The University of Chicago

Multiple Currency Paradigm: An analysis from the perspective of consumers

By

Wei Zhang

A thesis submitted in partial fulfillment of the requirement for the Master of Art degree in the Master of Arts Program in the Social Sciences

Faculty Advisor: Harald Uhlig
Preceptor: Gina Pieters

Multiple Currency Paradigm: An analysis from the perspective of consumers

Abstract

This paper uses the New Keynesian framework designed in Uhlig and Xie [2021] to explore how consumer's adoption of multiple digital currencies can transform the dominant currency paradigm (DCP) into a multiple currency paradigm (MCP). The result shows that consumers' changing preference for currencies can change the relative demands across sectors and prompt firms to adjust their pricing strategy. The consumers, therefore, trigger a reallocation of the currency distribution across international markets at the aggregate level, transforming a DCP into an MCP, with implications for sectoral output gap and inflation.

Keywords— Digital Currencies, Parallel Currencies, New Keynesian Model

1 Introduction

This paper studies a transformation dynamic from Dominant Currency Paradigm (DCP) to Multiple Currency Paradigm (MCP) by analyzing the adoption of digital currencies from consumers' perspectives. Digital currencies can break the physical boundaries across borders. They provide consumers with more purchasing options, leading to a deeper engagement by consumers in the market. This dynamic transfers market power from producers to consumers. Consumers (households) thus have a strong incentive to choose and use different digital currencies to participate in cross-country economic activities through digital platforms. These digital platforms can enable consumers to find a trading partner that matches their personal needs at low cost, facilitating a match between demand and supply at a decentralized individual level (Greenstein [2020]). These fundamental changes in the behavioral pattern of consumers have implications for the changing behaviors of firms. Given the adoption of the new money by consumers, firms face a large market of buyers holding many kinds of currencies. They are obliged to adjust their pricing strategy to obtain an optimal profit level, reallocating the currency distribution and transforming the DCP.

To study this dynamic, this paper adopted and modified the New Keynesian framework in Uhlig and Xie [2021]. Uhlig and Xie [2021] have created a macroeconomic dynamic to study the distribution pattern of currencies in presence of multiple digital currencies. This paper has extended the topic. It uses a modified NK framework to study the impact of consumers' changing behavioral pattern on firms' pricing strategies. The NK framework in this paper includes two parts: households and firms. Consumers can choose to hold multiple currencies and firms can choose their optimal pricing strategy across countries to maximize the market value of profits. In the short term, consumers can change their behaviors to hold multiple currencies to maximize their consumption needs. In the medium to long run,firms can change their pricing plan based on customer value-based strategy (Toni et al. [2016]).

This model has a new addition, the preference parameter. It uses this variable to identify the level of desirability (preference) that consumers have for obtaining different kinds of currencies. The level of desirability depends on the households' expectations about how difficult to obtain these currencies and how efficiently they work as a medium of exchange. This assumption implies that there is an expected cost associated with the adoption of any single currency. Such an expected cost can be either a direct or indirect cost. An example could be a tax imposed on consumers or mining fees paid to cryptocurrency miners (Schilling and Uhlig [2019]). The preference for currencies varies from one to another from consumers' perspectives. Some specific elements of the new digital money can impact households' evaluation of their preference level. (Ching and Hayashi [2010]) has listed the elements of ease-of-use, affordability, security as the critical ones that can impact the popularity of digital currencies in public.

The result shows that a preference shock can reduce people's desirability for holding currencies. At the aggregate level, the perceived price level increases, and people's demand for output declines. This dynamic can lead to a relative change of demand in a two-sector economy at the sectoral level. This finding further implies that the changing preference for currencies by consumers can prompt firms to adjust their pricing strategies. Such a adjustment can reallocate the currency distribution and potentially transform the DCP into MCP.

This finding contributes to the academic literature by providing analysis about how the consumers' preference for digital currencies can transform the real sector of the economy. It implies that consumers as separate individuals can trigger a transformation of the real sector at the macro level. The reason is that digital currencies distributed through different digital platforms can encourage consumers to satisfy their consumption needs without conforming to a collective preference (Greenstein [2020]). Thus they drive individuals to act differently at the aggregate level. These decentralized consumption behaviors can dilute the monopolistic power of big firms. Small enterprises gain more access to the foreign market in this process. These trends lead to an efficient match between demand and supply.

A stream of literature has discussed the benefit that consumers can gain from an efficient match between demand and supply. Glaeser et al. [2018] has used empirical data to specify the efficiency of digital platforms in providing information to boost the growth of local economies. (Moshe A. Barach [2019]) studied how digital development has prompted the market to build a credibility system that can strengthen the trust between suppliers and buyers. These findings have confirmed the critical role that digital inventions play, but they stay at the individual level without further implications for the real sector of the economy. However, they have built a bedrock that verifies consumers' strong incentive in holding multiple currencies, which is the fundamental assumption on which this paper builds.

Some literature has also provided a detailed discussion on consumers' preferences for holding a currency and explained why they prefer one over the other. Molnar et al. have pointed out those attributes that impact consumers' preference for holding a currency, including their ease-of-use, affordability, and security. Ching and Hayashi [2010] has talked about the advantage of using cash, and Kosse et al. [2017] has discussed why cash still plays a more critical role than credit and debit in satisfying households' basic consumption needs. The reason is that they are easy to obtain, and they can support some nuanced practices in a sophisticated social context. In this sense, digital currency is similar to cash in providing convenient payment methods for consumers. Only if users have a phone can they make transactions with anyone at any place. This discussion on consumers' preference for modern monies has stayed at the micro-level, and it only has implications for individuals' behavioral patterns. By integrating these attributes, this paper aims to discuss the impact of consumers' preferences for currencies in a macroeconomic dynamic.

The two main streams of literature that this paper uses as a foundation to build on the model are Uhlig and Xie [2021], and Gopinath et al. [2020]. Uhlig and Xie [2021] have discussed in detail the parallel currencies and price stickiness in a domestic sphere. They have proposed the adoption of multiple currencies under the New Keynesian framework. In this scheme, firms are allowed to choose their pricing strategies freely. They are motivated to reset their optimal price setting at the beginning of each time. The finding is that the impact of an exogenous exchange rate shock on a domestic economy depends on the level of price rigidity. The macroeconomic framework in Uhlig and Xie [2021] is the bedrock that this paper uses to conduct this analysis from the consumers' perspective.

Another stream of literature that this research paper uses is the depiction of the DCP by Gopinath et al. [2020]. Gopinath et al. [2020] has used the empirical data to reveal some critical implications of DCP in international trade. DCP has implied a pass-through effect of exchange rate shocks across countries. It implies that international trade is sensitive to the fluctuation of the dominant currency's exchange rate but less sensitive to the bilateral exchange rates between trading partners (Gopinath et al. [2020]). The economic intuition is that firms commit to a complementary pricing strategy to maximize their presence in the international market given fast globalization (Gopinath [2015]). This trend has reinforced the monopolistic power of big firms and excluded small ones from the competition. It also passively forces consumers to choose from provided options only. Digital currencies distributed through online platforms have interfered in this power dynamic. It prompts consumers to take the initiative. By achieving a more accurate and efficient match between the demand and supply, digital currencies transform consumers' behavioral patterns at a micro-level aggregately. This paper compares the impulse response of preference shock against the pass-through impact of the exogenous exchange rate shocks to test whether households' desirability for holding digital currencies can transform the DCP.

Last but not least, it is essential to point out the type of digital currencies that this paper refers to before starting the discussion of the primary model. Based on the technical design, digital currencies can fall into different categories (Pieters). This paper has used digital currency as a general term to denote all kinds of digital and cryptocurrencies. It assumes that these new monies have decentralized attributes, which would enable households to participate in economic activities in different sectors of the international economy. Based on the distribution method, digital currencies can fall into two categories. They can be distributed to individual households through retail or large financial institutions through wholesale (Boar and Wehrl [2021]). This paper does not define who should be the distributor of the new money; it can be either a government authority or individual automata (Keister and Sanches [2019]). In reality, the majority of Central Bank Digital Currency (CBDC) projects have planned to issue their new digital currencies through the retail distribution according to Boar and Wehrl [2021], stressing a focus on targeting consumers as the primary users of digital currencies.

The following part of the paper consists of five sections. Section 2 discusses the basic set-up of the New Keynesian Model and introduces the new concepts of liquidity and transaction cost. The goal is to depict the departure from the model's classic design and shows how the new addition fits into the classic design. In section 3, it shows the log-linearization of the model and the fundamental propositions used for building up the impulse response analysis of newly introduced variables. In section 4, it presents the result of the IRF to analyze the changing behaviors of households in the short run. It also discusses the practical meanings associated with the analytical results. In section 5, it extends the discussion of this bottom-up distribution dynamic, depicting how the changing behaviors of households can transform profit-driven firms by changing their pricing strategies. Lastly, in section 6 of the conclusion, I briefly talk about the potential extension of this topic in other directions.

2 The model

This section has used the money-in-utility function in the New Keynesian Model (Galí [2015]) to conduct the impulse response analysis of the preference shock τ on the output gap and inflation at an aggregate level. The preference shock determines the desirability level for currencies from the perspective of consumers. The macroeconomic environment set-up is built on the framework in Uhlig and Xie [2021]. The goal is to prepare a baseline model to derive sectoral dynamics in a two-sector economy in the next section. This step also prepares us to compare the IRF of the preference parameter τ to the IRF of an exogenous exchange rate that is implied by the DCP (Gopinath et al. [2020]), examining a potential transformation from DCP to MCP.

The model consists of households and firms. Households obtain enough liquidity to satisfy their expected consumption needs in the future, and firms adjust their pricing strategies to maximize their market value of profits. The liquidity of households is a function of transaction cost τ_t. The total liquidity is the sum of sectoral liquidity of all sectors. This section will introduce notations and the modified concept of liquidity, which have provided a base for building up the NK framework.

2.1 Indexes and Variables

In an economy of multiple digital currencies, V_j denotes each currency with $j = 1, 2, 3...n$ and v_j denotes the partition of each currency within the pool. The exchange rate is denoted as $\epsilon_{j,t}$ and $\epsilon_{j,t}$ is the exchange rate of conversing currency j to the dollar at time t. Thus the exchange rate of the dollar is 1:

$$\epsilon_{1,t} = 1$$

and $\Delta\epsilon_t$ denotes the unexpected exchange rate shock, such that:

$$\Delta\epsilon_t = \epsilon_{j,t} - \epsilon_{j,t-1}$$

The money supply of each currency type is denoted as M_j. The total money balances that households keep is conversed into dollar values:

$$M_t = \sum_{j=1}^{J} \epsilon_{j,t} M_{j,t}$$

The preference level of holding a currency is denoted by $(1 - \tau)$. At the aggregate level, it is assumed that τ is the same across all sectors. At the sectoral level, τ_1 and τ_2 are different and they respectively denote the preference shock for V_1 and V_2. When this dynamic of the distribution of digital currencies is discussed in a simple economic environment consisting of two sectors. Sector one uses the V_1, the dollar (an official currency), to invoice its products. Sector two uses an invented new digital money V_2 to invoice its products. V_1 is the dominant currency as referred to in DCP .

2.2 Liquidity and Devalued liquidity

There are two concepts of liquidity presented in the model. Liquidity L_t denotes the real value of money balances in the budget constraint of consumers; liquidity D_t denotes the devalued real value of money balances in the money-in-utility function of consumers.

Liquidity is L_t defined as the real money balances in Uhlig and Xie [2021]:

$$L_{j,t} \equiv \frac{\epsilon_{j,t} M_{j,t}}{P_t}$$

The aggregate liquidity is the sum of sectoral liquidity of all sectors, which is:

$$L_t = \sum_{j=1}^{J} \frac{\epsilon_{j,t} M_{j,t}}{P_t} \tag{1}$$

The liquidity that is devalued based on households' preferences is:

$$D_{j,t} \equiv \frac{\epsilon_{j,t} M_{j,t}}{P_t}(1 - \tau_{j,t}) \tag{2}$$

Where $\epsilon_{j,t}$ denotes the exchange rate of converting currency V_j to V_1 and M_j is the amount of the balance of currency j. By multiplying the exchange rate $\epsilon_{j,t}$, we can converse the value of currencies into the dollar. By multiplying $(1-\tau_{j,t})$, we can obtain the devalued liquidity that counts in the perception of the relative desirability that households hold for each currency.

Households' desirability for holding a currency depends on their assessment of how valuable it is to hold a specific currency. The preference shock τ_j devalues the real balance of V_j. A higher τ_j implies a higher level of difficulty obtaining access to V_j and a lower preference for holding V_j by households. It can also imply a relatively higher preference for holding other currencies from the perspective of consumers.

Some factors have contributed to the increasing attraction to digital currencies. With the development and popularity of electronic payment systems and digital currencies, households have access to a larger market at a low cost. Households can participate in transactions in foreign markets that can use multiple digital currencies. This trend can increase the desirability for digital currencies by consumers at a large scale, accordingly implying a low τ for digital currencies from the perspective of domestic consumers.

On some occasions, the invention of digital currencies also aims to satisfy specific needs other than consumption. These digital currencies can better support nuanced practices in sophisticated social contexts, satisfying households' needs for using the new money in many aspects of life (Kow et al. [2017]). In this case, a digital currency that supports the nuanced practices of a group implies a low τ from the perspective of households.

2.3 Households

Households consume a bundle of products C at time state t. They provide an amount of labor N at time t to satisfy their consumption needs (Galí [2015]). The aggregate liquidity L denotes the real value of money balances; the devalued liquidity D denotes the real value of money balances devalued by the preference level $(1-\tau)$. τ denotes such a preference shock. The devalued liquidity is a function of τ and L. 2.2 has mentioned factors that contribute to the changing of τ.

At this stage of setting up the model at the aggregate level, we can assume that $\tau_{j,t}$ is identical across all sectors. This assumption will be changed in section 3 while discussing the analysis in a two-sector economy, where we can use τ_1 and τ_2 to denote the expected cost of holding two currencies, respectively.

