

# Reputation Acquisition in Debt Markets

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This paper studies reputation formation and the evolution over time of the incentive effects of reputation to mitigate conflicts of interest between borrowers and lenders. Borrowers use the proceeds of their loans to fund projects. In the absence of reputation effects, borrowers have incentives to select excessively risky projects. If there is sufficient adverse selection, reputation will not initially provide improved incentives to borrowers with short credit histories. Over time, if a good reputation is acquired, reputation will provide improved incentives. General characteristics of markets in which reputation takes time to work are identified.

## I. Introduction

This paper analyzes the dynamics of an incentive problem between borrowers and lenders. The main result is that incentive problems can be most severe for borrowers with very short track records and become less severe for borrowers who manage to acquire a “good reputation.” This explicit prediction about the evolution of incentives over time extends the existing work on reputation that focuses on the beneficial effect of a long horizon in the future.

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I follow Kreps and Wilson (1982*a*) and Milgrom and Roberts (1982) in viewing a reputation as arising from learning over time from observed behavior about some exogenous characteristics of agents. Reputation effects on decisions arise when an agent adjusts his or her behavior to influence data others use in learning about him.

Reputation is important when there is a diverse pool of relevant exogenous characteristics in an observationally equivalent group of agents because this implies that there is a substantial amount to learn about an agent. My model analyzes the joint influence of adverse selection and moral hazard on the ability of reputation to eliminate the conflict of interest between borrowers and lenders about the choice of risk in investment decisions. If, initially, there is widespread adverse selection (a large proportion of borrowers with undesirable characteristics), reputation effects will be too weak to eliminate the conflict of interest for borrowers with short track records. Adverse selection becomes less severe as time produces a longer track record, and a good reputation can eventually become strong enough to eliminate the conflict of interest for borrowers with a long record of repayment without a default. Alternatively, if there is not substantial initial adverse selection, reputation can begin to work immediately. Two examples in Section VI illustrate these points explicitly.

A reputation that takes time to begin to work implies that new borrowers (with short track records) will face more severe incentive problems and would be the ones most likely to utilize costly technologies for dealing with such problems, such as restrictive covenants in bond indentures (see Smith and Warner 1979) and additional monitoring by a financial intermediary (see Diamond 1984, 1988). The model focuses on the study of debt markets but has implications about the dynamics of reputation formation in general.<sup>1</sup> The general characteristics of markets in which reputation takes time to begin to work are discussed in the conclusion.

An implication of most existing models of reputation, especially Holmstrom (1982), is that the effect of reputation is strongest at the start of an agent's "career" and does not take time to begin to work. This occurs because the amount of information an action can reveal about one's type is highest in the beginning when there are few previous data about actions (and because the horizon can only get shorter).<sup>2</sup> This implies that there would be a strong effort put into

<sup>1</sup> For some early studies of reputation in debt markets, see Spatt (1983) and Stiglitz and Weiss (1983).

<sup>2</sup> There has been some previous analysis of reputation building. In the two-sided uncertainty version of Kreps and Wilson (1982*a*), "weak" types follow a mixed strategy between playing tough or weak, and enough repeated realizations of tough behavior convince others that one is probably not weak, building one's reputation. Milgrom and

having a good beginning to one's track record. This previous work focuses on the incentive effects of the prospect of having a reputation in the future rather than the effects of one's current reputation. Other models of reputation, such as Fama (1980) and Klein and Lefler (1981), are silent on the dynamics of the strength of reputation effects.

My model begins with an observationally equivalent cohort of risk-neutral borrowers with no track record. One type of borrower has available two projects (which arrive each period): a safe project with a high expected return and a negative net present value risky project with a low expected return (but a high maximum return). The incentive problem is that the debt contract might provide incentives to choose the risky, less valuable project. There are two other indistinguishable types of borrowers: those who have access only to a risky project and those who have access only to a safe project.

Imperfect information about borrowers leads to different types being lumped together and initially treated identically: all will be charged the same initial interest rate.<sup>3</sup> Interest rates charged in the future and the prospects for borrowing in the future will depend on the information that later becomes available; a borrower's repayment record (the "track record") will provide this information. Apart from information that is unrelated to repayment history (such as accounting information), all situations in which there is no default are indistinguishable. The investment project chosen by a borrower is not observed by lenders. The most favorable message a repayment history can provide is a lack of default. Because many borrowers who select risky projects do not default, it takes a long time to indicate that safe projects have been selected.

Ex post project returns are borrowers' private information: outsiders cannot observe the ex post profitability because the entrepreneur can appropriate some of the returns to himself. This implies that financial contracts cannot depend directly on this information, ruling out equity contracts. Lenders know the proportion of each borrower type. The interest rate charged in the initial period (for one-period zero-coupon bonds) is set such that, given the proportions, lenders receive a competitive expected return. The higher the proportion of borrowers who choose the risky project, the higher the rate. After one

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Roberts (1982) also allow a type that cannot play tough, and this can cause reputation building in the initial period of their model. Neither paper focuses on reputation acquisition, and because actions are assumed to be observable, neither is consistent with a given agent choosing the weak action in the beginning and then later switching to the strong action.

<sup>3</sup> In the model, ex ante separation by choice of contract before any borrowing takes place is not viable

period, some borrowers default and some do not. The class of non-defaulters is now a more select group: the proportion of those with only risky projects has declined. The second-period interest rate charged to this group will be less than the initial rate, and this decline continues over time for the class of nondefaulters. With a long time horizon, these reduced rates for a borrower who does not default imply that the present value of the borrower's rents for any constant investment decision rises over time: the reputation itself becomes a valuable asset, and a single default causes a large decline in its value. This loss of value arises because default leads to a cutoff of credit (or, more generally, to an increase in the interest rate charged).

The value of a good reputation rises over time, as does the cost of a default. Therefore, over time, the relative payoff of the risky project (a very large payoff when it has a favorable outcome) declines relative to a safe but profitable project. If there is sufficient adverse selection, then a typical equilibrium path for a borrower with access to both types of projects is to choose risky projects when "young" and, if able to survive long enough without a default, to switch to safe projects from that point forward. In this formulation, reputation is important because it becomes a valuable asset worth protecting.<sup>4</sup>

The balance of the paper is organized as follows: Section II describes the setup of the model. Section III outlines the borrowing and lending arrangements each period and studies the one-period horizon. Sections IV and V provide analysis of the details of the model's equilibrium. Section IV derives optimal project choice for given interest rates and interest rates for given project choice. Section V analyzes the endgame, the periods near the horizon. Section VI presents a special case of the model to make the main points about what determines the evolution of reputation effects over time. Section VII generalizes the characterization to the general model. Section VIII concludes the paper.

## **II. The Basic Model: Technology, Endowments, and Preferences**

All borrowers and lenders are risk neutral. Lenders receive an endowment of inputs each period, and each has access to a constant returns to scale technology for storing endowment within a period, converting it to a consumption good at the end of a period. This

<sup>4</sup> This feature of the collateral-like incentive value of an asset that depreciates in case of default is also present in Merton (1978), where it arises in the case of a bank that prepaies for many years of deposit insurance, and failure implies that it can issue insured deposits for only a few of those years. The difference is that it arises endogenously in this formulation and is not necessarily present in the initial time periods.

technology returns  $r$  units of output at the end of a period per unit of input at the beginning of the period. Inputs must be stored (or used as an input in a project described below) within a period before it is possible to consume them. In an attempt to model lenders as an anonymous capital market rather than a financial institution, I assume that a given lender exists for only one period, implying that borrowers face a new set of lenders each period. The implication of this assumption is that reputation in the form of a borrower's credit history is the only intertemporal linkage. There is an infinite number of potential lenders each period. There is no commitment technology available to lenders. Lenders cannot commit themselves to take actions in the future (or at the end of a given period) that are not then in their *ex post* interest, even if this would be beneficial *ex ante*.

Borrowers receive no endowment but have access to indivisible investment projects each period (and do not have storage technology). They are indistinguishable from lenders (but contracts for borrowers turn out to be unattractive to lenders, and vice versa, so this is an unnecessary assumption). There are three sorts of projects, and the set of projects available to each borrower is his private information. Borrowers can commit to some degree by writing contracts each period that depend on publicly observed variables.

There are three types of borrowers: Type G borrowers have one safe project each period. They can invest one (dollar) and receive  $G > r$  at the end of the period. Type B borrowers have one risky, low-expected-return project each period. They can invest one, and with probability  $\Pi_B < 1$ , the project returns  $B$  (where  $\Pi_B B < r$  and  $B > G$ ); with probability  $1 - \Pi_B$ , it returns zero. Type BG borrowers have their choice each period between either one of two projects, but not both. One project is identical to that of type G's and the other is similar to that of type B's, except the probability of its returning  $B$  is  $\Pi \leq \Pi_B$ .<sup>5</sup>

Let the initial population of borrowers contain a fraction  $f_G$  of type G's,  $f_B$  of type B's, and  $f_{BG}$  of type BG's. These fractions are public information. The returns on the risky projects are all independently distributed. A borrower's type is private information, and all borrowers initially appear identical. In addition, the realized output of a project is private information observed only by the borrower. This makes it difficult to make a borrower's payments depend on a project's realized return. There is, however, a costly liquidation technology that borrowers can commit to use in financial contracts contingent on any publicly observed action or outcome. This technology allows

<sup>5</sup> It is simplest to think about the case in which  $\Pi = \Pi_B$ , and the risky project is identical to that of type B borrowers. The extra generality is stated simply because all our results hold for  $\Pi \leq \Pi_B$  as well.

the output of the project to be seized before the borrower can consume it. It is extremely costly to do this: it destroys the output, so the value then observed by the public is always zero if the project is liquidated. This limits the liquidation and bankruptcy process to working as a contract enforcement device that is costly to utilize. This corresponds to the role of liquidation in the world; it is used as a threat rather than a universally used information service. If an alternative assumption were made that costly liquidation made the realized value of project output observable to the public, liquidation would be useful in determining the type of the borrower. In particular, it would then allow those with successful risky projects to be distinguished from those with safe projects. Even without such an informational role, liquidation is useful in providing incentives for repayment. It serves to prevent the borrower from following a policy of "take the money and run."

Lenders in each period observe each borrower's history of defaults. They know the dates on which a borrower has borrowed and on which there was liquidation. This is assumed to be the only information available about the past: the series of past interest rates paid is not observed. An earlier version of this paper (Diamond 1986) uses the more general assumption that all past interest rates are observed and obtains identical outcomes to those found under this assumption.<sup>6</sup>

Borrowers maximize discounted expected consumption over  $T$  periods, where  $T$  is finite but very large (we take the limit as  $T \rightarrow \infty$ ). Assume finite  $T$  because infinite  $T$  introduces many equilibria that are not limits as  $T \rightarrow \infty$  (see Dybvig and Spatt 1980). Borrowers make decisions to maximize discounted expected consumption, given by  $\sum_{t=1}^T E(\tilde{c}_t) \cdot d^t$ , where  $c_t$  is the realization of the consumption random variable  $\tilde{c}_t$  in period  $t$ , and  $d$  is the discount factor that discounts end-of-period expected consumption to its beginning-of-period value. Assume  $d < 1$ , implying that borrowers discount the future.

To focus on the importance of a long time horizon in providing incentives, assume that with a one-period horizon (with  $T = 1$ ), type BG borrowers would select risky projects. The restriction on parameter values that yield this result is discussed in Section IIIA.

