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RELATIONSHIP LENDING AND THE TRANSMISSION OF CREDIT SUPPLY SHOCKS

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To Steven N.S. Cheung, who led me into the bedazzling world of economics.

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Abstract

I study the role of bank-firm lending relationships in determining the aggregate effects of credit supply shocks. I show two facts using a unique dataset. First, the loan balance between a lender-firm pair increases in the length of the lending relationship. Second, loan balances of longer relationships respond less to lender-level credit supply shocks. I interpret these findings using a competitive search model of credit markets. Banks write optimal lending contracts subject to firms' limited commitment that prescribe increasing lending over time to incentivize firms to stay in the relationship. I calibrate the model to match features of the data including the slope of relationship lending with respect to the relationship length. Then I validate the calibrated model by showing that, as in the data, the responsiveness of a relationship's loan balance to bank-level credit supply shocks is decreasing in the relationship length. Finally, I perform a counterfactual exercise to show how the aggregate impact of a credit supply shock depends on the distribution of relationship lengths across bank-firm pairs.

Chapter 1

Introduction

Bank-firm relationships are an important source of credit supply for many firms. In this paper, motivated by the extensive literature on how the intensity of bank-firm lending relationships affects external financing to firms, I study the role of the lengths of such relationships in determining the bank's lending to the firm and its responsiveness to credit supply shocks. In particular, I aim to answer two key questions: Is the length of a lender-firm relationship important for determining the quantity of credit to the firm? Is the distribution of relationship lengths in the economy important for the aggregate response of small business lending to credit supply shocks?

I answer these questions with a novel loan-level dataset from PayNet, an Equifax Company which provides credit assessments to commercial lenders. The data features a large lender-borrower matched panel with quarterly observations of more than 11 million small business commercial and industrial (C&I) loans in the US from 1991 to 2019. To my knowledge, the PayNet data is the first and only source of data with a large, long and lender-borrower matched panel on small business lending in the US. By 2019, the total outstanding amount of C&I loans in the provided PayNet data has already exceeded 15% of the total outstanding amount of small business C&I in the US. The richness of the data allows me to overcome the data limitations of existing empirical research on relationship lending by disentangling the effect of the relationship length on relationship lending from many types of lender, borrower of match heterogeneity over complete business cycles.

I show two main empirical facts using this dataset. First, I find that the loan balance in a lender-firm relationship significantly increases with the relationship length. I perform a regression of the balance amount of each relationship normalized by its initial balance amount onto its length. I control for potential selection bias of relationships in the data by including a credit score for the firm, the initial balance of the lender-firm pair, firm industry fixed-effects, firm location fixed-effects, lender fixed-effects, firm fixed-effects and time fixed-effects. The regression estimates suggest a one-quarter increase in the relationship length on average raises the relationship's loan balance by 9.33% of the initial balance amount.

Second, I show that loan balances of longer lender-firm relationships are less sensitive to credit supply shocks. I construct proxies for credit supply shocks to lenders in the data following identification in Amiti and Weinstein (2018). Amiti and Weinstein (2018) decomposes each quarter's balance growth between a lender and a firm into a credit supply shock to the lender, a credit supply shock to the borrower and a term that is orthogonal to both the credit supply and demand shocks. I consider a decomposition of balance growths between a lender and an industry-geography cell instead, as the majority of firms in the PayNet data have only one lender. Then, I run a regression to study how the growth rate of each relationship's loan balance depends on the credit supply shock proxies, the relationship length and their interaction term. I find that an one-quarter increase in the relationship length can reduce the loan balance's responsiveness to a credit supply shock by about 7%. This result is driven by positive credit supply shocks, as the relationship length's effect on relationship lending's response to negative credit supply shocks is not significant.

In order to interpret the empirical results and draw implications of the relationship length distribution on the macro economy, I use a competitive search model with optimal contracts between banks and firms, which is a version of Boualam (2018). In the model, firms are completely reliant on external financing from banks who face an exogenous funding cost. Shocks to this funding cost are the only source of aggregate fluctuations, and I consider them as credit supply shocks. A funded firm can walk away from their relationship with the bank at any time and steal a fraction of the loan for their own consumption. Therefore, optimal contracts imply that loan balances increase over time to incentivize firms not to default. This prediction is consistent with my empirical finding that the loan balance of a lender-firm relationship increases with the relationship length.

I calibrate the model to match features of the data including the slope of lending in a relationship with respect to the relationship length. I validate the model by showing that loan balances of longer relationships in the model also respond less to credit supply shocks and in particular to positive credit supply shocks, as I find in the data. I simulate the model for 77 quarters, consistent with the sample period of 200Q01 to 2019Q1, using a high funding cost to characterize NBER recessions during this episode and a low funding cost the expansions. I then run a regression each relationship's balance growth onto the relationship length, a model analogy of the credit supply shock proxy estimated in the data and their interaction term, which is the same as the exercise with the data. The regression result is consistent with the data and is also driven by positive credit supply shocks. In the model, a positive credit supply shocks raises the continuation value of firms who walk away from their relationships. In response, banks increase firms' values from relationships by lending more to satisfy firms' incentive compatibility. The increase in the continuation value of walking away from the relationship is independent of the relationship length, so for a firm in a shorter relationship whose value from the relationship is lower, the value of walking away increases by more. As a result, banks compensate firms in shorter relationships more than firms in longer relationships by offering larger balance growths.

