What Was the Industrial Revolution?

Robert E. Lucas Jr.
University of Chicago

> The characteristic which distinguishes the modern period in world history from all past periods is the fact of economic growth. It began in western Europe and spread first to the overseas countries settled from Europe. . . . For the first time in human history it was possible to envisage a sustained increase in the volume of goods and services produced per unit of human effort or per unit of accessible resources.
>
> (Cole and Deane 1966)

At some point in the first half of the nineteenth century, average real incomes—per capita GDP—in the United Kingdom and the United States began to grow at something like 1–2 percent per year and have continued to do so up to the present. Two-percent growth means real incomes that multiply sevenfold every century. It does not take very good data to see that nothing like this could ever have been seen before the nineteenth century. Now incomes in many economies routinely grow at 2 percent per year, and some grow at much higher rates. But these "catch-up" economies eventually slow down, and so far no economy has attained income levels that surpass the current levels of the original English-speaking countries.

These events surely represent a historical watershed, separating a traditional world in which incomes of ordinary working people remained low and fairly stable over the centuries from a modern world where incomes increase for every new generation. I take these facts as a definition of the Industrial Revolution and try to think about the choices in individual behavior that brought these changes about.

It will be useful to begin with two sets of figures. The first of these—figures 1–3—illustrates the universal decline of what Theodore Schultz (1964) called "traditional agriculture." The second—figures 4–9—illustrates the demographic transition: the fact that the onset of productivity growth in a country initially leads to larger families and population growth, followed by reduced fertility and smaller families.

I am grateful for comments from Ufuk Atcigit, Jeremy Greenwood, Gene Grossman, Benjamin Moll, Kevin Murphy, Esteban Rossi-Hansberg, Nancy Stokey, Robert Tamura, and David Weil and for the valuable assistance of Nicole Gorton.

[*Journal of Human Capital*, 2018, vol. 12, no. 2]

Figure 1 is a cross section of 112 countries, plotting the fraction of the labor force engaged in agriculture in each against that country's per capita GDP. These data are World Bank numbers for 1980. The income figures are in logs: 6 means $400 1990 US dollars, and 10 means $22,000. Almost the same figure can be obtained for other years, except that with each increase in a year most countries will have moved down the curve, away from agriculture and with increased GDP. We should think of the people in the large agriculture sectors—80 or 90 percent of the work force—as largely illiterate, living on subsistence incomes, and using the methods employed by their grandparents.

Figures 2 and 3 show time series for four countries. Figure 2 plots the fraction of workers in agriculture against calendar time. The data are from Kuznets's (1971) monograph, which I have updated to 2004, using the *2004 Pocket World in Figures* put out by the *Economist* magazine. In figure 3, I replace calendar time with the corresponding income figures from Maddison, so that figure 3 is in the same units as figure 1. Thus, calendar time is absent from figures 1 and 3. One can see that the cross section of countries in 1980 also closely matches the time series over two centuries in a selection of four countries.

The poorest countries in the 1980 cross sections are at about $\exp(6.4) =$ US $600 (1990) per capita per year. The United States in 1810 was at

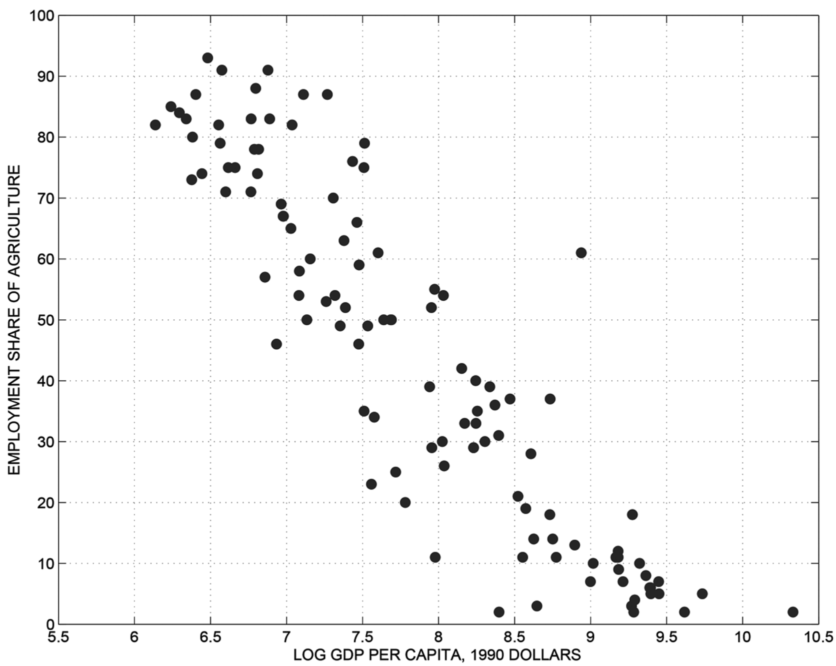

Figure 1.—Agricultural employment shares, 112 countries, 1980. Color version available as an online enhancement.

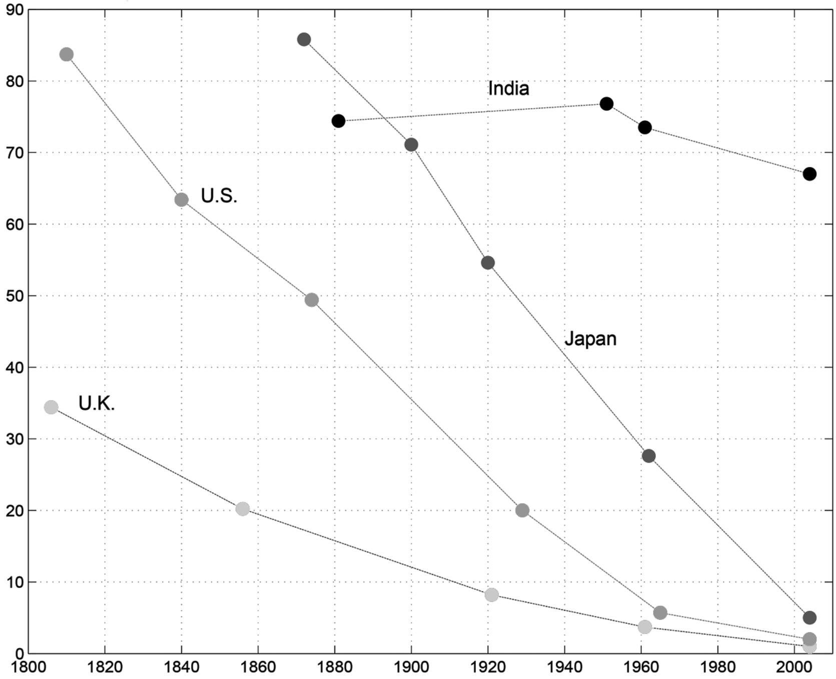

Figure 2.—Employment shares in agriculture in four countries across time. Color version available as an online enhancement.

about exp(6.6) = \$735. These are averages of rich and poor, so working people in the eighteenth century must have averaged something like \$500 or \$550. These figures require a good deal of guesswork, but the term "subsistence" is hard to avoid. Modern national-product accounting involves many subtleties, but in 1800, even in the richest countries, for most people income was a matter of having enough food to keep oneself alive and bring up a family. This is also the case for many people in Africa and South Asia today. At the rich end of the figures, the fitted curve begins to flatten out. The successful economies have 2 percent or less of their labor force in agriculture, yet many of these are exporters of agricultural goods. These economies have settled down to per capita growth rates of 1.5 or 2 percent annually.

