



## Original Article

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# A Circuit Core Economy

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**Abstract:** To decide how to buy or sell a private good is a task for an individual in the standard economics model. It would be better to amend even this simple case because the terms of a trade are seldom left entirely in the hands of the individual traders involved. Haggling about the terms for a single trade is uncommon in most economies today. Yet it is the staple of beginning economics courses. It portrays an individual free to buy a bundle of commodities, each available at a given price and in any amount at the pleasure of the buyer. A student is told consumers choose what to buy in order to maximize their utility subject to a budget constraint. Any objection to this picture raised by a student is typically brushed aside by saying it is a model not a realistic picture of consumer behavior. Evidence to test the model rarely comes up.

**Keywords:** circuit core economy, private goods, game theory, semi-private goods

## 1 Private Goods

To decide how to buy or sell a private good is a task for an individual in the standard economics model. It would be better to amend even this simple case because the terms of a trade are seldom left entirely in the hands of the individual traders involved. Haggling about the terms for a single trade is uncommon in most economies today. Yet it is the staple of beginning economics courses. It portrays an individual free to buy a bundle of commodities, each available at a given price and in any amount at the pleasure of the buyer. A student is told consumers choose what

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Professor Lester Telser passed away on September 3, 2022 at the age of 91. A fiercely independent scholar, Telser was a leading member of the old Chicago School of Economics and was recognized for his original contribution to what has become known as core theory. Professor Telser possessed an exceptionally warm and generous heart to generations of students at Chicago in the classroom and beyond. Professor Telser will be missed by those whose lives have been enriched by his ideas and personality – The Editor.

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to buy in order to maximize their utility subject to a budget constraint. Any objection to this picture raised by a student is typically brushed aside by saying it is a model not a realistic picture of consumer behavior. Evidence to test the model rarely comes up.

The typical description of a seller of private goods is often as remote from reality. It starts with a model of perfect competition. This model describes a situation in which a producer can sell any amount without any effect on the going market price of the product. It also assumes a producer can buy any quantities of inputs including labor without any effect on their prices. Firms get no profit when competition is perfect. Cost includes a given rate of return on capital inputs.

Departure from perfect competition is relegated to oligopoly where a few sellers recognize their results depends on actions of other sellers. At the extreme is monopoly with a single seller of a product in the market. The student learns that the profit maximizing output is where marginal revenue equals marginal cost. It follows that price is above marginal revenue. Under these conditions since more could be sold at a lower price, buyers would be better off and the seller would still cover cost plus a normal return below the monopoly profit.

While the term private good does capture some of the essential traits, it leaves out a host of others. A good is socially defined object even when it is private. What people make, sell and buy depends on their social environment. Articles of clothing illustrate this point. Even putting aside the forces of fashion, no one today wears the same types of clothing popular in the last one or two generations. Few wear clothing different than their close associates. One can predict what others wear from a very small sample.

Individuals belong to different social circles. These have common customs, traditions, and beliefs that affect an economy. Popular entertainment, sports, religion, politics, news of the day are aspects of the economic environment. The subject of economics as a science is the reality of the actual economy in society.

A thought experiment helps. Consider a simple consumer product, the banana, eventually a private good. The process begins with growing bananas in areas with suitable climate and land. Managers, agronomists, farm workers, wholesalers enter at the first stage. Next, shipping companies collect bananas from the different countries where they are grown and transport the bananas to the many other countries where they are consumed. By the time bananas reach retailers a formidable apparatus involving many people, specialists and capital has been involved. The demand side of the banana industry raises another set of challenging economic problems. What does economic science contribute toward understanding this?

An underlying theme of my approach replaces an individual with groups of individuals. To this end consider a commuter train system. A large city is surrounded by suburbs where some of its residents live in less densely populated areas. These

residents have jobs in the city. They commute daily from the suburb to the city and return. Commuter trains start in the morning from the suburbs, pick up passengers along its route, travels to the inner city, the passengers leave for their work. In the late afternoon the same process goes in reverse from the inner city to the outer suburbs. Because this happens daily, regular passengers typically buy train tickets for a month at a time. Some commuter trains are idle during the day while some make regular daily trips to and from the suburbs to the inner city. Commuter trains are durable and require maintenance. They are replaced when a new train is cheaper than running an old one. A round trip on a commuter train illustrates a simple circuit.