Finally, I perform a counterfactual exercise to study how the distribution of relationship lengths affects the response of aggregate lending to credit supply shocks. My counterfactual exercise increases the relationship separation rate by 42.86%, motivated by the fact the the firm exit rate in the US was 42.86% higher in 1980 than it is today. I study how this higher firm exit rate similar to that in 1980 affects the aggregate impact of a credit supply shock. I find that the aggregate lending growth due to a decrease in the bank funding cost is higher than in the benchmark case with the optimal contracts of the benchmark economy and the relationship length distribution of the counterfactual economy. This result is consistent with the model prediction that balances from longer relationships respond less to credit supply shocks, as the counterfactual relationship length distribution features a larger fraction of short relationship compared to the benchmark distribution. However, the shift in the relationship distribution also endogenously changes the optimal contract. So, looking at the overall effect of the relationship length distribution change, I find that the counterfactual economy actually experiences a smaller aggregate lending growth in response to the same funding cost decrease compared to the benchmark case. Therefore, the optimal contracting model is vital to fully evaluating the distributional effect of the relationship length on the transmission of credit supply shocks to aggregate lending.

Related Literature My paper is closely related to four strands of literature. The first strand empirically studies the significance of lender-firm relationships to both the quantity and the price of credit for the firms, to collateral requirements, and to firm dynamics. Studies conducted prior to the Great Recession mostly abstract from changes in credit supply conditions and focus on the how properties of a lending relationship such as the relationship length determine the evolution of a financing contract during the course of the relationship. The majority of them use survey data on either the bank side or the firm side, and the conclusions they draw are mixed. For example, Petersen and Rajan (1994) uses survey data on small businesses to show that the length of a firm's longest relationship has a positive effect on the availability of financing to the firm; Berger and Udell (1995) uses the same data and finds that borrowers with longer relationships pay less for financing and pledge less collaterals; Herrera and Minetti (2007) finds that longer relationships foster firms' innovation. Alternatively, Degryse and Cayseele (2000) obtains loan contracts collected from one Belgian bank, and finds that the price of credit actually increases over time in a relationship as the bank gathers more private information about the firm.

My paper relates to these earliest studies by revisiting the classic question of how the relationship length determines the amount of loans to the firm with new data. I am able to disentangle the role of the relationship length itself in affecting the lender-firm pair's loan balances from contributions of many types of lender, borrower, and match hetereogeneity, as the PayNet data covers a large number of lenders and borrowers with information on both their identities.

Since the Great Recession, empirical research on relationship lending is more concerned about the influence of lending relationships on the transmission of credit supply shocks onto lender-firm pairs. These studies mostly use lender-borrower matched data from credit registries of different countries with particular focus on the episode of the Great Recession. Using Spanish data, Jiménez et al. (2012) shows that banks are more likely to continue longer relationships when the GDP growth slows down. Iyer et al. (2013) uses exhaustive Portuguese loan-level data to find that banks cut lending less to relationships of larger loan balances during a credit crunch. Sette and Gobbi (2015) shows with Italian data that bank-firm relationships with older lengths and shorter distances in between are less impacted by the transmission of the Lehman Brothers default to the Italian economy, in terms of both the percentage change of the credit amount and the increase in the cost of loans. With a collection of European firm-level survey data of firms' financing constraints and locations of bank branches around these firms, Beck et al. (2018) argue that firms in the vicinity of more banks that consider themselves as relationship lenders benefit from alleviated credit constraints during an economic downturn, but the benefit of having close-by relationship lenders to a firm is unclear during a credit boom.

In this paper, I use a much longer panel of lender-firm relationships that covers two complete business cycles to measure how the relationship length metric affects the the loan balance growth in a relationship after both positive and negative credit supply shocks. My finding is in fact to the opposite of the results of Iyer et al. (2013) and Sette and Gobbi (2015) by showing that longer relationship lengths do not reduce the relationship loan balances' responsiveness to a negative credit supply shock; instead, they lead to greater loan balance growths when the credit supply conditions improve. A potential reason that my result disagrees with these two papers is that I identify proxies of quarterly credit supply shocks to each lender, while they use the Great Recession as an exogenous credit supply shocks to all lenders and perform an event study around the recession due to data constraints[1].

The second strand of literature to which this paper is related proposes theories about

1. Iyer et al. (2013) studies the changes in loan amounts and costs of lender-borrower relationships across two years, from 2007Q2 to 2009Q2; Sette and Gobbi (2015) alternatively considers changes in relationship lending from September 2008 to September 2009.

the formation of lending relationships. There are three types of relationship lending theories. I use a version of the model in Boualam (2018), where banks back-load contract values for firms to incentive firms of limited commitment to stay in relationships. This model belongs to the first type of relationship lending models where banks provide implicit insurance for the firms against changes in credit supply conditions through long-term contracts (Berlin and Mester (1999)). In this model, the extent of such insurance affects the response of a relationship's loan balance to credit supply shocks by the length of the relationship. The second type emphasizes how lenders monitor their borrowers' behaviours over lending relationships and design optimal contracts accordingly (Holmstrom and Tirole (1997), Boot and Thakor (2000)). The third type models how the lender gathers and uses soft or private information about the borrower quality, either through ex ante screening of firms or through gradual learning about the borrower over time (Hachem (2011), Bolton et al. (2016)).