Figures 1–3 do not show population growth, but we know that human population has grown quite steadily and expanded geographically since prehistoric times. Population growth as a whole has changed dramatically during the Industrial Revolution—just as Thomas Malthus and David Ricardo would have predicted—but it is a more puzzling fact that in the wealthier economies population growth has been declining. These economies have gone through a "demographic transition." Figures 4–9, all based on the Maddison data, are time-series plots of population and pro-

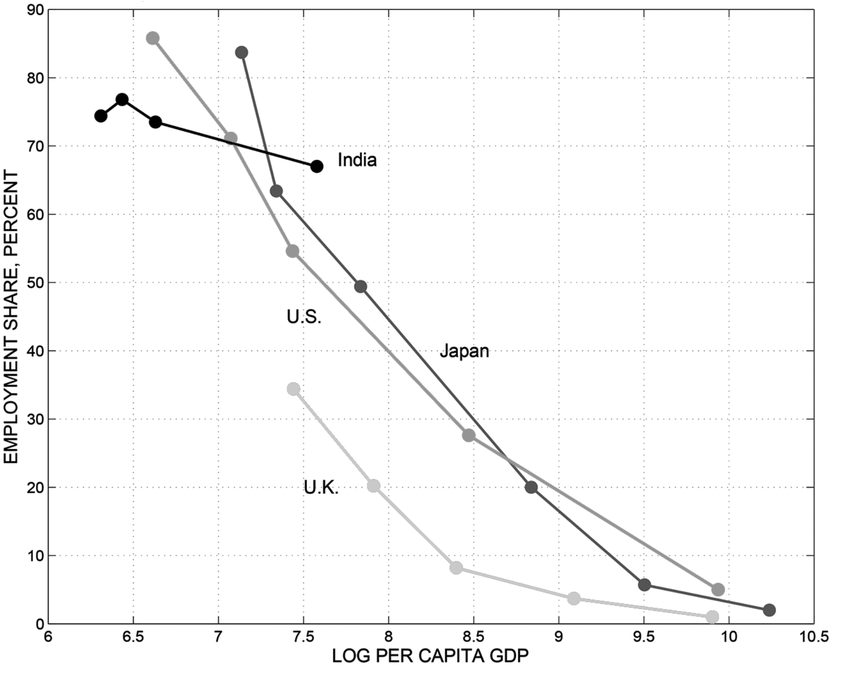

Figure 3.—Employment shares in agriculture in four countries by income. Color version available as an online enhancement.

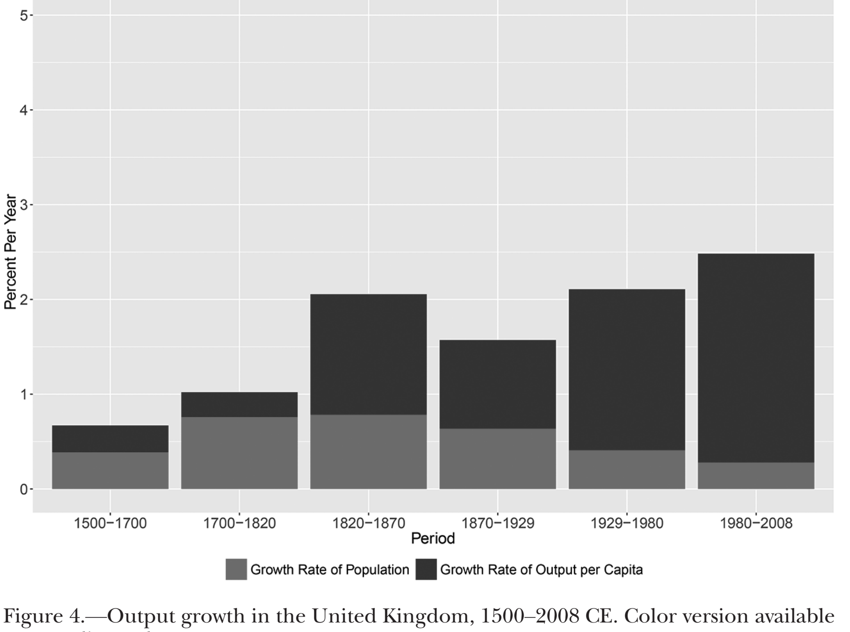

Figure 4.—Output growth in the United Kingdom, 1500–2008 CE. Color version available as an online enhancement.

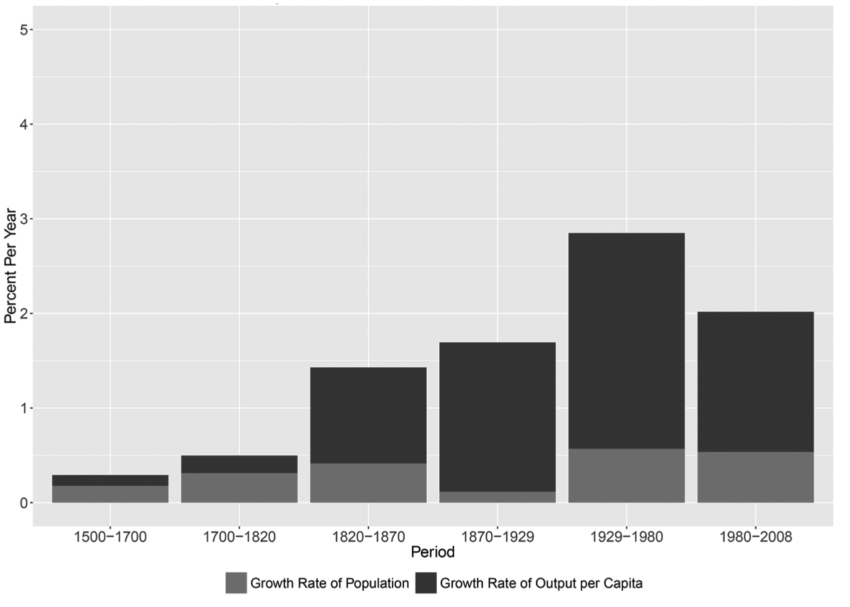

Figure 5.—Output growth in France, 1500–2008 CE. Color version available as an online enhancement.

