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Dynamics in the Interbank Market

By

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Abstract

This paper builds a model of interbank market with heterogeneous agents and search friction. Banks holding different amount of liquidity receive aggregate liquidity shock and trade liquidity in bilateral meeting. We prove the uniqueness of the stationary equilibrium and analyze the convergence path to the limit liquidity distribution. We find that with heterogeneous banks, the estimated trading volume is lower than that with a representative agent. Accessibility to the central bank standing facility will reduce the loss in efficiency. The dynamics can be obtained either with Krusell-Smith Algorithm or with linear approximation around the stationary distribution.

1 Introduction

The interbank market plays a vital role in the financial system. However, as is pointed out by Bianchi and Bigio (2022), the interbank market receives little attention in New-Keynesian models. Although New-Monetary theory has built theoretical framework to model the interbank market (Berentsen and Monnet, 2008), most of the researches are still limited. Especially dynamics in the short run is missing in the literature.

Most models only focus on the effect of policies in the steady state in the long run. (Berentsen and Monnet, 2008; Üslü, 2019; Farboodi et al., 2023) They build the micro-foundation of the interbank market to match the stationary stylized facts from data. And the very few papers which analyze the short run are based on property of aggregation (Bianchi and Bigio, 2022) or finite horizon (Afonso and Lagos, 2015). Actually, both the short run dynamics and heterogeneity matter, especially when analyzing the efficiency and financial risks.

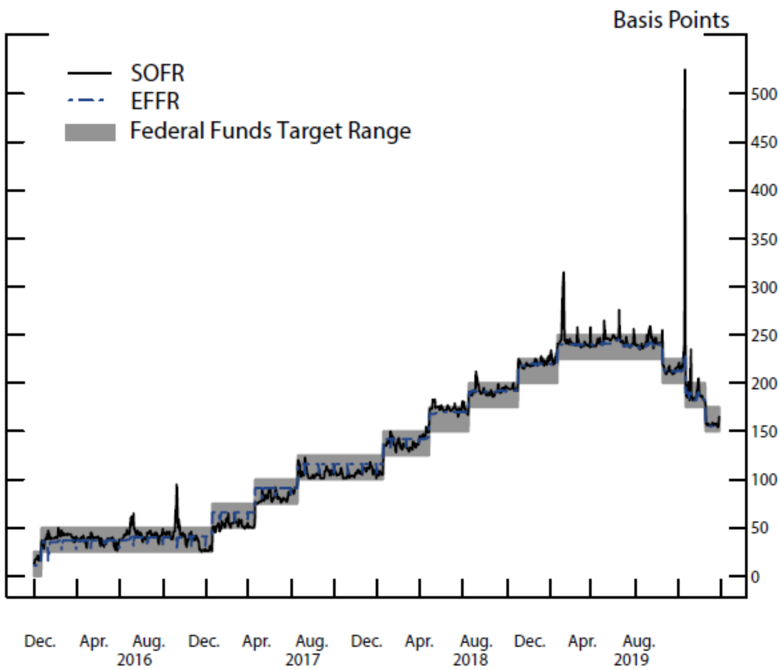

Figure 1: US EFFR Spike 2019

One of the possible reason for not investigating too much the interbank dynamics could be the interest rate corridor adopted by most central banks in the world. (Federal Reserve, European Central Bank, People's Bank of China...) After introducing the corridor, no arbitrage guarantees that the market rate is in the corridor. However, the volatility of the market could still push the interbank rate to the boundaries of the corridor. As is shown in Figure 1, the interest rate on the interbank market experienced a spike in 2019, far above the Federal Funds Target Range. This unexpected event further pointed out that sufficiently large shock to the financial system could lead to extreme response.