This paper also relates to a third strand of literature on optimal long-term contracts with incentive constraints, for example Thomas and Worrall (1988), Quadrini (2004), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), DeMarzo and Sannikov (2006) and Sannikov (2008). In particular, financing contracts between lenders and borrowers in Albuquerque and Hopenhayn (2004) are subject to the borrowers' limited liability and limited enforceability of debt repayment; contracts in Clementi and Hopenhayn (2006) are subject to asymmetric information about the borrowers' revenue realizations; in DeMarzo and Sannikov (2006), there exits both limited enforceability of borrowers' repayment and asymmetric information about the borrowers' effort into their projects. These limited commitment problems result in borrowing constraints of the firms that can be relaxed by increasing firm payoffs in future periods. This feature is also seen in the model in this paper, as funded firms' ability to walk away from the relationship and steal a proportion of the funding at any time limits the amount banks can safely lend to these borrowers.

Lastly, my paper is connected to the strand of literature on the resource allocation in a competitive search equilibrium. For instance, Guerrieri (2008) analyses how search frictions interact with limited worker commitment to influence (in)efficiency of wage contracts offered by the employers. Guerrieri et al. (2010) instead considers how the interaction of search frictions and private information of the agents' types affect the intensive and extensive margins of trading that can characterize different types of markets. Lamadon (2016) considers a competitive search framework in which search frictions matter for the transmission of idiosyncratic worker productivity shocks to earnings and employment in the economy. In this paper's model, search frictions influence the distribution of lending relationship lengths in different credit supply conditions. The competitive search environment is thus both qualitatively and quantitatively important for responses of individual and aggregate loan balances to credit supply shocks.

Chapter 2

Data

2.1 Data Description

I use a quarterly lender-firm matched panel of more than 25 million financing contracts spanning 1999Q1 to 2019Q1 acquired from PayNet. PayNet, an Equifax Company, provides the industry's largest commercial dataset that complements and validates internal analyses. The company was founded in 1999 on the mission to provide better information on private firms seeking credit. With access to all members' lending contracts, PayNet has constructed a large, proprietary database of the payment performances of these contracts and corresponding firm borrowers. In its twenty years of operations, PayNet has attracted more than 300 lenders to join, the majority of whose borrowers are small-sized. In the data provided by PayNet, almost 7.8 million firms are associated with financing contracts collected from these member lenders by PayNet. Of all contracts in the provided data, 44.23%, are leases, 21.31% are C&I loans, 19.77% are conditional sales, and the rest include credit cards, credit lines and trade credit. In this study, I focus on C&I loans in consideration of the way different contract types work. To be specific, with a C&I loan, the lender is likely not able to repossess the asset in case of default as easily as with a lease or a conditional sales contract. Thus, the loan financing market arguably suffers significantly more from commitment problems and may consequently appeal more to lending relationships.

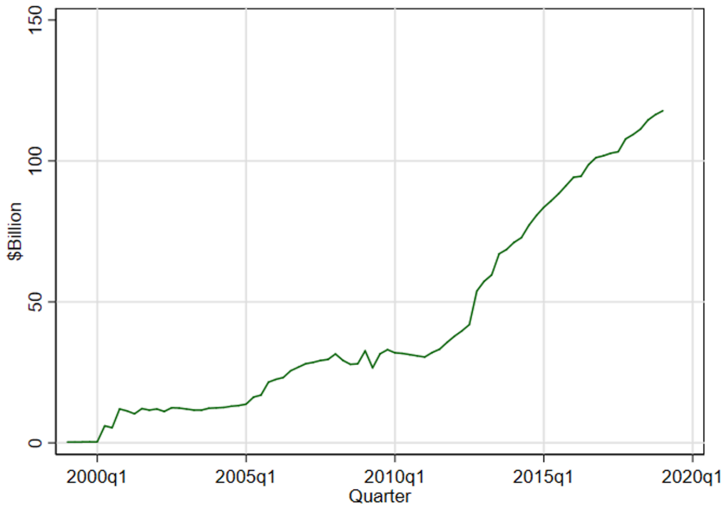
The PayNet data contains rich information of each bank-firm pair. At the contract level, observables include the type of financing, the start and end dates, whether collateral is used, the quarterly evolution of the outstanding balance, the amount of payment overdue for certain periods of time, and bad statuses like firm bankruptcy, repossession, or write-off. Additionally, each contract, lender and borrower are identified by unique internal IDs. These IDs allow me to link observations within the data to keep track of the performances of each bank-firm pair across time and compare the same lender or borrower's contracts with different partners. Moreover, I observe the type of the lender (e.g., banks, non-bank financial institutions) and a few borrower characteristics including its CBSA-level[1] geographic location, 4-digit SIC code, and the ownership type. Finally, I observe a time-varying credit score - "MasterScore v2" - for each firm generated by PayNet. The MasterScore v2 is developed from the largest pool of term debt leases and loans ever compiled to predict 90+ days past due on a 3-digit scale. This score is comparable across borrowers and time.

