Applications in the Secondary School Mathematics Curriculum: A Generation of Change

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In the 1960s, the ideal curriculum, as seen from recommendations in journals and reports, and the implemented curriculum, as viewed from textbooks, referred very little to applications of mathematics outside the subject. Yet today the teaching of real-world applications of mathematics is seen as a necessary component of a good mathematics education. A number of factors are responsible for this change: changing enrollment trends; changing theories toward how students learn and what they can learn; the arrivals of computers and calculators in schools; the public perception of performance of students on standardized tests; and recommendations of business and industry regarding what they would like to see in the people they hire. The change is manifested in various ways beyond the inclusion of problems that relate mathematics to the world outside the classroom. The most widely used of the newer curricula develops important application ideas from basic principles over many years. Newer influences on the thinking of mathematics educators come from advances in applied mathematics that have resulted in major changes in the workplace and a corresponding desire that no students be excluded from significant applied mathematics. As a result, some of the more recent curricula include entire courses based on units, each with a particular application theme, with the expectation that students will work both individually and in groups.

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In the 1930s, as a result of the Great Depression and the life-skills movement throughout education, there was an increase in the number of

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applications in mathematics textbooks (Butterweck 1937), a trend supported by leaders in mathematics education through the 1940s (Commission on Secondary School Curriculum of the Progressive Education Association 1940; Commission on Post-War Plans 1947). Then, in the decade from 1957 to 1967, responding to a demand for highly trained people in mathematics, science, and engineering, there was a cycle back toward pure mathematics. This decade was a time of major mathematics curriculum development. During that time most of the projects known then as now as the "new math" projects—the University of Illinois Committee on School Mathematics (UICSM), the School Mathematics Study Group (SMSG), the University of Maryland Mathematics Project, and others—tested their materials and saw these materials have their widest impact (Kinsella 1965). The new math projects were similar but not identical in philosophy, but they tended to downplay, if not altogether ignore, the uses of mathematics outside the field of mathematics itself—consumer applications, applications in business and other professions, and broad areas of applied mathematics such as statistics. One early description viewed the similarities of these programs as being essentially their common unifying mathematical themes: set theory, structure, measurement, systems of numeration, operations, logical deductions, graphical representation, and valid generalizations (National Council of Teachers of Mathematics 1961, pp. 22–27).

The recent decade from 1987 to 1997 has also been a time of large-scale mathematics curriculum development. Some of the major projects in this decade are described in this issue. Like the new math projects, these recent projects are not identical in philosophy, but they do have similarities. One of these similarities is a major emphasis on the uses of mathematics. The purpose of this article is to document some of the changes that have taken place and to examine the reasons for the changes in thinking that have brought applications of mathematics from a position of relative nonexistence in the 1960s to one of major importance in the 1990s.

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Views of the Mathematics Curriculum in 1966

Since the decades of curriculum reform are roughly 30 years apart, it is useful to compare thinking in 1996 with that of 1966. In that year, a description of the essence of the new math movement, first given by Meder in 1959 in the journal *Mathematics Teacher,* was considered so relevant that it was republished in the "Classics in Mathematics Education" department of the same journal.

> The objective is to develop mathematical insight, power, and understanding; to lead pupils to think as a mathematician would think; to develop the background of knowledge and meaning that will enable a pupil to solve problems that are not exactly like the type of problem that has been solved for him in the classroom or in the textbook; to help him to deal precisely with precise ideas and to express them correctly and, preferably, concisely; to deal with symbols, not like checkers on a board to be moved in accordance with predetermined and memorized rules, but as aids to thought, full of meaning and denoting ideas that can be understood; to develop the power to look at a line of mathematical symbols and interpret them, to tell what they mean. These are the things that have to be done if mathematical instruction is to meet the demands placed upon it in this day and age. (Meder 1966, p. 361)

Meder had been on the original executive committee of SMSG and also on its original advisory committee (Wooton 1965). So it is not surprising that Meder's view agreed with the forceful statements by the director of the first of the new math projects, Max Beberman, at the Ingalls lectures given at Harvard in 1958 (Beberman 1958). For Beberman, Meder, and many others at this time, thinking "as a mathematician would think" meant to use deduction and base one's logic on postulates or theorems. The collection of data and the use of experiments was considered not to be good mathematics and was discouraged. Not only was reliance on theory considered to be a better approach to traditional topics than the skill- and drill-oriented curriculum that was in most schools at this time, but it was also considered to have the potential for increasing the total amount of mathematics students might learn. Three years earlier, the Cambridge Conference on School Mathematics had issued a report, *Goals for School Mathematics* (1963), which stated, "The subject matter which we are proposing can be roughly described by saying that a student who has worked through the full thirteen years of mathematics in grades K to 12 should have a level of training comparable to three years of top-level college training today; that is, we shall expect him to have the equivalent of two years of calculus, and one semester each of modern algebra and probability theory" (p. 7).