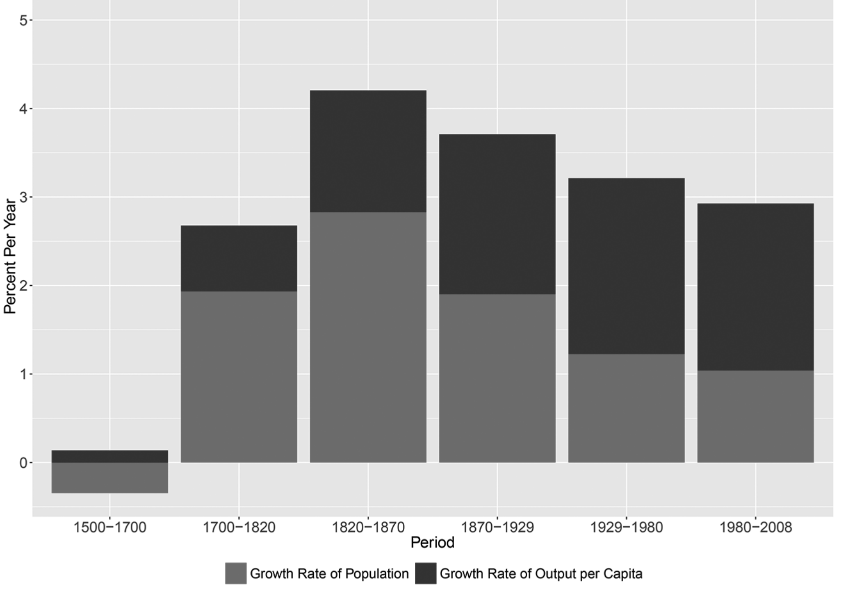

Figure 6.—Output growth in the United States, 1500–2008 CE. Color version available as an online enhancement.

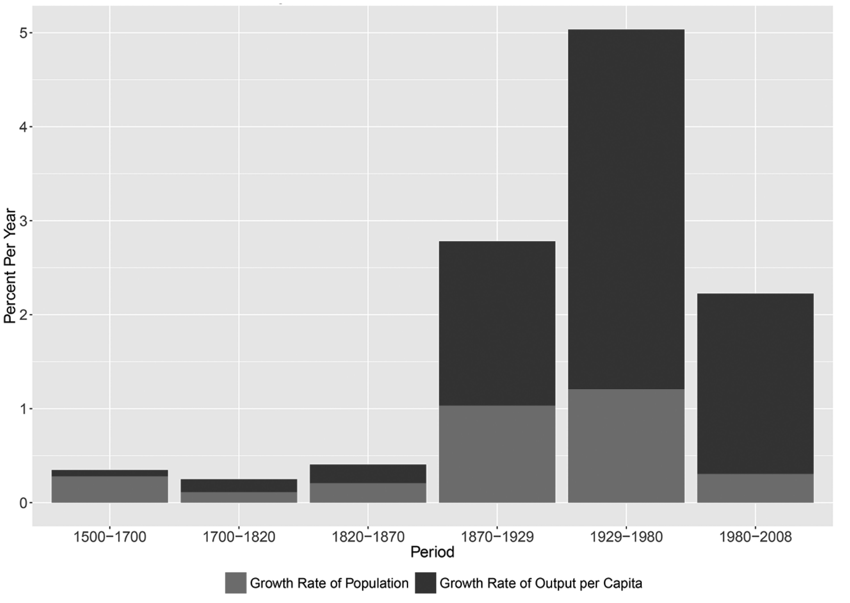

Figure 7.—Output growth in Japan, 1500–2008 CE. Color version available as an online enhancement.

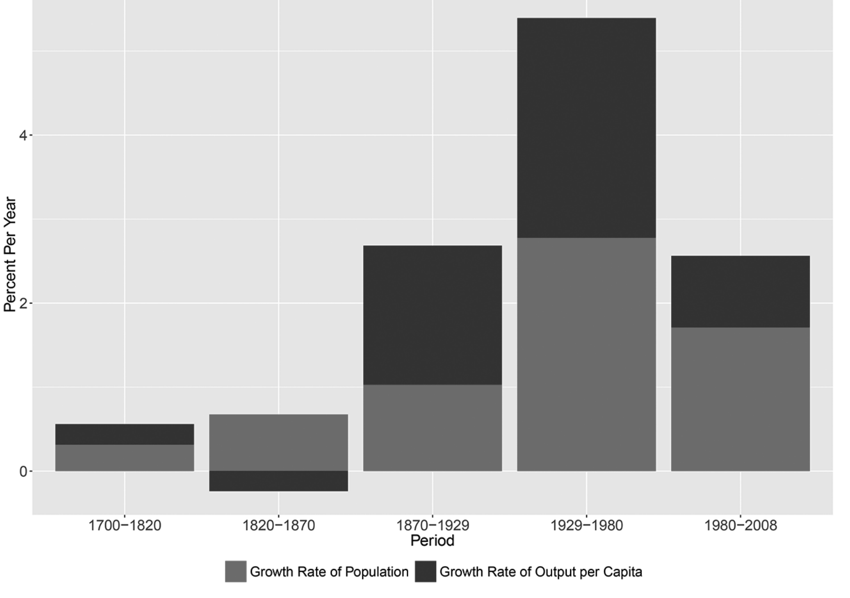

Figure 8.—Output growth in Mexico, 1700–2008 CE. Color version available as an online enhancement.

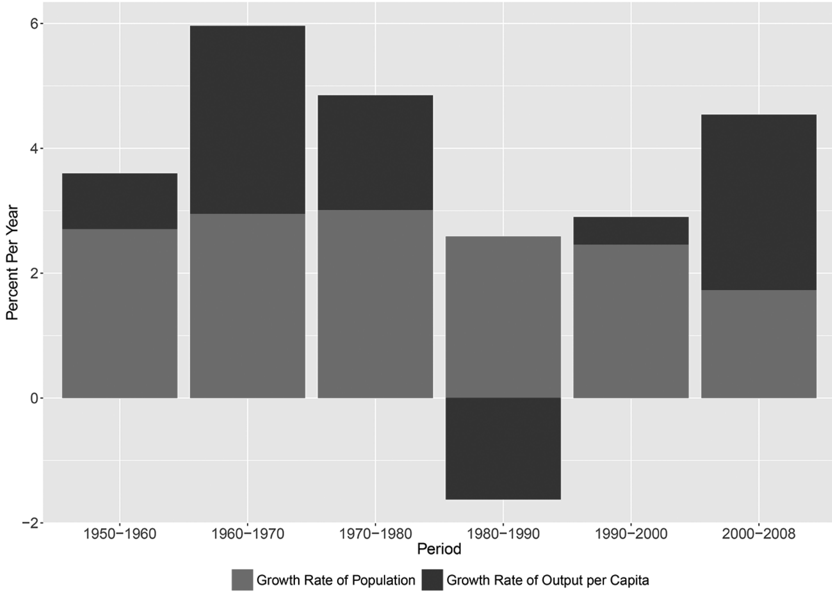

Figure 9.—Output growth in Nigeria, 1950–2008 CE. Color version available as an online enhancement.

duction per person superimposed.[1] Figure 4 shows the demographic transition in full steam in the United Kingdom in the first half of the nineteenth century and possibly visible earlier. For France, the timing is similar, but the transition was more sudden. For the United States, one can see a mixture of the transition combined with a large immigrant flow. For Japan, both sustained growth and the transition came much later. For Mexico, the transition came as late as the 1960s, if then. For the data from Nigeria, beginning with independence, it is hard to see anything but Malthusian growth.