The PayNet data represents a substantial share of the small business loan market. In June 2016, outstanding C&I loan balances in the provided data add up to $94 billion, which is more than 15% of the total outstanding amount of C&I loans to small businesses in the US, as reported by the Small Business Association. The evolution of PayNet's coverage of C&I loans in the provided data is presented by Figure 2.1.

The PayNet data has many advantages over data used by previous empirical research on relationship lending. A larger number of earlier papers use data from the National Survey of Small Business Finance of year 1987, 1993, 1998 and 2003 conducted jointly by The Federal Reserve System and the Small Business Administration (Petersen and Rajan (1994), Berger and Udell (1995), Guiso and Minetti (2010)). Each of these surveys contains information collected from 3000 to 5000 non-financial, non-farm small businesses for the corresponding financial year. Each firm was interviewed with roughly 200 questions concerning descriptive firm characteristics, use of credit, relationships with financial institutions and balance sheet information. Firms are stratified coarsely by size categories, length bins, census regions, and urban or rural location. Albeit rich in firm performance

1. CBSA stands for "core-based statistical areas" which consist of all metropolitan statistical areas (MSA) and micropolitan statistical areas (μSA).

Figure 2.1: Total Outstanding C&I Loans over Time



Notes: Total outstanding amount C&I loans in provided PayNet data from 1999Q1 to 2019Q in billion USD (nominal).

information, the data suffers from a small sample size; the coarse stratification of firms and the absence of information on each lender disable investigations of how borrower, lender, or match heterogeneity affects firm credit. The Federal Reserves Survey of Terms of Bank Lending to Business is another dataset commonly used in the relationship lending literature (Berger and Udell (1996)). This dataset contains only lender-side information and thus does not allow observing individual relationship's evolution as well. Many studies alternatively use syndicated loan data such as the DealScan data which shows both the borrower and the co-lenders of each loan contract. However, the contribution of each co-lender's relationship with the borrower to a syndicated loan's performance cannot be isolated. In recent years, the rise of a number of proprietary lender-borrower matched datasets of loan contracts in Europe[2] has enabled more comprehensive empirical research on the importance of relationship lending. Nonetheless, the PayNet data features a longer panel of relationships which allows understanding the dynamic impact of relationship lending. PayNet also provides a larger coverage of lenders, borrowers, and the amount of outstanding balances and is therefore more representative of the credit market. Last but not least, to my knowledge, PayNet is the only source of micro-data for small business C&I loans in the US.

2. For example, López-Espinosa et al. (2017) uses Spanish data of 20,000 loans between banks and small and medium enterprises; Bottero et al. (2018) use loan-level data from the Italian Credit Register; Schäfer (2018) uses an Armenian private credit registry.

The PayNet data also has its shortcomings. First, I am not able to observe the existence of lending relationships between firms in the data and lenders outside the data, since the data is collected from the lender side. Second, I use the quantity of credit between bank-firm pairs as the only measure of the bank-firm relationship quality, because data confidentiality and the fact that PayNet does not collect contract interest rates from member lenders preclude further knowledge of firm, lender or contract characteristics.

2.2 Relationship Patterns

The PayNet data comprises of loan contracts between lenders and firms that are of a variety of lengths. Each lender-borrower pair may have more than one contract ongoing at the same time. I define a relationship as a lender-borrower pair that has initiated at least one loan contract. The full sample I use is the set of all quarterly relationship observations in the provided PayNet data with at least one loan contract of a strictly positive balance from 1999Q1 to 2019Q1. Owing to the immense size of the data, for the majority of the exercises to be developed in this paper, I use a baseline sample comprised of two million firms randomly selected along with their complete profiles from the full sample.