The total inputs that can be utilized by all available projects (the sum of all three types) is less than the aggregate endowment of capital goods each period. The storage process is in use at the margin in any equilibrium, and competition among lenders in selecting debt contracts implies that a borrower can borrow by offering lenders a contract that provides an expected return of  $r$  per unit loaned. The

<sup>6</sup> Diamond (1986) found that the identical outcomes are a sequential equilibrium supported by the belief that for any interest rate offered in a period there is no information about a borrower's type.

projects are in economic scarcity, and any rents they generate in equilibrium will go to the agents endowed with the projects.

Consumption of each agent must be nonnegative ( $c_t \geq 0$  for all  $t$ ). This bit of realism is important. If this assumption were dropped, one might never need outside financing; one could simply consume a negative amount in early periods (producing goods) and consume positive amounts later (repaying one's "borrowed utility"). Alternatively, one could always issue riskless debt in that case, by producing enough goods through negative consumption to pay off any claim. In addition, no stronger punishment or nonpecuniary penalty can be imposed that is more severe than zero consumption. This is a form of limited liability, ruling out debtors' prisons, physical punishment, and similar phenomena.

The equilibrium concept used in this repeated game of incomplete information is sequential Nash equilibrium, defined in Kreps and Wilson (1982*b*). At each stage and for all possible actions, beliefs about the implied type of borrower are specified and all actions are a best response to these beliefs and the actions of all other players in the game. In addition, beliefs about all equilibrium actions are self-fulfilling.

*Review of assumptions.*—(1) There are four types of agents: lenders and three types of borrowers. All agents are risk neutral. A borrower's type is private information. (2) Inputs are endowed to each lender at the beginning of each period, none to borrowers. The endowment must be used as an input to storage or a project during a period to become a consumption good. Each lender lives only one period. The amount of each loan is one (dollar), the scale of a borrower's indivisible project. (3) There is no commitment technology for lenders. Borrowers can commit to use the liquidation technology conditional on some payments to lenders and not to use it given other payments. (4) There are  $T$  time periods;  $T$  is finite, but limiting behavior  $T \rightarrow \infty$  is used for most results. (5) Projects are in short supply, and as a result the storage technology is in use. Borrowers can borrow with any contract that offers an expected return of  $r$ , the return of storage. (6) Project selection and outcomes are private information observed only by the borrower. Each borrower's track record of repayment or default for all past periods is observed by all current lenders. (7) Consumption must be nonnegative, and nonpecuniary penalties are not feasible. (8) With a single-period horizon,  $T = 1$ , type BG borrowers choose risky projects.

### III. Borrowing and Lending with Debt Contracts

The contract between borrowers and lenders is assumed to be a debt contract. At the cost of longer arguments, debt can be shown to be the

optimal contract given the private information and unobservability of project returns that makes equity contracts (where lenders receive a share of realized returns) infeasible. Work on single-period contract design by Townsend (1979), Diamond (1984), and Gale and Hellwig (1985) shows that this unobservability implies that contracts are optimally of the debt form.

There are four stages each period. First borrowers offer contracts to lenders, then lenders decide which loan contracts to accept, then borrowers who get loans choose their projects, then they observe the return on their projects and decide how much to pay to lenders (facing liquidation for some possible payments). In the sequential equilibrium established below, the face value (one plus the interest rate) on debt in a given period,  $r_b$ , is the same for all types. If  $r_t$  is such that lenders get an expected return below  $r$ , given the proportion of types in a period, the contract is rejected in the second stage. If  $r_t$  provides an expected return of at least  $r$ , it is accepted. A debt contract at date  $t$  is parameterized by  $r_t$  and involves commitment to liquidation for all payments less than  $r_t$ . It will turn out that all borrowers who can pay  $r_t$  or more will pay  $r_b$ , and all others will be liquidated. Although  $r_t$  is a face value of a loan of one dollar, at times  $r_t$  will be referred to as the "interest rate," and parameters that result in higher or lower interest rates will be discussed. This will not lead to ambiguity because the interest rate on a loan is  $r_t - 1$ , a one-to-one function of the face value.

A. *One-Period Horizon: T = 1 (or the Final Period if T > 1)*

This is the case that has been analyzed previously in Fama and Miller (1972) and Jensen and Meckling (1976). The one-period model here is different because of the imperfect information about borrowers, and it is similar to Stiglitz and Weiss (1981). This is a necessary input to the multiple-period model because at the final period there will remain just one period.

Project outcomes are unobservable to lenders and cannot be used to directly specify debt repayments. Use of the liquidation technology can provide incentives for repayment by specifying a face value  $r_T$  for each loan of one unit, such that there is liquidation if less is repaid and liquidation is avoided if at least  $r_T$  is paid. Because liquidation implies the destruction of all output from the project, it also implies zero consumption by a borrower. If a project returns more than  $r_T$ , a borrower can repay  $r_T$  and consume the remainder of the project's return. No borrower would ever pay less than face value (and consume zero) if he could pay face value (and consume the excess of his project's return over the face value of debt). No borrower would pay more than  $r_T$  because this reduces the borrower's consumption (com-



pared to paying  $r_T$ ) and has no other benefit since liquidation is avoided by paying  $r_T$ . This implies that borrowers with projects returning  $r_T$  or more pay  $r_T$ , and all others are liquidated. In addition, if loans are made,  $r_T$  must be less than or equal to  $G$ , the maximum amount that type G borrowers can pay. A higher  $r_T$  would lead to liquidation for all borrowers with projects with an expected return greater than the opportunity cost  $r$ , implying that lenders would not lend.

The choice of project by type BG's facing a given face value of debt  $r_T$  needs to be analyzed to determine the equilibrium value of  $r_T$ . Facing an exogenous face value  $r_T$  implies that if BG's choose safe projects, their expected utility at the end of the period is  $G - r_T$ . If they choose risky projects, the expected utility is  $\Pi(B - r_T)$ .<sup>7</sup> The optimal selection is the one with the larger expected utility, implying that safe projects are best if and only if  $r_T \leq (G - \Pi B)/(1 - \Pi)$ , and risky projects are best if and only if the reverse inequality holds. This means that the level of interest rates can influence the optimal choice. Lower values of  $r_T$  improve the relative position of safe projects (because interest costs are paid only with probability  $\Pi$  with a risky project, but with certainty with a safe project). Figure 1 plots the beginning-of-period values,  $d(G - r_T)$  and  $d\Pi(B - r_T)$ , as functions of  $r_T$ .

The equilibrium face value  $r_T$  is set such that risk-neutral lenders receive an expected return of at least  $r$ , the riskless return on storage. The face value that will deliver an expected return of  $r$  depends on the investment decision that the type BG's are expected to make because more loans default if they select risky projects. Let  $r_T^g$  denote the face value that makes a lender's expected return equal to  $r$  if type BG's are assumed to select safe projects, and  $r_T^b$  the face value that provides that expected return if BG's are assumed to select risky projects. If safe projects are selected, then only type B's default, and this implies that  $r_T^g$  is given by  $r_T^g = r(f_B \Pi_B + f_{BG} + f_G)^{-1}$  because type B's repay with probability  $\Pi_B$ , all type G's and BG's will repay, and the probability of repayment is then  $f_B \Pi_B + f_{BG} + f_G$ . If risky projects are selected by all type BG's, then the face value  $r_T^b$  is given by  $r_T^b = r(f_B \Pi_B + f_{BG} \Pi + f_G)^{-1}$ .

The rate offered by borrowers depends on the policy that they anticipate lenders use to grant loans. The following policy is an equilibrium, supported by lenders' beliefs specified below. If  $r_T^g \leq (G - \Pi B)/(1 - \Pi)$ , then make loans if  $r_T \in [r_T^g, G]$ . If  $r_T^g \geq (G - \Pi B)/(1 - \Pi)$  (so it is not self-fulfilling for lenders to believe that BG's will select safe projects at  $r_T^g$ ), make loans if  $r_T \in [r_T^b, G]$ : in this case if  $r_T^b$  exceeds

<sup>7</sup> Beginning-of-period expected utility is obtained by multiplying each end-of-period expected utility by  $d < 1$ .

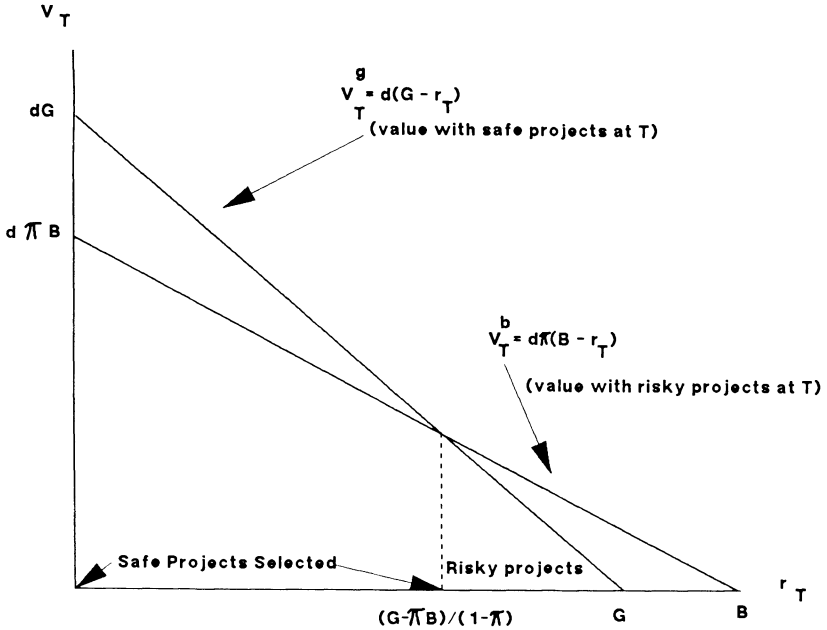


FIG. 1.—Value of safe and risky projects at the final date,  $T$ , as a function of the face value,  $r_T$ .

$G$ , no loans are made. In any case, if  $r_T^g > G$ , no loans are made. Given this policy, all borrowers offer the lowest interest rate that lenders will accept. Lenders' belief about borrower type as a function of the interest rate,  $r_T$ , offered is that for all feasible rates, the type is not a function of the rate. This is self-fulfilling because given it all borrowers offer the lowest rate that will lead to a loan:  $r_T^g$  is the equilibrium rate if it offers lenders an expected return of  $r$ .<sup>8</sup> The equilibrium rate in the final period is the lowest one that gives lenders an expected return of exactly  $r$ .

To focus on the role of a long horizon (as distinct from just imperfect information), we assume that the risky projects are sufficiently close to being profitable (have expected values  $\Pi B < r$ , but not close to

<sup>8</sup> This belief—that the implication of any rate offered is the pool of all current borrowers—satisfies various refinements of sequential Nash equilibrium (e.g., the Cho-Kreps [1987] intuitive criterion that disallows inferences from off-equilibrium actions that imply belief that some type took an action dominated by the proposed equilibrium payoff). This is true because all types prefer lower interest rates, and the only types that want to distinguish themselves are the type  $G$ 's, who never default and therefore face a cost of higher rates that is weakly higher than the cost for other types. There is no consistent interpretation of a deviation from all types offering the same rate, and this is true in any sequential equilibrium. This also implies that the pooling on the lowest rate is the unique equilibrium that satisfies the definition in Grossman and Perry (1986).

zero), such that even if financed at the lowest possible rate (the riskless interest rate  $r$ ), the optimal one-period choice for a type BG is to choose risky projects. That is, we know  $r_T \geq r$  and we assume

$$r > \frac{G - \Pi B}{1 - \Pi}. \quad (1)$$

Thus for *all* values of  $f_B$ ,  $f_{BG}$ , and  $f_G$  for which the market does not fail, the type BG borrowers will select risky projects and the interest rate  $r_T^b$  will prevail if there is a one-period horizon. The market is open at  $T$  if and only if  $r_T^b < G$  or, equivalently,  $f_G > [(r/G) - (\Pi f_{BG} + \Pi_B f_B)] > 0$ . One interpretation of this condition is that only a borrower with a track record that implies a strictly positive probability of being of type G will have a chance of borrowing at the final date  $T$ . Lemma 1 summarizes the results about a one-period horizon.

