* \in \mathbb{N}$ imported intermediate inputs,

$$Y_k = \left[\sum_{\{i \in \mathcal{K} : \kappa(i) = k\}} \xi_{Hy,i}^{\frac{1}{\sigma}} y_{H,i}^{\frac{\sigma-1}{\sigma}} + \sum_{\{i \in \mathcal{M}^* : \kappa(i) = k\}} \xi_{Fy,i}^{\frac{1}{\sigma}} y_{F,i}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{2.7}$$

where $y_{c,i}$ denotes the output of firm i located in country $c \in \{H, F\}$ and $\sigma \geq \epsilon$ is the elasticity of substitution across goods within a particular market k. The parameters

$\{\xi_{cy,i}\}_{i\in\mathcal{M}\cup\mathcal{M}^*,c\in\{H,F\}}$ and $\{\upsilon_{y,k}\}_{k\in\mathcal{K}}$ are non-negative demand shifters that reflect the preferences of the final good producer for inputs of one over another firm producing goods for different markets, such that,

$$\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \xi_{Hy,i} + \sum_{\{i\in\mathcal{M}^*:\kappa(i)=k\}} \xi_{Fy,i} = 1,$$

and

$$\sum_{k\in\mathcal{K}} \upsilon_{y,k} = 1.$$

Demand for Intermediate Inputs. Final good producers buy intermediate goods from Home producers at prices $p_{H,i}$ and from Foreign producers at prices $p_{F,i}$. A final good producer chooses intermediate inputs $y_{H,i}$ and $y_{F,i}$ to maximize profits,

$$Y - \sum_{k\in\mathcal{K}} \left(\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} p_{H,i} y_{H,i} + \sum_{\{i\in\mathcal{M}^*:\kappa(i)=k\}} p_{F,i} y_{F,i} \right),$$

subject to (2.6) and (2.7). The solution to this problem gives the demand functions for domestic and foreign inputs,

$$\begin{aligned} y_{H,i} &= \xi_{Hy,i}\upsilon_{y,\kappa(i)} \left(\frac{p_{H,i}}{P_{\kappa(i)}}\right)^{-\sigma} \left(\frac{P_{\kappa(i)}}{P}\right)^{-\epsilon} Y, \qquad (2.8) \\ y_{F,i} &= \xi_{Fy,i}\upsilon_{y,\kappa(i)} \left(\frac{p_{F,i}}{P_{\kappa(i)}}\right)^{-\sigma} \left(\frac{P_{\kappa(i)}}{P}\right)^{-\epsilon} Y, \end{aligned}$$

with $P_{\kappa(i)}$ and P the market and aggregate price index, and $\kappa(i) \in \mathcal{K}$ indicating the product market where firm i operates. These indexes follow the standard formulas with

$$P = \left[\sum_{k \in \mathcal{K}} \upsilon_{y,k} P_k^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \tag{2.9}$$

and

$$P_k = \left[\sum_{\{i \in \mathcal{M}:\kappa(i)=k\}} \xi_{Hy,i} p_{H,i}^{1-\sigma} + \sum_{\{i \in \mathcal{M}^*:\kappa(i)=k\}} \xi_{Fy,i} p_{F,i}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{2.10}$$

where we sum over all the firms that operates in market k, i.e. all the firms for which $\kappa(i) = k$ is true.

2.2.3 Intermediate Good Producers

Technology. It is defined by three properties: (i) the production technology; (ii) the identity of the labor market from where the firm hires her workers; and (iii) the identity of the product market where the firm sells her good. The first component is standard and determines how much output the firm can produce for a given amount of inputs. The other two determine the amount of competition the firm faces for workers and consumers.

The production technology of intermediate good producer i is a constant returns to scale production function that has labor as the only input. Firm i hires workers in labor market $\ell(i) \in \mathcal{L}$ to produce goods for product market $\kappa(i) \in \mathcal{K}$. The production function is given by,

$$y_i = z_i n_i, \tag{2.11}$$

where producer-level productivity z_i is the realization from the country specific distribution $F(z)$.[11]

11. Because we have a finite number of firms within each sector, market-level averages do not coincide with moments from the distribution $F(z)$. Therefore, the productivity parameters z_i are a source of granular

Trade Costs. The output of firm i can be traded under two types of trade costs. We assume there is a fixed cost of exporting that requires $f_k \geq 0$ units of the labor supply index for any firm that wishes to export any of its output to the other country.[12] We also assume there is an iceberg type cost of exporting $\tau_k \geq 1$. Let $y_{H,i}$ denote the amount sold by a Home intermediate good producer to the Home final good producer, and similarly let $y^*_{H,i}$ denote the amount sold by a Home intermediate good producer to the Foreign final good producers. The resource constraint for Home intermediate good producers is,

$$y_i = y_{H,i} + \tau_{\kappa(i)} y^*_{H,i}. \tag{2.12}$$

Market Structure. We assume that the intermediate good producing firms are engaged in imperfect competition both in the product and labor market playing a static game of quantity competition. Specifically, each firm i chooses its quantity $y_{H,i}$ and $y^*_{H,i}$ for the home and foreign market, respectively, taking as given the quantities chosen by the other firms in the economy, recognizing that sectoral prices vary when the firm changes its quantity. Simultaneously, each firm i chooses it demand for labor n_i taking as given the number of workers hired by the other firms operating in the same labor market $\ell(i)$, recognizing that market level wages vary when the firm changes its demand for labor. Although we model a finite number of product and labor markets, we assume that firms are atomistic with respect to the aggregate, and therefore take the aggregate price and aggregate wage as given.

comparative advantages between the two countries as in Gaubert and Itskhoki (2021).

12. The choice of the units of the fixed costs are not innocuous when we look at welfare and aggregate productivity (see for example Baqaee and Farhi, 2020; Gutierrez, 2022). We denote fixed costs in units of aggregate labor supply index N, as opposed to n_i for example, to avoid endowing the firm with market power over the fixed resources used for exporting.

We decide not to model the entry decision to the domestic market, and instead assume that the number of domestic firms within each product and labor market are exogenously given. Modeling entry is challenging because a decision to enter into a particular product and labor market needs to consider: i) which firms are already selling their output in product market k; and ii) which firms are hiring labor from labor market l. Both of these considerations opens the possibility of having multiplicity of equilibria, since the entry decision of the firm depends on the entry decisions of all the other firms.

However, the total number of firms in a product market, both domestic and foreign, that sell positive amounts of their goods in each country is determined endogenously in equilibrium - firms will choose to export if it is profitable for them to do so.[13]

The problem of the firm. A Home intermediate good producer chooses quantities $y_{H,i}$ and $y_{F,i}$ for the Home and Foreign market, labor units n_i, and whether to export or not $\phi_i \in \{0, 1\}$ in order to maximize profits,

$$\pi_i \equiv \max_{y_{H,i}, y_{F,i}, n_i} \left[y_{H,i} p_{H,i} + \phi_i y^*_{H,i} p^*_{H,i} - w_i n_i - \phi_i f_k W \right], \tag{2.13}$$

subject to the inverse labor supply curve, equations (2.3) and (2.4), the inverse product demand curve, equations (2.8) and (2.10), and the technology of the firm, equations (2.11) and (2.12), and taking as given the amount of output being produced by i's competitors in the product market as well as the number of workers being hired by i's competitors in the labor market.

A couple of things related to the characteristics describing a firm are worth emphasizing before describing the solution to the firms problem. The first thing to notice is that the set of

13. The discrete decision of whether to export or not also brings the possibility of multiple equilibria which compromises any counterfactual analysis of the model. To overcome this issue, in the quantitative section, we treat the observe data as an equilibrium of the model and then look at the counterfactual equilibrium with no international trade which is unique given our assumptions on firm entry.

competitors a firm faces differ across product and labor markets. Trivially, foreign producer do not hire workers from Home, but the generic assignment functions $\kappa(\cdot)$ and $\ell(\cdot)$ imposed at this point implies that even the domestic competitors of firm i in the product market do not necessarily coincide with those competing against i for workers in the labor market. These partial overlap across the set of competitors of a firm implies that different product markets are interconnected through labor markets, and therefore labor markets will play a role propagating shocks across product markets when one is suddenly more exposed to trade.

The second thing to notice is that labor market power interacts with the technology of the firm creating convexities in the cost function. The total cost a firm has to pay for a target value of output y she wishes to produce equals the amount of workers required to produce that amount, $\frac{y}{z_i}$, times the amount of wages she has to pay to those workers. Labor market power implies that the firm faces the labor supply equation (2.3), and therefore the amount of wages she has to pay depends on the amount of workers she wants to hire, $w_i\left(\frac{y}{z_i};\mathbf{n_{-i}},\mathbf{W}\right)$,

$$c_i\left(y\right)=w_i\left(\frac{y}{z_i};\mathbf{n_{-i}},\mathbf{W}\right)\frac{y}{z_i}.$$

Therefore, even though firms operate a linear technology, the decision of how much output to produce for the domestic and the foreign economy *cannot* be considered separately: The optimal amount of output produced for each country equates marginal revenue with marginal cost, however the marginal cost is a function of the total amount produced for both markets. Intuitively, if a firm decides to increase its scale of production to sell abroad it has to hire extra workers; but because of the firm's labor market power this can only be done by offering higher wages than its competitors. The higher wage, however, does not only apply to the marginal workers hired by the firm but to all previously hired workers which are used to produce goods for the domestic economy.[14] This feature of the model distinguish our framework from

14. This implies that absent fixed cost of exporting, if a firm were able to choose between operating as a unified firm selling to the foreign and domestic economy or to split into two firms, it would choose the latter. With positive cost of exporting the decision will depend on whether the exporting-establishment is able to

previous papers that study firm's export decisions, where perfectly competitive labor markets and linear technologies (i.e. constant marginal cost) allows them to treat the two markets separately (see for example Melitz, 2003; Chaney, 2008).[15]

Despite this extra difficulty in the firm's problem, optimal prices still satisfy standard formulas. Profit-maximization implies that a firm's optimal price is a markup over its marginal cost mc_i,

$$p_{H,i} = \mu_i^y mc_i, \qquad p_{H,i}^* = \mu_i^{y^*} \tau_{\kappa(i)} mc_i, \tag{2.14}$$

where the markups satisfy the usual Lerner formula,

$$\mu_i^y = \frac{\gamma_i^y}{\gamma_i^y - 1}, \qquad \mu_i^{y^*} = \frac{\gamma_i^{y^*}}{\gamma_i^{y^*} - 1}, \tag{2.15}$$

with $\gamma_i^y \equiv -\left[\frac{\partial \log p_{H,i}}{\partial \log y_{H,i}}\right]^{-1}$ and $\gamma_i^{y^*} \equiv -\left[\frac{\partial \log p_i^*}{\partial \log y_i^*}\right]^{-1}$ being the price elasticity of demand.

Profit-maximizing wages equal a markdown over a firm's marginal revenue product of labor $mrpl_i$,

$$w_i = \mu_i^n mrpl_i, \tag{2.16}$$

where the markdown depends inversely on the labor supply elasticity faced by the firm $\gamma_i^n \equiv \left[\frac{\partial \log w_i}{\partial \log n_i}\right]^{-1}$,

$$\mu_i^n = \frac{\gamma_i^n}{1 + \gamma_i^n}. \tag{2.17}$$

Under our specification of preferences, these elasticities have closed-form expressions and

cover the fixed cost by itself.

15. We go back to this point later in the text when we study the effect of lower trade costs. Because of this curvature in the marginal costs, firms do not increase their scale of production as much as they would do absent labor market power.

are given by,

$$\begin{aligned}\gamma_i^y &= \left[\frac{1}{\sigma} + \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_{Hi}^y\right]^{-1}, \\ \gamma_i^{y^*} &= \left[\frac{1}{\sigma} + \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_{Hi}^{y^*}\right]^{-1}, \\ \gamma_i^n &= \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right]^{-1},\end{aligned} \tag{2.18}$$

where s_{Hi}^y and $s_{Hi}^{y^*}$ are firm's i sales-share in the Home and Foreign economy, respectively, and s_i^n is the firm's payroll-share. Within a product (labor) market, firms with relatively high market shares have low demand (supply) elasticity and high markups (markdowns). Equations (2.14) to (2.18) are the open economy counterparts with oligopsonistic labor markets of Edmond et al. (2015) and Berger et al. (2022a).

The marginal cost for a firm is given by $mc_i = \frac{1}{\mu_i^n}\frac{w_i}{z_i}$, hence wages and price are related through the pricing equations,

$$p_{H,i} = \frac{\mu_i^y}{\mu_i^n}\frac{w_i}{z_i}, \qquad p_{H,i}^* = \frac{\mu_i^{y^*}}{\mu_i^n}\tau_{\kappa(i)}\frac{w_i}{z_i}. \tag{2.19}$$

Together, equations (2.18) and (2.19) highlight that prices and wages are jointly determined by the firm's choice level of employment and aggregate conditions in the labor and product market a point made also in Trottner (2022).

2.2.4 Equilibrium

In a decentralized equilibrium, households in each country maximize utility by choosing how much to consume of the final good and how many workers to supply to each firm, firms make output and employment decisions to maximize their profits, all the product and labor markets clear, and trade between Home and Foreign is balanced. Definition 2.2.1 formally defines the equilibrium, summarized by equations (2.2) to (2.20).

Definition 2.2.1. An equilibrium is an allocation $\{\{y_{c,i}, y^*_{c,i}\}, n_i, n^*_i\}, Y, Y^*, N, N^*\}$, prices $\{\{p_{c,i}, p^*_{c,i}\}_{c\in\{H,F\}}, w_i, w^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}$, and export decisions $\{\phi_i, \phi^*_i\}$ such that:

1. Given prices $\{\{p_{c,i}, p^*_{c,i}\}, w_i, w^*_i\}$, the quantities $\{\{n_i, n^*_i\}, Y, Y^*, N, N^*\}$ maximize the utility of the domestic and foreign household.

2. Given prices $\{p_{c,i}, p^*_{c,i}\}$, the quantities $\{\{y_{c,i}, y^*_{c,i}\}, Y, Y^*\}$ maximize the profit of the final good producer in the domestic and foreign country.

3. Prices $\{\{p_{c,i}, p^*_{c,i}\}, w_i, w^*_i\}$, quantities $\{\{y_{c,i}, y^*_{c,i}\}, n_i, n^*_i\}$, and export decisions $\{\phi_i, \phi^*_i\}$ maximize profits of the intermediate good producer in the domestic and foreign country.

4. Trade is balanced between countries, and therefore prices $\{\{p_{c,i}, p^*_{c,i}\}, w_i, w^*_i\}$ and quantities $\{y_{c,i}, y^*_{c,i}\}$ satisfy,

$$\sum_{i\in\mathcal{M}} p^*_{H,i} y^*_{H,i} = \sum_{i\in\mathcal{M}^*} p_{F,i} y_{F,i}. \tag{2.20}$$

5. All the product and labor markets clear.

Although, the equilibrium of the model cannot be solved for explicitly, it is insightful to go through the equations that characterize the firm-level market shares since they determine the resulting distribution of resources across firms and are tightly related with the amount of firm-level market power.

With that purpose in mind we define some useful notation moving forward. We let $\{S^y_k\}_{k\in\mathcal{K}}$ to denote product k's share of aggregate sales, and $\{S^n_l\}_{l\in\mathcal{L}}$ to denote labor market l's share of aggregate payrol. These are the market-level counterparts of our firm-level shares. We let $\{\lambda_k\}_{k\in\mathcal{K}}$ denote the share of imports in total sales in product market k.

An Expression for Firm-level Shares. The size of a firm in a product and in a labor market it is given by the share of the market that the firm captures. These shares summarize how resources are distributed across firms an determine the equilibrium level of firm market power, as it is evident from equations (2.15), (2.17), and (2.18).

In what follows, we impose two assumptions. Assumption 2.2.1 is a mathematical simplification that implies that the resulting equilibrium allocation does not depend on agents preference from one firm over the other but just on technological features of the firms. We note, however, that the product demand and labor supply shifters cannot be identify separately from the technology component z_i. So, alternatively, we can think of z_i as summarizing both technology and preferences. Assumption 2.2.2 is more consequential since it rules out selection into exporting based on firm's productivity levels.

Assumption 2.2.1. The product demand and labor supply parameters $\left\{\xi_{cy,i}, \xi^*_{cy,i}, \upsilon_{y,k}, \upsilon^*_{y,k}\right\}$ and $\left\{\xi_{n,i}, \upsilon_{n,k}, \xi^*_{n,i}, \upsilon^*_{n,k}\right\}$ all equal one.

Assumption 2.2.2. Trade between Home and Foreign is free.

Under Assumptions 2.2.1 and 2.2.2 firm optimality, the production technology of the intermediate good producers, and the labor supply and product demand equations that each firm faces we establish the following property of the equilibrium firm-level shares.[16]

Proposition 2.2.1. For given values of market-level sales and payroll shares, $\{S^y_k\}_{k\in\mathcal{K}}$ and $\{S^n_l\}_{l\in\mathcal{L}}$, and import expenditure shares$\{\lambda_k\}_{k\in\mathcal{K}}$, the equilibrium values of the firm-level shares $\{s^n_i, s^y_{H,i}\}_{i\in\mathcal{M}}$ solve the following system of equations. Payroll shares are given by:

$$s^n_i = \frac{\left(\frac{\mu^n\left(s^n_i\right)}{\mu^y\left(s^y_i\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(1+\eta)\sigma}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^y_{\kappa(i)}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \left(\frac{\mu^n\left(s^n_j\right)}{\mu^y\left(s^y_j\left(1-\lambda_{\kappa(j)}\right)\right)}\right)^{\frac{(1+\eta)\sigma}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^y_{\kappa(j)}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}, \tag{2.21}$$

16. The proof can be found in Appendix .1.

domestic sales share are $s_{Hi}^{y} = s_i^y \left(1 - \lambda_{\kappa(i)}\right)$ with,

$$s_i^y = \frac{\left(\frac{\mu^n\left(s_i^n\right)}{\mu^y\left(s_i^y\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(i)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \left(\frac{\mu^n\left(s_j^n\right)}{\mu^y\left(s_j^y\left(1-\lambda_{\kappa(j)}\right)\right)}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(j)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}, \tag{2.22}$$

and firm-level market power is given by,

$$\mu^n(s) = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)s\right]^{-1}, \quad \text{and} \quad \mu^y(s) = \left[\frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right)s\right]^{-1}.$$

Proof. See Appendix .1.2. □

Proposition 2.2.1 shows how to compute the firm-level shares. Different from the work of Berger et al. (2022a) and Deb et al. (2022), where firm-level shares are independent of economy aggregates, we do require knowledge of market-level aggregates to compute firm-level shares. For example, payroll shares do not depend on labor market aggregates, since those affect all firms in the market alike. However, they do depend on market level aggregates related to product markets since firms within a labor market produce different types of products, and so their scale of production is related to the market conditions in their respective product markets.[17]

Although Proposition 2.2.1 does not provide a characterization of the general equilibrium of the model, since the market-level shares are themselves equilibrium objects, it highlights the forces in the model that shape the allocation of resources across firms and the role that market power plays in it.

Missallocation and Market Power Dispersion. In this model, dispersion in firm-level market power reduces aggregate productivity, as in the work of Restuccia and Rogerson

17. With costly trade firms in a labor market have different export decisions, and so in this case the level of demand in the Foreign country also affects the distribution of payroll-shares.

(2008), Hsieh and Klenow (2009) ,and Edmond et al. (2015). To understand this effect, notice that total output produced in each product market can be written as

$$Y_k = A_k \times N_k,$$

where A_k is the endogenous level of labor productivity and N_k is the total amount of workers employed in the production of goods for product market k.[18] Using the optimality conditions and market clearing it can be show that productivity in market k is given by,

$$A_k = \left(\sum_{\{i \in \mathcal{M}: \kappa(i) = k\}} \frac{1}{z_i} \left(s_i^y \right)^{\frac{\sigma}{\sigma-1}} \left[\left(1 - \lambda_{\kappa(i)} \right)^{\frac{\sigma}{\sigma-1}} + \left(\lambda_{\kappa(i)} \right)^{\frac{\sigma}{\sigma-1}} \right] \right)^{-1}, \tag{2.23}$$

and so for a given amount of resources allocated to each market, how those resources get distributed across firms determines the labor productivity in each product market.

By contrast, the first-best level of labor productivity associated with an efficient allocation of resources satisfies an analogous expression to equation (2.23) with firm-level shares s_i^y given by equation (2.22) in Proposition 2.2.1 , but with $\mu_i^n = \mu_i^y = 1$. If there is no market power dispersion, i.e. the ratio μ_i^n / μ_i^y is constant across firms, then labor productivity is at its first-best level. Market power dispersion, whether it comes from heterogenous markups or markdowns, lowers labor productivity relative to the first-best because it induces an inefficient allocation of resources in product and labor markets: relative wages (prices) are not aligned with relative marginal revenue product of labor (marginal costs).[19]

18. The amount of workers employed in product market k is defined as:

$$N_k = \sum_{\{i \in \mathcal{K}: \kappa(i) = k\}} n_i.$$

Note that N_k does not coincide with any of the market-level labor supply indexes in the model since those aggregate employment taking into account the amount of substitution across firms, possibly located in different labor markets.

19. This dispersion in market powers comes from firms charging heterogeneous markups on marginal costs as well as heterogenous markdowns on wages. Therefore, the extreme cases where either $\sigma = \epsilon$, and therefore

Trade, and Firm-level Market Power. Trade affects the allocation of resources because it affects the amount of competition a firm faces in local and foreign markets. Higher levels of import competition reduces the share of the market that the domestic firm can capture, and because in this model market power is a function of the firm size, the markup the firm charges also falls. As Proposition 2.2.1 makes clear, the size of the firm in product and labor markets interact with each other through the endogenous markup and markdown of the firm, and so the higher level of import competition also affects the equilibrium share of the labor market that the firm captures. To compensate for the lower profitability in the product market the firm adjust in response her markdown on wages. Therefore, when markups are constant this effect is muted, and import competition cannot affect the allocation of resources within markets.[20]

Firm-level shares are also affected by the level of demand in the Foreign market. A higher demand in Foreign requires a larger scale of production by the firm. This can only be accomplished by hiring more workers through either an increase in wages or a reduction in markdowns.[21] Changes in trade costs then, shift the allocation of labor across firms to satisfy the higher foreign demand. Proposition 2.2.2 makes this point clear for the case where $\sigma = \epsilon$.[22] When $\sigma = \epsilon$ the model is one with constant markups across firms and firm shares are not affected by import competition, and so the dependence of payroll shares on iceberg

markups are constant across firms, or $\eta = \theta$, and so markdowns are constant across firms, do not achieve the first-best alone.

20. When $\sigma = \epsilon$ the model is one with constant markups across firms, with $\mu_i^y = \frac{\sigma}{\sigma-1}$. Proposition 2.2.2 defines a triangular system where equation (2.24) determines firm's payroll shares independent of s_i^y, and equation (2.22) determines the sales shares given the equilibrium values of s_i^n. Both sales and payroll shares are independent of the expenditure share on imported goods λ_k.

21. We note, however, that relative to the case with no markdowns the amount of workers the firms hires is lower. This is because relative to the no-markdown case where the marginal cost is constant, labor market power makes the marginal cost an increasing function of the total output produced.

22. Here we don't impose Assumption 2.2.2 to included costly trade. Iceberg trade cost are bigger than one, but fixed cost of exporting still equal zero.

cost is related to the firm's access to export markets.[23]

Proposition 2.2.2. Suppose that there are no fixed cost of exporting and that $\sigma = \epsilon$, and so firms charge a constant markup $\mu_i^y = \frac{\sigma}{\sigma-1}$. Then, for given values of market-level sales and payroll shares, $\{S_k^y\}_{k\in\mathcal{K}}$ and $\{S_l^n\}_{l\in\mathcal{L}}$, the equilibrium values of the firm-level shares $\{s_i^n, s_i^y\}_{i\in\mathcal{M}}$ solve the following triangular system of equations:

$$s_i^n = \frac{\mu^n\left(s_i^n\right)^{\frac{\sigma(1+\eta)}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[1+\tau_{\kappa(i)}^{-\sigma}\right]^{\frac{1+\eta}{\eta+\sigma}} \left(S_{\kappa(i)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \mu^n\left(s_j^n\right)^{\frac{\sigma(1+\eta)}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[1+\tau_{\kappa(i)}^{-\sigma}\right]^{\frac{1+\eta}{\eta+\sigma}} \left(S_{\kappa(i)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}, \tag{2.24}$$

$$s_i^y = \frac{\mu^n\left(s_i^n\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(i)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \mu^n\left(s_j^n\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(j)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}. \tag{2.25}$$

where $\mu^n(s) = \left[\frac{1+\eta}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)s\right]^{-1}$.

Proof. See Appendix .1.3. □

There are cases, however, where labor markets are insulated from trade shocks. This is the case when firms in a labor market produce a unique type of product and so the shock do not have a differential effect between firms in a labor market.[24] Payroll-shares are independent of any trade related variable, and therefore international trade cannot correct any distortions coming from firm's labor market power.[25] We summarize this result in Corollary 2.2.1.

Corollary 2.2.1. When firms in a labor market produce a single type of product, i.e. $\kappa(j) = \kappa(i)$ for all $i, j \in \mathcal{M}$ with $\ell(i) = \ell(j) = l$, the joint distribution of $\left\{\mu_i^n, s_i^n, z_i\right\}$

23. Import competition in product markets is captured by λ_k. When $\sigma = \epsilon$ the system of equations determining the values of s_i^n and s_i^y are independent of λ_k

24. We say that markets are overlap when firms in a labor market produce a unique type of product, and therefore compete against the same set of firm in both the product and labor market. We come back to this point in the quantitative section, Section 2.4.

25. Note that in labor markets where firms produce only one type of product equilibrium payroll-shares can be solved in equilibrium independent of the economy aggregates. In this cases, firm-level payroll shares satisfy the block recursive property of Berger et al. (2022a) in an open economy context.

in market l is determined under market equilibrium. The distribution of markdowns is independent of trade related variables. Therefore, trade cannot affect the competitiveness of labor market l.

Proof. It follows from Proposition 2.2.2. □

Market Competition, and Firm-level Market Power. In the model, firms in product markets compete against firms that operate in different labor markets. At the same time, firms in a labor market produce different types of goods, and so in the model, product and labor markets do not overlap with each other. In this type of situations, the amount of competition a firm faces in the labor (product) market becomes an attribute of the firm when competing in product (labor) markets. In Appendix .1.5 we show how the amount of competition in labor markets shape the allocation of resources in product markets, with firms located in less competitive labor markets capturing a larger share of the product market relative to their more productive competitors. This is because the lower level of competition allows the firm to charge higher markdowns and therefore produce at a lower cost despite having a smaller productivity level. Therefore, the partial overlap between markets creates an interesting effect where not necessarily the most productive firm captures the largest share. This distinguish our set-up from previous papers that have worked with the Atkeson and Burstein (2008) structure which predicts a correlation of one between productivity and market share.[26]

2.3 Data

We now describe the data we use. First we give a description of the Australian dataset. Because in the model, and therefore for the quantitative exercise, the definition of product

26. In Section 2.3 we show that partially overlapping markets in Australia is predominant feature in the data.

and labor markets is crucial we then explain how we construct these markets. We highlight facts about the degree of interconnectedness between product and labor markets, what we call *market overlap*, and facts about producer concentration in this data that are crucial for the calibration and the quantitative implications of the model.

2.3.1 Dataset

We use the Business Longitudinal Analysis Data Environment (BLADE) a product of the Australian Bureau of Statistics (ABS). BLADE is an economic data tool that combines tax, trade, and intellectual property data sourced from surveys conducted by the ABS Business Register and the Australian Taxation Office (ATO).[27]

Type of Activity Units. BLADE collects data on all active businesses in Australia between 2001 and 2019 and reports it at two different levels of aggregation. The first level is a firm or enterprise group defined according to Australia's Corporation Law and is standard to most datasets containing micro-level data. Enterprise Group are at the level at which financial and balance sheet accounts are maintained and from which a consolidated financial position can be derived. The second level of aggregation is called *Type of Activity Unit* or TAU and are related to the concept of establishment defined by the System of National Accounts. The main difference between the two is that producing units in BLADE are not restricted to a single physical location, and therefore there are cases in which multiple establishments are aggregated into a single TAU id. Whether a TAU is one or multiple establishments depends on whether the enterprise group can report productive and employment activities via a minimum set of data items such as sales, wages and salaries, and expenditures. For most cases a TAU id correspond to a single establishment, and for that reason in the paper

27. See Hansell and Rafi (2018) for a short overview of BLADE.

we use the word *establishments* to refer to BLADE *TAUs* id.

Information collected by BLADE. ATO requires all businesses registered for good and service taxes (GST) to submit a *Business Activity Statements* (BAS) where they report information on total sales, export sales, capital and non-capital purchases, and wages, salaries and other payments. We complement this information with the *Business Income Tax* (BIT) section of BLADE which provides data on wealth from balance sheet information, such as assets, liabilities, debts and the like. Data on firm-level employment is recovered from the *Pay As you Go* (PAYG) unit of BLADE which uses businesses income tax withholding to recover the total number of workers employed (head-count) and an estimate of the number of full-time equivalent employees. Finally, we use the *Business Location* unit and the *Indicative data items* sections of BLADE to recover information on business location and their industry classification. Both of these variables are crucial to define the boundaries of product and labor markets in Australia and to assign firms to each of these markets, which in turn determines the level of competition.

Import Shares. We supplement the data in BLADE with detailed import data from Australia's Department of Foreign Affairs and Trade (DFAT). DFAT reports sectoral imports at the 10-digit level of the Australian Customs Tariff classification. We use a correspondence index, provided by ABS, to translate those into the *Australian and New Zealand Standard Industry Classification* from 2006 (ANZSIC2006), which is the industry classification reported in BLADE. This match gives us dissagregated import penetration ratios for each product market category.

2.3.2 Product and Labor Markets.

Defining Product and Labor Markets. The amount of market concentration in the Australian data is crucial for our model's quantitative implications: They determine the amount of competition within each market, and therefore the distribution of resources across firms. In order to compute how much concentration there is in the data we not only need to define what constitutes a market, but also assign firms to each of them.[28]

Our definition of labor and product markets combines geographical information of Australia with information on the type of goods available in the country. The assignment of firms to markets uses instead the firm's location and industry classification contained in BLADE . To be more specific, we define a labor market as a region-sector pair. This definition is meant to capture two things. First, that certain skills are required in order to produce a particular type of good, and therefore workers cannot supply their labor services to any industry. Second, labor is not perfectly mobile and so even workers with the necessary skills cannot offer their services to firms located in distant places.[29]

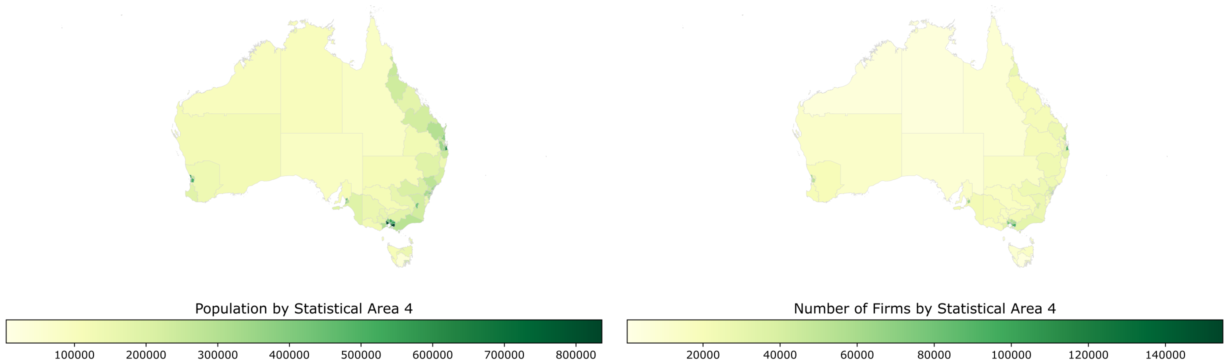
The choice of each component is not innocuous. We use a particular characteristic of our data to guide our choice. The Australian Statistical Geography Standard (ASGS) is a classification of Australia into a hierarchy of statistical areas, each designed to analyze a broad range of social, demographic, and economic statistics. The Statistical Area Level 4 (SA4s) classification are the largest sub-state regions in the ASGS and they are meant to represent labor markets or groups of labor markets within each state and territory.[30] They

28. An important dimension of heterogeneity across firms in the model is the identity of the product and labor market where the firm operates

29. For example, Manning and Petrongolo (2017) estimate that the attractiveness of jobs decays sharply with distance.

30. SA4s are designed to represent the labor markets of the largest regional cities such as Wollongong, Bendigo and Townsville. In outer regional and remote areas, labor markets tend to be small and localized around regional towns. SA4s in these areas represent aggregations of these labor markets based on travel to work interactions, as well as industry and regional similarities. Because SA4 are constructed with a minimum population requirement they are closer in spirit to labor market ares rather than commuting zones in the United States.

Figure 2.1: SA4s by Population and Number of Firms



Notes: The figure plots the different Statistical Area Level 4 (SA4) of the Australian Statistical Geography Standard (ASGS) classification. SA4s are designed to represent the labor markets of the largest regional cities. The left-hand panel plots each SA4s by its population size, while the right-hand side plots SA4s by the total number of firms. Most of the economic activity, both in terms of population an number of firms, is concentrated around the eastern side of Australia where the cities of Brisbane, Sydney, and Melbourne are located.

are designed to incorporate both where people live (labor supply) and where people work (labor demand), and therefore we use them for the regional component in our definition of labor markets. There are 87 SA4s covering the whole of Australia without gaps or overlaps.[31] Figure 2.1 displays the division of Australia into SA4. Most SA4s have a population above 100,000 people. In cities, SA4s tend to have larger populations of 300,000 to 500,000 people (left-panel), and in this cities is where most of the firms are located (right-panel). The sectoral component in our definition is a *Group* of the Australia and New Zealand Standard Industry Classification from 2006 (ANZSIC06). There are 55 Groups in ANZSIC06 that correspond to a classification of industries at the 3-digit level.

Our definition of product markets does not depart from the literature. As in Edmond et al. (2015) and Gaubert and Itskhoki (2021) we base our definition of product markets on good's characteristics only. A product market is a 4-digit industry (or *Class*) of the ANZSIC06

31. Australian Statistical Geography Standard (ASGS) Edition 3.

classification.[32]

Characteristics of Product and Labor Markets. We restrict our sample to the manufacturing industry.[33] In total, we divide the Australian economy into 143 distinct product markets, and around 3569 labor markets.[34] The median labor market in Australia has around 12 firms competing with each other for workers, while the median product market has around 493 firms.

Strong Market Interconnectedness. We build a measure of market overlap to understand the degree of interconnectedness across markets in the Australia data. Our notion of market overlaps measures the extent to which the set of firms competing for workers in labor markets coincide, partially or completely, with the set of firms competing in product markets, the relevant notion to understand the propagation of trade shocks across seemingly unrelated product markets.

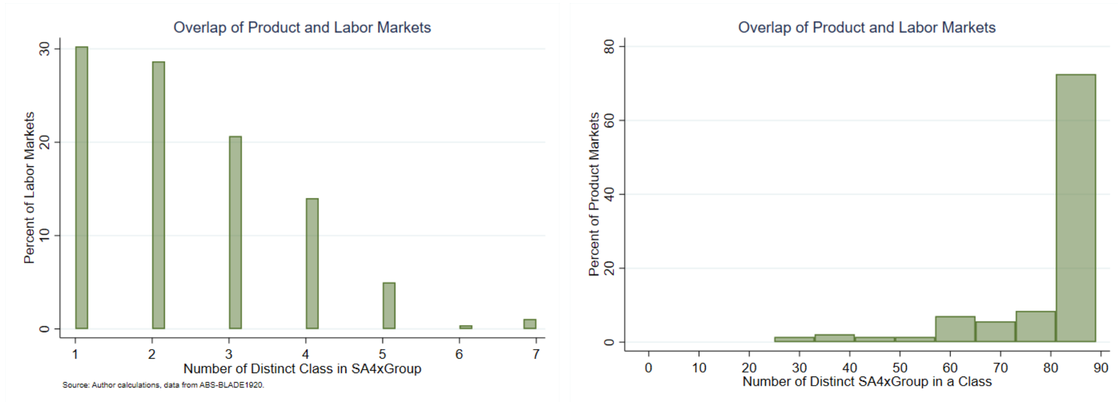
We say a labor market *overlaps* with a product market when all firms that operate in a labor market are also part of the same product market. Instead, we say that a labor market partially overlaps with a product market when the firms in a labor market belong to multiple product markets. Note that when a labor market overlaps with a particular product market we can infer the product the firm produces by looking at which labor market this firms belongs to. We can similarly define a notion of market overlap based on product markets, and say that a product market overlaps with a labor market when all firms selling a particular product operate within a single labor market. When both product and labor markets overlap

32. The 4-digit classification is the lowest level of aggregation at which we observed both the firm level data in BLADE and the import data in DFAT.

33. Manufacturing businesses correspond to the Sub-Divisions 11 to 25 of ANZSCI06. These industries account for 6% of Australia's GDP, and around 7% of aggregate employmeny.

34. There are 4785 possible SA4 times Group combination. In the data, only 3569 of those 4785 have a positive number of establishment.

Figure 2.2: Measuring Market Overlap in Australia - I



Notes: In the left-panel we plot the share of labor markets where the firms that operate in there belong to one, two, three and so forth, ***distinct product markets***. The right-panel plots the share of product markets where the firms that operates in there are located in only one, two and so forth labor markets. We say markets overlap when all firms that compete for workers (consumers) in a labor (product) market also compete for consumers (workers) in the product (labor) market.

with each other we say there is *complete overlap* between both markets.

We measure the amount of overlap by counting the total number of *distinct product markets* that we observe in the data operate within a particular labor market. The left panel in Figure 2.2 plots the share of labor markets with one (i.e. market overlap), two, three, and so forth distinct product markets.The resulting distribution shows high levels of market overlap between labor and product markets in the Australian economy. Roughly 70% of labor markets provide workers to at most two product markets (30% to only one product market). Similarly, the right panel in Figure 2.2 plots the share of product markets that are produce within one (i.e. market overlap), two, three, and so forth distinct labor markets. The figure shows that the amount of market overlap is even stronger with more than 90% of product being produced in multiple labor markets.

A potential problem with this figure is that it does not take into account for example cases where multiple product markets hire labor from a single labor market, but a significant share of the market is captured by only one sector. In this case, we would ruled out market overlap when in fact there is. To overcome this issue we also compute a measure of entropy within each category where we previously identified partial overlap. The entropy is defined for the event: *firm* $i \in \mathcal{M}$ *belongs to product market* $k \in \mathcal{K}$, *given that* i *belongs to labor market* $l \in \mathcal{L}$ *where there are* x *distinct products being produced*; and it is given by,

$$H\left(K \middle| L=l, X=x\right) = -\frac{1}{\log x} \sum_{k \in \mathcal{K}} p\left(K=k \middle| L=l, X=x\right) \log p\left(K=k \middle| L=l, X=x\right), \quad (2.26)$$

where $H(\cdot)$ denotes the entropy, $p(\cdot)$ the probability of the event, $\mathcal{M}$ denotes the set of firms, and $\mathcal{K}$ and $\mathcal{L}$ denotes the set of product and labor markets, respectively.[35] Note that $H \in [0,1]$ and takes the value zero when there is no uncertainty about the realization of the event. On the other hand, the entropy equals one when the uncertainty is maximum, and this happens when the events $\{K=k|\, L=l, X=x\}$ has equal probability across the different values of k.

Based on Figure 2.2 the concern of miss measuring market overlap is greater for labor markets and for that reason we only show here the entropy for labor markets. Figure 2.3 plots for each x, the distribution of $H\left(K|\, L=l, X=x\right)$ across labor markets for a given count of distinct product markets (x). The figure shows that our notion of market overlap is robust to this type of concern, and that market interconnectedness is strong in the data.

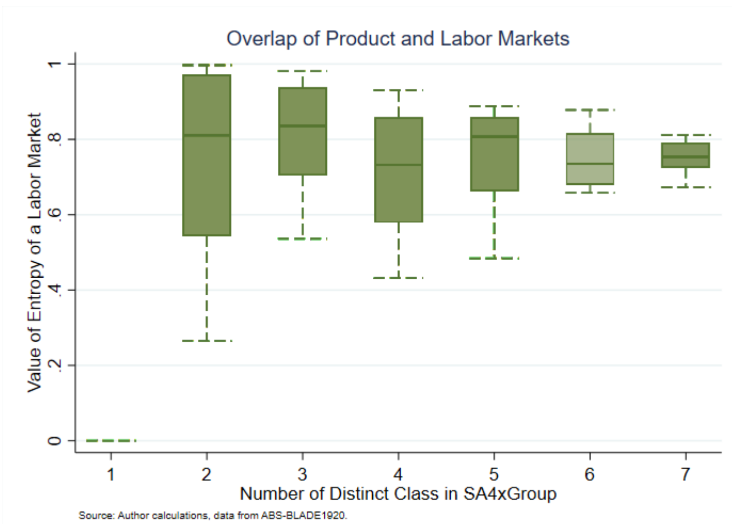
2.3.3 Concentration facts

The amount of concentration of sales and payrolls in product and labor markets is crucial for the model's quantitative implications. In this section we highlight facts about concentration for the manufacturing establishments in Australia.

Concentration within Labor Markets We measure a establishment's labor market share

35. We estimate $p(\cdot)$ with the proportion of firms in labor market l that produce goods in product market k.

Figure 2.3: Measuring Market Overlap in Australia - II



Notes: The figure shows, for a given count of distinct product markets x the box plot of the distribution of the entropy $H\left(K|\,L = l, X = x\right)$ across labor markets l. Entropy is defined as in equation (2.26). The lower and upper value in each box plot represent the 10th and 90th percentile of the distribution instead of the usual minimum and maximum value because of data disclosing rules by ABS. The figure shows that cases where partial overlap was correspond with cases where in a labor market a non trivial share of firms produce different products.

Table 2.1: Concentration Facts in BLADE

	Product Markets	Labor Markets
Within market concentration		
Mean inverse HHI	13.91	4.55
Median inverse HHI	3.42	2.52
Mean top share	0.555	0.576
Median top share	0.475	0.54
Distribution of market shares		
Mean share	0.007	0.158
Median share	0.001	0.089
p75 share	0.005	0.221
p95 share	0.038	0.475
SD share	0.033	0.205
Across markets concentration		
p10 inverse HHI	1.06	1.02
p50 inverse HHI	3.42	2.52
p90 inverse HHI	39.51	10.5
p25 top share	0.196	0.318
p50 top share	0.475	0.54
p75 top share	0.971	0.868
Market characteristics		
Number of markets	143	3569
p25 number of establishments	133	5
p50 number of establishments	518	12
p75 number of establishments	1786	31

Notes: The table shows descriptive statistcs on the amount of concentration in product and labor markets for the Australian manufacturing establishment, in 2018/2019 fiscal year. Observations in each labor market (product market) are weighted by the market's share of aggregate payrolls (aggregate sales).

by its share of total payroll within a Statistical Area 4 times a 3-digit sector of ANZSIC06. The column label *Labor Market* in Table 2.1 shows that establishments within labor market are highly concentrated. The top producer has on average a payroll share of of about 58 percent. The median inverse Herfhindahl index (HHI) measure of concentration is around 2.5 and 4.6, which implies that the average labor market is as concentrated as they would be if the market were divided evenly across 5 establishments. It is also much lower than the 12 number establishments that we observe in the data.

The distribution of labor market shares is skewed to the right and extremely fat-tailed. The median market share of a establishment is 9 percent while the average market share is 16 percent. While the ninety-fifth percentile accounts only for 48 percent of total payrolls the ratio of the ninety-ninth percentile to the median is well above the threshold of 5.6.[36]

The overall pattern that emerges is one of very strong concentration. Although quite a few establishments operate in any given labor market, most of these establishments are small and a few large establishments account for most of the market's payroll.

Concentration within Product Markets. The column label *Product Market* in Table 2.1 repeats our previous analysis of market concentration but looking at product instead of labor markets. We measure a establishment's market share by its share of domestic sales revenue within a given 4-digit sector of ANZSIC06. The pattern that emerges from the data is also one of concentrated product markets. The average 4-digit industry has a large number of establishments competing against each other, 518 establishments in product markets versus 12 establishments in labor markets. However, despite the large number of establishments in any given product market, the top producer has a market share of about 56 percent.

This disparity between the total number of establishments and the size of each of these establishments in the product market explain the low numbers for the inverse Herfhindahl Index. We find that the average inverse Herfhindahl index (HHI) measure of concentration is about 14 which implies that the average product market is as concentrated as they would be if the market were divided evenly across 14 establishments, much lower than the 493 producers that operates in a typical industry.

The overall pattern is one of strong concentration in product markets, with more concentrated labor markets. The difference between the levels of concentration in product and labor markets is in part explained by the less restrictive definition of product markets that do not

36. ABS disclosure rules prevent us from reporting the value of the ninety-ninth percentile.

consider any geographical restriction and therefore includes firms all around Australia.

2.4 Quantifying the model

To understand the role that international trade plays reducing distortions in product and labor markets we need to estimate our model. Table 2.2 summarizes the parameters in the model that need to be estimated to compute the pro-competitive gains from trade. On the production side we have five parameters. The two elasticity of substitution in the production technology of the final good: the elasticity of substitution across input varieties from a particular product market σ and the elasticity of substitution across bundles of output from different product markets ϵ. The other three parameters correspond to the two types of trade costs, the fixed costs of exporting f_k and the iceberg cost τ_k, and the firm-level technology z_i. There are also four parameters in the household's utility function. The disutility from labor $\bar{\varphi}$, the Frisch elasticity of labor supply φ, and the two elasticities of substitutions: across firms within a labor market θ, and across labor markets η.

Additionally, we need to recover the labor supply and product demand shifters at the firm $\{\xi_{n,i}, \xi_{cy,i}\}$, and market level $\{\upsilon_{n,l}, \upsilon_{y,k}\}$, and the distribution of productivities for the Australian producers, z_i. Finally, we need to define the boundaries of each product and labor market, as well of the composition of firms operating in each of those markets.

Our strategy for estimation treats each firm in the data as a firm in the model, and therefore rationalizes the observed data as an equilibrium outcome of our theoretical environment. To recover the parameters we use the model to derive a set of equation that relates the desired parameters with a set of moments that we can measure in the data. Parameters are estimated by inverting this system. However, to invert this system we required knowledge of other primitives such as the market structure and the elasticities for which we rely on BLADE and

Table 2.2: Parameters in the model

	Parameter	Source	Data	Model
Elasticities				
Elasticity of subs. labor market	$\eta = 1.02$, $\theta = 0.8$	Felix (2021)		
Elasticity of subs. product market	$\sigma = 2.5$, $\epsilon = 1.5$	Median sales to payroll ratio	3.369	3.313
Frisch elasticity labor supply	φ	Chetty (2012)	–	0.25
Disutility of labor	$\bar{\varphi}$	Normalized	–	1
Market Structure				
Number of domestic firms	$\mathcal{M}$	BLADE	34092	34092
Number of labor markets	$\mathcal{L}$	BLADE	3569	3569
Number of product markets	$\mathcal{K}$	BLADE	142	142
Mapping firms to markets	κ and ℓ	BLADE		
Preference Shifters				
Household pref	$\xi_{n,i}$, and $\upsilon_{n,l}$	Model Inversion		
Aggregate output	$\xi_{y,i}$ and $\upsilon_{y,k}$	Model Inversion		
Technology				
Productivity	z_i	Model Inversion		
Trade costs	τ_k and $f_{x,i}$	Policy Parameters		

Notes: The table presents a list of the parameters required to compute the Pro-competitive gains from trade. The last column, labeled *Source*, specificies the type of information used to recover the parameter. BLADE indicates that it is recover directly from data using the Business Longitudinal Analysis Data Environment.

estimates from the literature.[37] We use BLADE data, discussed in the previous section, to construct the set of moments for model. In particular we use information on firm-level sales, payroll, and wages, total imports by sector,and value of gross output.

The methodology we choose it does not come without costs. However, the advantages of this method outweights its constraints. In particular, it does not require to impose ad-hoc assumption on the fundamentals of the economy for either Australia or the rest of the world. Furthermore, since we treat the data as an equilibrium of the model we do not have to solve for the equilibrium in the open economy, which is not a trivial task, but only solve for the closed economy version of our model. The main issue with this approach is that it limits the type of questions that can be answered. For example we cannot consider counterfactual cases

37. In Section 2.5 we perform robustness to this choices.

with more or less degrees of openness to trade than the observed equilibrium, but instead look at limiting cases such as autarky.

2.4.1 Parametrization

Elasticities. We normalize the disutility of labor to $\bar{\varphi} = 1$ and we set the Frisch elasticity to $\varphi = 0.25$, a standard value in the literature (Chetty, 2012). For the four elasticities of substitutions our baseline results use estimates in the literature and information from BLADE, and then provide robustness to these choices.[38] We take the elasticity of substitution in labor markets from Felix (2021) which implies a within-market elasticity of $\eta = 1.02$ and an across-market elasticity of $\theta = 0.8$.

We chose the two elasticity in the product market so that the model matches the median value of the sales to payroll ratio in BLADE. The low levels of concentrations in sales shares implies that ϵ does not influence this ratio and therefore we set it to $\epsilon = 1.5$, a standard value in the literature (Atkeson and Burstein, 2008; Edmond et al., 2015; Amiti et al., 2019a; Gaubert and Itskhoki, 2021; Deb et al., 2022). In the data, the median firm has sales that are three times larger than its total labor costs. For the model to match this moment, it requires a low value of $\sigma = 2.5$. Our value of σ differs from those used in the previously cited papers, and therefore in Section 2.5 we perform robustness analysis to our choices of the parameters σ and η.[39]

38. Appendix .5 we discuss how the 4 elasticities of substitution could be estimated using our firm-level data from Australia. However, this is still work in progress.

39. We do not perform robustness analysis to ϵ and θ because there is less disagreement in the literature around these values. As we mentioned there is a long list of paper in the trade and macro literature that find an elasticity close to one for the upper nest in the CES goods aggregator. Similarly, Berger et al. (2022a), Felix (2021) and Deb et al. (2022) also finds that the elasticity in the upper nest is close to one for the CES aggregator for labor. In all these papers, the controversy is around the exact value of the elasticities in the inner nest.

Market Structure. We treat each manufacturing establishment in BLADE as a firm in the model. We use the definition of product and labor markets outlined in Section 2.3.2. Therefore, we calibrate the model to include around 30,000 manufacturing establishments operating in 143 different product markets and 3569 distinct local labor markets.

The dimension of the equilibrium is considerable given the granularity of the model which require us to solve for individual firm variables. However, in Appendix .3.1, we show that solving for the equilibrium can be recast in terms of firm-level and market-level shares. This does not reduce the dimension of the problem, but constraints the search to a vector of numbers strictly between zero and one. Furthermore, once the equilibrium is written in terms of these shares, a nested fixed point algorithm can be implemented in which given market-level shares we can solved for the equilibrium values of the firm-level shares, similar to Proposition 2.2.1. Then, given the firm-level shares we can aggregate them up and solve for the market-level shares. These properties of the model makes solving for the equilibrium computationally tractable.

The nested fixed point algorithm, builds on the works of Berger et al. (2022a), with one substantial difference. In Berger et al. (2022a), the equilibrium shares can be solved market by market. Here, instead, we have to solved for all firm-level shares jointly because markets are partially overlapped.[40]

2.4.2 Model Inversion: Firm-level productivity and demand and supply shifters

The last set of parameters that we need to recover are the firm-level productivity, and the product demand and labor supply shifters. We show here how to recover those given values

40. The solution technique outlined in Appendix .3.1, proved useful for solving even larger systems. The original version of this paper used to calibrate the model to the entire Australia economy with almost 700.000 firms, around 500 product markets, and close to 7000 labor markets.

of the parameters $\{\eta, \theta, \sigma, \epsilon, \varphi, \bar{\varphi}\}$, and a partition of the economy into product and labor markets.

Notation. Moving forward it is important to define some notation for the set of observables used to invert the model. As in the model, we use s_i^n to denote firm i's payroll share, and S_l^n for its market-level counterparts. In the same spirit, we use w_i to denote the average wage paid by firm i. We use d_i for firm i's domestic sales and s_{Hi}^y for the firm sales share. Finally, we use M_k to denote the total expenditure in imported inputs of k-type products, and S_k^y for the expenditure share in goods from product market k, this includes both domestic and imported goods.

Model inversion. Each of these observables have a counterpart in the model and are useful to pin-down the fundamentals for Australia. The information contained in firms' wages and their payroll shares allow us to recover the shifters in the labor supply equation for each firm,

$$\xi_{n,i} = \frac{s_i^n}{\left[\sum_{\{j \in \mathcal{M}: \ell(j)=\ell(i)\}} \left(\frac{w_i}{w_j}\right)^{1+\eta} s_j^n\right]}, \qquad \forall i \in \mathcal{M}, \tag{2.27}$$

and labor market,

$$\upsilon_{n,l} = \frac{S_l^n}{\left[\sum_{l' \in \mathcal{L}} \left(\frac{W_l}{W_{l'}}\right)^{1+\theta} S_{l'}^n\right]}, \qquad \forall l \in \mathcal{L}. \tag{2.28}$$

In the same way, information on firms' prices and their share in the domestic economy are informative of the product demand shifters for each firm,

$$\xi_{Hy,i} = \frac{s_{H,i}^y / p_{H,i}^{1-\sigma}}{\sum_{\{j \in \mathcal{M}: \kappa(j)=\kappa(i)\}} \frac{s_{H,j}^y}{p_{H,j}^{1-\sigma}} + \sum_{\{j \in \mathcal{M}^*: \kappa(j)=\kappa(i)\}} \frac{s_{F,j}^y}{p_{F,j}^{1-\sigma}}}, \tag{2.29}$$

and product market,

$$\upsilon_{y,k} = \frac{S_k^y / P_k^{1-\epsilon}}{\sum_{r \in \mathcal{K}} \frac{S_r^y}{P_r^{1-\epsilon}}}, \qquad \forall k \in \mathcal{K}. \tag{2.30}$$

Finally, the information on firms' shares, firms' wages, and firm's prices inform us about the firm's productivity level,

$$z_i = \frac{\mu^y \left(s_{H,i}^y\right)}{\mu^n \left(s_i^n\right)} \frac{w_i}{p_{H,i}}, \qquad \forall i \in \mathcal{M}. \tag{2.31}$$

However, we lack information on firm prices $(p_{H,i}, p_{F,i})$, and on foreign firms' shares $(s_{F,j}^y)$, and therefore we cannot identify all the parameters. Put it differently, there is more than one combination of firm's prices and demand shifters that are consistent with the same set of firm-level sales shares. One of those combinations has equilibrium prices that equal one for all sold goods in Australia, we labeled this economy the *Synthetic Australian Economy.*

The Synthetic Economy (se). The synthetic economy consists of a version of our original model except that they differs on the values of the firm-level technology parameters and on the values of the demand and supply shifters. These parameters are such that in the equilibrium of the synthetic economy all firm-level prices equal one. It follows that product market price indexes must also equal one,

$$P_k = \left[\sum_{i \in \mathcal{M}: \kappa(i) = k} \xi_{Hyi} + \sum_{i \in \mathcal{M}: \kappa(i) = k} \xi_{Fyi} \right]^{\frac{1}{1-\sigma}} = 1.$$

The second characteristic of our synthetic economy is that despite having unit prices, it still rationalizes the same set of observables as our original economy, and therefore it generates the same distribution of sales and payroll shares at the firm and market level.