Table 2.1: Relationship Length Distribution

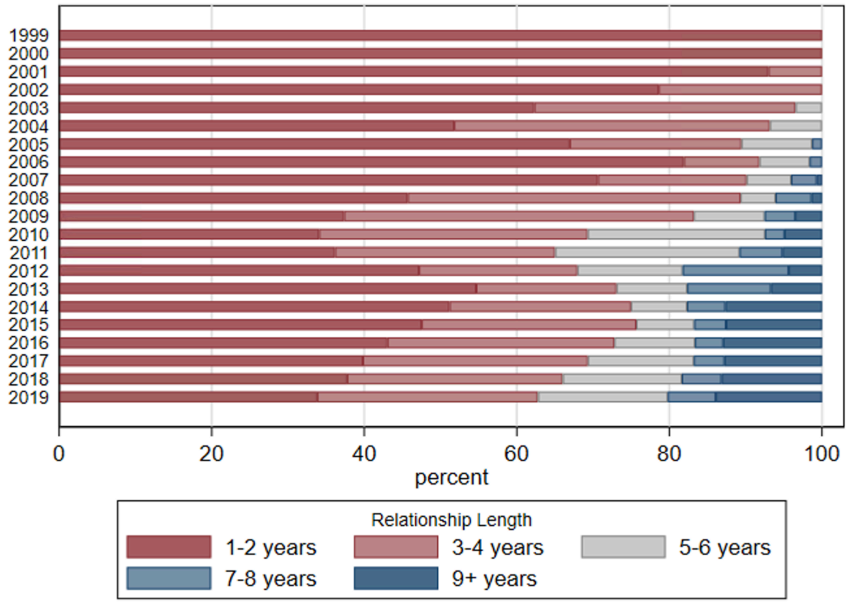
Unit: Quarters

	Mean	SD	25%	50%	75%	Count
Relationship length at observation	11.342	11.102	4	8	15	6,156,686
Total relationship length	12.116	11.551	6	11	17	495,163

I focus on the role of the length of a lender-firm relationship, namely the time since their first positive balance observation, in determining the pair's loan balance. The measurement of the relationship length is not subject to considerable left-censoring, since PayNet endeavours to acquire all current contracts as well as 1-3-year history of contracts from member lenders. Cross-sectional distribution statistics of relationship lengths across all observations and total lengths of relationship across all lender-borrower pairs in the baseline sample are summarized in Table 2.1. Note that the data is right-censored at

2019Q1, which implies the recorded total relationship length is the lower bound of the true relationship outcome. Figure 2.2 shows the composition of relationship length bins in the data by calendar year. The high percentage of very young relationships in the first few years is due to the fact no relationships were formed prior to 1999Q1.

Figure 2.2: Relationship Length Composition over Time

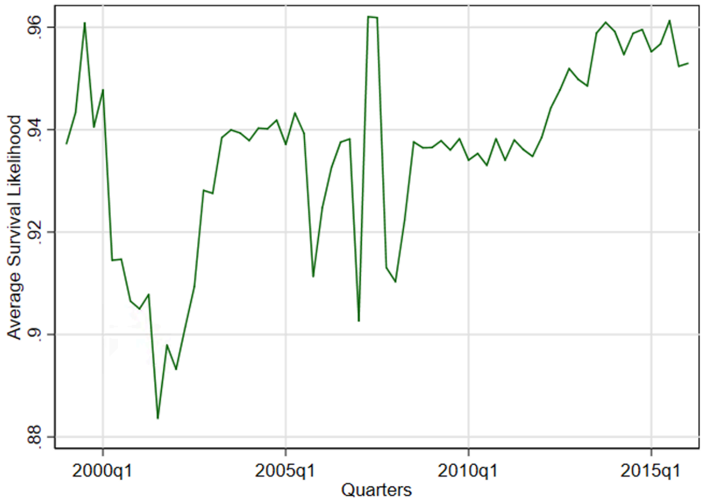


Notes: This figure shows the composition of relationship lengths measured in years in the PayNet data by calendar years. Relationships are categorized by length into "1-2 years", "3-4 years", "5-6 years", "7-8 years" and "9+ years".

Furthermore, relationships are notably sticky in the provided PayNet data. 90% of the two million firms in the baseline sample have only one lender. Also, the likelihood of a relationship surviving into the next quarter is high throughout the sample periods, as shown by Figure 2.3. I compute this time series only up to 2016Q1, because the data is right-censored at 2019Q1 and that the vast majority of relationships do not have a period of zero balance for over 3 years. The dramatic change in the average survival likelihood over the first few years is because of the small number of lenders with observations in these early years and that the distribution of lenders' total lending in the PayNet data becomes stable only after 2005Q1. During the Great Recession, relationships in the PayNet data saw increased volatility of the survival likelihood across quarters. The cross-sectional mean of the survival likelihood from 2005Q1 to 2016Q1 is 93.40%.

Table 2.2 shows the distribution of the loan balance in a relationship in the baseline

Figure 2.3: Relationship Survival Likelihood over Time



Notes: This figure shows the average likelihood that a relationship continues to have a positive loan balance in the next quarter over time.

sample, and the distribution of total lending by each lender in a randomly selected sample of half of the lenders covered. I deflate balance amounts using GDP deflators indexed by 2012Q1 USD to compare them across time.

Table 2.2: Pair-level and Lender-level Loan Balances

	Mean	SD	25%	50%	75%
Loan balance in a relationship ($1,000)	138.98	1008.59	10.35	27.20	71.76
Total lending by a lender ($1 billion)	0.81	1.58	0.02	0.12	0.73

Notes: Loan balances are deflated to reflect 2012Q1 USD using the quarterly GDP deflator from FRED.

Chapter 3

Empirical Results

I have two main empirical results: first, the loan balance between a lender-firm pair increases in the pair's relationship length; second, loan balances of longer lender-firm relationships respond less to credit supply shock to which the lenders are subject. For all empirical results as follows, I deflate balance amounts using GDP deflators indexed by 2012Q1 USD.