LEMMA 1. At the final period,  $T$ , (a) all borrowers offer the debt contract with the lowest interest rate that provides an expected return of  $r$ ; (b) all borrowers who can repay their debt do so, and all others default; and (c) only those borrowers with track records that imply a sufficiently high (and strictly positive) probability of being of type G are able to borrow.

### B. *Lending and Repayment Decisions with Multiple Periods*

The three properties of equilibrium at  $T$  in lemma 1 turn out to be properties of equilibrium at all previous dates. The stage that is of most interest in this study is the decision of borrowers about the choice of project. To focus on that decision, the policies followed at the other stages are briefly studied first, and analysis of the project selection decision is deferred to Section IV. Let us begin with the final strategic stage in a period: the decision of borrowers on how much to repay. This can be studied without a full analysis of the previous stages by establishing two useful properties of the second stage in which lenders decide whether to grant credit in a period. Given the characterization of the final (repayment) stage and the two properties of the second (loan-granting) stage, Section IIIC shows that at the first stage all borrowers offer the lowest possible interest rate, as in the one-period analysis.

Lenders observe the previous default record of each borrower. Borrowers face lenders who live for a single period, but lenders know that borrowers will want to continue borrowing in the future. Both take these anticipated future actions into account. One implication of this forecast of future actions is that lenders will not lend to a borrower unless they think there is some chance that he is of type G. We saw that no one would lend at  $T$  unless there was a strictly positive

probability that a borrower was of type G. No one known to be of type B would receive a loan because a type B's project has a total expected return below  $r$ . No one would lend to a known type BG by a familiar argument using backward induction: no one would lend in the final period (because risky projects with expected return below  $r$  would be selected), and successively earlier periods become the "last" borrowing opportunity, implying that no one would ever lend. This establishes the following lemma.

LEMMA 2. If a borrower is revealed at time  $t$  to be of either type B or type BG, no one will lend to him thereafter.

This cutoff of credit to those known not to be of type G requires no commitment by lenders; such lending is simply unprofitable.

Lemma 3 states a property of the face value of any loan that lenders would accept. It follows from the requirement that lenders get an expected return of at least  $r$  per dollar lent.

LEMMA 3. If a loan is made at date  $t$ , then the face value  $r_t \in [r, G]$ .

*Proof.* If  $r_t < r$ , then even if repaid with certainty, it provides a return below  $r$ . If  $r_t > G$ , then all borrowers with projects returning  $G$  default and are liquidated. All other borrowers have projects with expected returns below  $r$ , implying that lenders would get an expected return below  $r$ . Q.E.D.

These properties of the loan-granting stage are almost enough to characterize the repayment policy of borrowers. Lemma 4 states the equilibrium repayment each period by each type of borrower.

LEMMA 4. Borrowers repay face value  $r_t$  (and avoid liquidation) if their project returns at least  $r_t$ , and borrowers with projects that return less than  $r_t$  are liquidated.

*Proof.* See the Appendix.

The proof of the optimal repayment policy is almost the same as that in the one-period case. Suppose that credit is cut off on a single default. Then all borrowers who could avoid default and liquidation would do so, and because all situations in which there is not default are indistinguishable, no borrower would pay more than the minimum necessary to avoid default. Given that repayment policy, only those who cannot repay  $r_t \in [r, G]$  default, and such borrowers are not of type G. A default then does indeed cut off credit, by lemma 2, and the beliefs about the implications of a default are self-fulfilling.<sup>9</sup>

<sup>9</sup> The belief that those who have defaulted are those who were constrained to default and that those who did not are the pool of all who could avoid default satisfies the refinements to equilibrium mentioned in n. 8. This is because default is feasible for all types and has the lowest cost for the types who must default, implying that it is not self-fulfilling to believe that default conveys news that default could have been avoided. There can be no future favorable inference from current default, and there is a current benefit from avoiding liquidation, so all who can do so avoid default. This is the unique self-fulfilling belief about repayment and default of a loan actually granted in the past.

An immediate consequence of this lemma is a rule that lenders use in deciding if a loan should be made. This is lemma 5.

LEMMA 5. Any default (payment of less than  $r_t$ ) by a borrower at a date  $t$  leads to no lending for all future dates.

Because default will reveal that a borrower is not of type G, it will influence his future treatment by lenders: no more credit will be advanced.<sup>10</sup> This learning from observed behavior is key to the incentive value of reputation. One might actively avoid default (by choosing safe projects) to avoid the premature cutoff of one's credit, even though one will later choose risky projects (e.g., in the final period). This incentive effect is analyzed in Section IV.

### C. Rates Offered at the Beginning of Each Period

Borrowers who can repay and avoid liquidation will pay the minimum payment that avoids liquidation: this payment is  $r_t$ . This first stage is for borrowers to offer the debt contracts, and this is a choice of  $r_t$ . We saw in Section IIIA that in the final period all borrowers offer the lowest rate that provides lenders with an expected return of  $r$ . Borrowers offer the lowest possible rate in all periods. The supporting beliefs and actions of lenders are that they believe that the rate offered at  $T - 1$  reveals nothing new about a borrower's type, and they will lend at the lowest rate that gives an expected return of  $r$  given the belief that each borrower is a random draw from the pool of all borrowers who have not yet defaulted. Under this belief there is no current benefit from offering a higher than needed rate. The only possible motivation for such a higher current rate is a possible future benefit: for example, a borrower's attempt to signal his type by offering a higher than needed interest rate. If any borrower deviated and offered a rate higher than the minimum, he would be indistinguishable in the future from those who offered the lower rate and would achieve no future benefit.<sup>11</sup> This implies that all borrowers pool and offer the lowest rate that provides lenders with an expected return of

<sup>10</sup> It is not essential to our basic approach that credit be cut off. A similar effect occurs in a version of this model in which all projects have positive net present value but safe projects have higher net present value. This implies that the best response to a default is an increase in the interest rate.

<sup>11</sup> One more general implication of lemma 1 is that contract choice cannot separate the various types of borrowers, no matter how the beliefs are specified. Any contract that only a type other than G would offer would lead to no lending now or in the future and zero consumption for the borrower offering the contract. A contract that specifies a payment feasible for type G's (who can pay at most G) would allow the other types to have positive expected consumption, dominating the zero consumption implied by a separating contract that would be offered only by a type other than G.

$r$  at  $T - 1$ . The argument extends recursively to all previous  $t < T$ .<sup>12</sup> We have established the following lemma.

LEMMA 6. At all dates all borrowers offer the lowest interest rate that provides lenders with an expected return of  $r$ .

Within each period we have established some properties of a sequential equilibrium in which the interest rate offered at the beginning correctly takes into account a part of the rule that lenders use to grant loans and the rule that each borrower will use to repay at the end of each period. Given these properties, the project decisions of type BG borrowers can be analyzed. Only given that analysis can the “lowest interest rate that provides lenders with an expected return of  $r$ ” in lemma 6 be precisely defined. Section IV provides this analysis for given interest rates and an analysis of interest rates for given project decisions.

#### IV. Project Choice and Interest Rates

##### A. Project Decisions for Given Interest Rates

At the final period, type BG borrowers will select risky projects. The  $T - 1$  present value of continuing to borrow until period  $T$  (by not defaulting at  $T - 1$  because a default cuts off credit by lemma 5) is thus the value of choosing a risky project. Let  $V_t$  be the maximized value as of  $t$  of making optimal project decisions from  $t$  to  $T$ . At  $T$  this is given by  $V_T = d\Pi(B - r_T)$ .

Define  $V_t^b$  and  $V_t^s$  to be the value as of  $t$  of choosing, respectively, a risky and a safe project at  $t$  and making optimal decisions thereafter. Clearly,  $V_t = \max\{V_t^b, V_t^s\}$ , where  $V_t^b = d\Pi(B - r_t + V_{t+1})$  and  $V_t^s = d(G - r_t + V_{t+1})$ . A type BG borrower chooses safe projects if and only if  $V_t^s - V_t^b \geq 0$  or  $d\{G - \Pi B - [(1 - \Pi)(r_t - V_{t+1})]\} \geq 0$ , implying  $r_t - V_{t+1} \leq (G - \Pi B)/(1 - \Pi)$ . If and only if this condition holds,  $V_t = V_t^s$ , and this is then equivalent to  $V_t \geq d\{G - [(G - \Pi B)/(1 - \Pi)]\}$ . This establishes lemma 7.

LEMMA 7. Safe projects are the optimal choice at date  $t$  if and only if  $r_t - V_{t+1} \leq (G - \Pi B)/(1 - \Pi)$  or, equivalently,  $V_t \geq d\{G - [(G -$

<sup>12</sup> The belief that the rate offered reveals nothing new about type satisfies the refinements mentioned in n. 8, and pooling of rates at this rate is the only outcome that fulfills the equilibrium defined in Grossman and Perry (1986). The reason is the same as at the final period  $T$ : offering rates above the minimum needed has no differentially lower cost for type G (the only type that would want to differentiate itself), and the new wrinkle is that in later periods those who offer rates deviating from the proposed equilibrium rates are treated identically to those who offer the proposed equilibrium rates. Therefore, there is no consistent interpretation of a deviation from the proposed rates.

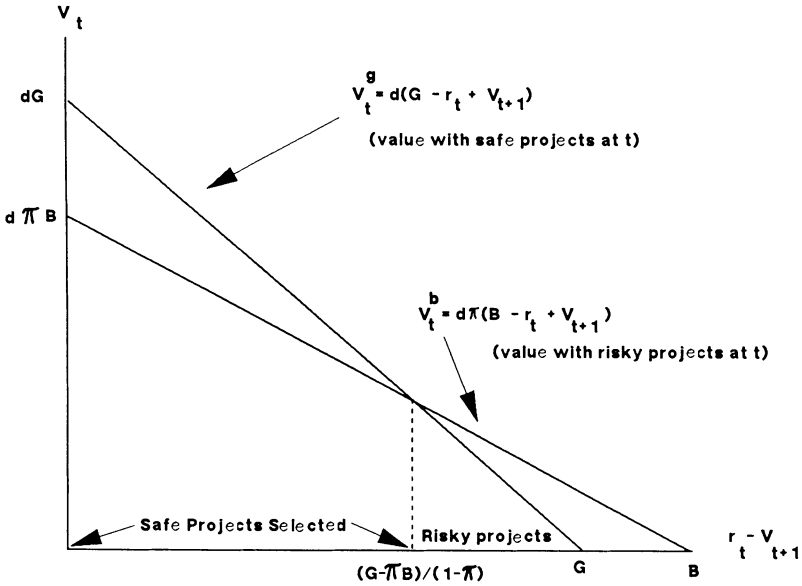


FIG. 2.—Value of safe and risky projects at date  $t$ , as a function of  $r_t - V_{t+1}$

$\Pi B) / (1 - \Pi)\}$ . Risky projects are the optimal choice if and only if the reverse inequalities hold.