Since the synthetic economy rationalizes the same data and in equilibrium prices equal one the parameters of the synthetic economy are uniquely determined by equations (2.27) to (2.31). We use upper script *se* to denote variables in our synthetic economy, and so we let $\left\{\xi^{se}_{n,i}, \xi^{se}_{Hy,i}, z^{se}_i\right\}_{\forall i\in\mathcal{M}}$, $\left\{\upsilon^{se}_{y,k},\right\}_{k\in\mathcal{K}}$, and $\left\{\upsilon^{se}_{n,l}\right\}_{l\in\mathcal{L}}$ be the recovered parameters in the synthetic economy. They satisfy,[41]

$$\begin{aligned}
\xi^{se}_{n,i} &= \frac{s^n_i/w_i^{1+\eta}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} s^n_j/w_j^{1+\eta}}, \qquad \forall i \in \mathcal{M},\\
\upsilon^{se}_{n,l} &= \frac{S^n_l/W_l^{1+\theta}}{\sum_{\{l'\in\mathcal{L}\}} S^n_{l'}/W_{l'}^{1+\theta}}, \quad \forall l \in \mathcal{L},\\
\xi^{se}_{Hy,i} &= s^y_{H,i}, \qquad \forall i \in \mathcal{M},\\
\upsilon^{se}_{y,k} &= S^y_k, \qquad \forall k \in \mathcal{K},\\
z^{se}_i &= \frac{\mu^y_i\left(s^y_{Hi}\right)}{\mu^n_i\left(s^n_i\right)} w_i, \qquad \forall i \in \mathcal{M}.
\end{aligned}$$

We note that, $\xi^{se}_{n,i} = \xi_{n,i}$ and $\upsilon^{se}_{n,l} = \upsilon_{n,l}$, because they are all constructed with the same data on firm wages and payroll shares.

Because all the relevant parameters for Australia are identified for the synthetic economy, it is possible to use the estimated parameters to compute the autarky equilibrium of this economy. The autarky equilibrium of the *se* economy is different from the autarky equilibrium we would like to compute. However, the two are related as stated in Proposition 2.4.1 where we used $\tilde{x}$ to denote variables in the autarky equilibrium. In particular, the two economies generate the same distribution of firm-level shares in autarky, and the same welfare levels.

Proposition 2.4.1. If $\left\{\tilde{w}^{se}_i, \tilde{p}^{se}_i, \tilde{s}^{n,se}_i, \tilde{s}^{y,se}_{H,i}\right\}_{i\in\mathcal{M}}$ is part of an autarky equilibrium in our artificial *se* economy, then, $\left\{\tilde{w}^{se}_i, \frac{z^{se}_i}{z_i}\tilde{p}^{se}_i, \tilde{s}^{n,se}_i, \tilde{s}^{y,se}_{H,i}\right\}_{i\in\mathcal{M}}$ is an equilibrium under autarky of the original economy. Furthermore, the synthetic economy and the true economy have the same welfare level in the autarky equilibrium.

41. The firm-level technology recovered in the synthetic economy is related to the true parameter z_i, by the equation,

$$z^{se}_i = p_{H,i} z_i,$$

and therefore measures the firm-level revenue productivity or TFPR (Hsieh and Klenow, 2009).

Proof. See Appendix .2.2. □

Furthermore, because the two economies rationalize the same initial equilibrium, the synthetic economy has the property that its autarky equilibrium predicts the same counterfactual changes in endogenous variables as the true economy, given the same proportional change in parameters. Therefore, although the two economies do not generate the same autarky equilibrium, this equilibrium is useful because it identifies counterfactual changes in the equilibrium of the original economy.[42] By solving the equilibrium of the synthetic economy we can study the welfare and efficiency gains, as well as the change in the distribution of market power, that result from closing Australia to international trade. We summarize this result in Proposition 2.4.2.

Proposition 2.4.2. Given the elasticity parameters $(\sigma, \epsilon, \eta, \theta, \varphi, \bar{\varphi})$, assignment functions $\kappa(\cdot)$ and $\ell(\cdot)$, and data on (i) firm level wages, (ii) firm-level payroll and domestic sales, and (iii) and expenditure shares on sectoral imports, the pro-competitive gains from closing the economy to international trade are identified.

Proof. See Appendix .2.2. □

2.4.3 Market Power Distribution

Table 2.3 reports moments of the distribution of firms' markups and markdowns in our model. We compare these to an economy that is identical except that we shut down international trade.

Initial Distribution. Panel A of Table 2.3 reports moments of the unconditional distribution pooling over all firms across the different product and labor markets in Australia. The benchmark model implies an average markup of 1.669, a median markup of 1.6669, and a standard deviation of log markups of 0.011. The low concentration in sales shares across

42. A similar procedure for identification is used in Adão et al. (2022).

Table 2.3: Market Power Distribution: Trade versus Autarky

	Markups (μ_i^y)		Markdowns (μ_i^n)		Market power (μ_i^y/μ_i^n)	
	Australia	Autarky	Australia	Autarky	Australia	Autarky
A. Firm-level moments						
Unconditional distribution						
Mean	1.669	1.671	0.497	0.497	3.358	3.362
Median	1.667	1.667	0.503	0.503	3.314	3.314
p75	1.667	1.668	0.504	0.504	3.347	3.350
p90	1.669	1.671	0.505	0.505	3.469	3.477
p95	1.672	1.677	0.505	0.505	3.637	3.651
Std. (log)	0.011	0.0148	0.029	0.029	0.033	0.036
$\log(p95/p50)$	0.002	0.003	0.001	0.002	0.040	0.042
B. Market-level moments						
Across-market distribution						
Mean	1.771	1.743	0.488	0.488		
Median	1.682	1.689	0.498	0.498		
p75	1.717	1.754	0.826	0.823		
p90	1.853	1.907	0.501	0.501		
p95	2.872	2.051	0.502	0.502		
Std. (log)	0.126	0.063	0.031	0.030		
$\log(p95/p50)$	0.232	0.084	0.007	0.008		
C. Aggregate Implications						
Average import share					0.330	0
Fraction of exporters					0.091	0
Gains from trade					–	-1.921
Gains from trade (planner)					–	-0.757

Notes: The table shows descriptive statistcs on the distribution of markups, markdowns, and total market power (the ratio of markups to markdowns) for the Australian manufacturing industry. The columns labeled *Australia* reports statistics for the observed trade equilibrium in the 2018/2019 fiscal year, while the columns labeled *Autarky* reports the same statistics at the counterfactual autarky equilibrium. Market-level markups are computed as the ratio of the quantity-weighted average marginal cost to the average price of the firms within a product market. Market-level markdowns are computed as the ratio of the employment-weighted average marginal revenue product of labor to the average wage of the firms within a labor market.

firms in product markets implies that the distribution of markdowns has low dispersion and that most of the observations are centered slightly above the theoretical lower bound $\sigma/(\sigma-1) = 1.6667$. In fact, the ninety-fifth percentile of the markup distribution equals 1.672 almost half of the markup a pure monopolist would charge in the model, $\theta/(\theta-1) = 3$.

The distribution of markdowns imply by the model also features low levels of dispersion, the standard deviation of log markdowns is 0.029 slightly above the dispersion in markups. The model imposes bounds on the distribution of markdowns: A monopsonist charges the highest markdown which equals $\theta/(\theta+1) = 0.444$, while at the other extreme we have atomistic firms with markdowns equal to $\eta/(\eta+1) = 0.505$. Since in our calibration η and θ are very close to each other the model predicts a low dispersion in markdowns despite the high levels of concentration in payroll shares within labor markets. The concentration in labor markets affects, however, where this distribution is centered. The average markdown equal to 0.4974, the median is 0.5031, and the ninety-fifth percentile is 0.5048 close to the monopsonist limit.

However, as we show in Section 2.2.4 what matters for the distribution of resources across firms is not the individual distribution of markups or markdowns, but the distribution of the ratio of the two. We refer to the ratio of a firm markup to its markdown as the total amount of market power the firm exerts, the last columns of Table 2.3 reports moments of this distribution. The model implies an average market power of 3.358, a median value of 3.314, and a standard deviation of log market power of 0.033. The distribution of market power exhibits more variation than the markups and the markdowns alone, and it is this dispersion what causes the misallocation of resources across firms.

We aggregate firm-level markups (markdowns) at the market-level using the sales (payroll) shares in the market as weights. Definition 2.4.1 makes explicit our aggregation procedure.[43]

43. We do not observe firm shares of foreign producers selling in Australia, and therefore we cannot use the model to back out the markups they charge to consumers. For this reason, our market-level markup only takes into account domestic producer. Since our counterfactual looks at the autarky equilibrium, where there are zero foreign sells, our measure of product market markup is still informative.

Panel B of Table 2.3 reports moments of the distribution of markups and markdowns across the different product and labor markets, respectively. Because large producers charge higher markups (markdowns), the average market-level markup (markdown) is 1.7707 (0.4878) much higher than the simple average. These market-level measures of market power are both larger and more disperse than their firm-level counterparts. The median labor market markdown is 0.4984 as opposed to 0.5031 while while the ninety-fifth percentile is 0.5018 as opposed to 0.5048. For product market markups the difference if even higher. The median product market markup is 1.6819 as opposed to 1.6669 while the ninety-fifth percentile is 2.8715 as opposed to 1.6719.

Definition 2.4.1. We let μ_l^n denote labor market l's markdown, and mu_k^y denote product market k's markup across domestic producers. The quantities μ_l^n and μ_k^y are a weighted average of their firm-level counterparts using payroll and domestic sales as weights. Formally,

$$\frac{1}{\mu_l^n} = \sum_{\{i \in \mathcal{M}:\ell(i)=l\}} s_i^n \frac{1}{\mu_i^n}, \qquad \text{for all } l \in \mathcal{L},$$

and

$$\frac{1}{\mu_k^y} = \sum_{\{i \in \mathcal{M}:\kappa(i)=k\}} s_i^y \frac{1}{\mu_i^y}, \qquad \text{for all } K \in \mathcal{K}.$$

As shown in Felix (2021), under the nested-CES structure of labor supply, this aggregation procedures implies that our market-level markdown equals the ratio between the employment-weighted average of a firm's wage and its marginal revenue product of labor. The nested-CES structure in product demand implies a similar result for the markup at the product market level, which in turn equals the ratio between the output-weighted average of a firm's price and its marginal cost of production. Furthermore, these market-level measures of market power are tightly linked with the amount of concentration in the market measured by its Herfindahl index. Proposition 2.4.3 summarize these results. Because our measured markup focus only on domestic producers the amount of concentration in the market is adjusted by the amount of import competition. The higher the level of concentration in a labor (product) market the larger it is the average markdown (markup) on wages (marginal cost). Thus, by exposing the economy more or less to international trade we affect the degree of market power in each market by inducing changes in the relative sizes of the firms.

Proposition 2.4.3. When labor supply is nested CES, and firms compete à la for Cournot for workers, the average wage markdown at labor market $l \in \mathcal{L}$ is given by:

$$\frac{1}{\mu_l^n} = \frac{\bar{r}_l}{\bar{w}_l} = 1 + \frac{1}{\theta} HHI_l + \frac{1}{\eta}\left(1 - HHI_l\right), \tag{2.32}$$

where $\bar{r}_l$ and $\bar{w}_l$ are market $l's$ employment-weighted average marginal revenue product of labor and wage, respectively, and $HHI_l = \sum_{\{i \in \mathcal{M}:\ell(i)=l\}} \left(s_i^n\right)^2$ the corresponding payroll Herfindahl index.

Similarly, when product demand is nested CES, and firms compete à la Cournot, the average markup charged by domestic producers at product market $k \in \mathcal{K}$ is given by:

$$\frac{1}{\mu_k^y} = \frac{\bar{mc}_k}{\bar{p}_{H,k}} = 1 - \frac{1}{\epsilon}\left(1 - \lambda_k\right) HHI_k^d - \frac{1}{\sigma}\left(1 - \left(1 - \lambda_k\right) HHI_k^d\right), \tag{2.33}$$

where $\bar{mc}_k$ and $\bar{p}_{Hk}$ are market $k's$ quantity-weighted average marginal cost and price, respectively, $HHI_k^d = \sum_{\{i \in \mathcal{M}:\kappa(i)=k\}} \left(s_i^y\right)^2$ is the corresponding domestic sales Herfindahl index, and λ_k is the expenditure share on imported goods.

Proof. The first part of the statement it is proven in Felix (2021). To prove the second part of the statement a similar set of steps are required. For completeness we prove the proposition in Appendix .4. □

Distribution under autarky. Consider now what happens when we shut down all international trade, which we report in Table 2.3, column labeled Autarky. The unconditional distribution markdowns remains almost invariant between the two equilibrium, while the distribution of markups is slightly shifted to the right. The average markdown is unchanged, the average markup increases from 1.669 to 1.6706, and there is a slight increase in the unconditional markup dispersion. Despite this, there is substantially more misallocation under autarky. As shown in panel C of Table 2.3, the benchmark economy implies welfare losses that are 2.5 higher than the welfare losses associated with the planning allocation. Hence the extent of misallocation is considerably worse under autarky.

As emphasized by Arkolakis et al. (2018), in several important theoretical benchmarks, the unconditional markup distribution is invariant to the level of trade costs. Therefore, if the changes in the moments of the unconditional markup distribution are used to infer the effects of trade on welfare and aggregate efficiency we can reach to wrong conclusions. While there is not an equivalent theoretical treatment for the relationship between trade and the distribution of markdowns, we show in Corollary 2.2.1 that there is a special case of our model where the same result holds true for the unconditional distribution of firm-level markdowns. What matters is the joint distribution of market power and employment across producers.

We exemplify these forces by studying what happens to the distribution of market power and employment to a particular set of firms. We look at distribution of markups for the product market that before the shock was at the 95% of the distribution of import competition across the different product markets. For future reference we label this product market $k_{\lambda_{95}}$. We also look at the distribution of markdowns across all the firms that operate in product market $k_{\lambda_{95}}$. Because firms within a product market are heterogeneous in terms of the labor market from where they hire workers, we also look at the the markdown distribution in the labor market where the top producer in market $k_{\lambda_{95}}$, measured by its sales share, operates, and the markdown distribution in the labor market where the majority of the firms in market

$k_{\lambda_{95}}$ operate.[44]

In our benchmark model, closing the economy to trade dramatically shifts the markup distribution in product market $k_{\lambda_{95}}$. As these firms face lower competition given the absense of foreign competitors they gain market share and increase the markup charged to consumers in the product market. This gives these firms an advantage in their labor markets: All of these firms also increase their markdowns on wages in their respective labor markets.

However, unlike product markets, where a significant share of the market is left for domestic producers to take, firms in labor markets are all operated by domestic producers. For this reason, after the shocks, not all shares can go up and so after closing the economy to trade there is a redistribution within the labor markets. While the most benefited firms manage to increase their size in labor markets, they do so at the expenses of their competitors. Closing the economy to international trade dramatically increases the markups of the largest producers where most employment is concentrated. As a result of this, misallocation increases resulting in larger welfare losses relative to the efficient allocation.

2.5 Gains From Trade

We focus on the gains due to a permanent reduction in trade costs and study the limit when these costs become prohibited.[45] We then ask our key question: to what extent does international trade reduce misallocation due to market power?

44. Low count of observations for each of these distributions preclude us from showing these plots.

45. This require us to solve for the autarky equilibrium. In Appendix .3.1 we discuss the relevant equations and present an algorithm to solved for the equilibrium.

Table 2.4: Welfare losses of closing the economy to international trade

	Market structure in Product and Labor Markets				
	Oligopoly Oligopsony	Oligopoly Perf. Comp.	Perf. Comp. Oligopsony	Perf. Comp.	Monopoly Monopsony
A. Substitution in product mkt.					
Baseline	1.921	1.064	1.17	0.757	1.658
$\sigma = 3$	1.586	0.789	0.967	0.556	1.325
$\sigma = 5$	1.038	0.364	0.644	0.236	0.795
$\sigma = 7$	0.829	0.208	0.515	0.109	0.594
$\sigma = 10$	0.678	0.096	0.416	0.012	0.448
B. Substitution in labor mkt.					
Baseline	1.921	1.064	1.17	0.757	1.658
$\eta = 1.5$	3.171	2.359	2.418	2.042	2.741
$\eta = 2$	4.327	3.525	3.572	3.199	3.785
$\eta = 5$	10.123	9.259	9.332	8.90	9.28
$\eta = 10$	14.627	13.69	13.803	13.313	13.621

Notes: The table reports the welfare losses of moving Australia from the observed trade equilibrium in the 2018/2019 fiscal year to the counterfactual autarky equilibrium under different classes of models nested in our main specification. The different columns specify the type of competition within product and labor markets, respectively. For example the column labeled *Perf. Comp. - Oligopsony* correspond to a version of our model with perfectly competitive product markets and oligopsonistic labor markets. The baseline calibration has within markets elasticity of substitution equal to $\eta = 1.02$ and $\sigma = 2.5$, and across markets elasticity of substitution equal to $\theta = 0.8$ and $\epsilon = 1.5$. The two panel in the table show sensitivity with respect to the substitution within labor and product markets, respectively.

2.5.1 Total Gains from Trade

We measure the gains from trade by the percentage change in household's welfare from one equilibrium to another, which includes both the change in aggregate consumption as well as the change in the total number of hours worked. As reported in Table 2.4, for our baseline calibration, closing the economy to international trade causes a 2% decrease in the household utility. The low value of welfare losses we find in our baseline calibration are associated with the low substitutability across firms within labor markets implied by the low value of the parameter η. Because firms are not perceived as close substitutes by the household, after closing the economy to trade there is not much mobility of workers across firms and so closing the economy to international trade generates low reallocative gains.[46]

The lost on welfare from moving the economy to autarky combines the effect of trade on product market power, labor market power, and on the interaction between the two. To decompose the contribution of each of these forces, we quantify the welfare losses under different classes of models that are nested in our main specification, and report the results in Table 2.4.

We first look at the case when both product and labor markets are perfectly competitive, column *Perf. Comp.*. In a perfectly competitive economy, the allocation of resources is efficient and there is no firm market power, therefore this case sets the relevant benchmark to understand the role of imperfect competition for our welfare numbers. The overall effect of trade on welfare is small in the perfectly competitive environment as it was under the full model. However, welfare losses in the former account for only 60% of the total effect. This highlights that, although the overall effects of trade are small, they are entirely the result of the imperfectly competitive nature of product and labor markets. Therefore, trade is an useful tool for correcting distortions caused by firm market power.

In columns *Oligopoly - Perf. Comp.* and *Perf. Comp. - Oligopsony* we works with a model where there are strategic complementarities in price- and wage-setting, respectively, but where we impose perfect competition in the other market. For example, the welfare losses reported under *Oligopoly - Perf. Comp.* are computed under a model where labor markets are perfectly competitive while firms are oligopolistic in product markets as in Atkeson and Burstein (2008) and Edmond et al. (2015). Both model with imperfect competition in only one market predict welfare losses from closing the economy to trade, and again these are considerably when compared to the efficient allocation. However, both of these models

46. Table 2.4 presents robustness exercises to the value of η. Welfare gains are increasing in the amount of substitutability across firms within labor markets, i.e., welfare gains are increasing in η.

underestimate the welfare losses of closing the economy to trade, on average, by 42%.[47] Furthermore, the welfare losses are systematically larger when we look at the role of labor market power alone relative to the case with only product market power, and therefore, in our data, labor market power is quantitatively more important to understand the welfare gains from trade.

What is behind these welfare numbers? The lack of foreign competition increases the amount of market power domestic producers can exert in product markets distorting relative prices. Low-productivity firms grow larger than what they should be at the expenses of their high-productivity competitor. However, contrary to common belief, this is not followed by a decrease in firm's labor market power that could mitigate the previous effect. As we show in the previous section, the distribution of markdowns remains almost invariant as some firms expand while others contract after the shock. This reallocation follows the previous pattern with low-productivity firms growing too large. which explains why we find even larger negative welfare losses from closing the economy to international trade. Product and labor market are inherently connected, and the distortions in the former amplify the distortions present in the latter doubling the welfare losses in the full model. To be precise, labor market power and its interaction with product market power account for 45 percent of the total welfare losses from the absence of trade.

Finally, in column *Monopoly - Monopsony*, we remove the strategic complementarities in price- and wage-setting. In this case, firms are monopolist in product markets and monopsonist in labor markets, and therefore, they charge a constant markup on marginal cost equal to $\mu^y = \frac{\sigma}{\sigma-1}$ and a constant markdown for wages equal to $\mu^n = \frac{\eta}{\eta+1}$. We find that in this case welfare losses are only 14 percent lower. Arkolakis et al. (2018) shows that for a large class of demand functions the gains from trade predicted by models with variables markups are equal to those predicted by models with constant markups. While the demand

47. Our sensitivity analysis does suggest, however, that the possible range for this bias goes from 6 to 90 percent depending on the values of the elasticity of substitution within markets.

system in this paper does not fall within the class of models studied by Arkolakis et al. (2018), our results suggest that this specification with constant levels of market power accounts for a significant share of the total effect. This model, however, hides the mechanism through which the gains from trade materialize, a point to which we come back in the next section.

Table 2.4 also reports sensitivity analysis to different values of the elasticity of substitution within product and labor markets. In particular, we compute the welfare gains for different combinations of the parameters η and σ. The values we consider covers the range of estimates find in Felix (2021) and Berger et al. (2022a) for the elasticities in the labor market, and Edmond et al. (2015) and Gaubert and Itskhoki (2021) for the elasticities in the product market. We find that our results are not specific to the set of parameters chosen for the baseline specification. The direction of the effects is consistent across specifications, however, the magnitude of the forces do vary with the value of these two parameter.

2.5.2 Pro-Competitive Gains from Trade

We finally look at the effectiveness of international trade to reduce market distortions, i.e., the amount of misallocation induced by markups and markdowns. We answer this question by studying the extent to which the losses from misallocation change as the economy is less exposed to trade.

We defined aggregate labor productivity in the model to be the quantity A_{μ^n,μ^y} that in equilibrium satisfies the equation

$$Y = A_{\mu^n,\mu^y} \times N.$$

Aggregate productivity is an endogenous outcome of the model that depends on the distribution of the firm's productivities level, z_i, as well as in the allocation of resources across

Table 2.5: Allocative efficiency losses of closing the economy to international trade

	Market structure in Product and Labor Markets				
	Oligopoly Oligopsony	Oligopoly Perf. Comp.	Perf. Comp. Oligopsony	Perf. Comp.	Monopoly Monopsony
A. Substitution in product mkt.					
Baseline	0.668	0.413	0.068	0	0
$\sigma = 3$	0.731	0.465	0.07	0	0
$\sigma = 5$	0.798	0.526	0.075	0	0
$\sigma = 7$	0.83	0.558	0.077	0	0
$\sigma = 10$	0.861	0.589	0.078	0	0
B. Substitution in labor mkt.					
Baseline	0.668	0.413	0.068	0	0
$\eta = 1.5$	1.296	0.451	0.391	0	0
$\eta = 2$	1.843	0.478	0.738	0	0
$\eta = 5$	3.546	0.543	1.962	0	0
$\eta = 10$	4.485	0.574	2.684	0	0

Notes: The table reports allocative efficiency losses of moving Australia from the observed trade equilibrium in 2018/2019 fiscal year to the counterfactual autarky equilibrium under different classes of models nested in our main specification. The different columns specify the type of competition within product and labor markets, respectively. For example the column labeled *Perf. Comp. - Oligopsony* correspond to a version of our model with perfectly competitive product markets and oligopsonistic labor markets. The baseline calibration has within markets elasticity of substitution equal to $\eta = 1.02$ and $\sigma = 2.5$, and across markets elasticity of substitution equal to $\theta = 0.8$ and $\epsilon = 1.5$. The two panel in the table show sensitivity with respect to the substitution within labor and product markets, respectively.

firms (see Section 2.2.4). Because firms exert market power in product and labor markets we usually have that $A_{\mu^n,\mu^y} \neq A_{\text{efficient}}$, where $A_{\text{efficient}}$ is the aggregate productivity attained under the efficient allocation of resources.

We measure the pro-competitive gains by looking at changes in allocative efficiency, defined as the change in aggregate labor productivity between the market equilibrium relative to the change in aggregate productivity under the efficient allocation. Formally, we defined the gains/losses in allocative efficiency as

$$\underbrace{\Delta \log A_{\mu^n,\mu^y}}_{\text{Total Change in TFP}} = \underbrace{\Delta \log A_{\text{efficient}}}_{\text{Productivity Gains}} + \underbrace{\Delta \log A_{\mu^n,\mu^y} - \Delta \log A_{\text{efficient}}}_{\text{Allocative efficiency gains}}.$$

The first term on the right-hand side of this expression gives the change in the first-best level of TFP, while the second term gives the change in misallocation, our measure of pro-competitive effects. The pro-competitive gains will be positive (negative) if increased trade affects misallocation so that the change in aggregate productivity is larger (smaller) than the change in first-best productivity.

Table 2.5 reports the pro-competitive gains under the different classes of models discussed in the previous section. In the efficient allocation there is no misallocation of resources and pro-competitive gains are zero, as it is reflected in the column *Perf. Comp.*. Furthermore, in a model with constant levels of market power, by construction, there is no dispersion in firms' markups and markdowns. Resources are distributed across firms as in the efficient allocation, aggregate productivity equals it first-best value, and therefore in this case there are also zero pro-competitive gains, column *Monopoly - Monopsony*.[48]

Pro-competitive losses from closing the economy to trade are 0.67%, as reported in Table 2.5. Our baseline estimate are small in magnitude, but the number is sensitive to the parameters of the model and they could be as high as 7 percent. As with our estimate of the welfare losses, models that incorporate in their analysis market power in only one market underestimate substantially the total effect.

The allocative efficiency gains allows for a further decomposition to understand the role in product and labor market power driving the results. In particular, we decompose allocative

48. The allocation of resources across firms is efficient in the sense that each firm captures the efficient share of each market. However, the total amount of resources produced in the economy is not efficient. The constant firm market power manifest as a wedge in the optimality condition between aggregate consumption and aggregate labor supply.

Table 2.6: Contribution of labor market power to the allocative efficiency losses

	Contribution Labor mkt power	Percentage of total effect
Baseline	0.255	38.2
$\eta = 1.5$	0.845	65.2
$\eta = 2$	1.365	74.1
$\eta = 5$	3.003	84.7
$\eta = 10$	3.938	87.8
Baseline	0.255	38.2
$\sigma = 3$	0.266	36.4
$\sigma = 5$	0.272	34.1
$\sigma = 7$	0.272	32.8
$\sigma = 10$	0.272	31.6

Notes: The table reports the contribution of labor market power to the allocative efficiency losses from moving Australia from the observed trade levels in the 2018/2019 fiscal year to the counterfactual autarky equilibrium. Allocative efficiency is calculated as the change between these two equilibrium in aggregate labor productivity between the market equilibrium relative to the change under the efficient allocation. The baseline calibration has within markets elasticity of substitution equal to $\eta = 1.02$ and $\sigma = 2.5$, and across markets elasticity of substitution equal to $\theta = 0.8$ and $\epsilon = 1.5$. The two panel in the table show sensitivity with respect to the substitution within labor and product markets, respectively.

efficiency gains into,

$$\Delta \log A_{\mu^n,\mu^y} - \Delta \log A_{\text{efficient}} = \Big(\Delta \log A_{\mu^y} - \Delta \log A_{\text{efficient}}\Big) + \Big(\Delta \log A_{\mu^n,\mu^y} - \Delta \log A_{\mu^y}\Big),$$

where the first term represents the allocative gains when only product market power is considered, and so the second term measures the contribution of labor market power and the interaction between the two.

Table 2.6 reports the contribution of labor market power to the allocative efficiency gains. While product market power accounts for the majority of the efficiency losses, labor market power is not a negligible force for aggregate productivity either. In the baseline model, labor market power is responsible for 38 percent of the decrease in labor productivity. The importance of labor market power grows with the amount of substitution within labor markets and under possible parametrization it accounts for more than 80 percent of the total effect. In short, our results indicate that opening to trade is a useful tool to reduce market power induced misallocation and that labor market power is an economically significant mechanism behind our results.

2.6 Conclusion

We build a quantitative trade model with imperfect competition, strategic complementarities in price- and wage-setting, and a rich structure of product and labor markets to quantify the pro-competitive gains from international trade. The model is able to perform trade policy evaluations in a parsimonious way.

We combine our model with confidential firm-level data from Australia and study the gains/losses of moving Australia to autarky. The model matches exactly facts on import share dispersion, distribution of domestic market shares, and the cross-sectional relationship between import penetration and domestic concentration. The Australian data is characterized by a large amount of dispersion and concentration in firm payroll shares, and small concentration in sales shares which implies extensive missallocation through the lens of the model.

Using the model, we decompose the different channels by which the gains from trade materialize. We find that closing the economy to trade reduces welfare by 2% and increase resource misallocation by around 1% in our benchmark calibration. While the former are well approximated by models with constant levels of market power these models do not capture the losses in aggregate productive caused by misallocation. The size of these effects vary with the key parameters of the model and the structure of product and labor markets and our robustness exercise indicate that losses can go as high as 7%. In this sense, we find that accounting for imperfect competition in product and labor markets is quantitatively and economically meaningful.

From a policy viewpoint, our model suggests that obtaining large welfare gains from an improved allocation of resources may not require a detailed scheme of producer-specific subsidies and taxes that reduce the distortions associated with variable market power. Instead, simply opening an economy to trade may provide an excellent practical alternative that substantially improves productivity and welfare.

Looking forward, our results imply that the size of the pro-competitive gains depends on the values of four key parameters the two elasticity of substitution in product and in labor markets. Therefore, there is value in estimating those in Australia using our data. In Appendix .5 we took a first step on estimation, however, there is much to be done. Second, our results looked only at the aggregate effects of trade liberalization. Gains, however, are not ubiquitous, and therefore it is important to look at distributional effects as well.

Appendix

.1 Proof and Derivations

.1.1 *Cost function of the firm*

In this section we derived the cost function of the firm. We show that the marginal cost of the firm is not constant as a function of firm level output despite the firm's linear technology. Hence, labor market power is a non-technological source of returns to scale.

The variable cost function of the firm is define as the solution to the problem:

$$c_i(y) \equiv \min_{n_i} w_i n_i,$$

subject to the technology of the firm $y = z_i n_i$ and the inverse labor supply equation (2.3) as the firm internalizes the market power she exerts over the labor input. Note that y represents the total amount of output produced by the firm and does not distinguish whether it is sell in the domestic economy or abroad.

Because of the linear technology, given an amount of desired output y, optimal employment is just $n_i = \frac{y}{z_i}$. Plugging this into equation (2.3), and then into the objective we can compute the variable cost of the firm,

$$c(y) w_i \left(\frac{y}{z_i}\right) \frac{y}{z_i}. \tag{34}$$

The marginal cost of the firm implied by equation (34) is the product of two terms,

$$\frac{\partial c(y)}{\partial y} = \frac{w_i}{z_i} \times \left[\frac{\gamma_i^n + 1}{\gamma_i^n}\right].$$

As it is standard in models with perfectly competitive markets the marginal cost depend on the wage per efficiency units w_i/z_i. Differently from those models is the second term, related to the elasticity of labor supply $\gamma_i^n \equiv \left[\frac{\partial \log w_i}{\partial \log n_i}\right]^{-1}$, that appears because of the labor market power exerted by the firm. Since the elasticity of labor supply and the wages offer by the firm both depend on the scale of production y, the resulting marginal cost is not constant as a function of total output. Intuitively, if a firm decides to increase its scale of production it has to hire extra workers, but because of the firm's labor market power this can only be done by offering a higher wage wage than its competitors. Since the firm has to offer a unique wage level to all its workers, this higher wage does not apply only to the marginal unit, but to all previously hired workers.

.1.2 *Proof Proposition 2.2.1*

Definition Symmetric Countries. To be concrete we start with a definition of what the symmetry countries assumptions implies.

Definition .1.1. In a model with granular firms, two countries are say to be symmetric if one is a replica of the other..

Under Definition .1.1, the Foreign country is partitioned into the same set of product and labor markets, with the same number of firms per market. Furthermore, each firm in Foreign has a counterpart in Home with the same level of productivity z_i.

An expression for firm-level shares. We start by noting that the share of domestic sales of firm i can be written as:

$$s_{Hi}^{y} = \frac{p_{Hi} y_{Hi}}{\sum_{\{j \in \mathcal{M} : \kappa(j) = \kappa(i)\}} p_{Hj} y_{Hj} + \sum_{\{j \in \mathcal{M}^* : \kappa(j) = \kappa(i)\}} p_{Fj} y_{Fj}} = s_i^y \left(1 - \lambda_{\kappa(i)}\right),$$

where $\lambda_{\kappa(i)}$ is the import share in product market $\kappa(i)$, and s_i^y is the share of domestic sales among domestic producers. Similarly, the sales share in the foreign country of firm i, $s_{H,i}^{y^*}$, can be written as:

$$s_{Hi}^{y^*} = \lambda^*_{\kappa(i)} s_i^{y^*},$$

where $\lambda^*_{\kappa(i)}$ is the import share of the Foreign country, and $s_i^{y^*}$ is the foreign sales shares of the domestic firms among exporters.

The three relevant shares for firm i are then $\left\{s_i^n, s_i^y, s_i^{y*}\right\}_{i\in\mathcal{M}}$. These are given by:

$$s_i^n = \frac{\xi_{n,i} w_i^{1+\eta}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \xi_{n,j} w_j^{1+\eta}}, \tag{35}$$
$$s_i^y = \frac{\xi_{Hy,i} p_{Hi}^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \xi_{Hy,j} p_{Hj}^{1-\sigma}},$$
$$s_i^{y*} = \frac{\xi_{Hy,i}^* p_{Hi}^{*1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \xi_{Hy,j}^* p_{Hj}^{*1-\sigma}}.$$

We use the following equations: (i) the production technology of the intermediate good producers; (ii) firm $i \in \mathcal{M}$ labor supply and product demand equations ; and (iii) firm optimality summarize by the firm's the optimal pricing equation, to derive an expression for firm-level prices and wages. We start with the technology of the firm and use the labor supply equation to eliminate n_i, and the demand equation to eliminate quantitiesy_{Hi} and y_{Hi}^*:

$$\xi_{Hy,i}\left(\frac{p_{Hi}}{P_{\kappa(i)}}\right)^{-\sigma} Y_{\kappa(i)} + \xi_{Hy,i}^*\left(\frac{p_{Hi}^*}{P_{\kappa(i)}^*}\right)^{-\sigma} Y_{\kappa(i)}^* = z_i \xi_{n,i}\left(\frac{w_i}{W_{\ell(i)}}\right)^{\eta} N_{\ell(i)}.$$

We then use the optimal pricing equation of the firm $p_{H,i} = \frac{\mu_i^y}{\mu_i^n}\frac{w_i}{z_i}$ and $p_{Hi}^* = \frac{\mu_i^y}{\mu_i^n}\tau_{\kappa(i)}\frac{w_i}{z_i}$ to express everything in term of firm-level wages. We then solve for wages to get the equation:

$$w_i = z_i^{\frac{\sigma-1}{\eta+\sigma}}\left[\frac{\xi_{Hy,i}}{\xi_{n,i}}\left(\frac{\mu_i^y}{\mu_i^n}\right)^{-\sigma}\frac{P_{\kappa(i)}Y_{\kappa(i)}/P_{\kappa(i)}^{1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}} + \phi_i\frac{\xi_{Hy,i}^*}{\xi_{n,i}}\left(\frac{\mu_i^{y^*}}{\mu_i^n}\tau_{\kappa(i)}\right)^{-\sigma}\frac{P_{\kappa(i)}^*Y_{\kappa(i)}^*/P_{\kappa(i)}^{*1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}}\right]^{\frac{1}{\eta+\sigma}}, \tag{36}$$

which implies prices are:

$$p_{Hi}^* = \tau_{\kappa(i)}\frac{\mu_i^{y^*}}{\mu_i^y}p_{Hi}. \tag{37}$$
$$p_{Hi} = \frac{\mu_i^y}{\mu_i^n}z_i^{-\frac{1+\eta}{\eta+\sigma}}\left[\frac{\xi_{Hy,i}}{\xi_{n,i}}\left(\frac{\mu_i^y}{\mu_i^n}\right)^{-\sigma}\frac{P_{\kappa(i)}Y_{\kappa(i)}/P_{\kappa(i)}^{1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}} + \phi_i\frac{\xi_{Hy,i}^*}{\xi_{n,i}}\left(\frac{\mu_i^{y^*}}{\mu_i^n}\tau_{\kappa(i)}\right)^{-\sigma}\frac{P_{\kappa(i)}^*Y_{\kappa(i)}^*/P_{\kappa(i)}^{*1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}}\right]^{\frac{1}{\eta+\sigma}},$$

We can plug equation (36) into equation (35) to derive our expressions for the firm's share.

We start computing the firm-level payroll shares. We first use equations (35) and (36) to write s_i^n as:

$$s_i^n = \frac{\xi_{n,i} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[\frac{\xi_{Hy,i}}{\xi_{n,i}} \left(\frac{\mu_i^n}{\mu_i^y}\right)^\sigma \frac{P_{\kappa(i)} Y_{\kappa(i)}}{P_{\kappa(i)}^{1-\sigma}} + \phi_i \frac{\xi^*_{Hy,i}}{\xi_{n,i}} \left(\frac{\mu_i^n}{\tau_{\kappa(i)} \mu_i^{y*}}\right)^\sigma \frac{P^*_{\kappa(i)} Y^*_{\kappa(i)}}{P^{*1-\sigma}_{\kappa(i)}} \right]^{\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j:\ell(j)=\ell(i)\}} \xi_{n,j} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[\frac{\xi_{Hy,j}}{\xi_{n,j}} \left(\frac{\mu_j^n}{\mu_j^y}\right)^\sigma \frac{P_{\kappa(j)} Y_{\kappa(j)}}{P_{\kappa(j)}^{1-\sigma}} + \phi_j \frac{\xi^*_{Hy,j}}{\xi_{n,j}} \left(\frac{\mu_j^n}{\tau_{\kappa(j)} \mu_j^{y*}}\right)^\sigma \frac{P^*_{\kappa(j)} Y^*_{\kappa(j)}}{P^{*1-\sigma}_{\kappa(j)}} \right]^{\frac{1+\eta}{\eta+\sigma}}}. \quad (38)$$

Note that similar to Berger et al. (2022a) labor market aggregates cancel out since those affect all the firms in the market alike. Different from Berger et al. (2022a) firm-level shares still depend on market level aggregates related to product markets. This is because firms in a labor market may or not participate in foreign trade and therefore the scale of production of some firms are affected by the level of demand in the Foreign economy. Second, firms in a labor market produce different types of products within the Home economy and therefore are differentially exposed to the demand conditions across these different product markets.

The next step is to express the market-level aggregates in terms of market-level shares. To do that we note that in the model the following relationship holds true for all product markets:

$$Y_k = \upsilon_{yk} \left(\frac{P_k}{P}\right)^{-\epsilon} Y, \quad \text{for all } k \in \mathcal{K},$$

and therefore, product market k sales share of total revenue, defined as $S_k^y \equiv \frac{P_k Y_k}{PY}$, in equilibrium equals

$$S_k^y = \upsilon_{y,k} \left(\frac{P_k}{P}\right)^{1-\epsilon}.$$

Putting these two equations together, and using the symmetric countries assumption, we get,

$$\frac{P_{\kappa(i)} Y_{\kappa(i)}}{P_{\kappa(i)}^{1-\sigma}} = \frac{P^*_{\kappa(i)} Y^*_{\kappa(i)}}{\left(P^*_{\kappa(i)}\right)^{1-\sigma}} = \upsilon_{y,\kappa(i)} \left(\frac{S^y_{\kappa(i)}}{\upsilon_{y,\kappa(i)}}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}} \frac{PY}{P^{1-\sigma}}. \quad (39)$$

Combining equations (38) with equation (39) and using again the symmetric countries property payroll-shares are:

$$s_i^n = \frac{\xi_{n,i}\left(z_i \upsilon_{y,\kappa(i)}^{\frac{1}{(\epsilon-1)}}\right)^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}\left[\frac{\xi_{Hy,i}}{\xi_{n,i}}\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\sigma}+\phi_i\frac{\xi_{Hy,i}^*}{\xi_{n,i}}\left(\frac{\mu_i^n}{\tau_{\kappa(i)}\mu_i^{y^*}}\right)^{\sigma}\right]^{\frac{1+\eta}{\eta+\sigma}}\left(S_{\kappa(i)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{\ell(j)=\ell(i)\}}\xi_{n,j}\left(z_j \upsilon_{y,\kappa(j)}^{\frac{1}{\epsilon-1}}\right)^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}\left[\frac{\xi_{Hy,j}}{\xi_{n,j}}\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\sigma}+\phi_j\frac{\xi_{Hy,j}^*}{\xi_{n,j}}\left(\frac{\mu_j^n}{\tau_{\kappa(j)}\mu_j^{y^*}}\right)^{\sigma}\right]^{\frac{1+\eta}{\eta+\sigma}}\left(S_{\kappa(j)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}. \quad (40)$$

To compute the domestic sales share we first note use equation (35) and (37) to substitute prices out of the definition of s_i^y, and use equation (39) to deal with product market level aggregates. For labor market level aggregates we note that they satisfy an analogous form to equation (39):

$$\frac{W_{\ell(j)}N_{\ell(j)}}{W_{\ell(j)}^{1+\eta}} = \upsilon_{n,\ell(j)}\left(\frac{S_{\ell(j)}^n}{\upsilon_{n,\ell(j)}}\right)^{\frac{\theta-\eta}{1+\theta}}\frac{WN}{W^{1+\eta}}. \quad (41)$$

Domestic sales share are then,

$$s_i^y = \frac{\xi_{Hy,i}\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\sigma-1}\left(z_i \upsilon_{n,\ell(i)}^{\frac{1}{1+\theta}}\right)^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}\left[\frac{\xi_{Hy,i}}{\xi_{n,i}}\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\sigma}+\phi_i\frac{\xi_{Hy,i}^*}{\xi_{n,i}}\left(\frac{\mu_i^n}{\tau_{\kappa(i)}\mu_i^{y^*}}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}}\left(S_{\ell(i)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum\xi_{Hy,j}\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\sigma-1}\left(z_j \upsilon_{n,\ell(j)}^{\frac{1}{1+\theta}}\right)^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}\left[\frac{\xi_{Hy,j}}{\xi_{n,j}}\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\sigma}+\phi_j\frac{\xi_{Hy,j}^*}{\xi_{n,j}}\left(\frac{\mu_j^n}{\tau_{\kappa(j)}\mu_j^{y^*}}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}}\left(S_{\ell(j)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}, \quad (42)$$

$$s_i^{y*} = \frac{\phi_i\xi_{Hy,i}^*\left(z_i \upsilon_{n,\ell(j)}^{\frac{1}{1+\theta}}\right)^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}\left(\frac{\mu_i^n}{\mu_i^{y^*}}\right)^{(\sigma-1)}\left[\frac{\xi_{Hy,i}}{\xi_{n,i}}\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\sigma}+\phi_i\frac{\xi_{Hy,i}^*}{\xi_{n,i}}\left(\frac{\mu_i^n}{\tau_{\kappa(i)}\mu_i^{y^*}}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}}\left(S_{\ell(i)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum\phi_j\xi_{Hy,j}^*\left(z_j \upsilon_{n,\ell(j)}^{\frac{1}{1+\theta}}\right)^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}\left(\frac{\mu_j^n}{\mu_j^{y^*}}\right)^{(\sigma-1)}\left[\frac{\xi_{Hy,j}}{\xi_{n,j}}\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\sigma}+\phi_j\frac{\xi_{Hy,j}^*}{\xi_{n,j}}\left(\frac{\mu_j^n}{\tau_{\kappa(j)}\mu_j^{y^*}}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}}\left(S_{\ell(j)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}.$$

Now we use Assumption 2.2.1 (all demand and supply shifters equal one) and Assumption 2.2.2 (trade is free between the two countries)in the main text. Under these assumptions we have that payroll shares are,

$$s_i^n = \frac{z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}\left[\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\sigma}+\left(\frac{\mu_i^n}{\mu_i^{y^*}}\right)^{\sigma}\right]^{\frac{1+\eta}{\eta+\sigma}}\left(S_{\kappa(i)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}}z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}\left[\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\sigma}+\left(\frac{\mu_j^n}{\mu_j^{y^*}}\right)^{\sigma}\right]^{\frac{1+\eta}{\eta+\sigma}}\left(S_{\kappa(j)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}, \quad (43)$$

domestic sales share are $s^y_{Hi} = s^y_i \left(1 - \lambda_{\kappa(i)}\right)$ with,

$$s_i^y = \frac{\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\sigma-1} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\sigma} + \left(\frac{\mu_i^n}{\mu_i^{y^*}}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}} \left(S^n_{\ell(i)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \left(\frac{\mu_j^n}{\mu_j^y}\right)^{\sigma-1} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\sigma} + \left(\frac{\mu_j^n}{\mu_j^{y^*}}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}} \left(S^n_{\ell(j)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}, \tag{44}$$

and sales share in Foreign are $s^{y^*}_{Hi} = s^{y^*}_i \lambda^*_{\kappa(i)}$ with,

$$s_i^{y*} = \frac{z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(\frac{\mu_i^n}{\mu_i^{y^*}}\right)^{(\sigma-1)} \left[\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\sigma} + \left(\frac{\mu_i^n}{\mu_i^{y^*}}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}} \left(S^n_{\ell(i)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(\frac{\mu_j^n}{\mu_j^{y^*}}\right)^{(\sigma-1)} \left[\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\sigma} + \left(\frac{\mu_j^n}{\mu_j^{y^*}}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}} \left(S^n_{\ell(j)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}. \tag{45}$$

Furthermore, symmetry between countries with free trade implies that in equilibrium $(1 - \lambda_k)$ must equal λ^*_k for all $k \in \mathcal{K}$, and therefore from equations (44) and (45) that $s^y_i = s^{y^*}_i$ is a solution of the system, for any set of values of firm payroll shares $\{s^n_i\}_{i\in\mathcal{M}}$, and market-level shares. Replacing this back into equation (43) to (45) we get the result in Proposition 2.2.1:

$$s_i^n = \frac{\left(\frac{\mu^n\left(s_i^n\right)}{\mu^y\left(s_i^y\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(1+\eta)\sigma}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^y_{\kappa(i)}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \left(\frac{\mu^n\left(s_j^n\right)}{\mu^y\left(s_j^y\left(1-\lambda_{\kappa(j)}\right)\right)}\right)^{\frac{(1+\eta)\sigma}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^y_{\kappa(j)}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}},$$

$$s_i^y = \frac{\left(\frac{\mu^n\left(s_i^n\right)}{\mu^y\left(s_i^y\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^n_{\ell(i)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(\frac{\mu^n\left(s_j^n\right)}{\mu^y\left(s_j^y\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} \left(S^n_{\ell(j)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}$$

with,

$$\mu_i^n = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right]^{-1}, \quad \text{and} \quad \mu_i^y = \left[\frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_i^y \left(1 - \lambda_{\kappa(i)}\right)\right]^{-1}.$$

.1.3 Proof Proposition 2.2.1

Here we don't impose Assumption 2.2.2.[49] When $\sigma = \epsilon$, the markup charged by the firm in both Home and Foreign is constant and equals $\mu^y = \frac{\sigma}{\sigma-1}$. Then from equation (40), we have that payroll-shares are given by:

$$s_i^n = \frac{\left(\mu_i^n\right)^{\frac{\sigma(\eta+1)}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(1+\tau_{\kappa(i)}^{-\sigma}\right)^{\frac{1+\eta}{\eta+\sigma}} \left(S_{\kappa(i)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \left(\mu_i^n\right)^{\frac{\sigma(\eta+1)}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(1+\tau_{\kappa(j)}^{-\sigma}\right)^{\frac{1+\eta}{\eta+\sigma}} \left(S_{\kappa(j)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}. \tag{46}$$

A reduction in trade costs changes the allocation of labor, however this reallocation of labor across firms does not come from an increase in import competition since markups are constants, and as it is clear from equation (46) neither λ_k or s_i^y determines the level of s_i^n. The reallocation of labor across firms comes instead from the lower costs that the firm face to export its goods the Foreign country. Because of the smaller iceberg costs firm requires more labor to satisfy the Foreign demand. We note, however, that relative to a case with no markdowns the amount of labor that firms hires is lower because to satisfy the demand implies increasing total cost due to labor market power. From equations 42, sales shares are:

$$s_i^{y*} = s_i^y = \frac{\left(\mu_i^n\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(i)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \left(\mu_j^n\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(j)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}. \tag{47}$$

49. We keep the iceberg cost but impose no fixed cost of exporting.

.1.4 Proof Corollary 2.2.1

To show the Corollary, notice that when all firms produce for the same product market the term in the numerator $\left(\left(1+\tau_{\kappa(i)}^{-\sigma}\right) S_{\kappa(i)}^{y\frac{\sigma-\epsilon}{1-\epsilon}}\right)^{\frac{1+\eta}{\eta+\sigma}}$ cancels out and we get,

$$s_i^n = \frac{\left(\mu_i^n\right)^{\frac{\sigma(\eta+1)}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \left(\mu_i^n\right)^{\frac{\sigma(\eta+1)}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}}.$$

which is independent of any trade related variable, and therefore so it is s_i^y.

.1.5 Market Competition and Firm Market Power

We focus on three particular markets, two labor markets l_1 and l_2, and one product market k. These three markets are interconnected because all the firms in labor market l_1 and l_2 produce for product market k. Furthermore, they are the only producers of market k goods, and therefore there is no foreign competition nor competition from other domestic firms outside labor markets l_1 and l_2.

Firms in labor market l_1 differ from firms in labor market l_2 in two dimensions. First, the productivity of the representative firm in labor market l_1, z_1, differs from the productivity level of the representative firm in labor market l_2, z_2. They also differ in the total number of firms operating in each market M_1 and M_2, respectively. This implies that in product market k, there are two type of firms : There are M_1 firms with productivity z_1 and M_2 firms with productivity z_2, all competing against each other in product market k.

The allocation of resources across these three markets is described by the equations:

- The labor supply equation:

$$w_i = \bar{\omega}_n \left(n_i\right)^{\frac{1}{\eta}} N_{\ell(i)}^{\frac{1}{\theta}-\frac{1}{\eta}}, \quad N_{\ell(i)} = \left[\sum_{\{j:\ell(j)=\ell(i)\}} \left(n_j\right)^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{1+\eta}}, \tag{48}$$

where $\bar{\omega}_n \equiv W N^{-\frac{1}{\theta}}$.

- The product demand equation:

$$p_i = \bar{\omega}_y y_i^{-\frac{1}{\sigma}} Y_{\kappa(i)}^{\frac{1}{\sigma}-\frac{1}{\epsilon}}, \qquad Y_{\kappa(i)} = \left[\sum_{\{j:\kappa(j)=\kappa(i)\}} y_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{49}$$

with $\bar{\omega}_y \equiv Y^{\frac{1}{\epsilon}} P$.

- The optimal pricing equation:

$$p_i = \frac{\mu_i^y}{\mu_i^n} \frac{w_i}{z_i}, \tag{50}$$

where, $\mu_i^y = \left[\frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_i^y\right]^{-1}$ and $\mu_i^n = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right]^{-1}$.

- Firm shares are:

$$s_i^y = \frac{y_i^{\frac{\sigma-1}{\sigma}}}{\sum_{j\in\mathcal{M}} y_j^{\frac{\sigma-1}{\sigma}}}, \qquad s_i^n = \frac{n_i^{\frac{1+\eta}{\eta}}}{\sum_{j\in l} n_j^{\frac{\eta+1}{\eta}}}. \tag{51}$$

Equilibrium Allocation. Because firm are symmetric within a labor market we have that firms split the market equally,

$$s_i^n = \frac{1}{M_{\ell(i)}}, \quad \ell(i) \in \{l_1, l_2\}, \tag{52}$$

and so markdowns are a function of the number of firms in the market only,

$$\mu_i^n = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\frac{1}{M_i}\right]^{-1}. \tag{53}$$

Absent firm heterogeneity within labor markets the amount of competition in the market is completely capture by the number of competitors in the market. Furthermore, firms within a labor market hire the same number of workers, which simplifies the labor supply equation to,

$$w_i = \bar{\omega}_n (n_i)^{\frac{1}{\theta}} M_i^{\frac{\eta-\theta}{(1+\eta)\theta}}. \tag{54}$$

Now to get an expression for the sales share we start with the pricing equation, equation (50), and use equation (54) to eliminate firm level wages,

$$p_i = \frac{\mu_i^y}{\mu_i^n}\frac{1}{z_i}\bar{\omega}_n\left(n_i\right)^{\frac{1}{\theta}} M_i^{\frac{\eta}{1+\eta}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)}.$$

Then, we use the production function to eliminate firm level employment and express it in terms of firm level output, and use the inverse demand function to substitute out p_i to get,

$$\bar{\omega}_y y_i^{-\frac{1}{\sigma}} Y_{\kappa(i)}^{\frac{1}{\sigma}-\frac{1}{\epsilon}} = \frac{\mu_i^y}{\mu_i^n}\frac{1}{z_i}\bar{\omega}_n\left(\frac{1}{z_i}y_i\right)^{\frac{1}{\theta}} M_i^{\frac{\eta}{1+\eta}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)}.$$

Solving for y_i,

$$y_i = \left(\frac{\mu_i^n}{\mu_i^y}\right)^{\frac{\sigma\theta}{\theta+\sigma}}\left(\frac{\bar{\omega}_n}{\bar{\omega}_y}\right)^{-\frac{\sigma\theta}{\theta+\sigma}}\left(z_i\right)^{\frac{\sigma(\theta+1)}{\theta+\sigma}} M_i^{-\frac{\sigma\theta}{\theta+\sigma}\frac{\eta}{1+\eta}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)} Y_{\kappa(i)}^{\frac{\sigma\theta}{\theta+\sigma}\left(\frac{1}{\sigma}-\frac{1}{\epsilon}\right)}. \tag{55}$$

Finally, we plug equation (55) into equation (51),

$$s_i^y = \frac{\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\frac{(\sigma-1)\theta}{\theta+\sigma}}\left(z_i\right)^{\frac{(\sigma-1)(\theta+1)}{\theta+\sigma}} M_i^{\frac{(\sigma-1)(\eta-\theta)}{(\theta+\sigma)(1+\eta)}}}{\sum_{\{j:\kappa(j)=\kappa(i)\}}\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\frac{(\sigma-1)\theta}{\theta+\sigma}}\left(z_j\right)^{\frac{(\sigma-1)(\theta+1)}{\theta+\sigma}} M_j^{\frac{(\sigma-1)(\eta-\theta)}{(\theta+\sigma)(1+\eta)}}}, \tag{56}$$

with,

$$\mu_i^n = \left[\frac{\eta+1}{\eta}+\left(\frac{1}{\theta}-\frac{1}{\eta}\right)\frac{1}{M_i}\right]^{-1}, \quad \text{and} \quad \mu_i^y = \left[\frac{\eta+1}{\eta}+\left(\frac{1}{\theta}-\frac{1}{\eta}\right)s_i^y\right]^{-1}. \tag{57}$$

Equations (56) and (57) form a system of non-linear equations that determine the equilibrium values of the sales shares. Although we cannot solve the system with pen and paper, it is possible to determine whether s_1^y is less, bigger or equal than s_2^y. We define the function,

$$f(s) \equiv (s)^{\frac{\theta+\sigma}{(\sigma-1)\theta}} \mu^y(s) = \frac{(s)^{\frac{\theta+\sigma}{(\sigma-1)\theta}}}{\frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s}.$$

The function $f(s)$ satisfies the properties that $f(0) = 0$, $f(1) = \frac{\epsilon}{\epsilon-1}$, and $f(\cdot)$ is strictly increasing in s ($f'(s) > 0$). From equation (14) we have,

$$f\left(s_i^y\right) = \frac{\mu_i^n M_i^{-\frac{(\eta-\theta)}{\theta(1+\eta)}} z_i^{\frac{1+\theta}{\theta}}}{\left[\sum_{j\in\mathcal{M}} \left(\frac{\mu_j^n}{\mu_j^y}\right)^{\frac{(\sigma-1)\theta}{\theta+\sigma}} M_j^{-\frac{(\sigma-1)(\eta-\theta)}{(\theta+\sigma)(1+\eta)}} \left(z_j\right)^{\frac{(\sigma-1)(1+\theta)}{\theta+\sigma}}\right]^{\frac{\theta+\sigma}{(\sigma-1)\theta}}},$$

and therefore,

$$\frac{f\left(s_1^y\right)}{f\left(s_2^y\right)} = \frac{\mu_1^n M_1^{-\frac{(\eta-\theta)}{\theta(1+\eta)}}}{\mu_2^n M_2^{-\frac{(\eta-\theta)}{\theta(1+\eta)}}} \left(\frac{z_1}{z_2}\right)^{\frac{\theta+1}{\theta}}. \tag{58}$$

Define then the function $h(m) = \left(\frac{1}{m}\right)^{\frac{(\eta-\theta)}{\theta(1+\eta)}} \mu_i^n \left(\frac{1}{m}\right)$.This function satisfies that $h(1) = \frac{\theta}{\theta+1}$, $\lim_{m\to\infty} h(m) = 0$, and $h(\cdot)$ is strictly decreasing in m ($h'(m) < 0$). We can express equation (58) in terms of the function $h(\cdot)$ as,

$$\frac{f\left(s_1^y\right)}{f\left(s_1^y\right)} = \left(\frac{z_1}{z_2}\right)^{\frac{1+\theta}{\theta}} \frac{h(M_1)}{h(M_2)}.$$

Since $f(\cdot)$ is increasing we know that $s_1^y > s_2^y$ if:

$$\left(\frac{z_1}{z_2}\right)^{\frac{1+\theta}{\theta}} \frac{h(M_1)}{h(M_2)} > 1. \tag{59}$$

Condition (59) is satisfied for different combinations of the parameters. When the two markets have the same amount of competition, i.e. $M_1 = M_2$, then which type of firms gets the largest share is dictated by difference in productivity. Therefore, as in Atkeson and Burstein (2008) and Edmond et al. (2015), the most productive firm gets the largest share. When the two markets differ in the amount of competitions, this relationship can be reverted and the less productive firm could gets the largest share. Take the case with $z_1 = z_2$, then it follows from $h(\cdot)$ being a decreasing function, that $s_1^y > s_2^y$ if $M_1 < M_2$, and so the the firm in the less competitive labor market captures the largest share despite both firms being equally productive.