The money-in-utility function, written in terms of C, N and D, is shown in the below:

$$u(C_t, N_t, D_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{D_t^{1-\xi} - 1}{1-\xi} - \frac{N_t^{1+\psi} + 1}{1+\psi} \tag{3}$$

While written in terms of the C, N, L and , it is:

$$u(C_t, N_t, L_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{[L_t(1-\tau_t)]^{1-\xi} - 1}{1-\xi} - \frac{N_t^{1+\psi} + 1}{1+\psi} \tag{4}$$

where σ is the parameter of relative risk aversion associated with consumption, ξ is the relative aversion of holding liquidity, and ψ is the Fisher elasticity of labor supply. A higher ξ implies a general aversion towards holding extra balances, which denotes a baseline cost of holding currencies applied to all. The initial values given to those parameters in the calibrated model are in Appendix D.

Assume the continuum of goods with the interval $[0, 1]$. The total consumption of goods by households is denoted as:

$$C \equiv \Big[\int_0^1 \Upsilon(i) C_t(i)^{1-\frac{1}{\eta}} di\Big]^{\frac{\eta}{\eta-1}} = \Big[\sum_{j=1}^{J} \int_{V_{j,t}} \Upsilon(i) C_t(i)^{1-\frac{1}{\eta}} di\Big]^{\frac{\eta}{\eta-1}} = 1 \tag{5}$$

The demand system simulates the framework set-up in Galí [2015]. η is the elasticity of substitution among products. $\Upsilon(i)$ is the measure of preference over products by consumers. A higher $\Upsilon(i)$ implies a high preference for products made by firm i, which implies that they are less likely to be substituted. $\Upsilon(.)$ as a preference parameter is adopted from Kimball [1995].

Households face a dynamic optimization problem:

$$E_t \sum_{t=0}^{\infty} \beta^t u(C_t, D_t, N_t) \tag{6}$$

Where E_t is the expectation operator, calculating the utility of household based on the information obtained at a time state t.

Households are facing a period budget constraint:

$$\sum_{j=1}^{J} \int_{V_{j,t}} \epsilon_{j,t} P_t(i) C_t(i) di + B_{t+1} + \sum_{j=1}^{J} \epsilon_{j,t+1} M_{j,t+1} + \varsigma_t P_t = W_t N_t + B_t(1 + i_t) + \sum_{j=1}^{J} \epsilon_{j,t} M_{j,t} \tag{7}$$

Where P_t denotes the price level at time t, W_t is the nominal wage level, and households receive wages denominated in the official currency dollar. In this setup, consumers only receive wages denominated in the main official currency. B_t is the amount of financial wealth that consumers hold at the beginning of time t, denominated in the dollar value. ς_t can be a lump-sum addition or subtraction of government tax or dividend of firms in its real value. The money balances of all digital currencies are converted into the main official currency by multiplying the exchange rate $\epsilon_{j,t}$.

By solving the dynamic maximization problem of households, the good-specific demand function from the perspective of households is:

$$C_t(i) = \left[\Upsilon(i) \frac{\epsilon_{j,t} P_t(i)}{P_t}\right]^{-\eta} C_t \tag{8}$$

In an economy of multi-goods, an aggregate price index is defined as:

$$P_t \equiv \left[\int_{V_j} (\epsilon_{j,t} P_t(i))^{1-\eta}\right]^{\frac{1}{1-\eta} di} \tag{9}$$

To achieve the optimal level of utility, it demands the relationship between the setoral price and the aggregate price to satisfy:

$$\int_{V_j} \epsilon_{j,t} P_t(i) C_t(i) di = P_t C_t \tag{10}$$

where the total expenditure in each sector is equal to the sum of all sectoral expenditures.

By plugging in the optimal solution to the budget constraint, we can obtain a budget constraint that households face in an aggregate economy:

$$P_tC_t + B_{t+1} + \sum_{j=1}^{J} \epsilon_{j,t}M_{j,t+1} + \varsigma_t P_t = W_tN_t + B_t(1+i_t) + \sum_{j=1}^{J} \epsilon_{j,t}M_{j,t} \tag{11}$$

Then, by converting the budget constraint into the official currency and divided it by the price level of the official currency, we can obtain a budget equation in the real term:

$$C_t + B_{t+1} + \sum_{j=1}^{J} L_{j,t+1} + \varsigma_t = W_tN_t + \frac{B_t(1+i_t)}{\Pi_t} + \sum_{j=1}^{J} \frac{L_{j,t}}{\Pi_t}\frac{\epsilon_{j,t+1}}{\epsilon_{j,t}} \tag{12}$$

2.4 Firms

Firms can set their pricing strategy at the beginning of each time state to obtain an optimal profit level. Calvo [1983] denotes the pricing friction as a natural index of price stickiness θ. The concept of pricing friction implies that firms face a possibility of $(1-\theta)$ to change their pricing strategy at each time state t. The price stickiness index θ is independent of time t.

To denote the infinite number of firms that produce differentiated goods in the monopolistic competitive market as $i = 1, 2, 3...n$. The production function of a firm is a function of the technology and labor:

$$Y_{j,t}(i) = A_tN_{j,t}(i)^{1-\alpha} \tag{13}$$

where α denotes the share of labor input in the production (parameters with initial values in Appendix D). Firms face a dynamic maximization problem:

$$\max \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}(\epsilon_t P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\} \tag{14}$$

The objective function above is constrained by the demand function (8), where $Y_t(i) = C_t(i)$. Q is the discount factor, $\frac{1}{\beta^t}$ that converses the future value of the profit into the present value. $\Psi(.)$ denotes the marginal cost of the production.

2.5 Equilibrium

In the equilibrium, while assuming that all produced goods were consumed by households, the production equals to the demand, as shown in the equation (8):

$$Y_t = C_t \tag{15}$$

Households adjust their consumption plan and manage the balances of their currencies to maximize their utility. The money balances that households keep denotes the amount of liquidity that consumers wish to maintain. Consumers maintain a certain level of liquidity to satisfy the consumption plans that can happen in the near future. While keeping all other variables constant and satisfying the budget constraint , the optimal condition satisfies the following FOC condition, such that:

$$U_{c,t}dC_t + U_{L,t}dL_t = 0$$

The calculation of the FOC of $U_{c,t}$ and $U_{L,t}$ is shown in the Appendix A.2.

Combing the optimal condition above with the equilibrium equation (15), we can obtain:

$$y_t = \frac{\beta}{\sigma} E_t\{C_{t+1} + E_t[\pi_{t+1}] - E_t[\Delta\epsilon_{t+1}]\} + \frac{\xi(1-\beta)}{\sigma} L_t - \frac{(1-\xi)}{\sigma}\tau_t \tag{16}$$

where the level of liquidity at the equilibrium is a function of liquidity excluding the devalued amount written in terms of the preference shock τ_t. As mentioned in the beginning of this section, τ_t is assumed to be the same across sectors. It will be assumed to be different across sectors in the analysis of a two-sector economy in the next section.

The clearing of the liquidity demands that:

$$D_t = \int_{V_{j,t}} D_t(j)dj \tag{17}$$

3 Linearization of a two-sector economy model

This section elaborates on the linearization of the model, and the development of a dynamic IS curve. It first shows the function of the aggregate output, aggregate inflation, and nominal rates that respond to those two variables. Then it shows the sectoral output, inflation, and nominal rates. The steady-state is at the zero inflation state.

This section starts with the introduction of perceived relative price. It then discusses the optimal pricing strategy by firms. The goal is to set up a bedrock for establishing a dynamic environment in which we can study the impact of the changing behaviors of households in the short term and explore its influence on firms' pricing strategy in the medium to long run.

3.1 The relative price

The perceived relative price s_t denotes the perceived purchasing power of one currency in terms of another. By including the cost of holding currencies, s_t is to represent the perceived real purchasing power of digital currency. This concept is adopted and modified from Uhlig and Xie [2021]:

$$s_t \equiv \frac{\epsilon_{2,t} P_{2,t}(1+\tau_{2,t})}{P_{1,t}(1+\tau_{1,t})} \tag{18}$$

In the set-up of a two-sector economy, the preference variable τ_1 and τ_2 helps to identify the relative desirability of holding an official currency and a digital currency, respectively, from the perspective of households. The real value of money balances is devalued based on these preference variables. Holding one currency to consume in one sector bears the opportunity cost of not consuming in the other sector. The sector that uses a main official currency is named the dollar sector; the other is the non-dollar sector.

Many reasons can contribute to the changing desirability for currencies by consumers. When an unexpected production increase occurs, households would have a more substantial consumption capability and consume more. Additionally, When some sanctioned regions have no access to the main currency, they would resort to digital currencies as an alternative, which can be obtained easily by households at the domestic level. The assumption is that consumers change their behavioral patterns in the short term; a surge in consumption trend urges firms to reset their pricing strategies.

Another assumption is that firms can obtain comprehensive information about consumers' desirability to hold each currency. A higher τ_1, t denotes a declined desirability for holding the dollar V_1. A relative change of demands in these two sectors results in a higher price level in sector one. Comparatively, tau_2 increases in a relative term, denoting a relative increase in sector two's price level.

3.2 The optimal pricing strategy

To obtain the optimal market value of profits, firms face a production function (13) and an objective maximization function (14):

$$\max \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}(\epsilon_t P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\}$$

To derive the FOC of the objective function above at the steady-state of zero inflation, we can obtain:

$$\sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k} Y_{t+k|t}(\epsilon_t P_t^* - \mathcal{M}\Psi_{t+k|t}\} = 0 \tag{19}$$

Where $\mathcal{M} = \frac{\eta}{1-\eta}$, denoting the frictionless markup. Accordingly, the optimal price level is written as:

$$P_t^* = \mathcal{M}\Psi_{t|t} = \mathcal{M}mc_t \tag{20}$$

Plugging in the inflation function of $\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t}$, the FOC of the objective function can be written as:

$$\sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k} Y_{t+k|t}(\epsilon_t \frac{P_t^*}{P_{t-1}} - \mathcal{M}mc_{t+k|t})\Pi_{t-1,t+k}\} = 0 \tag{21}$$

The function above yields a pricing path:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{mc}_{t+k|t} + p_{t+k} - p_{t-1} + e_{j,t}\} \tag{22}$$

where $\hat{mc}_{t+k|t}$ denotes the deviation from the steady state marginal cost. Assume that the steady state value of the marginal cost is determined, which is μ_{mc}, we can re-write the equation of the price above as:

$$p_t^* = \mu_{mc} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{mc}_{t+k|t} + p_{t+k}\} \tag{23}$$

Equation (23) implies that firms set their prices by adding a desired markup over a weighted average of the currency and expected marginal cost. The weighting is determined by the price stickiness index θ.

3.3 Aggregate inflation

This section discusses the set-up of the aggregate inflation. Given that firms can reset their pricing strategies in each period, inflation is defined as:

$$\Pi_t \equiv \frac{P_{t+1}}{P_t} \tag{24}$$

All firms in a sector j will invoice their products in currency j.The aggregate price level is represented by the equation (8), rewritten as the sum of sectoral prices:

$$P_t \equiv \left[\sum_{j=1}^{J} \int_{V_{j,t}} (\epsilon_{j,t} P_t(i))^{1-\eta} di \right]^{\frac{1}{1-\eta}} \tag{25}$$

As discussed in 2.4, price stickiness index is denoted as θ, implying that firms face a possibility of $(1-\theta)$ to change their price setting. The aggregate dynamic of inflation is written as a function of the price stickiness θ:

$$\Pi_t^{1-\eta} = \theta + (1-\theta)(\frac{\epsilon_{j,t} P_t^*}{\epsilon_{j,t-1} P_{t-1}})^{1-\eta} \tag{26}$$

where P_t' is the optimal pricing that firms adopt at time t.

In the steady state of 0 inflation, $\Pi = 1$, which implies that $\Pi_t = 1, p_t^* = p_t = p_{t+1}$. Take the log-linearization of equation (26) to obtain the aggregation inflation dynamic:

$$\pi_t = (1-\theta)(p_{t+1}^* - p_t + \Delta\epsilon_t) \tag{27}$$

The details about how to derive the inflation is in the Appendix A.1. By substituting the solution of marginal cost (75) to the pricing equation (23) in the appendix A.1, it yields that:

$$p_t^* - p_{t-1} = \beta\theta E_t[p_{t+1}^* - p_t + \Delta\epsilon_{j,t}] + (1-\beta\theta)\Theta\hat{mc}_t + \pi_t \tag{28}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\eta}$. Plugging in equation (27) into equation (28) to obtain the inflation function:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \hat{mc}_t \tag{29}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$, and $\hat{mc}_t = (\sigma + \frac{\alpha+\eta}{1-\alpha})(y_t - y_{n})$, as shown in the Appendix A.1.

By substituting $\hat{mc}_t$ in equation (29), the expression of inflation is written as:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - y_t^n) = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t \tag{30}$$

Where $\kappa \equiv \lambda(\sigma + \frac{\alpha+\varphi}{1-\alpha})$.