And the heterogeneity also matters. Two sources of heterogeneity exist in the interbank market, heterogeneity of liquidity holding and heterogeneity from search friction. When deciding the asset portfolio and best interbank terms of trade, banks consider not only their own states and aggregate risk, but the states of their potential trading partners as well. The price of liquidity is contingent on the liquidity demand of the buyer and the seller, and thus, aggregation doesn't necessarily hold in this scenario. After taking bankruptcy into consideration, extreme shortage of liquidity in one bank could lead to bank run of the whole system, even though the aggregate liquidity holding is above the safe line. And because the interbank market is not a competitive market cleared with one price, failure of finding a trading partner means being unable to smooth liquidity through time, which may enlarge the heterogeneity of liquidity holding.

In this paper, we build a general equilibrium model of a frictional interbank market with heterogeneity. In the model, banks hold different level of liquidity and smooth their liquidity holding through time by trading with bilateral search and matching. At the beginning of the day, banks receive the liquidity shock and seek to trade with another bank to maximize their utility from liquidity holding at the end of day. In our full model, banks also have access to the central bank standing facility to borrow liquidity at a higher penalty rate. To further illustrate the unique stationary distribution of asset holding and the convergence path, we simplify the model to a deterministic model without central bank lending.

From the model we find the following results. The distribution will converge to a unique stationary degenerate point. Based on the concavity of the bargaining solution, aggregation doesn't hold in this problem. The expected trading volume in the heterogeneous market is lower than that traded with a representative agent, which means the market is less efficient in pooling when liquidity holding differs from bank to bank.

Related Literature

This paper is related to 2 trends of literature. The first is the literature on interbank market. Berentsen and Monnet (2008) build a New Monetary model of a channel system with both an interbank market and a goods market.Banks confront random trading shocks and insure the shocks with both the money market and the central bank standing facility. To make their model tractable, they assume the utility function is quasi-linear in their consumption and thus their model has the memoryless property. Compared with their paper, we don't have the memoryless property and show the dynamics of asset holding distribution. Afonso and Lagos (2015) build a continuous time model of one single trading day to describe the behavior of banks throughout the trading session. They iterate over the distribution to get the dynamics path of the trading behaviors. However, in their model, banks only care about their payoff in one period and thus even splitting of liquidity between partners maximize the utility of banks. Hugonnier et al. (2014), Üslü (2019), and Farboodi et al. (2022) are all based on search-and-matching theory. They focus on the endogeneous intermediation of the interbank market with heterogeneous agents. Bianchi and Bigio (2022) integrate the Afonso and Lagos (2015) OTC market into a dynamic general equilibrium. They also show that their model features aggregation and is thus analytically tractable.

The second is on heterogeneous agent model and mean field game. Krusell and Smith (1998) provide an algorithm to approximate distribution of assets with the first moment. Given the functional form of the law of motion of the moments, the iteration on value function converges to the equilibrium. Miao (2006) proves the existence of the recursive solution used by Krusell and Smith (1998). Mean field game theory analyzes the problem in which agents decide their action based on the distribution of all agents which is the result of their actions. Lasry and Lions (2006) prove such problems have unique solutions. This paper study a standard heterogeneous agent problem: the terms of trade are determined based on the distribution of liquidity and the distribution of liquidity is determined by their choice. Our model can be solved either with K-S algorithm or with the MFG theory.

The rest of the paper is structured as follows: section 2 introduces the model environment. Section 3 defines the bargaining problem and the general equilibrium. Section 4 proves the unique stationary distribution of a simplified model and briefly covers the simulation algorithm. Section 5 concludes the paper.

2 Model

The model is defined in discrete time and infinite horizon. Each period is divided into 3 stages: first, there is an OTC market for interbank trading; second, agents have access to central bank lending facility and save their liquidity as excess reserve; third, there is a clearing market. The timeline of the model is shown in Figure 2.