Love for variety effects. The choice in preferences for labor supply introduces love-for-variety effects into the model.To show that the previous result is not affected by these forces we modify the labor supply as follows:

$$w_i = \bar{\omega}_n (n_i)^{\frac{1}{\eta}} N_{\ell(i)}^{\frac{1}{\theta}-\frac{1}{\eta}}, \quad N_{\ell(i)} = M_{\ell(i)}^{\chi} \left[\sum_{\{j:\ell(j)=\ell(i)\}} \left(n_j\right)^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{1+\eta}}.$$

In the standard set-up the parameter η controls both the elasticity of substitution across firms and the love-for-variety effects in the model[50]. By introducing χ into the model the parameter η controls the former effect but not the latter. The parameter χ controls the love-for-variety effects, when $\chi = \frac{1}{1+\eta}$ there are no love-for-variety effects. When $\chi = 0$ we go back to the original model.

Now, in the symmetric equilibrium, we have that $N_{\ell(i)} = M_{\ell(i)}^{\chi+\frac{\eta}{\eta+1}} n_i$, and therefore sales share satisfy the equation:

$$\frac{f\left(s_1^y\right)}{f\left(s_1^y\right)} = \left(\frac{z_1}{z_2}\right)^{\frac{1+\theta}{\theta}} \frac{h_\chi (M_1)}{h_\chi (M_2)}, \tag{60}$$

where

$$h_\chi (m) = m^{-\left(\chi+\frac{\eta}{1+\eta}\right)\left(\frac{1}{\theta}-\frac{1}{\eta}\right)} \mu_i^n \left(\frac{1}{m}\right).$$

For the case with no love-for-variety effects, i.e. $\chi = \frac{1}{1+\eta}$, we still have that: $h_{\frac{1}{1+\eta}}(1) = \frac{\theta}{\theta+1}$

50. For more on the topic see Benassy (1998)

, $\lim_{m\to\infty} h_{\frac{1}{1+\eta}}(m) = 0$, and $h_{\frac{1}{1+\eta}}(\cdot)$ is strictly decreasing in m . Therefore, the previous analysis does not change once we remove the love-for-variety effects from the mode.

.2 Identification of Gains From Trade

In this section we describe how to compute the pro-competitive gains from trade. As we mentioned in the main text the key to compute the pro-competitive gains from trade is to be able to compute the counterfactual autarky equilibrium given the observed trade equilibrium. The main discussion in this section is then about the autarky equilibrium. In this section we assume that we know the values of the parameters $\{\eta, \theta, \sigma, \epsilon, \varphi, \bar{\varphi}\}$, the partition of the economy into labor and product markets and the assignment functions $\kappa(\cdot)$ and $\ell(\cdot)$, and so this section is about identifying the shifters and the productivities of the firm in Home economy.

In terms of data we assume we observed firm level wages, domestic sales, exports and payroll. At the aggregate level we have information on sectoral imports and gross output. Moving forward is important to define some notation:

- We use $\{M_k\}_{k\in\mathcal{K}}$ to denote the total expenditure in imported inputs of k-type products.
- We use s_i^n to denote the payroll share of firm i and w_i the wage it pays its workers in the equilibrium.
- We use $s_{H,i}^y$ to denote the sales share of firm i in its domestic market in the equilibrium, and d_i for the value of those domestic sales.
- We use S_k^y to denote the expenditure share in goods from product market k, this includes both domestic sales and imports.

.2.1 Part I: The synthetic economy.

As reminder, the sysnthetic economy has a different set of parameters as the original economy, but rationalizes the same data and prices in equilibrium equal one. The parameters of this economy are given by,

$$
\begin{aligned}
\xi^{se}_{n,i} &= \frac{s^n_i}{\left[\sum_{\{j:\ell(j)=\ell(i)\}}\left(\frac{w_i}{w_j}\right)^{1+\eta} s^n_j\right]}, \qquad \forall i \in \mathcal{M}, \\
\xi^{se}_{Hy,i} &= s^y_{H,i}, \qquad \forall i \in \mathcal{M}, \\
z^{se}_i &= \frac{\mu^y_i\left(s^y_{Hi}\right)}{\mu^n_i\left(s^n_i\right)} w_i, \qquad \forall i \in \mathcal{M}, \\
\upsilon^{se}_{n,l} &= \frac{S^n_l / W_l^{1+\theta}}{\sum_{\{l':\ell(j)=l\}} S^n_{l'}/W_{l'}^{1+\theta}}, \quad \forall l \in \mathcal{L}, \\
\upsilon^{se}_{y,k} &= S^y_k, \qquad \forall k \in \mathcal{K}.
\end{aligned}
$$

We note that, $\xi^{se}_{n,i} = \xi_{n,i}$ and $\upsilon^{se}_{n,l} = \upsilon_{n,l}$ since both economy rationalizes the same firm-level wages and payroll shares.

.2.2 Part II: Counterfactual equilibrium equivalence.

Since we know the fundamentals in the autarky equilibrium we also know the autarky equilibrium in the synthetic economy. The next step is to show how this synthetic economy inform us about the autarky equilibrium of the true economy, which is the object we are interested in computing. The result is summarize in the following proposition were we used $\tilde{x}$ to denote variables in the autarky equilibrium.

Proposition .2.1. If $\left\{\tilde{w}^{se}_i, \tilde{p}^{se}_i, \tilde{s}^{n,se}_i, \tilde{s}^{y,se}_{H,i}\right\}_{i\in\mathcal{M}}$ is part of an autarky equilibrium in our artificial *se* economy, then, $\left\{\tilde{w}^{se}_i, \frac{z^{se}_i}{z_i}\tilde{p}^{se}_i, \tilde{s}^{n,se}_i, \tilde{s}^{y,se}_{H,i}\right\}_{i\in\mathcal{M}}$ is an equilibrium under autarky of the true economy. Furthermore, the synthetic economy and the true economy have the same welfare level in the autarky equilibrium.

Proof. We'll prove this result in several steps, showing relationships between the quantities in the two economies in the autarky equilibrium and then using this relationship to conclude the proof.

Claim 1: If $\tilde{w}_i = \tilde{w}_i^{se}$, then $\tilde{s}_i^n = \tilde{s}_i^{n,se}$.

To show this we start with our equation for payroll shares in the true economy. These shares need to satisfy,

$$\begin{aligned}
\tilde{s}_i^n &= \xi_{n,i} \frac{(\tilde{w}_i)^{1+\eta}}{\sum_{\{j \in \mathcal{M}:\ell(j)=\ell(i)\}} \xi_{n,j} \left(\tilde{w}_j\right)^{1+\eta}} \\
&= \xi_{n,i} \frac{\left(\tilde{w}_i^{se}\right)^{1+\eta}}{\sum_{\{j \in \mathcal{M}:\ell(j)=\ell(i)\}} \xi_{n,j} \left(\tilde{w}_j^{se}\right)^{1+\eta}} \\
&= \xi_{n,i}^{se} \frac{\left(\tilde{w}_i^{se}\right)^{1+\eta}}{\sum_{\{j \in \mathcal{M}:\ell(j)=\ell(i)\}} \xi_{n,j}^{se} \left(\tilde{w}_j^{se}\right)^{1+\eta}} \\
&= \tilde{s}_i^{n,se},
\end{aligned}$$

where we used that $\tilde{w}_i = \tilde{w}_i^{se}$ and $\xi_{n,i} = \xi_{n,i}^{se}$ in the second and third equality. Note that is trivial to show that the following is also true between the two economies,

$$\tilde{W}_l^{se} = \tilde{W}_l, \quad \forall l \in \mathcal{L},$$

and therefore,

$$\tilde{W}^{se} = \tilde{W}.$$

Claim 2: If $\tilde{w}_i = \tilde{w}_i^{se}$, then $\tilde{p}_{H,i} = \tilde{p}_{H,i}^{se} p_{H,i}$.

Prices need to satisfy the first order condition of the firm. Hence,

$$\tilde{p}_{H,i} = \frac{\mu^y\left(\tilde{s}^y_{H,i}\right)}{\mu^n\left(\tilde{s}^n_{H,i}\right)}\frac{\tilde{w}_i}{z_i}.$$

Therefore,

$$\begin{aligned}
\frac{\tilde{p}_{H,i}}{\tilde{p}^{se}_{H,i}} &= \frac{\mu^y\left(\tilde{s}^y_{H,i}\right)}{\mu^n\left(\tilde{s}^n_{H,i}\right)}\frac{\tilde{w}_i}{z_i}\frac{1}{\frac{\mu^y\left(\tilde{s}^{y,se}_{H,i}\right)}{\mu^n\left(\tilde{s}^{n,se}_{H,i}\right)}\frac{\tilde{w}^{se}_i}{z^{se}_i}} \\
&= \frac{\mu^y\left(\tilde{s}^y_{H,i}\right)}{\mu^y\left(\tilde{s}^{y,se}_{H,i}\right)}\frac{z^{se}_i}{z_i}.
\end{aligned}$$

Now, since the synthetic and the true economy rationalize the same set of observables we have,

$$z_i = \frac{w_i}{p_{H,i}}\frac{\mu^y\left(s^y_{H,i}\right)}{\mu^n\left(s^n_i\right)}, \quad z^{se}_i = w_i\frac{\mu^y\left(s^y_{H,i}\right)}{\mu^n\left(s^n_i\right)},$$

and so

$$\frac{\tilde{p}_{H,i}}{\tilde{p}^{se}_{H,i}} = \frac{\mu^y\left(\tilde{s}^y_{H,i}\right)}{\mu^y\left(\tilde{s}^{y,se}_{H,i}\right)}p_{H,i}.$$

We need to show now that for all $i \in \mathcal{M}$, $\tilde{s}^{y,se}_{H,i} = \tilde{s}^y_{H,i}$. To do this we start with the equation for sales share,

$$\begin{aligned}
\tilde{s}^y_{H,i} &= \frac{\xi_{Hy,i}\left(\tilde{p}_{H,i}\right)^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}}\xi_{Hy,j}\left(\tilde{p}_{H,j}\right)^{1-\sigma}} \\
&= \frac{\xi_{Hy,i}\left(\frac{\mu^y\left(\tilde{s}^y_{H,i}\right)}{\mu^y\left(\tilde{s}^{y,se}_{H,i}\right)}\tilde{p}^{se}_{H,i}p_{H,i}\right)^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}}\xi_{Hy,j}\left(\frac{\mu^y\left(\tilde{s}^y_{H,j}\right)}{\mu^y\left(\tilde{s}^{y,se}_{H,j}\right)}\tilde{p}^{se}_{H,j}p_{H,j}\right)^{1-\sigma}}.
\end{aligned}$$

At the initial equilibrium we know that, $\xi_{Hy,i}\left(p_{H,i}\right)^{1-\sigma}=s^{y}_{H,i}\left(P_{k}\right)^{1-\sigma}$. Hence,

$$\tilde{s}^{y}_{H,i}=\frac{s^{y}_{H,i}\left(\frac{\mu^{y}\left(\tilde{s}^{y}_{H,i}\right)}{\mu^{y}\left(\tilde{s}^{y,se}_{H,i}\right)}\tilde{p}^{se}_{H,i}\right)^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}}s^{y}_{H,j}\left(\frac{\mu^{y}\left(\tilde{s}^{y}_{H,j}\right)}{\mu^{y}\left(\tilde{s}^{y,se}_{H,j}\right)}\tilde{p}^{se}_{H,j}\right)^{1-\sigma}}. \tag{61}$$

From the synthetic economy we have a similar equation,

$$\tilde{s}^{y,se}_{H,i}=\frac{\xi^{se}_{Hy,i}\left(\tilde{p}^{se}_{H,i}\right)^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}}\xi^{se}_{Hy,j}\left(\tilde{p}^{se}_{H,j}\right)^{1-\sigma}},\quad \xi^{se}_{Hy,i}=s^{y}_{H,i},$$

or

$$\tilde{s}^{y,se}_{H,i}=\frac{s^{y}_{H,i}\left(\tilde{p}^{se}_{H,i}\right)^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}}s^{y}_{H,i}\left(\tilde{p}^{se}_{H,j}\right)^{1-\sigma}}. \tag{62}$$

Then, $\left\{\tilde{s}^{y,se}_{H,i},\tilde{s}^{y}_{H,i}\right\}$ need to satisfy equations (61) and (62). It is straightforward to verify that $\tilde{s}^{y,se}_{H,i}=\tilde{s}^{y}_{H,i}$ satisfies the system of equation. It follows then that $\tilde{p}_{H,i}=\tilde{p}^{se}_{H,i}p_{H,i}$ and so $\tilde{p}^{se}_{H,i}$ give us the change in firm-level prices from the true to the autarky equilibrium. Note that this also implies that, $\tilde{p}_{H,i}=\frac{z^{se}_{i}}{z_{i}}\tilde{p}^{se}_{H,i}$.

Claim 3: If $\tilde{w}_{i}=\tilde{w}^{se}_{i}$, then $\tilde{P}_{k}=\tilde{P}^{se}_{k}P_{k}$ and $\tilde{P}=\tilde{P}^{se}P$.

We start from product k price index equation,

$$\begin{aligned}
\tilde{P}_k &= \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \xi_{Hy,i}\left(\tilde{p}_{H,i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\
&= \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \xi_{Hy,i}\left(p_{H,i}\right)^{1-\sigma}\left(\tilde{p}^{se}_{H,i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\
&= \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s^{y}_{H,i}\left(\tilde{p}^{se}_{H,i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} P_k \\
&= \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \xi^{se}_{Hy,i}\left(\tilde{p}^{se}_{H,i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} P_k \\
&= \tilde{P}^{se}_k P_k.
\end{aligned}$$

Similarly, starting for the formula for the price index of the economy,

$$\begin{aligned}
\tilde{P} &= \left[\sum_{k\in\mathcal{K}} \upsilon_{y,k}\tilde{P}_k^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\
&= \left[\sum_{k\in\mathcal{K}} \upsilon_{Hy,k}\left(\tilde{P}^{se}_k P_k\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\
&= \left[\sum_{k\in\mathcal{K}} S^{y}_k\left(\tilde{P}^{se}_k\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} P \\
&= \left[\sum_{k\in\mathcal{K}} \upsilon^{se}_{y,k}\left(\tilde{P}^{se}_k\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} P \\
&= \tilde{P}^{se} P.
\end{aligned}$$

It follows then that $\tilde{P}_k = \tilde{P}^{se}_k P_k$ and so $\tilde{P}^{se}_k$ give us the change in market-level prices from the true to the autarky equilibrium. similarly, $\tilde{P}^{se}$ give us the change in the aggregate price index. Since we normalized $P = 1$ this result also implies that $\tilde{P} = \tilde{P}^{se}$.

Claim 4: If $\tilde{w}_i = \tilde{w}_i^{se}$ then $\tilde{\pi}_i = \tilde{\pi}_i^{se}\frac{\tilde{N}}{\tilde{N}^{se}}$.

We noticed that in either of the two equilibriums, labor supply and product demand can be written as:

$$n_i = \xi_{ni}\upsilon_{n\ell(i)}\left(\frac{s_i^n}{\xi_{ni}}\right)^{\frac{\eta}{1+\eta}}\left(\frac{S_{\ell(i)}^n}{\upsilon_{n,\ell(i)}}\right)^{\frac{\theta}{1+\theta}}N,$$

$$p_{Hi} = \left(\frac{s_i^y}{\xi_{Hyi}}\right)^{\frac{1}{1-\sigma}}\left(\frac{S_{\kappa(i)}^y}{\upsilon_{y\kappa(i)}}\right)^{\frac{1}{1-\epsilon}}.$$

With these two equations we can now write firm profits as:

$$\begin{aligned}
\pi_i &= \left(p_{Hi}z_i - w_i\right)n_i \\
&= z_i p_{Hi}\left(1 - \frac{\mu^n\left(s_i^n\right)}{\mu^y\left(s_{H,i}^y\right)}\right)n_i \\
&= z_i\left(\frac{s_i^y}{\xi_{Hyi}}\right)^{\frac{1}{1-\sigma}}\left(\frac{S_{\kappa(i)}^y}{\upsilon_{y\kappa(i)}}\right)^{\frac{1}{1-\epsilon}}\left(1 - \frac{\mu^n\left(s_i^n\right)}{\mu^y\left(s_{H,i}^y\right)}\right)\xi_{ni}\upsilon_{n\ell(i)}\left(\frac{s_i^n}{\xi_{ni}}\right)^{\frac{\eta}{1+\eta}}\left(\frac{S_{\ell(i)}^n}{\upsilon_{n,\ell(i)}}\right)^{\frac{\theta}{1+\theta}}N \\
&= \pi_i\left(s_i^n, S_{\ell(i)}^n, s_{Hi}^y, S_{\kappa(i)}^y\right)N.
\end{aligned}$$

So far we showed that if $\tilde{w}_i = \tilde{w}_i^{se}$, then $\tilde{s}_i^n = \tilde{s}_i^{nse}$ and $\tilde{S}_l^n = \tilde{S}_{\ell(i)}^{se,n}$ (claim 1), $\tilde{s}_{Hi}^y = \tilde{s}_{Hi}^{se,y}$ and $\tilde{S}_k^y = \tilde{S}_k^{se,y}$(claim 2). Therefore,

$$\tilde{\pi}_i = \tilde{\pi}_i^{se}\frac{\tilde{N}}{\tilde{N}^{se}}.$$

This implies,

$$\tilde{\Pi} = \tilde{\Pi}^{se}\frac{\tilde{N}}{\tilde{N}^{se}}.$$

Claim 5: If $\tilde{w}_i = \tilde{w}_i^{se}$ then $\tilde{C} = \tilde{C}^{se}$ and $\tilde{N} = \tilde{N}^{se}$.

The budget constraint give us the relationship:

$$C = WN + \Pi,$$

and from Claim 4 we have:

$$\tilde{C} = \left(\tilde{W} + \tilde{\Pi}\left(\tilde{s}^n, \tilde{S}^n, \tilde{s}^y, \tilde{S}^y\right)\right)\tilde{N}, \quad \tilde{C}^{se} = \left(\tilde{W} + \tilde{\Pi}\left(\tilde{s}^n, \tilde{S}^n, \tilde{s}^y, \tilde{S}^y\right)\right)\tilde{N}^{se},$$

and so

$$\frac{\tilde{C}}{\tilde{C}^{se}} = \frac{\tilde{N}}{\tilde{N}^{se}}.$$

The second equation is the first order condition between aggregate consumption and aggregate labor. When utility is linear in consumption this equation is $\bar{\varphi}^{\frac{1}{\varphi}} N^{\frac{1}{\varphi}} = W$. Since both economy have the same level of aggregate wages it is straightforward to show that $\tilde{N} = \tilde{N}^{se}$ and $\tilde{C} = \tilde{C}^{se}$. With log utility in consumption the relevant equation is $\bar{\varphi}^{\frac{1}{\varphi}} N^{\frac{1}{\varphi}} C = W$. In this case we get the system

$$\tilde{N}^{\frac{1}{\varphi}}\tilde{C} = \left(\tilde{N}^{se}\right)^{\frac{1}{\varphi}}\tilde{C}^{se}, \qquad \frac{\tilde{C}}{\tilde{C}^{se}} = \frac{\tilde{N}}{\tilde{N}^{se}}.$$

The only solution for this is again $\tilde{N} = \tilde{N}^{se}$ and $\tilde{C} = \tilde{C}^{se}$.

Claim 6: If $\tilde{w}_i = \tilde{w}_i^{se}$, then $\tilde{n}_j = \tilde{n}_j^{se}$ and $\tilde{\pi}_i = \tilde{\pi}_i^{se}$.

This follows from Claim 4 and Claim 5.

Claim 7: If $\tilde{w}_i = \tilde{w}_i^{se}$ then $\tilde{y}_{H,i} = \frac{z_i}{z_i^{se}}\tilde{y}_{H,i}^{se}$.

We start from the demand for inputs and use the previous claims to get,

$$
\begin{aligned}
\tilde{y}_{H,i} &= \xi_{Hy,i} \upsilon_{y,k} \left(\frac{\tilde{p}_{H,i}}{\tilde{P}_k} \right)^{-\sigma} \left(\frac{\tilde{P}_k}{\tilde{P}} \right)^{-\epsilon} \tilde{Y} \\
&= \xi_{Hy,i} \upsilon_{y,k} \left(\frac{\tilde{p}^{se}_{H,i} p_{H,i}}{\tilde{P}^{se}_k P_k} \right)^{-\sigma} \left(\frac{\tilde{P}^{se}_k P_k}{\tilde{P}^{se}} \right)^{-\epsilon} \tilde{Y} \\
&= \frac{1}{s^y_{H,i}} \frac{1}{S^y_k} \frac{1}{Y} y_{Hi} \tilde{y}^{se}_{H,i} \\
&= \frac{1}{p_{Hi}} \tilde{y}^{se}_{H,i}.
\end{aligned}
$$

Now from the pricing equation at the original equilibrium we have,

$$
p_{H,i} = \frac{\mu^y \left(s^y_{H,i} \right)}{\mu^n \left(s^n_i \right)} \frac{w_i}{z_i} = \frac{z^{se}_i}{z_i} \frac{\mu^y \left(s^y_{H,i} \right)}{\mu^n \left(s^n_i \right)} \frac{w_i}{z^{se}_i} = \frac{z^{se}_i}{z_i}.
$$

Putting the two equations together yields the result.

Putting together all this results we can now prove the result. We start with $\left\{ \tilde{w}^{se}_i, \tilde{p}^{se}_i, \tilde{y}^{se}_{H,i}, \tilde{n}^{se}_i \right\}$ being part of an equilibrium in the se economy. Then $\left\{ \tilde{w}^{se}_i, \frac{z^{se}_i}{z_i} \tilde{p}^{se}_i, \frac{z_i}{z^{se}_i} \tilde{y}^{se}_{H,i}, \tilde{n}^{se}_i \right\}$ is an equilibrium under autarky of the true economy:

- $\left\{ \tilde{w}^{se}_i, \frac{z^{se}_i}{z_i} \tilde{p}^{se}_i \right\}_{i \in \mathcal{M}}$ satisfy the pricing equation.
- Given $\left\{ \tilde{w}^{se}_i, \tilde{\Pi}^{se} \right\}_{i \in \mathcal{M}}$, $\left\{ \tilde{n}^{se}_i \right\}_{i \in \mathcal{M}}$ and $\tilde{C}^{se}$ is the utility maximizing choice of the household.
- Given $\left\{ \frac{z^{se}_i}{z_i} \tilde{p}^{se}_i \right\}_{i \in \mathcal{M}}$, $\left\{ \frac{z_i}{z^{se}_i} \tilde{y}^{se}_{H,i} \right\}_{i \in \mathcal{M}}$ is the profit maximizing choice of the final good consumer.
- The allocation is feasible.
- Since $\tilde{C} = \tilde{C}^{se}$ and $\tilde{N} = \tilde{N}^{se}$, the two equilibrium have the same utility levels.

Furthermore, the equilibrium of the synthetic economy has the same payroll and sales shares as the true economy. $\square$

Proposition .2.1 tell us that we can use the synthetic economy to construct the autarky values of firm-level shares, market-level shares and welfare. The next result proves the identification of welfare gains and the change in the distribution of market power.

Proposition .2.2. Given the elasticity parameters $(\sigma, \epsilon, \eta, \theta, \varphi, \bar{\varphi})$, assignment functions $\kappa(\cdot)$ and $\ell(\cdot)$, and data on (i) firm level wages, (ii) firm-level payroll and domestic sales, and (iii) and expenditure shares on sectoral imports, we can identified the gains from trade of moving from the observed trade equilibrium to the autarky equilibrium is identified, and the change in the distribution of firm-level market power.

Proof. From Proposition .2.1 we know how to compute welfare and firm shares in the autarky equilibrium. Because the shares are observed in the trade equilibrium it follows that the change in firm market power is identified.We also know welfare in autarky equilibrium, so the only thing that remains to be shown is how to compute welfare in the observed trade equilibrium.

Welfare in the trade equilibrium is given by:

$$U = \begin{cases} C - \bar{\varphi}^{\frac{1}{\varphi}} \frac{1}{1+\frac{1}{\varphi}} N^{1+\frac{1}{\varphi}} & \text{linear utility in consumption} \\ \log C - \bar{\varphi}^{\frac{1}{\varphi}} \frac{1}{1+\frac{1}{\varphi}} N^{1+\frac{1}{\varphi}} & \text{log utility in consumption} \end{cases},$$

where in both cases aggregate consumption satisfies:

$$C = Y = \sum_{i \in \mathcal{M}} p_{H,i} y_{H,i} + \sum_{i \in \mathcal{M}^*} p_{F,i} y_{F,i} = \sum_{i \in \mathcal{M}} d_i^y + \sum_{k \in \mathcal{K}} M_k,$$

that is the sum of total domestic sales plus total imports. Aggregate employment is given instead by:

$$N = \begin{cases} \frac{1}{\bar{\varphi}} W^{\varphi} & \text{linear utility in consumption} \\ \frac{1}{\bar{\varphi}} \left(\frac{W}{C}\right)^{\varphi} & \text{log utility in consumption} \end{cases},$$

where in both cases we can compute W from the information on firm wages. $\square$

.3 Algorithm to solve the Closed-economy version

In this section: (i) we described the closed economy version of the mode; and (ii) described the fixed point algorithm used to solve for the autarky equilibrium of the model.

.3.1 Closed Economy

Environment and Optimal Choices

Final Good Producer. Relative to the open economy model the main distinction here is that the production function of the final good producer uses only intermediate goods produced in the domestic economy. Total production is then,

$$Y = \left[\sum_{k \in \mathcal{K}} \upsilon_{y,k}^{\frac{1}{\epsilon}} Y_k^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}},$$

where now Y_k is given by,

$$Y_k = \left[\sum_{\{i \in \mathcal{M} : \kappa(i) = k\}} \xi_{y,i}^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

The demand for each individual variety is then described by the equation,

$$y_i = \xi_{y,i} \left(\frac{p_i}{P_{\kappa(i)}} \right)^{-\sigma} Y_{\kappa(i)}, \quad Y_{\kappa(i)} = \upsilon_{y,\kappa(i)} \left(\frac{P_{\kappa(i)}}{P} \right)^{-\epsilon} Y, \tag{63}$$

where p_i is the unit price of variety i, and $P_{\kappa(i)}$ and P are the market and aggregate price indexes given by,

$$P_k = \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \xi_{y,i} p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \quad \text{and} \quad P = \left[\sum_{k\in\mathcal{K}} \upsilon_{y,k} P_k^{1-\epsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{64}$$

Preferences. The labor supply decision of the household and demand for aggregate consumption are still given by equations (2.3), (2.2), and (2.5),

$$\begin{aligned} w_i &= \left(\frac{n_i}{N_{\ell(i)}}\right)^{\frac{1}{\eta}} \left(\frac{N_{\ell(i)}}{N}\right)^{\frac{1}{\theta}} W, \\ W_{\ell(i)}^{1+\eta} &= \sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \xi_{n,j} w_j^{1+\eta}, \\ W^{1+\theta} &= \sum_{l\in\mathcal{L}} \upsilon_{n,l} W_l^{1+\eta}, \\ N &= \frac{1}{\bar{\varphi}} \left(\frac{W}{P}\right)^{\varphi}. \end{aligned} \tag{65}$$

Intermediate Good producer. These firms engage in imperfect competition both in the product and labor market playing a static game of quantity competition. All of the firm's competitors are located in the domestic economy since trade is not allow. Each firm i chooses its quantity y_i and its employment level n_i, taking as given the quantities and employment decisions made by the other firms in the economy, but recognizing that market prices and wages vary with the firm's decision. Then, an intermediate good producer i operating in labor market $\ell(i) = l$ and selling its good in product market $\kappa(i) = k$ chooses quantity y_i and employment n_i to maximize profits. The optimal price and wage charged by the firm are related to each other according to the equation,

$$p_i = \mu_i^y \times \frac{1}{\mu_i^n} \frac{w_i}{z_i},$$

where μ_i^y and μ_i^n are the markup and the markdown of the firm, respectively, given by the equations,

$$\frac{1}{\mu_i^y} = \left[\frac{\sigma - 1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_i^y\right]^{-1}, \quad \frac{1}{\mu_i^n} = \left[\frac{\eta + 1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right],$$

where s_i^y and s_i^n are the sales and payroll share of the firm in the product and labor market where she operates.

Equilibrium in term of firm and market level shares

In this section we show how to solved for the Autarky equilibrium of the model. We start with a definition of the equilibrium and then show that it can be simplified by expressing it as function of firm and market level shares only. This transformation does not reduce the dimension of the system, however it is useful for two reasons. First, it allows for a nested fixed point algorithm. where the inner nest solves for firm level shares for given values of market level shares. The outer nest uses the firm level shares from the previous step to construct the equilibrium value of output and employment for each firm and updates the values of the sales and payroll shares at the market level. The second reason why this transformation is useful is because sales and payroll shares are by definition between zero and one and this provides stability to the nested fixed point algorithm.

Autarky equilibrium

Definition .3.1. The equilibrium of the closed economy consist of the following: a set of firm level prices $\{p_i\}_{i\in\mathcal{M}}$ and wages $\{w_i\}_{i\in\mathcal{M}}$; a set of price indexes for each product market $\{P_k\}_{k\in\mathcal{K}}$ and a set of wage indexes for each labor market $\{W_l\}_{l\in\mathcal{L}}$; an aggregate price and wage index $\{P, W\}$ and aggregate profits Π; a set of firm level quantities $\{y_i\}_{i\in\mathcal{M}}$ and employment $\{n_i\}_{i\in\mathcal{M}}$, and aggregate level of consumption of the final good Y and employment N such that:

- Given prices, wages and aggregate profits, the quantities $\{n_i, Y\}$ maximize the utility of the household.
- Given prices, the quantities $\{y_i, Y\}$ maximize profits for the final good producer.
- Given aggregates $\{W, P, N, Y\}$, $\{n_i, y_i, p_i, w_i\}$ maximize profits of the intermediate good producer.
- All product and labor markets clear.

A recursion for firm level shares

In this section, we derived the system of equations that has as solution firm level sales and payroll shares $\{s_i^y, s_i^n\}_{i\in\mathcal{M}}$ as functions of market level sales and payroll shares $\{\{S_k^y\}_{k\in\mathcal{K}}, \{S_l^n\}_{l\in\mathcal{L}}\}$. The market level shares are defined analogously to their firm level counterparts,

$$S_k^y \equiv \frac{P_k Y_k}{\sum_{k'\in\mathcal{K}} P_{k'} Y_{k'}}, \qquad S_l^n \equiv \frac{W_l N_l}{\sum_{l'\in\mathcal{L}} W_{l'} N_{l'}}.$$

We start with the following set of equations for the firm and market level shares that are standard in model with CES structures,

$$s_i^y = \xi_{y,i} \left(\frac{p_i}{P_{\kappa(i)}} \right)^{1-\sigma}, \qquad \text{and} \quad s_i^n = \xi_{n,i} \left(\frac{w_i}{W_{\ell(i)}} \right)^{1+\eta}, \tag{66}$$

where $P_{\kappa(i)}$ and $W_{\ell(i)}$ are defined in equations (64) and (65). To derive our expression for the firm level shares, we combine equation (66) with: (i) the production technology of the firm; (ii) firm i's labor supply; (iii) firm i's inverse demand function; and (iv) the optimal pricing equation of the firm. These are given by,

$$
\begin{aligned}
y_i &= z_i n_i, \\
n_i &= \xi_{n,i} \left(\frac{w_i}{W_{\ell(i)}} \right)^{\eta} N_{\ell(i)}, \\
p_i &= \left(\frac{1}{\xi_{y,i}} \frac{y_i}{Y_{\kappa(i)}} \right)^{-\frac{1}{\sigma}} P_{\kappa(i)}, \\
p_i &= \frac{\mu_i^y}{\mu_i^n} \frac{w_i}{z_i}.
\end{aligned}
$$

Step 1: An expression for firm level prices and wages. Starting with the inverse demand function, we can use the production function to eliminate y_i and then the labor supply equation to substitute out n_i,

$$
p_i = \left(\frac{z_i \xi_{n,i}}{\xi_{y,i}} \right)^{-\frac{1}{\sigma}} (w_i)^{-\frac{\eta}{\sigma}} \left(\frac{W_{\ell(i)} N_{\ell(i)}}{\left(W_{\ell(i)}\right)^{1+\eta}} \frac{\left(P_{\kappa(i)}\right)^{1-\sigma}}{P_{\kappa(i)} Y_{\kappa(i)}} \right)^{-\frac{1}{\sigma}}.
$$

Using the pricing equation to substitute out firm level wages w_i and solving for prices p_i yields,

$$
p_i = \left(\frac{\xi_{y,i}}{\xi_{n,i}} \right)^{\frac{1}{\sigma+\eta}} (z_i)^{-\frac{1+\eta}{\sigma+\eta}} \left(\frac{\mu_i^n}{\mu_i^y} \right)^{-\frac{\eta}{\sigma+\eta}} \left[\frac{P_{\kappa(i)} Y_{\kappa(i)} / P_{\kappa(i)}^{1-\sigma}}{W_{\ell(i)} N_{\ell(i)} / W_{\ell(i)}^{1+\eta}} \right]^{\frac{1}{\sigma+\eta}}, \tag{67}
$$

which implies that firm level wages are,

$$
w_i = \left(\frac{\xi_{y,i}}{\xi_{n,i}} \right)^{\frac{1}{\sigma+\eta}} (z_i)^{\frac{\sigma-1}{\sigma+\eta}} \left(\frac{\mu_i^n}{\mu_i^y} \right)^{\frac{\sigma}{\sigma+\eta}} \left[\frac{P_{\kappa(i)} Y_{\kappa(i)} / P_{\kappa(i)}^{1-\sigma}}{W_{\ell(i)} N_{\ell(i)} / W_{\ell(i)}^{1+\eta}} \right]^{\frac{1}{\sigma+\eta}}. \tag{68}
$$

Step 2: Expressing market level variables in terms of market level shares. The next step is to eliminate market level variables using instead market level shares and aggregate quantities. For this we use again that the CES structure in preference allow us to express market level quantities and market level shares as,

$$Y_k = \upsilon_{y,k} \left(\frac{P_k}{P} \right)^{-\epsilon} Y, \qquad S_k^y \equiv \frac{P_k Y_k}{PY} = \upsilon_{y,k} \left(\frac{P_k}{P} \right)^{1-\epsilon} \quad \text{for all } k \in \mathcal{K}.$$

Therefore,

$$\frac{P_{\kappa(i)} Y_{\kappa(i)}}{P_{\kappa(i)}^{1-\sigma_{\kappa(i)}}} = S_{\kappa(i)}^y \frac{PY}{P_{\kappa(i)}^{1-\sigma}} = \upsilon_{y,\kappa(i)} \left(\frac{S_{\kappa(i)}^y}{\upsilon_{y,\kappa(i)}} \right)^{\frac{\sigma-\epsilon}{1-\epsilon}} \frac{PY}{P^{1-\sigma}}. \tag{69}$$

A similar relationship between market level quantities and market level shares holds in labor markets and therefore we also have that,

$$\frac{N_{\ell(i)} W_{\ell(i)}}{W_{\ell(i)}^{1+\eta}} = \upsilon_{n,\ell(i)} \left(\frac{S_{\ell(i)}^n}{\upsilon_{n,\ell(i)}} \right)^{\frac{\theta-\eta}{1+\theta}} \frac{WN}{W^{1+\eta}}. \tag{70}$$

Step 3: Compute firm level shares. The last step involves substituting equations (67) to (70) into (66). Firm level payroll shares are,

$$s_i^n = \frac{\xi_{n,i} \left(\frac{\xi_{y,i}}{\xi_{n,i}} \right)^{\frac{1+\eta}{\sigma+\eta}} \left(\upsilon_{y,\kappa(i)} \right)^{\frac{(\sigma-1)(1+\eta)}{(\epsilon-1)(\sigma+\eta)}} \left(z_i \right)^{\frac{(\sigma-1)(1+\eta)}{\sigma+\eta}} \left(\frac{\mu_i^n}{\mu_i^y} \right)^{\frac{\sigma(1+\eta)}{\sigma+\eta}} \left(S_{\kappa(i)}^y \right)^{\frac{(\sigma-\epsilon)(1+\eta)}{(1-\epsilon)(\sigma+\eta)}}}{\sum_{\{j \in \mathcal{M}: \ell(j)=\ell(i)\}} \xi_{n,j} \left(\frac{\xi_{y,j}}{\xi_{n,j}} \right)^{\frac{1+\eta}{\sigma+\eta}} \left(\upsilon_{y,\kappa(j)} \right)^{\frac{(\sigma-1)(1+\eta)}{(\epsilon-1)(\sigma+\eta)}} \left(z_j \right)^{\frac{(\sigma-1)(1+\eta)}{\sigma+\eta}} \left(\frac{\mu_j^n}{\mu_j^y} \right)^{\frac{\sigma(1+\eta)}{\sigma+\eta}} \left(S_{\kappa(j)}^y \right)^{\frac{(\sigma-\epsilon)(1+\eta)}{(1-\epsilon)(\sigma+\eta)}}}. \tag{71}$$

Note that the denominator sums across all firm $j \in \mathcal{M}$ that share the labor market with firm i and therefore labor market aggregates cancel out. Furthermore, aggregate level variables also cancel out since those affect all firm in the same way. Finally, product market aggregates do not cancel out from our expression since firms in a labor market might be producing goods for different product markets.

Sales shares are given by,

$$s_i^y = \frac{\xi_{y,i} \left(\frac{\xi_{n,i}}{\xi_{y,i}} \right)^{\frac{\sigma-1}{\eta+\sigma}} \left(z_i \right)^{\frac{(\sigma-1)(1+\eta)}{\eta+\sigma}} \left(\frac{\mu_i^n}{\mu_i^y} \right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} \left(\upsilon_{n,\ell(i)} \right)^{\frac{(\sigma-1)(1+\eta)}{(1+\theta)(\eta+\sigma)}} \left(S_{\ell(i)}^n \right)^{\frac{(\sigma-1)(\theta-\eta)}{(1+\theta)(\eta+\sigma)}}}{\sum_{\{j \in \mathcal{M}: \kappa(j)=\kappa(i)\}} \xi_{y,i} \left(\frac{\xi_{n,j}}{\xi_{y,j}} \right)^{\frac{\sigma-1}{\eta+\sigma}} \left(z_j \right)^{\frac{(\sigma-1)(1+\eta)}{\eta+\sigma}} \left(\frac{\mu_j^n}{\mu_j^y} \right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} \left(\upsilon_{n,\ell(j)} \right)^{\frac{(\sigma-1)(1+\eta)}{(1+\theta)(\eta+\sigma)}} \left(S_{\ell(j)}^n \right)^{\frac{(\sigma-1)(\theta-\eta)}{(1+\theta)(\eta+\sigma)}}} \tag{72}$$

As before, since the denominator sums across all firm $j \in \mathcal{M}$ that share the product market with firm i product market aggregates cancel out. Labor market aggregates instead do not cancel from our expression since firms in a product market could be operating in different labor markets.

Finally, remember that firm level market power, the ratio of markups to markdowns, are only function of sales and payroll shares. So given values for market level shares $\{\{S_k^y\}_{k\in\mathcal{K}}, \{S_l^n\}_{l\in\mathcal{L}}\}$, we can use the recursion defined by the equations (71) and (72), to solved for the sales and payroll shares at the firm level.

Prices as functions of sales and payroll shares

Before describing the algorithm we use to solved for the equilibrium is convenient to express firm level prices and wages as functions of shares only. We start with the labor supply equation for firm i. We use that $s_i^n = \xi_{n,i}\left(\frac{w_i}{W_{\ell(i)}}\right)^{1+\eta}$ and $s_i^n = \upsilon_{n,\ell(i)}\left(\frac{W_{\ell(i)}}{W}\right)^{1+\theta}$ to express n_i as,

$$n_i = \tilde{n}_i N,$$

where we defined,

$$\tilde{n}_i \equiv \left(\xi_{n,i}\left(s_i^n\right)^{\eta}\right)^{\frac{1}{1+\eta}}\left(\upsilon_{n,\ell(i)}\left(S_{\ell(i)}^n\right)^{\theta}\right)^{\frac{1}{\theta+1}}.$$

Note that $\tilde{n}_i$ is only a function sale and payroll shares.

Using the production function of the intermediate good producer we can write firm level output as,

$$y_i = \tilde{y}_i N,$$

where $\tilde{y}_i \equiv z_i \tilde{n}_i$ is again only a function of shares. Therefore, market level and aggregate output are simply,

$$Y_k = \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \xi_{y,i}^{\frac{1}{\sigma}} \tilde{y}_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} N = \tilde{Y}_k N,$$
$$Y = \left[\sum_{k\in\mathcal{K}} \upsilon_{y,k}^{\frac{1}{\epsilon}} \tilde{Y}_k^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} N = \tilde{Y} N.$$

Finally, we can use the inverse demand equation to express prices as functions of tilde variables which only depend on sale and payroll shares,

$$p_i = \left(\frac{1}{\xi_{y,i}}\frac{y_i}{Y_{\kappa(i)}}\right)^{-\frac{1}{\sigma}} \left(\frac{1}{\upsilon_{y,\kappa(i)}}\frac{Y_{\kappa(i)}}{Y}\right)^{-\frac{1}{\epsilon}} P = \left(\frac{1}{\xi_{y,i}}\frac{\tilde{y}_i}{\tilde{Y}_{\kappa(i)}}\right)^{-\frac{1}{\sigma}} \left(\frac{1}{\upsilon_{y,\kappa(i)}}\frac{\tilde{Y}_{\kappa(i)}}{\tilde{Y}}\right)^{-\frac{1}{\epsilon}} \equiv \tilde{p}_i, \quad (73)$$

where the last equality uses that aggregate consumption is the numeraire of the economy.

Firm level wages can be computed through the optimal pricing equation of the firm,

$$w_i = z_i \frac{\mu_i^n}{\mu_i^y} \tilde{p}_i. \quad (74)$$

Since firm market power only depends on firm level share, and we showed prices can be written as functions of shares as well, wages are itself a function of shares only. Equation 65 can then be used to compute for each labor market the market level wage index,$\{W_l\}_{l\in\mathcal{L}}$ and the aggregate wage index $\{W\}$.

Algorithm: Autarky Equilibrium

A nested fixed point algorithm can be used to solved for the autarky equilibrium. In this section we describe the procedure. We use t^i and t^o to denote iterations for the inner and the outer nest, respectively. The outer nest start with a guess of market level payroll shares for each labor market $\left\{S_{l,t^o}^n\right\}_{l\in\mathcal{L}}$ and a guess of market level sales shares for each product market $\left\{S_{k,t^o}^y\right\}_{k\in\mathcal{K}}$. Given the parameters of the model and the market level shares,

$\left\{S^n_{l,t^o}\right\}_{l\in\mathcal{L}}, \left\{S^y_{k,t^o}\right\}_{k\in\mathcal{K}}$, we construct the firm level shares that correspond to the efficient allocation for the given values of the market level shares, and denote them as $\{s^n_{i,t^i}, s^y_{i,t^i}\}_{i\in\mathcal{M}}$. To construct these pseudo-efficient firm level shares we use equations (71) and (72), but we impose that firm level markups and markdowns are both equal to one as in the efficient allocation.

Step 1. Inner Nest. We use $\{s^n_{i,t^i}, s^y_{i,t^i}\}_{i\in\mathcal{M}}$ as the initial guess. Then:

1. We construct firm level market power as,

$$\frac{1}{\mu^y_{i,t^i}} = \frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s^y_{i,t^i}, \quad \text{and} \quad \frac{1}{\mu^n_{i,t^i}} = \frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^n_{i,t^i},$$

   and use the right-hand side of equations (71) and (72) to update the guessed value of the firm-level shares, $\{s^n_{i,t^i+1}, s^y_{i,t^i+1}\}_{i\in\mathcal{M}}$.

2. We iterate over this procedure until the resulting shares from two consecutive iterations are indistinguishable from one another. That is, we stop when the following condition is satisfy,

$$\max\left\{\max_{j\in\mathcal{M}}\left\{\left|s^n_{j,t^i+1} - s^n_{j,t^i}\right|\right\}, \max_{j\in\mathcal{M}}\left\{\left|s^y_{j,t^i+1} - s^y_{j,t^i}\right|\right\}\right\} \leq 1e^{-08}$$

3. In practice, we find useful to move from one iteration to the next one in small steps. So the updated guess is constructed as, $s_{i,t^i+1} = (1-\psi)s_{i,t^i+1} + \psi s_{i,t^i}$. We use $\psi = 0.9$.

Step 2. Outer Nest. Once Step 1 is completed we end up with our initial guess for the market level shares $\left\{S^n_{l,t^o}\right\}_{l\in\mathcal{L}}, \left\{S^y_{k,t^o}\right\}_{k\in\mathcal{K}}$, and the equilibrium values of the firm level shares that are consistent with these market level shares. We denote them as, $\{s^n_{i,out}, s^y_{i,out}\}_{i\in\mathcal{M}}$. Then,

1. We use the initial guess for the market level shares and the resulting firm level shares to construct firm level prices and firm level wages using equations (73) and (74).

2. We then construct the market and aggregate level price and wage index using the firm level prices and wages and the definition of theses indexes.

3. We update the market level shares using the formulas,

$$S^n_{l,t^o+1} = \upsilon_{n,l}\left(\frac{W_{l,t^o}}{W_{t^o}}\right)^{1+\theta}, \quad \text{and} \quad S^y_{k,t^o+1} = \upsilon_{y,k}\left(\frac{P_{k,t^o}}{P_{t^o}}\right)^{1-\epsilon}$$

4. If

$$\max\left\{\max_{l\in\mathcal{L}}\left\{\left|S^n_{l,t^o+1} - S^n_{l,t^o}\right|\right\}, \max_{k\in\mathcal{K}}\left\{\left|S^y_{k,t^o+1} - S^y_{k,t^o}\right|\right\}\right\} \leq 1e^{-08},$$

   we have found the equilibrium and the algorithm stops. If this condition does not hold, then we update the guess for the market level shares with $S_{t^o+1} = (1-\psi)S_{t^o+1} + \psi S_{t^o}$ and $\psi = 0.9$ and use it to initiate Step 1 and Step 2 again.

Step 3. After Step 1 and Step 2 are completed we end up with the market and firm level shares and all the prices and wages of the economy. The last step uses this information to construct the quantities.

Once we know the equilibrium value of the aggregate wage we can compute aggregate employment using the first order condition of the household,

$$N = \frac{1}{\bar{\varphi}}W^{\varphi}.$$

Then, from the labor supply equation we can construct firm level employment,

$$n_i = \left(\xi_{n,i}\left(s^n_i\right)^{\eta}\right)^{\frac{1}{1+\eta}}\left(\upsilon_{n,\ell(i)}\left(S^n_{\ell(i)}\right)^{\theta}\right)^{\frac{1}{\theta+1}} N,$$

and the production function give us firm level output, $y_i = z_i n_i$. We recover market level and aggregate quantities by aggregating the firm level employment and output.

.4 Market aggregation of firm market power

.4.1 Market level markdown

Proposition .4.1. When labor supply is nested CES, and firms compete for workers a la Cournot, the average wage markdown at labor market l is given by:

$$\mu_l^n \equiv \left[\frac{\bar{r}_l}{\bar{w}_l}\right]^{-1} = \left[1 + (\gamma_l^n)^{-1}\right]^{-1} = \left[1 + \frac{1}{\theta} HHI_l + \frac{1}{\eta}(1 - HHI_l)\right]^{-1},$$

where $\bar{r}_l$ and $\bar{w}_l$ are market $l's$ (employment-weighted) average marginal revenue product of labor and average wage, respectively, $\left(\gamma_l^n\right)^{-1}$ is the (payroll-weighted) average inverse elasticity of firm-specific residual labor supply across firms in market l, and $HHI_l = \sum_{\{i \in \mathcal{M}:\ell(i)=l\}} \left(s_i^n\right)^2$ is the market's payroll Herfindahl.

Proof. First we show that $1 + \left(\gamma_l^n\right)^{-1} = 1 + \frac{1}{\theta} HHI_l + \frac{1}{\eta}(1 - HHI_l)$:

$$\begin{aligned}
1 + (\gamma_l^n)^{-1} &\equiv \sum_{\{i \in \mathcal{M}:\ell(i)=l\}} s_i^n \left(1 + (\gamma_i^n)^{-1}\right) \\
&= \sum_{\{i \in \mathcal{M}:\ell(i)=l\}} s_i^n \left(\frac{\eta + 1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right) \\
&= \frac{\eta + 1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \sum_{\{i \in \mathcal{M}:\ell(i)=l\}} (s_i^n)^2 \\
&= 1 + \frac{1}{\eta}(1 - HHI_l) + \frac{1}{\theta} HHI_l.
\end{aligned}$$

Second we show that $\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(1+\left(\gamma_i^n\right)^{-1}\right) = \frac{\bar{r}_l}{\bar{w}_l}$. To see this note that,

$$\begin{aligned}
\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(1+(\gamma_i^n)^{-1}\right) &= \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(\frac{1}{\mu_i^n}\right) \\
&= \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(\frac{r_i}{w_i}\right) \\
&= \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} \frac{w_i n_i}{\sum_{\{j\in\mathcal{M}:\ell(j)=l\}} w_j n_j} \left(\frac{r_i}{w_i}\right) \\
&= \frac{\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} n_i r_i}{\sum_{\{j\in\mathcal{M}:\ell(j)=l\}} w_j n_j} \\
&= \frac{\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} \frac{n_i}{\sum_{\{h\in\mathcal{M}:\ell(h)=l\}} n_h} r_i}{\sum_{\{j\in\mathcal{M}:\ell(j)=l\}} \frac{n_j}{\sum_{\{h\in\mathcal{M}:\ell(h)=l\}} n_h} w_j} \\
&= \frac{\bar{r}_l}{\bar{w}_l}.
\end{aligned}$$

Putting the two together,

$$\frac{1}{\mu_l^n} = 1 + (\gamma_l^n)^{-1} = \frac{\bar{r}_l}{\bar{w}_l}.$$

□

Proposition .4.2. When product demand is nested CES, and firms compete a la Cournot, the average markup at product market k is given by:

$$\mu_k^y \equiv \left[\frac{\bar{mc}_k}{\bar{p}_{H,k}}\right]^{-1} = \left[1-\left(\gamma_k^y\right)^{-1}\right]^{-1} = \left[1-\frac{1}{\epsilon}\left(1-\lambda_k\right)HHI_k^d - \frac{1}{\sigma}\left(1-\left(1-\lambda_k\right)HHI_k^d\right)\right]^{-1},$$

where $\bar{mc}_k$ and $\bar{p}_{Hk}$ are market $k's$ (quantity-weighted) average marginal cost and average price, respectively, $\left(\gamma_k^y\right)^{-1}$ is the (sales-weighted) average inverse elasticity of firm-specific residual product demand across firms in market k, $HHI_k^d = \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} \left(s_i^y\right)^2$ is the market's domestic sales Herfindahl, and λ_k is expenditure share on imported goods.

Proof. First we show that $1-\left(\gamma_k^y\right)^{-1}=1-\frac{1}{\epsilon}\left(1-\lambda_k\right)HHI_k^d-\frac{1}{\sigma}\left(1-\left(1-\lambda_k\right)HHI_k^d\right)$.:

$$\begin{aligned}
1-\left(\gamma_k^y\right)^{-1} &\equiv \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(1-\left(\gamma_i^y\right)^{-1}\right)\\
&= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(\frac{1}{\mu_i^y}\right)\\
&= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(\frac{\sigma-1}{\sigma}-\left(\frac{1}{\epsilon}-\frac{1}{\sigma}\right)s_i^y\left(1-\lambda_k\right)\right)\\
&= 1-\frac{1}{\sigma}-\left(1-\lambda_k\right)\left(\frac{1}{\epsilon}-\frac{1}{\sigma}\right)\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}}\left(s_i^y\right)^2\\
&= 1-\frac{1}{\epsilon}\left(1-\lambda_k\right)HHI_k^d-\frac{1}{\sigma}\left(1-\left(1-\lambda_k\right)HHI_k^d\right).
\end{aligned}$$

Second we show that $\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n\left(1+\left(\gamma_i^n\right)^{-1}\right)=\frac{\bar{r}_l}{\bar{w}_l}$. To see this note that,

$$\begin{aligned}
\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(1-\left(\gamma_i^y\right)^{-1}\right) &= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(\frac{1}{\mu_i^y}\right)\\
&= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^n\left(\frac{mc_i}{p_{Hi}}\right)\\
&= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \frac{p_{Hi}y_{Hi}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}} p_{Hj}y_{Hj}}\frac{mc_i}{p_{Hi}}\\
&= \frac{\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}}\frac{y_{Hi}}{\sum_{\{h\in\mathcal{M}:\kappa(h)=k\}} y_{Hh}}mc_i}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}}\frac{y_{Hj}}{\sum_{\{h\in\mathcal{M}:\kappa(h)=k\}} y_{Hh}}p_{Hj}}\\
&= \frac{\bar{mc}_k}{\bar{p}_{H,k}}.
\end{aligned}$$

Putting the two together,

$$\frac{1}{\mu_l^y}\equiv 1+\left(\gamma_k^y\right)^{-1}=\frac{\bar{mc}_k}{\bar{p}_{H,k}}.$$

□

.5 Estimating the elasticities of substitution

In this section we described our strategy to estimate the four elasticities of substitutions: The two elasticities in the labor market (η and θ), and the two elasticities in the product market (σ and ϵ). The strategy discussed in this note is preliminary and it is in the process of being implemented.

.5.1 *Elasticities of substitution in Labor Markets*

Amiti et al. (2019a) provide direct empirical estimate's of firms' price responses to changes in competitor prices to answer the question: *How strong are strategic complementarities in price setting across firms?*. In this note we show how to adapt their methodology to study strategic complementarities in wage setting across firms.

Notation. We use i, j and k to denote individual firms, and l to denote labor markets. M_l represents the total number of firms in labor market l, while $\mathcal{L}$ represents the set of labor markets. We use t to denote time. We will derive the equations for a particular labor market l, so in most cases we won't index variables by l unless we have to.

We denote with $\mathbf{w}_t = \left(w_{1t}, \ldots, w_{M_l,t}\right)'$ the vector of wages paid by firms in labor market l, and use the notation $\mathbf{w}_{-it} = \left\{w_{jt}\right\}_{j \neq i}$ to denote the vector of wages that exclude firm's i. Hence, $\mathbf{w_t} = (w_{it}, \mathbf{w}_{-it})$.

Deriving our main equation.