The sectoral inflation can be written as a deviation from the optimal price level under the homogeneity of price rigidity. The law of motion of inflation in a two-sector economy is:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t + v\Delta\epsilon_t \tag{31}$$

3.4 Nominal interest rate

The nominal interest rate responds to the aggregate inflation and aggregate output under the baseline monetary policy. The function denotes the formation of interest is from Uhlig and Xie [2021]. In particular, $\phi_p i$ denotes the response of nominal rates to the changing in inflation; ϕ_y denotes the response of nominal rates to the changing in the output gap. The nominal interest rate is a function of output gap and inflation:

$$i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t \tag{32}$$

Where ϕ_π and ϕ_y respectively denotes the impact of π_t and $\tilde{y_t}$ on nominal rates. ϕ_π and ϕ_y are non-negative. The real interest rate is a function of technology:

$$\hat{r_t}^n = -\sigma\varphi_{ya}(1-\rho_a)a_t \tag{33}$$

In a baseline calibrated condition, $\varphi_{ya} = 1$, the set-up is consistent with the economic intuition such that a technology increase would result in a decline in employment. The relationship between the nominal interest rate and real interest rate is defined as:

$$\hat{i_t} = \hat{r_t} + E[\pi_t] \tag{34}$$

3.5 The dynamic of output gap

The dynamic IS curve which constituted the macroeconomic environment is a function of liquidity, the detailed calculation is in the appendix A.2

$$\tilde{y_t} = \frac{\beta}{\theta}\{E_t[C_{t+1}] + E_t[\pi_{t+1}] - E_t[\Delta\epsilon_{t+1}]\} + \frac{\xi(1-\beta)}{\sigma}L_t - \frac{(1-\xi)}{\sigma}\tau_t \tag{35}$$

Take the log-linearization of the money balance divided by the price level of the dollar, yielding the real value of the money balances(1):

$$\begin{aligned} L_t = \tfrac{MB_t}{P_t} &= \Delta\epsilon_t + m_t - p_t \\ &= \Delta\epsilon_t + \tilde{y_t} - \eta_m i_t \end{aligned} \tag{36}$$

Where v_t denotes the partition of the dollar (currency 1). The money demand is derived from the quantity theory of money, where $\frac{M}{P} = \frac{Y}{V} \Rightarrow m_t - p_t = y_t - (\eta_m i_t)$. η_m is the semi-elasticity of money demand (Galí [2015]). In the calibrated model, $\eta = 4$. The result is derived from the OLS regression of M2 inverse velocity by using 3 month treasury bills.
Inflation follows the law of motion of:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\tilde{y_t} + v_t\Delta\epsilon_t \tag{37}$$

When price is flexible, which is $\theta = 0$, the sectoral natural level of output is equal to the natural level of the output:

$$y^n_{j,t} = y^n_t \tag{38}$$

When price is rigid, the price stickiness index $\theta \in (0.1]$, The sectoral output gap depends also on η, the elasticity of substitution among products:

$$\tilde{y_{j,t}} = \tilde{y_t} - \eta\hat{p}_{j,t} \tag{39}$$

4 Impulse Response Function - Result Analysis

At the sectoral level, we assume that the preference shock $\tau_{j,t}$ is identical for holding any currency. The aggregate output gap and inflation reacts to the changing preferences for currencies. At the sectoral level, the analysis is conducted in a two-sector economy. $\tau_{1,t}$ and $\tau_{2,t}$ respectively represents the preference shock for V_1 and V_2. The sectoral output and inflation react to $\tau_{1,t}$ and $\tau_{2,t}$ differently.

We assume that the dollar sector mainly uses V_1 as the medium of exchange. The dollar sector accounts for a large share of the market, and most households hold V_1. At the sectoral level, a positive shock of $\tau_{j,t}$ implies an increased cost of holding V_1 and a declined desirability for obtaining it at time t. $\tau_{2,t}$ decreases in the relative term, and it implies higher desirability for holding V_2 at time t. The changing of τ_1 and τ_2 implies the changing preferences for V_1 and V_2 from consumers' perspective, resulting in the reallocation of the currencies when firms change their pricing strategies to obtain higher revenue.

4.1 Aggregate dynamic IS Curve

This section discusses the dynamic model at the aggregate level by looking into the impulse response function of aggregate output, aggregate inflation, and nominal rate. Both exchange rate shocks and acquisition shocks (preference shocks) influence the path of these economic indicators. Intuitively, a preference shock cause the demand for products decreases and both the aggregate output and inflation respond to τ_1 and τ_2 negatively. This step of the analysis at the aggregate level sets up a fundamental for the sectoral analysis.

To focus on the shocks of the preference τ and the exchange rate shock $\Delta\epsilon_t$, this section has excluded the productivity from the log-linearized utility function of households. It assumes that the exchange rate follows a random-walk process. This assumption coincides with the law of motion of $\Delta\epsilon_t$ in Manuelli and Peck [1990], Benigno and Uhlig [2019], Uhlig and Xie [2021] and Gopinath et al. [2020] (*Proposition 1* is in Appendix B.1).

Proposition 1. The exchange rate is a martingale and follows a random-walk process.

Proof in Appendix B.1.

The law of motion in *Proposition 1* implies that:

$$e_{j,t} = E_t[e_{j,t+1}] \tag{40}$$

The forecast of endogenous variables are:

$$E_t[e_{j,t+1}] = 0; E_t[\tilde{y}_{t+1}] = 0; E_t[\pi_{t+1}] = 0$$

In a two-sector economy, τ_t is calculated as the sum of weighted $\tau_{1,t}$ and τ_2, weighted by the partition of V_1 and V_2 correspondingly, such that:

$$\tau = v_1\tau_1 + v_2\tau_2 = v\tau_1 + (1-v)\tau_2$$

where $v_1 = 1$.

The constructed dynamic IS curve is obtained through combining equations (35), (36) and (37). The functions of aggregate indicators are written as the impulse response function in terms of $\Delta\epsilon_t$, $\Delta\tau_{j,t}$:

$$\begin{aligned} \tilde{y}_t &= \xi(1-\beta)(1-\eta_m\phi_\pi v)\Delta\epsilon_t/\Omega \\ &- (1-\xi)v\Delta\tau_{1,t}/\Omega \\ &- (1-\xi)(1-v)\Delta\tau_{2,t}/\Omega \end{aligned} \tag{41}$$

$$\begin{aligned} \tilde{\pi}_t &= [\kappa\xi(1-\beta)(1-\eta_m\phi_\pi v)/\Omega + v]\Delta\epsilon_t \\ &- \kappa(1-\xi)v\Delta\tau_{1,t}/\Omega \\ &- \kappa(1-\xi)(1-v)\Delta\tau_{2,t}/\Omega \end{aligned} \tag{42}$$

$$
\begin{aligned}
\tilde{i}_t &= \phi_y \xi(1-\beta)(1-\eta_m \phi_\pi v)\Delta\epsilon_t/\Omega + \phi_\pi[\kappa\xi(1-\beta)(1-\eta_m\phi_\pi v)/\Omega + v]\Delta\epsilon_t \\
&- \phi_y(1-\xi)v\Delta\tau_{1,t}/\Omega - \phi_\pi\kappa(1-\xi)v\Delta\tau_{1,t}/\Omega \\
&- \phi_y(1-\xi)(1-v)\Delta\tau_{2,t}/\Omega - \phi_\pi\kappa(1-\xi)(1-v)\Delta\tau_{2,t}/\Omega
\end{aligned} \tag{43}
$$

Where Ω is:

$$
\Omega = \sigma - \xi(1-\beta)(1-\eta_m\phi_y - \eta_m\phi_\pi\kappa) \tag{44}
$$

The impact of exogenous exchange rate shocks on output gap is φ_{ye}, and the value depends on the following equation:

$$
\varphi_{ye} = \xi(1-\beta)(1-\eta_m\phi_\pi v)\Delta\epsilon_t/\Omega \tag{45}
$$

In the calibrated model, ϕ_π is the reaction of nominal rates to inflation with an initial value of 0.5/4; ϕ_y is the reaction of nominal rates to output gap with an initial value 1.5. ξ denotes households' preference for holding currencies over other assets. When the preference for holding currencies is high, households tend to have higher desirability to obtain liquid assets to consume. It has an initial value 0.99.

At the aggregate level, a higher preference shock that causes a decline in the desirability to consumer can dampen the aggregate output. The analysis at this level has not yet differentiate the different impact of τ_1 and τ_2. The general conclusion is that when it is expensive to hold currencies, people's desirability for holding extra amount of liquidity declines and reduces the demand and aggregate output at the macro level. A lower demand results in a deflation accordingly. At this stage, the model has presented the mechanism about how consumers change their desirability for holding currencies facing their perceived preference for them.

4.2 Sectoral dynamics

This section derives the relationship between the dollar sector and the non-dollar sector's output gap and inflation. The dollar sector mainly uses the official currency, which faces a preference shock of $\tau_{1,t}$ at time t; the non-dollar sector mainly uses a digital currency, which faces a preference shock of $\tau_{2,t}$. τ_1 and τ_2 denotes consumers' preference for holding each currency, respectively, and they work differently on the two sectors' output and inflation.

Assuming the homogeneity of price stickiness across the two sectors, the sectoral dynamics of output gap and inflation follows a linear relationship in terms of the perceived relative price. The perceived relative price s_t follows an autoregressive process, which help to derive sectoral impulse response functions. These relationships are elaborated through the following propositions.

Proposition 2. Under the assumption of homogeneity of price stickiness, the bilateral relative price between the two sectors of economy follows an autoregressive process, as proved in Appendix B.1.

$$
s_t = \theta(s_{t-1} + \Delta\epsilon_{2,t} + \Delta\tau_{2,t} - \Delta\tau_{1,t}) \tag{46}
$$

Proposition 3. There is a linear relationship between the inflation of the two sectors of economy, as proved in Appendix B.1.

$$
\tilde{y}_{1,t} - \tilde{y}_{2,t} = -\eta s_t \tag{47}
$$

Proposition 4. There is a linear relationship between the output gap of the two sectors of economy, as proved in Appendix B.1.

$$
\pi_{1,t} - \pi_{2,t} = \frac{\theta - 1}{\theta} s_t \tag{48}
$$

Proposition 2 implies that s_t denotes the perceived purchasing power of currencies. An appreciation of the dollar and a preference shock of digital currency will cause an increase in the price level in the non-dollar sector. The perceived relative price has determined the linear relationship between two sectors' output and inflation. The implication is that the output gap and inflation in the two sectors change with the preference shock τ_1 and τ_2 in a relative term.

Proposition 3 is derived from the demand function (8). It says that the elasticity of substitution determines the discrepancy between the output gap of two sectors. A high level of elasticity implies an easy substitution, leading to a relative change in the two sectors' demand. When the substitution effect is low, it is less likely to see the relative change in the two sectors' demand because consumers find it hard to find substitutions elsewhere. Their loyalty to the currency that they hold is high.

Proposition 4 says that the inflation is a function of the price stickiness θ. When $\theta = 1$, the price is fully sticky across sectors, and the price level does not respond to the changing of perceived relative price s_t. When the price is flexible, the price level is sensitive to the changing of perceived relative price. In this case, consumers' changing preference for currencies is likely to cause a relative change in inflation in both sectors.

The calculation for the sectoral dynamic function is in A.4. The sectoral dynamic of output gap and inflation in the dollar sector are represented by the function:

$$\tilde{y}_{1,t} = \tilde{y}_t + v\eta s_t \tag{49}$$

$$\tilde{\pi}_{1,t} = \pi_t + v(1-\theta)s_{t-1} \tag{50}$$

The relative change in demand for output in the dollar sector consists of two parts. One is determined by the changing of the income at the aggregate level, and the other is determined by the substitution effects. The relative change in inflation in the dollar sector also consists of two part. One is determined by the changing of inflation at the aggregate level, and the other is determined by the level of price stickiness.

The sectoral dynamics of output gap and inflation in the non-dollar sector are represented by the function:

$$\tilde{y}_{2,t} = \tilde{y}_t - (1-v)\eta s_t \tag{51}$$

$$\tilde{\pi}_{2,t} = \pi_{1,t} - \frac{1-\theta}{\theta} s_t \tag{52}$$

The dynamic mechanism based on which the output and inflation changes is the same across two sectors. The degree of discrepancy is determined by the partition of currencies v and the price friction θ. In the rest of the analysis, it is assumed that the elasticity of substitution and price stickiness is the same in both sectors of the economy.

4.3 Uncovered Interest Rate Parity

The nominal rates across sectors follow the assumption of Uncovered Interest Rate Parity (UIP). UIP implies that

$$\frac{1+i_{1,t}}{1+i_{2,t}} = \frac{\epsilon^e_{2,t+1} - \epsilon_{2,t+1}}{\epsilon_{2,t}} \tag{53}$$

The log-linearization of UIP yield that:

$$i_{1,t} - i_{2,t} = \Delta\epsilon_{2,t} \tag{54}$$

By combing the sectoral inflation equation (37), the sectoral inflation in the dollar sector is written as:

$$\pi_1 = \beta E[\pi_1, t+1] + \kappa y_1 + v\Delta\epsilon_{2,t} \tag{55}$$

The nominal rates in the dollar and non-dollar sector are presented in terms of undetermined coefficients as:

$$i_1 = \phi_y y_1 + \phi_\pi[\beta E[\beta_1, t+1] + \kappa y_1 + v\Delta\epsilon_{1,2,t}] \tag{56}$$

$$i_2 = i_1 + S_t/\theta - S_{t-1} + \Delta\tau_{2,t} - \Delta\tau_{1,t} \tag{57}$$

4.4 IRF Analysis

This section presents the impulse response analysis of the preference shock τ in a two-sector economy. τ_1 and τ_2 denote the preference shock for V_1 and V_2, respectively. These two variables measure how difficult it is for households to obtain and use the currency V_1 and V_2. The goal is to measure how consumers' level of desirability for holding the two currencies impacts the output and inflation in the two sectors of the economy. This preference shock can be a higher cost of obtaining a currency, impacting people's desirability to hold the currency. It assumes that households tend to maintain an optimal level of liquidity to satisfy their needs for using multiple currencies to consume.

4.4.1 Dollar Sector

In the dollar sector, consumers and producers mainly use the currency V_1 (the dollar) to engage in economic activities. By assuming that V_1 is the dominant currency, it implies that the partition of the dollar v_1 is bigger or equal to 0.5. In the calibrated model, v_1 is set up with an initial value of 0.8. Under homogeneity of price stickiness in the two-sector economy, θ is given a value of $\frac{3}{4}$. The cost of holding V_1 is τ_1, denoting a level of desirability for holding the dollar from the perspective of consumers.

With the rise of electronic payment and the invention of blockchain technology, households' consumption options are no longer limited to the options provided domestically. They can access a broader market by directly engaging in transactions that surpass the physical territory border. These digital platforms can build up a direct match between producers and consumers, diluting the monopolistic power of big firms across countries and providing small enterprises access to foreign markets.

The following section provides an analysis of IRF analysis with a negative preference shock for V_1 and a positive preference shock for V_2. This simulates a scenario in which consumers tends to use digital currency more frequently than the official one. The goal is to explore how this trends of adopting digital currencies can induce a dynamic change in the relative demand of two sectors and lead to a reallocation of currencies.