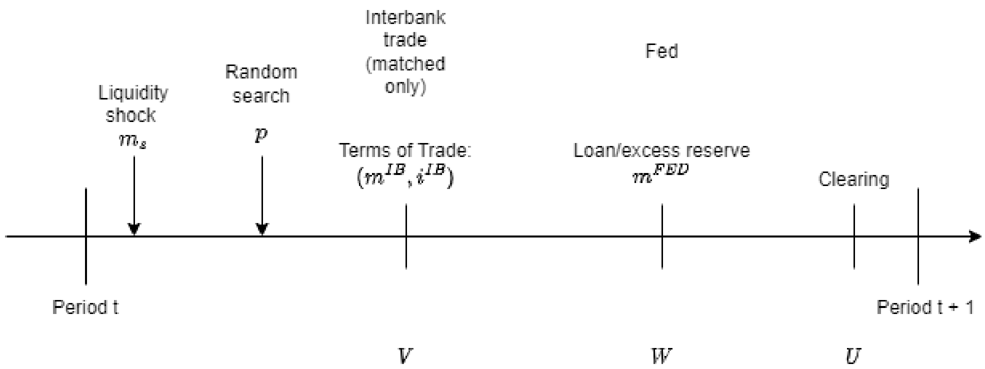

Figure 2: Timeline

Before a new period (we will call each period a day) begins, financial institutions (we will call them banks hereafter) have liquidity holding m and we use $D_m(t)$ to denote the distribution of m in period t. At the beginning of a new period, banks receive an aggregate liquidity shock m_s. m_s follows a Markov Process. The total liquidity holding is then $m + m_s$.

The initial liquidity holding is funded by equity, and at the end of each day the bank has to pay dividend ϕm back to the shareholders. The dividend can also be interpreted as the opportunity cost because in most times holding liquidity is costly. The exogeneous liquidity shock can be understood as liquidity from the deposit market and should be paid back at the end of the period. Different levels of m_s represent different state of the economy. At the end of the day, banks will save their liquidity holding as excess reserve and get overnight interest rate i_d. Banks will also get utility $u(d)$ from holding liquid asset where $d = m + m_s$ is the excess reserve. We assume $u(\cdot)$ is concave. These assumptions are used to model the incentive of banks to minimize the liquidity risk at the cost of holding idle liquidity.

The liquidity holding at the beginning of the next period m_+ satisfies:

$$m_+ = (1 + i_d)(m + m_s) - \phi m - m_s$$

Under the assumption of $\phi > i_d$, m_t converges to $\frac{i_d}{\phi - i_d} m_s$, which means the divergence in liquidity holding is decreasing throughout the time. Therefore with rational expectation of the future, banks maximize their utility by smoothing their liquidity holding.

To smooth their liquidity holding through time, banks trade liquidity with each other in the Over-The-Counter (OTC) market. Every bank will enter the OTC market, because banks with liquidity demand want to borrow and banks with sufficient liquidity can lend at an interest rate higher than the excess reserve rate. With probability p, one bank will be matched with a trading partner; with the rest probability $(1-p)$, the bank will be unmatched. Successfully matched partners will trade with each other. The bank in demand of liquidity will borrow m^{IB} from the bank with idle liquidity and pay $(1+i^{IB})m^{IB}$ back next period. The terms of trade (m^{IB}, i^{IB}) are determined by bilateral bargaining.

No matter being matched or not, banks will have access to trade with central bank standing facility. Banks can borrow from the central bank at an ex-ante posted interest rate i_{fed}. Not only unmatched banks borrow from the central bank, but matched banks are also able to adjust their liquidity holding if they failed to get sufficient money from their partners.

3 Equilibrium

3.1 Bellman Equations

Considering a bank at the end of a day with liquidity holding m, liability ℓ, and shock m_s, given the distribution of liquidity holding D_m and liability D_ℓ, the value function is defined as:

$$U(m, \ell, m_s; D_m, D_\ell) = u(m + m_s) + \beta E[V^{IB}(m', \ell' = 0, m_s'; D_m', D_\ell' = 0)|m_s] \tag{1}$$

s.t.

$$m' = (1 + i_d - \phi)m + i_d m_s - \ell \tag{2}$$

$$D_m' = H(m_s, D_m, D_\ell) \tag{3}$$

The liquidity holding of the next period is the liquidity this period plus interest from excess reserve deposit minus dividend and liability. H is the aggregation of the law of motion of individual liquidity holding. We assume $\phi > i_d$. V is the continuation value in the next period. Because all liability are repaid in the clearing market, the liability carried to the next period is 0 and the distribution is also 0.