3.1 Relationship Lengths and Loan Balances

I take advantage of the PayNet data's panel structure to investigate whether the quantity of credit borrowed by a firm from a lender varies with the length of their lending relationship:

$$\frac{L_{i,j,t}}{L0_{i,j}} = \alpha + \beta T_{i,j,t} + \mathbf{\Gamma}'\mathbf{Z}_{i,j,t} + \varepsilon_{i,j,t} \tag{3.1}$$

where $L_{i,j,t}$ is the amount of balance between firm i and lender j at time t, and $L0_{i,j}$ is the initial loan balance between the pair. I normalize the relationship's loan balance amount by its initial loan balance so that the regression equally represents how relationships of all loan balance sizes are affected by their relationship lengths. $T_{i,j,t}$ is the length of their relationship at time t measured in integer quarters. $\mathbf{Z}_{i,j,t}$ is a vector of control variables including the "MasterScore v2" of the firm at the time, the pair's initial balance amount, the firm dummy variables, the lender dummy variables, the time dummy variables, dummy variables for a firm's 4-digit SIC industry at the time, and dummy variables for a firm's

CBSA location at the time. The coefficients of interest in this regression is β, which indicates the increase in the pair's loan balance relative to their initial balance due to a one-quarter increase in the relationship length. The standard errors are clustered by two ways – firm or bank – to account for within-firm-or-bank correlation of the error term.

The goal of adding the control variables is to address the potential selection bias of the sample due to endogeneity of the relationship length with the loan balance. In particular, relationships of long lengths or large balances are likely over-represented in the sample. For example, productive firms, lenders, and high-quality lender-borrower matches are likely associated with larger loan balances relative to their initial balance amounts, and are also more likely to successfully survive for a long period of time. In contrast, low-quality firms, lenders, and matches who are likely associated with smaller normalized loan balances might have a harder time finding a match or maintaining a long and stable lending relationship. Therefore, I control for the PayNet "MasterScore v2" of firms to absorb the correlation between the relationship length and the firm quality proxies by firms' payment overdue probabilities. The initial amount of balance captures the heterogeneity in lender-firm match quality. The firm and lender fixed-effects control for permanent differences in matching and separation probabilities owing to unobserved borrower or lender heterogeneity. Furthermore, the time fixed-effects explain the components of the loan balance that vary by aggregate economic conditions. Finally, I include fixed-effects of the firm location and industry to control for differences in loan balance and matching outcomes caused by lender specializations or credit market localization.

I perform a robustness check with firm-time fixed-effects added to Regression (3.1) to address further concerns about firm decisions and performances causing endogeneity of the relationship length variable. The firm-time fixed-effects absorb all factors particular to each firm at each time that might influence both the relationship length and the lending outcome. These firm-time fixed-effects are not included in the main regression since most firms, as explained earlier, have only one observation per quarter in the data and their corresponding firm-time fixed-effects cannot be identified.

The regression suggests that a relationship length has a significantly positive effect on the relationship's loan balance. Column (1) of Table 3.1 summarizes the result of

Regression (3.1) without any control variable or fixed-effect. Column (2) shows the full result with time fixed-effects, firm fixed-effects, and lender fixed-effects included in the regressor. Column (1) features a significant and positive coefficient to the relationship length, indicating that an increase in the relationship length is associated with a loan balance increase. Column (2) is consistent with Column (1) and shows that, controlling for potential endogeneity of the relationship length, a one-quarter increase in the relationship length leads to a loan balance increase of 9.33% of the initial balance amount. Column (3) demonstrates the result of the robustness check with firm-time fixed-effects, confirming that the positive influence of the relationship length on the loan balance is robust and actually larger when controlling for variations of firm decisions and performances. It is also notable that, since most firms do not have multiple observations per period, the number of observations for this robustness check is considerably smaller than that of the main regression. In summary, the relationship length plays an important role in determining the quantity of credit for a lending relationship.

Table 3.1: Normalized Loan Balance Amount vs. Relationship Length

Dependent variable: $L_{i,j,t}/L0_{i,j}$ (%)

	(1)	(2)	(3)
$T_{i,j,t}$	6.80***	9.33***	13.70***
	(0.64)	(1.71)	(2.03)
MasterScore		0.23	0
		(0.58)	(.)
Initial balance ($100,000)		-2.00***	-10.50***
		(0.1.8)	(2.05)
Observations	2902106	1010348	234263
R^2	0.000	0.360	0.495
Fixed-effects	No	Time + firm + lender + SIC + CBSA	Firm-time + lender + SIC + CBSA

Notes: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. Results from regression (3.1). The unit of the initial balance is 100,000 in 2012Q1 dollars. The standard errors are clustered by two ways.