The processes illustrated in these figures—the onset of sustained growth following centuries of subsistence labor, the gradual migration away from traditional agriculture toward urban living and working, the demographic transition—are all at the center of the Industrial Revolution. And all of them are still underway.

What were the decisions taken by individual families that led to these changes? What motivated these decisions? There are surely many ways to approach these broad questions, but I will begin with Gary Becker's idea of a "quantity/quality trade-off" in fertility. Becker first addressed the fertility decision in a 1960 paper that accepted the view that more children are desirable, other things equal, but then added a second dimension that he called child "quality" to account for the fact that increases in wealth

[1] These plots are adapted from Galor and Weil (2000).

are sometimes associated with smaller family sizes. This original application was designed to account for cross-sectional evidence, but over the years the quantity/quality idea has been developed in many directions, including time series. Important later developments include Becker and Lewis (1973), Becker and Tomes (1976), Becker (1981), Becker and Barro (1988), Barro and Becker (1989), and Becker, Murphy, and Tamura (1990). The more recent of these papers are directly focused on the demographic transition. I do not see how any account of the Industrial Revolution can succeed without including some form of this two-dimensional quantity/quality trade-off.

My strategy for this paper is built on the observed correlation of sustained growth and the decline of traditional agriculture. Section I outlines a Malthusian model of an economy without sustained growth with a quantity/quality trade-off that is based most closely on the joint work by Becker and Barro. Many of the details are taken from Lucas (2002). Sections II and III then describe a pure human capital model that encompasses both stagnation and sustained growth as possibilities.

I treat these two model types as though they existed in separate, unrelated worlds. This is a device—certainly artificial—to establish some important features of both stagnation and sustained growth at their simplest levels. In Sections IV and V, I put the pieces together by admitting migration of labor between the two worlds. Again there are many possibilities. In Section IV, some of the illiterate farm workers move to the city and work as servants to the increasingly wealthy urban population. In Section V, farm workers invest in enough schooling for their children, who then migrate and join the urban population as equals. Section VI discusses some of the implications of these two models and others.

I. A Quantity/Quality Trade-Off: Traditional

Modern growth theory has mostly followed Robert Solow (1956) in treating fertility and population as externally fixed features of the environment. This simplification has facilitated progress in many directions, but it obviously will not help in understanding the demographic transition. For the classical economists—notably Adam Smith, Thomas Malthus, and David Ricardo—fertility decisions and population were central to explaining production and incomes. For my purposes, Ricardo's work is a good place to start.

In each of the examples below, I begin with the fertility decision of a newly formed family, endowed with a stream of resources x. The parents value three things: the goods they consume themselves, the number of children they will have, and the resources (if any) that they pass on to their children. We can express this as a Bellman equation,

$$v(x) = \max_{c,n,z} W(c, n, v(z/n)),$$

based on the recursive assumption that parents value resources passed on to each of their children as they value their own.[2] As in Becker's (1960) original formulation, this family values both "quantity" n and "quality" z/n per child. The Bellman approach is based on Becker and Barro (1988). This general setup has a lot of possibilities, depending on the family's abilities to transfer goods z to their children. In illustrations below, I use the more specific log-linear preferences

$$v(x) = \max_{c,n,z} c^{1-\beta} n^{\eta} v(z/n)^{\beta}. \tag{1}$$

Suppose, to begin with, that this family is part of a hunter-gatherer economy without ownership of land or anything else, so there is nothing tangible to pass on to the children. The family has available x units of a nonstorable consumption good, which it divides into kn units of children's goods and c units of adult consumption:

$$c + kn \leq x. \tag{2}$$

The child-raising cost k is taken as given. This family will choose the fertility level n so as to solve

$$\max_{n} (x - kn)^{1-\beta} n^{\eta} v^{\beta} .$$

These parents take pleasure in their children and are happier the larger is the utility level v per child, but in this society there is nothing they can do that affects the well-being of their children beyond the basic child-raising cost k. The first-order condition for this problem thus implies

$$n = \frac{\eta}{1 - \beta + \eta} \frac{x}{k}. \tag{3}$$

To get the equilibrium of the economy as a whole, we add the assumption that total available goods, xN when the population is N, is a Cobb-Douglas function of population N and the richness of the hunting territory, L:

$$xN = AL^{\alpha} N^{1-\alpha}. \tag{4}$$

If all families are treated equally, equations (3) and (4) together imply

$$n = \frac{\eta}{1 - \beta + \eta} \frac{A}{k} \left(\frac{L}{N}\right)^{\alpha}. \tag{5}$$

[2] Of course, there are enormous differences in many aspects of family life that are abstracted from here. See Clark and Cummins (2016).

Since at any date t, $n_t = N_{t+1}/N_t$, equation (3) implies the dynamics

$$N_{t+1} = \frac{\eta}{1-\beta+\eta}\frac{A}{k}L^{\alpha}N_t^{1-\alpha}.$$

Population thus converges for any initial size N_0 to the constant level

$$N_{ss} = \left(\frac{\eta}{1-\beta+\eta}\frac{A}{k}\right)^{1/\alpha}L,$$

and at this level per capita income is given by

$$1 = \frac{\eta}{1-\beta+\eta}\frac{x}{k}.$$

Note that none of the production-side parameters L, A, and α appear in this steady state expression for x. If the available land L were to double, so would the population, though this would take some time. This is the most basic form of the Malthusian model.

As a second example, consider a settled agriculture economy in which land is privately owned. Preferences remain as in equation (1). Each household now has x units of privately owned land (not goods) and 1 unit of labor. Land and labor can be combined to produce the consumption good, according to the production function $f(x) = Ax^{\alpha}$. Each child requires k units of goods, so

$$c + kn \leq Ax^{\alpha}. \tag{6}$$

Each family allocates an equal share of land to each child.

These assumptions lead to the Bellman equation

$$v(x) = \max_{n}(Ax^{\alpha} - kn)^{1-\beta}n^{\eta}v\left(\frac{x}{n}\right)^{\beta}, \tag{7}$$

where $v(x)$ is the adult utility of a family with landholdings x that behaves optimally. It can be shown that equation (7) has the unique solution

$$n = \frac{\eta-\beta\alpha}{1-\beta+\eta}\frac{A}{k}x^{\alpha}. \tag{8}$$

(Of course, all this makes sense only if $\eta > \beta\alpha$.) Note that the fertility function (eq. [8]) implies lower fertility at each income level than does the function (eq. [5]) for the hunter-gatherer economy. The private ownership of nonlabor resources—land, in this case—permits adults to affect the utility $v(x/n)$ of their children. They understand that increases in n dilute the bequests to each child. Here the quantity/quality trade-off is made explicit as a parental choice.