The sectoral equations of output gap and inflation are derived from aggregate equations and the sectoral linear relationship. The detailed calculation is in A.4. The result yields:

$$
\begin{aligned}
\tilde{y}_{1,t} &= v\eta s_t \\
&+ \xi(1-\beta)(1-\eta_m\phi_\pi v)\Delta\epsilon_t/\Omega \\
&- (1-\xi)v\Delta\tau_{1,t}/\Omega \\
&- (1-\xi)(1-v)\Delta\tau_{2,t}/\Omega
\end{aligned} \tag{58}
$$

$$
\begin{aligned}
\tilde{\pi}_{1,t} &= \tfrac{v(1-\theta)}{\theta} s_t \\
&+ [\kappa\xi(1-\beta)(1-\eta_m\phi_\pi v)/\Omega + v]\Delta\epsilon_t \\
&- \kappa(1-\xi)v\Delta\tau_{1,t}/\Omega \\
&- \kappa(1-\xi)(1-v)\Delta\tau_{2,t}/\Omega
\end{aligned} \tag{59}
$$

The impulse responses function is a function of the shock of exchange rate and preference shock. $\tau_{1,t}$ is a preference shock to V_1, the main official currency. $\tau_{2,t}$ is preference shock to V_2, the digital currency. A high $\tau_{1,t}$ implies lower desirability for obtaining V_1, thus a less preference for holding V_1 to satisfy consumer needs in the near future. This trend can signify a deviation from the DCP because consumers tend to hold less V_1. A low $\tau_{2,t}$ implies high desirability for obtaining V_2, thus a higher preference for using V_2 to consume. This trend denotes a transformation into MCP. The following part of the analysis looks into the detail of undetermined coefficients of these exogenous shocks, exploring their impacts on output and inflation. These indicators will help us examine whether the transformation from the DCP to MCP is plausible when consumers change their preference for currencie when they can hold multiple currenciess.

The impact of $\tau_{1,t}$ is represented by the undetermined coefficient $\varphi_{y_1\tau_1}$:

$$
\varphi_{y_1\tau_1} = -v\eta\theta - (1-\xi)v/\Omega \tag{60}
$$

In the calibrated model, $\sigma = 1$ and $\eta_m = 4$ (parameters in the Appendix D). Ω is a coefficient with a positive value, as referred to in equation (44). The absolute value of $\varphi_{y_1\tau_1}$ increases with ξ. ξ denotes consumer's willingness to hold liquidity instead of bonds or other assets. In the notation (60), $\varphi_{y_1\tau_1}$ consists of two parts. The first part $v\eta\theta$ implies a tendency to hold less total liquidity to consume due to a decline in total income at the aggregate level. The degree of change depends on the currency partition v and the elasticity of substitution η. When the substitution level is high across markets and consumers have less loyalty to products, a preference shock for V_1 can cause a large relative decline in demand for the dollar sector's production. The second part $(1-\xi)v/\Omega$ implies a tendency to substitute a currency from the perspective of consumers. When ξ is large, consumers prefer holding liquidity over other assets. A preference shock for currency V_1 can cause less decline in the dollar sector's demand.

Notice that, $\varphi_{y_1\tau_1}$ is positive only when:

$$
\eta < \frac{1-\xi}{\Omega} = \frac{1-\xi}{\sigma - \xi(1-\beta)(1-\eta_m\phi_y - \eta_m\phi_\pi\kappa)} \tag{61}
$$

In other conditions, $\varphi_{y_1\tau_1}$ will always be negative, implying a negative impact on the dollar sector's output under a negative preference shock for V_1, which is denoted by a positive shock of τ_1. Assuming ξ, the preference for holding liquidity over other assets, remains unchanged, the elasticity of substitution between two sectors need to be small to dampen consumers' incentive to exchange V_1 for V_2 to consume (leaving the dollar sector). A low level of elasticity implies that consumers will find it hard to find substitutions in other markets, and it can cause large costs for consumers to convert from one sector to the other.

Then we look into the impact of a negative preference shock for V_1 on inflation. It is the same as a positive shock of τ_1. The undetermined coefficient $\varphi_{\pi_1\tau_1}$ is written as:

$$\varphi_{\pi_1\tau_1} = -v(1-\theta) - \kappa(1-\xi)v/\Omega \tag{62}$$

The impact on dollar sector's inflation consists of two parts. The first part is determined by the price friction θ. When the price is flexible, the inflation is more sensitive to a positive τ_1 shock. The second part is induced by the change in demand. When a positive shock of τ_1 causes a negative impact on dollar sector's output, the inflation would encounter the same degree of decline multiplied by κ. The intuition is that when the cost of holding V_1 increases, the demand for products in the dollar sector declines. A lower demand results in a lower price level.

Therefore, a declined desirability for holding V_1 dampens the dollar sector's demand and lead to a lower price level, prompting consumers to convert to digital currency as an alternative option to consume. When the elasticity of substitution is high, consumers can move from one market to another at low cost. This trend will result in a reallocation of currencies with a smaller partition in V_1, leading to a transformation from the DCP to MCP.

The rest of this section discusses the impact of a negative shock τ_2 on output and inflation in the dollar sector. A negative shock of τ_2 denotes a higher preference for holding V_2. An increased desirability for holding V_2 can also result in the relative change of demand in two sectors.

The undetermined coefficient $\varphi_{y_1\tau_2}$ is written as:

$$\varphi_{y_1\tau_2} = v\eta\theta - \kappa(1-\xi)(1-v)/\Omega \tag{63}$$

This coefficient can also be decomposed into two parts. The first part depends on the change in demand, which is determined by the elasticity of substitution and the price friction. When the elasticity of substitution is high, consumers assume low cost to transfer from one market to the other, implying that they can obtain the same products everywhere easily. In this case, when the preference shock of V_2 decreases in comparison to that of V_1, consumers' desirability for holding V_2 to consume increases because of their easy access to it. Thus the demand in the non-dollar sector increases. The second part measures the substitution effects, which is determined by how easily consumers can substitute one currency for the other. When people have a high preference for holding liquidity over other assets, consumers tend to hold more of both V_2 and V_1. If the official currency dollar has a large partition v_1 to start with, the increase of output in the non-dollar sector caused by the substitution effect is relatively small, as shown in equation (63). Both impacts lead to an increased demand for output in the non-dollar sector. This trend will trigger a transformation to the MCP, implying a reallocation of currency with more partition distributed to V_2.

The coefficient that denotes the impact of a preference shock for V_2 on the dollar sector's inflation is $\varphi_{y_2\pi_2}$, written as:

$$\varphi_{\pi_1\tau_2} = v(1-\theta) - \kappa(1-\xi)(1-v)/\Omega \tag{64}$$

The inflation in the dollar sector also responds to a positive preference shock for V_2 in two ways. The level of price friction and the relative change in demand. When price is flexible, the price level is more sensitive to the change in the perceived price level. When consumers are encouraged to obtain V_2 to consume, the perceived price in the dollar sector increases, prompting consumers to leave for the non-dollar sector. In the calibrated model, both effects lead to a decline of the price level in the dollar sector.

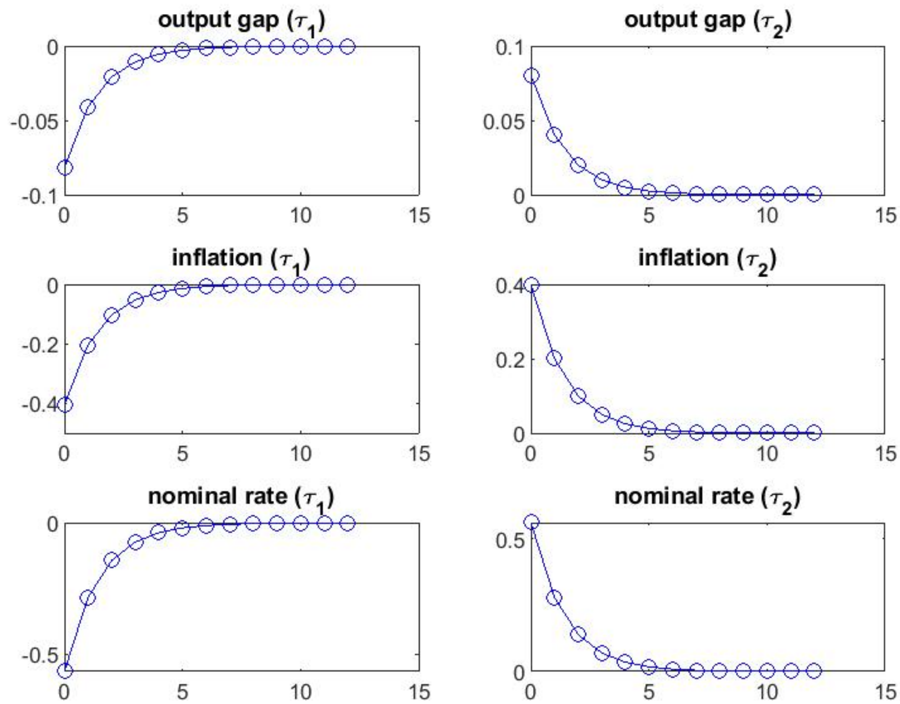

Figure 4.1. In the dollar sector, a positive shock of τ_1 implies a higher preference for the currency $V2$. It prompts consumers to switch to the non-dollar sector for consumption, causing a relative decline in the dollar sector's output. Inflation declines by responding to the change in output and the level of price flexibility. A positive shock of τ_2 works at the opposite direction.

Different initial values of v results in different IRF results in output, inflation and nominal rates. Appendix C.1 shows the degree of changes based on different initial values of the partition of a currency. In C.1, v is set as parameters with different values to show these results.

4.4.2 Non-dollar Sector

The non-dollar sector mainly uses a non-official digital currency V_2. The partition of currency V_2 is defined as $v_2 \equiv 1 - v$. Because this is the non-dominant sector, v_2 is some number smaller than 0.5. In the calibrated model, v_2 is given an initial value of 0.2. The preference shock to the currency V_2 is τ_2. A positive preference shock to V_2 implies that consumers have an easy access to the digital currency. It correspondingly results in a relative change in the expected pricing level in both sectors. Similarly, this section focuses on an IRF analysis with a negative preference shock for V_1 and a positive preference shock for V_2.

By using the linear relationship between two sectors' output, inflation and nominal rates, by shown in equations (46), (47) and (48), we can obtain the following sectoral equations :

$$\begin{aligned}\tilde{y}_{2,t} &= (v-1)\eta s_t \\ &+ \xi(1-\beta)(1-\eta_m\phi_\pi v)\Delta\epsilon_t/\Omega \\ &- (1-\xi)v\Delta\tau_{1,t}/\Omega \\ &- (1-\xi)(1-v)\Delta\tau_{2,t}/\Omega\end{aligned} \tag{65}$$

$$\begin{aligned}\tilde{\pi}_{2,t} &= \tfrac{(v-1)(1-\theta)}{\theta}s_t \\ &+ [\kappa\xi(1-\beta)(1-\eta_m\phi_\pi v)/\Omega + v]\Delta\epsilon_t \\ &- \kappa(1-\xi)v\Delta\tau_{1,t}/\Omega \\ &- \kappa(1-\xi)(1-v)\Delta\tau_{2,t}/\Omega\end{aligned} \tag{66}$$

To examine whether the adoption of digital currency can transform a DCP to MCP, we have to observe how the output and inflation in the non-dollar sector change with the preference shock to V_1 and V_2. The analysis in both the dollar and non-dollar sectors will depict a dynamic of currency reallocation.

To assess the impact of τ_1 on the non-dollar sector of the economy, we expand the undetermined coefficient of τ_1, which is $\varphi_{y_2\tau_1}$, yielding that:

$$\varphi_{y_2\tau_1} = -(v-1)\eta\theta - (1-\xi)v/\Omega \tag{67}$$

How the output in the non-dollar sector responds to a positive shock of τ_1 is twofold. The substitution effects induce the first one. When consumers exchange V_1 to V_2 to consume, the output increases in the non-dollar sector. The degree of the change depends on the elasticity of substitution η. When consumers can buy similar products across different markets, a declined preference for V_1, which is induced by a positive shock of τ_1, can prompt consumers to leave for the non-dollar sector. This trend can lead to a relative change in demand. The second one is the decline in output at the aggregate level. The degree of change depends on consumers' preference for holding liquidity over other assets, ξ. When ξ remains unchanged, the increase of the output induced by the substitution effect takes the lead. A relative increase in demand for the non-dollar sectors' products leads to a reallocation of currencies, with V_2 taking a more significant portion of the currency allocation. It denotes a transformation from DCP to MCP.

The impact of a negative preference shock to V_1 on the non-dollar sector's inflation is presented below. The undetermined coefficient of it is written as $\varphi_{\pi_2\tau_1}$:

$$\varphi_{\pi_2\tau_1} = (1-v)(1-\theta) - \kappa(1-\xi)v/\Omega \tag{68}$$

The inflation in the non-dollar sector responds to a positive shock to $\tau_{2,t}$ through two paths. The first part of equation (68) implies that the changing value depends on the price flexibility. The second part of the equation implies that the changing value depends on the relative change in demand. A negative preference shock to V_1 denotes a declined desirability for using V_1 to consume in the dollar sector. It leads to a price drop in the dollar sector and a price increase in the non-dollar sector. A higher level of demand in the non-dollar sector can trigger the price level to increase in the non-dollar sector. Both impacts can trigger inflation to increase in the non-dollar sector.

After examining the impact of a negative preference shock to V_1, it then looks into the impact of a positive preference shock to the digital currency V_2. The assumption is that digital currency has easy access to obtain from the perspective of consumers. The undetermined coefficient $\varphi_{y_2\tau_2}$ denotes the impulse response of preference shock on the output gap, and we write it as:

$$\varphi_{y_2\tau_2} = (v-1)\eta\theta - (1-\xi)(1-v)/\Omega \tag{69}$$

As noted earlier, a higher level of substitution, η, can cause the loyalty to hold one specific currency to decrease. In this case, a decrease in τ_2 triggers higher desirability for holding the non-dollar digital currency. The incentive for exchanging V_1 for V_2 to consume increases. The first part of the equation (69) denotes such a substitution effect. The second part of the equation represents the increase in output at the aggregate level. A surge in consumption capability can cause an increase in the output gap at the aggregate level. When households prefer to hold more currencies to consume, the demand for products in both sectors increases. Both the income and substitution effects contribute to a higher output gap in the non-dollar sector. This trend will lead to a transformation to an MCP with more households holding a digital currency other than the official one.