A more complicated case is an electric grid. The grid is a network of transmission lines from generators to customers. There are many alternative routes between generators and their customers. Each path has a limited capacity and can be used by different generators. This poses difficult problems to find the best route from source to destination under continuously changing conditions. Also, a new fact enters. It is very costly to stop and start electric generators. Because the demand for electricity varies during the day and over the days of a week, it is cheaper for an electricity producer to keep spinning generators on reserve to satisfy peaks of demand. The terms of transactions for electric power recognize these conditions that make an electric grid more complicated than a grocery.

In a monetary economy payments and receipts are in monetary units. Transactions have two components, things and money. The amount of money paid by buyers equals the amount of money received by sellers. It is an identity. This identity does not make a monetary economy into a zero-sum game. The parties to a voluntary transaction are better off. This happens although the monetary payments by buyers reduce their monetary assets and the monetary receipts of the sellers increase their monetary assets. Eventually buyers must also be sellers and sellers must also be buyers. Otherwise, buyers could not obtain the means to make their purchases and sellers would accumulate monetary assets indefinitely. Buyers and sellers are in an intricate network of arrangements amenable to analysis by means of circuits.

A monetary economy where budgets balance still creates monetary value. An economy has many circuits. Each circuit has connections to other circuits. Circuits can export to other circuits and import from other circuits. Exports must pay for imports. A network of connected circuits has the same problem as a single circuit. The monetary value of the commodities to the buyers may exceed the monetary value that they pay the sellers even if the monetary receipts of the sellers equals the cost of making commodities. Invoking the assumption that individuals derive benefits measured in money from their consumption of goods and services solves the problem provided the receipts of the producers suffice to cover just the cost of their inputs. An economy is not a zero-sum game.

## 2 Game Theory in Economic Theory

### 2.1 Genesis

A game is a recreational contest for amateurs as winners and losers. It is an economic activity only for professional players. No one would have thought of games as a model for economics before 1928 when John von Neumann published a mathematical article proving the Minimax Theorem for a 2-person zero-sum game. Three years later he gave a lecture in German at Princeton University that applied his Minimax Theorem to the growth of an economy. No firm operates at a profit in his economy. Active firms break even. His growth theorem asserts that the maximum rate of growth of the economy equals the minimum real rate of interest. This lecture is the first hint of marriage between game theory and economics. The marriage was consummated in 1944 when the *Theory of Games and Economic Behavior* was presented to the public by its proud parents, von Neumann and Oskar Morgenstern. This is the first appearance in English of game theory and economics. The second appearance in English in 1945 was the translation of V. Neumann's theory of economic growth in the *Review of Economic Studies*. Champernowne (1945) explains this theory for economists because few could follow the mathematics of von Neumann.

Reconciling the public welfare with the private profit motive was an early challenge for scholars in economics. *The Wealth of Nations* published by Adam Smith in 1776 is the first and most famous attempt to reconcile the two.

Smith proposed the existence of an Invisible Hand that leads a profit seeking business man to advance the public interest though it was no part of his intention. The baker, the butcher, the candle maker seeks profit yet this advances the social interest. The mechanism behind this is competition among the sellers that drives prices down to cover their costs. This brings about harmony between the private and the social interest. Economists labored for more than two centuries to put flesh on this proposition. Assuming atomistic participants, constant returns to scale, and rational consumers, economists brought forth a general equilibrium for an environment with internal and external order financed by taxes imposed by government.

Collective behavior by firms is criticized because it prevents competition. Collective behavior by consumers is ignored. Yet important work by some economists describe exceptions. Alfred Marshall explains that some goods could be produced more efficiently by a natural monopoly. Thorstein Veblen in his *Theory of the Leisure Class* put conspicuous consumption into the economic tool box.

Condorcet discovered the paradox of majority voting that casts doubt on the standard assumption in economics that preferences are transitive. You may prefer  $A$  over  $B$  and  $B$  over  $C$  but not necessarily  $A$  over  $C$ . The three alternatives are in a contest like rival baseball teams in the American League. The New York Yankees beat the Boston Red Sox, the Boston Red Sox beat the Chicago White Sox but the Chicago White Sox beat the Yankees. No transitivity is present in sporting contests. What people consume depends on what other people consume. What firms produce affects the costs of other firms even those with whom they have no direct contact. These factors must enter economic theory that intends to improve understanding how an actual economy functions.