At the beginning of the next period, we have

$$V(m_i, m_s; D_m) = p \int_{\underline{m}}^{\overline{m}} W(m_i + m_{ij}^{IB}, (1 + i_{ij}^{IB}) m_{ij}^{IB}, m_s) dF(m_j) + (1 - p) W(m, 0, m_s) \tag{4}$$

where $W(\cdot)$ is the continuation value in stage 2. $\{m_{ij}^{IB}, i_{ij}^{IB}\}$ are the terms of trade between agent i and agent j. $F(m_j)$ is the CDF of D_m. p is the probability of being matched. The first term is the value of being matched and the second is the value of skipping stage 1 and going straight forward to the central bank. After introducing the value function of stage 2, we will go back to the determination of the terms of trade through bargaining.

The Bellman function of stage 2 is:

$$W(m, \ell, m_s) = \max_{m^{fed}} U(m + m^{fed}, \ell + (1 + i_{fed}) m^{fed}, m_s) \tag{5}$$

s.t.

$$m^{fed} \geq 0$$

3.2 Bargaining

Two banks determine the terms of trade through bilateral Nash bargaining for $m_i \leq m_j$:

$$\{m_{ij}^{IB*}, i_{ij}^{IB*}\} = \arg \max_{m^{IB}, i^{IB}} S_i S_j$$

s.t.

$$S_i(m_i, m_j, m^{IB}, i^{IB}) = W(m_i + m_{ij}^{IB}, (1 + i_{ij}^{IB}) m_{ij}^{IB}, m_s) - W(m_i, 0, m_s) \geq 0$$

$$S_j(m_i, m_j, m^{IB}, i^{IB}) = W(m_j - m_{ij}^{IB}, -(1 + i_{ij}^{IB}) m_{ij}^{IB}, m_s) - W(m_j, 0, m_s) \geq 0$$

S_i and S_j are surpluses from trading. Notice that $S \geq 0$ has to hold. This condition makes sure that the interest rate $i^{IB} \in (i_d, i_{fed})$, in other words, the market rate is bounded in the interest rate corridor.

The first order conditions are:

$$\frac{W_1(m_i + m_{ij}^{IB}, (1 + i_{ij}^{IB})m_{ij}^{IB}, m_s)}{S_i} = \frac{W_1(m_j - m_{ij}^{IB}, -(1 + i_{ij}^{IB})m_{ij}^{IB}, m_s)}{S_j} \quad (6)$$

$$\frac{W_2(m_i + m_{ij}^{IB}, (1 + i_{ij}^{IB})m_{ij}^{IB}, m_s)}{S_i} = \frac{W_2(m_j - m_{ij}^{IB}, -(1 + i_{ij}^{IB})m_{ij}^{IB}, m_s)}{S_j} \quad (7)$$

Here we use W_1 and W_2 to notate the partial derivative with regard to the first and the second arguments of W.

The optimal borrowing from the standing facility is determined by:

$$U_1(m+m^{fed}, \ell+(1+i_{fed})m^{fed}, m_s)+(1+i_{fed})U_2(m+m^{fed}, \ell+(1+i_{fed})m^{fed}, m_s) = 0 \quad (8)$$

3.3 General Equilibrium

Notice that all the information in the distribution of liability D_ℓ can be fully covered by D_m so we have $D_\ell = G(D_m)$. And the law of motion H can be written as an operator $TD_m = H(m_s, D_m, G(D_m))$.