The undetermined coefficient $\varphi_{\pi_2\tau_2}$ denotes the impact of $\tau_{2,t}$ on the non-dollar sector's inflation, we write it as:

$$\varphi_{\pi_2\tau_2} = (v-1)(1-\theta) - \kappa(1-\xi)(1-v)/\Omega \tag{70}$$

Inflation responds to both the aggregate level change in demand and behavioral shift in currency holding. As the perceived price level in the dollar sector increases, there is a relative demand increase in the dollar sector. The inflation surges accordingly. At the same time, when more consumers choose to hold V_2, the substitution effects cause a demand increase in the non-dollar sector. Higher demand implies a higher price level. The two results together contribute to the price surging in the dollar sector.

The scenario above has depicted the dynamic changing of the currency distribution under the adoption of digital currencies. It assumes a negative preference shock to V_1 (a higher τ_1) and a positive preference shock to V_2 (a lower τ_2). Many reasons can contribute to the popularity of digital currencies: consumers have easy access to obtain them; they are easy to carry around; they achieve an efficient match between the demand and supply. Digital currencies thus encourage individuals to participate in economic activities with an active role.

Figure 4.2 below shows the impulse response of a negative preference shock to both V_1 and V_2 (a higher τ_1 and τ_2), and their impact on economic indicators in the non-dollar sector. The goal is to show how inflation and output gap respond to a negative preference shock to different currencies. C.1 presents the degree of changes under the different values of v.

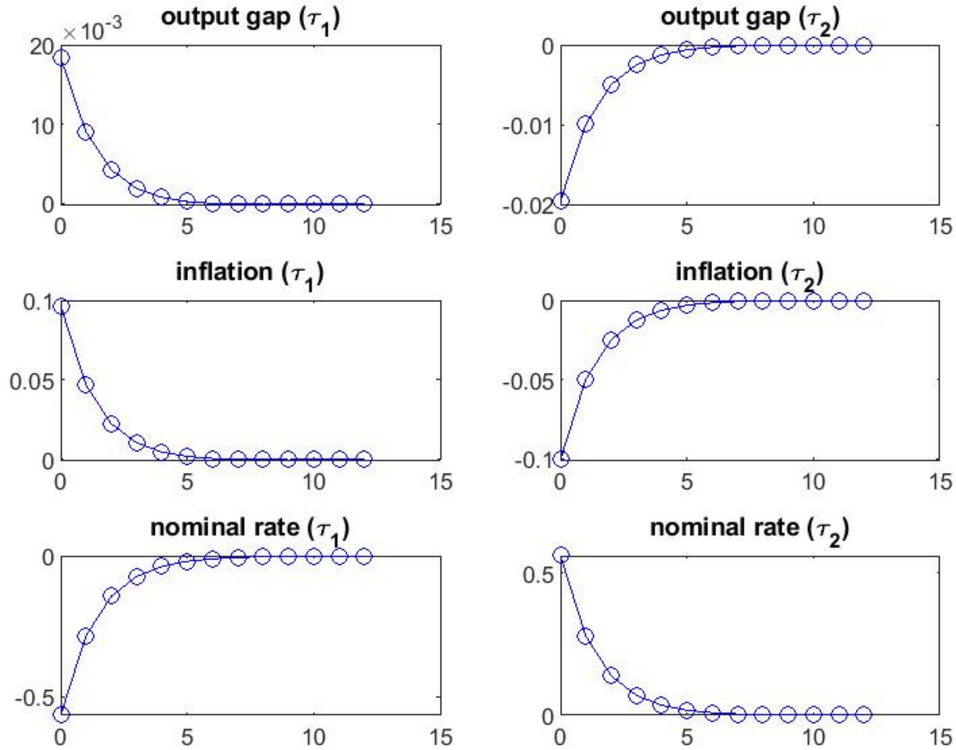

Figure 4.2. In the non-dollar sector, a positive shock to τ_2 implies lower desirability to use the digital currency, V_2. It prompts consumers to switch to the dollar sector for consumption, causing a relative decline in output. Inflation declines by responding to the change in output and the level of price flexibility. A positive shock to τ_1 works in the opposite direction.

4.4.3 From DCP to MCP

DCP implies that a dollar depreciation has a negative impulse response impact on the output in the non-dollar sector. The undetermined coefficient φ_{y_2e} denotes the impact of exogenous exchange rate on non-dollar sector's output, we write it as:

$$\varphi_{y_2e} = (v-1)\eta\theta + \xi(1-\beta)(1-\eta_m\phi_\pi v)/\Omega \tag{71}$$

In the calibrated model, v denotes the partition of the dollar. The coefficient represented by the equation (71) always yields a negative value. The result implies that a depreciation of the main currency can cause a decline in the non-dollar sector's output. Appendix C.3 shows the impact of the exogenous exchange rate on both sectors of the economy.

Under comparison, equation (67) and equation (69) have shown an increase in the non-dollar sectors' output given preference shocks for both currencies. Consumers tend to have increased desirability for holding digital currency and a declined preference for holding official currency because digital ones allow them to participate in economic activities with a more active role. The popularity of digital currencies can dilute the monopolistic power of large-scale companies and provide small companies with easy access to foreign markets. This trend can potentially transfer market power from firms to consumers. The following section will discuss how the currency partition between the official and the digital currency changes when firms adopt consumer value-based pricing strategies.

5 The Pricing Strategy of Firms

The pricing strategy that firms adopt is important to achieve their long-term goal (K.B.Monroe [2003]). It is closely related to how they identify the intrinsic value of their products, how they understand the cost structure of their institution, and how they perceive and design long-run goals to confront the market competitiveness. Factors that firms take into account for making the pricing strategies fall into three categories (Toni et al. [2016]): consumer value-based, cost-based, and competition-based factors.

A consumer-based strategy requires firms to identify the intrinsic value that a firm's products can bring to its consumers. Thus the development of new products is important. It is the key to develop and maintain consumers' loyalty to products. A cost-based strategy requires firms to set price based on its cost-structure (Toni et al. [2016],P. Ingenbleek and Verhallen [2003]). In this case, a firm needs to be aware of the consequence of exogenous exchange rate impact on its cost and benefit structure. Facing a surge of international trade, the market value of firms' profit can be vulnerable to the exchange rate fluctuations (Crockett and Goldberg [1998]). A competition-based strategy says that firms need to depend on competitors' information for their decision-making (Toni et al. [2016]). The side effect of the matter is that such an information structure is usually unreliable (Paul T. M. Ingenbleek [2010]) and risky. The cost and competition strategy had dominated the market (P. Ingenbleek and Verhallen [2003]) before the invention of digital currencies. The invention of digital currency has strengthened the role that consumers play in the decision-making process of the price setting. While consumers change their behaviors at an aggregate level, firms need to adjust their pricing strategy to maintain an optimal profit level.
Under the DCP, firms can obtain a higher market value of profit by invoicing their product on the most commonly used currency. Such a strategy can help them participate in global trade at the largest scale. Many literature have contributed to the discussion of this phenomenon (Gopinath [2015], Devereuz et al. [2008]). They depicted the empirical reality when firms adopt complementary strategies to facilitate trade. However, the invention of digital currencies has prompted the participation of households directly into cross-country transactions. When consumers have more options for consuming and making payments, their consumption is no longer limited to those domestic productions and selected imports chosen by firms or central authorities. Their independence has contributed to the firms adopting a consumer-based pricing strategy.

A consumer-based pricing strategy has significant meaning in achieving a firm's long-run goal (Toni et al. [2016]). It accentuates a stronger connection between consumers and firms, relating firms' strategy to those who will impact their market value of profit most. In particular, while facing a surge of consumption capability in the non-dollar sector supported by the invention of digital payment, firms tend to adopt a consumer-based pricing strategy.

5.1 The optimal pricing choice

This section uses the modified discrete choice model (McFadden [1974]) to relate the decision-making process of firms directly to the value of firms' objectives. When firms set their prices based on a consumer-based strategy, the preference for currencies directly impacts their optimal profit level by causing a relative change in demand across sectors. To derive the relationship, we first write the value function of the firm's objective function. The value function of the firm is:

$$\mathcal{V}_{|,} = \Xi(p^*_{s,2} + e_{2,t+k}|x_{t+l})] + \theta E_t Q_{t,t+1}\mathcal{V}_{|,+} \tag{72}$$

The firm is facing the maximisation problem in equation:

$$\max \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}(\epsilon_t P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\}$$

It is subject to the sectoral demand function equation (8):

$$C_t(i) = \left[\Upsilon(i)\frac{\epsilon_{j,t}P_t(i)}{P_t}\right]^{-\eta} C_t$$

The derived pattern of the currency distribution is depicted as (Uhlig and Xie [2021])

$$v_{j,t} = \theta v_{j,t-1} + Pr_{j,t}\sum_{j'=1}^{K}(1-\theta)(v_{j',t-1}) \tag{73}$$

Where $Pr_{j,t}$ denotes the possibility of a firm changing its pricing setting from using currency j to currency j':

$$Pr_{j,t} = \frac{\gamma_1 \mathcal{V}_{j,t}}{\gamma_1 \mathcal{V}_{j,t} + \gamma_2 \mathcal{V}_{j',t}} \tag{74}$$

Proposition 5. Under the assumption of homogeneity, $\gamma_1 = 1$ *and* $\gamma_2 = b$*.* The approximation of the equation yields that:

$$Pr_{j,t} = \frac{1}{1+\gamma} + \frac{\gamma}{(1+\gamma)^2\tilde{\mathcal{V}}_t}(\mathcal{V}_{1,t} - \mathcal{V}_{2,t}) \tag{75}$$

Substitute γ_2 with a newly created parameter b to represent the relative price of the transaction cost. Define b as:

$$b \equiv \frac{\tau_1}{\tau_2} \tag{76}$$

A higher b implies a relatively higher τ_1 compared to τ_2. The intuition is that a preference shock in the dollar sector reduces people's desirability to hold the dollar, thus diminishing the weighting on pricing on dollars. As a consequence, the possibility of choosing V_1 to set the price declines. As shown in 5.1, the value of b varies with the relative change of τ_1 and τ_2. As τ_2 decreases, preference for V_2 increases, and γ_2 has a higher value, implying that the possibility of invoicing in V_2 climes, the possibility of invoicing in V_1 declines. In a calibrated model, we assume that the homogeneity of price in both sectors and let $\theta = 0.8$. The partition of V_1 has an initial value of 0.6. When b increases, it denotes a situation in which people's preference for V_1 decreases and the preference for V_2 increases. v_1 declines with the changing of b, implying a smaller partition of the main currency and a transformation from DCP to MCP.

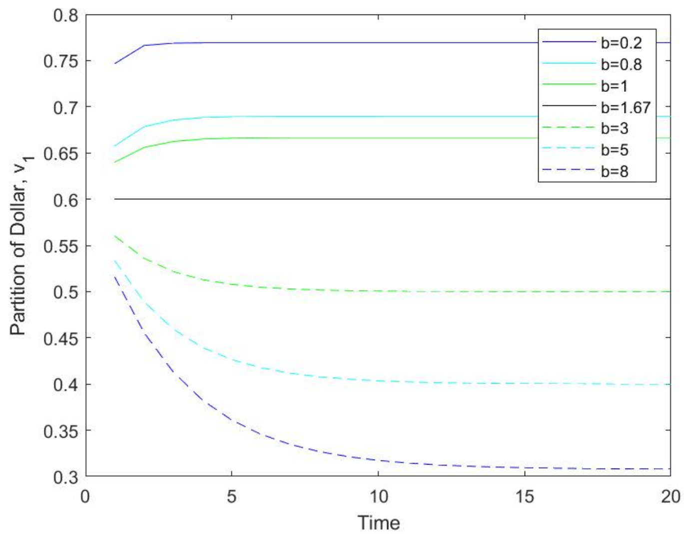

Figure 5.1. The chart shows the changing partition of v_1 with changing preferences for currencies. A high b implies a lower preference for holding the dollar. Given the initial values in the calibrated model, the partition v_1 decreases along the time when $b = \frac{\tau_1}{\tau_2} > 1.67$, which denotes a transformation from DCP to MCP.

6 Conclusion

The invention of digital currencies and electrical payment systems have lifted the constraints that a domestic market can impose on their local consumers. Before then, globalization has reinforced the monopolistic power of big firms, and consumers were limited to domestic production and imports offered by those big firms. Small businesses face obstacles to joining the international market, and households are in a passive position while choosing which products to buy and use what currencies to make transactions. The dominant currency paradigm further implies a pass-through effect of the exchange rate fluctuation into small economies at the international level.

The adoption of digital currencies has gradually transformed the passive position of consumers through two methods. First, digital platforms have broken the physical territorial boundaries among countries. Digital currencies are easy to obtain at a low cost. Thus individuals can use them to reach out to foreign markets easily. The algorithm of digital platforms can also achieve an efficient match between the demand and supply internationally at a very low cost. Consumers thus face less necessity to conform to a uniformed preference provided by big firms. Second, digital currencies have given much more freedom to the population mobilizing across borders. They are no longer confined to the limit of using a single currency. The implication is that, given enough consumption capabilities, individuals are no longer confined to a specific predetermined selection pool. Only if being equipped with a mobile phone can they spend at their willingness in any foreign country without concerning about the amount of physical currency to carry on. The desirability of consumers to hold currencies is different from one currency to another. This desirability depends on an expectation about how difficult or costly it is to hold a currency from consumers' perspective. They can have real significant impacreallyn the real sector of the economy if digital currencies can fundamentally transform consumers' purchasing behaviors.

This paper has shown that the changing behavioral pattern of consumers can transform a DCP to MCP. When households lack access to the main official currency or have been excluded from the economic activities governed by an official currency, they turn to those easily accessible ones that can satisfy their consumption needs. When households have a strong consumption capability, they are open to exploring new markets that can satisfy their demands. This situation has already happened in some developing countries. When less-developed regions cannot participate in or are excluded from the developed regions, they make transactions locally by using local digital currencies. When this trend spread, the popularity of a digital currency can contribute to a declined preference shock of obtaining it, implying higher desirability to hold them from consumers' perspective.

This assumption can be valid when production capabilities do not vary too much from one market to another, implying that the elasticity of substitution among consumption goods is high. Consumers can easily find substitutions of one product in a different market. A high level of substitution can potentially decrease the loyalty of consumers to any single market and any single currency. By following this angle to extend the discussion, we may continue our research to explore the role of substitution in this macro-level dynamic setting. Another interesting angle to extend the discussion is introducing a variable to denote an exogenous preference shock, which can be related to the ease of substitution between the different sectors of the economy.