I additionally run a regression that allows the influence of the relationship length on the loan balance to vary by phases of the relationship. Consider a variation of Regression

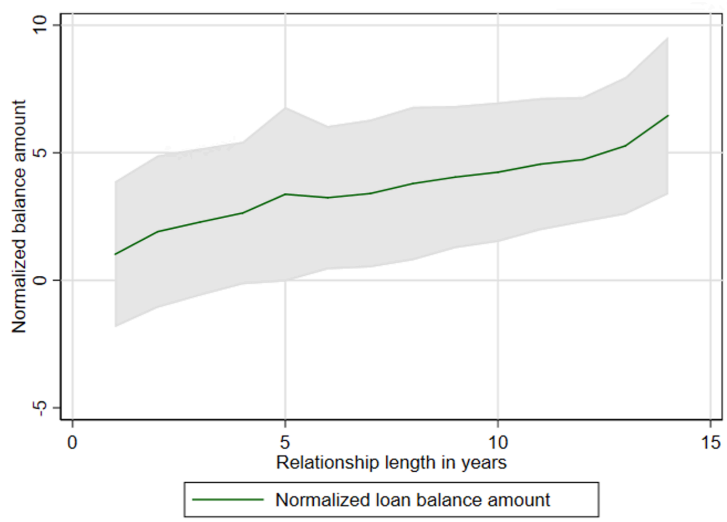
(3.1) as follows:

$$\frac{L_{i,j,t}}{L0_{i,j}} = \alpha + \sum_{\tau=2}^{14} \beta_{\tau} \mathbb{1}\{T^{Y}_{i,j,t} = \tau\} + \mathbf{\Gamma}'\mathbf{Z}_{i,j,t} + \varepsilon_{i,j,t} \tag{3.2}$$

where $T^{Y}_{i,j,t}$ is the length of the lender-firm relationship at time t measured in integer years, $\mathbb{1}\{T^{Y}_{i,j,t} = \tau\}$ is a dummy variable of whether the relationship is in the τth year, and $\alpha_{\mathcal{T}}$ is the normalized loan balance generated by being in the τth year of the relationship. The coefficients of interest in this regression are $\alpha_{\mathcal{T}}$ for each $\tau \in [1, 2, \ldots, 14]$, which indicates how the relationship length affects the pair's loan balance relative to the initial amount as the relationship develops. In this regression, I round relationship lengths to integer years because the the majority of the loan contracts are signed for an integer number of years.I also pool observations of 14-year-and-above relationships together due to their small number of observations.

The result of Regression (3.2), as illustrated by Figure 3.1, predicts no significant difference between each relationship year's influence on the relationship's normalized loan balance. Therefore, it is reasonable to assume that each quarter's increase in the relationship length changes the normalized loan balance by the same amount.

Figure 3.1: Normalized Loan Balance Amount vs. Relationship Length



Notes: Results from Regression (3.2). The solid line shows the point estimates for a relationship's loan balance amount relative to its initial loan balance in each year of the relationship. The shade demonstrates the standard errors of the estimation with 2-way clustering.

Lastly, I perform a robustness check to show that the positive influence of the relationship length on the loan balance estimated in Regression (3.1) is not due to over-representation of long relationships with large normalized balances. I regress the indicator of whether a relationship continues into the next quarter respectively onto the relationship length and the balance amount with the same set of control variables as in Regression (3.1). In this regression, I use observations from the baseline sample up to 2016Q1, because the data is right-censored at 2019Q1 and that the vast majority of relationships do not have an inactive period of over 3 years. As shown by Table 3.2, conditional on the control variables, while the normalized balance amount of a relationship has no significant effect on the relationship's survival, a longer relationship is in fact more likely to end. This result rules out the possibility that long relationships with large normalized balances are over-represented in the data to give rise to overestimated impact of the relationship length on the long balance.

Table 3.2: Robustness Check: Selection Bias

Dependent variable: survival indicator

	(1)	(2)	(3)	(4)
Normalized balance amount	1.15×10^{-5}	1.43×10^{-4}		
	(8.63×10^{-6})	(8.91×10^{-5})		
Relationship length (quarter)			2.63×10^{-4}	$-3.06\times10^{-3***}$
			(1.05×10^{-3})	(5.34×10^{-4})
MasterScore		$5.88\times10^{-4***}$		$6.88\times10^{-4***}$
		(7.46×10^{-5})		(8.10×10^{-5})
Initial balance		9.20×10^{-10}		$4.76\times10^{-9***}$
		(5.77×10^{-10})		(1.01×10^{-9})
Observations	2195192	658957	4638492	1169424
R^2	0.001	0.134	0.000	0.138
Time FE	No	Yes	No	Yes
Firm FE	No	Yes	No	Yes
Lender FE	No	Yes	No	Yes
Other FE	No	SIC + CBSA	No	SIC + CBSA

Notes: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. The standard errors are clustered by two ways.

3.2 Relationship Lengths and Credit Supply Shocks

I estimate the influence of relationship lengths on the responsiveness of a lender-firm pair's balance to a credit supply shock in two steps. First, I randomly select 50% of the observations in the baseline sample and use these selected observations to find a proxy for credit supply shocks to the lenders following the empirical methodology developed by Amiti and Weinstein (2018). Second, using the rest 50% of the baseline sample, I regress each pair's quarterly loan balance growth onto the relationship length, the credit supply shock proxy, their interaction and a number of control variables. I split the sample to reduce errors from the two-step estimation.