The implication of lemma 7 is that repaying a loan has the short-run cost of  $r_t$  and the long-run benefit of  $V_{t+1}$ , and the term  $r_t - V_{t+1}$  plays an identical role to  $r_t$  in the single-period case because a default cuts off a borrower's credit. Figure 2 shows  $V_t^b$  and  $V_t^g$  as a function of  $r_t - V_{t+1}$ . Risky projects are selected when  $r_t - V_{t+1}$  is high. The optimality of risky projects also implies that  $V_t$  is low, so type BG's always prefer to have reason to select the safe project. When type BG's are indifferent between safe and risky projects,  $V_t = d\{G - [(G - \Pi B) / (1 - \Pi)]\}$ . Further results on periods of indifference by type BG's are presented below in lemmas 11 and 12.

Reducing the interest rate charged to a borrower at any date makes safe projects become relatively more attractive on that date. Reduced interest rates at date  $t$  also increase  $V_t^b$  and  $V_t^g$ , implying that  $V_t = \max\{V_t^b, V_t^g\}$  is increased. This makes safe projects more attractive at  $t - 1$ . By a similar argument,  $V_{t-1}$  is increasing in  $V_t$ , implying that a reduction in  $r_t$  increases  $V_{t-1}$ , which makes safe projects relatively more attractive on date  $t - 2$  and, by recursion, on all dates before  $t$ .

The most favorable situation for the selection of safe projects is one in which "interest rates" are at their minimum possible value,  $r_t = r$ . This provides a necessary condition for reputation effects to have

value and induce type BG's to select safe projects on some date. It will later be shown to be a sufficient condition when  $T \rightarrow \infty$ . This is a necessary condition for there to exist a date  $t$  on which  $V_t^s \geq V_t^r$ .

LEMMA 8. Type BG borrowers will select safe projects on some date only if  $d(G - r)/(1 - d) > d\Pi(B - r)/(1 - d\Pi)$ , implying that the present value of financing the safe project at the riskless rate of interest for an infinite number of periods exceeds the present value of selecting the risky project for an infinite number of periods (until the first default).

*Proof.* From (1), risky projects are optimal at  $T$ . From lemma 7, safe projects are best if and only if  $r_t - V_{t-1} \leq (G - \Pi B)/(1 - \Pi)$ . Because risky projects are selected at  $T$  and if safe projects are ever optimal, there will be a date  $\hat{t}$  when safe projects are selected, followed by risky ones until  $T$ . On such a date  $\hat{t}$ , we know

$$V_{\hat{t}+1} < \frac{(d\Pi)(B - r)}{1 - d\Pi} = \lim_{T \rightarrow \infty} \sum_{t=1}^T (B - r)(d\Pi)^t$$

because this is the upper bound on the value of selecting risky projects until the first default for an infinite number of periods, and  $r_t \geq r$ . Thus safe project optimality at  $t$  requires

$$d \left[ G - r + \frac{d\Pi(B - r)}{1 - d\Pi} \right] \leq d\Pi \left[ B - r + \frac{d\Pi(B - r)}{1 - d\Pi} \right]$$

or

$$r - \frac{(d\Pi)(B - r)}{1 - d\Pi} \leq \frac{G - \Pi B}{1 - \Pi}.$$

Rearranging terms produces the following equivalent conditions that are necessary for reputation to ever have value:

$$\frac{d\Pi(B - r)}{1 - d\Pi} > r - \frac{G - \Pi B}{1 - \Pi}, \tag{2}$$

$$\frac{d(G - r)}{1 - d} > \frac{d\Pi(B - r)}{1 - d\Pi}, \tag{3}$$

and

$$\frac{d\Pi(B - G)}{1 - \Pi} > r - \frac{G - \Pi B}{1 - \Pi}. \tag{4}$$

Q.E.D.

Lemma 8 makes sense because the loss of reputation from a default at worst results in a cutoff of credit, and if the rents on the safe project are low enough that they are exceeded by those of choosing risky



projects at the low rate  $r$  (and conditions [2]–[4] fail), loss of reputation is not a potent enough weapon to induce cooperative behavior. This condition is also sufficient if  $T \rightarrow \infty$ , but proof of this is deferred to Section V.

Lemma 9 provides a sufficient condition for the selection of safe projects at a date if the horizon is long. It states that if future interest rates are below a given level at all dates in the future, then safe projects are currently optimal. This level of future interest rates will specify feasible future rates (i.e., rates greater than the riskless rate) only if conditions in lemma 8 hold.

LEMMA 9. If, for all  $t \in [\hat{t}, T]$ ,  $r_t < dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ , then there exists  $T < \infty$  such that safe projects are the optimal choice at date  $\hat{t}$ :  $T$  such that  $r_{\hat{t}} - V_{\hat{t}+1} \geq (G - \Pi B)/(1 - \Pi)$ . This bound on future interest rates specifies feasible rates—that is,  $dG + (1 - d)(G - \Pi B)/(1 - \Pi) > r$ —if and only if the necessary conditions (in lemma 8) for reputation to have value are true.

*Proof.* See the Appendix.

The conditions for safe projects to be selected at some date in lemmas 8 and 9 are stated in terms of interest rates, which are endogenous. The next subsection describes equilibrium interest rates for given project decisions. Section V uses these results to provide general necessary and sufficient conditions for reputation effects to be strong enough on some date to imply the selection of safe projects on some date. The remainder of the characterization in the general case is deferred to Section VII.

### B. Interest Rates for Given Project Decisions

For any equilibrium there is a range that bounds the equilibrium interest rate (face value),  $r_t$ . The lowest value that it can attain is the value that provides a normal expected return,  $r$ , to lenders under the assumption that type BG borrowers choose safe projects. As in our discussion in Section IIIA, we call this rate  $r_1^g$ . The largest possible value of the interest rate is  $r_1^b$ , the one that gives lenders a normal expected return if all type BG's choose risky projects. At  $t = 1$ , these rates are given by

$$r_1^g = \frac{r}{f_G + f_{BG} + \Pi_B f_B}$$

and

$$r_1^b = \frac{r}{f_G + (\Pi f_{BG}) + (\Pi_B f_B)}.$$

The bounds on the rates in periods after  $t = 1$  depend on the population of borrowers with a given track record. Because those who default are denied future credit, all those who continue to borrow have perfect records of no default. Define  $f_{\Theta t}$  as the fraction of the original pool of all borrowers who are of type  $\Theta$  and have not yet defaulted by the beginning of period  $t$ . For example,  $f_{B1} = f_B$ . Because type B borrowers always select risky projects,  $f_{Bt} = \Pi_B^{-1} f_B$ .<sup>13</sup> Type G borrowers always select safe projects, implying  $f_{Gt} = f_G$  for all  $t$ . The fraction of type BG borrowers in the pool of those with a reputation of no default depends on the decisions made each period: if safe projects are selected at date  $t - 1$  by all BG's, then  $f_{BGt} = f_{BGt-1}$ . If all BG's select risky projects at  $t - 1$ , then  $f_{BGt} = \Pi f_{BGt-1}$ . At date  $t$ , the pool of borrowers is a fraction  $f_G + f_{Bt} + f_{BGt}$  of the original pool of borrowers, and out of the original pool, the fraction of loans repaid at  $t$  is  $f_G + (\Pi_B f_{Bt}) + (\Pi f_{BGt})$ . The bounds on  $r_t$  at date  $t$  that give lenders an expected return of  $r$  are therefore given by

$$r_t^g = r \frac{f_G + f_{BGt} + f_{Bt}}{f_G + f_{BGt} + (\Pi_B f_{Bt})}$$

and

$$r_t^b = r \frac{f_G + f_{BGt} + f_{Bt}}{f_G + (\Pi f_{BGt}) + (\Pi_B f_{Bt})}.$$

Note that if  $f_{BG} = 0$ , then  $r_t^b = r_t^g$ . In the case of  $f_{BG} = 0$  the interest rates can be specified without knowing the equilibrium actions of the finite number of type BG's (because there are an infinite number of borrowers).

Characterization of the choice between  $r_t^g$  and  $r_t^b$  for all  $t$  is presented in Section V. This choice is first analyzed for the periods near the horizon,  $T$ .

### V. The Endgame: Analysis of Decisions near the Horizon, $T$

Lemmas 8 and 9 provide necessary and sufficient conditions, in terms of interest rates, for the selection of safe projects at some date if  $T \rightarrow \infty$ . This section extends these to conditions in terms of exogenous parameters.

Define the "endgame" as the period until  $T$  that begins when a type

<sup>13</sup> I use a version of the law of large numbers here by equating the fraction of type B's who actually repay with its expected value. This can be made rigorous (see Feldman and Gilles 1985).

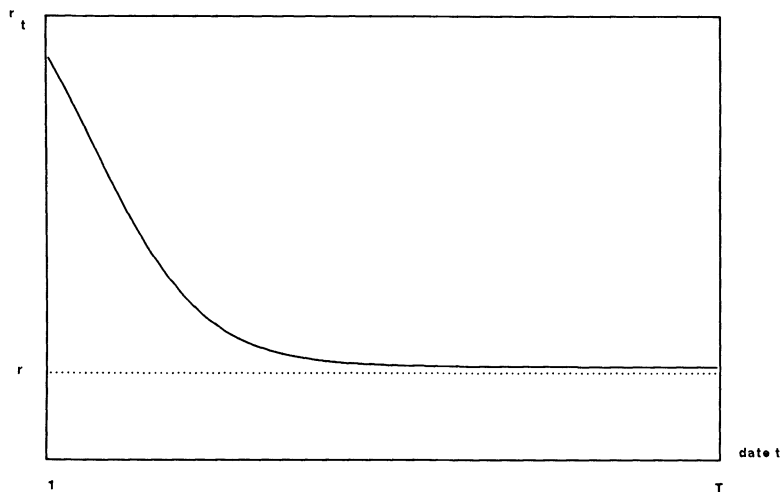


FIG. 3.—Face values,  $r_t$ , given no default up to date  $t$ . It is assumed that  $f_B > 0$  and  $f_{BG} = 0$ .

BG switches to (or back to) risky projects. Formally, the endgame begins on date  $\tau$ , where  $\tau$  is the smallest  $t$  with  $V_t^g \leq V_t^b$  that occurs after some date  $\hat{t}$  where  $V_{\hat{t}}^g > V_{\hat{t}}^b$ . If no date  $\hat{t}$  exists, then  $V_t^g \leq V_t^b$  for all  $t$ , and we define the endgame as the entire game from  $t = 1$  to  $t = T$ .

The endgame is *bounded* if there exists  $K < \infty$  such that  $T - \tau < K$  as  $T \rightarrow \infty$ . A bounded endgame implies that  $\tau \rightarrow \infty$  as  $T \rightarrow \infty$ .