The changing behaviors of consumers also have a significant impact on the role of financial institutions. When consumers can hold multiple currencies, banks need to reassess their role in providing lending and borrowing services. In particular, they need to re-identify their relationship with the real sector of the economy. Todd Keister [2019] has discussed the increasing cost of lending and higher competition for deposits among banks. Keister and Sanches [2019] has studied the competition among private monies, government monies, and automate issued monies. Some literature in this field has also discussed the risk of the bank run, including Brunnermeier and Landau [2019] and Benigno et al.. Those new angels listed above can contribute to further development of the model to explore how digital currencies can impact the real sector of the economy.

A Linearization

A.1 Firms

Firms face a production function:

$$Y_{j,t}(i) = A_t N_{j,t}(i)^{1-\alpha}$$

Given that firms produce at marginal cost, the FOC of the production function is equal to the marginal cost, such that:

$$MC_t(i) = \frac{W_t}{(1-\alpha)A_t N_t(i)^{-\alpha}}$$

The log-linearized form is denoted as:

$$mc_t = w_t + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t \tag{77}$$

In the medium-long run, the demand for products is equal to the supply. The market clearing condition of the labor is:

$$N_t = \int_0^1 N_t(i) di$$

By substituting the production function, yielding that:

$$\begin{aligned} N_t &= \int_0^1 N_t(i) di \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P-t(i)}{P_t}\right)^{-\frac{\eta}{1-\alpha}} di \end{aligned}$$

The log-linearized FOC of the labor clearing condition yields a function of output in terms of labor input and technology:

$$(1-\alpha)n_t = y_t - a_t + d_t \tag{78}$$

where d_t is the log of the price dispersion:

$$d \equiv (1-\alpha)\log \int_0^1 \left(\frac{\epsilon_{j,t} P_t(i)}{P_t}\right)^{-\frac{\eta}{1-\alpha}}$$

In a steady state of zero inflation, the price dispersion across firms is equal to zero under FOC, rewrite the equation (**??**) to obtain:

$$(1-\alpha)n_t = y_t - a_t$$

The log-linearized labor market clearing condition is:

$$n_t = \frac{y_t - a_t}{1-\alpha} \tag{79}$$

To write the firm's marginal cost as a function of the average real marginal cost in an economy:

$$mc_{t+k|t} = (\omega_{t+k} - p_{t+k}) - mpn_{t+k|t} \tag{80}$$

Plugging in the the function of aggregate output in terms of labor input:

$$\begin{aligned} mc_{t+k|t} &= (\omega_{t+k} - p_{t+k}) - \tfrac{1}{1-\alpha}(a_{t+k} - \alpha y_{t+k|t}) - log(1-\alpha) \\ &= (\sigma y_t + \varphi n_t) - (y_t - n_t) - log(1-\alpha) \\ &= \left(\sigma + \tfrac{\varphi+\alpha}{1-\alpha}\right) - \tfrac{1+\varphi}{1-\alpha} - log(1-\alpha) \end{aligned} \tag{81}$$

Which further implies that

$$\hat{mc}_t = \Big(\sigma + \frac{\varphi+\alpha}{1-\alpha}\Big)(y_t - y_t^n) \tag{82}$$

Where y_t^n is the natural level of output and $y_t - y_t^n$ is the output gap, which is denoted as $\tilde{y}_t \equiv y_t - y_t^n$. Firms set up prices at the optimal level when the real terms of the wage equals to the real marginal cost:

$$\begin{aligned} mc_{t+k|t} &= mc_{t+k} + \tfrac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= mc_{t+k} - \tfrac{\alpha\eta}{1-\alpha}(p_t^* - p_{t+k} + \epsilon_{j,t}) \end{aligned} \tag{83}$$

To maximize the profit, firms set up their prices such that the marginal revenue is equal to the marginal cost. By taking the derivative of the demand function (8) and pricing function (28), the marginal cost function can be rewritten as:

$$MC(i) = \frac{\eta-1}{\eta}\frac{\epsilon_{j,t}\tilde{P}_{j,t}}{P_t} \tag{84}$$

where $\tilde{P}_{j,t}$ is the desired price, such that $\tilde{P}_{j,t} = P_t(i)$.

After the log-linearization, the desired price is derived as:

$$\tilde{p}_{j,t} = \Theta mc_t + p_t - \epsilon_{j,t} \tag{85}$$

where $\Theta = \frac{1-\alpha+\alpha\eta}{1-\alpha}$.

A.2 Households

Households face a dynamic maximization problem with an objective function:

$$E_t \sum_{t=0}^{\infty} \beta^t u(C_t, D_t, N_t)$$

and money-in-utility function :

$$u(C_t, N_t, D_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma} + \frac{D_t^{1-\xi}-1}{1-\xi} - \frac{N^{1+\psi}+1}{1+\psi}$$

Substitute D_t with L_t yields that:

$$u(C_t, N_t, L_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma} + \frac{[L_t(1-\tau_t)]^{1-\xi}-1}{1-\xi} - \frac{N^{1+\psi}+1}{1+\psi}$$

The periodic budge constraint that households face is:

$$\sum_{j=1}^{J}\int_{V_{j,t}} \epsilon_{j,t}P_t(i)C_t(i)di + B_{t+1} + \sum_{j=1}^{J}\epsilon_{j,t+1}M_{j,t+1} = W_tN_t + B_t(1+i_t) + \sum_{j=1}^{J}\epsilon_{j,t}M_{j,t} \tag{86}$$

By conversing the budget constraint into the official currency and divided it by the price level of the official currency, we can obtain:

$$C_t + B_{t+1} + \sum_{j=1}^{J} L_{j,t+1} = W_tN_t + \frac{B_t(1+i_t)}{\Pi_t} + \sum_{j=1}^{J}\frac{L_{j,t}}{\Pi_t}\frac{\epsilon_{j,t+1}}{\epsilon_{j,t}} \tag{87}$$

To set the langarangian:

$$L = \sum_{t=0}^{T} \Big[\beta^t u(C_t, N_t, L_t) + \lambda_t(C_t + B_{t+1} + \sum_{j=1}^{J} L_{j,t+1} - W_t N_t - \frac{B_t(1+i_t)}{\Pi_t} - \sum_{j=1}^{J} \frac{L_{j,t}}{\Pi_t}\frac{\epsilon_{j,t+1}}{\epsilon_{j,t}}\Big] \quad (88)$$

The FOC in terms of N_t, B_t and $L_{j,t}$ is respectively:

$$[N_t] : W_t = \frac{N_t^{\varphi}}{C_t^{-\sigma}} \quad (89)$$

$$[B_{t+1}] : C_t^{-\sigma} = \beta(1+i_{t+1})E_t[C_{t+1}^{-\sigma}\frac{1}{\Pi_t}] \quad (90)$$

$$[L_{j,t}] : \frac{(1-\tau_t)^{1-\xi}}{1-\xi}\frac{L_t^{-\epsilon}}{C_t^{-\sigma}} = 1 - \beta E_t[\frac{C_{t-1}^{-\sigma}}{C_t^{-\sigma}}\frac{1}{\Pi_t}\frac{\epsilon_{j,t+1}}{\epsilon_{j,t}}] \quad (91)$$

Take the log-linearization of the FOC in terms of $L_{j,t}$ to create a dynamic IS curve, which yields that:

$$C_t = \frac{\beta}{\sigma}E_t\{C_{t+1} + E_t[\pi_{t+1}] - E_t[\Delta\epsilon_{t+1}]\} + \frac{\xi(1-\beta)}{\sigma}L_t - \frac{(1-\xi)}{\sigma}\tau_t \quad (92)$$

A.3 Aggregate Dynamics

The aggregate dynamics are depicted by functions represented by the undetermined coefficients:

$$\tilde{y}_t = \phi_{y_e}\Delta\epsilon_t + \phi_{yve}v\Delta\epsilon_t + \phi_{y_{\tau 1}}\Delta\tau_{1,t} + \phi_{y_{\tau 2}}\Delta\tau_{2,t} \quad (93)$$

$$\pi_t = \psi_{\pi_e}\Delta\epsilon_t + \phi_{\pi ve}v\Delta\epsilon_t + \phi_{\pi_{\tau 1}}\Delta\tau_{1,t} + \phi_{\pi_{\tau 2}}\Delta\tau_{2,t} \quad (94)$$

$$i_t = \phi_{i_e}\Delta\epsilon_t + \phi_{ive}v\Delta\epsilon_t + \phi_{i_{\tau 1}}\Delta\tau_{1,t} + \phi_{i_{\tau 2}}\Delta\tau_{2,t} \quad (95)$$

Where ψ with subtitles are coefficients to be determined.

A.4 Sectoral Dynamics

The sectoral dynamics under the aggregate inflation and aggregate output gap (AIAO): $\phi'_\pi = \phi_\pi v'$ and $\phi'_y = \phi_y v'$. The sectoral dynamics of inflation are:

$$\pi_{1,t} = \phi_{\pi_{1s}} s_{t-1} + \phi_{\pi_{1e}} \triangle e_t + \phi_{\pi_{1t1}} \triangle \tau_{1,t} + \phi_{\pi_{1t2}} \triangle \tau_{2,t}$$

$$\pi_{2,t} = \phi_{\pi_{2s}} s_{t-1} + \phi_{\pi_{2e}} \triangle e_t + \phi_{\pi_{2t1}} \triangle \tau_{1,t} + \phi_{\pi_{2t2}} \triangle \tau_{2,t}$$

The forecast for the sectoral inflation are:

$$E_t\pi_{1,t+1} = \phi_{\pi_{1s}} s_t$$

$$E_t\pi_{2,t+1} = \phi_{\pi_{2s}} s_t$$

Impulse Response Functions based on two sectors: dollar sector 1, non-dollar sector 2. Dollar-sector Inflation:

$$\begin{aligned}\pi_{1,t} &= \beta\phi_{\pi_{1s}} s_t + \kappa(\phi_{ye} \triangle e_t + \phi_{y\tau 1} \triangle \tau_{1,t} + \phi_{y\tau 2} \triangle \tau_{2,t}) + \lambda v s_t \\ &= (\beta\phi_{\pi_{1s}} + \lambda v)s_t + \kappa\phi_{ye} \triangle e_t + \kappa\phi_{y\tau 1} \triangle \tau_{1,t} + \kappa\phi_{y\tau 2} \triangle \tau_{2,t}\end{aligned}$$

From comparison with the unknown coefficients:

$$\phi_{\pi 1s} = (\beta\phi_{\pi 1s} + \lambda v)\theta$$

$$\phi_{\pi 1e} = (\beta\phi_{\pi 1s} + \lambda v)\theta + \kappa\ phi_{ye}$$

Yield that:

$$\phi_{\pi_{1s}} = (\beta\ phi_{\pi_{1s}} + \lambda v)\theta \Rightarrow\ phi_{\pi_{1s}} = v(1-\theta)[s_{t-1}]$$

$$\phi_{\pi_{1e}} = (\beta\ phi_{\pi_{1s}} + \lambda v)\theta + \kappa\phi_{ye} \Rightarrow \phi_{\pi_{1e}} = v(1-\theta)\Delta philon_t + \kappa\xi(1-\beta)(1-\eta_m\phi_\pi v)\Delta_t/\Omega + +\$$$

$$\phi_{\pi_{1\tau 1}} = \kappa\phi_{y\tau 1} \Rightarrow \phi_{\pi_{1\tau 1}} = v(1-\theta)\Delta\tau_{1,t} + \kappa[\xi(1-\beta)\eta_m\phi_\pi - 1 + \xi]v\Delta\tau_{1,t}/\Omega$$

$$\phi_{\pi_{1\tau 2}} = \kappa\phi_{y\tau 2} \Rightarrow \phi_{\pi_{1\tau 2}} = v(1-\theta)\Delta\tau_{2,t} + \kappa[\xi(1-\beta)\eta_m\phi_\pi - 1 + \xi](1-v)\Delta\tau_{2,t}/\Omega$$

By proposition B.3, yielding that:

$$\pi_{2,t} = \frac{\theta - 1}{\theta} s_t + \pi_{1,t} \tag{96}$$

Therefore, for the calculation of $\pi_{2,t}$:

$$\phi_{\pi_{2s}} = (\beta\phi_{\pi_{1s}} + \lambda v)\theta \Rightarrow \phi_{\pi_{2s}} = (v-1)(1-\theta)s_{t-1}$$

$$\phi_{\pi_{2e}} = (\beta\phi_{\pi_{1s}} + \lambda v)\theta + \kappa\phi_{ye} \Rightarrow \phi_{\pi_{1e}} = (v-1)(1-\theta)\Delta\epsilon_t\kappa\xi(1-\beta)(1-\eta_m\phi_\pi v)\Delta\epsilon_t/\Omega$$

$$\phi_{\pi_{2\tau 1}} = \kappa\phi_{y\tau 1} \Rightarrow \phi_{\pi_{1\tau 1}} = (v-1)(1-\theta)\Delta\tau_{1,t} + \kappa[\xi(1-\beta)\eta_m\phi_\pi - 1 + \xi]v\Delta\tau_{1,t}/\Omega$$

$$\phi_{\pi_{2\tau 2}} = \kappa\phi_{y\tau 2} \Rightarrow \phi_{\pi_{1\tau 2}} = (v-1)(1-\theta)\Delta\tau_{2,t} + \kappa[\xi(1-\beta)\eta_m\phi_\pi - 1 + \xi](1-v)\Delta\tau_{2,t}/\Omega$$

B Propositions

B.1 Proposition 1

Proposition1. A The exchange rate follows the random-walk process.

$$e_{j,t} - e_{j',t} = E_t[e_{j,t+1} - e_{j',t+1}]$$

Proposition1. B. The equivalent optimal price Given equation A1.4, the optimal currency j in each sector is presented as:

$$p_j^* + e_{j,t} = (1-\beta\theta_j)\sum_{k=0}^{\infty}(\beta\theta_j)^k E_t[\tilde{p}_{t+k}] \tag{97}$$

Assume that firms have perfect information about consumers' preference for holding currencies, the optimal price setting equation (97) can be extended and rewritten as:

$$p_j^* + e_{j,t} + \tau_{j,t} - \tau_{j',t} = (1-\beta\theta_j)\sum_{k=0}^{\infty}(\beta\theta_j)^k E_t[\tilde{p}_{t+k}] \tag{98}$$

Under the assumption of homogeneous price rigidity across sectors, θ_j is the same for all sectors $j = 0, 1, 2 \ldots n$, such that $\theta_j = \mu_\theta$. For any given μ_θ, $(1-\beta\mu_\theta)\sum_{k=0}^{\infty}(\beta\mu_\theta)^k E_t[\tilde{p}_{t+k}]$ is the same for all j, yielding that:

$$p_j^* + e_{j,t} + \tau_{j,t} = p_{j'}^* + e_{j',t} + \tau_{j',t} \tag{99}$$

for any given j and j'.