Now we can define the general equilibrium:

Definition 3.1 *The general equilibrium is an operator T, distributions D_m, D_ℓ, value functions U, V, W, and policy functions m^{IB}, i^{IB}, m^{fed}, s.t.*

1. *Value functions U, V, W and policy functions m^{IB}, i^{IB}, m^{fed} solve the banks' problem given T, D_m, D_ℓ.*

2. *T is the aggregation of law of motion of individual m.*

3. *The initial distribution $D_m(0)$ is given.*

This dynamic system is a heterogenous-agent problem and thus the existence and the uniqueness of the equilibrium is non-trivial. In the next section, we will introduce a simplified deterministic version of the model in which the central bank lending facility is shut down and liquidity shock is constant. In the simplified model, we will prove the existence and uniqueness of a stationary distribution. We will also briefly talk about the algorithm to simulate the dynamics of the system.

4 Stationary Distribution and Dynamics

Because the number of agents is large, the effect of the behavior of an individual agent can be neglected. We assume agents take $D_m(t)$ as given.

After shutting down the second stage, the system can be simplified to a Bellman equation and an equation of law of motion.

$$\begin{aligned} V(m, m_s) = (1-p)\{&u(m+m_s) + \beta E[V((1+i_d-\phi)m + i_d m_s, m_s')|m_s]\} \\ &+ p \int f(m_j)\Big\{u(m+m_s+m^{IB}) \\ &+ \beta E[V((1+i_d-\phi)m + i_d m_s - (i^{IB}-i_d)m^{IB}, m_s')|m_s]\Big\} dm_j \end{aligned} \tag{9}$$

$$m_i' \begin{cases} = (1+i_d-\phi)m_i + i_d m_s & \text{if unmatched} \\ \sim (1+i_d-\phi)m_i + i_d m_s - dist[(i_{ij}^{IB} - i_d) z_{ij}^{IB}] & \text{if matched} \end{cases} \tag{10}$$

To precisely define the law of motion, here we introduce the strict system of notation. Suppose $(M, \mathcal{M})$ is a measurable space of liquidity holding and λ be a probability measure on $(M, \mathcal{M})$. λ_{D_m} is the probability measure based on distribution D_m. $Q : M * \mathcal{M} \to [0,1]$ is the transition matrix corresponding to (10).

Then the law of motion is: for $\forall A \in \mathcal{M}$,

$$\lambda_{D_m'}(A) = (T\lambda)(A) = \int Q_{D_m}(m, A)\lambda_{D_m}(dm) \tag{11}$$

The initial distribution is $D_m(0)$.

We first focus on the stationary distribution. It's equivalent to finding the fixed point of operator T.

Theorem 4.1 *(11) converges to a unique fixed point if $m^{IB}(m_i, m_j)$ is concave and $m^{IB} \in [0, |m_j - m_i|]$.*

Proof

(D, d) is a complete metric space. For $\forall D_m,\ D_m \in D$ and $d(D, D') := \int |f - f'|dm$ where f and f' are PDFs of D_m and D'_m.

Define T^*f as the PDF of TD_m. T_mD_m is the distribution of liquidity holding if all agents are matched and T^*_mf is the corresponding PDF. T_uD_m is the distribution of liquidity holding if all agents are unmatched and T^*_uf is the corresponding PDF. We will guess and verify that $T^{*n}f \to \overline{D}_m$ as $n \to \infty$ where $\overline{D}_m$ is the distribution in which everyone holds $\overline{m} = \frac{i_d}{\phi - i_d}m_s$.

It's equivalent to prove that

$$d(TD_m, \overline{D}_m) \leq \gamma d(D_m, \overline{D}_m) \text{ for } \forall D_m \in D$$

where $\gamma \in [0, 1)$.