The eight appendices attached to the report reflect both the incredibly optimistic tone of these goals and the lack of attention to applications: "Probability and Statistics"; "Logarithms in Elementary School"; "The Introduction of Formal Geometry"; "Exploration"; "Elementary Modern Mathematics from the Advanced Standpoint"; "Opportunities for Proof-Making in the Elementary School"; "The Use of Units"; and "Remarks on Significant Figures." The only mention of specific applications is in the first of these appendices, with the encouragement to do statistical experiments. The general view, very much like that of Meder, is captured in the fifth appendix: "We find no justification, in our own thinking, for the view that, at the high school level, at any rate, the needs of the potential professional mathematician are different from those of the potential professional user of mathematics and even more different from those of the intelligent citizen of the 21st century" (Cambridge Conference on School Mathematics 1963, p. 85).

The lone major voice in opposition to the new math movement within the profession was that of Morris Kline, a professor of mathematics at New York University. Kline was a noted author of several books on mathematics and culture and later would write an oft-cited history of mathematics (Kline 1972). Universally acknowledged for his expertise in these areas, he was considered by mathematicians not to be reputable as a practicing research mathematician and by mathematics educators to be a dilettante who had no business sticking his nose into matters of the teaching and learning of mathematics.

Kline was persistent and seemed to enjoy the notoriety that he generated every time he appeared at a meeting of mathematics teachers. He often engaged in debates because organizers of conferences were afraid to let him speak without some retort. On at least three occasions, including the last two times articles of his appeared in *Mathematics Teacher* (Kline 1956, 1966), they were followed by responses (Meder 1959; Zant 1966), a pairing seldom found in this journal at the time. A summary of Kline's views and the responses of the community can be found in Dupre (1986).

On one of these occasions Kline put forth seven principles underlying the curriculum he would like to see in schools.

> 1. Mathematics must be developed, not deductively, but constructively. . . .
> 2. Instead of presenting mathematics as rigorously as possible, present it as intuitively as possible. . . .
> 3. Mathematics is not an isolated, self-sufficient body of knowl-

edge. Mathematics exists primarily to help man understand and master the physical world and, to a slight extent, the economic and social worlds. . . . We must constantly show what mathematics accomplishes in domains outside of mathematics. If our subject did not have this value, it would not get any place at all in the curriculum. . . .

4. Elementary mathematics is not self-generating. The significant mathematical concepts, operations, theorems, and even methods of proof were suggested by real situations, needs, problems, and phenomena. Mathematics grew out of experience in the physical world. . . .

5. In place of abstractions, we must, as far as possible, present concrete material. . . .

6. Introduce as few terms as possible. . . .

7. Use as few symbols as possible. (Kline 1966, p. 324)

Kline goes on to explain in some depth that he would develop concepts of ninth-grade algebra keeping the traditional content, but approaching it through applications and formulas. He sketches out a geometry course in which "what is most significant . . . is that each theorem or group of a few theorems is motivated by one or more physical or real problems which cause us to investigate the theorem or theorems" (Kline 1966, p. 328).

The response of James Zant, a mathematics educator at Oklahoma State University, in the last of the feature articles, mocked Kline.

> Is Dr. Kline finally coming around to the point of proposing a new mathematics curriculum and doing something about it? . . . If he would now find what mathematics is being taught in grades 1–8 and obtain the help of some good high school teachers who know how students of this age level learn, he could write a book, and then all of us could find out what he is talking about.
>
> Though this might prove to be a humbling experience for him, as it has been for many of us, it is the only way teachers and others interested in secondary mathematics can see what his proposal really involves. (Zant 1966, p. 331)

A third view of the mathematics curriculum at that time was given by Francis Mueller, who studied articles written from 1956 to 1965 about mathematics education in popular magazines indexed in the *Reader's Guide.* Mueller identified this era as "happy years for 'new math'" and concluded, "As these years pass, less and less is said about mathematics being a highly disliked subject; more and more is said about the brightness of the future along these new mathematical tracks" (Mueller 1966, p. 620). Mueller notes that beginning in 1965 there appear articles in

Time and *Newsweek* questioning the ideas behind the new math and wonders whether they will indeed "mark a point of transition at which the public began to revise its perception of 'new math'" (p. 621).

Applications in *Mathematics Teacher* Feature Articles

Mueller was prophetic: the popular view of the new math did change, from one that was positive toward the movement to one that is for the most part negative. Ironically, the principles of Kline represent contemporary thought more than the objectives of Meder. The buildup of mathematical ideas through applications, as Kline proposed, rather than through deduction, is a feature of many of the newer curricula described in other articles in this issue. The pendulum has swung so far in the direction of teaching applications of mathematics that some mathematicians are worried about the loss of proof and structure from the curriculum (Wu 1996; Addington and Roitman 1996). The view of Zant, who represented the views of a majority of mathematics educators at the time, should give zealots for today's ideas some pause; the pendulum may turn again.