#### A. A Low Fraction of Type BG's

To develop the basic points about the endgame, consider the case in which, of the infinite number of borrowers, a fraction  $f_B$  are of type B, a fraction  $f_G = 1 - f_B$  of type G, and a finite number (representing a zero fraction) of borrowers are of type BG. The series of the lowest interest rate that provides lenders an expected return of  $r$  (given the current population of borrowers with a given track record) is exogenous, and  $r_t^g = r_t^b$ . By lemma 6, the face value series,  $r_t$ , is given by (and shown in fig. 3)

$$r_t = r \frac{\prod_B^{t-1} f_B + 1 - f_B}{\prod_B^t f_B + 1 - f_B}. \quad (5)$$

Provided that  $r_1 \leq G$ , (5) states the interest rate charged over time to those who do not default up to time  $t$ . All those who default are revealed to be types other than G and thus have their credit cut off from that point forward. If  $r_1 > G$ , then the capital market fails and no one can borrow at any interest rate.

Note that  $r_t$  strictly falls over time and converges to  $r$  (the riskless rate) as  $t \rightarrow \infty$ . This occurs because the class of borrowers who have not yet defaulted contains a decreasing proportion of borrowers with risky projects (and  $f_{Bt} \rightarrow 0$  as  $t \rightarrow \infty$ ). The decision facing the finite number of type BG borrowers is to choose between the two projects available to them given the anticipated time path of interest rates available to them specified in (5).

Because  $r_t \rightarrow r$  as  $t \rightarrow \infty$ , if the horizon  $T \rightarrow \infty$ , there will be an arbitrarily large number of periods in which, for all  $\varepsilon > 0$ ,  $r_t < r + \varepsilon$ . Lemma 9 then implies that the necessary conditions for the selection of safe projects on some  $t$  (e.g., condition [2]) are necessary and sufficient for the selection of safe projects at some date and for the endgame to have bounded length as  $T \rightarrow \infty$ . This is lemma 10.

LEMMA 10. If  $f_{BG} = 0$ , then the necessary conditions stated in lemma 8 for selection of safe projects on some  $t$  as  $T \rightarrow \infty$  are necessary and sufficient for a bounded endgame as  $T \rightarrow \infty$ .

This result can be directly extended to  $f_{BG} > 0$ , but not too large. On any date,  $r_t \leq r_t^b$ . Suppose that  $f_{BG}$  is low enough so that if  $f_{Bt} = 0$ , then  $r_t^b < dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ . As  $t \rightarrow \infty$ ,  $f_{Bt} \rightarrow 0$ , implying that there is a  $t < \infty$  such that  $r_t^b < dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ . If the horizon  $T \rightarrow \infty$ , then an immediate application of lemma 9 implies that the conditions in lemma 8 are necessary and sufficient for a bounded length endgame if  $f_{BG}$  is not too large.

If  $f_{BG}$  is large, implying that  $r_t^b$  might exceed  $dG + (1 - d)(G - \Pi B)/(1 - \Pi)$  even with a low  $f_{Bb}$ , the direct application of lemma 9 to an upper bound on future interest rates cannot be made. The next subsection analyzes the case of large  $f_{BG}$ .

### B. A Bounded Endgame as $T \rightarrow \infty$ when $f_{BG} > 0$

To establish that (2) and  $r_1 \leq G$  are necessary and sufficient for a bounded length endgame (i.e.,  $\tau \rightarrow \infty$  as  $T \rightarrow \infty$ ), we proceed in two steps. First, an interest rate path  $r_t$  that provides lenders with an expected return of  $r$  each period and yields a bounded length endgame is established. Then it is shown that borrowers will offer interest rates that result in a bounded length endgame.

When  $f_{BG} > 0$ , the possible increase in interest rates on a date  $\tau$  when a switch is made to risky projects implies that there may need to be some periods in which some type BG's select safe projects and others select risky projects. The interest rate  $r_\tau^b$  that prevails if all BG's select risky projects could be too high if  $f_{BG\tau}$ , the fraction of the original pool of borrowers who are of type BG and who have not defaulted by date  $\tau$ , is too high. For example,  $r_\tau^b$  could exceed  $G$  (and be inconsistent with an open market), or it could be less than  $G$  but be high

enough to imply such a low  $V_\tau$  that it would be inconsistent with selection of safe projects at  $\tau - 1$ . In either case a period of time is needed in which some type BG's select safe projects and others select risky projects. This would require that they be indifferent between the two projects on those dates.

If  $f_{BG}$  is large,  $r_t^b$  might exceed the level specified in lemma 9, even if  $f_{Bt} = 0$ . The following lemmas show that a bounded period of mixed strategy project selection will allow  $f_{BGt}$  to be reduced sufficiently to guarantee that after the mixed strategy period,  $r_t^b \leq dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ , allowing application of lemma 9 and the result that the period after the mixed strategy period is bounded. In addition we show that directly before the mixed strategy period, type BG's select safe projects in pure strategy.

A period of mixed strategy indifference obviously requires that on all dates of it, including the final date of indifference, BG's be willing to select safe projects. Lemma 11 gives a bound on  $V_{t+1}$  that is required for indifference at  $t$ . At some period  $t = t'$  after an indifference period, risky projects will be selected in pure strategy (e.g., at  $t = T$ ), and the interest rates  $r_{t'}^b$  will prevail. At any date  $t'$ , we know that  $r_t \leq r_{t'}^b$  for  $t > t'$ . Lemmas 12 and 13 show that a bounded period of mixed strategy before  $t'$  can reduce  $f_{BGt'}$  sufficiently to allow  $r_{t'}^b$  to be less than or equal to the level (given in lemma 9) that implies that safe projects will be selected at  $t'$ . The result, proposition 1, is that if the market does not fail, then the necessary conditions (in lemma 8) for reputation to have value are necessary and sufficient for a bounded length endgame.

LEMMA 11. There exists an interest rate  $r_t \in [r, G]$ , such that type BG's are indifferent between safe and risky projects, implying  $V_t^\xi = V_t^b$  if and only if  $V_{t+1} \in [L, H]$ , where  $L \equiv r - [(G - \Pi B)/(1 - \Pi)]$  and  $H \equiv G - [(G - \Pi B)/(1 - \Pi)]$ .

*Proof.*  $V_t^\xi = V_t^b$  implies (by lemma 7)  $V_{t+1} = r_t - [(G - \Pi B)/(1 - \Pi)]$ , and replacing  $r_t$  by  $r$  and by  $G$  provides the bounds  $L$  and  $H$ . Q.E.D.

Lemma 11 implies that if  $V_t \in [L, H]$ , indifference between projects is feasible (as is strict preference for either type of project in the open interval  $(L, H)$ ) as long as  $f_{Bt}$  is low enough to allow a low  $r_t^\xi$  and  $f_{BGt}$  is not too low for an  $r_t^\xi$  sufficiently above  $r$ . Lemma 12 provides a characterization of repeated periods of mixed strategy indifference between projects.

LEMMA 12. If  $V_t^\xi = V_t^b$ , with  $r_t \in [r, G]$ , implying indifference between safe and risky projects at  $t$ , then

- (a)  $V_t = d\{G - [(G - \Pi B)/(1 - \Pi)]\}$ ;
- (b)  $V_t \in [L, H]$ ; and
- (c) there exists  $r_{t-1} \in [r, G]$  such that  $V_{t-1}^\xi = V_{t-1}^b$  and  $r_{t-1} = dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ .

*Proof.*  $V_t^g = V_t^b$  implies, by lemma 7,

$$V_t = d\left(G - \frac{G - \Pi B}{1 - \Pi}\right) = dH < H,$$

$$V_t = d\left(G - \frac{G - \Pi B}{1 - \Pi}\right) = \frac{1}{1 - \Pi} [d\Pi(B - G)] > L$$

by (4). By lemma 11,  $V_t \in (L, H)$  implies that there exists  $r_{t-1} \in (r, G)$  with  $V_{t-1}^g = V_{t-1}^b$  and substitution of  $V_t$  into  $r_{t-1} = V_t + [(G - \Pi B)/(1 - \Pi)]$  produces  $r_{t-1} = dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ . Q.E.D.

Lemma 12 shows that repeated periods of mixed strategy indifference are possible as long as  $f_{B_t}$  and  $f_{BG_t}$  are such that  $r_t = dG + (1 - d)(G - \Pi B)/(1 - \Pi)$  is feasible, implying that  $r_t^b \geq dG + (1 - d)(G - \Pi B)/(1 - \Pi) \geq r_t^g$ . Note that the same interest rate,  $dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ , is also the bound specified in lemma 9 such that if there is the ability to borrow at future rates less than the bound, then a finite number of remaining periods before  $T$  guarantees that safe projects are optimal at the current date. A mixed strategy period ending at date  $t'$  can thus reduce  $f_{BG_{t'}}$  sufficiently to imply  $V_{t'+1} > L$ , to use the notation of lemma 11. To support a preceding repeated mixed strategy period, lemma 11 implies that one needs  $V_{t'+1} \in [L, H]$ . Lemma 13 provides this stronger result.

LEMMA 13. If the necessary conditions for reputation to have value in lemma 8 are true, then there exists a bounded mixed strategy period ending on a date  $t'$ , such that  $V_{t'} \in [L, H)$ .

*Proof.* See the Appendix.

The previous results imply that there exists an interest rate path such that the length of the endgame has an upper bound as  $T \rightarrow \infty$ : a mixed strategy period of bounded length can guarantee that thereafter  $r_t^b \leq dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ , and there exists a bounded length period thereafter that begins with  $V_t \in [L, H)$ ; this supports the mixed strategy period. This implies that there exists an interest rate path that leads to a bounded endgame. The result of lemma 6, that borrowers offer the lowest feasible rates each period, implies the stronger result that the interest rates actually offered by borrowers imply a bounded length endgame. This is because at any date before the shortest feasible endgame, it is feasible for all borrowers to offer the interest rate  $r_t^g$ , and by lemma 6, this is the rate that is offered. During the endgame, rates above  $r_t^g$  will be offered, but these are the lowest feasible rates on those dates.

Overall, we have established the following proposition.

PROPOSITION 1. If the loan market is active, then the endgame is of bounded length as  $T \rightarrow \infty$  if and only if (2) is true.

With this result that the endgame comprises a bounded number of periods near  $T$ , the focus on the few periods near the end is complete.

## VI. The Dynamics of Reputation with a Long Horizon

Far enough from the horizon,  $T$ , safe projects will be selected if and only if (2) holds. If (2) is false, there is never an incentive effect of reputation. We now focus on the case in which (2) is true and reputation eventually has value. To see how the incentive value of reputation evolves over time and develop intuition into how it evolves, two special cases of the model are developed. The contrast between the two cases will develop the relevant ideas, and Section VII shows that these intuitive ideas are true in the general model. There are two special cases: (1) near absence of adverse selection ( $f_B = 0$ ), with near absence of moral hazard ( $f_{BG} = 0$ ), and (2) significant adverse selection ( $f_B > 0$ ), with near absence of moral hazard ( $f_{BG} = 0$ ).

In both cases,  $f_{BG} = 0$  is assumed to allow the simple determination of interest rates to ease exposition. The two cases illustrate the importance of adverse selection in the dynamics of reputation by a comparison of  $f_B = 0$  with  $f_B > 0$ .