Now consider equilibrium in an economy with L units of land and N such families, each with $x = L/N$ units of land. In this economy, the implied the population dynamics are

$$N_{t+1} = \frac{\eta - \beta\alpha}{1 - \beta + \eta}\frac{A}{k}L^{\alpha}N_t^{1-\alpha}. \tag{9}$$

In the steady state to which equation (9) converges, the population is

$$N_{ss} = \left(\frac{\eta - \beta\alpha}{1 - \beta + \eta}\frac{A}{k}\right)^{1/\alpha}L.$$

Additional land or technologies that enhance productivity induce proportional population increases but have no other long-run effects.

I think these two models, the hunter-gatherers and the landowners, pretty well cover the possibilities of traditional societies. The situation of hunter-gatherers carries over to any family without land to pass on to its children. The models encompass egalitarian societies of family farms as well as the vastly unequal Egypt of the pharaohs and many possibilities in between. There are, of course, enormous differences among these societies, but all share the common feature that increases in the available land or exogenous technical progress that makes land more productive ultimately result in population growth alone.

This is not a model of a species breeding itself into starvation or extinction. It describes a population settling down to a sustainable steady state, determined by available resources on the one hand and standards of child care on the other. The predicted zero population growth does not rest on the availability of modern contraceptive technology. The model applies to traditional human societies and indeed is routinely and successfully applied to animal populations every day.

Ricardo put this striking prediction at the center of classical economics. It places the determination of the living standards of working people—the real wage, if you like—entirely on the standards people have about child raising, on what they accept as "subsistence." Once these attitudes are set, no improvement in technology or resources will improve living standards: only the population reacts to such changes.

The Malthusian prediction that populations will vary in proportion to available resources but that living standards will not was successful empirical social science. Preindustrial societies in the lush environments of Java or South China had about the same average living standards as societies on the fringes of the Sahara Desert or the Arctic Ocean, and this continues to be largely the case for traditional agricultural societies today. The per capita income estimates reported for 1700 by Maddison range from about \$1,000 for Western Europe to \$500 for the rest of the world, and this factor-of-2 difference is at the high end of available figures. Compared to the cross-economy income inequality we see today, these differences are minor.

II. A Human Capital Economy

What have we learned since Ricardo's time that helps us understand the onset of sustained growth in living standards? We now have a theory of physical capital that serves as a centerpiece of most growth theories. This is a big step forward, but as Solow showed in 1956, diminishing returns prevent physical capital accumulation from generating sustained growth by itself. The substantial growth residual has been called "technology," "total factor productivity," and "human capital." Some economists view these terms as describing distinct forces, operating in concert, but to me they are just different labels for the same thing. I include them all under the term "human capital" because it invites inquiry into the actions of individual people that bring about increases in productivity.

Who inhabits this human capital economy? Here we focus on educated, literate people, urban, possibly landless, living in cities. Scientists, of course, but this group is much too narrow. Think of terms like "bourgeoisie," "intelligentsia," "traders," "merchants," and "middlemen": the people we read about in Landes (1969), Mokyr (1990, 2016), Greif (2006), Botticini and Eckstein (2012), McCloskey (2016), and others. Where did these people come from, and how did they create an ongoing revolution? Something beyond new technology is needed to account for this. We need an explanation of what went wrong with the theory of fertility that had worked so well over earlier centuries.[3]

The increased spending of high-income families on children that stimulated Becker's (1960) paper was not limited to, or even primarily focused on, bequests of land or other tangible capital. The "quality" aspect of spending on children also involves education and other forms of investment in human capital. But in contrast to the straightforward inheritance of land or other, given resources in the previous examples, a bequest of human capital can take many forms, and the returns it yields can vary, depending on the actions of others.

In this section, I develop a model of individual earnings, dividing agents' time into the usual categories of schooling and on-the-job learning. A distinctive feature of the model, as of reality, is the social character of work and creativity. The higher the skill levels of the people around you, the more you improve your own skills. In this section, all of the discoveries, new technology, and anything that affects anyone's productivity are assumed to result from some individual's on-the-job activity. Growth is generated only through the stimulus of others.

We begin by describing an environment very different from the land-based world described in the previous section. Think of a city consisting

[3] See, e.g., Wrigley (2004).

of agents who produce consumption goods with labor only. (Throughout this section and the next, we set land wealth aside.) Identify each agent by the triple (z, s, t), where z is his skill or productivity, s is his age, and t is calendar time. Let $G(z, s, t)$ be the fraction of agents at a given date t who are of age s and have skill less than z. Assume a constant demography, where the probability density function $\pi(s)$ denotes the number of people of age s. It is convenient to assume that the cumulative distribution function G has the Frechet distribution

$$G(z, s, t) = \exp\left(-\mu(s, t) z^{-1/\theta}\right), \tag{10}$$

where $\mu(s, t)$ is an endogenous location parameter that traces changes in skill levels and θ is a constant that measures both the variance of skill levels and the size of the Pareto tail.

There is a learning technology: each agent continuously meets others at a rate $\alpha(S)$, where S is years of schooling. At each meeting, an agent compares his own productivity z to the productivity z' of person he meets and emerges with $\max(z, z')$. An agent of age s has been meeting others throughout his working years $u = (S, s)$, and at each date along the way he meets other working people of all ages with equal probability. An agent begins at $u = S$ with no knowledge at all. Schooling enables him to learn from others who are already working at a rate $\alpha(S)$. His knowledge by age $s \geq S$ is then given by

$$\log G(z, s, t) = \alpha(S) \int_S^s \left(\int_S^\infty \log G(z, \tau, t - s + u) \pi(\tau) d\tau \right) du. \tag{11}$$

The inner bracket describes the best match at each age u, and the outer bracket sums up the best match over the career to date.[4] Combining equations (10) and (11) and canceling the terms $z^{-1/\theta}$ gives

$$\mu(s, t) = \alpha(S) \int_0^s \left(\int_S^\infty \mu(\tau, t - s + u) \pi(\tau) d\tau \right) du. \tag{12}$$

In this context, consider a balanced-growth path (BGP) along which all quantiles grow at a common, constant rate γ, to be determined. In this case, $\mu(s, t)$ takes the form

$$\mu(s, t) = \lambda(s) e^{\gamma t}$$

for some $\lambda(s)$, and in place of equation (12) we have

$$\lambda(s) = \alpha(S) \frac{1}{\gamma} (1 - e^{-\gamma s}) \left(\int_S^\infty \lambda(\tau) \pi(\tau) d\tau \right) du. \tag{13}$$

[4] This development is taken from Caicedo, Lucas, and Rossi-Hansberg (2016). This combination of an initial Frechet distribution and continuous arrivals maintains the Frechet assumption.