B.2 Proposition 2

The relative price of products from the perspective of consumers includes the effect of transaction cost τ. Assuming the homogeneous price rigidity across sectors,for currency j and j', the price at time $t+1$ can be written as a recursive function in terms of θ:

$$P_{j,t+1} = (1-\theta)P_{j,t+1}^* + \theta P_{j,t+1}' + \tau_{j,t} \tag{100}$$

Likely,

$$P_{j',t+1} = (1-\theta)P_{j',t+1}^* + \theta P_{j',t} - \tau_{j,t} \tag{101}$$

Take the log of the relative price yield that:

$$S_{jj',t} = p_{j,t} +_{j,t} +\tau_{j,t} - (p_{j',t} +_{j',t} +\tau_{j',t}) \tag{102}$$

By substituting $P_{j,t}$ and $P_{j',t}$ with equation (100) and (101), yielding that:

$$\begin{aligned} S_{jj',t} &= (1-\theta)(P_{j,t}^* - P_{j',t}) + \theta(P_{j,t-1} - P_{j \cdot e,t-1}) \\ &= \theta S_{jj\prime,t-1} + \theta(\Delta\epsilon_{j,t} - \Delta\epsilon_{j',t}) + \theta(\Delta\tau_{j,t} - \Delta\tau_{j,t}) \end{aligned} \tag{103}$$

B.3 Proposition 3

From proposition 2 in B.2, the relative price can be written as a recursive equation, such that:

$$S_{j,j',t} = \theta S_{j,j',t-1} + \theta(\Delta\epsilon_{j,t} - \Delta\epsilon_{j',t}) + \theta(\Delta\tau_{j,t} - \Delta\tau_{j,t}) \tag{104}$$

$$S_{jj',t-1} = \frac{1}{\theta} S_{jj',t} - \Delta\epsilon_{j,t} + \Delta\epsilon_{j',t} - \Delta\tau_{j,t} + \Delta\tau_{j',t} \tag{105}$$

The law of motion of S_t follows the pattern:

$$S_{jj',t} = S_{jj',t-1} + \pi_{j,t} + \Delta\epsilon_{j,t} + \Delta\tau_{j,t} - \pi_{j',t} - \Delta\epsilon_{j',t} - \Delta\tau_{j',t} \tag{106}$$

which can be written as:

$$\pi_{j,t} - \pi_{j',t} = S_{jj',t} - S_{jj',t-1} - \Delta\epsilon_{j,t} - \Delta\tau_{j,t} + \Delta\epsilon_{j',t} + \Delta\tau_{j',t} \tag{107}$$

Equation representing the form of the relative price (103) implies that:

$$S_{jj',t} - S_{jj',t-1} = (\theta-1)S_{jj',t-1} + \theta(\Delta\epsilon_{j,t} - \Delta\epsilon_{j',t}) + \theta(\Delta\tau_{j,t} - \Delta\tau_{j',t}) \tag{108}$$

By combining (107) and (108), yielding that:

$$\pi_{j,t} - \pi_{j',t} = S_{jj',t} - S_{jj',t-1} - \Delta\epsilon_{j,t} - \Delta\tau_{j,t} + \Delta\epsilon_{j',t} + \Delta\tau_{j',t} = \frac{\theta-1}{\theta} S_{jj',t} \tag{109}$$

B.4 Proposition 4

The FOC of the demand function (8) yields that $\tilde{y}_{j,t} = -\eta \hat{P}_t + y_t$. For any two sectors, the output gap follows the pattern:

$$\tilde{y}_{j,t} - \tilde{y}_{j',t} = (-\eta \hat{P}_{j,t} + y_t) - (\eta \hat{P}_{j't} + y_t) = \eta s_t \tag{110}$$

C Additional Charts

C.1 Impulse response of transaction cost shock τ_1 in both sectors

- The chart below shows the consequence of a 25% transaction cost shock in the dollar section on output. With calibrated parameters $v = 0.8, 0.6, 0.4, 0.2$.

τ_1 on output (dollar) **τ_1 on output (non-dollar)**

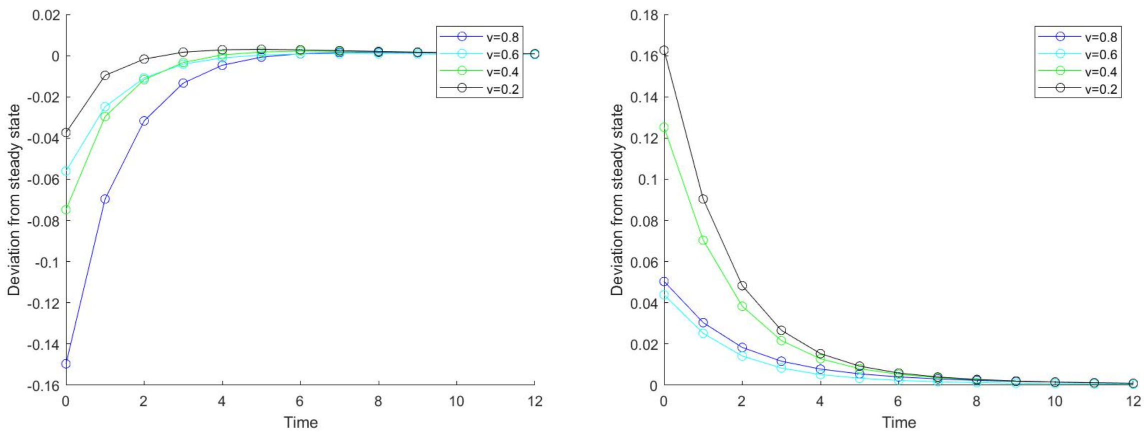

Figure C1.1. Under a 25% shock of transaction cost in $v = 0.8, 0.6, 0.4, 0.2$, the output increases in the dollar sector and decreases in the non-dollar sector. When the partition of the dollar sector is much bigger than that of the non-dollar sector, aggregate output decreases.

- The chart below shows the consequence of a 25% transaction cost shock on inflation. With calibrated parameters $v = 0.8, 0.6, 0.4, 0.2$.

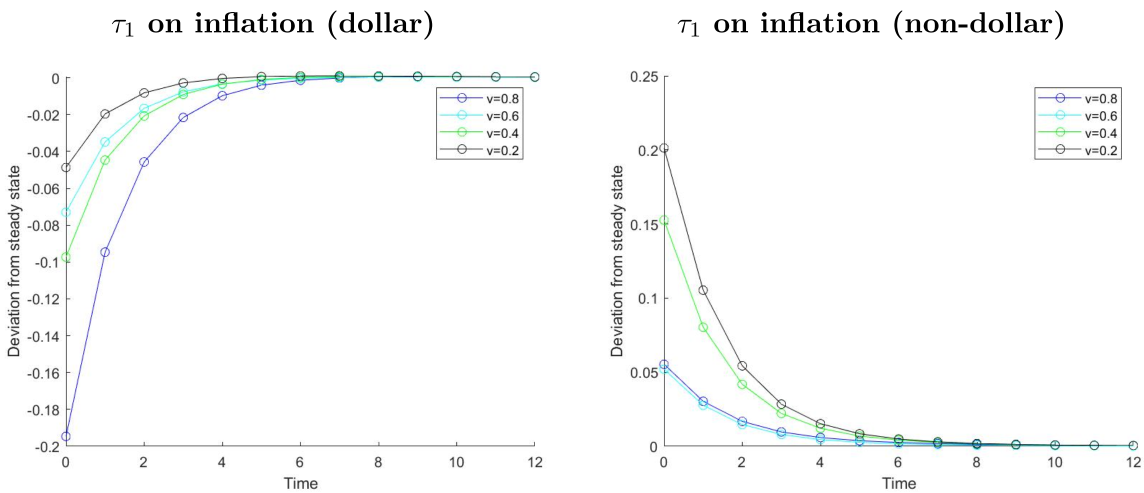

Figure C1.2 Under a 25% shock of transaction cost in $v = 0.8, 0.6, 0.4, 0.2$, the output increases in the dollar sector and decreases in the non-dollar sector. A lower demand of products in dollar sector induced by a higher transaction cost results in a lower inflation.

- The chart below shows the consequence of a 25% transaction cost shock on nominal interest rate. With calibrated parameters $v = 0.8, 0.6, 0.4, 0.2$.

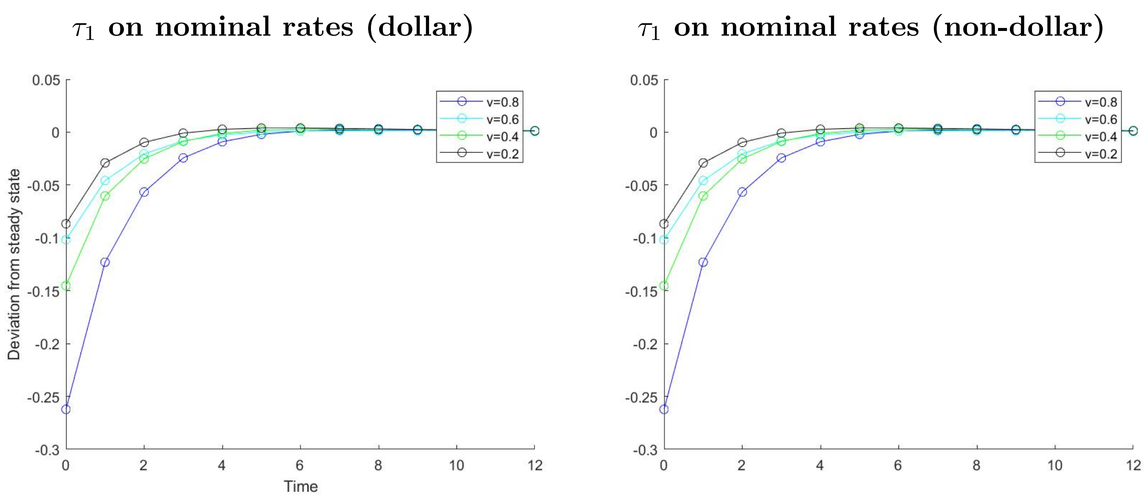

Figure C1.3 Under a 25% shock of transaction cost in $v = 0.8, 0.6, 0.4, 0.2$. The interest rates in both sectors respond to the aggregate output and aggregate inflation.

C.2 Impulse response of transaction cost shock τ_2 in the non-dollar sector

- The chart below shows the consequence of a 25% transaction cost shock in the non- dollar sector on output. With calibrated parameters $v = 0.8, 0.6, 0.4, 0.2$.

τ_2 **on output (dollar)** τ_2 **on output (non-dollar)**

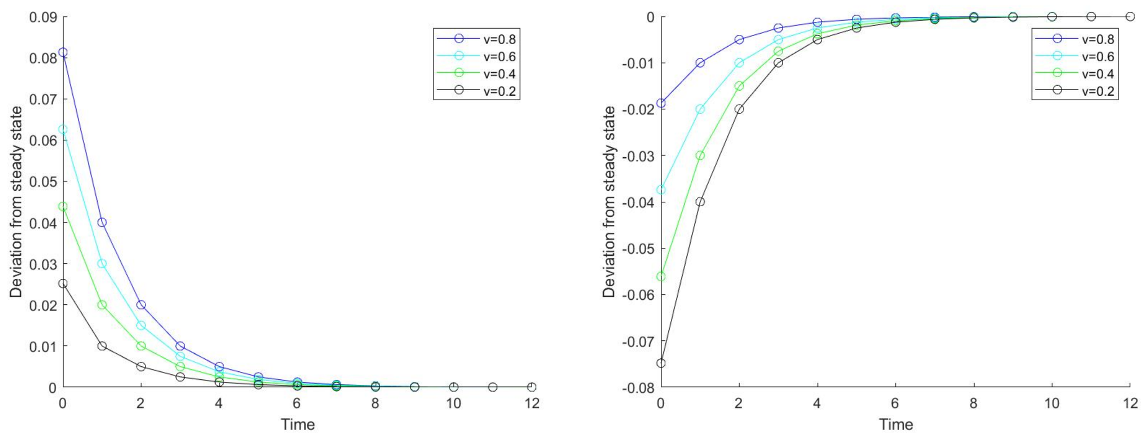

Figure C1.1. Under a 25% shock of transaction cost in $v = 0.8, 0.6, 0.4, 0.2$, the output increases in the dollar sector and decreases in the non-dollar sector. When the partition of dollar sector is much bigger than that of the non-dollar sector, aggregate output decreases.

- The charts beow shows the consequence of a 25% transaction cost shock on inflation. With calibrated parameter $v = 0.8, 0.6, 0.4, 0.2$.

τ_2 **on inflation (dollar)** τ_2 **on inflation (non-dollar)**

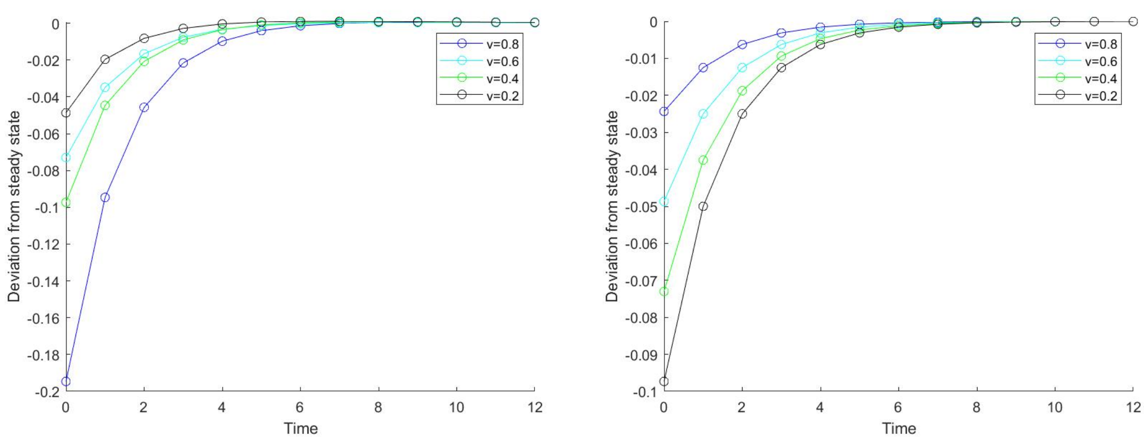

Figure C1.2 Under a 25% shock of transaction cost in $v = 0.8, 0.6, 0.4, 0.2$, the output increases in the dollar sector and decreases in the non-dollar sector. A lower demand of products in dollar sector induced by a higher transaction cost results in a lower inflation.