We have

$$\begin{aligned} d(TD_m, \overline{D}_m) &= \int |T^*f(m) - \overline{f}(m)|dm \\ &= \int |p(T^*_mf - \overline{f}) + (1-p)(T^*_uf - \overline{f})|dm \\ &\leq \int p|T^*_mf - \overline{f}| + (1-p)|T^*_uf - \overline{f}|dm \end{aligned}$$

and

$$d(D_m, \overline{D}_m) = \int |f - \overline{f}|dm = \int p|f - \overline{f}| + (1-p)|f - \overline{f}|dm$$

For the rest of the proof, we will show that

$$\int p|T^*_mf - \overline{f}|dm \leq \gamma \int p|f - \overline{f}|dm$$

and

$$\int (1-p)|T^*_uf - \overline{f}|dm \leq \gamma \int (1-p)|f - \overline{f}|dm$$

To prove the two inequalities above, we first prove a lemma:

Lemma 4.2
$$\int |T^{*n} f - \overline{f}| dm \to 0$$
is true if $|E(\mathcal{T}m) - \overline{m}| \le \gamma |E(m) - \overline{m}|$ *is true for* $\forall m$.

Proof

$$\begin{aligned}
\int |T^* f - \overline{f}| dm &= \int_{m \neq \overline{m}} |T^* f - \overline{f}| dm + \int_{m = \overline{m}} |T^* f - \overline{f}| dm \\
&= \int_{m \neq \overline{m}} T^* f dm + \int_{m = \overline{m}} \overline{f} - T^* f dm \\
&= \int T^* f dm + \int_{m = \overline{m}} \overline{f} dm - 2 \int_{m = \overline{m}} T^* f dm \\
&= 2 - 2 \int_{m = \overline{m}} T^* f dm
\end{aligned}$$

From Markov's Inequality, we have
$$\int_{m \in (\overline{m} - \epsilon, \overline{m} + \epsilon)} T^* f dm \ge 1 - \frac{E(\mathcal{T}m - \overline{m})}{\epsilon^2}$$

So
$$\int |T^* f - \overline{f}| dm \le 2 \frac{|E(\mathcal{T}m - \overline{m})|}{\epsilon^2}$$

$$\begin{aligned}
\int |T^{*n} f - \overline{f}| dm &\le 2 \frac{|E(\mathcal{T}^n (m - \overline{m}))|}{\epsilon^2} \\
&= 2 \frac{|E(\mathcal{T}((\mathcal{T}^{(n-1)} m) - \overline{m}))|}{\epsilon^2} \\
&\le 2\gamma \frac{|E(\mathcal{T}^{(n-1)} m) - \overline{m}))|}{\epsilon^2} \\
&\le 2\gamma^n |E(m) - \overline{m}|
\end{aligned}$$

Then we come back to the proof of Theorem 4.1. $E(\mathcal{T}_u m) = \mathcal{T}_u m = (1 + i_d - \phi) m + i_d m_s$. So we have $|E(\mathcal{T}_u m) - \overline{m}| = |(i_d - \phi) E(m) + i_d m_s| = |(\phi - i_d)(\overline{m} - E(m))| < |\overline{m} - E(m)|$. So the second inequality holds.

As for inequality 1, Notice that

$$\begin{aligned}|E(\mathcal{T}_m m) - \overline{m}| &= |E(E(\mathcal{T}_m m|m)) - \overline{m}| \\ &\leq \gamma |E(m) - \overline{m}|\end{aligned}$$

holds if

$$|E(\mathcal{T}_m m|m) - \overline{m}| \leq \gamma |m - \overline{m}|$$

is true for $\forall m$.