Since 1945 after the most destructive war in human history and the Great Depression, the world has seen unprecedented economic growth and prosperity. How to explain and understand the reasons for this is the major challenge facing the science of economics. Let us begin with a summary of the past that led to the new economy in the present.

Two major advances in science began early in the 20th century, quantum physics, originated by Max Planck and relativity theory, the work of Albert Einstein. Enrico Fermi's famous experiment demonstrating self sustaining nuclear reaction not only led to the atom bomb but also to atomic energy as a source of electric power. Ongoing research continues to reveal more about its potential. The relation between mass and energy the famous formula  $E = MC^2$ , announced by Einstein in 1905 remains a foundation in the new physics. Of equal importance was the discovery of the secret of biological life in DNA by Watson and Crick in 1953 has revolutionised biology and medicine. We live in a world in which the cost of communication has shrunk to zero thanks to Steve Jobs and the I phone. A powerful computer is available at low cost to a vast market. The power of computers has gone past the predictions of the most imaginative science fiction. Now within seconds one can find on the internet an answer to nearly any question. The internet is the creation of Google, the firm founded by Larry Page and Sergey Brin in 1998. The software company Oracle founded by Larry Ellison and two others in 1977 is a major firm pointing the way to a new industry apparently overlooked by the computer pioneer, IBM. The computer age is a major force in retailing. It began with books and phonograph records sold by Jeff Bezos on the internet by the company he founded about 30 years ago, Amazon. It now is an assembly of a wide variety of retailers selling almost anything you can think of.

Vannevar Bush wrote an article in 1945 "As We May Think" that predicted everything in the future computer age with incredible accuracy. Bush was a prophet and a leader who chose most of the scientific research projects during World War II.

### 3 The Challenge to Economic Science

The goal of an economy is to advance the economic well being of its inhabitants. The purpose of economic science is to study how well the actual economy advances toward this goal. The present economy is very different than it was 80 years ago. An economic model must take this into account. Turning our attention to public well being in a material sense leads to measures of total output, gross national product, national income and so forth. Actors in the economy are both producers and consumers. A better procedure makes more output from the same amount of inputs than an inferior one.

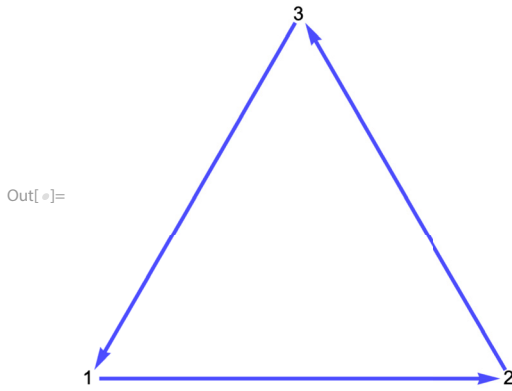
The simplest economy has one individual alone on an island, Robinson Crusoe. His primary task is survival. His means are primitive. The next economy has 2 individuals upon the arrival of Friday. Cooperation is necessary for their survival. Voluntary trade begins, barter. Even bargaining becomes feasible. Offers are made and rejected. We leave the island to study the next economy with three individuals.

An economy with three individuals can include barter and something else more important, a medium of exchange, money. Two individuals can bargain over the terms of trade between goods and money. You may not have what I want, but I can use the money you pay me to buy something from someone who has what I want and will accept money for it. Thus begins a market where goods and money are exchanged.

Markets facilitate the real growth of an economy. A triangle can represent the simplest market. The vertexes represent individuals, the arrows joining the vertexes represent their relations. Since a purchase is a sale, a sale from  $A$  to  $B$  is the same as a purchase by  $B$  from  $A$ . In this simple 3-person economy a round trip can begin from any vertex. This is shown by the arrow  $a[i, j]$  going from vertex  $i$ , the source of the arrow, to vertex  $j$ , the destination of the arrow. A round trip starting from vertex 1 is written  $\{a[1, 2], a[2, 3], a[3, 1]\}$ . The destination of each arrow is the source of the next arrow. Each vertex appears twice in a circuit, once as source and once as a destination. Each arrow appears only once in the circuit. These three arrows form a partition a major element in my model of an economy (Graph 1).