Integrating both sides against $\int_0^\infty \pi(s)\,ds$ and canceling gives

$$\gamma = \alpha(S)\int_S^\infty (1 - e^{-\gamma s})\pi(s)\,ds. \tag{14}$$

One solution γ for equation (14) is $\gamma = 0$, which implies stagnation. The right-hand side of equation (14) is concave in γ, so there is a second solution that can be either negative or positive (see fig. 10). In the case $\gamma < 0$, zero is the only steady state. In the case $\gamma > 0$, stagnation and sustained growth are both solutions. For positive growth we need

$$\frac{d(\text{rhs})}{d\gamma} = \alpha(S)\int_S^\infty s\pi(s)\,ds > 1.$$

Sustained growth requires some combination of a high frequency of search, $\alpha(S)$, or a high level of longevity. The learning process involves young learning from elders. It is limited by low schooling levels, early death, or retirement.[5]

The distinctive feature of this model is the social character of work and creativity: learning from others. The role of schooling here serves only to prepare people for actual work, improving their ability to process new ideas. The class of literate merchants, traders, shippers—the "bourgeoisie"—can coexist with either traditional Malthusian societies or modern, sustained-growth societies. The model here thus admits both stagnation and growth as possible equilibria. It has the feature—promising for understanding the Industrial Revolution—that small or gradual changes in individual behavior can transform a stagnating economy into an economy of sustained growth.

III. A Schooling Choice Problem

To this point we have specified a role for schooling, but we have not given parents any control over the schooling their children receive. Now we introduce a quantity/quality trade-off for urban parents that is analogous to that for the land-owning parents described in Section I. Instead of a decision on the number of children n and the land x each child inherits, the urban parent chooses n and a schooling level S for his children. Just as in Section I, we assume that parents have exactly one choice to make per lifetime, a single quantity/quality choice that determines the number of children they have and the utility level available to each child. We restrict the analysis to a BGP where everyone has a common schooling level S. That is, we maintain the assumption that equation (14) holds for some S.

[5] A similar theoretical connection of growth rates and longevity can be traced back to Ehrlich and Lui (1991) and Ehrlich and Kim (2015).

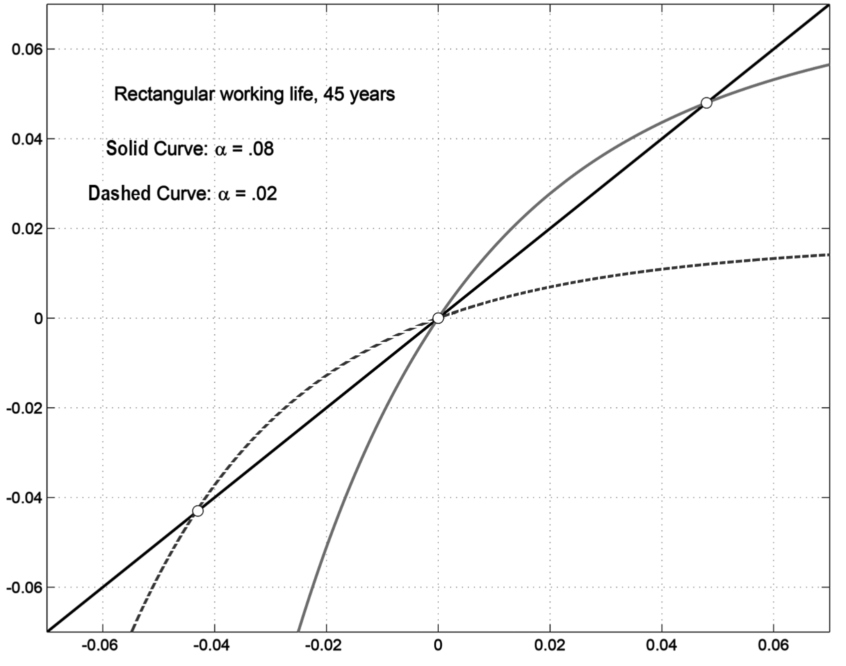

Figure 10.—Equilibrium gamma possibilities. Color version available as an online enhancement.

Within this framework, we spell out the options parents have, the choices they make, and the implications for equilibrium.

Suppose that each child born at calendar date t and with S years of schooling will have the expected present value

$$V = \int_S^{\infty} e^{-rs}\Pi(s)W(s, S, t)\,ds,$$

where r is a given interest rate, $\Pi(s) = \int_0^s \pi(\tau)\,d\tau$, and $W(s, S, t)$ is expected earnings. We maintain the assumptions of balanced growth and a Frechet distribution with parameter θ. On a BGP, income grows at rate γ and everything else is constant, so the expected present value of the earnings stream is

$$V = e^{\gamma t}A\alpha(S)^{\theta}e^{-rS}\int_S^{\infty} e^{-r(s-S)}\Pi(s)\left(1 - e^{-\gamma(s-S)}\right)^{\theta} ds \tag{15}$$

for some constant A. Death and retirement are far off when S is decided, so a reasonable and convenient approximation is to set $\Pi(s) = 1$. We can then replace equation (15) with

$$V = Be^{\gamma t}\alpha(S)^{\theta}e^{-rS}, \tag{16}$$

where B is another constant.

Now assume in addition that parents who choose to have n children and school them all at a level z must give up the fraction δnz of their time, where δ is a fixed parameter. The Bellman equation they face is then

$$v(S) = \max_{c,n,z}\left(c^{1-\beta} n^{\eta} (e^{\gamma} v(z))^{\beta}\right),$$

subject to[6]

$$c \leq B\alpha(S)^{\theta} e^{-rS}(1 - \delta nz).$$

The next generation's goods production capacity relative to their parents' is e^{γ}, which adds $e^{\beta\gamma}$ to parents' utility.

The Bellman equation for this problem can be written in logs as

$$\log v(S) = (1 - \beta)\log\left(B\alpha(S)^{\theta} e^{-rS}\right) + \beta\theta\gamma$$
$$+ \max_{n,z}((1 - \beta)\log(1 - \delta nz) + \eta\log(n) + \beta\log v(z)).$$

Along the BGP, $z = S$, and the first-order conditions imply that n and S must satisfy

$$\frac{\eta}{S} = \beta\frac{d\log\left(\alpha(S)^{\theta} e^{-rS}\right)}{dS}$$

and

$$\delta nS = \frac{\eta}{1 - \beta + \eta}.$$

The fraction of time devoted to adult consumption is

$$\frac{1 - \beta}{1 - \beta + \eta}.$$

Their optimal schooling must satisfy

$$\frac{\alpha'(S)}{\alpha(S)} = \frac{r}{\theta}.$$

Here $\alpha(\cdot)$ (and thus $\alpha'(\cdot)$) is a given function that describes the effect of schooling S on a person's ability to learn from others. If, for example, $\alpha(S)$ takes the form KS^{ϕ}, where K and ϕ are known parameters, the optimal schooling level is

$$S = \frac{\beta\theta\phi - \eta}{\beta r}.$$

[6] This cost in terms of time (as opposed to goods) is adapted from Becker, Murphy, and Tamura (1990).