We have

$$\begin{aligned}|E(\mathcal{T}_m m|m) - \overline{m}| &= |E[(1 + i_d - \phi)m + i_d m_s - (i^{IB} - i_d)m^{IB}|m] - \overline{m}| \\ &= |(1 + i_d - \phi)m + i_d m_s - E((i^{IB} - i_d)m^{IB}|m) - \overline{m}|\end{aligned}$$

$|E(\mathcal{T}_m m|m) - \overline{m}| \leq \gamma |m - \overline{m}|$ is equivalent to $|E(m^{IB}|m)| < \frac{\phi - i_d}{max\{i^{IB} - i_d\}} |m - \overline{m}|$

We can write $m^{IB}|m$ as $G(m_j)$ where m_j is the liquidity holding of the trading opponent. From the assumption that $G(m_j)$ is concave, from Jensen's Inequality we have:

$$E(G(m_j)) \leq G(E(m_j))$$

$$\lim_{n\to\infty} E(\mathcal{T}_m^n m_j) = \lim_{n\to\infty} E(\mathcal{T}_u^n m_j) = \overline{m}$$

Inserting $\tilde{m} = \mathcal{T}_m^n m$,

$$\begin{aligned}|E(\tilde{m}^{IB}|\tilde{m})| &= |E(G(\tilde{m}_j)|\tilde{m})| \\ &\leq |G(E(\tilde{m}_j))| = |G(\overline{m})| \\ &\leq |\tilde{m} - \overline{m}| \\ &< \frac{\phi - i_d}{max\{i^{IB} - i_d\}} |\tilde{m} - \overline{m}| \text{ if } \phi \text{ is sufficiently large}\end{aligned}$$

Thus the condition in Lemma 4.2 holds for both cases.

Q.E.D.

From the proof, we notice that the distribution of liquidity holding will converge to a degenerate distribution where everyone holds $\overline{m} = \frac{i_d}{\phi - i_d} m_s$.

To shed more light on the converging path to the stationary distribution, we can either approximate the dynamic system linearly around the stationary point or solve the system globally with Krusell-Smith algorithm. In the Krusell-Smith approximation, taking the first 2 moments could be sufficient and the law of motion can take the following functional form:

$$E(m_+) = \alpha E(m) + (1 - \alpha)\overline{m}$$

$$var(m_+) = \beta var(m)$$

where $\alpha, \beta \in (0, 1)$ are parameters to be estimated. It's obvious that a distribution following such a law of motion will converge to the degenerate distribution described above.

The condition for the unique stationary distribution is the concavity of the trading quantity. Compared with a model without heterogeneity from search friction, the expected volume of liquidity traded on the interbank market is smaller, which leads to tightening of the interbank market. Except for the aggregate shock, if we consider an idiosyncratic shock to the initial distribution of liquidity holding, the reversion to the stationary distribution will be slowed down by the uncertainty from heterogeneous potential trading partners. Availability of central bank standing facility provides a backup choice for failing to get sufficient level of liquidity, which alleviate the tightening of the interbank market by adjusting the expectation of the future.

5 Conclusion and Future Work

This paper has developed a framework to study dynamics in the interbank market and is one of the first papers to describe the dynamics under a search-and-matching macro finance model. Heterogeneity of agents and search frictions are incorporated in the framework which is different from the new-monetary literature (Berentsen and Monnet, 2008). Not only do we prove the existence and uniqueness of the stationary distribution, but we also provide comments on the numerical simulation of the dynamics both with perturbation method and with projection method.

After 2008, stricter monitoring on systematically important financial institutions marked an important step to not recognize the financial system as a whole but focus on the structure of the system. The model from this paper provides another theory that the heterogeneity itself in the interbank market deserves notice. A system with more dispersed liquidity holding is less efficient in interbank trading and is under higher financial risk.

Even though the FED has introduced the corridor system, the uncertainty from interbank trading is greatly reduced, the interbank rate spike in 2019 still implies the potential failure of the system.

Currently, this paper is purely theoretical and the next step is realizing the numerical simulation and implementing related counter-factual impulse response to financial shocks.

The model is closely related to the Mean Field Game theory. In the future, we will also rewrite our model as a continuous time control problem with idiosyncratic shock and define the mean field game in this circumstance.

Our model is also open to multiple extensions, for instance, adding bank run, describing the network structure of the interbank market, further endogenize of the liquidity supply...

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