The change in attitude from 1966, when mathematics educators felt that school mathematics should not be approached through applications, to a view that is far more supportive of applications today, can be documented by an examination of articles in the *Mathematics Teacher,* a journal edited at the offices of the National Council of Teachers of Mathematics, which, with a current circulation of about 50,000, makes it the most widely circulated journal among mathematics educators. I make the assumption that its articles tend to represent the views of leaders in mathematics education from the college and senior high school levels.

An article in the *Mathematics Teacher* was considered to be a "feature article" if it was in the "Features" section of the journal (in 1966) or the "Articles" section (in 1996). These sections always have begun the journal, and most feature articles run consecutively from beginning to end. Table 1 shows that the number of feature articles was about the same in the two years studied, and there was a tendency for more authors in 1996 to be high school teachers.

It is very common for articles in the *Mathematics Teacher* to contain examples of problems for use in the classroom. I defined an "application" to be a situation treated mathematically that uses real-world data or realistically contrived situations. This is in line with the view of Niss (1989), one of the leaders of the movement toward applications in European mathematics education:

> What do we mean by an "application" of mathematics . . . ? We shall have to content ourselves with giving a few *working definitions.*
>
> Let us imagine an arbitrary area of extra-mathematical reality (a segment of real life, whatever that is, or of another discipline). If the area is submitted to *any* kind of treatment which involves either mathematical notions or concepts, methods, results, topics or theories, we shall speak of the process of **applying** mathematics to that area. For the result of the process we shall use the term an **application** of mathematics. (Niss 1989, p. 27)

I first counted the number of articles that contained at least one application. That yielded most of the data in table 2 and confirmed my view that attention to applications had increased. But as I examined the articles, I realized that the differences in spirit were not captured by these counts. So I went back and counted those articles in which application examples constituted the theme around which the entire article was

TABLE 1

Overall Data about Authors of "Mathematics Teacher" Feature Articles in the Years 1966 and 1996

Year	1966	1996
Number of issues	8	9
Number of feature articles	67	63
Author:		
At least one at college level	47	37
At least one at high school level	15	23
None a teacher	9	7

TABLE 2

Number and Percentage of Articles in "Mathematics Teacher" with Applications, in the Years 1966 and 1996

	1966	1996
Number of articles	67	63
Number with examples	50 (75)	55 (87)
Number with application examples	11 (22)	33 (60)
Number with all applications contrived	3 (4)	5 (8)
Number with application theme	8 (16)	30 (55)

NOTE.—Numbers in parentheses are percentages.

developed. Then we see the magnitude of the change: the percentage of articles with examples that had an application theme more than tripled, from 16% to 55%.

Rethinking the Role of Applications

A variety of phenomena initiated the process of rethinking the role of applications in school mathematics: enrollment trends; psychological theories; the arrivals of the first computers and calculators in schools; and the public perception of performance of students on standardized tests.

Test scores.—By the 1970s, there was a widespread perception that the new math was a failure. Mean scores on the mathematics section of the Scholastic Aptitude Test (SAT) peaked in 1963, from which there was a rather steady decline until the trend began to reverse itself in 1981. A panel of the College Board convened to examine the decline concluded that the decline from 1963 to 1970 was due to an increase in the number of students taking the SAT exams, but that the later decline was real (Wirtz et al. 1977). Although it was and is common to attribute this later decline to the new math curricula, the causal connection is not clear (Bracey 1997). The decline in mean verbal scores was even more severe than those in mathematics. This suggests influences more general than just those that relate to mathematics, and the possibility that, given the verbal nature of many of the mathematics questions, the mathematics decline might have been partially or wholly due to the decrease in verbal performance. It is also interesting to note that both the high mean scores earlier and the lower mean scores later were achieved by students who had essentially the same mathematics curricula quite strongly based on new math principles, for through the entire decline of the 1960s and the 1970s the dominating textbooks in use in high schools were those of the Dolciani series published by Houghton Mifflin (Dolciani et al. 1962, 1963; Jurgensen et al. 1963).

Still, the decline of mean SAT scores and the widespread perception that the decline was the result of the new math curricula led to a "back-to-basics" backlash against the new math, the reemergence of the view that skill and drill paper-and-pencil skills should again constitute the core of activities in the mathematics classroom, at least in arithmetic and algebra. This movement seemed to have little or no support within the mathematics or mathematics education communities—even Kline was against it—until the appearance in the middle 1970s of a complete K–12 mathematics textbook series published by a major publisher, Holt, Rinehart & Winston, which was unabashedly skill-dominated (Nichols et al. 1974*a,* 1974*b,* 1974*c*). As I describe below, the back-to-basics response to the perceived decline was the impetus for a new look at the mathematics curriculum, and in that new examination application ideas increase in importance.