The mathematical framework for my model of an economy is a polygon with  $m$  vertexes joined by selected one-way arrows. The triangle is the simplest case. A vertex represents an individual. A relation between two individuals is represented by a one-way arrow linking the two vertexes that represent them. The basic economic unit is not an individual, it is a one-way arrow that represents a relation between a pair of individuals. In an  $m$ -polygon in my model there are  $m(m - 1)/2$  one way arrows. Only an odd  $m$ -polygon has a partition of these arrows. A partition is a collection of these arrows arranged into non overlapping circuits. Each arrow appears only once in one of these circuits. However, there is another important feature

In[ #]:= makeGrph [A, 2]



**Graph 1:** Three-person circuit.

of the model. Each vertex has  $m - 1$  arrows, half outgoing and half incoming. This means the individual is represented by a vertex having  $(m - 1)/2$  arrows that are a source for that many circuits and an equal number,  $(m - 1)/2$  that are a destination for that many circuits. Thus if vertex  $i$  is a carpenter, then the carpenter belongs to  $(m - 1)/2$  circuits that use a carpenter to make something and to  $(m - 1)/2$  circuits that supply something to the carpenter. A carpenter who sits alone as a hermit does not belong to any economy in my model. A reader should keep in mind there is a reality underlying the abstract level of discourse involving polygons, arrows, circuits and partitions.

The industry producing bananas is a sub economy composed of many different commercial and non commercial enterprises. An economic model of all the organizations in the banana industry must reckon with the complex relations in this industry. An economy is a collection of sub economies performing a variety of economic activities organized in many ways. Each sub economy has many circuits of many sizes connected to other circuits of many sizes, not necessarily all in the same sub economy. A carpenter in a circuit sells his services to an enterprise in the banana industry. He could also belong to a circuit in an industry making furniture.

Limited resources begets competition among producers and consumers. This conflict is one aspect of competition but not its sole aspect. The rivals for the limited resources are circuits not individuals. Within circuits there is cooperation. My model incorporates both aspects owing to the existence of limited resources in the actual economy. Indeed, circuits compete for members to become bigger circuits under suitable conditions.

Pairs of producers join these circuits. A pair may join another circuit into which it can fit. This depends on the circuit structure. Some pairs can fit into a circuit, some

cannot. The problem of obtaining the best outcome for a sub economy requires organizing circuits into a suitable structure using the variety of the feasible alternatives. The members of a circuit in a partition are individuals. The variety of partitions for a sub economy are its feasible alternatives. It may be that only one partition is best or that the best partition is not unique. A partition is an arrangement of all the individuals in the sub economy into a set of non overlapping circuits leaving no one out. A circuit with  $m$  members represented by  $m$  vertexes,  $v[i]$ , vertex  $i$ ,  $i = 1, 2, \dots, m$  is written

$$\{v[1] \rightarrow v[2]\}, \{v[2] \rightarrow v[3]\}, \{v[3] \rightarrow v[4]\}, \dots, \{v[m] \rightarrow v[1]\}.$$

The next case is a square, A square has no partition. This means it has structural unemployment (Graph 2).

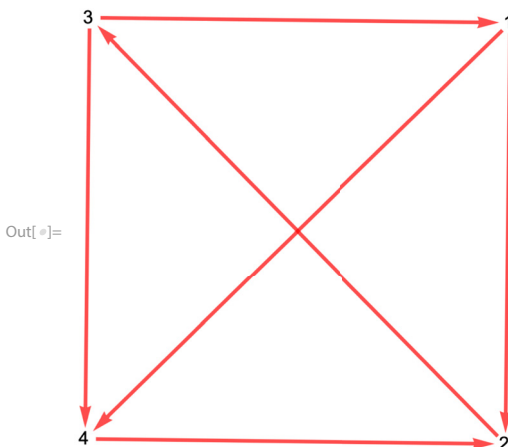
The 6 arrows in the square are

$$\begin{array}{ccc} \{1, 2\} & \{1, 4\} & \{3, 1\} \\ \{2, 3\} & - & \{4, 2\} \\ \{3, 4\} & - & - \end{array}$$