A higher-level θ or ϕ means increased benefit from schooling on the margin, and a higher r means higher opportunity cost of schooling. The constant implied fertility rate on the BGP, which can be on either side of 1, is

$$n = \frac{\eta\beta r}{\delta(1+\eta)(\beta\theta\phi - \eta)}.$$

Here we need to assume that $\beta\theta\phi > \eta$. The product of altruism β toward children, quality θ of interaction with others, and efficacy ϕ of schooling must be enough to override the desire η for more children. In practice, child quality involves not simply a wish for better life for children but also an environment that enables a parent to bring this about.

IV. Migration Dynamics 1

We have set out descriptions of two distinct economies: a land-based Malthusian economy with landless people living at subsistence levels and a human capital–based economy undergoing sustained productivity growth at a constant rate. Now we view these types—an urban population N_{ct} at t and an unskilled rural population N_{ft}—as coexisting in the same economy. The dynamics will involve migration from the rural to the urban economy and the fertility choices of each type. We need to spell out what the options are for agents of each type.

Assume, to begin with, that $N_{ct} = N_c$ (a constant, assuming $n = 1$ and no new entry) and that productivity in the urban sector grows as in the previous section,

$$v_t(S) = v_0(S)e^{\beta\gamma t}.$$

We next modify the utility function of urban people to include a demand for low-skilled services and let rural people migrate to the city. Retaining the log utility used above, this demand function for low-skilled services a will take the form

$$e^{\beta\gamma t}v_0(S)\max_{a,n,z}\left((1 - \delta nz - wa)^{1-\beta}n^{\eta}a^{\xi}v(z)^{\beta}\right).$$

The first-order condition for a_t is

$$a_t w_t = e^{\beta\gamma t}v_0(S)\frac{\xi}{1-\beta+\eta+\xi} = Ce^{\beta\gamma t},$$

where C is a constant, a_t is hours of service, and w_t is the wage rate. In this situation, the low-skilled N_{ft} now have two sources of employment: services N_{at} and farm work $N_{ft} - N_{at}$. Markets will clear when both

$$w_t = \alpha L^{1-\alpha}\left(N_{ft} - N_{at}\right)^{\alpha-1}$$

and

$$a_t = \frac{N_{at}}{N_c}$$

hold.

These are three equations in a_t, w_t, and N_{at}, given the populations N_c and N_{ft}. We can solve for

$$\frac{N_{at}}{\left(N_{ft} - N_{at}\right)^{1-\alpha}} = De^{\beta\gamma t}.$$

It follows that $N_{at} \to N_{ft}$ (unskilled labor becomes increasingly urban) as $t \to \infty$ and that $w_t \to \infty$ and $a_t \to N_{ft}/N_c$ as $t \to \infty$. This is all we need to get the emptying out of traditional agriculture.

But in this scenario landless farmers are still Malthusians, even after moving to city. They still have nothing to pass on to their children, and so their fertility choice is still

$$n_t = \frac{1}{k}\frac{\eta w_t}{1 - \beta + \eta},$$

just as in Section I. The implied evolution of N_{ft} is just

$$N_{f,t+1} = n_t N_{ft},$$

and since $w_t \to \infty$, it follows that

$$\lim_{t \to \infty} \frac{N_{f,t+1}}{N_{ft}} \to \infty .$$

A proletariat blindly multiplying itself toward subsistence income levels: impossible? Well, sooner or later, yes, but we are familiar with cities where educated and uneducated coexist with the latter still expanding. Think of Mumbai or Rio de Janeiro or the Pakistani population in the Emirates.

V. Migration Dynamics 2

Let us then go to an opposite extreme: drop the possibility of unskilled urban jobs and admit people to cities only if they match up to city standards. And what are they? Consider an unskilled parent earning wage w_t. If he chooses to raise n unskilled children, he solves

$$\max_n (w_t - kn)^{1-\beta} n^{\eta} w_{t+1}{}^{\beta}.$$

He has no control of the children's wages next period, though he is pleased if they do well. The chosen number of children is

$$n = \frac{\eta w_t}{(1 - \beta + \eta)k}.$$

The parent's utility (including altruistic pleasure) is

$$\left(\frac{1-\beta}{1-\beta+\eta}\right)^{1-\beta}\left(\frac{\eta}{1-\beta+\eta}\right)^{\eta}k^{-\eta}{w_{t+1}}^{\beta}w_t^{1-\beta+\eta}. \tag{17}$$

Alternatively, this same unskilled parent can decide to put his children through school. If so, we assume that he is required to meet the school costs,

$$v_t(S) = e^{\gamma t}B\alpha(S)^{\theta}e^{-rS}, \tag{18}$$

and so ensure that each child attains $z = S$ years of school.[7] Then the only choice left is n, so he solves

$$\max_n (w_t - v_t(S)\delta nS)^{1-\beta}n^{\eta}e^{\beta\gamma}v_t(S)^{\beta}.$$

The first-order condition in this case is

$$v_t(S)\delta nS = \frac{\eta w_t}{1-\beta+\eta}.$$

The implied utility is

$$\left(\frac{1-\beta}{1-\beta+\eta}\right)^{1-\beta}\left(\frac{\eta}{1-\beta+\eta}\right)^{\eta}\left(\frac{1}{v_t(S)\delta S}\right)^{\eta}e^{\beta\gamma}v_t(S)^{\beta}w_t^{1-\beta+\eta}. \tag{19}$$

Everyone has both options, so the equality of equations (17) and (19) must hold:

$$w_{t+1} = \left(\frac{k}{\delta S}\right)^{\eta/\beta}e^{\gamma}v_t(S)^{1-\eta/\beta}. \tag{20}$$

Now w_t is just the marginal value of unskilled labor,

$$w_t = \alpha L^{1-\alpha}N_{ft}^{\alpha-1},$$

so equation (20) implies

$$\alpha L^{1-\alpha}N_{f,t+1}^{\alpha-1} = \left(\frac{k}{\delta S}\right)^{\eta/\beta}e^{\gamma}v_t(S)^{1-\eta/\beta},$$

which, using equation (18), reduces to

$$N_{ft} = Me^{-\gamma[(1-\eta/\beta)/(1-\alpha)]t},$$

where M is a constant. We assume that $\beta > \eta$ ("quality" > "quantity"), which ensures that $N_{ft} \to 0$.