Enrollment trends.—Through the 1960s enrollment in high school mathematics courses increased (National Advisory Committee on Mathematical Education [NACOME] 1975, pp. 5–7). One result of this increased enrollment was that there was a population of students in algebra and geometry courses who, according to the critics, seemed neither to appreciate nor to understand the logical development and structure in the courses and either failed or limped through them (Kline 1973, chap. 4). Applications were not viewed as an appropriate substitute because of a general feeling (still prevalent today) that applied mathematics was messier and more difficult than pure mathematics, a view disputed even then by some applied mathematicians (Pollak 1968).

Students not ready for algebra at ninth grade tended to be placed in a general mathematics course in ninth grade, a course that concentrated on remediation of arithmetic skills (NACOME 1975, p. 33). If they continued in mathematics as tenth graders, consumer mathematics was the most popular second course. Ironically, the consumer mathematics course in particular tended to include large numbers of applications and often had units based entirely on applications (e.g., see Bolster et al. 1978), and its relative success led to the view that appropriate applications of mathematics were suitable for low-achieving students.

Psychological theory.—The theories of Piaget, which were beginning to become popular at these times, were interpreted to imply that some of these students were *unable* to learn the formal and abstract content of algebra and geometry. "In the third stage [of logical reasoning according to Piaget], formal operational, which occurs only after the age of eleven or twelve, the physical world of reality can be left and formal thought is freed from beliefs of the moment or experiments based on contact with the physical world" (Copeland 1970, p. 143). At all levels, teachers generally found that low-achieving students found the theory in new math to be quite difficult and not useful in developing either skill or concepts. Piagetian theory of the biological development of the organism and Brunerian theory about the learning of concepts (Bruner 1966) suggest that students need concrete hands-on experiences before they can understand the formal abstract concepts. As a result, schools bought hands-on materials such as Cuisenaire rods (for early number sense, addition, and subtraction), base 10 blocks (for numeration), balance scales (for algebra), and geoboards (for geometry). Where they were used, these materials tended to be used by all students in elementary and some junior high schools within their classrooms, and by the lower-performing students in other junior high schools and senior high schools in specially equipped math labs. While these materials were not associated with applications per se, they supported the view that experiences other than drill and practice of skills were appropriate for young or struggling students.

Technology.—The appearance in the early 1970s of handheld calculators was felt by some mathematics leaders to be a further justification for the de-emphasis of paper-and-pencil drill work in favor of discovery and laboratory approaches. A major question then as now first appeared: If the paper-and-pencil algorithms are not to be the backbone of the arithmetic curriculum, what will take their place? One answer given was to move toward applications as the backbone (Usiskin and Bell 1976), and later, with National Science Foundation support, materials were created to help in developing that backbone (Usiskin and Bell 1984).

The appearance of computers in schools also worked to influence the examination of the role of statistics and the new discipline of computer science in K–12 education. A joint committee of the American Statistical Association and the National Council of Teachers of Mathematics was organized to develop statistical materials appropriate for the schools. A report of the Conference Board of the Mathematical Sciences recommended that all students have experience with computers in junior high school, that programs be developed at the high school level that would integrate computing into high school mathematics courses, and that other programs be developed to make vocational computer training more generally available (Conference Board of the Mathematical Sciences, Committee on Computer Education 1972).

Curricular Beginnings

Despite their opposition in public to the views of Kline regarding applications, some mathematics educators were beginning to agree with him that the curriculum should be centered around or give significant attention to applications of mathematics. In the late 1960s, SMSG had begun to prepare a substantially different curriculum beginning in junior high school, called informally the "second round." This curriculum was organized around units rather than yearlong courses and included stronger attention to applications. To help teachers with these new approaches, SMSG commissioned the writing of two volumes collecting applications appropriate for school mathematics (Bell 1967, 1972). And in the early 1970s, the UICSM, the first of the new mathematics projects, received a grant from the National Science Foundation to develop a first-year algebra course from an applied mathematics point of view.

In the middle 1970s, the perceived failure of the new curricula in mathematics and a political controversy over the social studies curriculum *Man: A Course of Study* (see Kraus 1977) led the National Science Foundation to curtail its support for curriculum development, so that by 1975 in the creation of materials for school mathematics only a teacher-development project directed by Alan Hoffer and a curriculum-development project I directed were being supported.