We consider an arbitrary invertible supply system $n_i = n_i\left(\mathbf{w}, \xi\right)$ for $i \in \{1, 2, \ldots, M_l\}$ which constitutes a one-to one mapping between any vector of wages $\mathbf{w}_t$ and a corresponding vector of quantities supplied $\mathbf{n}_t$, given the vector of supply shifters $\xi_t = \left\{\xi_{tj}\right\}_{j=1}^{M^l}$. The supply shifters summarize all variables that move the quantity supplied for a given wage vector. The invertibility assumption rules out the case of perfect substitutes, where multiple allocations of quantities across individuals are consistent with the same common wage.

Under these assumptions on supply and market structure, we prove the existence of a markdown function, which characterizes the firm's optimal wage-setting strategy in a given labor market equilibrium.

Proposition .5.1. For any given invertible supply system, there exists a markdown function $\mu_{it} = \mathcal{M}_i\left(w_{it}, \mathbf{w}_{-it}; \xi_t\right)$ such that the firm's static profit minimization wage $\tilde{w}_{it}$ is the solution to the following fixed point equation, for any given wage vector of the competitors $\mathbf{w}_{-it}$:

$$\tilde{w}_{it} = mrpl_{it} + \mathcal{M}_i\left(w_{it}, \mathbf{w}_{-it}, \xi_t\right).$$

Proof. Consider the profit maximization problem of the firm

$$\max_{n}\left\{p\left(y\right)y - w\left(n\right)n|\text{s.t. } y = f\left(n\right)\right\},$$

where the firm chooses the amount of labor n and inputs $\mathbf{x}$ to maximize profits. The firm internalize the labor supply $w\left(n\right)$ and the demand for its products $p\left(y\right)$. This case accommodates cases where the firm has market power in product markets as well. The first-order condition for labor are given by,

$$\left[p'\left(y\right)y + p\left(y\right)\right]\frac{\partial y}{\partial n} = w\left(n\right)\left[\frac{\gamma\left(\mathbf{w},\xi\right)+1}{\gamma\left(\mathbf{w},\xi\right)}\right]. \tag{75}$$

The left-hand side of this equation represents the marginal revenue product of labor ($MRPL$) of the firm. Note that $MRPL$ incorporates the effect that more output decreases the price of the firm. From (75) we obtain the expression for the optimal markdown of the firm. Defining the markdown as $\mu_i \equiv \log w - \log MRPL$, we get,

$$\mu_i = \log \frac{\gamma(\mathbf{w}, \xi)}{\gamma(\mathbf{w}, \xi) + 1},$$

where $\varepsilon(\mathbf{w}, \xi)$ is the perceived elasticity of supply. Therefore, the optimal wage of the firm solves the following fixed point equation:

$$w_i = mrpl_i + \mu_i.$$

$\square$

Proposition .5.1 does not require that competitor prices are equilibrium outcomes, as ti holds for any possible vector $\mathbf{w}_{-it}$, and corresponds to the firm's best response schedule. The full market equilibrium is achieved when equation (75) holds for every firm in the market, that is all firms are on their response schedules.

From now on we focus on equation (75) at the equilibrium,

$$w_{it} = mrpl_{it} + \mathcal{M}_i(w_{it}, \mathbf{w}_{-it}, \xi_t). \tag{76}$$

Wage change decomposition

Totally differentiating (in time) the previous expression around the equilibrium point give us the following equation for the change in wages,

$$\mathrm{d}w_{it} = \frac{1}{1 + \Gamma_{it}} \mathrm{d}mrpl_{it} + \frac{\Gamma_{-it}}{1 + \Gamma_{it}} \mathrm{d}w_{-it} + \varepsilon_{it}, \tag{77}$$

where Γ_{it} and Γ_{-it} where defined as:

$$\Gamma_{it} \equiv -\frac{\partial \mathcal{M}_i(\mathbf{w}_t, \xi_t)}{\partial w_{it}} \quad \text{and} \quad \Gamma_{-it} \equiv \sum_{\{j \neq i, j \in l\}} \frac{\partial \mathcal{M}_i(\mathbf{w}_t, \xi_t)}{\partial w_{jt}}, \tag{78}$$

and represent the own and (cumulative) competitor markdown elasticity, respectively, measuring the slope of the optimal markdown function. 'The term ε_i is firm's i effective labor supply shock and it is given by,

$$\varepsilon_{it} \equiv \frac{1}{1+\Gamma_{it}} \sum_{j=1}^{M_l} \frac{\partial \mathcal{M}_i\left(\mathbf{w}_t, \xi_t\right)}{\partial \xi_{jt}} \mathrm{d}\xi_{jt} \tag{79}$$

Finally, $\mathrm{d}w_{-it}$ is an index of competitor wage changes:

$$\mathrm{d}w_{-it} \equiv \sum_{\{j \neq i, j \in l\}} \omega_{ijt} \mathrm{d}w_{jt}, \quad \text{where} \quad \omega_{ijt} \equiv \frac{\partial \mathcal{M}_i\left(\mathbf{w}_t, \xi_t\right) / \partial w_{jt}}{\sum_{\{k \neq i, k \in l\}} \partial \mathcal{M}_i\left(\mathbf{w}_t, \xi_t\right) / \partial w_{kt}}. \tag{80}$$

Our main equation is then,

$$\mathrm{d}w_{it} = \alpha_{it} \mathrm{d}mrpl_{it} + \gamma_{it} \mathrm{d}w_{-it} + \varepsilon_{it}, \tag{81}$$

which decomposes firm i's wage change into responses to its own marginal revenue product of labor, to its competitor's wage change, and to the supply shifters captured by the residual. In practice, however, we can estimate,[51]

$$\Delta w_{it} = \alpha \Delta mrpl_{it} + \gamma \Delta w_{-it} + \varepsilon_{it}, \tag{82}$$

where $\Delta w_{it} = w_{i,t+1} - w_{it}$. The parameter α captures the mean pass-through of shocks from the marginal-revenue product of labor to wages. The parameter γ captures the degree of strategic complementarity in wage setting. These two parameters are shaped by the markdown elasticities: a higher own-markdown elasticity reduces the own pass-through, as markdowns are more accommodative of shocks, while a higher competitor markdown elasticity increases the strategic complementarity elasticity.

51. The estimated parameter γ is a measure of strategic complementarities in wage setting. A positive value for γ is evidence of oligopsonistic labor markets.

In order to empirically estimate the coefficients in the theoretical wage decomposition (82), we need to measure the competitor price index $\mathbf{d}w_{-it}$ and the marginal revenue product of labor of the firm. We now provide conditions under which both can be measure from data.

Constructing the equation regressors

Identifying dw_{-it}. Let W_{lt} denote the log payroll function of labor market l, which quantifies the cost of purchasing one unit of aggregate labor in a given labor market. Formally, it is defined as,

$$W_t \equiv \log \max_{n_{it}} \left\{ \sum_{i=1}^{M_l} \exp w_{it} + n_{it} \middle| U\left(\{n_{it}\}, N_t, \xi_t\right) = 1, N_t = 1 \right\},$$

where $U(\cdot)$ is the preference aggregator which defines the unit of market-level employment N_t.

Definition .5.1. We say that W_{lt} is a sufficient statistic for a firm's competitors wage if we can write the labor supply face by the firm as $n_{it} = n_i\left(w_{it}, W_t, \xi_t\right)$.

Proposition .5.2. If W_t is a sufficient statistic for a firm's competitors wage then the weights in the competitor wage index are proportional to the competitor payroll shares s_{jt}^n,for $j \neq i$, and given by $\omega_{ijt} \equiv \frac{s_{jt}^n}{1-s_{it}^n}$. Therefore, the index of competitor price changes simplifies to:

$$\mathbf{d}w_{-it} \equiv \sum_{\{j \neq i, j \in l\}} \frac{s_{jt}^n}{1 - s_{it}^n} \mathrm{d}w_{jt}.$$

Under the strong assumption that the perceived supply elasticity is a function of the wage of the firm relative to the market payroll function, $\gamma_{it} = \gamma_i\left(w_{it} - W_t; \xi_t\right)$, the two markdown elasticities are equal,

$$\Gamma_{-it} = \Gamma_{it}.$$

Proof. If $n_i = n_i(w_i, z; \xi)$ then we can follow the same steps as in Proposition .5.1, we can show there exists a markdown function:

$$\mu_i = \mathcal{M}_i(w_i, z; \xi) \equiv -\log \frac{\gamma_i(w_i, z; \xi) + 1}{\gamma_i(w_i, z; \xi)},$$

such that the profit maximizing wage of the firm solves $w_i = mrpl_i + \mu_i$. Now using the definition of the competitor wage change index and the properties of the log payroll function $W = W(w, \xi)$, we have:

$$\begin{aligned}
\omega_{ijt} &= \frac{\partial \mathcal{M}_i(\mathbf{w}_t, \xi_t)/\partial w_{jt}}{\sum_{\{k \neq i, k \in l\}} \partial \mathcal{M}_i(\mathbf{w}_t, \xi_t)/\partial w_{kt}} \\
&= \frac{(\partial \mathcal{M}_i(w_i, W; \xi)/\partial W) \times \left(\partial W/\partial w_j\right)}{\sum_{\{k \neq i, k \in l\}} (\partial \mathcal{M}_i(w_i, W; \xi)/\partial W) \times (\partial W/\partial w_k)} \\
&= \frac{(\partial \mathcal{M}_i(w_i, W; \xi)/\partial W) \times s_j^n}{\sum_{\{k \neq i, k \in l\}} (\partial \mathcal{M}_i(w_i, W; \xi)/\partial W) \times s_k^n} \\
&= \frac{s_j^n}{\sum_{\{k \neq i, k \in l\}} \times s_k^n} \\
&= \frac{s_j^n}{1 - s_i^n}.
\end{aligned}$$

where we use the envelope condition to conclude that,

$$\frac{\partial W}{\partial w_j} = \frac{W_j N_j}{\sum_{\{k \in l\}} W_k N_k} = s_j^n.$$

If a stronger condition $\gamma_i = \gamma_i(w_i - z; \xi)$ is satisfied, then:

$$\mu_i = \mathcal{M}_i(w_i - W; \xi) \equiv \log \frac{\gamma_i(w_i - W; \xi)}{\gamma_i(w_i - W; \xi) - 1},$$

and using the definition of Γ_i and Γ_{-i} we have:

$$\Gamma_i = -\frac{\partial \mathcal{M}_i \left(\mathbf{w}_t - W, \xi_t\right)}{\partial w_i}\left(1 - \frac{\partial W}{\partial w_{it}}\right) = -\frac{\partial \mathcal{M}_i \left(\mathbf{w}_t - W, \xi_t\right)}{\partial w_i}\left(1 - s_i^n\right)$$
$$\Gamma_{-i} = -\frac{\partial \mathcal{M}_i \left(w_i - W, \xi_t\right)}{\partial w_{it}} \sum_{\{j \neq i, j \in l\}} \left(s_j^n\right) = -\frac{\partial \mathcal{M}_i \left(w_i - W, \xi_t\right)}{\partial w_{it}}\left(1 - s_i^n\right).$$

□

Definition .5.1 may seem strange at first but it holds exactly for the nested-CES structure as it show in the example below.

Nested CES. In the oligoponistic model of Berger et al. (2022a) we have:

$$N_i = \xi_i \upsilon_l \frac{N}{W^\theta}\left(\frac{W_i}{\tilde{W}_l}\right)^\eta \tilde{W}_l^\theta,$$

where $\tilde{W}_l$ is the payroll function and therefore $W_l = \log \tilde{W}_l$. Then,

$$n_i = \log\left(\xi_i \upsilon_l \frac{N}{W^\theta}\right) + \eta w_i + (\theta - \eta) W.$$

Note then that $W$is a sufficient statistic for a firm's competitors wage. Furthermore, the inverse supply curve is given by,

$$w_i = \left(\frac{1}{\xi_i}\frac{N_i}{N_l}\right)^{\frac{1}{\eta}}\left(\frac{1}{\upsilon_l}\frac{N_l}{N}\right)^{\frac{1}{\theta}} W,$$

and so the labor supply elasticity is,

$$\gamma_i \equiv \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right]^{-1},$$

which implies that $\gamma_i = \gamma_i \left(w_i - W; \xi\right)$ since,

$$s_i^n \equiv \frac{W_i N_i}{\sum W_j N_j} = \frac{\xi_i \left(W_i\right)^{1+\eta}}{\sum \xi_j \left(W_j\right)^{1+\eta}} = \xi_i e^{(1+\eta)(w_i - W)}.$$

Finally, we have

$$\mu_i = \log \frac{\gamma_i}{\gamma_i + 1} = -\log \left(\frac{\eta + 1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta} \right) s_i^n \right)$$

So the model implied markdown elasticities is:

$$\begin{aligned}
\Gamma_i &\equiv -\frac{\partial \mu_i}{\partial w_i} \\
&\equiv \frac{1}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n} \left(\frac{1}{\theta} - \frac{1}{\eta} \right) \frac{\partial s_i^n}{\partial w_i} \\
&\equiv \frac{1}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n} \left(\frac{1}{\theta} - \frac{1}{\eta} \right) s_i^n \left(1 + \eta\right) \left(1 - s_i^n\right).
\end{aligned}$$

Rearranging terms:

$$\Gamma_i = \frac{\left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n \left(1 + \eta\right) \left(1 - s_i^n\right)}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n}.$$

Also,

$$\begin{aligned}
\frac{\partial \mu_i}{\partial w_j} &= -\frac{1}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n} \left(\frac{1}{\theta} - \frac{1}{\eta} \right) \frac{\partial s_i^n}{\partial w_j} \\
\implies \Gamma_{-i} &= \frac{1}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n} \left(\frac{1}{\theta} - \frac{1}{\eta} \right) s_i^n \left(1 + \eta\right) \left(1 - s_i^n\right)
\end{aligned}$$

The main additional insight from this example is that $\Gamma_i \approx 0$ for the very small firms with $s_i^n \approx 0$. Such small firms behave nearly as constant-markup monopolistic competitors, with a complete own marginal product pass-through and no strategic complementarities. In contrast, firms with positive market shares have $\Gamma_{-it} = \Gamma_{it} > 0$ and hence exhibit incomplete pass-through and positive strategic complementarities.

Since proposition .5.2 holds true in our model we can recover dw_{-it} directly from our data. Furthermore, since $\Gamma_i = \Gamma_{-i}$ Proposition .5.2 offers a useful way to empirically test the implication of its assumptions: the condition on markdown elasticities implies that the two coefficients in the wage decomposition sum to one:

$$\alpha_{it} + \gamma_{it} = 1.$$

We do not impose this condition in our estimation, but instead test it empirically.

Identifying $d\ mrplw_{it}$**.** In order to identify the firm's marginal revenue product of labor we need to impose assumptions on the firm's production function and on firm's behavior.[52]

Assumption .5.1. Let θ_i^X denote firm $i's$ output elasticity with respect to input X. The output elasticity θ_i^X is firm specific but **time-invariant,**

$$\theta_i^X \equiv \frac{\partial \log Y_{it}}{\partial \log X_{it}}.$$

Assumption .5.2. There is at least one input (V_{it}), different from labor, that is "flexible", i.e. V_{it} is a variable input for which the firm does not have market power.

Proposition .5.3. Under assumptions .5.1 and .5.2 firm $i's$ log marginal revenue product of labor is given by,

$$mrpl_{it} = \log\left(P_{vt} V_{it}\right) - n_{it} + \log\left(\frac{\theta_i^n}{\theta_i^v}\right),$$

where P_{vt} is the price of input V, and n_{it} is firm $i's$ log employment. Furthermore,

$$d\ mrpl_{it} = d \log\left(P_{vt} V_{it}\right) - dn_{it}.$$

52. Amiti, Itskhoki and Konings (2019) impose a functional form for the marginal cost of the firm. This restriction allows them to recover changes in the marginal cost from changes in the total variable cost relative to changes in the total amount of output produced. The variable cost is taken from the data while the output is constructed from firm revenues deflated by a firm price index.

Proof. Profit maximization of the firm implies:

$$\mu_i^y + p_i + \log \frac{\partial Y_i}{\partial N_i} = w - \mu_i^n.$$

At the same time, static cost minimization of the firm implies:

$$p_v = mc_i + \log \theta_i^v + (y_i - v_i)\,, \qquad \text{and } w_i - \mu_i^n = mc_i + \log \theta_i^n + (y_i - n_i)\,.$$

Combining these three equations we get:

$$mrpl_i = p_v + v_i - n_i + \log \frac{\theta_i^n}{\theta_i^v}.$$

It follows that:

$$d\ mrpl_{it} = d \log (P_{vt} V_{it}) - dn_{it}.$$

$\square$

Proposition .5.3 tell us that we can recover changes in marginal revenue product of labor from changes in employment and firms total payments to a flexible input, both directly observed in our data.[53]

Instrumental Variables

Equation (82) has two major empirical challenge left: (i) endogeneity of the competitors wages, which are determined simultaneously with the wage of the firm in equilibrium of the wage-setting game; (ii) measurement error in the marginal revenue product of labor.

53. There are some similarities between our assumptions .5.1 and .5.2 and the assumptions imposed by the literature that estimates firm-level markups/markdowns. However, note that we don't have to fully parameterized the production function, nor we need to specify a process for firm-level productivity. This is because we care about changes in *mrpl* instead of its level, and therefore we don't have to go through the process of estimating the parameters θ_i^n and θ_i^v.

Amiti et al. (2019a) exploits rare features of their data to construct instruments to address these issues in the context of price setting behavior. Their identification strategy exploits the idiosyncratic variation in firms' marginal costs, which arises as firms, even within the same industry, source their intermediate inputs from different countries and suppliers. Then, using information on firm import prices they construct a proxy for each firms marginal cost, and construct instruments based on these proxy variables. Our identification strategy mimics their construction of instruments, but here we instrument for the marginal revenue product of labor instead of the marginal cost. We exploits the idiosyncratic variation in firms' marginal revenue product of labor, which arises as firms, even within the same labor market, produce goods for different product markets.

Baseline instruments. We delay the discussion of the instrument for now and denote it with $\mathrm{d}mrpl^*_{it}$. To address the endogeneity of the competitor wage index Δw_{-it}, we construct instruments that proxy for the marginal revenue product of labor of the different competitors faced by Ausralian firms. We construct a weighted average of the proxied $mrpl$ of firm i's competitors using the weights defined in Proposition .5.2:

$$\Delta mrpl^*_{-it} \equiv \sum_{j \neq i} \frac{s^n_{jt}}{1 - s^n_{it}} \Delta mrpl^*_{jt}.$$

Recovering the elasticities.

The parameters estimated in equation (82), α and γ, are related to the elasticity of substitution within (η) and across labor markets (θ). We have the following relationships:[54]

$$\alpha_i = \frac{1}{1 + \Gamma_{it}}, \quad \text{and} \quad \Gamma_{-it} = \Gamma_{it} = \frac{\left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^n_i \left(1 + \eta\right) \left(1 - s^n_i\right)}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^n_i}.$$

54. We do not impose in the estimation the restriction $\alpha + \gamma = 1$. Therefore, this is a testable implication and provides evidence on whether our theoretical model is a good description of Australia's labor markets.

Because our model implies a relationship between α and γ we have one estimate to recover two parameters (η, θ). We fixed a value for θ and chose η to match our estimate of α using BLADE data to construct the average value for Γ_i.

Our choice of fixing θ instead of η is because papers that have estimated a similar labor supply structure have found similar values for θ, but there is a larger variation in the point estimates of η (see for example Berger et al., 2022a; Felix, 2021).

.5.2 *Elasticity of substitution in Product Markets*

To recover the two parameters in the product markets, we note that our model implies the following non-linear relationship between the share of firm's revenue that is pay to workers, and the sales and payroll shares of the firm for non-exporters:

$$\frac{w_i n_i}{p_{Hi} y_{Hi}} = \frac{\mu_i^n}{\mu_i^y} = \frac{\frac{\sigma-1}{\sigma}}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n} - \frac{\left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_i^y}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n}.$$

Given knowledge of the parameters η and θ we can use this relationship in the data to recover σ and ϵ.

CHAPTER 3

TRADE POLICY AND GLOBAL SOURCING: AN EFFICIENCY RATIONALE FOR TARIFF ESCALATION

Joint work with Pol Antràs, Teresa C. Fort, and Felix Tintelnot

Abstract. Import tariffs tend to be higher for final goods than for inputs, a phenomenon commonly referred to as tariff escalation. We show that tariff escalation can be rationalized on efficiency grounds in the presence of scale economies. When both downstream and upstream sectors produce under increasing returns to scale, a unilateral tariff in either sector boosts the size and productivity of that sector, raising welfare. While these forces are reinforced up the chain for final-good tariffs, input tariffs may drive final-good producers to relocate abroad, mitigating their potential productivity benefits. The welfare benefits of final-good tariffs thus tend to be larger, with the optimal degree of tariff escalation increasing in the extent of downstream returns to scale. A quantitative evaluation of the US-China trade war demonstrates that any welfare gains from the increase in US tariffs are overwhelmingly driven by final-good tariffs.

3.1 Introduction

Import tariffs tend to be lower on intermediate inputs than on final goods. This pattern has been documented in multiple studies spanning numerous countries across five decades (Travis, 1964; Balassa, 1965; Bown and Crowley, 2016; Shapiro, 2020), and is commonly referred to as 'tariff escalation,' a term that captures the fact that tariffs 'escalate' down the production chain. Figure 3.1 illustrates the prevalence of tariff escalation across trading partners in 2007: for almost every country-pair, the simple average of final-good tariffs is higher than average input tariffs.

Empirical research suggests that these relatively low input tariffs improve downstream firm and worker outcomes. Early papers in this area document significant productivity gains from lower input tariffs (Amiti and Konings, 2007; Goldberg et al., 2010; Topalova and Khandelwal, 2011), while new evidence shows that recent US input tariff hikes harmed US manufacturing employment (Flaaen and Pierce, 2019) and exports (Handley et al., 2020).[1]

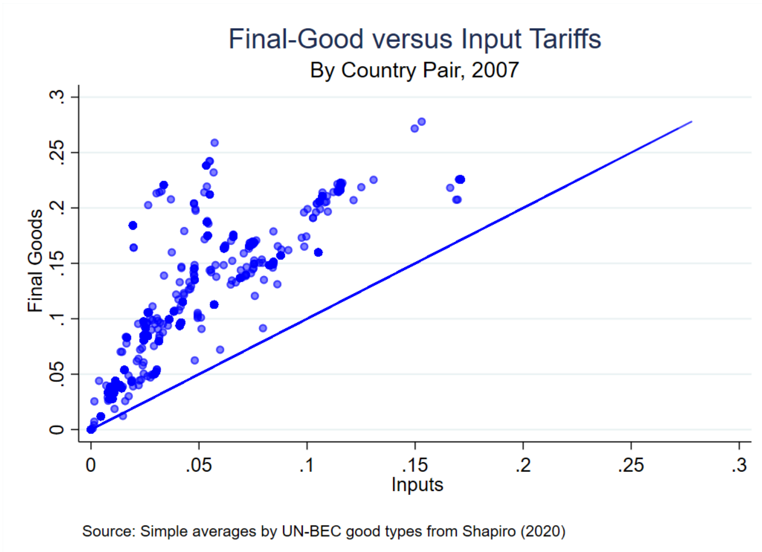
Despite the ubiquity of tariff escalation and mounting evidence on the benefits of relatively low input tariffs, existing trade theory provides little guidance for why tariff escalation might be welfare enhancing. Early neoclassical models with homogeneous goods analyze input versus final-good tariffs explicitly, but do not show that optimal tariffs should be lower for inputs. Modern Ricardian trade models stress that optimal tariffs should be uniform across sectors, and that tariff *de*-escalation may maximize welfare in second-best settings without export taxes.

The leading explanation for tariff escalation relies on political counter-lobbying (Cadot et al., 2004; Gawande et al., 2012), in which all firms lobby for protection of their output, but final-good producers counter-lobby against tariffs on their imported inputs. Although consumers would also prefer low final-good tariffs, this explanation assumes that collective action disincentives preclude them from lobbying. The assumption that final-good importers face a lobbying disadvantage is often cited, however it is increasingly at odds with the well-documented and rising concentration of consumer-good imports by a few large wholesale and retail firms (Basker and Van, 2010; Ganapati, 2018; Smith and Díaz, 2020).[2] Moreover, the welfare implications are similar to those from modern-Ricardian trade models: uniform tariffs across sectors raise welfare.

1. Evidence on the costs of raising prices on imported inputs is also documented for anti-dumping duties (Bown et al., 2020; Barattieri and Cacciatore, 2020) and the Bush steel tariffs (Cox, 2021). Relatedly, reductions in US trade policy uncertainty with China on firms' inputs led to relatively higher US export growth (Breinlich et al., 2021).

2. For example, in September 2018 Walmart responded to the Trump tariffs on consumer goods with a direct letter to US Trade Representative Robert Lighthizer warning of price hikes and consumer harm.

Figure 3.1: Average Final-Good versus Input Tariffs



In this paper, we analyze the market structures under which tariff escalation emerges as a social-welfare maximizing policy. Since neoclassical models with constant returns to scale do not predict that lower input tariffs are optimal, we use a general-equilibrium framework with an upstream and downstream sector that produce differentiated intermediate and final goods, respectively, both under increasing returns to scale. These scale economies provide an efficiency motive for shifting expenditure towards domestic varieties in each sector, but final-good versus input tariffs differ in their ability to do so and raise welfare. While a final-good tariff is unique in that it can be used to achieve the first-best allocation without any other downstream instruments, an input tariff is never sufficient. Although input tariffs shift expenditure towards domestic varieties, they also tend to drive final-good producers to relocate abroad. This relocation reduces the size of the downstream sector, which is detrimental to welfare when downstream production features increasing returns to scale. As a result, we find that optimal trade policy generally involves higher tariffs on final goods than inputs, with the degree of tariff escalation closely related to the downstream scale elasticity (i.e., the elasticity of industry-level productivity to scale).

We model both sectors as monopolistically competitive with scale economies that are internal to firms, but show that there is an isomorphic model with perfect competition and external economies of scale that generates identical tariff motives. In both cases, the presence of increasing returns in the *upstream* sector results in a domestic inefficiency whenever final goods are produced using labor and inputs. The upstream sector is too small, and a production subsidy to inputs shifts labor upstream, thereby expanding that sector and raising its efficiency. The optimal size of this subsidy depends *only* on the extent of increasing returns upstream, and despite pulling labor from the downstream sector, also increases final-good production. This downstream growth arises because the expanded availability of inputs raises final-good producers' labor efficiency. Intuitively, the market fails to deliver efficiency because upstream firms do not internalize their impact on downstream production, and this has a bigger impact when the increasing returns to scale upstream are larger.[3] In the closed-economy, this is the only inefficiency and it is only present when both sectors use labor. There is no motive for a downstream subsidy because there is no misallocation in consumers' expenditure allocation.

To analyze optimal tariffs, we first consider a small, open economy with a Home and Foreign country, and trade in both inputs and final goods. Home has market power over its exported varieties, but takes import prices (net of tariffs) as given. By holding Foreign price indices fixed, we can derive analytic solutions for optimal trade policy following the primal approach (Lucas and Stokey, 1983; Costinot et al., 2015).

In the open-economy, the social planner has the well-known motives to exploit her country's market power abroad (Gros, 1987), and to shift expenditure towards Home varieties to benefit from increasing returns to scale in production (Venables, 1987; Ossa, 2011). The social planner can always achieve the first-best allocation with a combination of production subsidies and export taxes in both sectors (e.g., as in Lashkaripour and Lugovskyy, 2021).

3. In the monopolistically competitive model, the optimal subsidy is increasing in the elasticity of substitution across inputs. The relationship between this elasticity and the degree of returns to scale is pinned down in our isomorphic model such that the optimal subsidy is identical in both settings.

This implementation is useful for understanding policy motives. The subsidies increase the sizes of each sector, which raises their efficiency (due to increasing returns to scale), and thus also welfare. The sizes of the optimal subsidies depend only on the degree of increasing returns (i.e., the scale elasticity) in each sector. Export taxes allow Home to exercise its market power abroad, and thus optimal taxes depend only on Foreign's elasticity of demand for each sector. There is no welfare-motive for a tariff in either sector under this implementation, though Lerner Symmetry (see Costinot and Werning, 2019) implies that they could be set to any (arbitrary) uniform level with an offsetting adjustment to the subsidies and taxes.

A key distinction between final-good versus input tariffs is that a final-good tariff is a perfect substitute for the combined downstream production subsidy and export tax, whereas an input tariff is not. A final-good tariff that depends only on the extent of scale economies downstream shifts domestic expenditures towards Home varieties, thereby increasing the size of the sector and constraining exports such that Home exploits it market power abroad optimally. By contrast, an input tariff is *never* sufficient to achieve the first-best allocation. Even when the input tariff raises upstream productivity and thus lowers Home input prices, it is less efficient than the combined subsidy and export tax, since it necessarily pulls labor from downstream firms to do so through higher wages. The first-best can thus also be achieved using a tariff as the sole downstream instrument, but always requires an upstream export tax. When the downstream sector uses labor, the first-best also requires an upstream production subsidy that depends only on the degree of increasing returns to scale in that sector.

A main goal of this paper is to assess whether real-world tariffs reflect social welfare-maximizing policies. Since export taxes are often illegal and production subsidies face strict World Trade Organization (WTO) limitations, analyzing optimal policies that rely solely on import tariffs is crucial.[4] When the downstream sector uses only inputs to produce, all labor is already employed upstream, so there is no scope to generate efficiency gains in upstream

4. For example, Article I, Section 9, Clause 5 of the US Constitution explicitly bans export taxes.

production. The Home government now exploits its market power abroad using an input tariff, but we show that tariff escalation persists, and is in fact larger, than it would have been if the upstream export tax were available. This increased escalation arises because the input tariff raises downstream firms' costs, which would lead them to relocate to Foreign without an offsetting higher downstream tariff.

When the downstream sector uses both labor and inputs to produce, the social planner now uses an input tariff to alleviate (imperfectly) the misallocation of labor identified in the closed-economy. As a result, tariff escalation is no longer necessarily optimal. Although we cannot derive a simple characterization of tariff escalation in this setting, we discuss how it is shaped by the relative sizes of increasing returns in the two sectors, and the share of labor used in downstream production. First, the optimal input tariff tends to be larger when the downstream labor share is high, because the difference between the market allocation of labor upstream versus the planner's allocation is increasing in the labor share. Second the relative size of upstream versus downstream returns to scale is now also relevant, since a tariff serves to increase a sector's size and thus its efficiency. To demonstrate the importance of these distinct channels, we solve for optimal, second-best tariffs as a function of a wide range of values for these three key parameters. In our grid search, tariff escalation is optimal in more than 90 percent of cases, with de-escalation being optimal only when the downstream labor share is high, and the input sector's scale elasticity is equal to or greater than the final-good sector's scale elasticity. For all empirically plausible parameter combinations of the downstream labor share and the scale elasticities, we always obtain tariff escalation as a solution to the planner's second-best problem in which tariffs are the only policy instruments.

We further highlight the key role of increasing returns to scale in downstream production for explaining tariff escalation by characterizing optimal policy when production features constant returns to scale. When inputs are produced under constant returns to scale but final-good production features increasing returns, optimal tariffs feature escalation across all the cases described above, including in second-best settings without subsidies or export taxes. By contrast, when downstream goods are produced under constant returns to scale, tariff escalation is only optimal when demand for downstream goods is more elastic than upstream demand.

The calibrated model is particularly helpful for analyzing second-best policies that rely solely on tariffs. In this setting, the optimal final-good tariff is 30.6 percent, versus an optimal input tariff of only 17.0 percent.[5] Tariff escalation is decreasing in the extent to which the downstream sector uses labor in production, consistent with it mimicking the upstream subsidy that is not available under a second-best implementation. As before, the solution to the social planner's second-best problem features tariff escalation for all empirically plausible parameter combinations for the downstream labor share and the scale elasticities.

Finally, we use the model to study the welfare effects of the US trade war in 2018 to 2019. Approximately 60 percent of the 2018 tariffs were on inputs, affecting nearly 20 percent of all US imports of intermediate inputs (Bown and Zhang, 2019). Our quantitative results indicate that, absent any foreign retaliation, these tariff increases would have raised US welfare by 0.12 percent, with the positive effect overwhelmingly driven by higher final-good tariffs. Once we include foreign retaliatory tariffs, the increase in US welfare shrinks to 0.02 percent. By comparison, the welfare effects net of retaliation would have been negative if input tariffs alone had been used.

Our paper contributes to the literature in several ways. First, we provide an efficiency rationale for tariff escalation in an environment with a benevolent social planner.[6] Early neoclassical models with homogeneous final goods and inputs explicitly modeled both types

5. The finding that optimal tariffs are much larger than tariff rates in the data is a well-known feature of the quantitative trade policy literature (see Costinot and Rodriguez-Clare 2014; Ossa 2014).

6. Our result on tariff escalation differs from the celebrated production efficiency result in Diamond and Mirrlees (1971), since they study a closed-economy setting in which a planner seeks to raise government revenue at the minimum efficiency cost. Tariff escalation is also distinct from the 'cascading trade protection' in Erbahar and Zi (2017), who study the effects of upstream tariffs on the *demand* for downstream tariffs, regardless of efficiency considerations.

of tariffs, but did not show that relatively lower input tariffs would raise welfare, despite their effects on downstream costs (Ruffin, 1969; Casas, 1973; Das, 1983). Recent work on optimal trade policy in multi-sector competitive Ricardian models predicts that optimal tariffs should be uniform across sectors (Costinot et al., 2015; Beshkar and Lashkaripour, 2020).[7] Blanchard et al. (2021) demonstrate that the terms-of-trade motive for final-good tariffs persists, but is dampened, in a competitive model when a country's final-good imports contain its domestic value added. We contribute to this work by analyzing input and output tariffs separately under different market structures. While uniform tariffs are optimal when both sectors produce under constant returns to scale, increasing returns to scale in downstream production provide an efficiency rationale tariff escalation, with the extent of optimal escalation increasing in the degree of these returns.[8]

Our departure from perfect competition and constant returns to scale highlights the potential for final-good and input tariffs to affect welfare by changing the mass of firms in both sectors. These production relocation effects are studied in Venables (1987) and Ossa (2011), who show that with imperfect competition and scale economies, an increase in a final-good import tariff attracts firms to a country, which in turn lowers prices and thus raises welfare. We extend the analysis by adding input trade and moving from a partial- to a general-equilibrium framework. Prior work finds that in general equilibrium with roundabout production, agglomeration forces may lead countries to specialize in manufacturing production (Krugman and Venables, 1995; Puga and Venables, 1999). Amiti (2004) introduces a two-sector model with agglomeration forces and uses numerical simulations to argue that tariff de-

7. Since the terms-of-trade motive is decreasing in a sector's export supply elasticity, the tariff escalation present in real-world trade policy might appear to be consistent with existing neoclassical theory if export supply elasticities were to be higher for intermediate inputs than for final goods. However, the data used in Shapiro (2020) indicate a weak positive correlation of 0.049 between the measure of upstreamness in Antràs et al. (2012) and the inverse export supply elasticities in Soderbery (2015).

8. Recent papers also study the effects of input tariffs in frameworks with *relational* GVCs and incomplete contracts (Ornelas and Turner, 2008; Antràs and Staiger, 2012; Ornelas and Turner, 2012; Grossman and Helpman, 2020).

escalation is optimal. Our contribution is to model input versus final-good sectors separately, to provide closed-form solutions for the first-best welfare allocations that demonstrate why final-good versus input tariff motives differ, and to demonstrate that tariff escalation is present for all empirically plausible combinations of the model's parameter space.

Finally, we add to a growing body of work that studies optimal trade policy in the presence of market power and domestic distortions. Early work shows that market power provides an incentive for final-good tariffs, even when countries are too small to affect world prices (Gros, 1987; Demidova and Rodiguez-Clare, 2009).[9] Caliendo et al. (2021) demonstrate that when domestic subsidies are unavailable, the optimal tariff tends to be lower when imports are used in production in a roundabout setting because the lower tariff mitigates a double marginalization inefficiency that arises from markups in the tradable sector. Our results differ because we model trade in both inputs and final-goods and show that tariff escalation *decreases* when an upstream subsidy is not available. We also exploit an isomorphism between the model with market power and one with external economies of scale, similar to Kucheryavyy et al. (2017), to show analytically that the first-best allocation using the minimal number of instruments features tariff escalation. This may seem at odds with Lashkaripour and Lugovskyy (2021), who characterize optimal tariffs as independent of the economy's input-output structure when optimal production subsidies are available. While we also find that a downstream production subsidy can be used instead of a downstream tariff, its use then requires an additional downstream export tax. Crucially, a final-good tariff is a perfect substitute for those instruments, while an input tariff is not. In sum, we show that optimal tariffs on inputs are lower than those on final-goods when implementing the first-best with the minimum number of domestic instruments. Extensive quantitative exploration also indicates that second-best import tariffs (which rule out all subsidies and

9. See Campolmi et al. (2014, 2018) for other recent work on optimal trade policy in the presence of domestic distortions. Other papers study the desirability of tariff escalation under various market structures (e.g., Spencer and Jones, 1991, 1992; McCorriston and Sheldon, 2011; Hwang et al., 2017).

export taxes) feature tariff escalation for all empirically plausible parameter combinations.[10]

The rest of the paper is structured as follows. In Section 3.2, we use a closed-economy version of our Krugman-style model expanded to include an intermediate-input sector to analyze domestic distortions and optimal policy. In Section 3.3, we develop the open-economy version of the model, as well as the isomorphism to a competitive economy with external economies of scale. Optimal policy for a small open economy is developed in sections 3.4 and 3.5, and we provide quantitative results for a large open economy in Section 3.6. In Section 3.7, we study the impact of small input and final-good tariffs for a large open economy, we perform a counterfactual analysis of the welfare impacts of the Trump tariffs in Section 3.8, and we conclude in Section 3.9.

3.2 A Krugman Economy with an Input Sector

In this section, we introduce a closed-economy model with an upstream (input) sector and a downstream (final-good) sector, both featuring increasing returns to scale, product differentiation, and monopolistic competition. Our framework is a simple extension of the closed-economy version of the classical model in Krugman (1980), expanded to include an intermediate-good sector. We begin with this relatively simple framework because we can derive analytical solutions to both the market equilibrium and the social planner's problem, which provide intuition for how imperfect competition and scale economies in vertical sectors lead to welfare-reducing distortions.

10. In this sense, our paper contributes to an active literature studying the quantitative implications of trade policy (Eaton and Kortum, 2002; Alvarez and Lucas Jr, 2007; Costinot and Rodriguez-Clare, 2014; Ossa, 2014).

3.2.1 Environment

Consider an economy in which the representative consumer values the consumption of differentiated varieties of manufacturing goods according to the utility function

$$U = \left(\int_0^{M^d} q^d\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1, \tag{3.1}$$

where M^d is the endogenous measure of final-good varieties produced in the economy, $q^d\left(\omega\right)$ is the quantity consumed of variety ω, and σ is the elasticity of substitution across varieties. Individuals supply one unit of labor inelastically, with L denoting the total labor force. There are no other factors of production, so labor should be interpreted as representing "equipped" labor.

Labor is used for the production of intermediate inputs (the upstream sector) and (possibly) in producing final goods (the downstream sector). More specifically, we represent technologies in the upstream and downstream sectors with

$$f^u + x^u(\varpi) = A^u \ell^u(\varpi), \qquad \varpi \in [0, M^u], \tag{3.2}$$

and

$$f^d + x^d(\omega) = A^d \ell^d(\omega)^\alpha Q^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M^d], \quad \alpha \in [0,1], \tag{3.3}$$

respectively. In these expressions, f^s denotes the fixed output requirements for entry in sector $s \in \{D, U\}$, $x^s(\omega)$ is the output produced for sale by variety ω in sector $s \in \{D, U\}$, A^s is a sector-specific technology parameter, and $Q^u(\omega)$ is a composite of all intermediate goods, which is in turn given by

$$Q^u(\omega) = \left(\int_0^{M^u} q^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi \right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \tag{3.4}$$

where $q^u(\varpi)$ is the quantity consumed of input variety ϖ. In words, the upstream sector uses only labor, and technology features increasing returns to scale due to the presence of a fixed overhead cost. The downstream sector combines labor with a continuum of intermediate inputs of measure M^u, where M^u is endogenous, and technology again exhibits increasing returns stemming from a fixed overhead cost. Notice that $\theta > 1$ governs the degree of substitutability across inputs, while $\alpha \in [0, 1]$ corresponds to the downstream labor (or value-added) intensity in production.[11]

There is an endogenous measure, M^d, of manufacturing firms in the downstream sector, each producing a single final-good variety. Analogously, there is an endogenous measure, M^u, of manufacturing firms in the upstream sector, each producing a single intermediate-input variety. All entrants have access to the same technologies in (3.2), (3.3) and (3.4). Market structure in both sectors is characterized by monopolistic competition and free entry.

3.2.2 Equilibrium and Efficiency

Given the CES assumptions built into our framework and the lack of strategic interactions, firms in both sectors charge a constant markup over their marginal cost, which combined with free entry, pins down firm size according to (see Appendix .1 for details):

$$x^u = (\theta - 1)f^u, \qquad x^d = (\sigma - 1)f^d. \tag{3.5}$$

Naturally, in equilibrium we must have $x^d = q^d$ and $x^u = M^d q^u$. Invoking households' demand for downstream goods and labor-market clearing (see Appendix .1), we can determine the measure of upstream and downstream firms in the economy:

11. Note that we specify the fixed costs of production in terms of output rather than labor. This assumption is immaterial for our main results, and it avoids introducing additional sources of inefficiency into our framework (see also Costinot and Rodriguez-Clare, 2014, footnote 20).

$$M^u = \frac{(1-\alpha)A^u L}{f^u \theta}; \tag{3.6}$$

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((\theta-1)\, f^u\right)^{1-\alpha} \left(\frac{(1-\alpha)A^u}{f^u \theta}\right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}}. \tag{3.7}$$

Finally, welfare of the representative consumer is simply given by $U = \left(M^d\right)^{\frac{\sigma}{\sigma-1}} q^d$, where M^d is given in (3.7) and $q^d = x^d$ in (3.5). When $\alpha \to 1$, we obtain

$$U = \left(\frac{A^d}{f^d \sigma} L\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) f^d,$$

which is the standard formula in Krugman (1980).[12] Welfare is increasing in market size with an elasticity equal to $\sigma/(\sigma-1) > 1$, reflecting the variety gains associated with living in an economy that provides a larger number of final-good varieties.

As in the "Krugman" benchmark, our model features scale effects, with welfare increasing in the size of the labor force L. In addition, these scale effects are even larger when the upstream sector is active (i.e., $\alpha < 1$). To see this, we can write welfare as

$$U = \left(\frac{(\sigma-1)A^d/\sigma}{\left((\sigma-1)f^d\right)^{\frac{1}{\sigma}}} \left(\frac{(\theta-1)\, A^u/\theta}{\left((\sigma-1)f^u\right)^{1/\theta}}\right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}} \right)^{\frac{\sigma}{\sigma-1}} \xi_\alpha, \tag{3.8}$$

where ξ_α is a function of only α and θ. Note that $\frac{\theta-\alpha}{\theta-1} \geq 1$, and thus the elasticity of welfare with respect to L is larger when $\alpha < 1$.

To gain a better understanding of the role of imperfect competition and increasing returns to scale in shaping welfare in our closed economy, in Appendix .1.2 we characterize the social optimum in our model, and explore conditions under which the above market equilibrium is efficient. There, we prove the following result:

12. A small and largely immaterial point of departure from Krugman (1980) is the fact that we have modeled the productivity terms A^d and A^u as shaping both the marginal and fixed costs of production. As a result, firm size is independent of these productivity parameters, but these parameters affect welfare directly.

Proposition 3.2.1. In the decentralized equilibrium, firm-level output is at its socially optimal level in both sectors, but the market equilibrium features too little entry into both the downstream and upstream sectors unless $\alpha = 1$ (so the upstream sector is shut down) or $\alpha = 0$ (so the downstream sector does not use labor directly in production).

Why is the market equilibrium typically inefficient? It might seem intuitive that this inefficiency is associated with upstream markups leading to a double-marginalization inefficiency. However, combining equations (3.5), (3.6) and (3.7), one can show that the aggregate market allocation of labor to the upstream sector is given by:

$$M^u \ell^u = (1 - \alpha)L; \tag{3.9}$$

and is in fact independent of the degree of input substitutability (θ) and thus of the level of upstream markups. In other words, lower input substitutability – and thus higher markups – do *not* depress the market allocation of labor to the upstream sector. Intuitively, although markups reduce the demand for intermediate inputs, they also induce entry of new firms into the upstream sector, and these two effects cancel each other, as often occurs with CES preferences.

Instead, lower input substitutability and higher markups matter for welfare because they *increase* the social-welfare maximizing allocation of labor to that sector. More specifically, in Appendix .1.2 we show that the social planner would allocate a share of labor to that sector equal to:

$$M^u \ell^u = \frac{\theta}{\theta - \alpha}(1 - \alpha)L > (1 - \alpha)L, \tag{3.10}$$

which is decreasing in θ. The intuition for this result is as follows: in the market equilibrium, upstream firms do not internalize the fact that their entry generates positive spillovers for firms in the downstream sector, with the size of this vertical spillover decreasing in the degree of input substitutability θ.

To reinforce this interpretation, in Appendix .1.4, we show that the equilibrium of our vertical Krugman economy is isomorphic to that of a competitive vertical economy with external economies of scale under specific relationships between the elasticities of substitution and the external economies of scale parameters (cf. Kucheryavyy et al., 2017). In this variant of our model, there are no markups and it is clear that the market inefficiency is due only to upstream suppliers failing to internalize the positive productivity effects of their entry on downstream firms. This isomorphism will also prove useful in characterizing optimal trade policy in the open economy (see, in particular, Section 3.3.2).

Although this vertical closed economy is generically inefficient, Proposition 3.2.1 highlights the fact that efficiency is achieved when $\alpha = 1$ or $\alpha = 0$. The intuition for this result is straightforward: in those cases, all labor is allocated to either the downstream sector (when $\alpha = 1$) or to the upstream sector (when $\alpha = 0$), and because firm-level output is always efficient (see Proposition 3.2.1), there is no scope for a market inefficiency.

3.2.3 Optimal Policy

To analyze how a government can restore efficiency, suppose that it has the ability to provide production subsidies (or charge production taxes). We denote these taxes by s^d and s^u in the downstream and upstream sectors, respectively, and assume that subsidy proceeds are extracted from households (or tax revenue is rebated to households) in a lump-sum manner.

In Appendix .1.3 we show that downstream subsidies s^d have no impact on the market allocation, while the following implementation result applies:

Proposition 3.2.2. The social planner can restore efficiency in the market equilibrium by subsidizing upstream production at a rate $(s^u)^* = 1/\theta$.

This upstream subsidy serves the role of increasing the size of the upstream sector. To do so, it must reallocate labor upstream, and in our two-sector model, this necessarily draws labor from downstream firms. While increasing labor upstream clearly raises upstream output, notice that the optimal upstream subsidy also increases downstream output, despite the fact that the sector now employs less labor. This is due to the increasing returns to scale upstream. By increasing the size of the upstream sector, the optimal subsidy also raises its efficiency, which provides the downstream sector with more inputs such that it also grows.

Crucially, the optimal amount of labor reallocation from the downstream to the upstream sector depends only on the upstream elasticity of substitution. Intuitively, the only 'net' source of inefficiency in our framework is the vertical spillover from entry upstream to productivity downstream, and this effect is mediated by upstream substitutability (see also Appendix .1.4). Because our downstream sector captures all consumer spending regardless of downstream substitutability, downstream subsidies are redundant instruments in our framework, and the upstream subsidy is independent of the downstream elasticity of substitution, σ.

In Appendix .1.5, we briefly develop two extensions of the simple model in this section. First, we allow the upstream sector to use the same bundle of inputs Q^u used in the final-good sector, while letting labor intensity upstream (denoted by β) differ from that downstream. Second, we outline a multi-stage extension of the model, in which the input bundle used in upstream production aggregates varieties from a yet more upstream sector, which in turns uses inputs from an even more upstream sector, and so on. In both extensions, we show that efficiency again calls for the use of subsidies in all input sectors, but not in the most downstream sector.[13]

Having explored the equilibrium and efficiency properties of a "Krugman" closed economy with an input sector, we next turn to exploring the open-economy implications of this framework. We are particularly interested in shedding light on the consequences of trade protection in both upstream and downstream sectors, and also on the optimal design of

13. Unlike in the work of Liu (2019), we do not find that optimal subsidies should necessarily be monotonically increasing in the upstreamness of a sector. The reason for this is that, unlike in Liu's work, we solve for the first-best *vector* of subsidies: when the government can set subsidies in only one sector, the size of the subsidy is indeed higher, the more upstream the sector.

these trade policies. Beyond the double-marginalization and vertical-spillovers mechanism discussed above, the determination of trade policies in our framework is shaped by both terms-of-trade considerations (as in Neoclassical Trade Theory), as well as relocation effects (as in New Trade Theory).

3.3 Open Economy Equilibrium: A Useful Isomorphism

We now consider a two-country extension of our two-sector model, which allows for (costly) international trade in both final goods and intermediate inputs. Throughout the rest of the paper, whenever the downstream sector uses labor and inputs (i.e., $\alpha \in (0,1)$), we focus on equilibria with incomplete specialization.

3.3.1 Environment with Internal Economies of Scale

There are now two countries (Home and Foreign), indexed by i or j (and sometimes by H and F), each populated by L_i consumers/workers. Trade is costly due to the presence of both iceberg trade costs and import tariffs. We denote the symmetric iceberg trade costs that apply to final goods and inputs by $\tau^d > 1$ and $\tau^u > 1$, respectively, and we denote the tariffs set by country i on imports of final goods and intermediate inputs by t_i^d and t_i^u, respectively.

We also consider additional instruments, namely domestic production subsidies (s_i^d and s_i^u) and export taxes (v_i^d and v_i^u) in both sectors. Prior work has shown how these combined instruments can be used to maximize social welfare, and we show that the same is true in our setting. Since our goal is to analyze if and when tariff escalation may arise as a real-world, welfare-maximizing policy, however, we are particularly interested in implementations that use instruments that are actually available to governments. As such, considering cases without export taxes is important, since they are rarely used in practice, and in fact are expressly prohibited in some countries, such as the United States (see U.S. Constitution, Article 1, Section 9, Clause 5). Similarly, studying situations without production subsidies is important for understanding observed policies since the WTO Agreement on Subsidies and Countervailing Measures ('SCM Agreement') significantly limits governments use of such instruments without punishment. We also note that, perhaps due to these institutional constraints, ruling out production subsidies is a widespread practice in theoretical analyses of trade policy determination in which production subsidies could well improve welfare, as exemplified in the work of Grossman and Helpman (1994) or Ossa (2011), among many others.

Denoting country $i \in \{H, F\}$ variables with i subindices, technologies upstream and downstream are now characterized by equations

$$f_i^u + x_i^u(\varpi) = A_i^u \ell_i^u(\varpi), \qquad \varpi \in [0, M_i^u], \quad i \in \{H, F\},$$

and

$$f_i^d + x_i^d(\omega) = A_i^d (\ell_i^d(\omega))^{\alpha} Q_i^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M_i^d], \quad \alpha \in [0, 1], \quad i \in \{H, F\}.$$

Because intermediate inputs are tradable, the bundle of inputs now includes both domestic and foreign input varieties:

$$Q_i^u(\omega) = \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^u} q_{ji}^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi \right) \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \quad i \in \{H, F\}.$$

The representative consumer in country i derives utility according to:

$$U_i = \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^d} q_{ji}^d(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1, \quad i \in \{H, F\}, \tag{3.11}$$

where M_j^d is the endogenous measure of firms in country j. Implicit in these equations is the fact that because trade costs are all ad-valorem and preferences are CES, all firms in all sectors find it profitable to sell in both markets. As in our closed-economy model, market structure in both sectors and both countries is characterized by monopolistic competition and free entry.

As mentioned above, the government imposes tariffs on imports of both final goods and inputs. Given symmetry across firms, the tariff revenue collected by the government is rebated to households via lump-sum transfers in an amount

$$R_i = \frac{t_i^d}{1+t_i^d} M_j^d p_{ji}^d q_{ji}^d + \frac{t_i^u}{1+t_i^u} M_i^d M_j^u p_{ji}^u q_{ji}^u + \frac{v_i^d}{1-v_i^d} M_i^d \tilde{p}_{ij}^d q_{ij}^d + \frac{v_i^u}{1-v_i^u} M_j^d M_i^u \tilde{p}_{ij}^u q_{ij}^u, \quad (3.12)$$

where p_{ji}^d and p_{ji}^u are the prices paid by consumers in i for final goods and by firms in i for inputs, and where $\tilde{p}_{ij}^d$ and $\tilde{p}_{ij}^u$ are the prices collected by producers in i when selling final goods and inputs in country j. When the government also levies production subsidies, this government balance condition needs to be modified in a straightforward manner.

3.3.2 An Isomorphic Competitive Economy with External Economies of Scale

It is not complicated to derive the equations characterizing the equilibrium of the above two-country economy as a function of the parameters of the model and the policy choices of a given country i, which we associate with Home. In our analysis of optimal policy, however, it is much more convenient and tractable to work with the equilibrium conditions of an isomorphic competitive economy with external rather than internal economies of scale.[14] In the remainder of this section, we develop such an isomorphic characterization. We provide a derivation of the equilibrium conditions of the above 'Krugman' economy with internal economies of scale in Appendix .2.1.

Consider a simpler economy in which there are only four goods: a Home final good, a

14. This isomorphism is inspired by the work of Kucheryavyy et al. (2017). We thank Steve Redding and Iván Werning for the helpful suggestion to pursue this direction.

Foreign final good, a Home intermediate input, and a Foreign intermediate input. Preferences in country $i = \{H, F\}$ are given by

$$U\left(Q_{ii}^{d}, Q_{ji}^{d}\right) = \left(\left(Q_{ii}^{d}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{ji}^{d}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{3.13}$$

where Q_{ii}^{d} is i's consumption of its local good, and Q_{ji}^{d} are imports by i of country j's good. As in our baseline model, the parameter σ governs the substitutability between the Home and Foreign goods.

The final good in each country is produced combining local labor (ℓ_i^d), the Home intermediate input (q_{ii}^u), and the Foreign intermediate input (q_{ji}^u). Technology is given by

$$x_i^d = \hat{A}_i^d F^d\left(\ell_i^d, q_{ii}^u, q_{ji}^u\right) = \hat{A}_i^d\left(\ell_i^d\right)^{\alpha}\left(\left(q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}(1-\alpha)},$$

where $\hat{A}_i^d$ is downstream productivity, α determines the labor intensity of final-good production, and θ governs the substitutability between the Home and the Foreign inputs. Downstream productivity $\hat{A}_i^d$ is in turn given by

$$\hat{A}_i^d = \bar{A}_i^d\left(F^d\left(L_i^d, Q_{ii}^u, Q_{ji}^u\right)\right)^{\gamma^d} = \bar{A}_i^d\left(\left(L_i^d\right)^{\alpha}\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\right)^{\gamma^d}, \tag{3.14}$$

and is thus an endogenous function of (i) country i's aggregate allocation of labor L_i^d to the downstream sector, (ii) its aggregate use of country i's intermediate input, and (iii) its aggregate use of country j's intermediate input. The parameter γ^d governs the degree of external economies of scale in the downstream sector, and is often referred to as the scale elasticity of this sector.

The intermediate input in each country is produced using local labor ℓ_i^u according to

$$x_i^u = \hat{A}_i^u F_i^u \left(\ell_i^u\right) = \hat{A}_i^u \ell_i^u,$$

where upstream productivity is also endogenous and given by

$$\hat{A}_i^u = \bar{A}_i^u \left(L_i^u\right)^{\gamma^u}, \tag{3.15}$$

where L_i^u is country i's aggregate allocation of labor to the upstream sector, and where the parameter γ^u governs the scale elasticity of the upstream sector.

We assume that the above technologies are available to a competitive fringe of producers in each country and sector. These producers take prices of all goods as given, and do not internalize the effects of their choices on the productivity terms $\hat{A}_i^d$ and $\hat{A}_i^u$. Given symmetry, it should be clear that in equilibrium, $\ell_i^d = L_i^d$, $\ell_i^u = L_i^u$, $q_{ii}^u = Q_{ii}^u$, and $q_{ji}^u = Q_{ji}^u$ for all firms. We can thus ignore lower-case variables hereafter.