### A. Near Absence of Adverse Selection ( $f_B = 0$ ), with $f_{BG} = 0$

With  $f_G = 1$  and a finite number of other types, (5) shows that  $r_t = r$  for all  $t$ . At  $T$ , risky projects will be selected, and thus  $V_T = d\Pi(B - r)$  while  $r_{T-1} - V_T = r - d\Pi(B - r)$ . By lemma 7, safe projects are selected at  $t$  if and only if  $r - V_{t+1}$  greater than  $(G - \Pi B)/(1 - \Pi)$ , and for  $t'$  sufficiently near  $T$ ,

$$V_{t'} = \sum_{t=t'}^T (B - r)d\Pi^{1-t'+t},$$

which is a strictly decreasing function of  $t'$ . On some date  $t' < T$ , safe projects will be selected because the sum will exceed the critical value  $r - [(G - \Pi B)/(1 - \Pi)]$  and will exceed this value for all  $t < t'$ . This implies that on all dates  $t < t'$  safe projects will be selected because at any date  $t'$  the value of continuing to borrow is at least the value of borrowing and choosing the risky project each period. Figure 4 shows  $r - V_{t+1}$ , when interest rates are constant at the riskless rate,  $r$ .

The implication of this special case is that *if there is no significant adverse selection, so interest rates do not change as a function of one's reputation, then reputation works immediately if it ever works*: safe projects are selected at  $t = 1$  unless risky projects are selected for all  $t$ . This is consistent with the basic intuition from previous models of reputations.

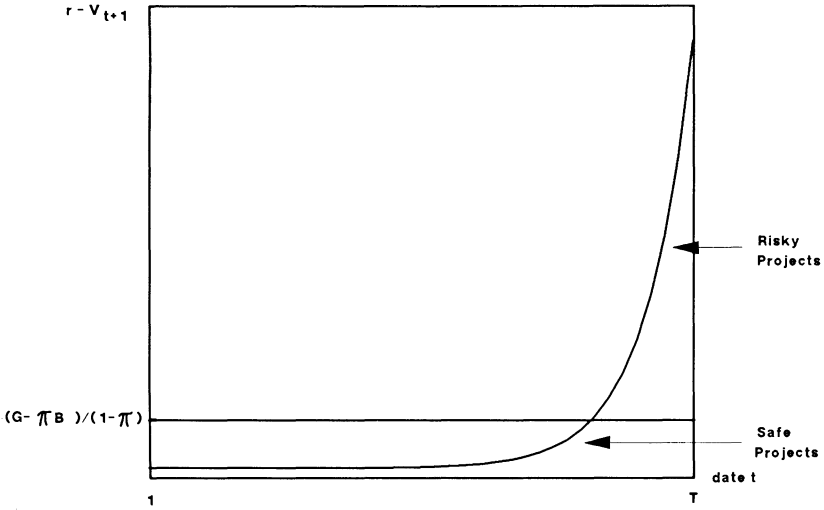


FIG. 4.—The time series of  $r_t - V_{t+1}$  assuming  $f_B = 0$ . Assuming that  $f_B = 0$  implies that interest rates are constant at the riskless rate, i.e.,  $r_t = r$ .

B. Significant Adverse Selection ( $f_B > 0$ ), with  $f_{BG} = 0$

With  $f_{BG} = 0$ ,  $r_t$  is given by (5). For large  $t$ ,  $r_t \rightarrow r$ , so if  $T \rightarrow \infty$ , then near  $T$  the analysis is similar to the last subsection, where  $r_t = r$ . However, for borrowers with short track records (small  $t$ ), the higher rates,  $r_t > r$ , have two effects because the decision between safe and risky projects depends only on  $r_t - V_{t+1}$ . Higher rates make safe projects relatively less attractive for given  $V_{t'+1}$ , and higher current rates reduce  $V_{t'}$ , making safe projects less attractive for  $t < t'$ . This implies that if  $f_B$  is high enough (initial interest rates are high enough), then the finite number of type BG's will select risky projects at some early dates, even though those who do not default will later select safe projects. In principle they might switch back and forth between the two projects. Proposition 2 provides a characterization of the scope for project switching assuming only that  $r_t$  falls over time. Under the assumption that  $f_{BG} = 0$ , (5) shows that  $r_t$  does fall over time. In Section VII, proposition 2 will be shown to apply with  $f_{BG} > 0$  if the remaining time before the horizon is sufficiently long.

PROPOSITION 2. If  $r_t$  falls over time and a type BG borrower optimally selects safe projects at  $t^+$  and selects risky projects at some  $t' < t^+$ , then risky projects are optimal for all  $t < t'$ . This implies that if safe projects are best on two dates  $t_1$  and  $t^+$  ( $t_1 < t^+$ ), then the optimal project selection is safe projects for all  $t \in [t_1, t^+]$ .

*Proof.* Let date  $\hat{t} \geq t'$  be the largest date that is less than  $t^+$  on which



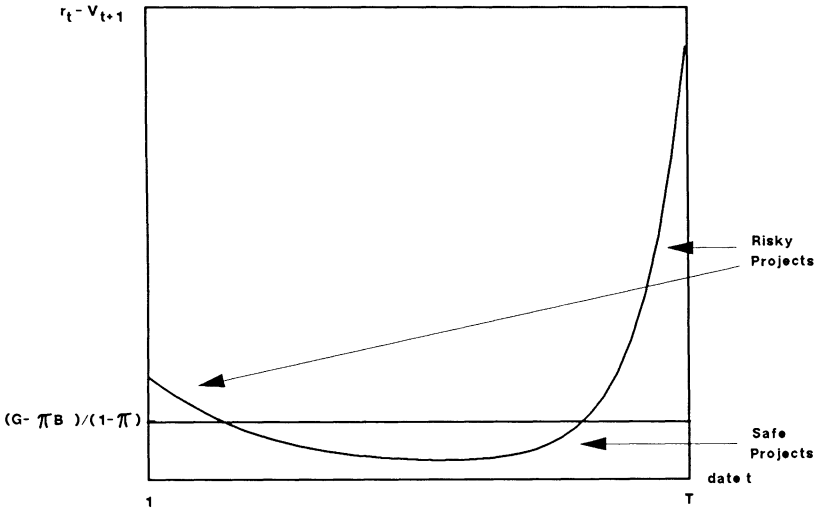


FIG. 5.—The time series of  $r_t - V_{t+1}$ , assuming  $f_B > 0$ ;  $f_B > 0$  implies that interest rates fall as a longer track record is acquired. Sufficiently high  $f_B$  implies that risky projects are best at  $t = 1$ .

risky projects are selected. Safe projects are best at  $\hat{t} + 1$ , and by lemma 7,  $V_{\hat{t}+1} \geq d\{G - [(G - \Pi B)/(1 - \Pi)]\}$ . Because risky projects are best at  $\hat{t}$ ,  $V_{\hat{t}} < d\{G - [(G - \Pi B)/(1 - \Pi)]\} \leq V_{\hat{t}+1}$ , and  $r_{\hat{t}} - V_{\hat{t}+1} > (G - \Pi B)/(1 - \Pi)$ . Interest rates fall, implying  $r_{\hat{t}} \leq r_{\hat{t}-1}$ , and combined with  $V_{\hat{t}} < V_{\hat{t}+1}$  we have

$$\begin{aligned}
 r_{\hat{t}-1} - V_{\hat{t}} &> r_{\hat{t}} - V_{\hat{t}+1} \\
 &> \frac{G - \Pi B}{1 - \Pi}.
 \end{aligned}$$

This implies that risky projects are best at  $\hat{t} - 1$ , and because  $r_{t'} \leq r_{\hat{t}}$  for all  $t' < \hat{t}$ , recursion implies that because  $V_{\hat{t}-1} < d\{G - [(G - \Pi B)/(1 - \Pi)]\} \leq V_{\hat{t}+1}$ , risky projects are best for all  $t < t'$ . Q.E.D.

Proposition 2 makes no use of the length of the time horizon. There are only two reasons for choosing risky projects: a short horizon or high current interest rates. Because interest rates fall over time conditional on not defaulting, the only reason for switching to risky projects once rates are low enough to justify safe projects is a short horizon. If there is a date on which safe projects are selected, then on such a date, or any earlier date, a short horizon is not the problem. If risky projects are selected on such an earlier date, the problem is high interest rates. If one goes back further in time, rates are higher, so risky projects are again best. Figure 5 shows the time path of  $r_t - V_{t+1}$ , computed using the interest rates in figure 3 (rates that decline over

time). Proposition 2 implies that if rates decline with longer periods of lack of default, then  $r_t - V_{t+1}$  plotted as a function of  $t$  can cross the threshold for choice of safe projects,  $(G - \Pi B)/(1 - \Pi)$ , at most twice. If the rates in the early periods are sufficiently above  $r$  because of a large fraction of type B's (but are less than  $G$ ), then the optimal choice on those dates will be risky projects, again by the reasoning of lemma 7. If the horizon is long, the reason is simpler. If the horizon is long enough,  $V_{t+1} \geq V_t$  because rates fall while the horizon is not significantly reduced (the proof is left to the reader).

The result of this example is that *when there is significant adverse selection (a heterogeneous pool of borrower qualities) in the initial pool of borrowers with no track record, then reputation initially will not deal with moral hazard problems, but instead a period of "reputation acquisition" will be required.* If the adverse selection is severe enough, so there are high interest rates (from [5]) in early periods for those with short track records and resulting low  $V_t$  for low  $t$ , reputation will not work in early periods, and risky projects will be selected for  $t = 1, \dots, \bar{t}$  until rates fall enough for  $r_t - V_{t+1}$  to be less than  $(G - \Pi B)/(1 - \Pi)$ . Borrowers without track records will select risky projects. Only some of those who select risky projects will default, and the others will get a good reputation. This will eventually lead to such a good credit rating and such low interest rates that type BG borrowers who have not defaulted will want to avoid loss of this valuable credit rating and will switch to safe projects for an unbounded number of periods until the endgame. That is, there will be a period of reputation acquisition, but eventually reputation will work to provide incentives. The next section shows that this result applies in general, with  $f_{BG} > 0$ , and provides a full characterization of equilibrium.

## VII. Equilibrium Path of Project Choices with $f_{BG} > 0$

Under the assumption  $f_{BG} = 0$ , characterization of equilibrium for any horizon length was a matter of computing the functions  $V_t^b$  and  $V_t^g$  using the interest rates in (5). With  $f_{BG} > 0$ , we need to determine whether the appropriate rate each period is  $r_t^g$  or  $r_t^b$ . The result of proposition 1, that all borrowers offer the lowest feasible interest rate, suggests the following characterization, which is established below. First, try a conjecture in which, from  $t = 1$  until the endgame, type BG's choose safe projects (and face interest rates  $r_t = r_t^g$  every period until the endgame). If this leads to interest rates that imply the conjectured behavior by type BG's, it is the equilibrium rate path, by proposition 1. On the other hand, if the interest rate series conditioned on the assumption that type BG's select safe projects leads type BG's to

find risky projects more profitable (because there are many type B's and thus high rates), then the equilibrium at  $t = 1$  involves their selecting risky projects (and paying higher rates that reflect this). If  $r_1 < G$  and (2) holds, eventually the fraction of type B's who have not defaulted gets low enough, and BG's will switch to safe projects for an unbounded number of periods ( $\tau \rightarrow \infty$  as  $T \rightarrow \infty$ ). If (2) is false and  $r_1 < G$  so the market does not fail, then reputation never has value and BG's select risky projects every period.