Thus, when mainline publishers began publishing back-to-basics curricula in the mid-1970s, the response of mathematics education professionals was not with materials but with conferences. A conference sponsored by the National Institute of Education (NIE) was convened to discuss basic skills in mathematics. Because of the perception regarding the new mathematics, there was no sentiment for going back to a structure-based curriculum. There was also great sentiment to provide curricula that would be meaningful for less successful students. As a result, discussions turned toward the use of applications. The report of this conference (NIE 1975) recommended that the curriculum have 10 basic skill areas, of which all but the first were unabashedly tied to the real world: appropriate computational skills, links between mathematical ideas and physical situations, estimation and approximation, organization and interpretation of numerical data, measurement, alertness to reasonableness of results, qualitative understanding and drawing inferences from functions and rates of change, probability, computer uses, and problem solving. Thus, in response to a countermovement, the notion that basic skills included being able to *apply* mathematics was born.

Reports of two major professional organizations helped to popularize this notion. In 1977, the National Council of Supervisors of Mathematics (NCSM), then a small organization that included all the state mathematics supervisors and many supervisors from large city and county districts, produced a position paper on basic skills (NCSM 1977/1978) based on the NIE 10 areas. Not long after that, the National Council of Teachers of Mathematics (NCTM) published its first set of recommendations for school mathematics since just after World War II, in which it advocated that "problem solving must be the focus of school mathematics in the 1980s" (NCTM 1980, p. 2). Problem solving quickly became the rallying cry and focus of curriculum work and remained so until the publication of the first NCTM standards document in late 1989. Thus, through the 1980s, the principle was promulgated that a good school mathematics curriculum gave strong attention to applications. "The curriculum should maintain a balance between attention to the applications of mathematics and to fundamental concepts" (NCTM 1980, p. 3).

The task that remained was for mathematics educators to move the incorporation of applications from the ideal curriculum found in reports and their recommendations into the implemented curriculum of the classroom. This task is qualitatively different depending on the mathematics in the applications one wishes to incorporate. Applications of arithmetic abound in society and can be seen by students in their daily lives and in the media. Furthermore, applications had never been entirely removed from the arithmetic curriculum. Even during the height of the back-to-basics movement in the 1970s, the series *Math around Us* (Bolster et al. 1975) featured two-page lessons centered around a particular application theme, such as the size of whales, with brilliant photographs and other graphics to emphasize the theme. The success of *Math around Us*—it was by far the most-used series in the eighth-grade classes studied in the Second International Mathematics Study (Flanders 1992)—led others to copy the approach. The recommendations of NCSM and NCTM to center the curriculum around problem solving reinforced the importance and accelerated the inclusion of applications in K–8 textbooks.

The incorporation of applications into standard high school courses was a more difficult task. The traditional secondary school subjects of algebra, geometry, trigonometry, and elementary functions tend to be avoided by the media and are seldom found in a student's everyday life. Echoing the views of Thorndike (1923, pp. 137–65), those who wished to see applications in the curriculum viewed most "word problems" or "story problems" as contrived and not real applications. For instance, consider the "coin" problem, "I have 20 dimes and quarters whose total value is $4.40. How many dimes and how many quarters are there?" Given this information, why would anyone in the real world care about how many dimes and quarters there are? And how would anyone know the total value unless they had examined the dimes and quarters in the first place and could have counted them then? Consequently, this problem would not be viewed as a real-world application. The best real-world applications were those that actually had been encountered by someone either as a consumer or on the job, and not as a student. Acceptable real-world applications were those that could have been encountered, or that were reasonably close, perhaps simplified variants of the best applications. Such applications were not likely to be encountered by practicing mathematicians or mathematics educators in their schooling, nor did their jobs naturally introduce them to these problems.

It may surprise the reader unacquainted with mathematics education that secondary school mathematics teachers would not know of applications of their subject. However, the education of mathematicians and mathematics teachers at the college level has always tended to focus more on mathematical theory than application. With the exception of velocity and acceleration problems in calculus, even in the recent past a college mathematics major might not see a single application in the mathematics courses required for the degree. Courses in advanced calculus, real or complex analysis, college-level geometry, linear algebra, abstract algebra, and differential equations tended to have few if any applications in the sense of dealing with a real situation with actual or simplified data. Although other areas of mathematics, such as statistics and "discrete mathematics" (the name given to a collection of mathematical ideas applied in computer science and distinguished by their reliance on properties of integers more than properties of real numbers), have many applications, these were typically not required courses for mathematics majors. Thus even well-trained teachers and teacher educators of the 1960s and 1970s might know of few applications of school mathematics other than applications of arithmetic. In fact, one of the leading advocates for increased attention to applied mathematics in the school curriculum, at an international conference on "the teaching of mathematics so as to be useful," felt the need to call for an international effort to collect a sufficient variety of examples of applications of mathematics for use in classrooms (Pollak 1968).

The development of materials for the incorporation of statistics into the curriculum was aided by the existence of a professional organization that mathematics educators could turn to, the American Statistical Association (ASA), but hindered by the size and scope of this area of applied mathematics. In a curriculum that already seemed overcrowded, where was there room for statistics? The first solution was to discuss statistics wherever you could. This was the conclusion of a national committee organized by the National Science Foundation, which recommended that "instructional units dealing with statistical ideas be fitted throughout the elementary and secondary school curriculum" (NACOME 1975, p. 139). It was also the strategy of the joint ASA-NCTM committee, whose first materials, *Statistics by Example,* were published in four soft-cover volumes intended to be used beginning in junior high school and ending late in the high school curriculum (Mosteller et al. 1973).