The square has three circuits using these 6 arrows. Circuit 1 and circuit 2 have three pairs. Circuit 3 has 4 pairs and leaves out  $a[1, 2]$  and  $a[3, 4]$ . A square has no partition. The two pairs,  $a[1, 2]$  and  $a[3, 4]$  are left out because they cannot fit into the two 3-circuits. They are structurally unemployed.

$$\begin{array}{l} \text{circuit1 } \{\{v[1] \rightarrow v[2]\}, \{v[2] \rightarrow v[3]\}, \{v[3] \rightarrow v[1]\}\}; \\ \text{circuit2 } \{\{v[2] \rightarrow v[3]\}, \{v[3] \rightarrow v[4]\}, \{v[4] \rightarrow v[2]\}\}; \\ \text{circuit3 } \{\{v[1] \rightarrow v[4]\}, \{v[4] \rightarrow v[2]\}, \{v[2] \rightarrow v[3]\}, \\ \{v[3] \rightarrow v[1]\}\}; \end{array}$$

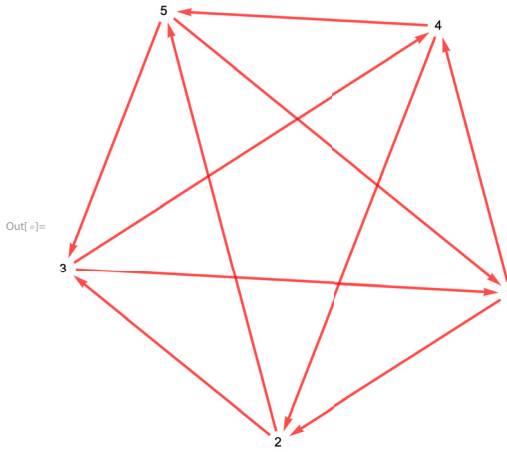
In[ ]:= `makeA [4]; makeGrph [A, 3]`



**Graph 2:** Four-person circuit.



In[10]:= makeA [5]; makeGrph [A, 3]



**Graph 3:** Five-person circuit.

Only polygons with an odd number of vertexes can have partitions. Polygons with an even number of vertexes cannot have partitions.

However, even  $m$ -polygons can have quasi-partitions that leave out selected pairs (Graph 3).

## 4 Pentagon

The pentagon has 10 pairs as follows. This concise notation shows only the vertex indexes and omits the arrows. Vertex indexes are Mod 5.

- $a[1, 2]$     $a[1, 4]$
- $a[2, 3]$     $a[2, 5]$
- $a[3, 4]$     $a[3, 1]$
- $a[4, 5]$     $a[4, 2]$
- $a[5, 1]$     $a[5, 3]$

The pentagon five 3-circuits, five 4-circuits and two 5-circuits. With concise notation that shows the vertexes without the arrows, the circuits are

3 – circuits	4 – circuits	5 – circuits
1, 2, 3	1, 4, 2, 3	
2, 3, 4	2, 5, 3, 4	1, 4, 2, 5, 3
3, 4, 5	3, 1, 4, 5	1, 2, 3, 4, 5
4, 5, 1	4, 2, 5, 1	
5, 1, 2	5, 3, 1, 2	

The next table for the 3-circuits in the pentagon shows the arrows.

$$\begin{aligned} & \{\{v[1] \rightarrow v[2]\}, \{v[2] \rightarrow v[3]\}, \{v[3] \rightarrow v[1]\}\}; \\ & \{\{v[2] \rightarrow v[3]\}, \{v[3] \rightarrow v[4]\}, \{v[4] \rightarrow v[2]\}\}; \\ & \{\{v[3] \rightarrow v[4]\}, \{v[4] \rightarrow v[5]\}, \{v[5] \rightarrow v[3]\}\}; \\ & \{\{v[4] \rightarrow v[5]\}, \{v[5] \rightarrow v[1]\}, \{v[1] \rightarrow v[4]\}\}; \\ & \{\{v[5] \rightarrow v[1]\}, \{v[1] \rightarrow v[2]\}, \{v[2] \rightarrow v[5]\}\}; \end{aligned}$$

Next are the three partitions for the pentagon. The circuits in a partition do not overlap.