The key conditions characterizing the decentralized market equilibrium in a given country i consist of four resource constraints and four optimality conditions. The four resource constraints are (i) an aggregate labor market constraint

$$L_i = L_i^u + L_i^d, \tag{3.16}$$

(ii)-(iii) two equations equating output produced in each sector to its use (for domestic consumption or for export)

$$\hat{A}_i^u L_i^u = Q_{ii}^u + Q_{ij}^u; \tag{3.17}$$

$$\hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} = Q_{ii}^d + Q_{ij}^d, \tag{3.18}$$

and (iv) a trade-balance condition

$$P_{ji}^{d}Q_{ji}^{d} + P_{ji}^{u}Q_{ji}^{u} = P_{ij}^{d}Q_{ij}^{d} + P_{ij}^{u}Q_{ij}^{u}. \tag{3.19}$$

In this last condition, the prices P_{ji}^{d} and P_{ji}^{u} reflect import prices collected by foreign exporters, so the domestic (country i) prices paid by the buyers of those goods are $(1+t_{i}^{d})P_{ji}^{d}$ and $(1+t_{i}^{u})P_{ji}^{u}$, respectively, where remember that t_{i}^{d} and t_{i}^{u} are the import tariffs set by country i. Similarly, the export prices P_{ji}^{d} and P_{ji}^{u} in the trade balance condition (3.19) correspond to the prices paid by foreign buyers, so the domestic (country i) price collected by the sellers of those goods are $(1-v_{i}^{d})P_{ij}^{d}$ and $(1-v_{i}^{u})P_{ij}^{u}$, respectively, where remember that v_{i}^{d} and v_{i}^{u} denote country i's downstream and upstream export taxes.

Turning to the four optimality conditions characterizing the decentralized equilibrium, the first two equations simply equate the marginal rate of substitution in final-good and intermediate-input consumption to the domestic (country i) relative price faced by the buyers of these goods, or

$$\frac{U_{Q_{ii}^{d}}\left(Q_{ii}^{d},Q_{ji}^{d}\right)}{U_{Q_{ji}^{d}}\left(Q_{ii}^{d},Q_{ji}^{d}\right)} = \frac{\left(1-v_{i}^{d}\right)}{\left(1+t_{i}^{d}\right)}\frac{P_{ij}^{d}}{P_{ji}^{d}}; \tag{3.20}$$

$$\frac{F_{Q_{ii}^{u}}^{d}\left(L_{i}^{d},Q_{ii}^{u},Q_{ji}^{u}\right)}{F_{Q_{ji}^{u}}^{d}\left(L_{i}^{d},Q_{ii}^{u},Q_{ji}^{u}\right)} = \frac{\left(1-v_{i}^{u}\right)}{\left(1+t_{i}^{u}\right)}\frac{P_{ij}^{u}}{P_{ji}^{u}}. \tag{3.21}$$

In these equations, subindices on the functions U and F^{d} denote partial derivatives of these functions with respect to the argument in the denominator. The next optimality condition ensures the equality between the benefits of exporting the domestic intermediate input to the benefits of using that amount of domestic inputs to produce an additional amount of the final good that is in turn exported:

$$\hat{A}_i^d F_{Q_{ii}^u}^d \left(L_i^d, Q_{ii}^u, Q_{ji}^u\right) = \frac{\left(1 - v_i^u\right) P_{ij}^u}{\left(1 - v_i^d\right) P_{ij}^d}. \tag{3.22}$$

The final efficiency condition equates the marginal product of labor in both sectors in terms of a common good (i.e., the final good)

$$F_{L_i^d}^d \left(L_i^d, Q_{ii}^u, Q_{ji}^u\right) = \hat{A}_i^u F_{L_i^u}^u \left(L_i^u\right) F_{Q_{ii}^u}^d \left(L_i^d, Q_{ii}^u, Q_{ji}^u\right). \tag{3.23}$$

We have developed equilibrium conditions (3.16) through (3.23) in a competitive model without meaningful firm-level decisions on entry, exporting, importing and pricing. Nevertheless, as anticipated above, in Appendix .2.2 we prove that our baseline Krugman-style model with internal economies of scale and imperfect competition delivers the exact same set of equilibrium conditions for an appropriate choice of the primitive productivity terms $\bar{A}_i^d$ and $\bar{A}_i^u$ in equations (3.14) and (3.15), and as long as the scale elasticities γ^d and γ^u are set to $\gamma^d = 1/\left(\sigma - 1\right)$ and $\gamma^u = 1/\left(\theta - 1\right)$, respectively.

We summarize this discussion as follows (the proof is in Appendix .2.2):

Proposition 3.3.1. The decentralized equilibrium of the two-country model in Section 3.3.1 featuring internal scale economies, product differentiation, and monopolistic competition can be reduced to a set of equations identical to equations (3.16) through (3.23) applying to the competitive model with external economies of scale developed in this section.

3.4 Optimal Trade Policy for a Small Open Economy with No Domestic Distortions

In this section, we consider the impact and optimal design of trade policies upstream and downstream for the special case in which the Home country is a small open economy, and in which the downstream sector does not employ labor (i.e., $\alpha = 0$). The first assumption allows us to ignore any impact of Home policies on aggregate, world price indices (though note that Home still faces a downward sloping demand curve for its differentiated products). The second assumption implies that the allocation of labor across sectors is necessarily efficient, since it is all employed upstream (see Proposition 3.2.1) and independent of trade-policy choices. Furthermore, because firm-level output levels are also socially efficient (see Appendix .2.1), trade policies can only affect the measure of final-good producers that enter in each country, and countries' relative wages. This allows us to compare our results more cleanly to those in the important contributions of Gros (1987), Venables (1987), and Ossa (2011), which feature "horizontal" models without an input sector. As in those frameworks, a combination of trade taxes applied to imports or exports is sufficient to implement the first-best allocation, so domestic subsidies are redundant, as we show later in this section.

We focus on a small, open economy because, despite the simplification on the labor side, characterizing optimal trade policy is still involved. This relatively simple example allows us to derive analytic solutions for the first- and second-best optimal policies, and to highlight the key role of the degree of scale economies in the downstream sector for our results. In addition, the solutions we derive here are an extremely good approximation for our quantitative findings in Section 3.6.

We proceed in two steps. First, we consider the (unrestricted) set of trade policies that implement the first-best in our small open economy. Because such an optimal mix necessarily involves export taxes, and often these are not available to governments (e.g., they are forbidden in the US Constitution), we also analyze the choice of second-best policies when only import tariffs are allowed. As explained in Section 3.3.2, we conduct our analysis based on our isomorphic competitive economy featuring external economies of scale because this greatly simplifies the derivations.

3.4.1 First-Best Policies

To solve for the first-best policies, we closely follow the primal approach in Costinot et al. (2015).[15] More specifically, we first consider an environment in which a (fictitious) social planner directly controls consumption and output decisions, and we derive three key conditions characterizing the structure of the optimal allocation. We then compare these conditions to those derived from a decentralized market equilibrium in which the government imposes trade taxes (see Section 3.3.2), and we finally show how the optimal allocation can be implemented through a simple combination of these taxes. We later consider the case in which the government also has access to domestic policy instruments, such as production or consumption subsidies.

A. Optimal Allocation

Consider the problem of a Home social planner who seeks to maximize welfare in equation (3.13), subject to the labor-market constraint (3.16), the output-market constraints (3.17) and (3.18), and the trade balance condition (3.19). The planner is assumed to control domestic consumption (Q^d_{HH}, Q^u_{HH}), imports (Q^d_{FH}, Q^u_{FH}) and exports (Q^d_{HF}, Q^u_{HF}) of both final goods and intermediate inputs. Based on the recasting of our model in Section 3.3.2, for the case $\alpha = 0$, this problem reduces to choosing $\left\{Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}\right\}$ to

$$\max U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

15. See Costinot et al. (2020) and Kortum and Weisbach (2021) for other recent applications of this approach.

subject to the constraints,

$$\hat{A}^u_H(L_H)\,L_H = Q^u_{HH} + Q^u_{HF}$$
$$\hat{A}^d_H\left(F^d(Q^u_{HH}, Q^u_{FH})\right)F^d(Q^u_{HH}, Q^u_{FH}) = Q^d_{HH} + Q^d_{HF}$$
$$Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}}P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}(Q^u_{HF})^{-\frac{1}{\theta}}P^u_{FF}(Q^u_{FF})^{\frac{1}{\theta}} = P^d_{FH}Q^d_{FH} + P^u_{FH}Q^u_{FH},$$

where $\hat{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)$ and $\hat{A}^u_H(L_H)$ are given in equations (3.14) and (3.15) for $\alpha = 0$, respectively, and where

$$F^d(Q^u_{HH}, Q^u_{FH}) = \left((Q^u_{HH})^{\frac{\theta-1}{\theta}} + (Q^u_{FH})^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}. \tag{3.24}$$

The first two constraints in the program above simply equate the output of each sector to its uses (domestic consumption or exports). The third constraint, a trade balance constraint, requires a bit more explanation. First, notice that this equation is derived from (3.19), after substituting $P^d_{HF} = P^d_{FF}\left(Q^d_{HF}/Q^d_{FF}\right)^{-1/\sigma}$ and $P^u_{HF} = P^u_{FF}\left(Q^u_{HF}/Q^u_{FF}\right)^{-1/\theta}$, which correspond to Foreign's inverse demand for the Home final and intermediate-input goods, respectively.[16] Although we have assumed that Home is a small open economy, the fact that it produces *differentiated* final goods and *differentiated* intermediate inputs still confers some market power to the Home government, since it perceives a downward sloping demand for its goods. Second, it may seem non-standard to introduce prices in the constraint of a planner problem, but this is precisely where our assumption of Home being a small open economy is useful. More specifically, we assume that Home is small in the sense that its policy choices have *no impact* on Foreign's domestic prices P^d_{FF} and P^u_{FF}, or on the prices P^d_{FH} and P^u_{FH} (before import tariffs) collected by Foreign exporters. As a result, the Home government treats these prices as parameters in the planner problem above.

16. These equations can in turn be derived based on the optimality conditions (3.20) and (3.21) applying when $i = F$ and $j = H$.

Working with the first-order conditions of this problem (see Appendix .3.1), we characterize the first-best allocations via the three following conditions. First, on the consumption side, the Home social planner seeks to equate the representative consumers' marginal rate of substitution with the *social* relative cost of domestic versus foreign goods

$$\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \frac{\frac{\sigma-1}{\sigma} P^d_{HF}}{P^d_{FH}}. \tag{3.25}$$

Note that the private cost of consuming domestic goods (which is equal to the opportunity cost P^d_{HF} of exporting these goods) exceeds its social cost. This wedge reflects a fairly standard rationale for terms-of-trade manipulation. In particular, for a given level of final-good production, an increase in domestic consumption Q^d_{HH} necessarily reduces exports, and this in turn raises Home's export prices and thus improves its terms of trade, even when Home is a small open economy (see Gros, 1987). Raising the private cost of imported goods or decreasing the private benefit of exporting goods by a factor $\sigma/\left(\sigma-1\right)$ restores the equality of the *relative* private and social costs of domestic and foreign goods.[17]

The second key efficiency condition is analogous to equation (3.25) and equates the marginal rate of substitution between domestic and foreign inputs in the production of final goods to the ratio of social costs of these inputs, or

$$\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} = \frac{\frac{\theta-1}{\theta} P^u_{HF}}{P^u_{FH}}. \tag{3.26}$$

The ratio of social costs of these inputs is again distinct from the ratio of their private

17. One may wonder whether an alternative interpretation of this result relates it to the fact that the markups charged by domestic firms on domestic consumers generate profits that remain in the Home country (or, more precisely, they lead to increased labor demand via firm entry seeking to dissipate those profits), while for imported goods, markups are collected by foreign firms, and thus the private and social marginal cost of those foreign goods coincide. Since we derive our results in a competitive economy with no markups, our preferred intuition is that this condition reflects market power in export markets driven by product differentiation, and has little to do with domestic market structure or scale economies. Appendix Section .3.2 provides a formal illustration of this intuition.

costs. The reason for this is analogous to the one in equation (3.25): the social cost of domestic inputs is lower than the private cost because the Home government perceives a downward sloping demand for its goods, and thus raising the private cost of imported inputs or decreasing the private benefit of exporting them is socially desirable because it allows Home to exploit its market power abroad.

The final efficiency condition is given by

$$\left(1+\gamma^{d}\right)\hat{A}_{H}^{d}F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)=\frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}}\frac{P_{HF}^{u}}{P_{HF}^{d}}, \tag{3.27}$$

and equates the benefits of exporting domestic inputs to the benefits of using them to produce final goods that are in turn exported. This third equation takes into account both the productivity enhancing effects of boosting domestic production of final goods (the first term $1+\gamma^{d}$ on the left-hand side), as well as the relative potential for Home to exploit market power in its inputs versus final goods, which is mediated by the ratio $\left(\sigma/\left(\sigma-1\right)\right)/\left(\theta/\left(\theta-1\right)\right)$ on the right-hand-side of equation (3.27). Note that when σ and θ go to infinity, Home's market power disappears, and Home becomes a small open economy in the traditional sense, i.e, in the sense of being unable to affect its terms of trade through relative price effects.

B. First-Best Trade Policies

We now compare these optimal allocations to those from the decentralized equilibrium in which the government can set import tariffs or export taxes, as derived in Section 3.3.2. It should be clear that there is a close connection between equations (3.20)–(3.22) for the decentralized equilibrium, and equations (3.25)–(3.27) characterizing the socially optimal allocations.[18]

18. In Section 3.3.2, we identified a fourth optimality condition – equation (3.23) – associated with the allocation of labor across sectors, but this condition is irrelevant when the downstream sector does not use labor (i.e. $\alpha=0$).

There are two key differences between these two sets of equations. First, the market equilibrium conditions naturally incorporate the effect of taxes in shaping individual consumers' and firms' private choices. Second, these decentralized-market equations do *not* incorporate the positive impact of downstream output expansion on productivity, or the positive effect of curtailing exports of final goods or inputs on Home's terms of trade.

A simple comparison of these sets of equations indicates that a combination of import tariffs and export taxes can achieve the first-best allocation as long as it satisfies:

$$1 + t_H^d = \left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \tag{3.28}$$

$$1 + t_H^u = 1 + \bar{T}; \tag{3.29}$$

$$1 - v_H^d = \frac{\sigma - 1}{\sigma}\left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \tag{3.30}$$

$$1 - v_H^u = \frac{\theta - 1}{\theta}\left(1 + \bar{T}\right), \tag{3.31}$$

for any arbitrary constant such that $1 + \bar{T} \geq 0$.

A few comments are in order. First, note that the level of optimal import tariffs and export taxes is indeterminate in our setting. This is a manifestation of Lerner's symmetry: optimal policies featuring a common ratio of gross import tariffs and export taxes (i.e., $\left(1 + t_H^s\right) / \left(1 - v_H^s\right)$ for $s = \{d, u\}$) deliver the exact same market allocations. Second, notice that the ratio of optimal gross import tariffs on final goods and on inputs, or the "tariff escalation wedge" *is* pinned down in our model and given by

$$\frac{1 + t_H^d}{1 + t_H^u} = 1 + \gamma^d > 1.$$

Third, the optimal allocation *cannot* be achieved with only import tariffs, since implementing the first-best requires distinct export taxes downstream and upstream.[19] Fourth, noting that

19. By Lerner symmetry, export taxes are redundant only if they can be set at the same level in all sectors.

our isomorphism applies only when $1+\gamma^d = \sigma/(\sigma-1)$, setting $\bar{T}=0$ minimizes the set of instruments necessary to achieve the first-best. In such a case, the first-best policies involve only two instruments: a downstream import tariff at a level $t_H^d = \gamma^d = 1/(\sigma-1)$ and an upstream export tax v_H^u equal to $1/\theta$. In sum, we have derived the following result:

Proposition 3.4.1. When $\alpha = 0$, the first-best allocation can be achieved with a combination of import and export trade taxes. Although, the levels of trade taxes are not uniquely pinned down, the tariff escalation wedge is necessarily given by $\left(1+t_H^d\right)/\left(1+t_H^u\right) = 1+\gamma^d > 1$. Furthermore, under the isomorphism condition $\gamma^d = 1/(\sigma-1)$ the first-best can be achieved with just a downstream import tariff t_H^d equal to $1/(\sigma-1)$ and an upstream export tax v_H^u equal to $1/\theta$.

Why do optimal policies involve higher import tariffs on final goods than on inputs? And why does the government choose to tax imports of final goods while taxing exports of inputs when using the minimum set of instruments? The key distinction between trade taxes on final goods and on inputs is as follows. A downstream import tariff or export tax shifts consumers' expenditures towards Home final-good varieties, thereby improving Home's terms of trade (Gros, 1987). The increased demand for Home's final goods in turn raises downstream productivity by expanding the size of the sector. While this mechanism is similar to prior work on relocation effects (Venables, 1987; Ossa, 2011), when $\alpha = 0$ the expansion of the downstream sector is due to increased input expenditures rather than a reallocation of labor to that sector.

Although an upstream tariff similarly redirects expenditure towards Home inputs and improves its terms of trade, it also raises Home's downstream producers' costs, which reduces their output and thus efficiency. This asymmetry between final-good and input tariffs arises because inputs are sold to firms that produce under increasing returns to scale and can relocate to Foreign, whereas final-goods are sold to consumers who do not produce under increasing returns and cannot move. As a result, the Home government has a disproportionate incentive to manipulate its terms of trade in the input sector via an export tax because it shifts expenditure on inputs towards Home firms without raising downstream firms' input costs. This is clear from equations (3.30) and (3.31), which show that the incentive to use upstream export taxes is magnified by a factor $1 + \gamma^d$ relative to the incentive to use downstream export taxes. In other words, the returns to scale in downstream production govern the benefits from increasing the size of that sector. Because the relative size of sectoral import tariffs and export taxes is constrained by (3.20) and (3.21), this in turn manifests itself in the form of a lower import tariff upstream than downstream.

This special case of our model highlights the combined importance of three key factors in the optimality of tariff escalation: (i) downstream firms' costs depend on input costs, and thus on input tariffs, (ii) downstream production features increasing returns to scale, and (iii) downstream firms can relocate to Foreign while consumers cannot. As a result, input tariffs raise downstream firms' costs, which decreases the size of the downstream sector (as firms relocate), and thus lowers its efficiency. By contrast, final-good tariffs raise consumer costs, but do not decrease the number of consumers or affect their production efficiency (in fact, downstream tariffs raise downstream efficiency by expanding the size of the sector).[20] In the next subsection, we further demonstrate why increasing returns to scale downstream are essential for optimal policy to feature escalation by analyzing a comparable model with constant returns to scale.

C. The Case of No Scale Economies Downstream

A key advantage of our isomorphic competitive economy with external economies of scale is that when $\gamma^d \to 0$, this economy converges to a competitive economy with no scale

20. In this special case of the model, input tariffs cannot increase upstream efficiency because the sector produces using only labor and is already employing all of it.

economies.[21] Using the expressions above, it is straightforward to derive first-best trade policies in this case. Specifically, equations (3.28)–(3.31) now reduce to

$$\begin{aligned}
1+t_H^d &= 1+\bar{T}; \\
1+t_H^u &= 1+\bar{T}; \\
1-v_H^d &= \frac{\sigma-1}{\sigma}\left(1+\bar{T}\right); \\
1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right),
\end{aligned}$$

for any arbitrary constant such that $1+\bar{T}\geq 0$. It is then immediate that:

Proposition 3.4.2. When $\alpha = 0$, in the absence of scale economies downstream, the first-best can be attained with a combination of import and export taxes. Although, the levels of trade taxes are not uniquely pinned down, the tariff escalation wedge $\left(1+t_H^d\right)/\left(1+t_H^u\right)$ necessarily equals 1. Furthermore, the first-best can be achieved with just a downstream export tax at a level v_H^d equal to $1/\sigma$ and an upstream export tax v_H^u equal to $1/\theta$.

This result shows that the emergence of tariff escalation is directly tied to the existence of scale economies in the downstream sector. In their absence, we obtain a result analogous to that derived by Costinot et al. (2015) and by Beshkar and Lashkaripour (2020), namely that optimal trade policy involves uniform import tariffs across sectors (regardless of their differentiation or whether they are inputs or final goods) and differential export taxes that optimally exploit Home's market power. Conversely, the presence or absence of scale economies in the upstream sector is irrelevant for the desirability of trade policies featuring tariff escalation in this setting.

We should briefly mention a caveat with the above argument. In particular, the isomorphism between our economies with internal economies of scale and with external economies

21. In this setting with all labor employed upstream by assumption, there is no scope for efficiency gains in the upstream sector, and thus increasing returns upstream are irrelevant.

imposes $\gamma^d \to 0 = 1/(\sigma - 1)$. So it would appear that as $\gamma^d \to 0$, we must necessarily have $\sigma \to \infty$, which would imply that the Home economy has no market power in downstream markets. Note, however, that even in such a case, the model without scale economies would still not generate tariff escalation.

D. General Functional Forms

It is interesting to note that even when $\sigma \to \infty$ and $\theta \to \infty$, so that Home ceases to have any market power in exports, our model continues to rationalize tariff escalation as long as $\gamma^d > 0$. The reason for this is that in deriving the first-best policies in equations (3.28)-(3.31) , we have not invoked the fact that Home preferences $U_H\left(Q^d_{HH}, Q^d_{FH}\right)$ in (3.13) or that the aggregator $F^d\left(Q^u_{HH}, Q^u_{FH}\right)$ of inputs in (3.24) are CES aggregators governed by σ and θ, respectively. As a result, the parameters σ and θ in the first-best policies are solely related to parameters governing preferences and technology *in Foreign*, not at Home; only the scale elasticity parameter γ^d in these formulas is associated with features of the Home economy. This reinforces our interpretation that the appearance of σ and θ in the formulas above is associated with standard terms-of-trade-manipulation incentives, rather than with the markups faced by domestic buyers.[22]

E. Alternative First-Best Implementations

A natural question about our results is whether and how domestic tax instruments, such as consumption or production subsidies, affect optimal tariff escalation. It is well understood that, in some settings, it is straightforward to replicate the effects of an import tariff using a combination of consumption taxes and production subsidies. In Appendix .3.3, we analyze

22. Although optimal trade policies seem to depend only on foreign demand elasticities and the extent of increasing returns in Home's downstream production, we acknowledge that the CES functional forms for $U_H\left(Q^d_{HH}, Q^d_{FH}\right)$ and $F^d\left(Q^u_{HH}, Q^u_{FH}\right)$ are crucial for the isomorphism in Section 3.3.2. As a result, the role of γ^d implies a role for the Home elasticity σ_H, since $\gamma^d = 1/\left(\sigma_H - 1\right))$ in that model (see Appendix .3.2 for more details).

first-best policies when the set of instruments is expanded to include these instruments. We summarize the results here.

First, the only way to implement the first-best using *only* two domestic instruments (analogously to the two trade policy instruments in Proposition 3.4.1) is via the use of discriminatory consumption subsidies on domestic purchases of final goods and inputs. More specifically, the first-best can be achieved via a subsidy to the consumption of domestic intermediate inputs at a rate equal to $s^u_{HH} = 1/\theta$ (which coincides with the level that implements the first-best in the closed economy), and a subsidy to the consumption of domestic final goods equal to $s^d_{HH} = 1/\sigma$. These subsidies shift domestic consumption towards Home varieties, which boosts the sectors' sizes and thus their efficiencies. At the same time, the discriminatory nature of the subsidies effectively constrains Home sales in Foreign, such that firms in both sectors optimally exploit their market power abroad.

Second, an import tariff downstream is a perfect substitute for a discriminatory consumption subsidy downstream, while an export tax upstream is a perfect substitute for a discriminatory consumption subsidy upstream. It then follows that the first-best can also be achieved with a combination of a subsidy in one sector and a trade instrument in the other sector. Whether tariff escalation remains a feature of the first-best policies is sensitive to the instruments used in the implementation.

Third, when discriminatory consumption subsidies are not available – as they typically are not – the first-best *cannot* be achieved with just two instruments, unless these two instruments are trade policy instruments, as in our implementation in Proposition 3.4.1. If the government chooses to rely on non-discriminatory production or consumption subsidies, it can achieve the first-best combining an appropriate level of these non-discriminatory subsidies with either export taxes or import tariffs in both sectors to offset the portion of the subsidy that accrues to Foreign firms or consumers. In particular, production subsidies require export taxes to offset the portion of the subsidy that accrues to Foreign, while consumption subsidies require import tariffs to offset the subsidy to Home consumers' and firms' purchases of Foreign goods. The implied tariff escalation level is naturally sensitive to which precise instruments are used in the implementation of the first-best. To reiterate, however, any implementation of the first-best involving a domestic subsidy requires at least three tax instruments, rather than just two, as in Proposition 3.4.1. In addition, it is the use of those redundant subsidies themselves that motivates the use of tariffs that do not feature escalation, rather than the underlying structure of the economy.

3.4.2 Second-Best Import Tariffs

We now consider an environment in which the only policies available to the Home government are import tariffs on final goods and on inputs. As shown in the previous sections, these tariffs are not sufficient to achieve the first-best allocation, so a natural question is whether second-best import tariffs continue to feature tariff escalation.

A. Second-Best Import Tariffs with Scale Economies

For the case of a small open economy, the primal approach developed in Costinot et al. (2015) is again very useful to characterize the second-best allocation and how they can be implemented with only import tariffs. The second-best optimal allocation seeks to solve the same problem laid out in Section 3.4.1, expanded to include an additional constraint. Specifically, while the Home planner can always ensure that the optimality conditions (3.25) and (3.26) are satisfied via an appropriate choice of import tariffs, the same is not true about the optimality condition (3.27). To see this, note that absent export taxes ($v_H^d = v_H^d = 0$), equation (3.22) reduces to

$$\hat{A}_H^d F_{Q_{HH}^u}^d \left(Q_{HH}^u, Q_{FH}^u\right) = \frac{P_{HF}^d}{P_{HF}^u} = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{(Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}, \tag{3.32}$$

which cannot be affected directly via import tariffs. As in the first-best case, the Home government will internalize (but private agents will not) the fact that the ratio of export prices is shaped by Home's relative export supply of inputs and final goods.

In Appendix .3.4, we work with the first-order conditions of the planner problem, compare them those applying to a decentralized market equilibrium with only import tariffs – which corresponds to equations (3.20)–(3.22) setting $v_H^d = v_H^u = 0$ – and establish that:

Proposition 3.4.3. When $\alpha = 0$, the second-best optimal combination of import tariffs involves an import tariff on final goods t_H^d higher than $1/(\sigma - 1)$ and a tariff escalation wedge larger than the first-best one, so $(1 + t_H^d)/(1 + t_H^u) > 1 + \gamma^d = \sigma/(\sigma - 1) > 1$.

To understand the intuition behind this result, it is useful to remember the role that export taxes serve in the set of first-best trade policies. First, export taxes are essential for manipulating differential terms-of-trade effects in final-good versus input markets. Second, upstream export taxes also provide a tool to manipulate the terms of trade for inputs in a less distortionary way (with respect to the size and productivity of the downstream sector) than upstream import tariffs. In the absence of export taxes, the Home government finds it optimal to use import tariffs on inputs to manipulate its terms of trade upstream (since it cannot rely on export taxes to do so), but the motive to do so is attenuated for inputs relative to final goods. This is for precisely the same reason that tariff escalation is present in optimal first-best trade policy. Input tariffs raise final-good producers' costs, which shrinks the size of the sector, and thus lowers its efficiency. As a result, even in the second-best without access to export taxes or subsidies, the tariff escalation wedge $(1 + t_H^d)/(1 + t_H^u)$ remains above one, and is in fact higher than in the first-best, as a higher downstream tariff is now necessary to compensate for the negative impact of upstream import tariffs on entry in the Home downstream sector.

B. Second-Best Import Tariffs with No Scale Economies

It is also instructive to characterize second-best import tariffs in the absence of scale effects. Characterizing these policies is again quite straightforward, since we need only consider the case when $\gamma^d \to 0$ in our competitive economy with external scale economies. In Appendix .3.4, we prove the following result:

Proposition 3.4.4. In the absence of scale economies, the second-best optimal combination of import tariffs involves tariff escalation (i.e., $\left(1+t_H^d\right)/\left(1+t_H^u\right) > 1$) if and only if $\sigma > \theta$.

To understand this result, it is useful to focus on the case $\sigma = \theta$. In a competitive Ricardian model, as long as optimal export taxes are common across sectors (i.e., $\sigma = \theta$ in our setting), the first-best can be implemented via either export taxes or import tariffs (Costinot et al., 2015; Beshkar and Lashkaripour, 2020). As a result, second-best import tariffs are sufficient to achieve the first-best allocations, and will necessarily be equal across sectors (regardless of their upstreamness). In such a case, this competitive model with no scale effects deliver *no* tariff escalation.

Starting from this benchmark, when $\sigma \neq \theta$, the first-best can no longer be implemented with only import tariffs. Beshkar and Lashkaripour (2017) show that in a horizontal economy without vertical links across sectors, second-best import tariffs continue to be common across sectors. In the presence of vertical links, Beshkar and Lashkaripour (2020) instead point out that import tariffs can partly mimic the effects export taxes by raising the relative price of downstream sectors. If the planner would like to set a higher export tax in one sector relative to another sector, it can adjust the relative size of the second-best import tariff on inputs to achieve the desired differential terms-of-trade manipulation. When $\sigma < \theta$, the desired export tax is higher downstream, so the government will actually implement tariffs featuring higher tariffs upstream (i.e., tariff de-escalation). Conversely, when $\sigma > \theta$, the planner would prefer a lower export tax downstream, which can be partly achieved by setting a lower tariff on intermediate inputs (i.e., tariff escalation), since this reduces the relative price of exports of the downstream good.[23]

3.5 Optimal Trade Policy for a Small Open Economy with Domestic Distortions

In this section, we consider environments in which final-good production uses both inputs and labor. As a result, the two sectors compete for workers, and as we show in Section 3.2, the intersectoral allocation of labor in the decentralized equilibrium is inefficient because too little labor is allocated to the upstream sector. This labor misallocation naturally has ramifications for the set of first-best policies – as trade taxes are no longer sufficient to achieve the optimal allocation – and also for the second-best import tariffs, since tariffs will now be used to alleviate this inefficiency.

3.5.1 First-Best Policies

We begin by studying the optimal structure of first-best policies. As in Section 3.4, we follow the primal approach in Costinot et al. (2015) and first characterize the optimal allocation, which we then show how to implement via trade taxes and domestic instruments. Because many of the derivations are analogous to those in Section 3.4, we relegate details to Appendix .4.1.

Determining the optimal allocation in this setting is analogous to the problem in Section 3.4.1, except that (i) the planner also controls the allocation of labor across sectors subject to a labor-market constraint, and (ii) productivity in *both* sectors is endogenous and shaped by the allocation of labor to each sector. More precisely, the planner chooses

23. Interestingly, Proposition 3.4.4 continues to hold unaltered when $\alpha > 0$, which is the reason why its statement does not impose the proviso $\alpha = 0$ (see Section 3.5).

$\left\{L_H^u, L_H^d, Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, Q_{HF}^u\right\}$ to

$$\max \qquad U\left(Q_{HH}^d, Q_{FH}^d\right) = \left(\left(Q_{HH}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{FH}^d\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

subject to the constraints,

$$\begin{aligned}
L_H^u + L_H^d &= L_H \\
\hat{A}_H^u\left(L_H^u\right) L_H^u &= Q_{HH}^u + Q_{HF}^u \\
\hat{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right) F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= Q_{HH}^d + Q_{HF}^d \\
Q_{HF}^d (Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} &= P_{FH}^d Q_{FH}^d + P_{FH}^u Q_{FH}^u,
\end{aligned}$$

where $\hat{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{HF}^u\right)\right)$ and $\hat{A}_H^u\left(L_H^u\right)$ are given in (3.14) and (3.15), respectively, and where

$$F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \left(L_H^d\right)^{\alpha} \left(\left(Q_{HH}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}.$$

As we show in Appendix .4.1, manipulating the first-order conditions of this problem produce three optimality conditions identical to those in equations (3.25)-(3.27), except that L_H^d now appears as an argument of the partial derivative terms associated with the function $F^d(\cdot)$ in equations (3.26) and (3.27). More substantively, the optimal allocation now also includes the following fourth optimality condition,

$$F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \left(1+\gamma^u\right) \hat{A}^u\left(L_H^u\right) F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right), \tag{3.33}$$

which equates the social value of the marginal product of labor in both sectors in terms of a common good (i.e., the final good). More specifically, the left-hand-side includes terms associated with the social marginal product of directly allocating labor to the production of final goods, while the right-hand-side contains terms related to the social marginal product of allocating labor to the upstream sector, and then using the resulting intermediate inputs to increase the production of final goods.[24]

We next compare these optimal allocations to those from a decentralized equilibrium in which the government can set taxes or subsidies on all transactions. In Section 3.4.1, and in particular, in equations (3.20)–(3.22), we show how trade taxes affect the market-equilibrium analogues of conditions (3.25)–(3.27). Because condition (3.33) is an internal optimality condition involving only domestic transactions, trade taxes cannot possibly affect it. In fact, the *only* type of policy instruments that can affect it are taxes or subsidies affecting the production or consumption of domestic inputs. In particular, denoting the subsidy for intermediate inputs by s_H^u, the market-equilibrium analogue of equation (3.33) is

$$F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \frac{1}{1-s^u_H}\hat{A}^u\left(L^u_H\right)F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right). \tag{3.34}$$

Comparing equations (3.33) and (3.34), it is clear that the implementation of the optimal allocation necessarily requires an upstream subsidy equal to

$$s^u_H = \frac{\gamma^u}{1+\gamma^u} = \frac{1}{\theta},$$

which is identical to the optimal subsidy in the closed-economy version of our model (see Proposition 3.2.2).

How does the use of this upstream domestic subsidy affect the nature of the other first-best policies? As in our analysis in Section 3.4, we first focus on the case in which the government minimizes the use of non-trade taxes. Because downstream domestic subsidies are redundant instruments in our model, we initially rule them out (though we will consider them below). If

24. The left-hand-side and right-hand-side of (3.33) do not *exactly* capture the social marginal return to labor because we have cancelled terms capturing the endogenous increase in productivity associated with the expansion of the final-good sector (see Appendix .4.1).

the government has access to an upstream production subsidy s_H^u, we show in Appendix .4.2 that the first-best instruments implementing the social optimality conditions (3.25)–(3.27) must satisfy the exact same conditions (3.28)–(3.31) as in the case when $\alpha = 0$, namely

$$\begin{aligned}
1 + t_H^d &= \left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
1 + t_H^u &= 1 + \bar{T}; \\
1 - v_H^d &= \frac{\sigma - 1}{\sigma}\left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
1 - v_H^u &= \frac{\theta - 1}{\theta}\left(1 + \bar{T}\right),
\end{aligned}$$

for any arbitrary constant such that $1 + \bar{T} \geq 0$. As a result, we can conclude that:

Proposition 3.5.1. When $\alpha > 0$, the first-best allocation can be achieved with a production subsidy for inputs, and (at least two) trade taxes associated with a tariff escalation wedge $(1 + t_H^d)/(1 + t_H^u) = 1 + \gamma^d = \sigma/\left(\sigma - 1\right) > 1$. Furthermore, the first-best can be achieved with just an upstream production subsidy s_H^u equal to $1/\theta$, a downstream import tariff at a level t_H^d equal to $1/\left(\sigma - 1\right)$, and an upstream export tax v_H^u equal to $1/\theta$.

In words, once the domestic distortion identified in the closed-economy version of our model is corrected using an upstream production subsidy, the first-best can be attained with the same trade instruments used in the simpler case in which all labor is employed upstream and there are no domestic distortions ($\alpha = 0$). As a result, first-best policies continue to feature tariff escalation for the same reasons explained in Section 3.4.

Alternative Implementations As described in the introduction, the first-best can also be achieved using domestic subsidies. To provide further intuition on the differing forces behind final-good versus input tariffs, we analyze the range of domestic policies that can achieve the first-best in Appendix .4.2 and summarize the results here.

First, the simplest way to achieve the first-best allocation is by using a *discriminatory* upstream consumption subsidy equal to $1/\theta$, and a comparable discriminatory downstream subsidy equal to $1/\sigma$. Although discriminatory subsidies are generally illegal under WTO rules precisely because they act as trade barriers, this implementation highlights the government's objective to shift Home consumption towards its own varieties, which boosts each sector's size and thus its productivity. In this implementation, there is no need for tariffs and thus no measure of escalation.

Second, and crucially for understanding the distinct motives for upstream versus downstream tariffs, a downstream tariff is a perfect substitute for this discriminatory subsidy, whereas an upstream tariff is *not*. In other words, Home can maximize social welfare using a final-good tariff equal to $1/(\sigma - 1)$, and a discriminatory consumption subsidy on inputs equal to $1/\theta$. A comparable input tariff will not achieve the first-best. Although it also shifts expenditure towards Home inputs, which increases the sector's size and efficiency, in our GE framework it does so via increased labor demand upstream and thus higher wages. These higher wages raise downstream firms' costs, which leads them to relocate to Foreign thereby reducing the size and efficiency of the downstream sector, and thus welfare.[25] In this implementation, tariff escalation is again $1 + \gamma^d = \sigma / (\sigma - 1) > 1$, as in Proposition 3.5.1 above.[26]

Third, if the government cannot use discriminatory subsidies, the first-best can also be achieved using a combination of production subsidies *and* export taxes. This implementation is studied in Lashkaripour and Lugovskyy (2021), who argue that optimal tariffs are uniform, regardless of scale economies and input-output relationships. Their argument also applies in our setting. A downstream production subsidy equal to $1/\sigma$ and export tax equal to $1/\sigma$, along with an upstream production subsidy equal to $1/\theta$ and an export tax equal to $1/\theta$ are

25. Note that the final-good tariff not only increases labor demand, but also increases input demand, which counterbalances its higher wage effects.

26. Note also that an upstream export tax can no longer replicate the effects of an upstream domestic discriminatory subsidy when $\alpha > 0$. Only the latter instrument can ensure that condition (3.33) is satisfied in the decentralized equilibrium.

sufficient to achieve the first-best, and there is in fact no welfare motive for tariffs of any size, and thus no measure of escalation.[27] Crucially, and in-line with the intuition above, a final-good tariff is a perfect substitute for the combined final-good subsidy and export tax. By contrast, an input tariff is never sufficient to satisfy the constraints in equation (3.34). In other words, and as captured in Proposition 3.5.1, the first-best allocation requires a combined production subsidy and export tax upstream, while a downstream tariff is sufficient. In this implementation, tariff escalation is again as defined in Proposition 3.5.1. Even in cases in which the government has access to a full range of instruments, the set of required instruments is always minimized when downstream domestic subsidies are *not* used, in which case achieving the first-best entails optimal tariffs that feature tariff escalation.

The Case of No Scale Economies As in Section 3.4, we analyze optimal policy in the absence of scale economies. This simply amounts to setting $\gamma^d = \gamma^u = 0$. In such a case, it is straightforward to verify that Proposition 3.4.2 continues to apply: although, the levels of first-best trade taxes are not uniquely pinned down, the tariff escalation wedge $\left(1 + t_H^d\right) / \left(1 + t_H^u\right)$ necessarily equals one, and the first-best can be achieved with just two export taxes. In fact, for optimal tariffs to be uniform, it is sufficient to set $\gamma^d = 0$, again highlighting the crucial role of downstream scale economies for generating tariff escalation.

3.5.2 Second-Best Trade Policies

We now analyze the more realistic case in which the Home government only has access to import tariffs. All implementations of the first-best in the previous section involved at least an upstream subsidy and an export tax, so it follows that the first-best *cannot* be achieved using tariffs alone. Furthermore, in the absence of subsidies and export taxes, import tariffs will seek to mimic the role they played in the first-best implementation.

27. By Lerner Symmetry, they can be set to any uniform level as long as the production subsidies and export taxes are adjusted to cancel out the tariffs' impact.

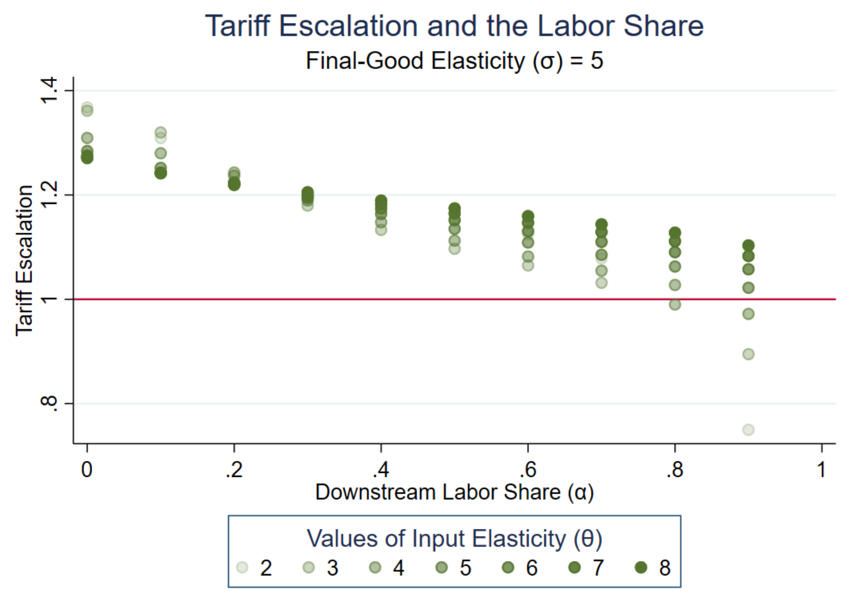
As in Section 3.4, it is straightforward to see that when upstream export taxes are ruled out, the Home government seeks to manipulate its terms of trade via upstream import tariffs. Second-best policies thus involve positive upstream import tariffs. Nevertheless, and as formalized in Proposition 3.4.3, this force is not sufficient on its own to undo the desirability of tariff escalation.

When both sectors use labor ($\alpha > 0$), however, the upstream sector's size and thus productivity are directly affected by the amount of labor it employs. As a result, the Home government can increase upstream efficiency, and potentially welfare, by shifting domestic expenditure towards Home inputs using an input tariff. The size of the downstream labor share generates two opposing forces on the welfare effects of this efficiency gain. On the one hand, the difference between the amount of labor the social planner would like to allocate upstream versus the amount used upstream in the competitive market is increasing in the final-good sector's labor share, as reflected by the difference between equations (3.9) and (3.10). A high labor share thus generates a stronger subsidy (and hence input tariff in the second-best) motive. On the other hand, as the downstream labor share rises, inputs are relatively less important for final-good output, such that the gains from a larger and thus more efficient input sector are smaller.

These countervailing forces have precluded us from obtaining an analytical solution to the second-best policies. We therefore assess the prevalence of tariff escalation as a welfare-maximizing policy in this more realistic, second-best setting by computing numerical solutions to the planner problem for a wide range of parameter values. For the key elasticities of substitution across final goods (σ) and inputs (θ), we consider discrete values from 2 to 8. Similarly, we solve the model using a labor share for downstream production (α) ranging from 0 to 0.9. We obtain solutions for 92 percent of these 490 cases.[28] The cases for which

28. Solving for optimal tariffs in this second-best setting is quite involved since we need to provide values for import prices, export demand shifters, and productivities in both sectors. We construct these using guidance from our quantitative analysis in Section 3.6 (see Appendix .5.1 for details).

Figure 3.2: Second-Best Tariff Escalation and α



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the downstream labor share (α) and upstream elasticity of substitution (θ). The downstream elasticity of substitution (σ) is fixed at 5. Appendix .5.1 shows similar patterns for additional values of σ.

we cannot obtain a solution tend to have a high labor share ($\alpha \approx 0.90$) or low values of the downstream elasticity ($\sigma \approx 2$). Technical details and a full discussion of this analysis are presented in Appendix .5.1.

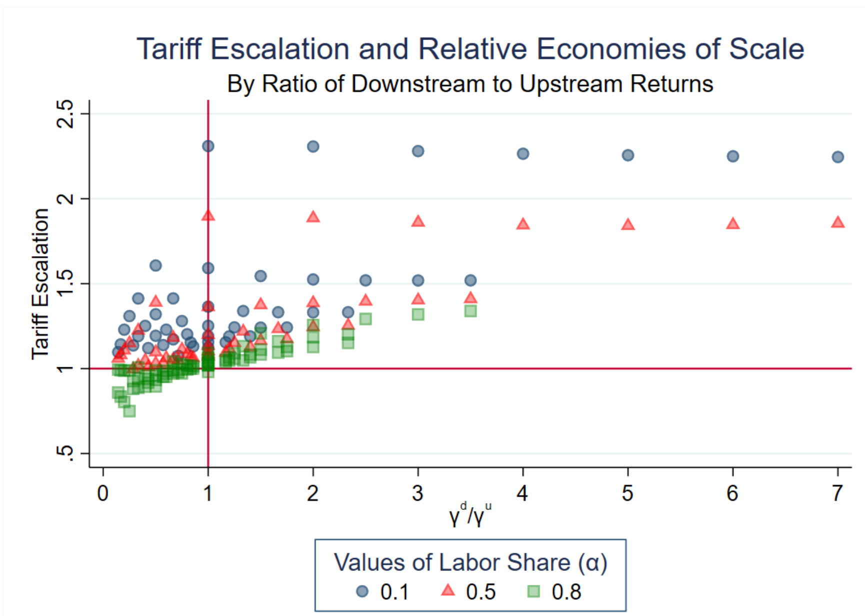
The model delivers tariff escalation in 91 percent of the solved cases. The average value of tariff escalation is 1.25 and the median is 1.16. To understand why tariff escalation tends to maximize welfare in this second-best setting, we analyze how it varies across different values of the downstream labor share and the upstream and downstream elasticities.

Figure 3.2 plots the resulting tariff escalation in this second-best setting as a function of the labor share for the range of upstream elasticities, and delivers two key messages. First, there is a strong, negative relationship between tariff escalation and the downstream labor share. This is due to the fact that the difference between the social planner's first-best labor allocation upstream versus the competitive market's allocation is increasing in α (see equations (3.9) and (3.10) in Section 3.2). When the planner cannot address this inefficiency directly with an upstream subsidy, she instead relies on an input tariff. The tariff shifts Home expenditure towards domestic inputs, which increases the upstream sector's size and thus its efficiency. Notice that unlike for the downstream sector, which can grow by accessing more/cheaper inputs, the upstream tariff can only expand via increased use of labor.[29] Second, and directly related to the efficiency gains from reallocating labor upstream, tariff escalation is less likely when the upstream elasticity is lower, which in our isomorphic model implies more returns to scale upstream. This result is precisely in-line with the fact that in the second-best, an input tariff now substitutes for an input subsidy by increasing the size of the upstream sector. When the returns to scale upstream are higher, this motive for an input tariff increases. When $\sigma = 5$, $\theta =$, 2, 3, or 4 and $\alpha = 0.9$, the model indicates that tariff de-escalation is optimal. Though we hold the downstream elasticity in this figure constant at 5, the same patterns are evident using the full range of values from 2 to 8 (see Appendix Figure 6).

Figure 3.3 depicts the importance of the relative sizes of the downstream versus upstream elasticities, again for different values of the downstream labor share. Consistent with the role of downstream returns as a motive for tariff escalation, the extent of optimal escalation is increasing in the relative size of downstream versus upstream returns to scale. In fact, tariff escalation is always optimal when the downstream returns are higher than the upstream returns. By contrast, tariff de-escalation may be optimal when the upstream returns are larger. As explained above, this is because an input tariff shifts Home expenditure towards domestic inputs, which raises the size of the sector and thus its efficiency. As evident in Figure 3.3, this efficiency motive is strongest for higher values of α. In this case, the social

29. Although we do not model roundabout production in inputs for simplicity, the results from Caliendo et al. (2021) suggest that tariff escalation would be even more prevalent in that case. Those authors analyze optimal second-best tariffs when only inputs are traded, and find that they tend to be lower when input production is roundabout.

Figure 3.3: Second-Best Tariff Escalation and Relative Scale Economies



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the relative returns to scale in downstream versus upstream production (γ^d/γ^u) and the downstream labor share (α).

planner has a larger motive to reallocate labor from downstream to upstream production, and a higher input tariff can help with this objective in the absence of subsidies.[30]

We also evaluate tariff escalation for different values of A_d, A_u, τ_d, and τ_u, while holding the downstream and upstream elasticities fixed at 5 and the downstream labor share at 0.55.[31] We obtain solutions in 92 percent of the 625 cases we consider. The average value of the ratio of optimal downstream to upstream tariffs is 1.12 and the median is also 1.12. These optimal tariffs feature escalation in 96 percent of the solved cases (see Appendix .5.1).

The Case of No Scale Economies Finally, we provide additional intuition for the key role played by increasing returns to scale in determining optimal tariffs by analyzing special cases of our model with no scale economies. When the upstream sector does not feature increasing returns to scale (i.e., $\gamma^u = 0$), the model is much more tractable and we provide analytic results that show tariff escalation is always optimal in the second-best, even when $\alpha > 0$. In this case, there is no efficiency rationale for taxing inputs, whereas increasing returns to scale downstream continue to motivate a downstream import tariff. The result in Proposition 3.4.3 in fact continues to hold even when $\alpha > 0$, and thus import tariffs result in a tariff escalation wedge larger than the first-best one, or $(1 + t_H^d)/(1 + t_H^u) > 1 + \gamma^d = \sigma/(\sigma - 1) > 1$. This highlights again the role of downstream scale economies in generating tariff escalation. Indeed, when we further set $\gamma^d = 0$, so the model features constant returns to scale in both sectors, optimal tariffs only feature tariff escalation when $\sigma > \theta$, just as in our previous Proposition 3.4.4 for the case $\alpha = 0$. Details and derivations are in Appendix .4.3.

30. When the downstream sector does not employ much labor, the potential to increase upstream efficiency by reallocating labor is small, so there is less motive for a larger input tariff on efficiency grounds, and tariff escalation remains more likely. By contrast, when the downstream sector employs a high share of labor, there is more potential for the input sector to grow from reallocating this labor and thus become more efficient.

31. The median values of these parameters are chosen based on the results of the structural estimation of our model in Section 3.6.

3.6 Quantitative Results for a Large Open Economy

In this section, we relax the 'small open economy' assumption by allowing for Home and Foreign prices to change, and quantitatively solve for the social-welfare maximizing levels of input and final-good tariffs. We perform the analysis by mapping world data to our two-country model, interpreting the Home country as the United States, and the Foreign country as the Rest of the World (RoW, hereafter). We first analyze the implementation of the first-best allocation, which as in our theoretical analysis of the small-open economy case requires an upstream subsidy and at least one export tax. We then analyze optimal tariffs when only import tariffs are feasible. In this real-world setting, tariff escalation appears to be a robust feature of the structure of optimal import tariffs. In the first-best allocation, optimal tariffs continue to feature escalation as long as downstream subsidies are not used.

Although we provide quantitative results for a set of parameters anchored on US data, the qualitative nature of our results – most notably, the fact that second-best optimal tariffs on final goods are larger than on inputs – remains unaffected when exploring a wider range of parameter values as demonstrated in Section 3.6.3. These results line up well with our theoretical results in Sections 3.4 and 3.5 obtained for a 'small open economy.' Since the results in this section are quantitative, we revert to the 'Krugman' version of our model with internal economies of scale, monopolistic competition, and firm entry. This allows us to provide additional intuition on the role of firm reallocation in explaining tariff escalation, which we do in the next section using the estimated results from this section.

3.6.1 Data and Parameters

In order to discipline our model quantitatively, we need to take a stance on a number of parameters and ensure that they provide values for key equilibrium variables consistent with those observed in the data. The main parameters of the model are the elasticities of substitution upstream and downstream (θ and σ), the downstream labor share (α), iceberg trade costs upstream and downstream (τ^u and τ^d), productivity upstream and downstream in each country (A^u_{US}, A^u_{RoW}, A^d_{US} and A^d_{RoW}), fixed costs upstream and downstream (f^u and f^d), and the Home and Foreign labor endowments (L_{US} and L_{RoW}). Perhaps not too surprisingly given the isomorphism we develop in previous sections, the fixed cost parameters turn out to be irrelevant for our quantitative conclusions, so we do not discuss them below.

Our quantitative approach constitutes a blend of calibration and estimation. We first discuss various approaches to estimating the key elasticities of substitution θ and σ, we then back out the downstream labor share α and the labor forces L_{US} and L_{RoW} from readily available public data, and we finally estimate trade costs (τ^u and τ^d) and the productivity parameters (A^u_{US}, A^u_{RoW}, A^d_{US} and A^d_{RoW}) by minimizing the distance between our model and a series of moments obtained from standard sources.

Elasticities of Substitution (θ and σ) We consider four alternative approaches to quantifying the elasticities of substitution across varieties in the upstream and downstream sectors (θ and σ, respectively). We summarize these approaches here and provide additional details in Appendix .6.2. The first approach is to treat these elasticities as symmetric across sectors. In this first approach, we fix the values of the elasticities of substitution across varieties in each sector to 5 ($\sigma = \theta = 5$), as in Costinot and Rodriguez-Clare (2014). We first consider this symmetric case to rule out the possibility that differences in demand elasticities across good types are the only source of variation in the response of welfare to changes in input versus final-good tariffs.

The second approach is to calibrate these parameters from data on mark-ups. Recall that under monopolistic competition and CES preferences, the optimal firm-level mark-up is equal to $\theta/(\theta-1)$ upstream and $\sigma/(\sigma-1)$ downstream. Using sales and mark-up data from Baqaee and Farhi (2020) based on publicly listed firms in Compustat, we compute the sales-weighted average mark-ups of firms which we assigned to either upstream or downstream based on their primary sector. This approach leads to estimates of $\theta = 4.43$ for the elasticity of substitution upstream and of $\sigma = 6.44$ for the elasticity of substitution downstream.

The third approach is to estimate these parameters based on the response of trade flows to the US-China trade war in 2018 to 2019. Specifically, we follow Amiti et al. (2019b) and calculate 12-month changes in US imports and US import tariffs at the product-country level. Under the CES demand structure, regressing the changes in trade flows on the changes in tariffs provides estimates of the trade elasticity. Our preferred specification from this approach leads to estimates of $\theta = 2.35$ for the elasticity of substitution upstream, and $\sigma = 3.08$ for the elasticity of substitution downstream. The small magnitude of the trade elasticities is consistent with the findings in Amiti et al. (2020) and could reflect that the response in trade flows was diminished by uncertainty about the persistence of these tariff changes.

The fourth (and final) approach is to exploit the isomorphism of our model to a competitive model with external economies of scale. As discussed in Section 3.2, the isomorphism places the following restrictions between the external economies of scale parameters and the elasticities of substitution across varieties hold: $\gamma^u = 1/(\theta - 1)$ and $\gamma^d = 1/(\sigma - 1)$. We use estimates of scale elasticities from Bartelme et al. (2019). We note two important caveats. First, they estimate these parameters only for 15 manufacturing sectors (we classify nine of these as upstream and six as downstream). Second, their framework abstracts from intermediate inputs and therefore their estimates may not be perfectly compatible with our setup. With these caveats in mind, the average (unweighted) scale elasticities are 0.133 upstream and 0.135 downstream. Exploiting the isomorphism between this setup and our framework with monopolistic competition and free entry, we convert these to $\theta = 8.52$ and $\sigma = 8.41$ for this fourth approach.

Downstream Labor Intensity, Trade and Expenditure Shares and Labor Endowments We measure the share of inputs in production, $1 - \alpha = 0.45$, from usage of intermediate inputs by downstream sectors based on the WIOD database (see Appendix

.6.3 for details). Similarly, we calculate trade and expenditure shares for the upstream (intermediate-input) and downstream (final consumption) sectors based on trade flow data provided in the WIOD, taking into account whether a trade flow is used for final consumption or as an intermediate input).[32] We infer the labor endowment of each country from population data published by CEPII.[33]

Estimation of Productivity Parameters and of Trade Costs Finally, we normalize US productivity in both sectors to one, $A^d_{US} = A^u_{US} = 1$. This leaves us with four parameters to estimate: trade costs in each sector $\{\tau^d, \tau^u\}$, and sectoral productivity in the rest of the world $\{A^d_{RoW}, A^u_{RoW}\}$.[34] To estimate the model, we search for the vector of parameters $\{\tau^d, \tau^u, A^d_{RoW}, A^u_{RoW}\}$ that minimizes the sum of squares of the differences between model-generated and empirical moments, subject to our equilibrium constraints. Panel B of Table 3.1 lists the set of moments we target in the estimation. The moments correspond to those that are necessary to solve for the changes in equilibrium outcomes in response to a counterfactual change in tariffs (i.e., the hat algebra approach) and are all retrieved from the World Input-Output Database (WIOD).