The National Council of Teachers of Mathematics signaled its general approval of the movement to include applications as part of the standard curriculum with the appearance of a yearbook devoted to the teaching of applications (Sharron and Reys 1979), its first yearbook on this subject since 1942. At the same time, to prepare a collection of real-world applications beyond arithmetic to be simple enough to be useful in instructional situations in schools, a joint committee of the Mathematical Association of America and the NCTM was commissioned to edit a sourcebook of applications. The committee put a call in the newsletters of these organizations for members to send in their favorite applications but received very few responses; they were forced to write virtually the entire book themselves (Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics 1980).

The University of Chicago School Mathematics Project

Aside from elementary curricula that emphasized applications, the first significant multiyear curriculum that incorporated applications throughout was the six-year secondary curriculum of the University of Chicago School Mathematics Project (UCSMP) developed over the years 1983–90 (UCSMP 1990*a,* 1990*b,* 1990*c,* 1991, 1992*a,* 1992*b*). The importance of applications in the UCSMP materials is manifested in two ways. First, applications are given specific objectives in a manner analogous to that traditionally reserved for skills, and these objectives are reviewed and tested. For instance, one of the objectives in the first course, *Transition Mathematics,* is "Use the Comparison Model for Subtraction to form sentences involving subtraction" (UCSMP 1990*c,* p. 333). This explicit statement of a type of application of subtraction is in contrast to other materials that might say, "Apply equations involving subtraction to solve real-world problems." The applications are embedded in an overall scheme that considers algorithmic skills, mathematical properties, real-world uses, and representations (graphs, networks, etc.) as four important dimensions of understanding, and which tests on all four dimensions (Usiskin 1989).

A second characteristic of the UCSMP materials emanates from the view that students traditionally have difficulty applying mathematics because they encounter applications for which they have not studied the prerequisite applied ideas (Usiskin 1991). Consequently, applications are themselves sequenced over the content. Some sequences of applications appear only in one of the courses, while other applications are sequenced over many years. Most of these sequences are grounded in fundamental meanings of one or more arithmetic operations (Usiskin and Bell 1984) and begin in *Transition Mathematics,* the first course of the secondary curriculum.

For instance, in *Transition Mathematics* is the objective "Use the Rate Model for Division." One question is, "On a diet, some people lose 10 pounds in 30 days. What rate is this?" (UCSMP 1990*c,* p. 528). In the next course, *Algebra,* the idea of "rate of change" combines the fundamental meanings of subtraction and division that have been mentioned above. The development begins, "At age 9 Karen was 4′3″ tall. At the age of 11 she was 4′9″ tall. How fast did she grow from age 9 to age 11?"

(UCSMP 1990*b*, p. 366). Generalizing the operations used in this calculation of a rate of change leads to the expression

$$\frac{y_2 - y_1}{x_2 - x_1}$$

to represent the slope of the line through points (x_1, y_1) and (x_2, y_2). In the fourth course, *Advanced Algebra,* slope is extended to the concept of average rate of change of a function. In the last course, *Precalculus and Discrete Mathematics,* the limit of slopes is utilized in developing the idea of derivative. In contrast, in traditional curricula the only sequence of applications is found in the study of probability, which is based on counting, and that sequence typically starts and ends in a single chapter or unit of a text.

Another sequence begins with a meaning of multiplication called *size change.* This model covers problems such as "a part of" and "times as many" and receives its name from the fact that, if a geometric figure is placed on a coordinate plane, then multiplying the coordinates of all its points by the positive number k transforms the original figure into an image figure that is a similar figure whose linear dimensions are k times the size of the original. If k is negative, the figure is also rotated 180°, a property that nicely pictures multiplication by a negative number and makes it more reasonable to accept the result that the product of two negatives is positive. The geometric properties of size changes and their relationships to similar figures are discussed in all of the first four UCSMP courses: *Transition Mathematics, Algebra, Geometry,* and *Advanced Algebra.*