PartitionI

$$\begin{array}{cccc} (1, 2) & (2, 3) & (3, 1) & - \\ (3, 4) & (4, 5) & (5, 3) & - \\ (5, 1) & (1, 4) & (4, 2) & (2, 5) \end{array}$$

PartitionII

$$\begin{array}{cccc} (2, 3) & (3, 4) & (4, 2) & - \\ (4, 5) & (5, 1) & (1, 4) & - \\ (1, 2) & (2, 5) & (5, 3) & (3, 1) \end{array}$$

PartitionIII

$$\begin{array}{ccccc} (1, 4) & (4, 2) & (2, 5) & (5, 3) & (5, 3) \\ (1, 2) & (2, 3) & (3, 4) & (4, 5) & (5, 1) \end{array}$$

The circuit core model of a sub economy faces the problem of describing which feasible partition will prevail. It should be the one that yields the most value. Let  $w[i, j]$  denote the payoff to the owner of the pair of arrows  $a[i, j]$  This raises the problem of how to measure the value of a circuit. The relative position of the terms in a circuit is given. A correct measure of the value recognizes this fact. It is possible to do so thanks to an algebra created by Emma Noether that defines special matrixes which correctly measure the value of a circuit. I shall present the result without explaining the Noether algebra behind it.

**Definition.** The value of an  $m$ -circuit in Noether Algebra is the product of the terms in the circuit. Let  $v[i, j]$  denote  $v[i] \rightarrow v[j]$ . Let  $C[m]$  denote a circuit with  $m$  terms. The notation for the value of the circuit with  $m$  terms is

$$\prod_{\{i,j\} \in C[m]} a[i, j] \tag{1}$$

This formula is familiar to economists. It is the Cobb-Douglas production function. Indeed Noether algebra explains the wide spread use of this production function because it shows the output of a team of producers arranged in a circuit, for example, an assembly line. The sum of the payoffs to the producers who belong to  $C[m]$  is

$$\sum_{\{i,j\} \in C[m]} w[i, j] \tag{2}$$

Is the value of the output made by the circuit big enough to remunerate the producers of the output? A famous result enters the stage to answer this question. It shows the relation between the arithmetic mean and the geometric mean, probably known to the ancients. The arithmetic mean of  $n$  nonnegative numbers,  $x[1], x[2], \dots, x[n]$  is

$$\mu [n] = \sum_{i=1}^n x[i]/n \tag{3}$$

Their geometric mean,  $\gamma[n]$  is

$$\gamma [n] = \sqrt[n]{x [i]} \tag{4}$$

The theorem says that  $\mu[n] \geq \gamma[n]$  unless all the  $x$  are equal. When they are equal, the arithmetic and geometric means are equal. There is another more important implication. The geometric mean attains the maximum when all the  $x$  are equal. Therefore, the output of a homogenous circuit is bigger than the output of a heterogenous circuit of equal size! To analyze the implications, assume that all the circuits in a partition are homogeneous so that each attains its maximum output and circuit member obtains the same remuneration. Different circuits may have different maxima. The next table shows the payoffs to the members of the circuits in the three partitions.

PartitionI

(1, 2)	(2, 3)	(3, 1)	–	$\xi [I]$
(3, 4)	(4, 5)	(5, 3)		$\eta [I]$
(5, 1)	(1, 4)	(4, 2)	(2, 5)	$\zeta [I]$

PartitionII

(2, 3)	(3, 4)	(4, 2)	–	$\xi [II]$
(4, 5)	(5, 1)	(1, 4)		$\eta [II]$
(1, 2)	(2, 5)	(5, 3)	(3, 1)	$\zeta [II]$

PartitionIII

(1, 4)	(4, 2)	(2, 5)	(5, 3)	(3, 1)	$\xi [I]$
(1, 2)	(2, 3)	(3, 4)	(4, 5)	(5, 1)	$\eta [II]$

We see that in Partition I the payoff,  $w[1, 2]$ , is  $\xi[I]$ , in Partition II, it is  $\zeta[II]$  and in Partition III, it is  $\eta[III]$ . For  $w[3, 4]$  in the same partition, it is  $\eta[I], \xi[II], \eta[III]$ , a different result. A similar result can be written for each of the 10 arrows. There would be no unanimous decision to decide which partition is best for the individual. In the language of core theory, the core is empty. However, there is no problem about which partition is best. Add the payoffs in each and pick the biggest sum.