Panel A in Table 3.1 presents the estimated values of the RoW's productivities and iceberg trade costs in each sector obtained under symmetric elasticities upstream and downstream, $\theta = \sigma = 5$. Trade costs appear slightly higher in the downstream sector, but within the range of standard estimates of trade barriers. The estimates indicate that the United States is about three times more efficient in final-good production than the rest of the world, and seven times more efficient in terms of input production. Despite only estimating four parameters, the fit of the model is quite good for most moments, except for the ratio of total sales in

32. We use data for 2014 which is the latest available year in the WIOD.

33. Specifically, we set $L^{us} = 10 \times \frac{Pop^{us}}{Pop^{us}+Pop^{row}} = 0.45$ and $L^{row} = 10 \times \frac{Pop^{row}}{Pop^{us}+Pop^{row}} = 9.55$.

34. We restrict entry costs f^d and f^u to be symmetric across sectors and countries and fix those values to 1. As anticipated, this restriction is without loss of generality, as we find that both the model fit and counterfactuals are invariant to changing the entry costs to arbitrary (and possibly asymmetric) values.

Table 3.1: Calibrated Parameters and Moments

A. Calibrated Parameters	
Productivity in final-good sector, RoW relative to US, A^d_{row}	0.325
Productivity in input sector, RoW relative to US, A^u_{row}	0.142
Iceberg cost for final goods from US to RoW, τ^d	2.375
Iceberg cost for inputs from US to RoW, τ^u	2.032

B. Moments	Data	Model
Sales share to US from US in final goods	0.943	0.964
Sales share to RoW from RoW in final goods	0.988	0.985
Sales share to US from US in intermediate good	0.897	0.889
Sales share to RoW from Row in intermediate good	0.982	0.978
Expenditure share in US final goods for the US	0.960	0.946
Expenditure share in RoW final good for the RoW	0.981	0.989
Expenditure share in US int. good for the US	0.906	0.921
Expenditure share in RoW int. good for the RoW	0.980	0.967
Total US sales (int. goods) to total US expenditure (final goods)	0.771	0.466
Total RoW sales (int. goods) to total RoW expenditure (final goods)	1.242	0.446
Total US sales (final goods) to total US expenditure (final goods)	1.018	0.997
Total RoW sales (final goods) to total RoW expenditure (final goods)	0.993	0.999
Total expenditure in final goods by the US relative to RoW	0.303	0.285

Sources: World Input Output Database and authors' calculations. Notes: Panel A presents the estimated values of the RoW's productivities and iceberg trade costs in each sector obtained under symmetric elasticities upstream and downstream, $\theta = \sigma = 5$. Panel B presents the targeted moments in the estimation. Column 1 presents moments from the data and column 2 presents their estimated counterparts. Note that in the model, total sales upstream to total expenditure downstream cannot be larger than 1 since the upstream sector is pure value added.

the upstream sector to total expenditure in the downstream sector. Note that in the data, the upstream sector uses intermediate inputs in production as well – which for simplicity we abstract from in our framework.

3.6.2 Optimal Tariffs

In this section, we use the estimated parameters to compute the optimal tariff levels on final goods and intermediate inputs for the United States when the rest of the world sets a zero tariff on US goods.

Optimal Import Tariffs under First-Best Policies We begin by considering optimal policy in an environment in which the Home government has the necessary instruments to achieve the first-best allocation. We focus on the set of instruments discussed in Proposition 3.5.1, namely import tariffs and export taxes, as well as an upstream production subsidy. After normalizing the downstream export tax to zero (by Lerner's symmetry), we find that the optimal vector of policies is given by

$$\left(t_H^d, t_H^u, s_H^u, v_H^u\right) = (0.253, 0.003, 0.200, 0.200) .$$

In words, even when accounting for general equilibrium effects due to the United States not being a small open economy, we find that the first-best policies are remarkably consistent with the results from Proposition 3.5.1. Tariff escalation is close to $\sigma/(\sigma-1) = 5/4$, and the optimal domestic production subsidy upstream and the optimal upstream export tax are essentially indistinguishable from $1/\theta = 0.2$. Note also that the upstream import tariff is virtually zero.

Optimal Import Tariffs under Second-Best Policies Real world trade policies rarely feature export taxes—they are in fact outlawed by the US Constitution—and production subsidies are rarely used systematically. For explaining the observed tariff escalation, second-best policies that only feature import tariffs are therefore of particular interest. We again maximize US welfare, taking as given that the rest of the world places no tariffs on the United States. In this case, the vector of optimal import tariffs is given by

$$\left(t_H^d, t_H^u\right) = (0.306, 0.170) .$$

Tariff escalation thus prevails under second-best policies. Here, under $\alpha = 0.55$, tariff escalation is smaller in magnitude under second-best import tariffs compared to the first-best policy results above. Recall that in the $\alpha = 0$ case tariff escalation is larger under second-best policies as shown in Proposition 3.4.1, but this need not be the case for $\alpha > 0$. Interestingly, however, optimal tariff escalation under second-best policies (around 1.11) is a bit larger in magnitude than the observed tariff escalation of around 1.04 observed in the US in 2018 (see Figure 3.5 in Section 3.8).

3.6.3 Optimal Tariffs: Robustness

We next explore the robustness of our findings to alternative parameter values. We analyze optimal import tariffs under the three alternative procedures of estimating θ and σ as well as for different values for α (i.e., downstream value-added intensity). When changing these parameters, we re-calibrate the trade and productivity parameters to provide the best fit of the moments from Panel B of Table 3.1.

Table 3.2 presents the results of first-best policies that include an upstream production subsidy and export tariffs (panel A), and the second-best results using only import tariffs (panel B). As is clear, the level of the tax instruments is quite sensitive to changes in the parameters. However, for all parameter values, we have that $\frac{1+t^d}{1+t^u} > 1$, and therefore optimal final-good tariffs are higher than input tariffs. Under the elasticity parameters shown in column 4, the optimal second-best tariff escalation is close in magnitude to the observed tariff escalation from Figure 3.5 (though the level of the import tariffs is much larger). We thus conclude that for all empirically plausible parameter combinations for the downstream labor share and the scale elasticities, the solution to the social planner's second-best problem always features tariff escalation.

Panel A of Table 3.2 also reveal a systematic property of the tariff escalation consistent with the results from Proposition 3.5.1 derived for a small open economy. Comparing across columns, $\frac{1+t^d}{1+t^u} \approx \frac{\sigma}{\sigma-1}$. The pattern is striking, though note that this relationship is not exact,

Table 3.2: Optimal Tax Policy - Robustness for Various Parameter Values

Parameter Values	$\alpha = 0.55$					$\sigma = \theta = 5$		
	$\theta = 4.43$ $\sigma = 6.44$	$\theta = 2.35$ $\sigma = 3.08$	$\theta = 8.52$ $\sigma = 8.41$	$\theta = 2.5$ $\sigma = 4$	$\theta = 5.5$ $\sigma = 4$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0$
A.								
t^d	0.19	0.49	0.14	0.34	0.34	0.26	0.26	0.27
t^u	0	0	0	0	0	0	0	0
v^u	0.23	0.43	0.12	0.43	0.18	0.20	0.21	0.21
s^u	0.23	0.43	0.12	0.40	0.18	0.20	0.20	0
$\frac{1+t^d}{1+t^u}$	1.18	1.48	1.14	1.33	1.34	1.25	1.26	1.26
B.								
t^d	0.22	0.51	0.16	0.37	0.39	0.26	0.33	0.35
t^u	0.18	0.31	0.09	0.30	0.15	0.18	0.11	0.06
$\frac{1+t^d}{1+t^u}$	1.04	1.15	1.07	1.05	1.21	1.07	1.20	1.29

Notes: Each column presents optimal tariffs and taxes for alternative values of the parameters described in Section 3.6.1 and their corresponding, re-estimated values of τ^d, τ^u, A^d_{row} and A^u_{row}. The set of calibrated parameters that corresponds to each column is displayed in Table 11 in Appendix .5.3. Panels A and B present optimal tariffs and taxes for the cases of policy instruments in Section 3.6.2. Tariff escalation ($\frac{1+t^d}{1+t^u} > 1$) is a robust feature across all specifications.

and can vary for a given level of σ as other parameters (e.g., α) are changed.

3.7 Decomposing the Welfare Effects from Tariffs

In this section, we analyze the distinct welfare effects of input versus final-good tariffs both analytically and numerically. To do so, we decompose the first-order welfare effects of *small tariffs* levied by the 'Home' government on imported final goods or imported inputs into six, distinct channels. As in Section 3.6, we allow for endogenous prices in Home and Foreign, and use the 'Krugman' firm-level model so that we can analyze firm-level relocation effects. We first derive a theoretical decomposition of the welfare effects of small tariffs, and then evaluate this decomposition quantitatively.

3.7.1 First-Order Welfare Effects of Small Import Tariffs

The key innovation in this subsection relative to our theoretical results in Sections 3.4 and 3.5 is that we no longer restrict Home to be a small, open economy.

Because welfare at Home corresponds to the representative household's real income, we have

$$U_H = \frac{w_H L_H + R_H}{P_H^d},$$

where R_H is tariff revenue in equation (3.12), and where P_H^d is the ideal price index at Home.

We are interested in the change in Home's welfare associated with a change in the tariff schedule $\{t_H^d, t_H^u\}$ starting from an equilibrium with zero tariffs. For simplicity, and without loss of generality, we set the Home wage to be the numéraire, so we can focus on the effect of tariffs on tariff revenue and the price index. The change in Home's welfare, dU_H, around $t_H^d = 0$ and $t_H^u = 0$ (and thus $R_H = 0$), can then be written as:

$$\frac{dU_H}{U_H} = \left[-\frac{dP_H^d}{P_H^d} + \frac{dR_H}{w_H L_H} \right], \tag{3.35}$$

with

$$\frac{dR_H}{w_H L_H} = b_F^H \times dt_H^d + \lambda_H^d \times \Omega_{FH} \times dt_H^u, \tag{3.36}$$

where $b_F^H \equiv \frac{M_F^d p_{FH}^d q_{FH}^d}{w_H L_H}$ is the share of Home income spent on foreign varieties, $\lambda_H^d \equiv \frac{M_H^d p_H^d x_H^d}{w_H L_H}$ is the ratio of domestic final-good revenue to national income (with $R_H = 0$) in country H, and $\Omega_{FH} \equiv \frac{M_F^u M_H^d p_{FH}^u q_{FH}^u}{M_H^d p_H^d x_H^d}$ is the share of Home final-good revenue spent on intermediate input varieties from F.

Consider next the change in Home's ideal price index. Given the formula for this price index – see equations (61) and (63) in Appendix .2.1 – and given firm symmetry, we have:

$$\frac{dP_H^d}{P_H^d} = b_H^H \times \left(\frac{1}{1-\sigma} \frac{dM_H^d}{M_H^d} + \frac{dp_{HH}^d}{p_{HH}^d} \right) + b_F^H \times \left(\frac{dM_F^d}{M_F^d} \frac{1}{1-\sigma} + \frac{dp_{FH}^d}{p_{FH}^d} + dt_H^d \right). \tag{3.37}$$

The ideal (downstream) price index changes because in equilibrium the total measure of firms, in both Home and Foreign, responds to the change in tariff. At the same time, the change in relative prices also affects the price charged by downstream producers. Each factor's contribution to the change in the price index depends on the importance of foreign and domestic goods in the consumption basket, b_j^H. The change in the unit price of downstream goods is given by:

$$\frac{dp_{ii}^d}{p_{ii}^d} = \alpha \frac{dw_i}{w_i} + (1-\alpha)\frac{dP_i^u}{P_i^u}, \tag{3.38}$$

with

$$(1-\alpha)\frac{dP_i^u}{P_i^u} = \left(\frac{dM_i^u}{M_i^u}\frac{1}{1-\theta} + \frac{dp_{ii}^u}{p_{ii}^u}\right)\Omega_{i,i} + \left(\frac{dM_j^u}{M_j^u}\frac{1}{1-\theta} + \frac{dp_{ji}^u}{p_{ji}^u} + dt_i^u\right)\Omega_{ji}. \tag{3.39}$$

This latter equation captures the change in the upstream price index in each country, which is in turn shaped by the change in the measure of upstream firms in each country, the change in the price of individual input varieties, and the relative importance of domestic and foreign inputs in production, as captured by the terms Ω_{ii} and Ω_{ji}. Since we have set Home wages as the numéraire, we have $\frac{dw_H}{w_H} = 0$. Also, since we hold iceberg trade costs fixed in this exercise, we have $\frac{dp_{ji}^d}{p_{ji}^d} = \frac{dp_{ii}^d}{p_{ii}^d}$. Finally, since upstream goods only use labor in production, we have $\frac{dp_{FF}^u}{p_{FF}^u} = \frac{dp_{FH}^u}{p_{FH}^u} = \frac{dw_F}{w_F}$.

Putting all the pieces together – that is, combining equations (3.35)-(3.39) – we finally obtain the following expression for the first-order effect of tariffs on Home welfare:

$$
\begin{aligned}
\frac{dU_H}{U_H} = & -\left(b_H^H \Omega_{FH} + b_F^H \left(\Omega_{FF} + \alpha\right)\right) \frac{dw_F}{w_F} \\
& + \left(\frac{b_H^H \Omega_{HH} + b_F^H \Omega_{HF}}{\theta - 1}\right) \frac{dM_H^u}{M_H^u} + \left(\frac{b_H^H \Omega_{FH} + b_F^H \Omega_{FF}}{\theta - 1}\right) \frac{dM_F^u}{M_F^u} \\
& + \left(\frac{b_H^H}{\sigma - 1}\right) \frac{dM_H^d}{M_H^d} + \left(\frac{b_F^H}{\sigma - 1}\right) \frac{dM_F^d}{M_F^d} \\
& + \left(\lambda_H^d - b_H^H\right) \Omega_{FH} (dt_H^u) \mathbb{I}_{\{t = t^u\}}.
\end{aligned}
\tag{3.40}
$$

This expression contains six terms.[35] The first one captures 'factorial' *terms-of-trade* benefits from raising tariffs, which in our Ricardian model operate via changes in relative wages (or foreign wages, given our choice of numéraire). The next four terms capture *relocation effects* due to changes in the masses of domestic and foreign firms in both the upstream and downstream sectors. All terms enter positively, reflecting the positive effect of increased varieties upstream and downstream on welfare, but it should be clear that general-equilibrium constraints will preclude all these measures of firms from increasing in reaction to Home import tariffs. How each of these relocation effects influences welfare is in turn given by the relative importance of these four types of firms in the purchases of Home consumers and Home firms.

To provide intuition for the quantitative results to come, notice that due to home bias, we typically have $b_H^H > b_H^F$ and also $b_H^H > b_F^H$. Furthermore, $\Omega_{HH} \leq 1 - \alpha < 1$, and b_H^F will be small unless Home is a large economy. As a result, it will typically be the case that $b_H^H > b_H^H \Omega_{HH} + b_F^H \Omega_{HF}$. This carries two significant implications. First, a given percentage increase in the measure of domestic downstream firms (dM_H^d / M_H^d) has a larger impact on Home welfare than the same percentage increase in the measure of foreign downstream firms

35. Note that while the downstream tariff has direct effects on the price index and on tariff revenues, these two effects are exactly offsetting, so that the only net effects on welfare operate indirectly through equilibrium variables.

(dM_F^d/M_F^d). Second, a given percentage increase in the measure of domestic downstream firms (dM_H^d/M_H^d) has a larger impact on Home welfare than the same percentage increase in the measure of domestic upstream firms (dM_H^u/M_H^u). This suggests that, on account of relocation effects, (i) the Home government will have an incentive to levy import tariffs downstream to attract the entry of final-good producers into its economy – as highlighted in the work of Venables (1987) and Ossa (2011) –, and (ii) although such an incentive also exists with regard to entry of upstream firms, the net welfare effects of input producers' entry are smaller.

The sixth and final term in equation (3.40) is more subtle and relates to a key term identified in the work of Beshkar and Lashkaripour (2020). More specifically, notice that $\lambda_H^d - b_H^H$ represents the value of exported downstream goods as a share of Home's GDP. This last term then captures the extent to which an input tariff is passed on to foreign consumers, thereby mimicking an export tax, which also improves Home's terms of trade (Costinot et al., 2015; Beshkar and Lashkaripour, 2020). For this same reason, this last term *only* applies to changes in input tariffs.

Because changes in tariffs do not enter the other terms in equation (3.40) explicitly, it would appear that (small) intermediate-input import tariffs increase welfare by more than (small) final-good tariffs on account of this last extra term. Nevertheless, we have already indicated above that final-good and input tariffs generate differential effects on relative wages (dw_F/w_F) and on relocation effects (dM_i^s/M_i^s for $s = d, u$), and we will show in the next subsection that these channels are quantitatively dominant. More precisely, and anticipating the quantitative results to come, relocation effects seem to be the quantitatively dominant force in leading small final-good tariffs to generate larger welfare gains than small input tariffs.

3.7.2 Quantitatively Decomposing the Welfare Effects of Tariffs

To assess the relative magnitudes of the channels above, we use the calibrated model to quantify each of them. To do so, we first solve for the zero-tariff equilibrium, so that we can compute the statistics Ω_{ij}, b_j^i and λ_i^d in this environment.[36]

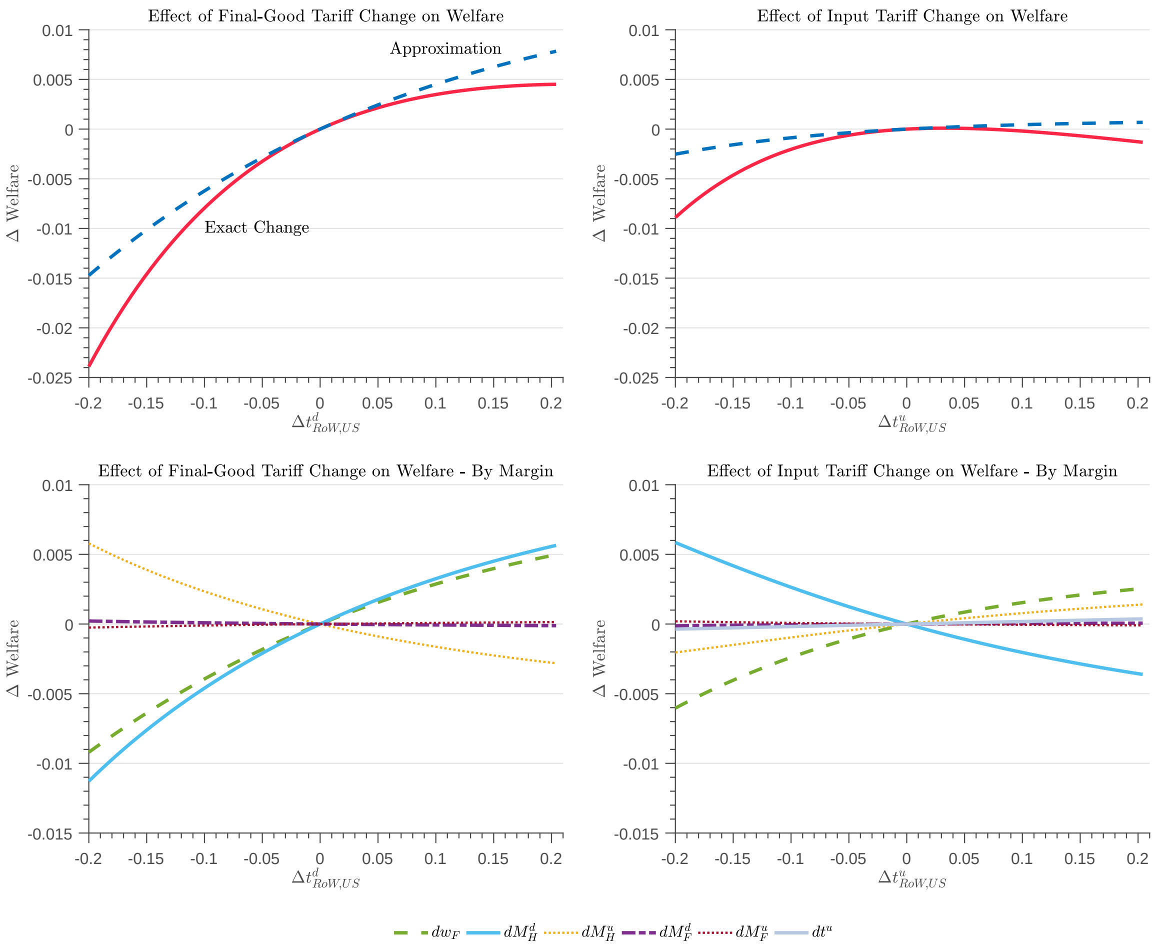
Figure 3.4 depicts the welfare effects of changes in a final-good tariff (left panels) versus changes in an input tariff (right panels). The top two panels compare the percentage changes in welfare starting from the zero-tariff equilibrium (solid red line) to the percentage changes predicted by our first-order approximations around zero (dashed-blue line). The first-order approximation works well for small changes in both final-good and intermediate-input tariffs. Starting from zero tariffs, the welfare effects of small import tariffs are positive for both types of goods, but turn negative for input tariffs at much lower rates than for final-good tariffs.

The bottom panels of Figure 3.4 decompose the approximation of the aggregate effects into their component parts, as shown in equation (3.40). Specifically, we decompose changes in welfare into changes due to: (i) changes in relative wages (dashed green); (ii) the relocation of final good producers to the United States (solid cyan); (iii) the relocation of input producers to the United States (dotted yellow); (iv) changes in the mass of final-good producers in the RoW (short-dash purple); (v) changes in the mass of input producers in the RoW (dash-dot magenta); and (vi) the gain from passing part of the input tariff onto final consumers in the RoW (solid gray). Although in the figure we label these by dw_F, dM_H^u, and so on, it should be understood that we are plotting the full value of each of the six terms in equation (3.40), with the labels identifying only one element of each term.

Several observations are in order. First, notice that by raising tariffs on final goods, US welfare increases not only because it tilts the factorial terms of trade in its favor – i.e., a reduction in w_F – but also because it induces a relocation of final-good producers into its

36. Under zero tariffs, these statistics take the values $\Omega_{H,H} = 0.41$, $\Omega_{F,H} = 0.04$, $\Omega_{F,F} = 0.44$, $\Omega_{H,F} = 0.02$, $b_H^H = 0.93$, $b_F^H = 0.07$, $\lambda_H^d = 0.98$.

Figure 3.4: First Order Decomposition of Welfare Changes



Notes: Figure depicts the welfare effects of changes in a final-good tariff (left panel) versus changes in an input tariff (right panel). The top panels compare the percentage changes in utility starting from the zero-tariff equilibrium (solid red line) to the percentage changes predicted by our first-order approximations around zero (dashed blue line). The bottom two panels decompose the approximation of the aggregate effects into the component parts in equation (3.40). These are changes in welfare due to: (i) changes in relative wages (dashed green); (ii) the relocation of final good producers to the United States (solid cyan); (iii) the relocation of intermediate producers to the United States (dotted yellow); (iv) changes in the mass of final-good producers (short-dash purple); (v) changes in the mass of intermediate good producers (short-dash magenta); and (vi) the gain from passing part of the tariff onto foreign consumers (solid gray).

own country. In addition, notice that the magnitude of the term associated with dM_H^d is on average as large as the term dw_F. Hence, this relocation effect is as important as the factorial terms-of-trade channel usually emphasized in the literature. Nevertheless, the net entry of final-good producers is accompanied by a net exit of input producers (yellow-dotted curve), with welfare effects that are about half as large as those associated with the relocation of final-good producers. The other three effects are largely negligible (the last one is exactly zero for the case of final-good tariffs). These results suggest that the United States is similar to a small, open economy relative to the RoW, and provides quantitative support for the similarity between our analytical solutions for a small, open economy and the quantitative solutions for a LOE.

Turning to the results for input tariffs, the relocation effects are again similar in magnitude to the factorial terms-of-trade effects, though these relocation effects now entail a net *negative* effect on welfare. Higher import tariffs on intermediate inputs increase entry upstream, but reduce entry of final-good producers. The negative welfare effect of the latter dominates quantitatively, which is intuitive based on a comparison of the terms multiplying dM_H^d and dM_H^u in equation (3.40), as explained in Section 3.7. These terms contain the exposure of the consumer price index to changes in the measure of downstream and upstream producers, respectively.[37] The effect on welfare from changes in the mass of firms in the rest of the world is largely negligible, while the last term – the gain from passing intermediate-input tariffs on to foreign consumers – is small in magnitude.

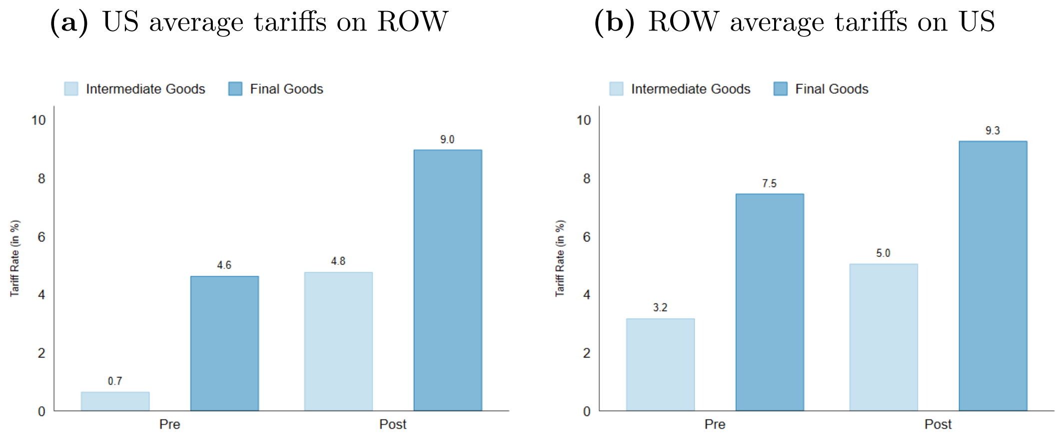
3.8 Counterfactuals: Evaluation of the 2018-2019 Tariff Increases

We close the paper by evaluating how the tariff increases during the 2018 to 2019 trade war affected US welfare. Figure 3.5 illustrates the general pattern of tariff escalation for both US tariffs on other countries and for foreign tariffs applied on the United States, both before and after the recent trade war. Due to their low initial levels, average input tariffs increased the

37. Note that $\Omega_{H,H}$ and $\Omega_{H,F}$ are bounded above by $1-\alpha$. The welfare effect associated with a percentage increase in downstream firms is 0.31, whereas the term multiplying the percentage change in upstream firms is only 0.13 in equation (3.40).

most on a percent basis, though tariff escalation remains a distinct feature of the data.[38]

Figure 3.5: Tariff Escalation Before and After the US-China Trade War



Notes: Pre: Tariffs in January 2018, Post: Tariffs in December 2019. Tariff data from WTO and USITC, 2015 annual import and export data for weighted averages from US Census Bureau. Goods are classified as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods start with BEC code 41, 521, 112, 321, 522, 61, 62, 63 (including capital goods). All other goods have no classification.

These tariff increases largely arose from the US-China trade war, but also include the US tariffs on washing machines, solar panels, aluminum, and steel, as well as the retaliatory tariffs from the rest of the world (RoW).

To evaluate the welfare effects of these tariff increases, we calculate US and RoW welfare under five scenarios. First, we use the observed 2017 tariffs and no other domestic instruments. Second, we apply the observed 2019 US tariffs. Third, we evaluate the distinct welfare effects of downstream versus upstream tariffs by changing tariffs in each sector individually. Fourth, we construct a counterfactual tariff that generates the same revenue as the 2019 US tariffs, but only applies to either the downstream or the upstream sector. Finally, we evaluate the welfare implications of optimal second-best tariffs and optimal first-best policies. For each of these scenarios, we first hold RoW tariffs constant at their 2017 values, and then re-calculate

38. Figure 3.5 displays the trade-weighted average tariffs on intermediate and final goods imposed by the United States on the Rest of the World (RoW) and by the RoW on US imports. Tariff data at the HTS 6-digit level for ROW are from the WTO Tariff Download Facility. Tariff data at the HTS 8-digit level for the United States are from the USITC. Import and export data are from the US Census Bureau. We classify goods into intermediate and final goods using the UN Broad Economic Categories (BEC). For details on data construction and for a version of Figure 3.5 without trade weights, see Appendix .6.1.

welfare using the observed 2019 retaliatory tariff implemented by the RoW.

To approximate the US economy as closely as possible, we use elasticity estimates from US data on mark-ups. These estimates correspond to an upstream elasticity of $\theta = 4.43$ and a downstream elasticity of $\sigma = 6.44$ (see Section 3.6.1). Under the isomorphic model, these imply values of $\gamma^u = 0.29$ and $\gamma^d = 0.18$, such that the upstream sector has greater increasing returns to scale.

Table 3 displays our results, with panels A and B showing the cases with and without retaliation, respectively. Average US import tariffs increased from 0.7 to 4.8 percentage points for intermediate goods, and from 4.6 to 9 percentage points for final goods. We find that without tariff retaliation by the RoW, US welfare would have increased by 0.12% from these tariff changes. This gain is consistent with our estimates of the sizable unilateral optimal tariff rates for the US in Section 3.6 (in the absence of any export taxes or domestic subsidies).

We next evaluate the extent to which this gain was due to larger final-good versus input tariffs. In the third row of Table 3.3, we consider only the increase in the final good tariff, whereas row four considers the case if only the tariffs on intermediate goods had increased. The comparison reveals that the US welfare gains are driven overwhelmingly by higher final-good tariffs.

The dominant role of final-good tariffs on welfare increases is even starker when considering a counterfactual increase in US final-good tariffs (row 5 of Table 3.3) that would have (naively) raised the same revenue as the observed tariff increases based on the average tariff rate change and the 2017 trade flows. In this case, US welfare – absent any foreign retaliation – would have risen by 0.11%. If instead the tariff increases had been adjusted so to apply only to intermediate inputs, US welfare would have increased by only 0.02%. foreign retaliation.

In Panel B of Table 3.3, we repeat these exercises but now take into consideration the observed retaliation by the RoW, which increased its import tariffs on US intermediate inputs from 3.2 to 5.0 percentage points and on US final goods from 7.5 to 9.3 percentage points.

Table 3.3: Counterfactual Welfare Effects of US-China Trade War

	A. RoW tariff at 2017 level			**B.** RoW tariff at 2019 level		
	U_{US}	U_{RoW}	$\frac{U_{US}}{U_{US,2017}}$	U_{US}	U_{RoW}	$\frac{U_{US}}{U_{US,2017}}$
1. US tariff - 2017 level	0.0576	0.1723				
2. US tariff - 2019 level	0.0577	0.1721	1.0012	0.0576	0.1722	1.0002
3. 2019 US tariff only Downstream	0.0576	0.1722	1.0008	0.0576	0.1722	0.9998
4. 2019 US tariff only Upstream	0.0576	0.1722	1.0003	0.0576	0.1723	0.9993
5. Counterfactual Tariff only Downstream	0.0577	0.1721	1.0011	0.0576	0.1721	1.0001
6. Counterfactual Tariff only Upstream	0.0576	0.1722	1.0002	0.0575	0.1722	0.9992
7. Optimal US Import Tariffs	0.0577	0.1717	1.0025	0.0577	0.1718	1.0015
8. Optimal US Trade & Tax Policies	0.0584	0.1717	1.0137	0.0583	0.1717	1.0126

Notes: Table presents US welfare (U_{US}), RoW welfare (U_{RoW}), and US welfare relative to its initial 2017 level ($\frac{U_{US}}{U_{US,2017}}$). Panel A computes welfare holding the RoW tariffs at their 2017 levels, while Panel B uses the observed 2019 RoW retaliatory tariffs. *1. US tariff - 2017 level* provides baseline welfare values using actual 2017 tariff values; *2. US tariff - 2019 level* uses actual 2019 US tariffs; *3. 2019 US tariff only Downstream* uses 2017 upstream but 2019 downstream tariffs; *4. 2019 US tariff only Upstream* uses 2017 downstream but 2019 upstream tariffs; *5. Counterfactual Tariff only Downstream (6. Upstream)* uses a counterfactual US downsteam (upstream) tariff that generates the same revenue as the actual 2019 US tariffs, based on the observed trade flows in 2017. Counterfactual tariffs are $\tilde{t}^d = 0.131$ or $\tilde{t}^u = 0.156$; *7. Optimal US Import Tariffs* uses the second-best optimal import tariffs in response to RoW's trade policy in 2017 (Panel A) or 2019 (Panel B). The optimal policy vector for panel A is $\left(t_H^d, t_H^u\right) = (0.231, 0.183)$ and $\left(t_H^d, t_H^u\right) = (0.232, 0.184)$ for panel B; *8. Optimal US Trade & Tax Policy*: US chooses optimal import tariffs, export tax, and production subsidy, as described in Section 4, in response to RoW's trade policy from 2017 (Panel A) or 2019 (Panel B). The optimal policy vector is $\left(t_H^d, t_H^u, v_H^u, s_H^u\right) = (0.186, 0.003, 0.227, 0.226)$ for panel A and $\left(t_H^d, t_H^u, v_H^u, s_H^u\right) = (0.186, 0.003, 0.227, 0.226)$ for panel B.

In this case, the US welfare gain from the tariff increases shrinks to only 0.02%. Therefore, tariff retaliation by the RoW largely undermines the US welfare gains from higher tariffs, which in turn are overwhelmingly driven by higher final-good (and not input) tariffs. If the US had only placed the tariffs on final goods, while (naively) raising the same revenue (row 5), US welfare would have increased by 0.01%, even accounting for retaliation. If instead those tariffs had only been placed on intermediate inputs (row 6), US welfare would have declined by 0.06%.

Rows seven and eight present the potential welfare gains from implementing optimal import tariffs with and without other US policy instruments. The gains from second-best, optimal import tariffs (without production subsidies or export taxes) are displayed in row seven of Table 3.3. Optimal US tariffs absent any foreign retaliation achieve a welfare gain of 0.25 percent, with a tariff escalation wedge of 1.04.[39] Row eight allows for a full set of instruments that includes both import and export taxes, as well as production subsidies.[40] The optimal trade policy with domestic subsidies and export taxes (and assuming no foreign retaliation) leads to a 1.4 percent increase in welfare, and features close to zero tariffs on inputs and an escalation wedge of 1.18. As in our analysis in the SOE, the tariff escalation wedge is now notably higher since the government uses the input subsidy rather than the tariff to address the domestic labor misallocation. We note that these calculations assume no foreign retaliation (or, in panel B, no retaliation above the observed changes from RoW tariffs from 2017 to 2019).

3.9 Conclusion

In this paper, we provide an efficiency rationale for the fact that import tariffs on final good are systematically higher than those on intermediate inputs. This so-called tariff escalation has been widely documented across time and space, but there is little support in the literature for the notion that the pattern constitutes a social welfare-maximizing policy.

We develop a two-sector model with a final-good sector and an intermediate input sector, both featuring increasing returns to scale, and show that (i) first-best trade policies are consistent with tariff escalation, and that (ii) second-best import tariffs generally feature tariff escalation. A key result is that tariff escalation is driven by the presence and extent of increasing returns to scale in downstream production. Access to cheaper inputs expands the downstream sector's size and thus raises its efficiency. Relatively higher input tariffs are

39. The levels of these tariffs are much higher than the ones observed in the data $(t_d^*, t_u^*) = (0.2313, 0.1831)$.

40. Optimal policy has import taxes $(t_d^*, t_u^*) = (0.1860, 0.0027)$ and export taxes upstream, $v_u^* = 0.2269$, as well as an input production subsidy $s_u = 0.2261$.

only optimal in a small set of second-best settings in which an upstream production subsidy cannot be used, and upstream efficiency and thus input prices are sufficiently sensitive to the upstream sector's size.

Although our model generically features domestic distortions related to the existence of scale economies upstream, the optimality of relatively lower input tariffs is *not* explained by a (second-best) correction of these domestic distortions. If anything, domestic distortions reduce the desirability of tariff escalation. Instead, input tariffs are less beneficial because they impact the size of the final-good sector *and* because the size of the final-good sector shapes its productivity under increasing returns to scale. It is thus scale economies downstream, rather than upstream, that shape the optimality of tariff escalation.

Our results are based on a parsimonious model featuring a single factor of production, only two sectors of production, and homogeneous firms. Future research should elucidate the robustness of our results to more realistic settings, which should also provide a helpful lens through which to interpret the drivers of tariff escalation in the data.

Appendix

.1 Closed-Economy Model: Details on Derivations

.1.1 *Equilibrium*

Given the CES assumptions built into our framework and the lack of strategic interactions, firms in both sectors charge a constant markup over their marginal cost, which delivers

$$p^u = \frac{\theta}{\theta - 1} \frac{w}{A^u} \tag{41}$$

and

$$p^d = \frac{\sigma}{\sigma - 1} \frac{1}{A^d} \frac{w^\alpha (P^u)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}, \tag{42}$$

where P^u is the price index of intermediate inputs associated with Q^u, or

$$P^u = \left(\int_0^{M^u} p^u(\varpi)^{1-\theta} d\varpi \right)^{\frac{1}{1-\theta}}.$$

Firms make zero profits due to free entry, which pins down firm size according to:

$$x^u = (\theta - 1) f^u, \qquad x^d = (\sigma - 1) f^d. \tag{43}$$

Naturally, in equilibrium we must have $x^d = q^d$ and $x^u = M^d q^u$. The total measure of firms in the economy can be determined as follows. First, note that from the household's demand for downstream goods we have

$$M^d p^d q^d = wL, \tag{44}$$

which plugging in (42) and (43) delivers

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((1-\alpha) \frac{\theta - 1}{\theta} A^u \right)^{1-\alpha} (M^u)^{\frac{1-\alpha}{\theta - 1}} L. \tag{45}$$

Next, note that labor-market clearing imposes

$$L = M^u \frac{(f^u + x^u)}{A^u} + M^d \frac{\alpha p^d x^d}{w}. \tag{46}$$

Plugging in equations (43) and (44), we can solve for the total measure of firms in the upstream sector:

$$M^u = \frac{(1-\alpha) A^u L}{f^u \theta}. \tag{47}$$

Then, equations (45) and (3.6) determine the measure of firms in the downstream sector:

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((\theta - 1) f^u\right)^{1-\alpha} \left(\frac{(1-\alpha) A^u}{f^u \theta}\right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}}. \tag{48}$$

Finally, aggregate welfare is simply given by $U = \left(M^d\right)^{\frac{\sigma}{\sigma-1}} q^d$, where M^d is given in (3.7) and $q^d = x^d$ in (43). When $\alpha \to 1$, we obtain

$$U = \left(\frac{A^d}{f^d \sigma} L\right)^{\frac{\sigma}{\sigma-1}} (\sigma - 1) f^d,$$

which is the standard formula in Krugman (1980).[41] Welfare is increasing in market size with an elasticity equal to $\frac{\sigma}{\sigma-1} > 1$, reflecting the variety gains associated with living in an economy that provides a larger number of final-good varieties.

Relative to this "Krugman" benchmark, in the presence of an active upstream sector (i.e., $\alpha < 1$), our model continues to feature scale effects, and in fact the elasticity of welfare to market size is larger than in the model with only a final-good sector. To see this, we can write welfare as

41. A small and immaterial point of departure from Krugman (1980) is the fact that we have modeled the productivity terms A^d and A^u as shaping both the marginal and fixed costs of production. As a result, firm size is independent of these productivity parameters, but these parameters affect welfare directly.

$$U=\left(\frac{(\sigma-1)A^d/\sigma}{\left((\sigma-1)f^d\right)^{\frac{1}{\sigma}}}\left(\frac{(\theta-1)\,A^u/\theta}{((\sigma-1)f^u)^{1/\theta}}\right)^{\frac{(1-\alpha)\theta}{\theta-1}}(L)^{\frac{\theta-\alpha}{\theta-1}}\right)^{\frac{\sigma}{\sigma-1}}\xi_\alpha, \tag{49}$$

where ξ_α is a function of only α and θ. Note that $\frac{\theta-\alpha}{\theta-1}\geq 1$, and thus this framework features larger scale effects than our model without an input sector.

To gain a better understanding of the role of imperfect competition and increasing returns to scale on welfare in our closed economy, we next turn to characterizing the social optimum in our model, and explore conditions under which the above market equilibrium is efficient.

.1.2 Social Planner Problem

The social planner maximizes the objective in (3.1), choosing M^d, M^u, ℓ^d, ℓ^u, x^d and x^u subject to feasibility, or:

$$\begin{aligned}
\max_{M^d,M^u,\ell^d,\ell^u,x^d,x^u}\quad & U=\left(M^d\right)^{\frac{\sigma}{\sigma-1}}x^d\\
s.t.\quad & L=\ell^uM^u+\ell^dM^d\\
& f^u+x^u=A^u\ell^u\\
& f^d+x^d=A^d\left(\ell^d\right)^\alpha\left((M^u)^{\frac{\theta}{\theta-1}}\frac{x^u}{M^d}\right)^{1-\alpha}.
\end{aligned}$$

Working with the first-order conditions of this problem, we find that

$$(x^u)^*=(\theta-1)f^u \tag{50}$$

and

$$(M^u)^*=\frac{\theta}{\theta-\alpha}\frac{(1-\alpha)A^uL}{\theta f^u} \tag{51}$$

in the upstream sector, and

$$\left(x^d\right)^* = (\sigma - 1) f^d \tag{52}$$

and

$$\left(M^d\right)^* = \left(\frac{\theta-1}{\theta-\alpha}\right)^{\alpha} \left(\frac{\theta}{\theta-\alpha}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{\alpha^\alpha A^d}{\sigma f^d} \left(\left(\theta-1\right) f^u\right)^{1-\alpha} \left(\frac{(1-\alpha)A^u}{\theta f^u}\right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}} \tag{53}$$

in the downstream sector. Comparing equations (50)-(53) to the corresponding ones in the market equilibrium, we conclude that:[42]

Proposition .1.1. In the decentralized equilibrium, firm-level output is at its socially optimal level in both sectors, but the market equilibrium features too little entry into both the downstream and upstream sectors unless $\alpha = 1$ (so the upstream sector is shut down) or $\alpha = 0$ (so the downstream sector does not use labor directly in production).

Why is the market equilibrium typically inefficient? At first glance, it may appear that the only source of inefficiency is the markup charged by upstream producers, which distorts the choice between labor and the bundle of input varieties for downstream firms. More specifically, this upstream markup makes inputs relatively more expensive and, in response, downstream firms substitute towards labor. At the same time, that markup also incentivizes entry upstream, which generates a variety-productivity effect downstream. To disentangle these two opposing forces, it is useful to compare the market allocation of labor to the social planner's optimal allocation.

Combining equations (3.2), (43), and (47), the aggregate decentralized market allocation of labor to the upstream sector is given by

$$M^u \ell^u = (1-\alpha)L,$$

while from equations (50) and (51), the social planner would allocate a share of labor to that sector

42. Notice that for $\theta > 1$, $\left(\frac{\theta-1}{\theta-\alpha}\right)^{\alpha}\left(\frac{\theta}{\theta-\alpha}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \geq 1$, with equality when α is either 0 or 1.

equal to

$$M^u \ell^u = \frac{\theta}{\theta - \alpha}(1-\alpha)L > (1-\alpha)L.$$

Thus, the market equilibrium is inefficient, in the sense that it allocates too little labor to the upstream sector. It might seem intuitive that this inefficiency is associated with upstream markups leading to a double-marginalization inefficiency. However, note that the market allocation of labor to the upstream sector is actually *independent* of the degree of input substitutability (θ), and thus, of the level of upstream markups. In other words, lower input substitutability – and thus higher markups – do *not* depress the market allocation of labor to the upstream sector; instead, they increase the social-welfare maximizing allocation of labor to that sector. This fact does not necessarily rule out the relevance of a double marginalization inefficiency, but it does suggest that the market inefficiency may alternatively be interpreted as reflecting that, in the market equilibrium, upstream firms do not internalize the fact that their entry generates positive spillovers for firms in the downstream sector, with the size of this spillover decreasing in the degree of input substitutability θ.[43]

When $\alpha = 1$ or $\alpha = 0$, all labor is allocated to either the downstream sector (when $\alpha = 1$) or to the upstream sector (when $\alpha = 0$), and because firm-level output is always efficient, there is no scope for a market inefficiency in those two cases.

.1.3 Optimal Policy

Suppose we endow a government with the ability to provide subsidies (or charge taxes) on the purchases of final goods or intermediate inputs. Denote these taxes by s^d and s^u in the downstream and upstream sectors, respectively. We assume that subsidy proceeds are extracted from households (or tax revenue is rebated to households) in a lump-sum manner.

Once we introduce taxes, price indexes become:

$$P^u = (M^u)^{\frac{1}{1-\theta}}(1-s^u)p^u, \qquad P^d = \left(M^d\right)^{\frac{1}{1-\sigma}}(1-s^d)p^d$$

43. This can be verified from the fourth constraint of the social planner problem above, which indicates that downstream productivity is proportional to $(M^u)^{\frac{\theta(1-\alpha)}{\theta-1}}$.

and household disposable income becomes,

$$I = wL - M^d s^d p^d x^d - M^u s^u p^u x^u.$$

It is straightforward to show that taxes and subsidies do not alter the equilibrium firm size, which is still pinned down by free entry at the (efficient) levels given in (43). Turning to the determination of the measure of firms in each sector, we first invoke households' demand for downstream goods combined with goods-market clearing and household total income to obtain

$$M^d = \frac{wL - s^u M^u p^u x^u}{p^d x^d}.$$

Next, labor market clearing ensures that equation (46) still holds. The equilibrium measure of firms, given subsidies s^d and s^u, is then:

$$\begin{aligned} M^u &= \frac{1}{1-\alpha s^u} \frac{(1-\alpha)A^u L}{\theta f^u} \\ M^d &= (1-s^u)^\alpha \left(\frac{1}{1-\alpha s^u}\right)^{\frac{\theta-\alpha}{\theta-1}} \frac{\alpha^\alpha A^d}{\sigma f^d} \left[\frac{(1-\alpha)A^u}{\theta f^u}\right]^{(1-\alpha)\frac{\theta}{\theta-1}} \left((\theta-1)\, f^u\right)^{1-\alpha} L^{\frac{\theta-\alpha}{\theta-1}}. \end{aligned}$$

Notice that downstream subsidies s^d have no impact on the market allocation. Because they are a *redundant* instrument, we can safely set them to 0. From the above expressions, it is then clear that:

Proposition .1.2. The social planner can restore efficiency in the market equilibrium by subsidizing upstream production at a rate $(s^u)^* = 1/\theta$.

Notice that the subsidy corresponds to the reciprocal of the elasticity of substitution across inputs. As a result, this subsidy encourages the entry of upstream suppliers especially when the inputs they produce are relatively less substitutable. There are two potential (and non-exclusive) explanations for this result. First, the lower is θ, the larger is the market power of and thus the markup charged by input suppliers, and thus the larger the subsidy required to undo this double marginalization inefficiency. Second, the lower is θ, the larger are the variety gains associated with upstream entry on the productivity of downstream firms, so to the extent that those gains are not internalized by input suppliers, again the larger is the required subsidy upstream.

.1.4 *Double Marginalization versus External Effects*

We next dig a little bit deeper into the source of the market inefficiency. More specifically, we show that our vertical Krugman economy is isomorphic to a competitive vertical economy with external economies of scale. In this variant of our model, it is clear that the market inefficiency is due only to upstream suppliers failing to internalize the positive productivity effects of their entry on downstream firms (since there are no markups), and an upstream subsidy is again sufficient to restore efficiency.

The vertical economy with external economies of scale features consumers that spend their income on a single homogeneous final good. On the production side, this final good is produced combining labor and a homogeneous intermediate input, which is in turn produced with labor. The homogeneous intermediate input and final good are produced according to the technologies

$$\begin{aligned} x^u &= A^u \ell^u \left(L^u\right)^{\gamma^u} \\ x^d &= A^d \left(\ell^d\right)^{\alpha} \left(q^u\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{\gamma^d}, \end{aligned}$$

where L^u and L^d are the aggregate allocations of labor to the upstream and downstream sector, Q^u is total production upstream, and γ^u and γ^d measure the magnitude of external economies of scale.

Individual firms are symmetric, competitive, and infinitesimal, so they take the aggregates as given and price at marginal cost. The resulting prices for the upstream and downstream sector are given by

$$P^u = \frac{w}{A^u} \left(L^u\right)^{-\gamma^u}$$

and

$$P^d = \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{P^u}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{-\gamma^d}.$$

Invoking $P^d Q^d = wL$, $Q^u = A^u \left(L^u\right)^{1+\gamma^u}$ and $Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^d}$, it is straightforward to infer that the equilibrium allocation of labor across sectors is given by

$$L^u = (1-\alpha)\, L, \quad \text{and} \qquad L^d = \alpha L,$$

just as in our "Krugman" vertical economy with internal economies of scale. In addition, one can also show that whenever $\gamma^u = 1/(\theta - 1)$ and $\gamma^d = 1/(\sigma - 1)$, with an appropriate choice of the technological parameters A^d and A^u, the equilibria of the two models are fully isomorphic, not just in terms of the allocation of labor across sectors, but also in terms of prices and welfare.

As in the case of internal economies of scale, this market equilibrium can easily be shown to be inefficient. In particular, setting up the planner problem,

$$\begin{aligned} \max_{L^u, L^d} \quad & Q^d = A^d \left(\left(L^d\right)^{\alpha} (Q^u)^{1-\alpha} \right)^{1+\gamma^d} \\ s.t. \quad & Q^u = A^u \left(L^u\right)^{1+\gamma^u} \\ & L^u + L^d = L, \end{aligned} \tag{54}$$

it is straightforward to see that this delivers

$$(L^u)^* = \frac{1+\gamma^u}{\gamma^u (1-\alpha) + 1} (1-\alpha)\, L, \quad \text{and} \qquad \left(L^d\right)^* = \frac{1}{\gamma^u (1-\alpha) + 1} \alpha L. \tag{55}$$

Clearly, the market equilibrium features too little labor allocated to the upstream sector whenever $\gamma^u > 0$ and $0 < \alpha < 1$. Finally, one can also verify that an upstream subsidy equal to $(s^u)^* = \gamma^u / (1 + \gamma^u)$ is sufficient to restore efficiency. While it is perhaps surprising that the planner need not make any correction for the external economies in the downstream sector, this result is due to the fact that all of that sector's output is sold to consumers. By contrast, the fact that inputs are all sold to firms means that their under provision requires a subsidy so that it does not distort final-good producers' purchases of inputs versus labor.

In sum, we have shown that a model with external economies of scales is isomorphic to our model with internal economies of scale as long as $\gamma^u = 1/(\theta - 1)$, and that the rationale for the use of upstream subsidies to restore efficiency can be tied to a love-for-variety productivity effect, rather than it being necessarily driven by a double marginalization inefficiency.

.1.5 Extensions

In this Appendix, we briefly develop two extensions of our closed-economy model, both featuring a more complex input sector.

I. Roundabout Production Upstream

We first allow the upstream sector to use not only labor in production, but also the same bundle of inputs Q^u used in the final-good sector. More specifically, and focusing on the isomorphic economy with perfect competition, homogeneous goods, and external economies of scale developed in Section .1.4, we now assume

$$
\begin{aligned}
x^u &= A^u \left(\ell^u\right)^\beta \left(q^u\right)^{1-\beta} \left(\left(L^u\right)^\beta \left(Q^u\right)^{1-\beta}\right)^{\gamma^u} \\
x^d &= A^d \left(\ell^d\right)^\alpha \left(q^u\right)^{1-\alpha} \left(\left(L^d\right)^\alpha \left(Q^u\right)^{1-\alpha}\right)^{\gamma^d},
\end{aligned}
$$

where β governs the labor intensity of production upstream. It is clear from the second of these expressions that firms in the downstream sector will spend a fraction of its costs on the upstream sector, or

$$P^u q^u = (1-\alpha) P^d x^d.$$

Noting that, due to symmetry, $x^u = Q^u$ and $x^d = Q^d$, and that the decentralized market prices for the downstream sector is given by

$$P^d = \frac{1}{A^d}\left(\frac{w}{\alpha}\right)^\alpha \left(\frac{P^u}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^\alpha \left(Q^u\right)^{1-\alpha}\right)^{-\gamma^d}.$$

Invoking $P^d Q^d = wL$ and $Q^d = A^d \left(\left(L^d\right)^\alpha \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^d}$ we thus obtain

$$\frac{1}{A^d}\left(\frac{w}{\alpha}\right)^\alpha \left(\frac{P^u Q^u}{1-\alpha}\right)^{1-\alpha} A^d \left(L^d\right)^\alpha = wL$$

or

$$\left(\frac{w}{\alpha}\right)^{\alpha}(wL)^{1-\alpha}\left(L^{d}\right)^{\alpha}=wL,$$

from which it is immediate that

$$L^{u}=(1-\alpha)\,L, \quad \text{and} \quad L^{d}=\alpha L,$$

just as in our baseline model

We next consider the planner problem,

$$\begin{aligned}
\max_{L^u,L^d} \quad & Q^{d}=A^{d}\left(\left(L^{d}\right)^{\alpha}(Q^{u})^{1-\alpha}\right)^{1+\gamma^{d}} \\
s.t. \quad & Q^{u}=A^{u}\left((L^{u})^{\beta}(Q^{u})^{1-\beta}\right)^{1+\gamma^{u}} \\
& L^{u}+L^{d}=L.
\end{aligned}$$

Noting that the second constraint can be written as

$$Q^{u}=\tilde{A}^{u}\left(L^{u}\right)^{1+\tilde{\gamma}^{u}},$$

where

$$\begin{aligned}
\tilde{A}^{u} &= (A^{u})^{\frac{1}{1-(1-\beta)(1+\gamma^{u})}} \\
\tilde{\gamma}^{u} &= \frac{\gamma^{u}}{1-(1-\beta)\left(1+\gamma^{u}\right)},
\end{aligned}$$

it then becomes clear that this program is identical to the one in our baseline model, except for the fact that the scale elasticity upstream is now not given by γ^u, but by $\tilde{\gamma}^u > \gamma^u$ (the program also features a rescaled upstream productivity, but that is immaterial). Note that the gap between $\tilde{\gamma}^u$ and γ^u is decreasing in β.

Analogously to equation (55), the socially optimal allocation of labor is given by

$$(L^u)^* = \frac{1+\tilde{\gamma}^u}{\tilde{\gamma}^u(1-\alpha)+1}(1-\alpha)L, \quad \text{and} \qquad \left(L^d\right)^* = \frac{1}{\tilde{\gamma}^u(1-\alpha)+1}\alpha L.$$

Clearly, the market equilibrium features too little labor allocated to the upstream sector whenever $\gamma^u > 0$ and $0 < \alpha < 1$, just as in our baseline model, but the inefficiency is now decreasing in β. Finally, one can also verify that an upstream subsidy equal to $(s^u)^* = \tilde{\gamma}^u / (1+\tilde{\gamma}^u)$ is sufficient to restore efficiency. Because $\tilde{\gamma}^u > \gamma^u$, this subsidy is now higher than in our baseline model, and it is decreasing in β.