The numerical properties of size change are also sequenced. A specific application is to the calculation of interest. If the annual interest rate is 5%, then multiplying the cost by the size change multiplier 1.05 yields the amount after one year. From this start follow two sequences of applications. A first sequence begins by repeated multiplication of this type, which yields the formula $A = P(1 + r)^n$ for the amount A in an account at the end of n years if the original principal is P and the interest rate is r. When the amount of compoundings is c per year, and the interest rate is divided by c each time, as is common practice, then the formula becomes

$$A = P\left(1 + \frac{r}{c}\right)^{cn}.$$

Increasing c while keeping r and n constant gives the formula for continuously compounded interest, namely, $A = Pe^{cn}$, where e is the base of natural logarithms. This sequence begins in *Transition Mathematics* and is extended in *Algebra, Advanced Algebra,* and *Functions, Statistics, and Trigo-*

nometry. A second sequence begins by considering situations where the compounding begins at different times, such as with retirement plans. If amounts $a_1, a_2, \ldots,$ are invested at the beginning of years 1, 2, . . . , and the interest rate remains the same so that the multiplier is a constant x, then at the beginning of year 2 there is $a_1x + a_2$, at the beginning of year 3 there is $a_1x^2 + a_2x + a_3$, and so on. This sequence leads to polynomials, geometric sequences, and their applications to annuities. These sequences begin in *Algebra* and are extended in the two later courses mentioned above.

The University of Chicago School Mathematics Project was also the first full curriculum to require technology in the way of calculators and computers. Almost from the beginning, all of its courses have required scientific calculators, and even before there existed function-graphing calculators for the secondary school, the course *Functions, Statistics, and Trigonometry* required computer software with function-graphing and statistics capabilities (Rubenstein et al. 1992). In the second edition, automatic graphers are required in three of the courses and strongly recommended in the other three, and drawing programs are strongly recommended in geometry. This requirement emanates from the practical consideration that numerical calculations, statistics, graphing, and drawing are done in the workplace using calculators and computers, and that not to use these technologies is to distort the process of applying mathematics. For the same reasons, the second edition of the last course in the curriculum, *Precalculus and Discrete Mathematics,* includes discussion of the symbol-manipulating technology that can be used to solve problems from algebra to calculus and beyond.

In the last two books of its first edition, UCSMP introduced projects, extended tasks that might take several days to complete, as tasks that might be done or assigned in each chapter. About half of the projects deal with applied themes. For example, for a project in *Precalculus and Discrete Mathematics,* a student is asked to determine the types of license plates that are available in that student's state, and for each type to count the number of possible license plates. The popularity of this idea led to its inclusion in all six of the second edition books. That real-world problem solving often takes extended periods of time underlies one of the ideas behind other contemporary projects, to which we now turn.

From Problems and Lessons to Units Based on Application Themes

In 1980, the Consortium for Applications of Mathematics (COMAP) was founded and adopted an approach to applications quite different from the individual problem or lesson, even if developed over many years. Beginning at the college level, units and courses centered around applications were developed. In the mid-1980s, similar units were developed for the high school level, and when the National Science Foundation announced its intention to fund curriculum projects at the high school level, COMAP received one of the major grants for its ARISE (Applications Reform in Secondary Education) project.

The ARISE curriculum, being published by South-Western Publishing Company, is organized around units, or modules, based on a problem theme. These are not the traditional problems of secondary school mathematics; for example, one grade 10 unit, "Fairness and Apportionment," begins with the question of dividing a cake or other object into *n* equal pieces and moves to the question of fairness in voting. It is not the only curriculum organized in this fashion. The Interactive Mathematics Program (IMP) curriculum, being published by Key Curriculum Press, described elsewhere in this issue, is built around units of five to eight weeks in length that are almost all motivated by real-world applications. The curriculum of the Core-Plus Mathematics Project, being published by Everyday Learning Corporation, "emphasizes mathematical modeling and modeling concepts of data collection, representation, interpretation, prediction, and simulation" (Core-Plus Mathematics Project 1996). The curriculum of the Systemic Initiative for Montana Mathematics & Science (SIMMS), being published by Simon & Schuster Custom Publishing, is similar except that the units are shorter, each being about two to two and a half weeks in length. Each of these curricula is designed to include the full range of mathematics students encounter in high school.

The titles of the grade 9 units of these curricula exhibit how far the first high school course has shifted from a structure based on algebraic properties.

ARISE (8 units): "Pick a Winner: Decision Making in a Democracy"; "Secret Codes and the Power of Algebra"; "Landsat"; "Prediction"; "Animation/Special Effects"; "Wildlife"; "Imperfect Testing"; "Testing 1,2,3." (ARISE 1998)

Core-Plus (8 units): "Patterns in Data"; "Patterns of Change"; "Linear Models"; "Graph Models"; "Patterns in Space and Visualization"; "Exponential Models"; "Simulation Models;" "Planning a Benefits Carnival." (Coxford et al. 1997)

IMP (5 units): "Patterns"; "The Game of Pig"; "Overland Trail"; "The Pit and the Pendulum"; "Shadows." (Fendel and Resek 1997)

SIMMS (16 units): "Reflect on This"; "So You Want to Buy a Car"; "Yesterday's Food Is Walking and Talking Today"; "A New Look at

> Boxing"; "What Will We Do When the Well Runs Dry?"; "Skeeters Are Overrunning the World"; "Oil: Black Gold"; "I'm Not So Sure Anymore"; "AIDS: The Preventable Epidemic"; "Are You Just a Small Giant?"; "Going in Circuits"; "One Step Beyond"; "From Rock Bands to Recursion"; "Under the Big Top but Above the Floor"; "Fair Is Fair"; "Digging into 3-D." (SIMMS Project 1993)

The phenomena behind this change in a view toward applications are not the same as the phenomena that brought applications themselves into the curriculum. The move toward entire chunks of the curriculum being devoted to applications has mainly come from the growth of applied mathematics as a discipline, the influence of business and industry, and the resulting pressures for inclusion of all students in sophisticated applied mathematics.