In this model, as in the previous one, the population engaged in traditional agriculture shrinks toward zero while the urban sector continues to

[7] The only reason for this assumption is to keep the number of types down to two.

grow in wealth and, possibly, in numbers. In the first case, urban growth consists of unskilled workers providing services to the ever-wealthier bourgeoisie. In the second, migration to the city is motivated by the possibility of raising educated, high–human capital children who then become full members of the bourgeoisie. In fact, both these migration models—based on employment opportunities for the unskilled or on the possibility of educating children—can operate at the same time, even in the same family.

VI. Conclusion

This paper began with two kinds of evidence. The first is the strong negative correlation of an economy's real income levels with the fraction of its workforce that is engaged in agriculture. The second, less clear, is the demographic transition—the fact that migration out of traditional agriculture typically induces increases in population followed by reduced growth. I have interpreted the movement out of traditional agriculture as a transition toward literacy and education more generally and the onset of an urban class that generates sustained productivity growth. A version of Becker's quantity/quality trade-off then unites the two.

The role of schooling in my interpretation is quite different from that in more familiar growth models, where the contribution of schooling is measured by years in school and where on-the-job learning is viewed as an age-specific fixed effect.[8] In this paper, schooling prepares people to take advantage of the ideas of others, and the knowledge they gain throughout their careers depends in part on the quality of those with whom they interact. It is the quality of these people—parents, teachers, fellow students, supervisors, coworkers, people we meet at work or at parties, people we observe from a distance, see on television, read about in books—that determines the direction and quality of our lives.

In the model I have outlined here, there are educated, urban families who put their children in schools. These children, interacting with others, continue to learn on their jobs. These are the people who discover new ways of doing things and get the rewards of success. There are also illiterate families raising illiterate children who will earn their living as farm workers or as unskilled servants to the wealthy urbanites. These families, too, benefit from the productivity growth generated by others, and that enables them to have more children than their parents had. Some of these families choose to sacrifice some of their own consumption and the number of their children in order to give their children the quality of schooling that they themselves did not have. I have kept these people in just two types, but only for clarity. In any actual society, many in-between types are also represented.

[8] See, for one example out of many, Hall and Jones (1999). My position here is closer to that of Manuelli and Seshadri (2014).

In such an economy, market forces do not give the right signals. Parents choose fertility and schooling levels that maximize their own well-being but place no value on the benefits that accrue to others with whom their children will interact. This inefficiency could be corrected by government-financed universal education (though finding the right level is not easy). All of the wealthy economies have done this, if imperfectly. When all economies have done so, the Industrial Revolution will be complete.

References

Barro, Robert, and Gary S. Becker. 1989. "Fertility Choice in a Model of Economic Growth." *Econometrica* 57 (2): 481–501.
Becker, Gary S. 1960. "An Economic Analysis of Fertility." In *Demographic and Economic Change in Developed Countries: A Conference of the Universities–National Bureau Committee for Economic Research*, 209–40. New York: NBER.
———. 1981. *A Treatise on the Family*. Cambridge, MA: NBER.
Becker, Gary S., and Robert Barro. 1988. "A Reformulation of the Economic Theory of Fertility." *Q.J.E.* 103:1–25.
Becker, Gary S., and H. Gregg Lewis. 1973. "On the Interaction between the Quantity and Quality of Children." *J.P.E.* 81 (2): S279–88.
Becker, Gary S., Kevin M. Murphy, and Robert S. Tamura. 1990. "Human Capital, Fertility, and Economic Growth." *J.P.E.* 98 (5): S12–S37.
Becker, Gary S., and Nigel Tomes. 1976. "Child Endowments and Quantity and Quality of Children." *J.P.E.* 84 (4): S143–62.
Botticini, Maristella, and Zvi Eckstein. 2012. *The Chosen Few: How Education Shaped Jewish History, 70–1492*. Princeton, NJ: Princeton Univ. Press.
Caicedo, Santiago, Robert E. Lucas, Jr., and Esteban Rossi-Hansberg. 2016. "Learning, Career Paths, and the Distribution of Wages." Working Paper no. 22151 (April), NBER, Cambridge, MA.
Clark, Gregory, and Neil Cummins. 2016. "The Child Quality-Quantity Tradeoff, England, 1780–1880: A Fundamental Component of the Economic Theory of Growth is Missing." CEPR Discussion Paper no. 11232, Centre for Economic Policy Research, London.
Cole, W. A., and Phyllis Deane. 1966. "The Origins of Sustained Growth." In *Cambridge Economic History of Europe from the Decline of the Roman Empire*, vol. 6, *The Industrial Revolutions and After: Incomes, Population and Technological Change*, edited by H. J. Habakkuk and M. Postan, 1–9. Cambridge: Cambridge Univ. Press.
Ehrlich, Isaac, and Jinyoung Kim. 2015. "Immigration, Human Capital Formation, and Endogenous Economic Growth." *J. Human Capital* 9 (4): 518–63.
Ehrlich, Isaac, and Francis T. Lui. 1991. "Intergenerational Trade, Longevity, and Economic Growth," *J.P.E.* 99 (5): 1029–59.
Galor, Oded, and David N. Weil. 2000. "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond." *A.E.R.* 90 (4): 806–28.
Greif, Avner. 2006. *Institutions and the Path to the Modern Economy: Lessons from Medieval Trade*. Cambridge: Cambridge Univ. Press.
Hall, Robert E., and Charles I. Jones. 1999. "Why Do Some Countries Produce So Much More Output per Worker Than Others?" *Q.J.E.* 114:83–116.
Kuznets, Simon. 1971. *Economic Growth of Nations: Total Output and Production Structure*. Cambridge, MA: Harvard Univ. Press.
Landes, David. S. 1969. *The Unbound Prometheus: Technological Change and Industrial Development in Western Europe from 1750 to the Present*. Cambridge: Cambridge Univ. Press.

Lucas, Robert E., Jr. 2002. *Lectures on Economic Growth*. Cambridge, MA: Harvard Univ. Press.
Manuelli, Rodolfo E., and Ananth Seshadri. 2014. "Human Capital and the Wealth of Nations." *A.E.R.* 104 (9): 2736–62.
McCloskey, Deirdre N. 2016. *Bourgeois Equality: How Ideas, Not Capital or Institutions, Enriched the World*. Chicago: Univ. Chicago Press.
Mokyr, Joel. 1990. *The Lever of Riches: Technological Creativity and Economic Progress*. Oxford: Oxford Univ. Press.
———. 2016. *A Culture of Growth: The Origins of the Modern Economy*. Princeton, NJ: Princeton Univ. Press.
Schultz, Theodore W. 1964. *Transforming Traditional Agriculture*. New Haven, CT: Yale Univ. Press.
Solow, Robert. 1956. "A Contribution to the Theory of Economic Growth." *Q.J.E.* 70 (1): 65–94.
Wrigley, E. A. 2004. *Poverty, Progress, and Population*. Cambridge: Cambridge Univ. Press.