- The charts beow shows the consequence of a 25% transaction cost shock on nominal interest rate. With calibrated parameter $v = 0.8, 0.6, 0.4, 0.2$.

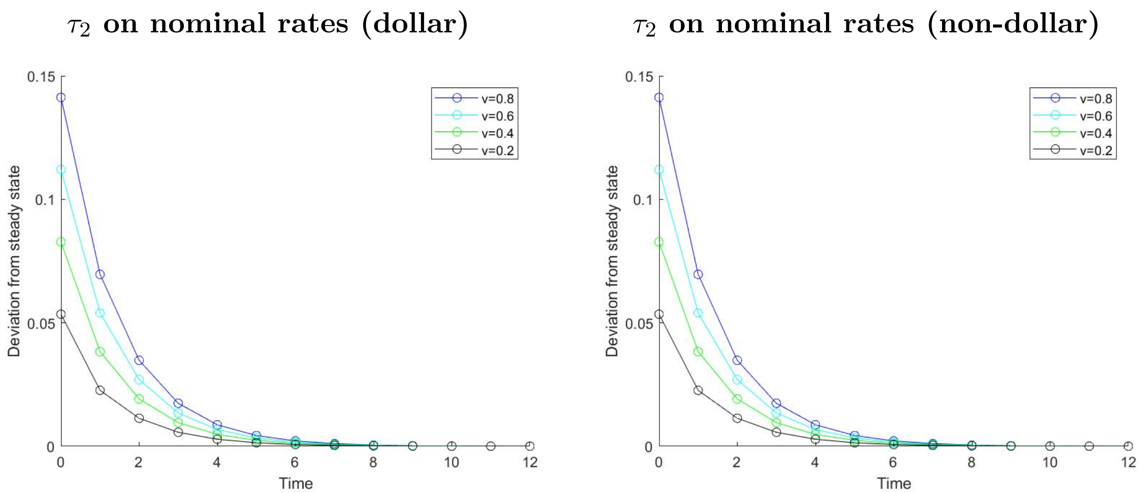

Figure C1.3 Under a 25% shock of transaction cost in $v = 0.8, 0.6, 0.4, 0.2$. The interest rates in both sectors respond to the aggregate output and aggregate inflation.

C.3 Impulse response of exchange rate shocks in both sectors

- The chart below shows the consequence of a 25% exchange rate shock in the dollar sector on output. With calibrated parameters $v = 0.8, 0.6, 0.4, 0.2$.

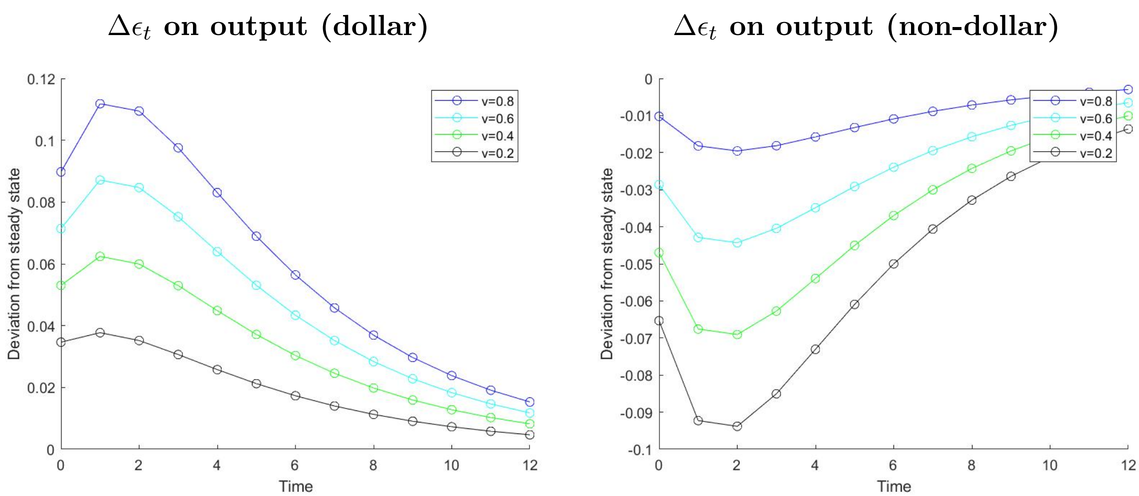

Figure C1.1. Under a 25% shock of exchange rate (depreciation of dollar) in $v = 0.8, 0.6, 0.4, 0.2$, the output increases in the dollar sector and decreases in the non-dollar sector. When the partition of dollar sector is much bigger than that of the non-dollar sector, aggregate output increases.

- The chart below shows the consequence of a 25% exchange rate shock on inflation. With calibrated parameters $v = 0.8, 0.6, 0.4, 0.2$.

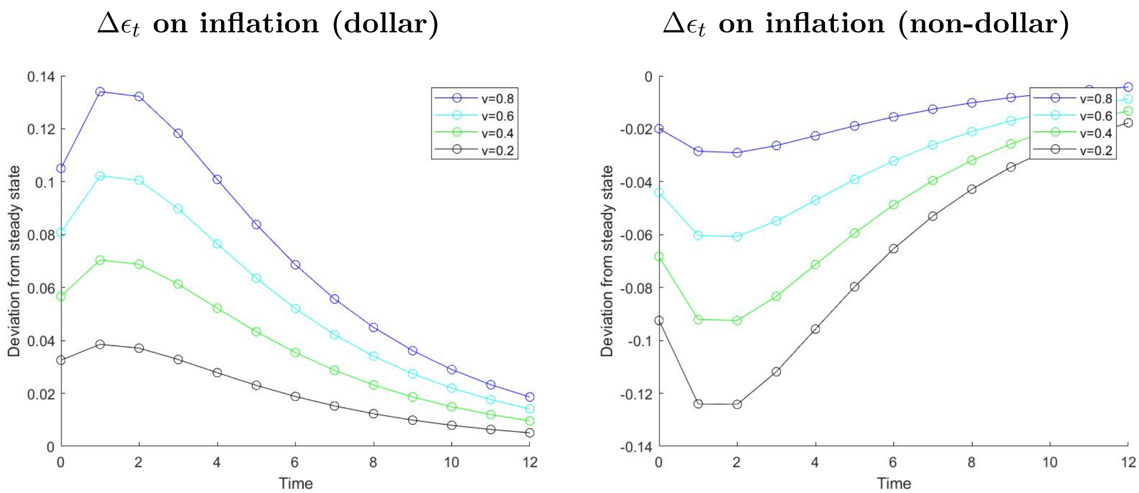

Figure C1.2 Under a 25% shock of exchange rate in $v = 0.8, 0.6, 0.4, 0.2$, the output increases in the dollar sector and decreases in the non-dollar sector. A lower demand of products in dollar sector induced by a higher transaction cost results in a lower inflation.

- The chart below shows the consequence of a 25% transaction cost shock on nominal interest rate. With calibrated parameters $v = 0.8, 0.6, 0.4, 0.2$.

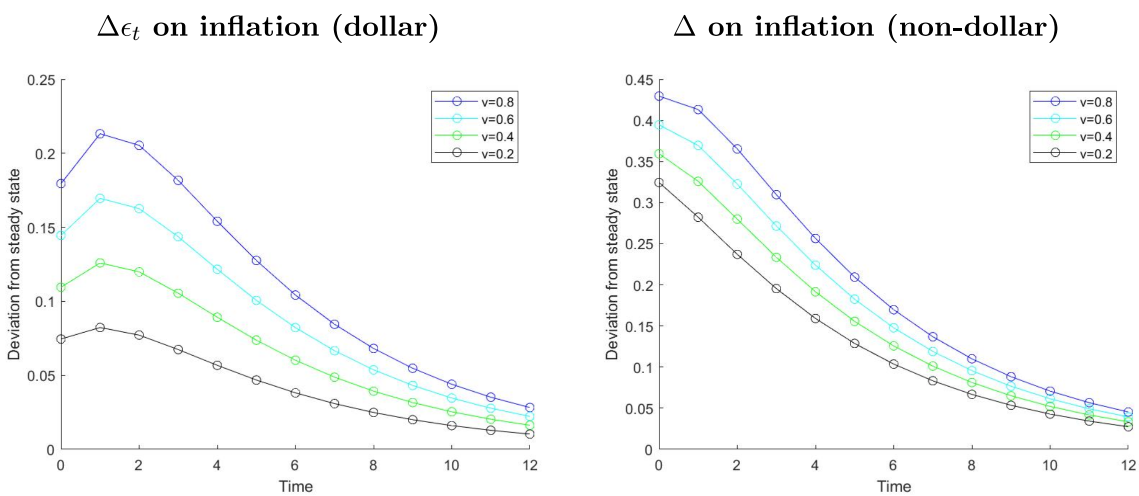

Figure C1.3 Under a 25% shock of exchange rate in $v = 0.8, 0.6, 0.4, 0.2$. The interest rates in both sectors respond to the aggregate output and aggregate inflation.

D Parameters

Parameter	Description	Initial Values
α	Share of Labor Input in the production function	1/3
σ	Coefficient of risk aversion	1
φ	A unitary Frisch elasticity of labor supply	1
β	Discount factor	0.99
θ	Probability of not adjusting prices (Price Stickiness)	0.5
η	Elasticity of substitution among consumption good	0.8
v	Size of dollar sector	0.8
η_m	Semi-elasticity of money demand	4

References

Harald Uhlig and Taojun Xie. Parallel digital currencies and sticky prices. *National Bureau of Economic Research*, 2021.

Shane Greenstein. The economics of digitization. *National Bureau of Economic Research*, 2020. URL https://www.nber.org/reporter/2020number2/economics-digitization.

Deonir De Toni, Gabriel Sperandio Milan, Evandro Busata Sacilotob, and Fabiano Larentis. Pricing strategies and levels and their impact on corporate profitability. 2016. Three strategies.

L. Schilling and H. Uhlig. Currency substitution under transaction costs. 2019.

Andrew T. Ching and Fumiko Hayashi. Payment card rewards programs and consumer payment choice. *Journal of Banking Finance*, 2010.

Edward L. Glaeser, Hyunjin Kim, and Michael Luca. Measuring gentrification: Using yelp data to quantify neighborhood change. *National Bureau of Economic research*, (24952), 2018.

Joseph M. Golden John J. Horton Moshe A. Barach. Steering in online markets: The role of platform incentives and credibility. *National Bureau of Economic research*, (25917), 2019.

Jozsef Molnar, Oleksandr Shcherbakov, Qinghui Yu, and Kim P. Huynh (corresponding author). Demand for payment services and consumer welfare: The introduction of a central bank digital currency.

Anneke Kosse, Heng Chen, Marie-Hélène Felt, Valéry Dongmo Jiongo, and Kerry Nield Angelika Welte. The costs of point-of-sale payments in canada. *Bank of Canada*, 2017.

Gita Gopinath, Emine Boz, Camila Casas, Federico J. Díez, Pierre-Olivier Gourinchas, and Mikkel Plagborg-Møller. Dominant currency paradigm. *American Economic Review*, 110(3):677–719, March 2020. doi: 10.1257/aer.20171201.

Gina Gopinath. The international price system. *Jackson Hole Symposium*, 27, 2015.

Gina Pieters. Digital currencies and central banks. *Palgrave Handbook of Alternative Finance.*

Codruta Boar and Andreas Wehrl. Ready, steady, go? results of the third bis survey on central bank digital currency. *Bank for International Settlements*, 2021.

T. Keister and D Sanches. Should central banks issue digital currency? *Federal Reserve Bank of Philadelphia*, (19-26), 2019.

Jordi Galí. Monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework and its applications - second edition. *Princeton University Press*, 2015.

Yong Kow, Xinning Gui, and Waikuen Cheng. *Human-Computer Interaction – INTERACT 2017.* 2017.

Miles S Kimball. The quantitative analytics of the basic neomonetarist mode. *Journal of Money, Credit and Banking*, 27, 1995.

G. A. Calvo. taggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 1983.

R. E. Manuelli and J. Peck. Exchange rate volatility in an equilibrium asset pricing model. *International Economic Review*, 1990.

L. Schilling Benigno, P. and H. Uhlig. Cryptocurrencies, currency competition, and the impossible trinity. *Becker Friedman Institute for Economics Working Paper*, 2019.

K.B.Monroe. *Pricing making profitable decisions*. McGraw-Hill/Irwin, New York, 2003.

R.T. Frambach P. Ingenbleek, M. Debruyne and T.M. Verhallen. Successful new product pricing strategies: A contingency approach. *Marketing Letters*, 2003.

Keith Crockett and Linda S. Goldberg. The dollar and us manufacturing. *Current Issues in Economics and Finance*, 1998.

Theo M. M. Verhallen Paul T. M. Ingenbleek, Ruud T. Frambach. The role of value-informed pricing in market-oriented product innovation management. *Journal of Product Innovation Management*, 2010.

M.B. Devereuz, B. Timlin, and Dong W. Vehicle currency. *Technical report*, 2008.

D. McFadden. Conditional logit analysis of qualitative choice behaviour. *Frontiers in Econometrics*, 1974.

Daniel R. Sanches Todd Keister. Should central banks issue digital currency? *Federal Reserve Bank of Philadelphia*, 2019.

H. James Brunnermeier, M. K. and J.-P. Landau. The digitalization of money. *Princeton University*, 2019.

P. Benigno, L. Schilling, and H. Uhlig. Cryptocurrencies, currency competition, and the impossible trinity.