The growth of applied mathematics.—Fifty years ago, the only major fields requiring college-level mathematics were engineering, the physical sciences, and actuarial science. The Allies' survival and ultimate victory in the Second World War were aided by developments in coding theory, mathematics now behind such everyday actions as the personal identification numbers savers have to protect them when they use automatic teller machines and partly accounting for the National Security Agency being the largest employer of Ph.D. mathematicians. Just after the war, new ways of solving linear programming problems (problems to find the most efficient way of handling many transportation and supply problems in business—not to be confused with computer programming) were developed, forming one part of a new discipline called operations research. Thirty years ago, computer science began to emerge as a separate discipline. With it statistics and economic theory became both easier to apply and more sophisticated. And, more recently, computers have fueled the use of robotics in manufacturing and the ubiquitous use of computer chips in cars, appliances, games, and other consumer items. It is natural that some of these newer ideas would filter down into school curricula, and this filtering has been accelerated by the presence of powerful computers in schools and homes that can handle the large amounts of memory that some of these applications require.

Influence of business and industry. The first and most obvious response of business and industry to the increasing sophistication of their workplace is to ask that schools teach more of this new content. "Put simply, students must go to school longer, study more, and pass more difficult tests covering more advanced subject matter" (Johnston and Packer 1987, p. 117).

In particular, the request is that students be asked to become more sophisticated, learn about technology, and spend time in decision making. "Workers are less and less expected to carry out mindless, repetitive chores. Instead they are engaged actively in team problem-solving, talking with their co-workers and seeking mutually acceptable solutions" (Wellspring Symposium Report 1988, p. 3). The call to train students in group work and cooperation is itself quite a change for mathematics, which has historically been seen as a solitary activity.

The Standards movement. The mathematics education community was the first to come out with national curriculum standards (NCTM 1989), and it followed these with teaching standards (NCTM 1991) and assessment standards (NCTM 1995). A theme of the curriculum standards is the notion of "mathematics for all" (NCTM 1989, p. 3), but not necessarily the same mathematics for all (pp. 124 ff.). However, the "Standards" movement has led to calls for everyone to study the same curriculum (Business Roundtable 1996; see also the article by Alper el al. in this issue), matched by a strong movement to move students away from vocational education programs into the mainstream academic subjects. "There is no excuse for vocational programs that 'warehouse' students who perform poorly in academic subjects" (Johnston and Packer 1987, p. xxvii). These movements particularly affect mathematics, historically the most tracked of the high school subjects. Units with application themes are seen by the recent curriculum projects as vehicles not only to capture the attention of these students, but also to use as concrete activities to help those students better learn the mathematics they encounter.

The Ideal versus the Implemented Curriculum

Although this article has focused almost entirely on journal articles, reports of national commissions, and materials of projects, a comparison of the most-used textbooks in 1996 with those most used in 1966 would be likely to show the same change in view toward applications as was found in the *Mathematics Teacher* feature articles. A cursory look at these books shows that there were no more than a handful of applications in the standard textbooks of 1966, while most of the textbooks of today have applications interspersed throughout. This is true of today's books even when they do not go to the extent of integrating the applications into the fabric of the course, such as basing lessons or units on them, or use application questions on tests.

Will there be another cycle? Tyack and Tobin (1994) point out how difficult it is to change the grammar of schooling. The most recent curricular projects particularly wish to change the grammar of mathematics education, from a subject whose organization in senior high schools and textbook adoption and college requirements in many places are based on the fields of algebra, geometry, and analysis, to one whose organization utilizes problem themes. As the article by Lappan in this issue illustrates, this change in organization is also being tried at the middle school level. As the titles of the units of the most recent secondary school courses demonstrate, the projects differ quite a bit from each other in content, certainly more than traditional textbooks for geometry, for example, have differed. These differences raise the question of whether it is possible for more than one of these projects to survive in a particular area in a climate of movement toward state and national standards, and if there is no more than one survivor, it is likely the movement toward basing a curriculum on applications will recede. But the long-term trend toward the increasing amount of mathematics in everyday life activities, and with it the increasing demand for knowledge of more and more relevant mathematics by greater numbers of people, seems quite likely to remain.

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