II. Multi-Stage Production

We next develop a multi-stage extension of the model. We begin with a simple three-stage production process with a downstream sector, a midstream sector, and an upstream sector. The technologies are given by

$$\begin{aligned} x^u &= A^u(\ell^u)(L^u)^{\gamma^u} \\ x^m &= A^d(\ell^m)^\beta (q^u)^{1-\beta}\left((L^m)^\beta (Q^u)^{1-\beta}\right)^{\gamma^m} \\ x^d &= A^d\left(\ell^d\right)^\alpha (q^m)^{1-\alpha}\left(\left(L^d\right)^\alpha (Q^m)^{1-\alpha}\right)^{\gamma^d}, \end{aligned}$$

Using the fact that, in a decentralized equilibrium, we have

$$\begin{aligned} P^d Q^d &= wL; \\ Q^d &= A^d\left(\left(L^d\right)^\alpha (Q^m)^{1-\alpha}\right)^{1+\gamma^d}; \\ P^d &= \frac{1}{A^d}\left(\frac{w}{\alpha}\right)^\alpha\left(\frac{P^m}{1-\alpha}\right)^{1-\alpha}\left(\left(L^d\right)^\alpha (Q^m)^{1-\alpha}\right)^{-\gamma^d}; \\ P^m Q^m &= (1-\alpha)P^d Q^d, \end{aligned}$$

we immediately obtain

$$L^d = \alpha L.$$

Next, because

$$
\begin{aligned}
P^m Q^m &= (1-\alpha)\, wL; \\
Q^m &= A^m \left((L^m)^\alpha (Q^u)^{1-\alpha} \right)^{1+\gamma^m}; \\
P^m &= \frac{1}{A^m} \left(\frac{w}{\beta} \right)^\beta \left(\frac{P^u}{1-\beta} \right)^{1-\beta} \left((L^m)^\beta (Q^u)^{1-\beta} \right)^{-\gamma^m}; \\
P^u Q^u &= (1-\beta)\, P^m Q^m,
\end{aligned}
$$

we obtain

$$L^m = \beta (1-\alpha) L, \quad \text{and} \qquad L^u = (1-\beta)(1-\alpha) L.$$

Now consider the planner problem

$$
\begin{aligned}
\max_{L^u, L^m, L^d} \quad & Q^d = A^d \left(\left(L^d \right)^\alpha (Q^m)^{1-\alpha} \right)^{1+\gamma^d} \\
s.t. \quad & Q^m = A^m \left((L^m)^\beta (Q^u)^{1-\beta} \right)^{1+\gamma^u} \\
& Q^u = A^u (L^u)^{1+\gamma^u} \\
& L^u + L^m + L^d = L.
\end{aligned}
$$

which delivers

$$
\begin{aligned}
(L^u)^* &= \frac{(1+\gamma^u)(1+\gamma^m)}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} (1-\beta)(1-\alpha) L \\
(L^m)^* &= \frac{1+\gamma^m}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \beta (1-\alpha) L \\
\left(L^d\right)^* &= \frac{1}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \alpha L.
\end{aligned}
$$

Notice that the gap between the socially optimal and the market allocation of labor is higher the more upstream the stage. Does that mean that subsidies are higher, the more upstream a sector? To answer this question, consider the following key conditions identify a market equilibrium with subsidies:

$$\begin{aligned}
P^d Q^d &= wL - s^m P^m Q^m - s^u P^u Q^u \\
Q^d &= A^d \left(\left(L^d \right)^{\alpha} \left(Q^m \right)^{1-\alpha} \right)^{1+\gamma^d}; \\
P^d &= \frac{1}{A^d} \left(\frac{w}{\alpha} \right)^{\alpha} \left(\frac{\left(1 - s^m\right) P^m}{1-\alpha} \right)^{1-\alpha} \left(\left(L^d \right)^{\alpha} \left(Q^m \right)^{1-\alpha} \right)^{-\gamma^d}; \\
\left(1 - s^m\right) P^m Q^m &= \left(1 - \alpha\right) P^d Q^d \\
Q^m &= A^m \left(\left(L^m \right)^{\alpha} \left(Q^u \right)^{1-\alpha} \right)^{1+\gamma^m} \\
P^m &= \frac{1}{A^m} \left(\frac{w}{\beta} \right)^{\beta} \left(\frac{\left(1 - s^u\right) P^u}{1-\beta} \right)^{1-\beta} \left(\left(L^m \right)^{\beta} \left(Q^u \right)^{1-\beta} \right)^{-\gamma^m} \\
\left(1 - s^u\right) P^u Q^u &= \left(1 - \beta\right) P^m Q^m
\end{aligned}$$

Note that

$$P^d Q^d = wL - \frac{s^m}{1-s^m} \left(1-\alpha\right) P^d Q^d - \frac{s^u}{1-s^u} \frac{\left(1-\beta\right)}{1-s^m} \left(1-\alpha\right) P^d Q^d$$

or

$$P^d Q^d = \frac{wL}{1 + \frac{s^m}{1-s^m}\left(1-\alpha\right) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}\left(1-\alpha\right)}.$$

Next

$$\begin{aligned}
P^d Q^d &= \left(\frac{wL^d}{\alpha} \right)^{\alpha} \left(\frac{\left(1 - s^m\right) P^m Q^m}{1-\alpha} \right)^{1-\alpha} \\
&= \left(\frac{wL^d}{\alpha} \right)^{\alpha} \left(P^d Q^d \right)^{1-\alpha},
\end{aligned}$$

so

$$\frac{L^d}{L} = \frac{\alpha}{1 + \frac{s^m}{1-s^m}\left(1-\alpha\right) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}\left(1-\alpha\right)}.$$

Next,

$$\begin{aligned}
P^m Q^m &= \left(\frac{wL^m}{\beta} \right)^{\beta} \left(\frac{\left(1 - s^u\right) P^u Q^u}{1-\beta} \right)^{1-\beta} \\
&= \left(\frac{wL^m}{\beta} \right)^{\beta} \left(P^m Q^m \right)^{1-\beta},
\end{aligned}$$

so

$$P^m Q^m = \frac{(1-\alpha)}{(1-s^m)} P^d Q^d = \frac{wL^m}{\beta}$$

or

$$\frac{L^m}{L} = \frac{\beta \frac{(1-\alpha)}{(1-s^m)}}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)}.$$

We thus have that the subsidies s^m and s^u need to satisfy

$$\frac{\alpha}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)} = \frac{1}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))}\alpha$$

and

$$\frac{\beta \frac{(1-\alpha)}{(1-s^m)}}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)} = \frac{1+\gamma^m}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))}\beta(1-\alpha),$$

which delivers

$$s^m = \frac{\gamma^m}{1+\gamma^m}; \quad s^u = \frac{\gamma^u}{1+\gamma^u}.$$

As is clear from this expression, subsidies in all sectors producing inputs are positive, but notice that subsidies are higher upstream relative to midstream only if $\gamma^u > \gamma^m$, i.e., only if the scale elasticity is higher upstream than midstream. This contrasts with the results of Liu (2019), who finds that optimal subsidies should necessarily be higher, the more upstream the sector. The reason is that, unlike in Liu's work, we solve for first-best subsidy policy: when the government can only set subsidies in one sector, the size of the subsidy would be higher, the more upstream the sector, because as we have seen above, the gap between the social optimal and market allocation of labor is highest in the upstream sector.

.2 Open Economy Model: Details on Derivations

.2.1 *Open Economy Equilibrium with Internal Economies of Scale*

In this Appendix, we outline the equilibrium conditions corresponding to the two-country model featuring internal scale economies, product differentiation and monopolistic competition outlined in Section 3.3.1. We will then work with these equations in Appendix to demonstrate the isomorphism claimed in Proposition 3.3.1.

Import tariffs on the downstream sector create a wedge between consumer prices in country i and producer prices in country j. More specifically, given CES preferences, consumer prices in i for goods originating in j are given by:

$$p_{ji}^{d} = \left(1 + t_{i}^{d}\right) \frac{\sigma}{\sigma - 1} \tau^{d} \frac{1}{A_{j}^{d}} \left(\frac{w_{j}}{\alpha}\right)^{\alpha} \left(\frac{P_{j}^{u}}{1-\alpha}\right)^{1-\alpha} = \frac{1 + t_{i}^{d}}{1 - v_{j}^{d}} \tilde{p}_{ji}^{d}. \tag{56}$$

Similarly, import tariffs on the upstream sector create a wedge between the price paid by final-good producers in i for inputs from j, and the producer price for those inputs obtained by suppliers in country j. In particular, we have

$$p_{ji}^{u} = \left(1 + t_{i}^{u}\right) \frac{\theta}{\theta - 1} \tau^{u} \frac{w_{j}}{A_{j}^{u}} = \frac{1 + t_{i}^{u}}{1 - v_{j}^{u}} \tilde{p}_{ji}^{u}. \tag{57}$$

In equation (56), the price index P_{i}^{u} is given by

$$P_{i}^{u} = \left[\sum_{j \in \{H,F\}} \left(P_{ji}^{u}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{58}$$

with

$$P_{ji}^{u} = \left[\int_{0}^{M_{j}^{u}} \left(p_{ji}^{u}(\varpi)\right)^{1-\theta} d\varpi\right]^{\frac{1}{1-\theta}}. \tag{59}$$

When setting $j = i$, the above pricing equations also characterize domestic prices in country j after setting $t_{i}^{d} = t_{i}^{u} = v_{i}^{d} = v_{i}^{u} = 0$ and $\tau^{d} = \tau^{u} = 1$. Note that $p_{ii}^{d} = \tilde{p}_{ii}^{d}$ and $p_{ii}^{u} = \tilde{p}_{ii}^{u}$.

Next, utility maximization implies that when consuming country j varieties, consumers in i

allocate to each variety ω a share of spending equal to

$$\frac{p_{ji}^{d}(\omega)\, q_{ji}^{d}(\omega)}{P_{ji}^{d} Q_{ji}^{d}} = \left(\frac{p_{ji}^{d}(\omega)}{P_{ji}^{d}}\right)^{1-\sigma}, \tag{60}$$

of their total spending on country j varieties, where

$$P_{ji}^{d} = \left[\int_{0}^{M_{j}^{d}} \left(p_{ji}^{d}(\omega)\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}. \tag{61}$$

Consumers' (aggregate) spending on Home and Foreign varieties is in turn determined by

$$P_{ji}^{d} Q_{ji}^{d} = \left(\frac{P_{ji}^{d}}{P_{i}^{d}}\right)^{1-\sigma} (w_i L_i + R_i), \tag{62}$$

where P_i^d is the aggregate consumer price index in i

$$P_{i}^{d} = \left[\sum_{j\in\{H,F\}} \left(P_{ji}^{d}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{63}$$

and R_i is tariff revenue, which we have defined in equation (3.12).

We now turn to profit maximization by downstream producers in country i. First note that free entry implies that firm revenue (net of tariffs) will equal total costs, and that a share of those costs will go to pay labor. As a result, labor compensation by each final-good producer in i is given by

$$w_i \ell_i^d = \alpha \left(\tilde{p}_{ii}^{d} q_{ii}^{d} + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^{d} q_{ij}^{d}\right). \tag{64}$$

Next, when purchasing inputs from upstream producers in country j, final-good producers in country i, will demand an amount of each variety ϖ from country j equal to

$$q_{ji}^{u}(\varpi) = Q_{ji}^{u}(\omega) \left(\frac{p_{ji}^{u}}{P_{ji}^{u}}\right)^{-\theta},$$

while aggregate spending on all country j's input varieties is given by

$$P_{ji}^{u}Q_{ji}^{u}=(1-\alpha)\left(\tilde{p}_{ii}^{d}q_{ii}^{d}+\frac{1-v_{i}^{d}}{1+t_{j}^{d}}\tilde{p}_{ij}^{d}q_{ij}^{d}\right)\left(\frac{P_{ji}^{u}}{P_{i}^{u}}\right)^{1-\theta}M_{i}^{d}. \tag{65}$$

Aggregate spending on Home and Foreign intermediate inputs in country i is then given by

$$P_{i}^{u}Q_{i}^{u}=(1-\alpha)\left(\tilde{p}_{ii}^{d}q_{ii}^{d}+\frac{1-v_{i}^{d}}{1+t_{j}^{d}}\tilde{p}_{ij}^{d}q_{ij}^{d}\right)M_{i}^{d}. \tag{66}$$

Our final set of equilibrium conditions impose market clearing. First, labor-market clearing in both countries implies that

$$L_{i}=M_{i}^{d}\ell_{i}^{d}+M_{i}^{u}\ell_{i}^{u}, \tag{67}$$

where ℓ_i^d is given in (64), and $\ell_i^u=\left(f_i^u+x_i^u\right)/A_i^u$.[44] Second, goods-market clearing imposes

$$q_{ii}^{d}+\tau^{d}q_{ij}^{d}=x_{i}^{d} \tag{68}$$

and

$$M_{i}^{d}q_{ii}^{u}+M_{j}^{d}\tau^{u}q_{ij}^{u}=x_{i}^{u}. \tag{69}$$

Note that free entry upstream and downstream implies that firm revenue is equal to total costs, which delivers

$$x_{i}^{d}=(\sigma-1)\,f_{i}^{d};\qquad x_{i}^{u}=(\theta-1)\,f_{i}^{u} \tag{70}$$

for $i=\{H,F\}$. Firm-level production levels are thus independent of tariff choices, and the only way in which tariffs can affect the allocation of labor across sectors is by changing the measure of firms in each of the two sectors. As a result, optimal trade policies will seek to achieve a social-welfare maximizing allocation of labor across sectors, with no concern for the allocation of labor within sectors (across fixed costs of entry versus marginal costs of production).

Despite the simple structure of the model and relatively simple equilibrium conditions, an

44. Naturally, equilibrium also requires trade balance, but this is ensured by the other equilibrium conditions outlined in this section.

analysis of how the market equilibrium is affected by input and final-good tariffs set by the Home country is complex, so we begin by considering the special case in which downstream production only uses inputs (and no labor) in production, or $\alpha = 0$.

.2.2 *Equilibrium of Isomorphic Competitive Economy with External Economies of Scale*

In this Appendix we prove the isomorphism claimed in Proposition 3.3.1. More specifically, our goal is to show that equilibrium conditions of the decentralized equilibrium of the two-country model in Section 3.3.1 featuring internal scale economies, product differentiation and monopolistic competition can be reduced to a set of equations identical to equations (3.16) through (3.23) applying to the competitive model with external economies of scale developed in this section.

Preferences We begin by noting that given symmetry in final-good production, we can express preferences as

$$\begin{aligned} U_i &= \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^d} q_{ji}^d(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[M_i^d (q_{ii}^d)^{\frac{\sigma-1}{\sigma}} + M_j^d (q_{ji}^d)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left(\left(Q_{ii}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{ji}^d\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where

$$Q_{ii}^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} q_{ii}^d; \quad Q_{ji}^d \equiv \left(M_j^d\right)^{\frac{\sigma}{\sigma-1}} q_{ji}^d. \tag{71}$$

Starting from (3.11), we have thus derived (3.13), which are preferences in the isomorphic economy with two final goods (a Home one and a Foreign one) and external economies of scale.

Labor-Market Clearing Next, remember that ℓ_i^d and ℓ_i^u are the firm-level amounts of labor used downstream and upstream to cover fixed and variable costs. Hence, defining

$$L_i^d \equiv M_i^d \ell_i^d; \quad L_i^u \equiv M_i^u \ell_i^u, \tag{72}$$

we have that equation (67) in the monopolistic competition model implies equation (3.16) in the external economies model, or

$$L_i = M_i^d \ell_i^d + M_i^u \ell_i^u = L_i^u + L_i^d.$$

Upstream Market Clearing and Upstream Endogenous Productivity Next let us define

$$Q_{ii}^u \equiv M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u; \quad Q_{ij}^u \equiv M_j^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ij}^u. \tag{73}$$

Given these definitions in (73), and given the definition of the input aggregate $Q_i^u(\omega)$ in the monopolistic competition model, that is

$$Q_i^u(\omega) = \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^u} q_{ji}^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi\right)\right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \quad i \in \{H,F\},$$

we have that the total usage of inputs by firms in country i is given by

$$\begin{aligned} Q_i^u &= M_i^d Q_i^u(\omega) = \left[M_i^u \left(q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + M_j^u \left(q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(M_i^d \left(M_j^u\right)^{\frac{\theta}{\theta-1}} \left(q_{ji}^u\right)\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \end{aligned} \tag{74}$$

and thus is analogous to a CES aggregator of only two inputs: a Home and a Foreign one, as defined in equation (73). These inputs are either produced domestically or are imported.

Now consider the domestic production of those inputs. Let us start from the definition of upstream technology in the monopolistic competition model, that is

$$f_i^u + x_i^u(\varpi) = A_i^u \ell_i^u(\varpi), \qquad \varpi \in [0, M_i^u], \quad i \in \{H, F\}.$$

Imposing symmetry and firm-level output in equation (70) – i.e., $x_i^u = (\theta - 1) f_i^u$ –, and invoking the definition of L_i^u in (72), we have

$$X_i^u \equiv (M_i^u)^{\frac{\theta}{\theta-1}} x_i^u = \left(\frac{A_i^u}{\theta f_i^u}\right)^{\frac{\theta}{\theta-1}} (\theta - 1) f_i^u (L_i^u)^{\frac{\theta}{\theta-1}} \tag{75}$$

or

$$X_i^u = \hat{A}_i^u F_i^u (\ell_i^u) = \bar{A}_i^u (L_i^u)^{1+\gamma^u},$$

where

$$\bar{A}_i^u \equiv \left(\frac{A_i^u}{\theta f_i^u}\right)^{\frac{\theta}{\theta-1}} (\theta - 1) f_i^u,$$

and

$$\gamma^u \equiv 1/(\theta - 1).$$

Because this domestic production X_i^u is sold domestically or exported, we have

$$\bar{A}_i^u (L_i^u)^{1+\gamma^u} = Q_{ii}^u + Q_{ij}^u,$$

which corresponds exactly to equation (3.17) in the external economies model.

Downstream Market Clearing and Downstream Endogenous Productivity We can proceed analogously for final-good production. We begin with the definition of technology in the downstream sector in the monopolistic competition model:

$$f_i^d + x_i^d(\omega) = A_i^d (\ell_i^d(\omega))^{\alpha} Q_i^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M_i^d], \quad \alpha \in [0,1], \quad i \in \{H, F\}.$$

Imposing symmetry and (70), we obtain

$$M_i^d = \frac{A_i^d}{\sigma f_i^d} (M_i^d \ell_i^d(\omega))^\alpha \left(M_i^d Q_i^u\right)^{1-\alpha}$$

or

$$X_i^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = \bar{A}_i^d \left[\left(L_i^d\right)^\alpha \left(\left(L_i^d\right)^\alpha \left((Q_{ii}^u)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}}. \tag{76}$$

where

$$\bar{A}_i^d \equiv \left(\frac{A_i^d}{\sigma f_i^d}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) f_i^d.$$

This aggregate output X_i^d is sold domestically or exported, and thus

$$\bar{A}_i^d \left(\left(L_i^d\right)^\alpha \left((Q_{ii}^u)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\right)^{\gamma^d} = Q_{ii}^d + Q_{ij}^d,$$

where

$$\gamma^d \equiv 1/(\sigma-1).$$

In sum, starting from the monopolistic competition model, we have derived equation (3.17) in the external economies model.

Trade Balance Consider next the trade balance condition. Starting from the monopolistic competition economy, we have

$$\frac{p_{ji}^d}{1+t_i^d} M_j^d q_{ji}^d + \frac{p_{ji}^u}{1+t_i^u} M_i^d M_j^u q_{ji}^u = \frac{\tilde{p}_{ij}^d}{1-v_i^d} M_i^d q_{ij}^d + \frac{\tilde{p}_{ij}^u}{1-v_i^u} M_j^d M_i^u q_{ij}^u, \tag{77}$$

which equates the import revenue in i paid to exporters in j with export revenue collected from j by producers in i.

Now from equations (60) and (61), notice that we have

$$\frac{p_{ji}^d(\omega)\, q_{ji}^d(\omega)}{P_{ji}^d Q_{ji}^d} = \left(\frac{p_{ji}^d(\omega)}{P_{ji}^d}\right)^{1-\sigma},$$

and

$$P_{ji}^{d}=\left[\int_{0}^{M_{j}^{d}}\left(p_{ji}^{d}\left(\omega\right)\right)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}},$$

so given symmetry, we have

$$P_{ji}^{d}=\left(M_{j}^{d}\right)^{\frac{1}{1-\sigma}}p_{ji}^{d} \tag{78}$$

and

$$P_{ji}^{d}Q_{ji}^{d}=\left(M_{j}^{d}\right)^{\frac{-1}{\sigma-1}}p_{ji}^{d}\times\left(M_{i}^{d}\right)^{\frac{\sigma}{\sigma-1}}q_{ii}^{d}=M_{j}^{d}p_{ji}^{d}q_{ii}^{d}.$$

Similarly, for inputs

$$P_{ji}^{u}Q_{ji}^{u}=\left(M_{j}^{u}\right)^{\frac{1}{1-\theta}}p_{ji}^{u}\times M_{i}^{d}\left(M_{i}^{u}\right)^{\frac{\theta}{\theta-1}}q_{ji}^{u}=p_{ji}^{u}M_{i}^{d}M_{j}^{u}q_{ji}^{u}.$$

This implies that we can write total imports in the trade balance condition (77) as

$$\frac{P_{ji}^{d}}{1+t_{i}^{d}}Q_{ji}^{d}+\frac{P_{ji}^{u}}{1+t_{i}^{u}}Q_{ji}^{u}=\bar{P}_{ji}^{d}Q_{ji}^{d}+\bar{P}_{ji}^{u}Q_{ji}^{u},$$

which corresponds to the left-hand-side of the trade balance condition (3.19) for the economy with external economies of scale after noting that $\bar{P}_{ji}^{d}$ and $\bar{P}_{ji}^{u}$ are the prices collected by country j (or Foreign) exporters (not those paid by domestic or country i consumers).

Now consider revenue from exporting final goods. Notice that, regardless of whether the Foreign government imposes import tariffs or not, we have that export revenue is

$$\frac{\tilde{p}_{ij}^{d}}{1-v_{i}^{d}}M_{i}^{d}q_{ij}^{d}+\frac{\tilde{p}_{ij}^{u}}{1-v_{i}^{u}}M_{j}^{d}M_{i}^{u}q_{ij}^{u}$$

Prices paid by country j are $\tilde{p}_{ij}^{d}/\left(1-v_{i}^{d}\right)$ and $\tilde{p}_{ij}^{u}/\left(1-v_{i}^{u}\right)$, so following analogous steps, the right-hand-side of (3.19) becomes

$$\frac{\tilde{P}_{ij}^{d}}{1-v_{i}^{d}}Q_{ij}^{d}+\frac{\tilde{P}_{ij}^{u}}{1-v_{i}^{u}}Q_{ij}^{u}=\bar{P}_{ij}^{d}Q_{ij}^{d}+\bar{P}_{ij}^{u}Q_{ij}^{u},$$

where $\bar{P}_{ij}^{d}$ and $\bar{P}_{ij}^{u}$ are the prices paid by country j (or Foreign) importers (not those paid collected by country i exporters).

Note: In the main text, we denote $\bar{P}^d_{ji}$, $\bar{P}^u_{ji}$, $\bar{P}^d_{ij}$, and $\bar{P}^u_{ij}$ as simply P^d_{ji}, P^u_{ji}, P^d_{ij}, and P^u_{ij}. We do so not to clutter the notation, but these are distinct from the price indices applying to the monopolistic competition model, which are always built based on prices paid by consumers, regardless of their country.

Optimality Conditions We have so far shown that the four 'resource' constraints (3.16) through (3.19) of our isomorphic economy can be derived from our baseline model with monopolistic competition and internal economies of scale. We next turn to an analogous derivation for the optimality conditions (3.20) through (3.23).

Given our functional forms for utility and technology, these optimality conditions in the model with external economies of scale are given by

$$\left(\frac{Q^d_{ii}}{Q^d_{ji}}\right)^{-\frac{1}{\sigma}} = \frac{\left(1-v^d_i\right)}{\left(1+t^d_i\right)}\frac{\bar{P}^d_{ij}}{\bar{P}^d_{ji}}; \tag{79}$$

$$\left(\frac{Q^u_{ii}}{Q^u_{ji}}\right)^{-\frac{1}{\theta}} = \frac{\left(1-v^u_i\right)}{\left(1+t^u_i\right)}\frac{\bar{P}^u_{ij}}{\bar{P}^u_{ji}}; \tag{80}$$

$$(1-\alpha)\,\hat{A}^d_i\left(L^d_i\right)^{\alpha}\left(\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}+\left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{1}{Q^u_{ii}}\frac{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}+\left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}} = \frac{\left(1-v^u_i\right)\bar{P}^u_{ij}}{\left(1-v^d_i\right)\bar{P}^d_{ij}}; \tag{81}$$

$$(1-\alpha)\,\hat{A}^u_i\frac{1}{Q^u_{ii}}\frac{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}+\left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}} = \alpha\frac{1}{L^d_i}. \tag{82}$$

Optimal in Final-Good Consumption Let us begin with the first one, equating the marginal rate of substitution in final-good consumption to relative prices. Given equation (62) in the model with monopolistic competition, we have

$$\frac{Q^d_{ii}}{Q^d_{ji}} = \left(\frac{P^d_{ii}}{P^d_{ji}}\right)^{-\sigma},$$

where Q^d_{ji}, Q^d_{ji}, P^d_{ji} and P^d_{ji} are defined in (73) and (78). Thus, we have

$$\left(\frac{Q^d_{ii}}{Q^d_{ji}}\right)^{-\frac{1}{\sigma}} = \frac{P^d_{ii}}{P^d_{ji}} = \frac{\left(1 - v^d_i\right) \bar{P}^d_{ij}}{\left(1 + t^d_i\right) \bar{P}^d_{ji}},$$

where $P^d_{ii} = \left(1 - v^d_i\right) \bar{P}^d_{ij}$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}^d_{ij}$ is the price paid by consumers in j for final goods from j). We have thus derived equation (79), which corresponds to (3.20) in the external economies of scale model.

Optimal in Input Consumption The derivation of equation (80), equating the marginal rate of substitution in input consumption to relative prices, is completely analogous. In particular, from equation (65) in the model with monopolistic competition, we have

$$\frac{Q^u_{ii}}{Q^u_{ji}} = \left(\frac{P^u_{ii}}{P^u_{ji}}\right)^{-\theta},$$

where Q^u_{ji}, Q^u_{ji}, P^u_{ji} and P^u_{ji} are defined in (73) and (78). Thus, we have

$$\left(\frac{Q^u_{ii}}{Q^u_{ji}}\right)^{-\frac{1}{\theta}} = \frac{P^u_{ii}}{P^u_{ji}} = \frac{\left(1 - v^u_i\right) \bar{P}^u_{ij}}{\left(1 + t^u_i\right) \bar{P}^u_{ji}},$$

where $P^u_{ii} = \left(1 - v^d_i\right) \bar{P}^u_{ij}$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}^u_{ij}$ is the price paid by consumers in j for final goods from j). We have thus derived equation (80), which corresponds to (3.21) in the external economies of scale model.

Optimality Domestic Input Allocation We next move to the third optimality condition (3.22), which equates the benefits of exporting domestic intermediate inputs with the benefits of using those additional domestic inputs to produce an additional amount of the final good that is in turn exported.

We begin with equation (66), and note that aggregate input use in country i in the monopolistic competition model is given by

$$P_i^u Q_i^u = (1-\alpha)\left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d q_{ij}^d\right) M_i^d. \tag{83}$$

To reiterate this, note from (57) that $\frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d = \tau^d p_{ii}^d$, and plugging in equation (68), we obtain

$$P_i^u Q_i^u = (1-\alpha)\, p_{ii}^d \left(q_{ii}^d + \tau^d q_{ij}^d\right) M_i^d = (1-\alpha)\, p_{ii}^d x_i^d M_i^d. \tag{84}$$

Next invoke equation (65) applied to $P_{ii}^u Q_{ii}^u$ to obtain (after plugging in (83)):

$$P_i^u Q_i^u = P_{ii}^u Q_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}. \tag{85}$$

Combining (84) and (85), we obtain:

$$(1-\alpha)\, p_{ii}^d x_i^d M_i^d = P_{ii}^u Q_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1},$$

which we decompose as

$$(1-\alpha) \times \left(M_i^d\right)^{\frac{-1}{\sigma-1}} p_{ii}^d \times \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = P_{ii}^u Q_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}, \tag{86}$$

Now remember from equation (76) derived above that

$$\left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = \bar{A}_i^d \left[\left(L_i^d\right)^{\alpha} \left(\left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}},$$

and also from (78) that $\left(M_i^d\right)^{\frac{-1}{\sigma-1}} p_{ii}^d = P_{ii}^d$, so we can write (86) as

$$P_{ii}^d (1-\alpha)\, \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} = P_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}.$$

Now invoke (65)

$$\frac{Q_{ii}^u}{Q_i^u} = \left(\frac{P_{ii}^u}{P_i^u}\right)^{-\theta},$$

to obtain

$$P_{ii}^d (1-\alpha)\hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} = P_{ii}^u \left(\frac{Q_{ii}^u}{Q_i^u}\right)^{-\frac{\theta-1}{\theta}},$$

which given the definition of Q_i^u in (56) delivers

$$P_{ii}^d (1-\alpha)\hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} \frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}} = P_{ii}^u.$$

The final step is to note, as we did above, that indifference between selling domestically and exporting, delivers $P_{ii}^d = \left(1-v_i^d\right)\bar{P}_{ij}^d$ and $P_{ii}^u = \left(1-v_i^d\right)\bar{P}_{ij}^u$, where remember that $\bar{P}_{ij}^d$ and $\bar{P}_{ij}^u$ are the prices paid by country j residents. In sum, we have derived equation (81), or

$$(1-\alpha)\hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} \frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}} = \frac{\left(1-v_i^d\right)\bar{P}_{ij}^u}{\left(1-v_i^d\right)\bar{P}_{ij}^d}.$$

Optimal Labor Market Allocation We finally tackle the fourth optimality condition, associated with the optimal allocation of labor across sectors. We begin with the firm-level monopolistic competition model, equating the wage paid in both sectors. Because of free entry, total revenue upstream must equal total wage payments, while in the downstream sector, wage payments are a share α of total revenue, as indicated in equation (64), or

$$\frac{\alpha\left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d q_{ij}^d\right)}{\ell_i^d} = \frac{\tilde{p}_{ii}^u M_i^d q_{ii}^u + \frac{1-v_i^u}{1+t_j^u}\tilde{p}_{ij}^u M_j^d q_{ij}^u}{\ell_i^u}.$$

Now noting that from (57), we have $\frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d = \tau^d p_{ii}^d$ (and analogously $\frac{1-v_i^u}{1+t_j^u}\tilde{p}_{ij}^u = \tau^u p_{ii}^u$), and plugging in equations (68) and (69), we have

$$\frac{\alpha p_{ii}^d x_i^d}{\ell_i^d} = \frac{p_{ii}^u x_i^u}{\ell_i^u}. \tag{87}$$

Next, invoke the price index definitions – see equation 78) – as well as the definitions $L_i^d = M_i^d \ell_i^d$ and $L_i^u = M_i^u \ell_i^u$, to write (87) as

$$\alpha P_{ii}^d \frac{x_i^d \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}}}{L_i^d} = P_{ii}^u \frac{\left(M_i^u\right)^{\frac{\theta}{\theta-1}} x_i^u}{L_i^u}.$$

Next, plugging (75) and (76), delivers

$$\frac{\alpha P_{ii}^d}{L_i^d} \hat{A}_i^d \left(L_i^d\right)^\alpha \left(\left(L_i^d\right)^\alpha \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha} = P_{ii}^u \hat{A}_i^u,$$

where remember that $\hat{A}_i^d$ and $\hat{A}_i^u$ are defined in equations (3.14) and (3.15) in the main text.

The next step is to note, as we did above, that indifference between selling domestically and exporting, delivers $P_{ii}^d = \left(1 - v_i^d\right) \bar{P}_{ij}^d$ and $P_{ii}^u = \left(1 - v_i^d\right) \bar{P}_{ij}^u$, where remember that $\bar{P}_{ij}^d$ and $\bar{P}_{ij}^u$ are the prices paid by country j residents, so we have

$$\frac{\alpha}{L_i^d} \hat{A}_i^d \left(L_i^d\right)^\alpha \left(\left(L_i^d\right)^\alpha \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha} = \hat{A}_i^u \frac{\left(1 - v_i^d\right)}{\left(1 - v_i^d\right)} \frac{\bar{P}_{ij}^u}{\bar{P}_{ij}^d}.$$

The final step is to plug optimality condition (81) and cancel terms to obtain

$$\frac{\alpha}{L_i^d} = (1 - \alpha) \hat{A}_i^u \frac{1}{Q_{ii}^u} \frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}},$$

which corresponds to the last optimality condition (81).

This completes the proof of the isomorphism claimed in Proposition 3.3.1.

.3 Optimal Trade Policy for a Small Open Economy with No Domestic Distortions: Derivations

.3.1 First-Best Policies

We begin by characterizing the solution to the program

$$\max \qquad U\left(Q^d_{HH}, Q^d_{FH}\right),$$

subject to the constraints,

$$\begin{aligned}
\hat{A}^u_H L_H &= Q^u_{HH} + Q^u_{HF} \\
\hat{A}^d_H F^d\left(Q^u_{HH}, Q^u_{FH}\right) &= Q^d_{HH} + Q^d_{HF} \\
P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} &= Q^d_{HF} (Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF} \left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}},
\end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H \left(L_H\right)^{\gamma^u},$$

respectively. We also note that

$$U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

and that

$$F^d\left(Q^u_{HH}, Q^u_{FH}\right) = \left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},$$

but this will prove immaterial for the derivations below.

We first write the Lagrangian of this problem

$$
\begin{aligned}
& U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_u \left[\bar{A}^u_H \left(L_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] \\
& + \mu_d \left[\bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
& + \mu_{TB} \left[Q^d_{HF} (Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF} \left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF} \left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF} \left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right].
\end{aligned}
$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, and Q^u_{HF} are as follows:

$$
\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_d && (88) \\
U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_{TB} P^d_{FH} && (89) \\
\mu_d &= \mu_{TB} \frac{\sigma - 1}{\sigma} P^d_{HF} && (90) \\
\mu_u &= \mu_d \left(1 + \gamma^d\right) \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) && (91) \\
\mu_{TB} P^u_{FH} &= \mu_d \left(1 + \gamma^d\right) \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right) && (92) \\
\mu_u &= \mu_{TB} \frac{\theta - 1}{\theta} P^u_{HF} && (93)
\end{aligned}
$$

Dividing equation (3.25) by equation (89), and plugging in (90), we obtain:

$$
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \frac{\frac{\sigma-1}{\sigma} P^d_{HF}}{P^d_{FH}},
$$

which corresponds to the first optimality condition (3.25) in the main text.

Next, we divide equation (91) by equation (92), and plugging in (93), delivers

$$
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} = \frac{\frac{\theta-1}{\theta} P^u_{HF}}{P^u_{FH}},
$$

which corresponds to the second optimality condition (3.26) in the main text.

Finally, combining equation (91) with the ratio of equations (90) and (93) produces

$$\left(1+\gamma^{d}\right)\bar{A}_{H}^{d}\left(F^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)\right)^{\gamma^{d}}F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)=\frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}}\frac{P_{HF}^{u}}{P_{HF}^{d}},$$

which corresponds to the third optimality condition (3.27) in the main text.

.3.2 *Generalizations*

As demonstrated in the derivations in the above Appendix .3.1, we have made no use of the properties of the functions $U\left(Q_{HH}^{d},Q_{FH}^{d}\right)$ and $F^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)$. In particular, we could assume that

$$U\left(Q_{HH}^{d},Q_{FH}^{d}\right)=\left(\left(Q_{HH}^{d}\right)^{\frac{\sigma_{H}-1}{\sigma_{H}}}+\left(Q_{FH}^{d}\right)^{\frac{\sigma_{H}-1}{\sigma_{H}}}\right)^{\frac{\sigma_{H}}{\sigma_{H}-1}}$$

and that

$$F^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)=\left(\left(Q_{HH}^{u}\right)^{\frac{\theta_{H}-1}{\theta_{H}}}+\left(Q_{FH}^{u}\right)^{\frac{\theta_{H}-1}{\theta_{H}}}\right)^{\frac{\theta_{H}}{\theta_{H}-1}},$$

with potentially $\sigma_{H}\neq\sigma$ and $\theta_{H}\neq\theta$. It is clear from the derivations in Section 3.4.1 that the first-best trade policies will continue to satisfy

$$\begin{aligned}
1+t_{H}^{d} &= \left(1+\gamma^{d}\right)\left(1+\bar{T}\right);\\
1+t_{H}^{u} &= 1+\bar{T};\\
1-v_{H}^{d} &= \frac{\sigma-1}{\sigma}\left(1+\gamma^{d}\right)\left(1+\bar{T}\right);\\
1-v_{H}^{u} &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

The only significant difference in this case is that if we want to invoke our isomorphism to claim that these policies also implement the first-best in the Krugman vertical economy with internal economies of scale, then we necessarily need to impose $\gamma^{d}=1/\left(\sigma_{H}-1\right)$, and thus the level of the tariff escalation is closely related to the degree of differentiation in the final-good sector. This is not particularly surprising, since love-for-variety effects will be more powerful, the lower the degree of substitutability across final goods.

.3.3 Alternative First-Best Implementations

In this Appendix, we explore the structure of first-best policies when the set of available instruments includes domestic production subsidies, domestic consumption subsidies, or domestic production/consumption subsidies that only apply to domestic transactions.

Discriminatory Domestic Subsidies

We first consider the case in which the Home government has access to discriminatory domestic subsidies s^d_{HH} and s^u_{HH} that apply only to purchases of final goods and of intermediate inputs involving only Home residents. The inclusion of these instruments alters the decentralized market equilibrium conditions (3.20), (3.21) and (3.22) as follows:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)} &= \left(1-s^d_{HH}\right)\frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}};\\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)} &= \left(1-s^u_{HH}\right)\frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}};\\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right) &= \left(1-s^u_{HH}\right)\frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (3.25), (3.26), and (3.27), it is clear that the first-best can be achieved by setting

$$\begin{aligned}
\left(1+t^d_H\right)\left(1-s^d_{HH}\right) &= \left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
1+t^u_H &= 1+\bar{T};\\
1-v^d_H &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
\left(1-s^u_{HH}\right)\left(1-v^u_H\right) &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

These equations illustrate that a downstream discriminatory subsidy is a perfect substitute for the downstream import tariff (only the product $\left(1+t^d_H\right)\left(1-s^d_{HH}\right)$ matters), while an upstream discriminatory subsidy is a perfect substitute for the upstream export tax (only the product $(1-s_{HH}^u)(1-v_H^u)$ matters). A straightforward implication of this result is that, whenever $1+\gamma^d = \sigma/(\sigma-1)$, as imposed by our isomorphism, the first-best can be attained by setting $\left(1+t_H^d\right)\left(1-s_{HH}^d\right) = \sigma/(\sigma-1)$ and $(1-s_{HH}^u)(1-v_H^u) = (\theta-1)/\theta$. Thus, the first-best can be achieved with only discriminatory subsidies, or with a combination of a subsidy in one sector and a trade instrument in the other sector. When only domestic instruments are used, we necessarily have $s_{HH}^d = 1/\sigma$ and $s_{HH}^u = 1/\theta$. Whether or not the resulting first-best policies entail tariff escalation depends on the level of the downstream domestic subsidy, since $\left(1+t_H^d\right)/(1+t_H^u) = \left(1+\gamma^d\right)/\left(1-s_{HH}^d\right)$.

Production Subsidies

We next consider the case of production subsidies s_H^d and s_H^u that apply to Home production of final goods and intermediate inputs, regardless of where those goods are sold (domestically or exported). The inclusion of these instruments alters the decentralized market equilibrium conditions (3.20), (3.21) and (3.22) as follows:

$$\begin{aligned}
\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} &= \frac{\left(1-v_H^d\right)}{\left(1+t_H^d\right)}\frac{P_{HF}^d}{P_{FH}^d};\\
\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)} &= \frac{\left(1-v_H^u\right)}{\left(1+t_H^u\right)}\frac{P_{HF}^u}{P_{FH}^u};\\
\hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) &= \left(1-s_H^d\right)\frac{\left(1-v_H^u\right)}{\left(1-v_H^d\right)}\frac{P_{HF}^u}{P_{HF}^d}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (3.25), (3.26), and (3.27), it is clear that the first-best can be achieved by setting

$$\begin{aligned}
1+t_H^d &= \left(1-s_H^d\right)\left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
1+t_H^u &= 1+\bar{T};\\
1-v_H^d &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1-s_H^d\right)\left(1+\bar{T}\right);\\
1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

Notice that, as long as $s_H^d > 0$, the set of first-best policies will entail this subsidy and at least two additional trade instruments. For instance, when setting $s_H^d = \gamma^d / \left(1+\gamma^d\right)$, the first-best can be achieved with this production subsidy and two export taxes ($1 - v_H^d = (\sigma - 1)/\sigma$ and $1 - v_H^u = (\theta - 1)/\theta$), while setting all import tariffs to zero. Alternatively, when setting $s_H^d = (1+\gamma^u)/\gamma^u = (\theta - 1)/\theta$, the first-best can be achieved with this production subsidy and two import tariffs ($t_H^d = 1/(\sigma - 1)$ and $t_H^u = 1/(\theta - 1)$).

Regardless of the actual implementation, the tariff escalation wedge is given by:

$$\frac{1+t_H^d}{1+t_H^u} = \left(1 - s_H^d\right)\left(1+\gamma^d\right).$$

Consumption Subsidies

We finally consider the case of consumption subsidies s_H^d and s_H^u that apply to Home consumption of final goods and of intermediate inputs, regardless of where those goods are purchased (domestically or imported). The inclusion of these instruments alters the decentralized market equilibrium conditions (3.20), (3.21) and (3.22) as follows:

$$\begin{aligned}
\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} &= \frac{\left(1 - v_H^d\right)}{\left(1+t_H^d\right)} \frac{P_{HF}^d}{P_{FH}^d}; \\
\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)} &= \frac{\left(1 - v_H^u\right)}{\left(1+t_H^u\right)} \frac{P_{HF}^u}{P_{FH}^u}; \\
\hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) &= \left(1 - s_H^u\right) \frac{\left(1 - v_H^u\right)}{\left(1 - v_H^d\right)} \frac{P_{HF}^u}{P_{HF}^d}.
\end{aligned}$$

These equations are completely analogous to those applying to the case of production subsidies, with s_H^u replacing s_H^d, so the conclusions that arise from it are also analogous.

.3.4 *Second-Best Import Tariffs*

In this Appendix we characterize the second-best import tariffs when the government only has access to import tariffs upstream and downstream.

A. Second-Best Import Tariffs with Scale Economies

As mentioned in the main text, the second-best optimal allocation will seek to solve the same problem laid out in Section 3.4.1 expanded to include the additional constraint:

$$\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) = \frac{P^d_{HF}}{P^u_{HF}} = \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}.$$

More specifically, the planner problem is now

$$\begin{aligned}
\max \quad & U\left(Q^d_{HH}, Q^d_{FH}\right) \\
s.t. \quad & \hat{A}^u_H L_H = Q^u_{HH} + Q^u_{HF} \\
& \hat{A}^d_H F^d\left(Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\
& P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} \\
& \hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) = \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}
\end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H \left(L_H\right)^{\gamma^u},$$

respectively.

We first write the Lagrangian of this problem

$$
\begin{aligned}
&U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_u\left[\bar{A}^u_H\left(L_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] \\
&+ \mu_d\left[\bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
&+ \mu_{TB}\left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}}P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}}P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH}Q^d_{FH} - P^u_{FH}Q^u_{FH}\right] \\
&+ \mu_{SB}\left[\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) - \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}}P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}}P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}\right]
\end{aligned}
$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, and Q^u_{HF} are as follows:

$$
\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_d && (94) \\
U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_{TB}P^d_{FH} && (95) \\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF} - \mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && (96) \\
\mu_u &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) \\
&\quad + \mu_{SB}\bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) \\
&\quad \times\left[\gamma^d\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d\left(Q^u_{HH}, Q^u_{FH}\right)} + \frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}\right] && (97) \\
\mu_{TB}P^u_{FH} &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right) \\
&\quad + \mu_{SB}\bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right) \\
&\quad \times\left[\gamma^d\frac{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d\left(Q^u_{HH}, Q^u_{FH}\right)} + \frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}\right] && (98) \\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF} + \mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && (99)
\end{aligned}
$$

In these derivations, note that we have used

$$
\begin{aligned}
\frac{\partial\left(\bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)\right)}{\partial Q^u_{HH}} &= \bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) \\
&\quad \times\left[\gamma^d\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d\left(Q^u_{HH}, Q^u_{FH}\right)} + \frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}\right]
\end{aligned}
$$

and

$$\frac{\partial\left(\bar{A}_H^d\left(F^d\left(Q_{HH}^u,Q_{FH}^u\right)\right)^{\gamma^d}F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)\right)}{\partial Q_{FH}^u} = \bar{A}_H^d\left(F^d\left(Q_{HH}^u,Q_{FH}^u\right)\right)^{\gamma^d}F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right) \times\left[\gamma^d\frac{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F^d\left(Q_{HH}^u,Q_{FH}^u\right)}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}\right].$$

From equations (94), (95), and (96), we obtain:

$$\frac{U_{Q_{FH}^d}\left(Q_{HH}^d,Q_{FH}^d\right)}{U_{Q_{HH}^d}\left(Q_{HH}^d,Q_{FH}^d\right)}=\frac{P_{FH}^d}{P_{HF}^d}\left(\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d}\right).$$

Because in a competitive equilibrium with import tariffs we have

$$\frac{U_{Q_{FH}^d}\left(Q_{HH}^d,Q_{FH}^d\right)}{U_{Q_{HH}^d}\left(Q_{HH}^d,Q_{FH}^d\right)}=\left(1+t_H^d\right)\frac{P_{FH}^d}{P_{HF}^d}, \tag{100}$$

we can establish that

$$1+t_H^d=\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d}. \tag{101}$$

Also, note from equations (94) and (95), as well as (100), that

$$1+t_H^d=\frac{\mu_{TB}P_{HF}^d}{\mu_d}. \tag{102}$$

In a competitive equilibrium with import tariffs, we also have that

$$\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}=\frac{P_{HF}^u}{\left(1+t_H^u\right)P_{FH}^u}. \tag{103}$$

Furthermore, the last constraint in the planner problem can be written as:

$$\bar{A}_H^d\left(F^d\left(Q_{HH}^u,Q_{FH}^u\right)\right)^{\gamma^d}F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)=\frac{P_{HF}^u}{P_{HF}^d}. \tag{104}$$

Now combine equations (98), (102), (103), and (104) to obtain

$$\frac{1+t_H^d}{1+t_H^u} = 1+\gamma^d + \frac{\mu_{SB}}{\mu_d}\left[\gamma^d \frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d\left(Q^u_{HH}, Q^u_{FH}\right)} + \frac{F^d_{Q^u_{HH}, Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}\right]. \tag{105}$$

We next work with equations (97) and plug in (98) and (104) to obtain

$$\mu_{TB} P^u_{FH} \frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} = \mu_u + \mu_{SB}\frac{P^u_{HF}}{P^d_{HF}}\left[\frac{F^d_{Q^u_{HH}, Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} - \frac{F^d_{Q^u_{HH}, Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}\right]$$

And plugging μ_u from equation (99), we get

$$\frac{\mu_{TB}}{\mu_{SB}\frac{P^u_{HF}}{P^d_{HF}}} = \frac{\frac{1}{\theta}\frac{1}{Q^u_{HF}} + \frac{F^d_{Q^u_{HH}, Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} - \frac{F^d_{Q^u_{HH}, Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}}{P^u_{FH}\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} - \frac{\theta-1}{\theta}P^u_{HF}}$$

Invoking equation (103) we can simplify this last expression further to

$$\frac{\mu_{TB}}{\mu_{SB}} P^d_{HF} = \frac{\frac{1}{\theta}\frac{1}{Q^u_{HF}} + \frac{F^d_{Q^u_{HH}, Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} - \frac{F^d_{Q^u_{HH}, Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}}{\frac{1}{\left(1+t^u_H\right)} - \frac{\theta-1}{\theta}} \tag{106}$$

The three equations (101), (105), and (106) are sufficient to characterize the properties of second-best import tariffs. In particular, these equations can be reduced to

$$1+t^d_H = \frac{\sigma}{\sigma-1} + \left[\frac{1+t^d_H}{1+t^u_H} - \frac{\theta-1}{\theta}\left(1+t^d_H\right)\right]\frac{A}{C} \tag{107}$$

$$\frac{1+t^d_H}{1+t^u_H} = 1+\gamma^d + \left[\frac{1+t^d_H}{1+t^u_H} - \frac{\theta-1}{\theta}\left(1+t^d_H\right)\right]\frac{B}{C}, \tag{108}$$

where

$$
\begin{aligned}
A &= \frac{1}{\sigma-1}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} > 0; \\
B &= \gamma^d \frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d\left(Q^u_{HH}, Q^u_{FH}\right)} + \frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}; \\
C &= \frac{1}{\theta}\frac{1}{Q^u_{HF}} + \frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} - \frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}.
\end{aligned}
$$

Using

$$
F^d\left(Q^u_{HH}, Q^u_{FH}\right) = \left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},
$$

it is easy to verify that

$$
\begin{aligned}
B &= \gamma^d \frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d\left(Q^u_{HH}, Q^u_{FH}\right)} + \frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} = \left(\gamma^d + \frac{1}{\theta}\right)\frac{1}{Q^u_{HH}}\frac{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}} > 0 \\
C &= \frac{1}{\theta}\frac{1}{Q^u_{HF}} + \frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} - \frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)} = \frac{1}{\theta}\left(\frac{1}{Q^u_{HH}} + \frac{1}{Q^u_{HF}}\right) > 0.
\end{aligned}
$$

Now note that, manipulating (107) and (108), we obtain

$$
\begin{aligned}
1 + t^d_H &= \frac{\sigma}{\sigma-1} + \left[\frac{1+t^d_H}{1+t^u_H} - \frac{\theta-1}{\theta}\left(1+t^d_H\right)\right]\frac{A}{C} \\
\frac{1+t^d_H}{1+t^u_H} &= 1 + \gamma^d + \left[\frac{1+t^d_H}{1+t^u_H} - \frac{\theta-1}{\theta}\left(1+t^d_H\right)\right]\frac{B}{C},
\end{aligned}
$$

Solving this system delivers

$$
1 + t^d_H = \frac{\sigma}{\sigma-1}\frac{1 - \frac{B}{C} + \frac{1+\gamma^d}{\frac{\sigma}{\sigma-1}}\frac{A}{C}}{1 - \frac{B}{C} + \frac{\theta-1}{\theta}\frac{A}{C}}
$$

and

$$
\frac{1+t^d_H}{1+t^u_H} = \left(1+\gamma^d\right)\frac{1 + \frac{\theta-1}{\theta}\frac{A}{C} - \frac{\theta-1}{\theta}\frac{\frac{\sigma}{\sigma-1}}{1+\gamma^d}\frac{B}{C}}{1 + \frac{\theta-1}{\theta}\frac{A}{C} - \frac{B}{C}}.
$$

Noting that $1+\gamma^d = \frac{\sigma}{\sigma-1}$ in our isomorphism, immediately implies

$$1+t_H^d > \frac{\sigma}{\sigma-1}$$

and

$$\frac{1+t_H^d}{1+t_H^u} > 1+\gamma^d = \frac{\sigma}{\sigma-1}.$$

This proves Proposition 3.4.3.

B. Second-Best Import Tariffs with No Scale Economies

Given the above derivations, it is straightforward to prove Proposition 3.4.4. Simply set $\gamma^d = 0$ in the system (107) and (108), and obtain

$$1+t_H^d = \frac{\sigma}{\sigma-1}\frac{1-\frac{B}{C}+\frac{\sigma-1}{\sigma}\frac{A}{C}}{1-\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}}$$

and

$$\frac{1+t_H^d}{1+t_H^u} = \frac{1+\frac{\theta-1}{\theta}\frac{A}{C}-\frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1}\frac{B}{C}}{1+\frac{\theta-1}{\theta}\frac{A}{C}-\frac{B}{C}}.$$

From the second equation, is clear that $\frac{1+t_H^d}{1+t_H^u} > 1$ if and only if $\frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1} < 1$, or $\sigma > \theta$. Furthermore, when $\theta = \sigma$, we have

$$1+t_H^d = 1+t_H^u = \frac{\sigma}{\sigma-1} = \frac{\theta}{\theta-1}.$$

.4 Optimal Trade Policy for a Small Open Economy with Domestic Distortions: Derivations

.4.1 First-Best Policies with an Upstream Production Subsidy

We begin by characterizing the solution to the program

$$
\begin{aligned}
\max \quad & U\left(Q_{HH}^{d}, Q_{FH}^{d}\right) = \left(\left(Q_{HH}^{d}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{FH}^{d}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
s.t. \quad & L_{H}^{u} + L_{H}^{d} = L_{H} \\
& \hat{A}_{H}^{u}\left(L_{H}^{u}\right) L_{H}^{u} = Q_{HH}^{u} + Q_{HF}^{u} \\
& \hat{A}_{H}^{d}\left(F^{d}\left(L_{H}^{d}, Q_{HH}^{u}, Q_{FH}^{u}\right)\right) F^{d}\left(L_{H}^{d}, Q_{HH}^{u}, Q_{FH}^{u}\right) = Q_{HH}^{d} + Q_{HF}^{d} \\
& P_{FH}^{d} Q_{FH}^{d} + P_{FH}^{u} Q_{FH}^{u} = Q_{HF}^{d} (Q_{HF}^{d})^{-\frac{1}{\sigma}} P_{FF}^{d}\left(Q_{FF}^{d}\right)^{\frac{1}{\sigma}} + Q_{HF}^{u}\left(Q_{HF}^{u}\right)^{-\frac{1}{\theta}} P_{FF}^{u}\left(Q_{FF}^{u}\right)^{\frac{1}{\theta}},
\end{aligned}
$$

where $\hat{A}_{H}^{u}$ and $\hat{A}_{H}^{d}$ are given by

$$
\hat{A}_{H}^{d} = \bar{A}_{H}^{d}\left(F^{d}\left(L_{H}^{d}, Q_{HH}^{u}, Q_{FH}^{u}\right)\right)^{\gamma^{d}}
$$

and

$$
\hat{A}_{H}^{u} = \bar{A}_{H}^{u}\left(L_{H}\right)^{\gamma^{u}},
$$

respectively. We also note that

$$
U\left(Q_{HH}^{d}, Q_{FH}^{d}\right) = \left(\left(Q_{HH}^{d}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{FH}^{d}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}
$$

and that

$$
F^{d}\left(L_{H}^{d}, Q_{HH}^{u}, Q_{FH}^{u}\right) = \left(L_{i}^{d}\right)^{\alpha}\left(\left(Q_{HH}^{u}\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^{u}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}},
$$

but this will prove immaterial for the derivations below.

We first write the Lagrangian of this problem

$$
\begin{aligned}
& U\left(Q_{HH}^d, Q_{FH}^d\right) + \mu_L\left[L_H - L_H^u - L_H^d\right] + \mu_u\left[\bar{A}_H^u\left(L_H\right)^{1+\gamma^u} - Q_{HH}^u - Q_{HF}^u\right] \\
& + \mu_d\left[\bar{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{1+\gamma^d} - Q_{HH}^d - Q_{HF}^d\right] \\
& + \mu_{TB}\left[Q_{HF}^d(Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d\left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u\left(Q_{FF}^u\right)^{\frac{1}{\theta}} - P_{FH}^d Q_{FH}^d - P_{FH}^u Q_{FH}^u\right].
\end{aligned}
$$

The first order conditions associated with the choices of Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, Q_{HF}^u, L_H^d, and L_H^u are as follows:

$$
\begin{aligned}
U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) &= \mu_d && (109) \\
U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) &= \mu_{TB} P_{FH}^d && (110) \\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma} P_{HF}^d && (111) \\
\mu_u &= \mu_d\left(1+\gamma^d\right)\bar{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) && (112) \\
\mu_{TB} P_{FH}^u &= \mu_d\left(1+\gamma^d\right)\bar{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) && (113) \\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta} P_{HF}^u && (114) \\
\mu_L &= \mu_u\left(1+\gamma^u\right)\bar{A}_H^u\left(L_H\right)^{\gamma^u} && (115) \\
\mu_L &= \mu_d\left(1+\gamma^d\right)\bar{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) && (116)
\end{aligned}
$$

Dividing equation (109) by equation (110), and plugging in (111), we obtain:

$$
\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} = \frac{\frac{\sigma-1}{\sigma} P_{HF}^d}{P_{FH}^d},
$$

which corresponds to the first optimality condition (3.25) in the main text.

Next, we divide equation (112) by equation (113), and plugging in (114), delivers

$$
\frac{F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)} = \frac{\frac{\theta-1}{\theta} P_{HF}^u}{P_{FH}^u},
$$

which corresponds to the second optimality condition (3.26) in the main text.