It can be shown that owing to the following inequality

$$\sum_{\{ij\} \in C[m]} w[i, j] > \prod_{\{ij\} \in C[m]} a[i, j] \tag{5}$$

the usual core constraints are not feasible. There is a way around this obstacle. It changes the variables to logs. The log of the value of a partition becomes

$$\text{Log} \left[ \prod_{\{ij\} \in C[m]} V[i,j] \right] = \sum_{\{ij\} \in C[m]} \text{Log}[V[i,j]] \quad (6)$$

Change the payoff to the owner of arrow  $a[i,j]$  from  $w[i,j]$  to  $\text{Log}[w[i,j]]$ . Let the payoff to the arrow  $a[i,j]$  be the log of its contribution to the value of the circuit. The payoff in logs is feasible as is evident from equation (6).

## 4.1 Semi Private Goods

Semi-private goods are financed and owned by a circuit. Their value increases with the homogeneity of its members owing to theorem in Noether algebra. The value of some semi-private goods to an individual increases with the number of users. Computer software is the leading case. It says the demand for software is unusual because its demand function is upward sloping, not downward sloping as is typical of private goods. Therefore, a circuit model is better for analysing most semi-private goods than a model of demand that ignores this fact. Indeed, this is not all that new. The theory of conspicuous consumption, more than a century ago, implies an upward sloping demand for fashion goods.

Both a country club and computer software are semi-private goods. Only members and their guests can use the facilities of their country club.

Only buyers of a software package can use it legally. Most users of the software did not create it. Those who bore the cost of creating software had estimated its potential market, knowing that it depends on how many will use it unlike private non fashion goods. In contrast to software, users of country clubs value it more, the more it restricts membership to certain classes.

Semi-private goods fall into two classes, unrestricted and restricted. Software belongs to the unrestricted class because its value to a user rises with the number of users. Country clubs, elite colleges and fashion goods belong to the restricted class because their value increases, the narrower the group of acceptable users.

## 5 Evidence for the Circuit Core Model

A circuit model of a sub economy unveils a surprising paradox. It describes a sub economy by a collection of related circuits with production functions linear in the logs of their inputs and outputs. For each circuit and given the means of inputs, total

outputs are bigger, the more equal their productivity inputs. Total output of every circuit would be maximal when all their inputs were equally productive. This result applies especially within circuits. It does not follow there is a tendency toward a uniform distribution of circuit values.

Some evidence supports several implications of the circuit model of the economy. Most nations have relatively homogenous populations constituting more than one half of a well-defined ethnic, racial and religious group. It would have been accurate to say this also held for the U.S. until recently. Now the predominant group in the U.S. accounts for a little less than half its population. This makes the U.S. unique among the nations in the world. Switzerland, Belgium and Canada are close rivals for this class but their diverse groups are nearly self governing political states. Only Western nations such as England, France and the Netherlands that had colonial empires in the recent past now resemble the U.S. with respect to heterogeneity owing to immigration from some of their former colonies. Residential data for the U.S. especially deserves attention as empirical confirmation of the circuit model. In the U.S. within urban residential neighborhoods there tends to be homogeneity but among neighborhoods heterogeneity prevails.

This essay has new results for partitions made of 3-circuits. They furnish a simple partition to demonstrate that a model of an economy using circuits has a non empty core if the pertinent variable are in logs. The pertinent variable describe the value of the circuits and the productivity of its terms

\* I have since learned that my matrixes are a special case of a total matrix algebra. It is a feature of a complete matrix ring in van der Waerden (1950, Vol II, pp. 133 ff). His two volume treatise, first published in German in 1931, is based on lectures by E. Nöther and E. Artin. Both had been forced to leave Germany early in the Nazi period having been fired from their academic positions at Göttingen University. Nöther and Weyl are famous students of David Hilbert. MacLane wrote his doctoral dissertation supervised by Weyl and was a student of Nöther.

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