Consider more precisely the case in which markets do not fail and the necessary conditions for reputation to have value (e.g., [2]) are true. If  $T \rightarrow \infty$ , then both  $t$  and  $T - t$  can be made arbitrarily large. On such a date,  $t^*$ , proposition 1 states that safe projects are best and  $r_{t^*} = r_t^g$ . Because safe projects are best at  $t^*$ , by lemma 7,  $V_{t^*} \geq d\{G - [(G - \Pi B)/(1 - \Pi)]\}$ . Note that  $r_t^g$  is a strictly decreasing function of  $t$  because  $f_{B,t}$  is strictly decreasing in  $t$ . When one moves back in time, if  $r_t^g$  gets sufficiently high that risky projects are best at some date  $t'$ , then on that date  $r_{t'} = r_{t'}^b$ . In addition, at any date  $t < t'$ ,  $r_t \geq r_{t'+1}^g$ , so safe projects will not be selected at  $t < t'$ . This implies that interest rates fall over time until the endgame and that the result of proposition 2 applies: if  $T \rightarrow \infty$  and one considers dates away from the bounded endgame, then if safe projects are selected at  $t^*$  and risky projects at  $t' < t^*$ , then risky projects are selected for all  $t \leq t'$ .

To actually compute the values  $V_t$ , values for  $r_t^g$  and  $r_t^b$  are needed, and these depend on decisions by type BG's at earlier periods. Because proposition 2 applies, the only information needed to calculate these interest rates is the single date  $\bar{t}$ , near  $t = 1$ , where BG's switch from risky to safe projects. Define the face value  $R_t^g[\bar{t}]$  to be the one that gives lenders an expected return of  $r$  at date  $t$  if BG's choose safe projects at  $t$ , and the projects selected for dates less than  $t$  are risky projects from dates 1 to  $\bar{t} - 1$  and safe projects for  $t \in [\bar{t}, t - 1]$  (if  $t < \bar{t} - 1$ , obviously there are no past dates on which safe projects were selected). The term  $R_t^b[\bar{t}]$  is the face value that provides an expected return of  $r$  if risky projects are selected at  $t$ , and the projects selected at dates less than  $t$  are as described in the previous sentence. These rates are easily computed using the definitions of  $r_t^b$  and  $r_t^g$  because  $f_{B,t} = \Pi_B^{t-1}$  and  $f_{BG,t} = \Pi^{t-1}$  if  $t \leq \bar{t}$ , with  $f_{BG,t} = \Pi^{t-1}$  for  $t > \bar{t}$ .

The face values  $R_t^b[\bar{t}]$  are given by

$$\begin{aligned}
 R_t^b[\bar{t}] &= \frac{\Pi^{t-1}f_{BG} + \Pi_B^{t-1}f_B + f_G}{\Pi^t f_{BG} + \Pi_B^t f_B + f_G} \cdot r \quad \text{if } t < \bar{t}, \\
 R_t^b[\bar{t}] &= \frac{\Pi^{\bar{t}-1}f_{BG} + \Pi_B^{\bar{t}-1}f_B + f_G}{\Pi^t f_{BG} + \Pi^t f_B + f_G} \cdot r \quad \text{if } t \in [\bar{t}, \tau].
 \end{aligned}
 \tag{6}$$

The face values  $R_t^g[\bar{t}]$  are given by

$$\begin{aligned}
 R_t^g[\bar{t}] &= \frac{\Pi^{t-1}f_{BG} + \Pi_B^{t-1}f_B + f_G}{\Pi^{t-1}f_{BG} + \Pi_B^t f_B + f_G} \cdot r \quad \text{if } t < \bar{t}, \\
 R_t^g[\bar{t}] &= \frac{\Pi^{\bar{t}-1}f_{BG} + \Pi_B^{\bar{t}-1}f_B + f_G}{\Pi^{\bar{t}-1}f_{BG} + \Pi_B^{\bar{t}} f_B + f_G} \cdot r \quad \text{if } t \in [\bar{t}, \tau].
 \end{aligned}
 \tag{7}$$

Define the  $\tau$  element vector  $\mathbf{R}[\bar{t}]$  to be the face value series from 1 to  $\tau$ , where the first  $\bar{t} - 1$  elements (for  $t = 1, \dots, \bar{t} - 1$ ) are given by  $R_t^b[\bar{t}]$  and the final elements (from  $\bar{t}$  to  $\tau$ ) are given by  $R_t^g[\bar{t}]$ . This is the interest rate series anticipated given the definition of  $\bar{t}$ . The equilibrium value of  $\bar{t}$  is the smallest self-fulfilling value of  $\bar{t}$ : if  $\bar{t} = 1$  is self-fulfilling in the sense that safe projects are best if  $\mathbf{R}[1]$  (i.e.,  $r_t = R_t^g[1]$  for  $t = 1, \dots, \tau$ ) is anticipated, then there is not a period of reputation acquisition required and  $\bar{t} = 1$ .

Denote the present value of choosing *safe* projects from  $\hat{t}$  to  $\tau$  facing rates  $\mathbf{R}[\bar{t}]$  as  $W_{\hat{t}}[\bar{t}]$ , given by

$$W_{\hat{t}}[\bar{t}] = \sum_{t=\hat{t}}^{\tau} (G - R_t^g[\bar{t}])d^{1+t-\hat{t}} + (V_{\tau} \cdot d^{1+\tau-\hat{t}}).$$

As  $\tau \rightarrow \infty$ , the final term approaches zero, and one can approximate  $W_{\hat{t}}[\bar{t}]$  arbitrarily closely by  $\bar{W}_{\hat{t}}[\bar{t}]$ :

$$\bar{W}_{\hat{t}}[\bar{t}] \equiv \sum_{t=\hat{t}}^{\infty} (G - R_t^g[\bar{t}])d^{1+t-\hat{t}}.$$

If  $R_1^g[1] - \bar{W}_2[1] \leq (G - \Pi B)/(1 - \Pi)$ , then  $r_t = R_t^g[1]$  for all  $t \in [1, \tau]$ , and  $\tau \rightarrow \infty$  as  $T \rightarrow \infty$ . In this case, reputation works immediately. Alternatively, if  $R_1^g[1] - \bar{W}_2[1] > (G - \Pi B)/(1 - \Pi)$ , then risky projects will be selected at  $t = 1$ . Under the assumption that markets do not fail and that (2) holds, then eventually safe projects will be selected. This will occur on the smallest  $\hat{t}$  such that  $R_{\hat{t}}^g[\bar{t}] - \bar{W}_{\hat{t}+1}[\bar{t}] \leq (G - \Pi B)/(1 - \Pi)$ . Safe projects will be selected from that date for an unbounded number of periods until  $\tau$ , as  $T \rightarrow \infty$ .

This provides a characterization of interest rates and project decisions of type BG's up to any fixed  $\tau < \infty$  as  $T \rightarrow \infty$ , under the assumption that (2) holds and that markets do not fail. If (2) is false, then risky projects are selected each period if markets do not fail. The condition for markets not to fail is  $r_1 \leq G$ . If safe projects are best at  $t = 1$ , there are open markets if and only if  $R_1^g[1] \leq G$ . If risky projects are best at  $t = 1$ , there are open markets if and only if  $R_1^b[1] \leq G$ .

It is straightforward to show that the condition that determines the

optimal project at  $t = 1$  for type BG's,  $R_1^g[1] - \bar{W}_2[1]$ , is a strictly increasing function of  $f_B$  because higher  $f_B$  implies higher interest rates at all dates. Because  $R_1^g[1] - \bar{W}_2[1]$  is computed using the interest rates  $r_t^g$ , the condition does not depend on  $f_{BG}$ . There are two conditions for open markets. The first,  $R_1^g[1] \leq G$ , does not depend on  $f_{BG}$  but requires  $f_B$  to be below some positive level. The second condition,  $R_1^b[1] \leq G$ , requires an upper bound on  $f_B$  and  $f_B + f_{BG}$ . These results are used to add conditions on  $f_B$  and  $f_{BG}$  to the characterization of equilibrium. Further comparative static properties of  $R_1^g[1] - \bar{W}_2[1]$  are presented in Section VIIA. Proposition 3 summarizes our characterization.

**PROPOSITION 3.** For any fixed date  $\tau < \infty$ , there exists a horizon  $T < \infty$  such that, for all  $t < \tau$ , the following conditions hold:

- (a) If  $R_1^g[1] - \bar{W}_2[1] \leq (G - \Pi B)/(1 - \Pi)$  and  $R_1^g[1] \leq G$  (i.e.,  $f_B$  is sufficiently low and [2] holds), then there is an *immediate reputation equilibrium* in which safe projects are selected for all  $t \leq \tau$ , and interest rates are  $\mathbf{R}[1]$ .
- (b) If  $R_1^g[1] - \bar{W}_2[1] > (G - \Pi B)/(1 - \Pi)$  and  $R_1^b[1] \leq G$  (i.e.,  $f_B$  is above a positive critical level and  $f_B + f_{BG}$  is not too high) and (2) holds, then there is a *reputation acquisition equilibrium*: BG's choose risky projects on dates  $t < \bar{t}$  and safe projects on  $t \in [\bar{t}, \tau]$ , where  $\bar{t}$  is the smallest  $t$  such that  $R_t^g[\bar{t}] - \bar{W}_{t+1}[\bar{t}] \leq (G - \Pi B)/(1 - \Pi)$ . The equilibrium  $r_t$  is  $\mathbf{R}[\bar{t}]$ .
- (c) If  $R_1^g[1] - \bar{W}_2[1] > (G - \Pi B)/(1 - \Pi)$  and  $R_1^g[1] \leq G$  (i.e.,  $f_B$  and  $f_{BG} + f_{BG}$  are each not too high) and (2) is false, then there is *no reputation effect* and BG's select risky projects for all  $t \leq T$  with interest rates given by  $\mathbf{R}[\tau]$ .
- (d) If either  $R_1^g[1] - \bar{W}_2[1] \leq (G - \Pi B)/(1 - \Pi)$  and  $R_1^g[1] > G$  (i.e.,  $f_B$  is too high) or  $R_1^g[1] - \bar{W}_2[1] > (G - \Pi B)/(1 - \Pi)$  and  $R_1^b[1] > G$  (i.e.,  $f_B$  and  $f_B + f_{BG}$  are each too high and [2] is false), then *markets fail* on all  $t \leq T$ .

The reason a period of reputation acquisition in which risky projects are selected may be necessary is the same as with  $f_{BG} = 0$ : type BG's are pooled together with type B's, which leads to high initial interest rates. If there were very few type B's, then the initial interest rates would be low, near  $r$ , because rates are low under the conjecture that type BG's choose safe projects. The many type B's lead to high initial rates for all borrowers that imply a lower present value of rents in the future, weakening the cementing force of a valuable reputation.

It is the type B's in the initial pool of borrowers that cause reputation initially to be too weak: adverse selection prevents immediate reputation effects. The consequences of a weak initial reputation de-

pend on the large number of type BG's. The larger is  $f_{BG}$ , the larger is the difference between the interest rate  $r^f$  that occurs if type BG's choose safe projects and  $r^b$ , the rate that occurs when they choose risky projects.

#### A. A Summary of Comparative Statics

Simple calculations show that each of the following comparative static changes decreases  $R_1^f[1] - \bar{W}_2[1]$ , improving the relative payoff of the risky project at  $t = 1$  and making it less likely that reputation will provide incentives to those with short track records: (1) increase the fraction of type B's,  $f_B$ ; (2) decrease the payment of the safe project,  $G$ ; (3) increase the payment of the risky project when successful,  $B$ ; (4) increase the riskless interest rate,  $r$ ; (5) decrease the discount factor,  $d$ .

The first four changes increase  $R_1^f[1]$  and (weakly) decrease  $\bar{W}_2[1]$ , both of which lemma 7 shows improve the current relative payoff of the risky project. A decrease in the discount factor,  $d$ , makes the present value of any fixed decision in the future lower and decreases  $\bar{W}_2[1]$ , and this improves the relative payoff to selecting the risky project.