Next, combining equation (112) with the ratio of equations (111) and (114) produces

$$\left(1+\gamma^{d}\right)\bar{A}_{H}^{d}\left(F^{d}\left(L_{H}^{d},Q_{HH}^{u},Q_{FH}^{u}\right)\right)^{\gamma^{d}}F_{Q_{HH}^{u}}^{d}\left(L_{H}^{d},Q_{HH}^{u},Q_{FH}^{u}\right)=\frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}}\frac{P_{HF}^{u}}{P_{HF}^{d}},$$

which corresponds to the third optimality condition (3.27) in the main text.

Finally, from equations (115) by equation (116), and plugging in (112), we obtain

$$F_{L_{H}^{d}}^{d}\left(L_{H}^{d},Q_{HH}^{u},Q_{FH}^{u}\right)=\left(1+\gamma^{u}\right)\bar{A}_{H}^{u}\left(L_{H}\right)^{\gamma^{u}}F_{Q_{HH}^{u}}^{d}\left(L_{H}^{d},Q_{HH}^{u},Q_{FH}^{u}\right),$$

which corresponds to equation (3.33) in the main text, after noting that $\bar{A}_{H}^{u}\left(L_{H}\right)^{\gamma^{u}}=\hat{A}_{H}^{u}$.

.4.2 First-Best Policies with Alternative Instruments

In this Appendix, we explore the structure of first-best policies when the set of available instruments includes instruments other than domestic upstream production subsidies and trade taxes.

Discriminatory Domestic Subsidies

Consider first the case in which the Home government has access to discriminatory domestic subsidies s_{HH}^{d} and s_{HH}^{u} that apply only to purchases of final goods and of intermediate inputs involving only Home residents. The inclusion of these instruments alters the decentralized market equilibrium conditions (3.20), (3.21), (3.22), and (3.23) as follows:

$$\begin{aligned}
\frac{U_{Q_{HH}^{d}}\left(Q_{HH}^{d},Q_{FH}^{d}\right)}{U_{Q_{FH}^{d}}\left(Q_{HH}^{d},Q_{FH}^{d}\right)} &= \left(1-s_{HH}^{d}\right)\frac{\left(1-v_{H}^{d}\right)}{\left(1+t_{H}^{d}\right)}\frac{P_{HF}^{d}}{P_{FH}^{d}};\\
\frac{F_{Q_{HH}^{u}}^{d}\left(L_{H}^{d},Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(L_{H}^{d},Q_{HH}^{u},Q_{FH}^{u}\right)} &= \left(1-s_{HH}^{u}\right)\frac{\left(1-v_{H}^{u}\right)}{\left(1+t_{H}^{u}\right)}\frac{P_{HF}^{u}}{P_{FH}^{u}};\\
\hat{A}_{H}^{d}F_{Q_{HH}^{u}}^{d}\left(L_{H}^{d},Q_{HH}^{u},Q_{FH}^{u}\right) &= \left(1-s_{HH}^{u}\right)\frac{\left(1-v_{H}^{u}\right)}{\left(1-v_{H}^{d}\right)}\frac{P_{HF}^{u}}{P_{HF}^{d}};\\
F_{L_{H}^{d}}^{d}\left(L_{H}^{d},Q_{HH}^{u},Q_{FH}^{u}\right) &= \frac{1}{1-s_{HH}^{u}}\hat{A}^{u}\left(L_{H}^{u}\right)F_{Q_{HH}^{u}}^{d}\left(L_{H}^{d},Q_{HH}^{u},Q_{FH}^{u}.\right)
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (3.25), (3.26), (3.27), and (3.33), it is clear that the first-best can be achieved by setting

$$\begin{aligned}
\left(1+t_H^d\right)\left(1-s_{HH}^d\right) &= \left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1+t_H^u &= 1+\bar{T}; \\
1-v_H^d &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
\left(1-s_{HH}^u\right)\left(1-v_H^u\right) &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right); \\
\frac{1}{1-s_{HH}^u} &= \frac{\theta}{\theta-1}.
\end{aligned}$$

These equations illustrate that a downstream discriminatory subsidy is a perfect substitute for the downstream import tariff (only the product $\left(1+t_H^d\right)\left(1-s_{HH}^d\right)$ matters). By contrast, an export tax is *no longer* a perfect substitute when $\alpha > 0$, then the first best requires an upstream subsidy More specifically, because the first-best calls for $s_{HH}^u = 1/\theta$, we must necessarily have

$$1-v_H^u = 1+t_H^u = 1+\bar{T}.$$

A straightforward implication of this result is that, whenever $1+\gamma^d = \sigma/\left(\sigma-1\right)$, as imposed by our isomorphism, the first-best can be attained by setting $\left(1+t_H^d\right)\left(1-s_{HH}^d\right) = \sigma/\left(\sigma-1\right)$, $1-s_{HH}^u = \left(\theta-1\right)/\theta$, and $v_H^u = v_H^d = t_H^u = 0$. Thus, the first-best can be achieved with only two discriminatory subsidies, or with a combination of an upstream subsidy and a downstream import tariff. When only domestic instruments are used, we necessarily have $s_{HH}^d = 1/\sigma$ and $s_{HH}^u = 1/\theta$. If the downstream subsidy is not used, then $1+t_H^d = 1+\gamma^d$, and thus $\left(1+t_H^d\right)/\left(1+t_H^u\right) = 1+\gamma^d$ as well.

Production Subsidies

We next consider the case in which the Home government uses a nondiscriminatory downstream production subsidy s_H^d in addition to a nondiscriminatory upstream production subsidy, as in our baseline implementation. The inclusion of this instrument does not affect the market equilibrium condition (3.23), while it alters the decentralized market equilibrium conditions (3.20), (3.21), (3.22) in a manner analogous to the case $\alpha = 0$, that is:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} &= \frac{\left(1 - v^d_H\right)}{\left(1 + t^d_H\right)} \frac{P^d_{HF}}{P^d_{FH}}; \\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} &= \frac{\left(1 - v^u_H\right)}{\left(1 + t^u_H\right)} \frac{P^u_{HF}}{P^u_{FH}}; \\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) &= \left(1 - s^d_H\right) \frac{\left(1 - v^u_H\right)}{\left(1 - v^d_H\right)} \frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (3.25), (3.26), and (3.27), it is clear that the first-best can be achieved by setting

$$\begin{aligned}
1 + t^d_H &= \left(1 - s^d_H\right)\left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
1 + t^u_H &= 1 + \bar{T}; \\
1 - v^d_H &= \frac{\sigma - 1}{\sigma}\left(1 + \gamma^d\right)\left(1 - s^d_H\right)\left(1 + \bar{T}\right); \\
1 - v^u_H &= \frac{\theta - 1}{\theta}\left(1 + \bar{T}\right).
\end{aligned}$$

Notice that, as long as $s^d_H > 0$, the set of first-best policies will entail this subsidy and at least two additional trade instruments. For instance, when setting $s^d_H = \gamma^d / \left(1 + \gamma^d\right)$, the first-best can be achieved with this production subsidy, the upstream production subsidy at a level $s^u_H = \gamma^u / \left(1 + \gamma^u\right)$ and two export taxes ($1 - v^d_H = \left(\sigma - 1\right)/\sigma$ and $1 - v^u_H = \left(\theta - 1\right)/\theta$), while setting all import tariffs to zero. Alternatively, when setting $s^d_H = \left(1 + \gamma^u\right)/\gamma^u = \left(\theta - 1\right)/\theta$, the first-best can be achieved with this production subsidy and two import tariffs ($t^d_H = 1/\left(\sigma - 1\right)$ and $t^u_H = 1/\left(\theta - 1\right)$).

Regardless of the actual implementation, the tariff escalation wedge is given by:

$$\frac{1 + t^d_H}{1 + t^u_H} = \left(1 - s^d_H\right)\left(1 + \gamma^d\right).$$

When only upstream production subsidies and trade taxes are used, the first-best policies continues to feature a tariff escalation wedge equal to $1 + \gamma^d = \sigma / \left(\sigma - 1\right)$.

Consumption Subsidies

We finally consider the case of consumption subsidies s_H^d and s_H^u that apply to Home consumption of final goods and intermediate inputs, regardless of where those goods are purchased (domestically or imported). The inclusion of these instruments does not affect the market equilibrium condition (3.23), as long as s_H^u is set at $s_H^u = 1/\theta$. Furthermore, the decentralized market equilibrium conditions (3.20), (3.21) and (3.22) become:

$$\begin{aligned}
\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} &= \frac{\left(1-v_H^d\right)}{\left(1+t_H^d\right)}\frac{P_{HF}^d}{P_{FH}^d}; \\
\frac{F^d_{Q_{HH}^u}\left(Q_{HH}^u, Q_{FH}^u\right)}{F^d_{Q_{FH}^u}\left(Q_{HH}^u, Q_{FH}^u\right)} &= \frac{\left(1-v_H^u\right)}{\left(1+t_H^u\right)}\frac{P_{HF}^u}{P_{FH}^u}; \\
\hat{A}_H^d F^d_{Q_{HH}^u}\left(Q_{HH}^u, Q_{FH}^u\right) &= \left(1-s_H^u\right)\frac{\left(1-v_H^u\right)}{\left(1-v_H^d\right)}\frac{P_{HF}^u}{P_{HF}^d}.
\end{aligned}$$

These equations are completely analogous to those applying to the case of production subsidies, with s_H^u replacing s_H^d, but note that we now necessarily have $s_H^u = 1/\theta$. As a result, replacing $1+\gamma^d = \sigma/\left(\sigma-1\right)$, we obtain

$$\begin{aligned}
1+t_H^d &= \frac{\theta-1}{\theta}\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1+t_H^u &= 1+\bar{T}; \\
1-v_H^d &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right); \\
1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

In such a case, it is clear that the relative size of $1+t_H^d$ and $1+t_H^u$ depends on $\frac{\theta-1}{\theta}\left(1+\gamma^d\right) = \frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1}$, and thus on the relative size of σ and θ.

.4.3 Second-Best Policies

In this Appendix we characterize the second-best import tariffs for the general case $\alpha \geq 0$ when the government only has access to import tariffs upstream and downstream. We first derive the key equations characterizing tariff levels and tariff escalation, and we later explore special cases.

A. Second-Best Import Tariffs with Scale Economies: Key Equations

The second-best optimal allocation will seek to solve the same problem laid out in Section 3.4.1 expanded to include the additional constraints:

$$\hat{A}_H^d F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) = \frac{P^d_{HF}}{P^u_{HF}} = \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}$$

and

$$F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \hat{A}^u\left(L^u_H\right) F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right).$$

Second-Best Planner Problem and First-Order Conditions More specifically, the planner sets $\left\{L^u_H, L^d_H, Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}\right\}$ to

$$\begin{aligned}
\max \quad & U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
s.t. \quad & L^u_H + L^d_H = L_H \\
& \hat{A}^u_H\left(L^u_H\right) L^u_H = Q^u_{HH} + Q^u_{HF} \\
& \hat{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right) F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\
& P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF}\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} \\
& \hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) = \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}} \\
& F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \hat{A}^u\left(L^u_H\right) F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)
\end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H\left(L^u_H\right)^{\gamma^u},$$

respectively.

We first write the Lagrangian of this problem

$$
\begin{aligned}
& U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_u \left[\bar{A}^u_H \left(L^u_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] \\
& + \mu_d \left[\bar{A}^d_H \left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
& + \mu_{TB} \left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right] \\
& + \mu_{SB} \left[\hat{A}^d_H F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) - \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}\right] \\
& + \mu_{LC} \left[\frac{F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)} - \hat{A}^u\left(L^u_H\right)\right]
\end{aligned}
$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}, L^d_H, and L^u_H are as follows:

$$
\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_d && (117)\\
U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_{TB} P^d_{FH} && (118)\\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF} - \mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && (119)\\
\mu_u &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot) \\
&\quad +\mu_{SB}\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot) \\
&\quad \times\left[\gamma^d\frac{F^d_{Q^u_{HH}}(\cdot)}{F^d(\cdot)} + \frac{F^d_{Q^u_{HH},Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] \\
&\quad +\mu_{LC}\left[\frac{F^d_{L^d_H,Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)} - \frac{F^d_{L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\frac{F^d_{Q^u_{HH},Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] && (120)\\
\mu_{TB}P^u_{FH} &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{FH}}(\cdot) \\
&\quad +\mu_{SB}\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot)\times\left[\gamma^d\frac{F^d_{Q^u_{FH}}(\cdot)}{F^d(\cdot)} + \frac{F^d_{Q^u_{HH},Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] \\
&\quad +\mu_{LC}\left[\frac{F^d_{L^d_H,Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)} - \frac{F^d_{L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\frac{F^d_{Q^u_{HH},Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] && (121)\\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF} + \mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && (122)\\
\mu_L &= \mu_u\left(1+\gamma^u\right)\bar{A}^u_H\left(L^u_H\right)^{\gamma^u} - \mu_{LC}\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1} && (123)\\
\mu_L &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{L^d_H}(\cdot) \\
&\quad +\mu_{SB}\hat{A}^d_H\left[\gamma^d\frac{F^d_{L^d_H}(\cdot)}{F^d(\cdot)}F^d_{Q^u_{HH}}(\cdot) + F^d_{Q^u_{HH},L^d_H}(\cdot)\right] \\
&\quad +\mu_{LC}\left[\frac{F^d_{L^d_H,L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)} - \frac{F^d_{L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\frac{F^d_{Q^u_{HH},L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] && (124)
\end{aligned}
$$

In these derivations, note that we have used

$$\frac{\partial\left(\bar{A}_H^d\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot)\right)}{\partial Q^u_{HH}} = \bar{A}_H^d\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot) \times\left[\gamma^d\frac{F^d_{Q^u_{HH}}(\cdot)}{F^d(\cdot)}+\frac{F^d_{Q^u_{HH},Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right]$$

and

$$\frac{\partial\left(\bar{A}_H^d\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot)\right)}{\partial Q^u_{FH}} = \bar{A}_H^d\left(F^d(\cdot)\right)^{\gamma^d}F^d_{Q^u_{HH}}(\cdot) \times\left[\gamma^d\frac{F^d_{Q^u_{FH}}(\cdot)}{F(\cdot)}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right].$$

Useful Expressions with Our Functional Forms Remember that technology is given by

$$F^d\left(L_H^d,Q^u_{HH},Q^u_{FH}\right)=\left(L_H^d\right)^{\alpha}\left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}}+\left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}},$$

and define

$$\pi^u_{HH}\equiv\frac{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}}+\left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}}$$

and

$$X_H^d\equiv\bar{A}_H^d\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{1+\gamma^d}.$$

We next note that:

$$
\begin{aligned}
F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \alpha\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{L^d_H} \\
F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= (1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{Q^u_{HH}}\pi^u_{HH} \\
F^d_{Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= (1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{Q^u_{FH}}\left(1-\pi^u_{HH}\right) \\
F^d_{L^d_H, L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \alpha(\alpha-1)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{L^d_H L^d_H} \\
F^d_{Q^u_{HH}, Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= (1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{Q^u_{HH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[-\alpha\pi^u_{HH} - \frac{1}{\theta}\left(1-\pi^u_{HH}\right)\right] \\
F^d_{Q^u_{HH}, Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= (1-\alpha)F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\frac{1-\pi^u_{HH}}{Q^u_{FH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\left(\frac{1}{\theta}-\alpha\right) \\
F^d_{L^d_H, Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \alpha(1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{L^d_H}\frac{\pi^u_{HH}}{Q^u_{HH}} \\
F^d_{L^d_H, Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) &= \alpha(1-\alpha)\frac{F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{L^d_H}\frac{1-\pi^u_{HH}}{Q^u_{FH}}
\end{aligned}
$$

First-Order Conditions with Functional Forms We can now plug some of the above expressions into our first-order conditions

$$
\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_d && (125)\\
U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_{TB} P^d_{FH} && (126)\\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF} - \mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && (127)\\
\mu_u &= \mu_d\left(1+\gamma^d\right)(1-\alpha)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}} \\
&\quad + \mu_{SB}(1-\alpha)\frac{X^d_H}{Q^u_{HH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\gamma^d(1-\alpha)\pi^u_{HH} - \alpha\pi^u_{HH} - \frac{1}{\theta}\pi^u_{FH}\right] \\
&\quad + \mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{(1-\alpha)}\left[1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right] && (128)\\
\mu_{TB}P^u_{FH} &= \mu_d\left(1+\gamma^d\right)(1-\alpha)X^d_H\frac{1-\pi^u_{HH}}{Q^u_{FH}} \\
&\quad + \mu_{SB}(1-\alpha)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{1-\pi^u_{HH}}{Q^u_{FH}}\left[\gamma^d(1-\alpha) + \left(\frac{1}{\theta}-\alpha\right)\right] \\
&\quad + \mu_{LC}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L^d_H}\frac{1-\pi^u_{HH}}{Q^u_{FH}}\frac{Q^u_{HH}}{\pi^u_{HH}} && (129)\\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF} + \mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && (130)\\
\mu_L &= \mu_u\left(1+\gamma^u\right)\bar{A}^u_H\left(L^u_H\right)^{\gamma^u} - \mu_{LC}\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1} && (131)\\
\mu_L &= \mu_d\left(1+\gamma^d\right)\frac{\alpha X^d_H}{L^d_H} + \mu_{SB}\left(1+\gamma^d\right)\frac{\alpha(1-\alpha)X^d_H}{L^d_H}\frac{\pi^u_{HH}}{Q^u_{HH}} \\
&\quad - \mu_{LC}\frac{\alpha}{1-\alpha}\frac{1}{L^d_H L^d_H}\frac{Q^u_{HH}}{\pi^u_{HH}} && (132)
\end{aligned}
$$

Manipulating the First-Order Conditions From equations (125), (126), and (127), we obtain:

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \frac{P^d_{FH}}{P^d_{HF}}\left(\frac{\sigma}{\sigma-1} + \frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}}\right).$$

Because in a competitive equilibrium with tariffs we have

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \left(1+t^d_H\right)\frac{P^d_{FH}}{P^d_{HF}}, \tag{133}$$

we can establish that

$$1+t_H^d=\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d}. \tag{134}$$

Also, note from equations (125), (126), and (133) that we have

$$1+t_H^d=\frac{\mu_{TB}P_{HF}^d}{\mu_d}, \tag{135}$$

and in a competitive equilibrium with import tariffs (but no export taxes)

$$\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}=\frac{P_{HF}^u}{\left(1+t_H^u\right)P_{FH}^u}. \tag{136}$$

Furthermore, the penultimate constraint in the initial optimization can be written as:

$$\frac{X_H^d}{F^d\left(Q_{HH}^u,Q_{FH}^u\right)}F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)=\frac{P_{HF}^u}{P_{HF}^d}, \tag{137}$$

and the last one as

$$\frac{\alpha}{1-\alpha}\frac{Q_{HH}^u}{\pi_{HH}^u L_H^d}=\frac{X_H^u}{L_H^u}. \tag{138}$$

Now combine equations (129), (135), (136), (137), and (138)

$$\frac{1+t_H^d}{1+t_H^u}=1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\gamma^d\left(1-\alpha\right)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L_H^d}\frac{P_{HF}^d}{P_{HF}^u}. \tag{139}$$

We next plug equation (129) into equation (128) to obtain

$$\mu_{TB}P_{FH}^u\frac{\pi_{HH}^u}{Q_{HH}^u}\frac{Q_{FH}^u}{1-\pi_{HH}^u}=\mu_u+\mu_{SB}\left(1-\alpha\right)\frac{X_H^d}{Q_{HH}^u}\frac{\pi_{HH}^u}{Q_{HH}^u}\frac{1}{\theta}-\mu_{LC}\frac{1}{L_H^d}\frac{\alpha}{1-\alpha}\frac{1}{\theta}\frac{1}{\pi_{HH}^u}$$

Next, plugging μ_u from equation (130), and invoking equation (136) and (137), we obtain

$$\mu_{TB}P_{HF}^u\left[\frac{1}{1+t_H^u}-\frac{\theta-1}{\theta}\right]=\mu_{SB}\frac{1}{\theta}\left(1-\alpha\right)X_H^d\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\right]-\mu_{LC}\frac{1}{L_H^d}\frac{\alpha}{\left(1-\alpha\right)}\frac{1}{\theta}\frac{1}{\pi_{HH}^u}$$

Next, invoking equation (136) and (137), we can simplify this to

$$\mu_{TB}P_{HF}^{u}\left[\frac{1}{1+t_{H}^{u}}-\frac{\theta-1}{\theta}\right]=\mu_{SB}\frac{1}{\theta}X_{H}^{d}\left(1-\alpha\right)\frac{\pi_{HH}^{u}}{Q_{HH}^{u}}\left[\frac{1}{Q_{HF}^{u}}+\frac{1}{Q_{HH}^{u}}\right]-\mu_{LC}\frac{\alpha}{1-\alpha}\frac{1}{L_{H}^{d}}\frac{1}{\theta}\frac{1}{\pi_{HH}^{u}}.$$

And, plugging in (135). this delivers

$$\frac{1+t_{H}^{d}}{1+t_{H}^{u}}-\frac{\theta-1}{\theta}\left(1+t_{H}^{d}\right)=\frac{\mu_{SB}}{\mu_{d}}\frac{1}{\theta}\left[\frac{1}{Q_{HF}^{u}}+\frac{1}{Q_{HH}^{u}}\right]-\frac{\mu_{LC}}{\mu_{d}}\frac{\alpha}{1-\alpha}\frac{1}{L_{H}^{d}}\frac{1}{\theta}\frac{1}{\pi_{HH}^{u}}\frac{P_{HF}^{d}}{P_{HF}^{u}}. \tag{140}$$

We finally seek to solve for μ_{LC} as a function of μ_{SB}. We begin with equation (123) and (132)

$$\begin{aligned}\frac{\mu_{u}}{\mu_{d}}\bar{A}_{H}^{u}\left(L_{H}^{u}\right)^{\gamma^{u}}=&\frac{\left(1+\gamma^{d}\right)}{\left(1+\gamma^{u}\right)}\frac{\alpha X_{H}^{d}}{L_{H}^{d}}+\frac{\mu_{SB}}{\mu_{d}}\frac{\left(1+\gamma^{d}\right)}{\left(1+\gamma^{u}\right)}\frac{\alpha\left(1-\alpha\right)X_{H}^{d}}{L_{H}^{d}}\frac{\pi_{HH}^{u}}{Q_{HH}^{u}}\\&+\frac{\mu_{LC}}{\mu_{d}}\frac{1}{1+\gamma^{u}}\left[\gamma^{u}\bar{A}_{H}^{u}\left(L_{H}^{u}\right)^{\gamma^{u}-1}-\frac{\alpha}{1-\alpha}\frac{1}{L_{H}^{d}L_{H}^{d}}\frac{Q_{HH}^{u}}{\pi_{HH}^{u}}\right].\end{aligned}$$

Next, plug equation (120) and using (138), we obtain

$$\begin{aligned}&\frac{\left(1+\gamma^{d}\right)\gamma^{u}}{\left(1+\gamma^{u}\right)}X_{H}^{d}+\frac{\mu_{SB}}{\mu_{d}}\left(1-\alpha\right)\frac{X_{H}^{d}\pi_{HH}^{u}}{Q_{HH}^{u}}\left[\frac{\left(1+\gamma^{d}\right)}{\left(1+\gamma^{u}\right)}\gamma^{u}-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{\left(1-\pi_{HH}^{u}\right)}{\pi_{HH}^{u}}\right)\right]\\=&\frac{\mu_{LC}}{\mu_{d}}\bar{A}_{H}^{u}\left(L_{H}^{u}\right)^{\gamma^{u}}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^{u}}\left(\frac{\gamma^{u}L_{H}^{d}}{L_{H}^{u}}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^{u}}{\pi_{HH}^{u}}\right)\right],\end{aligned}$$

which we can express as

$$\frac{\mu_{LC}}{\mu_{d}}=\frac{\frac{\left(1+\gamma^{d}\right)\gamma^{u}}{\left(1+\gamma^{u}\right)}X_{H}^{d}+\frac{\mu_{SB}}{\mu_{d}}\left(1-\alpha\right)\frac{X_{H}^{d}\pi_{HH}^{u}}{Q_{HH}^{u}}\left[\frac{\left(1+\gamma^{d}\right)}{\left(1+\gamma^{u}\right)}\gamma^{u}-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{\left(1-\pi_{HH}^{u}\right)}{\pi_{HH}^{u}}\right)\right]}{\bar{A}_{H}^{u}\left(L_{H}^{u}\right)^{\gamma^{u}}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^{u}}\left(\frac{\gamma^{u}L_{H}^{d}}{L_{H}^{u}}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^{u}}{\pi_{HH}^{u}}\right)\right]}. \tag{141}$$

Recap of Key Equations

$$1+t_H^d = \frac{\sigma}{\sigma-1} + \frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d}$$

$$\frac{1+t_H^d}{1+t_H^u} = 1+\gamma^d + \frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right] + \frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L_H^d}\frac{P_{HF}^d}{P_{HF}^u}$$

$$\frac{1+t_H^d}{1+t_H^u} - \frac{\theta-1}{\theta}\left(1+t_H^d\right) = \frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\right] - \frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{1}{L_H^d}\frac{1}{\theta}\frac{1}{\pi_{HH}^u}\frac{P_{HF}^d}{P_{HF}^u}$$

$$\frac{\mu_{LC}}{\mu_d} = \frac{\frac{(1+\gamma^d)\gamma^u}{(1+\gamma^u)}X_H^d + \frac{\mu_{SB}}{\mu_d}(1-\alpha)\frac{X_H^d\pi_{HH}^u}{Q_{HH}^u}\left[\frac{(1+\gamma^d)}{(1+\gamma^u)}\gamma^u - \frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left(L_H^u\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^u L_H^d}{L_H^u}-1\right) - \frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}$$

We have not been successful in proving any general results, so let us study some special cases.

B. Second-Best Import Tariffs with No Scale Economies in Either Sector

Given the above derivations, it is straightforward to prove that Proposition 3.4.4 applies even when $\alpha > 0$. Simply set $\gamma^d = \gamma^u = 0$ in equations (139), (140)and (141). First note, equation (141) becomes

$$\frac{\mu_{LC}}{\mu_d} = \frac{\mu_{SB}}{\mu_d}\frac{(1-\alpha)\frac{X_H^d\pi_{HH}^u}{Q_{HH}^u}\left[\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left(L_H^u\right)^{\gamma^u}\left[\frac{1}{\alpha}+\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]},$$

and plugging (138),

$$\frac{\mu_{LC}}{\mu_d} = \frac{\mu_{SB}}{\mu_d}(1-\alpha)^2 X_H^d L_H^d\left(\frac{\pi_{HH}^u}{Q_{HH}^u}\right)^2\frac{1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}}{1+\frac{\alpha}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}}.$$

Plugging this expression for $\frac{\mu_{LC}}{\mu_d}$ into (139), delivers

$$\frac{1+t_H^d}{1+t_H^u} = 1 + \frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\left[\frac{\alpha+\pi_{HH}^u(1-\alpha)}{\alpha+\pi_{HH}^u(\theta-\alpha)}\right]$$

And finally, plugging $\frac{\mu_{LC}}{\mu_d}$ into equation (140) delivers

$$\frac{1+t_H^d}{1+t_H^u} - \frac{\theta-1}{\theta}\left(1+t_H^d\right) = \frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q_{HF}^u} + \frac{1}{Q_{HH}^u}\frac{(1-\alpha)\,\theta\pi_{HH}^u}{\alpha+\pi_{HH}^u\,(\theta-\alpha)}\right].$$

In sum, we can write the system as

$$\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1} + \frac{\mu_{SB}}{\mu_d}A \\
\frac{1+t_H^d}{1+t_H^u} &= 1 + \frac{\mu_{SB}}{\mu_d}B \\
\frac{1+t_H^d}{1+t_H^u} - \frac{\theta-1}{\theta}\left(1+t_H^d\right) &= \frac{\mu_{SB}}{\mu_d}C
\end{aligned}$$

where

$$\begin{aligned}
A &= \frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d} > 0 \\
B &= \frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\left[\frac{\alpha+\pi_{HH}^u\,(1-\alpha)}{\alpha+\pi_{HH}^u\,(\theta-\alpha)}\right] > 0 \\
C &= \frac{1}{\theta}\left[\frac{1}{Q_{HF}^u} + \frac{1}{Q_{HH}^u}\frac{(1-\alpha)\,\theta\pi_{HH}^u}{\alpha+\pi_{HH}^u\,(\theta-\alpha)}\right] > 0
\end{aligned}$$

So we have

$$\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1} + \left[\frac{1+t_H^d}{1+t_H^u} - \frac{\theta-1}{\theta}\left(1+t_H^d\right)\right]\frac{A}{C} \\
\frac{1+t_H^d}{1+t_H^u} &= 1 + \left(\frac{1+t_H^d}{1+t_H^u} - \left(1+t_H^d\right)\frac{\theta-1}{\theta}\right)\frac{B}{C}
\end{aligned}$$

When solving this system, we obtain

$$\frac{1+t_H^d}{1+t_H^u} = \frac{1-\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta}\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}}{1-\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}},$$

which is higher or lower than 1 depending on the relative size of σ and θ. More specifically, when $\sigma > \theta$, $\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta} < 1$, and we have tariff escalation. But when $\sigma < \theta$, then $\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta} > 1$, and we have tariff de-escalation.

C. Second-Best Import Tariffs with No Scale Economies Upstream ($\gamma^u = 0$)

We next study the case in which $\gamma^d > 0$ but $\gamma^u = 0$. In that case, equation (141) reduces to

$$\frac{\mu_{LC}}{\mu_d} = \frac{\mu_{SB}}{\mu_d}\frac{(1-\alpha)\frac{X_H^d \pi_{HH}^u}{Q_{HH}^u}\left[\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left[\frac{1}{\alpha}+\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]},$$

and we can write

$$\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}A \\
\frac{1+t_H^d}{1+t_H^u} &= 1+\gamma^d+\frac{\mu_{SB}}{\mu_d}B \\
\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right) &= \frac{\mu_{SB}}{\mu_d}C
\end{aligned}$$

with

$$\begin{aligned}
B &= \frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{X_H^d\left[\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}\alpha\frac{\theta-1}{\theta}\frac{1}{L_H^d}\frac{P_{HF}^d}{P_{HF}^u}\right] \\
&= \frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\left[\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}\frac{\theta-1}{\theta}\right] \\
&= \frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\frac{\pi_{HH}^u(\theta-1)+\alpha\sigma+\pi_{HH}^u\sigma(1-\alpha)}{(\sigma-1)(\alpha(1-\pi_{HH}^u)+\pi_{HH}^u\theta)} > 0.
\end{aligned}$$

and

$$C = \frac{1}{\theta}\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\frac{1}{\theta}\left[1-\frac{\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)}{\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}\right] > 0.$$

Given $B > 0$ and $C > 0$, it is then straightforward to use the same steps as in the proof of the $\alpha = 0$ case in Appendix .3.4 to show that

$$1 + t_H^d > \frac{\sigma}{\sigma - 1}$$

and

$$\frac{1 + t_H^d}{1 + t_H^u} > 1 + \gamma^d.$$

We thus provide a simple analytic solution that shows tariff escalation is optimal when upstream goods are produced under constant returns to scale.

.5 Quantitative Analysis

.5.1 *Numerical Simulations of Second-Best Tariff Escalation*

In this Appendix, we describe how we solve numerically for second-best trade policies in the small, open economy (SOE), and report the results of solving this problem for various parameter values. First, we perform an extensive grid search over various values of the key parameters σ, θ and α. Second, we describe results of a second exercise in which we evaluate tariff escalation for different values of A_d, A_u, τ_d and τ_u. Table 4 provides the list of values we consider for each parameter.

Table 4: List of Parameters in Grid Search Exercise

Parameter	List of values
Elast. Subs. Downstream, σ	$\{2, 3, 4, \ldots, 8\}$
Elast. Subs. Upstream, θ	$\{2, 3, 4, \ldots, 8\}$
Labor share Downstream, α	$\{0, 0.1, 0.2, \ldots, 0.9\}$
Foreign Prod. Downstream, A_d	$\{0.16, 0.24, 0.32, 0.49, 0.65\}$
Foreign Prod. Upstream, A_u	$\{0.07, 0.14, 0.21, 0.28\}$
Iceberg cost Downstream, τ_d	$\{1.19, 1.78, 2.37, 3.56, 4.75\}$
Iceberg cost Upstream, τ_u	$\{1.02, 1.52, 2.03, 3.05, 4.07\}$

Notes: Each row presents the list of values considered for each parameter during the grid search exercise. For the two elasticities of substitution and the labor share we cover the entire range of reasonable values in the literature. For iceberg costs and the two productivities we start from their calibrated value, table 3.1 in section 3.6.1, and consider a 25% and 50% decrease, and a 50% and 100% increase.

Solving for optimal import tariff

Solving for optimal tariffs in this second-best setting requires providing values for import prices and export demand shifters, both of which are exogenously given in the SOE, and for productivities in both sectors. To recover the value of these parameters, we use the fact that the LOE approximates the SOE assumption from Home's perspective when its population is low relative to the population of the rest of the world. Therefore, import prices and export demand shifters in the SOE can be constructed from the equilibrium of the LOE under this limit according to,[45]

$$\begin{aligned}
P^d_{soe,FH} &= \left(M^d_F\right)^{\frac{1}{1-\sigma}} p^d_{FH} \\
P^u_{soe,FH} &= \left(M^u_F\right)^{\frac{1}{1-\theta}} p^u_{FH} \\
P^d_{soe,HF} &= \left(\frac{1}{\tau_d}\right)^{\frac{\sigma}{\sigma-1}} P^d_{HF} C^{\frac{1}{\sigma}}_{HF} \\
P^u_{soe,HF} &= \left(\frac{1}{\tau_u}\right)^{\frac{\theta}{\theta-1}} P^u_{HF} Q^{\frac{1}{\theta}}_{HF}.
\end{aligned}$$

Similarly, productivity levels in the two sectors can be constructed from the equilibrium of the LOE and the isomorphism in Proposition 3.3.1,

$$A^u_{soe} = \left(\frac{A^u_H}{f^\theta_H}\right)^{\frac{\theta}{\theta-1}} (\theta-1)\, f^u_H, \qquad A^d_{soe} = \left(\frac{A^d_H}{f^d_H \sigma}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1)\, f^d_H.$$

Given these values, it is straightforward to compute numerically the optimal import tariff of the problem described in Appendix .4.3. In practice, however, we solve for the optimal import tariff in the LOE when $L_H/L_F = 0.01$ for each combination of the parameters in Table 4 because this method is more numerically stable than the corresponding one in the SOE. Table 5 shows that this approximation works well for the calibrated values of Section 3.6.1 with $L_H/L_F = 0.01$.

Table 5: Optimal taxes in the large and small open-economy

	$1+t^d$	$1+t^u$	$1-v^u$	$1-s^u$
LOE - First-Best	0.249		0.200	0.200
SOE - First-Best	0.25		0.2	0.2
LOE - Second-Best	0.328	0.184		
SOE - Second-Best	0.332	0.191		

Notes: Table compares the first- and second-best policy in the LOE and SOE. We compute the optimal tariffs using the estimated parameters in section 3.6.1 with $L_H/L_F = 0.01$.

45. Note the export demand shifters are corrected by iceberg costs because of how we write the feasibility constraints in the SOE.

Grid over σ, θ and α

We first solve for optimal trade policy for values of σ and θ ranging from 2 to 8, and for values of α ranging from 0 to 0.9, as described in Table 4. We fix the values A_d, A_u, τ_d and τ_u to the values we estimate in Section 3.6.1. Overall, we explore $7 \times 7 \times 10 = 490$ configurations of parameters. We can successfully solve the model for 453 of these 490 cases. Most problematic cases are associated with very low values of σ, for which we find high tariff escalation values when we can solve the model.

Statistics tariff ratio wedge

The next three tables report statistics related to the tariff escalation wedge, the mean, the median, the standard deviation, the minimum and the maximum. The second column of Table 6 provides values of these statistics for the 453 cases for which we have a solution. On average, 'gross' downstream tariffs are 25 percent higher than 'gross' upstream tariffs, and the medians show a similar divergence (16 percent). Some cases feature tariff escalation levels as high as 2.31, while tariff de-escalation remains modest even in the most extreme cases (minimum of 0.75).

Table 6: Statistics of the tariff escalation wedge: all cases

	All cases	With escalation	With de-escalation
Mean	1.25	1.28	0.95
Median	1.16	1.18	0.97
Standard Deviation	0.28	0.28	0.006
Minimum value	0.75	1.001	0.75
Maximum value	2.31	2.31	0.99
N	453	412	41

Notes: Table reports statistics for the tariff escalation wedge for optimal import tariffs computed in the grid search exercise.

The third and fourth column of Table 6 recalculates the statistic but looking at the cases for which the tariff ratio is above and below 1, respectively. It is interesting to note that when our model predicts tariff de-escalation, it does so with fairly moderate levels (median of 0.97).

Tariff escalation and parameter space

We now present results about the parameter combinations that generate de-escalated tariffs. Table 7 reports the fraction of cases for which tariffs are de-escalated for a given parameter, for each possible value that this parameter can take. For example, we have 70 combinations with $\sigma = 4$, with 1 percent of these cases featuring de-escalated tariffs.[46]

Table 7: Tariff De-escalation across the parameter space, I

$\sigma =$	2	3	4	5	6	7	8			
Share:	0.00	0.00	0.01	0.07	0.11	0.17	0.23			
$\theta =$	2	3	4	5	6	7	8			
Share:	0.13	0.22	0.17	0.09	0.05	0.00	0.00			
$\alpha =$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
Share:	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.13	0.36	0.55

Notes: Table reports the share of cases in each cell for which optimal tariffs are de-escalated, i.e $\left(1 + t_H^d\right) / (1 + t_H^u) < 1$.

Table 8 presents the fraction of cases with de-escalated tariffs for the ten values of α, split into cases with $\sigma > \theta$, $\sigma < \theta$, and $\sigma = \theta$. In line with the intuition in the draft, optimal tariffs are more likely to be de-escalated when the returns to scale upstream are larger than those downstream, and when the downstream labor share is high.

Table 8: Tariff De-escalation across the parameter space, II

	Values of α									
	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$\sigma > \theta$	0.00	0.00	0.00	0.00	0.00	0.05	0.14	0.30	0.70	0.94
$\sigma < \theta$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = \theta$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20

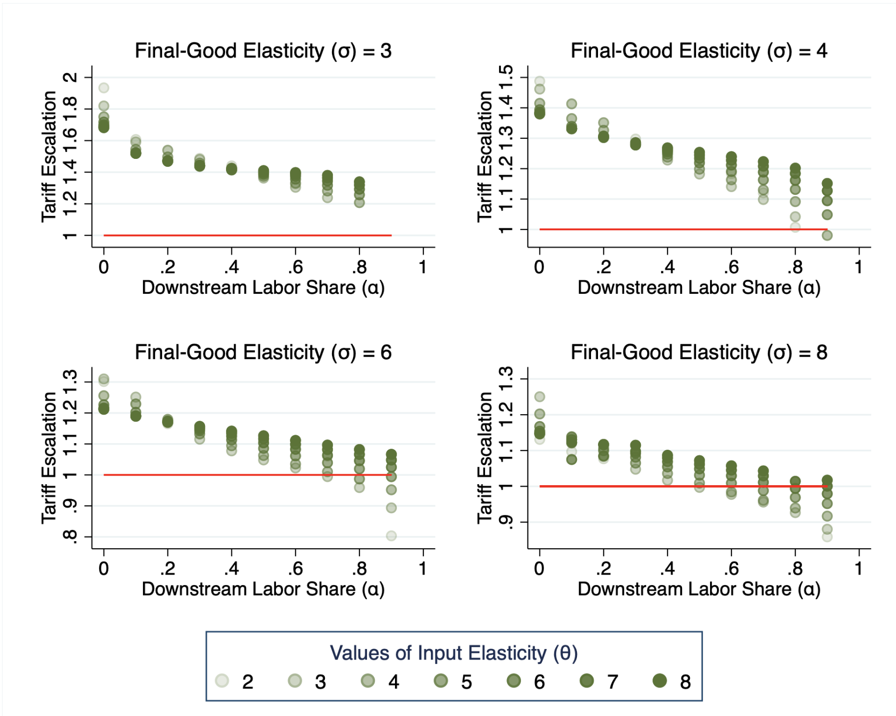
Notes: Table presents the share of cases in each column for which the optimal tariff are de-escalated.

46. From the 70 combinations with $\sigma = 4$, we cannot solve for the equilibrium for 2 cases. The 1 percent reported in the table is the share out of the 68 cases for which we have a solution.

Second-best tariff escalation and the labor share

We replicate Figure 3.2 for different values of the elasticity of substitution across downstream goods.

Figure 6: Second-Best Tariff Escalation and the Labor Share



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the downstream labor share (α) and upstream elasticity of substitution (θ) for different values of the elasticity of substitution downstream (σ).

.5.2 *Grid over A_d, A_u, τ_d and τ_u*

We now perform a similar analysis, but simulate the model for various values of the parameters A_d, A_u, τ_d and τ_u. We start from their calibrated value in Table 3.1 in Section 3.6.1, and consider a 25 percent and a 50 percent decrease, and a 50 percent and a 100 percent increase for each parameter. We fix $\sigma = \theta = 5$ and $\alpha = 0.55$, as in our baseline quantitative analysis. The grid has 625 different combinations in total, and we can solve for 575 cases. The following tables provide descriptive statistics for the tariff escalation wedge .

Table 9: Statistics of the tariff escalation wedge: all cases

	All cases	With escalation	With de-escalation
Mean	1.11	1.12	0.91
Median	1.12	1.12	0.91
Standard Deviation	0.04	0.01	0.00
Minimum value	0.91	1.10	0.91
Maximum value	1.13	1.13	0.91
N	575	552	23

Notes: Table reports statistics for the tariff escalation wedge for the optimal import tariff computed in the grid search exercise.

Tariff escalation and parameter space

Finally, we present results illustrating the combinations of parameters that generate de-escalated tariffs. Table 10 reports the fraction of cases for which tariffs are de-escalated for a given parameter.

Table 10: Tariff De-escalation across the parameter space, I

A_d	0.16	0.24	0.32	0.49	0.65
Share:	0.20	0.00	0.00	0.00	0.00
A_u	0.07	0.11	0.14	0.21	0.28
Share:	0.00	0.00	0.00	0.00	0.20
τ_d	1.19	1.78	2.37	3.56	4.75
Share:	0.04	0.04	0.04	0.04	0.04
τ_u	1.02	1.52	2.03	3.05	4.07
Freq:	0.04	0.04	0.04	0.04	0.04

Notes: Table reports the share of cases in each cell for which optimal tariff are de-escalated, i.e $\left(1+t_H^d\right)/\left(1+t_H^u\right) < 1$.

.5.3 Robustness for Calibrated Parameters

Table 11: Calibrated Parameters - Robustness

	τ^d	τ^u	A^d_{RoW}	A^u_{RoW}
$\theta = 4.43$ and $\sigma = 6.44$	1.787	2.2986	0.289	0.114
$\theta = 2.35$ and $\sigma = 3.08$	5.021	8.536	0.248	0.102
$\theta = 8.52$ and $\sigma = 8.41$	1.508	1.446	0.301	0.121
$\theta = 2.5$ and $\sigma = 4$	3.007	6.878	0.260	0.103
$\theta = 5.5$ and $\sigma = 4$	3.007	1.877	0.281	0.116
$\alpha = 0.75$	2.249	2.040	0.196	0.114
$\alpha = 0.25$	2.239	2.042	0.119	0.124
$\alpha = 0$	2.375	2.073	0.598	0.128

Notes: This table reports the re-calibrated parameters used in our robustness exercise in Table 3.2.

.6 Data Appendix

.6.1 Data Construction for Figure 3.5

US Tariff Data.

- We use US import tariff data at the 8-digit level from the US Harmonized Tariff Schedule (HTS) available at https://dataweb.usitc.gov/tariff/annual. We use the most-favored-nation (MFN) ad valorem tariff rate whenever possible. In approximately 25% of the cases, the MFN ad valorem rate is not available and instead a "specific" tariff rate is applied such as "68 cents/head", "1 cents/kg", "0.9 cents each" etc. In these cases we perform an imputation by calculating an ad valorem equivalent tariff rate using unit values obtained from the US Census Bureau.

- In a next step we use the imputed ad valorem tariff rate to calculate applied MFN ad valorem tariff rates for all goods, taking trade agreements between the US and other countries into account. That is, we calculate the applied MFN ad valorem tariff rate as an import weighted average of the MFN ad-valorem rate and the tariff rate that is paid by countries that are members of a trade agreement.[47] US import data for the year 2015 come from the US Census Bureau.

- Data on tariffs imposed in February and March 2018 on almost all countries (washers; solar panels; iron and steel; aluminum) come from Fajgelbaum et al. (2020) and all subsequent tariffs imposed on imports from China throughout 2018 and 2019 from Chad Bown (available here).

ROW Tariff Data.

47. We account for the following trade agreements: Generalized System of Preferences (GSP, 41 countries), The Agreement on Trade in Civil Aircraft (32 countries), NAFTA (3 countries), Caribbean Basin Initiative (CBI, 17 countries), African Growth and Opportunity Act (AGOA, 40 countries), Caribbean Basin Trade Partnership Act (CBTPA, 8 countries), Dominican Republic-Central America FTA (6 countries) and the Agreement on Trade in Pharmaceutical Products (7 countries).

- We use tariff data for 115 countries plus the European Union at the 6-digit HS code level from the WTO Tariff Download Facility available at http://tariffdata.wto.org/default.aspx. We use the most-favored-nation (MFN) ad valorem tariff rate which constitutes the simple average duty of all products within a 6-digit HS code classification.

- We use data on retaliatory tariffs imposed by China throughout 2018 and 2019 from Chad Bown (available here). Data on retaliatory tariffs imposed by the European Union, Canada, Mexico, India and Turkey stem from Li (2018). Using data on these tariff waves we adjust the MFN applied tariff rates taking 2015 US export value weighted averages with US export data coming from the US Census Bureau.

Intermediate and Final Goods Classification

- We classify goods into intermediate and final goods using the UN Broad Economic Categories (BEC). The cross-walk between HTS10 codes and end-use categories is available here. We classify goods as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods (including capital goods) start with BEC code 41, 521, 112, 321, 522, 61, 62, 63. All other goods have no classification.

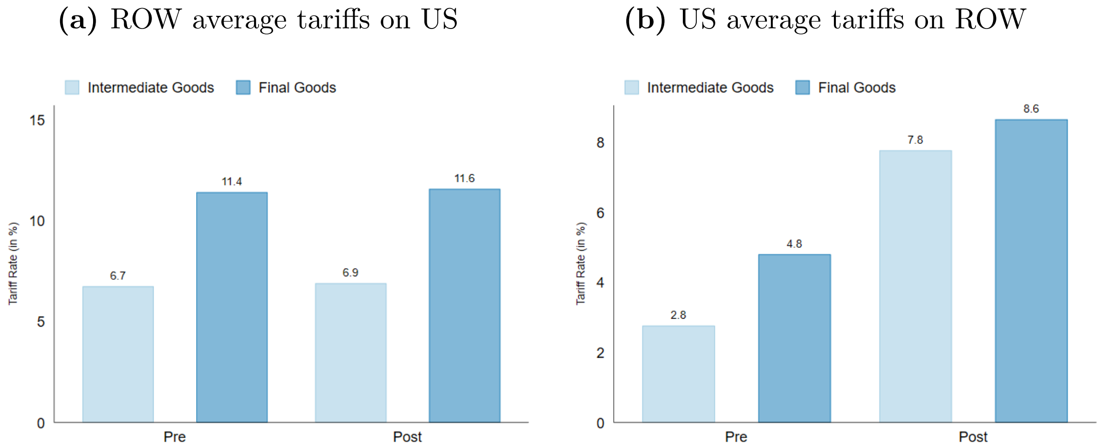
Tariff Escalation Unweighted

- As alternative to Figure 3.5 which shows trade-weighted tariff rates, Figure 7 displays an unweighted version of the tariff increase on intermediate and final goods by the ROW on imports from the US throughout the trade war and vice versa.

.6.2 Elasticity Estimation

Below we explain the estimation of the elasticities of substitution in the upstream and downstream sectors using three different approaches: the trade elasticity approach, the sectoral markup approach, and the scale elasticity approach. We present results for all three approaches and demonstrate how they differ.

Figure 7: Comparison of ROW and US Input & Final-Good Tariffs (Unweighted)



Notes: Pre: Tariffs in January 2018, Post: Tariffs in December 2019. Tariff data from WTO and USITC. Goods are classified as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods start with BEC code 41, 521, 112, 321, 522, 61, 62, 63 (including capital goods). All other goods have no classification.

Sectoral Markup Approach Our first elasticity estimation approach relies on sectoral markups. Information on firm-level markups allows us to derive elasticities in a straightforward manner since equations (41) and (42) illustrate that $markup = \frac{elasticity}{elasticity-1}$. We thus compile data for this exercise as follows:

We obtain upstream/downstream sector classifications using WIOD. We use 2014 sales of the US to the US and RoW to calculate the share of total sales per sector that goes to final consumers. We then classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median. This yields a dataset which shows upstream and downstream classifications for 87 sectors at the 2-digit NACE level (European industry classification). This 2-digit NACE data we combine with a NACE-NAICS concordance file that maps 4-digit NACE (we only use the first 2 digits) to 6-digit NAICS. If there are multiple NACE 2-digit codes for a NAICS 6-digit code, we choose the NACE 2-digit code that has larger total US sales. This yields a final dataset that shows upstream and downstream classifications for 1,175 different NAICS 6-digit codes. We combine these data with data kindly provided by Baqaee and Farhi (2020) (BF) based on 6-digit NAICS codes. The BF data list markups and sales for 31,683 different firms from 1978 – 2018. They provide three different types of markups calculated based on a user cost, a production function, or an accounting profits method. We select their data between 2012 and 2017 and focus on the markups calculated using the production function estimation approach. We further exclude firms that have markups smaller than 1 (14% of all firm-year observations).

Table 12: Elasticities

	mean	sd	min	p5	p25	p50	p75	p95	max	count
Upstream	4.43	4.26	1.10	1.15	1.60	2.75	5.04	16.50	16.50	11045
Downstream	6.44	6.05	1.29	1.46	2.44	4.03	7.49	22.24	22.24	14773

Notes: The table shows weighted mean elasticities for upstream and downstream sectors between 2012 and 2017 across all firms in the WIOD that have markups greater than 1. Elasticities stem from the production function estimation approach. Weights represent the share of firm sales in total sales. We winsorize elasticities and sales at the 5-95th percentile by sector.

We then calculate firm-level elasticities as $elasticity = \frac{markup}{markup-1}$ and winsorize elasticities and sales at the 5-95th percentile by sector. Finally, we calculate weighted mean elasticities for upstream and downstream sectors across all firms where weights represent the share of firm sales in total sales. Table 12 presents elasticities for upstream and downstream sectors pooling all years from 2012 to 2017.

Trade Elasticity Approach In our second elasticity estimation approach, we estimate elasticities in the upstream and downstream sectors by measuring the response of imports in the upstream and downstream sectors to changes in import tariffs. More specifically, we calculate the changes in US import values in both sectors during the US-China trade war (January 2018 to December 2019) that raised US import tariffs on upstream goods by 4.1 percentage points and downstream goods by 4.4 percentage points. We obtain data on import values at the country-HTS10-month level from the US Census Bureau's Application Programming Interface (API). Data on US import tariffs are constructed as described in Section .6.1.

We regress 12-month log changes in import values on 12-month log changes in tariff rates via the following regression specification:

$$\Delta ln(v_{ijt}) = \alpha_j + \tau_{it} + \beta \Delta ln(1 + Tariff_{ijt}) + \omega_{ijt}, \tag{142}$$

where i indicates foreign countries, j denotes products, and t corresponds to time; α_j is a product fixed effect; τ_{it} is a country-time fixed effect; and ω_{ijt} is a stochastic error. We denote import values by v_{ijt}. We estimate equation 142 separately for intermediate and final goods using both log differences and the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$.[48] The results are presented in Table 13.

Table 13: Impact of US Tariffs on Import Values

	Intermediate Goods		Final Goods	
	(1) Log Change Import Value $\Delta ln(v_{ijt})$	(2) Inv. Hyperb. Import Value $\Delta ln(v_{ijt})$	(3) Log Change Import Value $\Delta ln(v_{ijt})$	(4) Inv. Hyperb. Import Value $\Delta ln(v_{ijt})$
log change tariff				
$\Delta ln(1 + Tariff_{ijt})$	-1.05***	-2.35***	-1.81***	-3.08***
	(0.07)	(0.44)	(0.08)	(0.35)
N	1302744	2220920	1253577	2251844
R2	0.027	0.048	0.022	0.045

Notes: Observations are at the country-HTS10-month level for the period January 2018 to December 2019. Since the specification is in 12-month changes, the data includes observations from January 2017 onwards. Robust standard errors in parentheses. Variables are in twelve-month log change. All columns include product-level and country-time fixed effects. The dependent variables are the log change and the change in the inverse hyperbolic sine of US import values of intermediate and final goods, respectively. We use the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$. *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$.

Column 1 (3) suggests that a one percent increase in tariffs on intermediate (final) goods is associated with a 1.05 (1.81) percent decrease in import value. However, since tariffs can lead to zero imports, which will be dropped from the regression, columns 2 and 4 perform the same regression this time using the inverse hyperbolic sine instead of the log change. This adjustments leads to greater trade elasticities for both types of goods. A one percent increase in tariffs on intermediate (final) goods is associated with a 2.35 (3.08) percent decrease in import value. Note that the estimates from this specification correspond to an elasticity of substitution between intermediate (final) goods of 2.35 (2.08).

48. Note that regression coefficients based on the hyperbolic sine transformation are sensitive to the scale of the import values. This is, results vary depending on whether import values are measured in thousands, millions, etc. Following Amiti et al. (2019b), we measure import values in single US dollars.

Scale Elasticity Approach Our final elasticity estimation approach exploits the isomorphism of our model to a model with external economies of scale. As discussed in Section 3.2, these models are isomorphic provided that the following restrictions between the external economies of scale parameters and the elasticities of substitution across varieties hold: $\gamma^u = 1/(\theta - 1)$ and $\gamma^d = 1/(\sigma - 1)$. Data on γ^u and γ^d thus allow us to easily derive information on elasticities.

Data on scale elasticities comes from Bartelme et al. (2019). The authors provide 2SLS estimates on scale elasticities for 15 manufacturing industries presented in Table 14. We classify these industries into upstream and downstream industries following the same procedure as in the *Sectoral Markup Approach* and then calculate the average scale elasticity in those sectors.

Table 14: Scale Elasticities

Industry	**NACE Rev. 2**	**WIOD class.**	**Scale elast.**
Food products, beverages and tobacco	10, 11, 12	downstream	0.16
Textiles	13, 14, 15	downstream	0.12
Wood and products of wood and cork	16	upstream	0.11
Paper products and printing	17, 18	upstream	0.11
Coke and refined petroleum products	19	upstream	0.07
Chemicals and pharmaceutical products	20, 21	upstream	0.2
Rubber and plastic products	22	upstream	0.25
Other non-metallic mineral products	23	upstream	0.13
Basic metals	24	upstream	0.11
Fabricated metal products	25	upstream	0.13
Computer, electronic and optical products	26	downstream	0.13
Electrical equipment	27	upstream	0.09
Machinery and equipment, nec	28	downstream	0.09
Motor vehicles, trailers and semi-trailers	29	downstream	0.15
Other transport equipment	30	downstream	0.16

Notes: Industries and 2SLS scale elasticities stem from Bartelme et al. (2019). Upstream and downstream classifications stem from WIOD where we classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median.

For the upstream sector we obtain an average scale elasticity of 0.133 and for the downstream sector an average scale elasticity of 0.135. Exploiting the isomorphism between this setup and our framework with monopolistic competition and free entry, we convert these to $\theta = 8.52$ and $\sigma = 8.41$.

.6.3 Share of Inputs in the Downstream Sector

As in the "Sectoral Markup Approach," we classify sectors into upstream and downstream depending on whether the share of total sales to final consumers is below or above the median across all sectors. From the WIOD in 2014 we calculate the share of inputs in the downstream sector as the ratio of intermediate inputs to sales in the downstream sectors leading to an estimate of $1 - \alpha = 0.45$.

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