There is not an unambiguous comparative static effect on  $R_1^f[1] - \bar{W}_2[1]$  from reducing  $\Pi_B$ , the probability of type B's project succeeding, because a lower  $\Pi_B$  implies both that the initial face value  $R_1^f[1]$  is higher and also that  $R_1^f[1]$  falls at a more rapid rate. These two effects are weighted by the discount factor,  $d$ . Depending on the value of  $d$ , the net effect can go either direction. One simple result is that given pairs of  $f_B$  and  $\Pi_B$  that imply the same value of  $R_1^f[1]$ , higher  $f_B$  combined with higher  $\Pi_B$  improves the value of beginning with the risky project because that rate then falls less rapidly. This decreases  $\bar{W}_2[1]$ , the value of going into period 2 without a default. All the comparative static results above, except those dealing with  $f_B$  and  $\Pi_B$ , influence not only  $R_1^f[1] - \bar{W}_2[1]$  but also condition (2), the condition for reputation to have value at some date. Reputation acquisition requires  $R_1^f[1] - \bar{W}_2[1] > (G - \Pi B)/(1 - \Pi)$  and (2). The simplest explanation of when this is likely to be the case relates to the "amount of inequality" in (2), that is, by how much safe projects dominate risky projects when financed forever at the riskless rate of interest. If (2) is close to an equality, then values of  $f_B > 0$  and  $\Pi_B < 1$  that imply even a small increase in interest rates  $R_1^f[1]$  above  $r$  in early periods will tip the balance to risky projects because  $\bar{W}_2[1]$  needs to be near its maximum possible value,  $d(G - r)/(1 - d)$ , for safe projects to be best. If, instead, (2) is far from being an equality, then reputation acquisition will not be the equilibrium. This is because very large values of  $R_1^f[1]$  are necessary to tip the balance toward risky projects, implying high  $f_B$

and low  $\Pi_B$  that also imply rapidly falling rates. The value of  $R_1^g[1]$  necessary to make risky projects best at  $t = 1$  would exceed  $G$  and result instead in market failure.

In summary, if the maximal value of future rents is too small, so (2) is false, there will be no reputation effect. If there are sufficient rents so that (2) is true and is far enough from being an equality, then an active loan market implies immediate reputation. If the rents on safe projects are positive but not exceedingly large (and [2] is not too far from an equality), then there will be reputation acquisition in early periods with risky projects dominating if there is sufficient adverse selection: if  $f_B$  and  $\Pi_B$  imply that the face value  $R_1^g[1]$  is significantly above  $r$  for small  $t$ .

### VIII. Conclusion

The analysis of incentive problems in debt markets shows that it is likely that these problems will be most severe in early periods when new firms have short track records. If there is sufficiently widespread adverse selection, the initial pool of borrowers will be of low average quality and the interest rates for borrowers with short track records will be high. As a result, the present value of rents in the future from establishing a good reputation will start out very low. Rents can be sufficiently low that those with a choice of projects choose the short-run optimum, the risky low-value project. A fraction of those who select the risky project achieve success and are able to continually repay their loans, achieving a good reputation. As a borrower achieves a good reputation, the interest rate falls, and the present value of rents in the future from a good reputation rises. Eventually these rents become high enough for the borrower to switch to the long-run optimum, the safe high-value project, for an arbitrarily large number of periods until the endgame. Only if there is little adverse selection will reputation instead work to immediately provide incentives to new borrowers. The model specifies the reputation in terms of the credit rating, which is public information. Observable implications of the model then have empirical content.

A number of the model's conclusions are quite general and apply to the general study of reputation in markets. The key assumptions that differ from the reputation model in Kreps and Wilson (1982a) are that in my model, actions are not observed and there is a nontrivial fraction of all agent types. If actions (project choices) are directly observable, then unless there are incentives in the first period to take an action that is beneficial to one's reputation, there is never any incentive to take that action. In terms of my model, once observed selecting a risky project, a borrower can never credibly claim to be of

type  $G$ . In terms of the chain store in Kreps and Wilson (1982a), once it gives in to an entrant, it can never credibly claim to be "tough." The existence of nontrivial fractions of all types, that is, significant adverse selection, is important because otherwise the incentive effect of a reputation would be near its maximal value in the initial period. In my model, borrowing would begin at essentially the riskless rate of interest. In a more general setting, for example a market for goods or services of unobservable quality, if there is significant adverse selection, the market will have low expectations of initial quality, and the market will not pay very high prices for the output of agents without a long record. This implies that agents with short track records will have a low initial present value of rents in the future, and those with a choice will supply low quality. Only over time, with an acquired good record, will there be a large present value of rents in the future from maintaining a good reputation by providing high quality. In addition, although I have not modeled entry, low initial present value of rents is an appealing notion in a free-entry setting.

The model has direct applications to examinations of differential new project acceptance decisions: firms with certain reputations will turn down a given profitable project that others would accept. This can be interpreted as a well-defined cost of capital that is firm specific rather than project specific because of the private information about project decisions. In addition, the model can be used to explain, on the basis of public information, some determinants of which firms choose to borrow through financial intermediaries and use their delegated monitoring services (see Diamond 1988). If the intermediary (at a cost) can help control project decisions, then the model suggests that firms with short histories will do their reputation acquisition by borrowing from intermediaries. Firms with a long-standing high credit rating will borrow directly in the open market. These are just a few examples of what I hope will be a large harvest of extensions with strong empirical implications.

## Appendix

### *Proof of Lemma 4*

Lenders observe the past record of default/liquidation. Lenders believe the following: If there was a past liquidation, the project was assumed to return less than  $G$  that period, and type is assumed to be either  $B$  or  $BG$ , with probabilities specified by Bayes's law. If there were no liquidations in any previous period when credit was granted, a borrower's project returned at least  $r$  each past period (this is implied by lemma 3), and Bayes's law specifies conditional probabilities across the types that could produce such a record. The actions supported by these beliefs are that a default implies no future loans (this action is the important one) and a lack of past default leads to



future loans if they are at rates that offer an expected return of at least  $r$ . It turns out that the repayment decision does not depend on the specification of the action taken when there is no previous default.

Let  $V_t \geq 0$  be the beginning of period  $t$  present value of expected consumption of a borrower, making optimal project selection and loan repayment decisions from  $t$  to  $T$ . If a borrower repays less than  $r_t$ , then  $V_{t+1} = 0$  because those who default are not believed to be type G's. For all payments that avoid liquidation,  $V_{t+1}$  is a nonnegative constant. Let  $\theta_t$  be the realization of the project return of the borrower in period  $t$  (i.e.,  $\theta_t \in \{0, B, G\}$ ). The repayment selected at the end of period  $t$  by a borrower is  $Z_t$ ; it is subject to the constraints  $Z_t \leq \theta_t$  and  $Z_t \geq 0$ . The constraints imply that if  $\theta_t = 0$  then  $Z_t = 0$ .

Discounted expected consumption of the borrower with  $\theta_t \geq G \geq r$  at date  $t$  for each of the three actions  $Z_t = r_t$ ,  $Z_t > r_t$ , and  $Z_t < r_t$  is given by

$$\underbrace{\text{payoff from } Z_t = r_t}_{\theta_t - r_t + V_{t+1}} \quad > \quad \underbrace{\text{payoff from } Z_t > r_t}_{\theta_t - Z_t + V_{t+1}} \quad \geq \quad \underbrace{\text{payoff from } Z_t < r_t}_{0}.$$

All borrowers with  $\theta_t \geq r_t$  select  $Z_t = r_t$ . Only borrowers with project returns less than  $r_t$  default, and they pay  $Z_t = 0$ . This implies that the beliefs above are self-fulfilling when  $r_t \leq G$ .

For completeness, consider the case of  $r_t > G$ . It might appear that lenders' beliefs would be contradicted by the necessity of type G's defaulting if  $r_t > G$ . However, future lenders condition on the fact that a loan was made in a past period, and lemma 3 shows that lenders would not lend at date  $t$  if  $r_t > G$ .

*Proof of Lemma 9*

It is always feasible to select safe projects each period, and we know, for  $t \geq \hat{t}$ , that  $r_t < dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ , implying

$$V_{\hat{t}} \geq \sum_{t=\hat{t}}^T (G - r_t)d^{t+1-i} > \sum_{t=\hat{t}}^T \left\{ G - \left[ dG + (1 - d)\frac{G - \Pi B}{1 - \Pi} \right] \right\} d^{t+1-i}.$$

Taking the limit of the final expression as  $T \rightarrow \infty$  yields

$$\begin{aligned} & \frac{d}{1 - d} \left\{ G - \left[ dG + (1 - d)\frac{G - \Pi B}{1 - \Pi} \right] \right\} \\ &= \frac{d}{(1 - d)(1 - \Pi)} \{ [G(1 - d)(1 - \Pi)] + [(1 - d)(G - \Pi B)] \} \\ &= d \left( G - \frac{G - \Pi B}{1 - \Pi} \right). \end{aligned}$$

This implies that one can find  $T < \infty$  such that the  $V_i > d\{G - [(G - \Pi B)/(1 - \Pi)]\}$ . By lemma 7, one can then conclude that safe projects are optimal at date  $\hat{t}$ . To prove that  $dG + (1 - d)(G - \Pi B)/(1 - \Pi) > r$  if and only if (4) holds,

$$\begin{aligned} dG + (1 - d)\frac{G - \Pi B}{1 - \Pi} &= d \left( G - \frac{G - \Pi B}{1 - \Pi} \right) + \frac{G - \Pi B}{1 - \Pi} \\ &= \frac{d\Pi(B - G)}{1 - \Pi} + \frac{G - \Pi B}{1 - \Pi} \\ &> r \end{aligned}$$

if and only if (4) holds. Q.E.D.

*Proof of Lemma 13*

A mixed strategy period can be continued on a date on which  $f_{B_t}$  is close to zero if  $r_t^p > dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ , and continuing the mixed strategy period reduces  $f_{B_{G,t}}$  at an increasing rate. This implies that a finite number of indifference periods, ending on date  $t^0$ , will reduce  $f_{B_{G,t}}$  sufficiently so, for  $t \in [t^0, T]$ ,  $r_t \leq r_t^p < dG + (1 - d)(G - \Pi B)/(1 - \Pi)$ . By lemma 9, there exists  $T < \infty$  such that  $V_{t^0} \geq d\{G - [(G - \Pi B)/(1 - \Pi)]\} = H$ , implying that safe projects can be best at  $t^0$ . For such a fixed  $T$ , there exists  $t^1 \leq T$  such that  $V_{t^1} < L$  because  $V_T < L$  by (1). This implies that the mixed strategy period could end on date  $t^0$  such that  $V_{t^0} \geq H$  or on date  $t^1$  such that  $V_{t^1} \leq L$ . There then exists an ending date for mixed strategy period  $t' \in [t^0, t^1]$  such that  $V_{t'} \in [L, H]$ : choose the largest ending date  $t$  such that  $V_t < L$ , implying that  $V_{t-1} \geq L$ . If  $V_t \leq L$ , then risky projects are selected at  $t - 1$ , and by lemma 7,  $V_{t-1} \leq d\{G - [(G - \Pi B)/(1 - \Pi)]\} = dH < H$ . Thus  $V_{t-1} \in [L, H)$ . Q.E.D.

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