Amiti and Weinstein (2018) proposes an ideal way of separating firm-credit-demand shocks from lender-credit-supply shocks using a lender-borrower matched panel of loan balances. Their econometric approach is consistent with the formation of new relationships and the growth of aggregate lending. They consider the specification

$$\Delta L_{i,j,t} = \frac{L_{i,j,t} - L_{i,j,t-1}}{L_{i,j,t-1}} = D_{i,t} + S_{j,t} + \varepsilon_{i,j,t} \tag{3.3}$$

where $\Delta L_{i,j,t}$ stands for the loan growth between firm i and lender j from period $t-1$ to t, and $D_{i,t}$ is the credit demand shock to the firm, $S_{j,t}$ denotes the credit supply shock to the lender. With this specification, credit supply and demand shocks can be estimated using the following moments:

$$\mathbb{E}\left[\Delta L_{i,t}\right] = \mathbb{E}\left[\frac{\sum_j L_{i,j,t} - \sum_j L_{i,j,t-1}}{\sum_j L_{i,j,t-1}}\right] = D_{i,t} + \sum_j \theta_{i,j,t-1} S_{j,t} + e_t^d \tag{3.4}$$

$$\mathbb{E}\left[\Delta L_{j,t}\right] = \mathbb{E}\left[\frac{\sum_i L_{i,j,t} - \sum_i L_{i,j,t-1}}{\sum_i L_{i,j,t-1}}\right] = S_{j,t} + \sum_i \psi_{i,j,t-1} D_{i,t} + e_t^s \tag{3.5}$$

where $D_{i,t}$ and $S_{i,t}$ are the credit demand and supply shocks normalized respectively by shocks to the first firm and the first bank at the same time, $\theta_{i,j,t-1} = \frac{L_{i,j,t-1}}{\sum_j L_{i,j,t-1}}$ is the share of lender j in firm i's total borrowing at $t-1$, $\psi_{i,j,t-1} = \frac{L_{i,j,t-1}}{\sum_i L_{i,j,t-1}}$ is the share of borrower i in lender j's total lending at $t-1$, e_t^d is a demand-side time fixed-effect, and e_t^s is a supply-side time fixed effect. If there is no new relationship formed at $t-1$, this estimation is equivalent to a WLS regression of lender-firm pairs' loan growths onto firm-time dummies and lender-time dummies with lagged pair balances as weights. The estimation thus requires that each firm or lender in the data have multiple observations per period.

I approximate the Amiti and Weinstein approach at a more aggregate level due to the data constraint. In the PayNet data, each lender has many observations in each quarter; a firm, nevertheless, usually has only one observation per period, because most of them have contracts with only one lender. For this reason, I group firms in the data by their CBSA and 4-digit SIC code[1]. That is to say, I decompose the growth of lender j's lending to CBSA-SIC cell c from period $t-1$ to t as

$$\Delta L_{c,j,t} = \frac{L_{c,j,t} - L_{c,j,t-1}}{L_{c,j,t-1}} = D_{c,t} + S_{j,t} + \varepsilon_{c,j,t} \tag{3.6}$$

where $D_{c,t}$ is the demand shock to CBSA-SIC cell c and $S_{j,t}$ is a proxy for the credit supply shock to lender j. The assumption behind this approximation is that any idiosyncratic credit demand change to a firm is uncorrelated with the credit supply shock to its lender conditional on the CBSA-SIC cell credit demand shock. Given that the data focuses on small businesses, it is reasonable to assume that borrowers can only collectively but not individually affect their lender's conditions at the cell level.

I randomly select 50% of the observations in the baseline sample to estimate the credit supply shock proxy in Specification (3.6). The estimation uses modified Amiti and Weinstein moment conditions. The regression follows

$$\Delta L_{c,t} = \hat{D}_{c,t} + \sum_j \theta_{c,j,t-1} \hat{S}_{j,t} + \hat{e}_t^d \tag{3.7}$$

$$\Delta L_{j,t} = \hat{S}_{j,t} + \sum_c \psi_{c,j,t-1} \hat{D}_{c,t} + \hat{e}_t^s \tag{3.8}$$

where $\hat{D}_{c,t}$ is the estimate of the normalized cell credit demand shock and $\hat{S}_{j,t}$ is the estimate of the normalised proxy of the credit supply shock.

1. The data contains observations from 954 different CBSAs and 1156 different 4-digit SIC codes. A median CBSA-SIC cell has x observations per quarter.

With the credit supply shock proxy estimated, I run the following regression to investigate how the relationship length matters for the responsiveness of a lender-borrower pair's balance to these credit supply shocks:

$$\Delta L_{i,j,t} = \beta_0 + \beta_1 \hat{S}_{j,t} + \beta_2 \left(T_{i,j,t-1} \times \hat{S}_{j,t} \right) + \beta_3 T_{i,j,t-1} + \mathbf{\Gamma}' \mathbf{Z}_{i,j,t} + \varepsilon_{i,j,t} \tag{3.9}$$

where $\Delta L_{i,j,t}$ is the balance growth between firm i and lender j from period $t-1$ to t, $\hat{S}_{j,t}$ is the estimated proxy for the credit supply shock to the lender that hits at time $t-1$, $T_{i,j,t-1}$ is the relationship length of the lender-firm pair at time $t-1$ now measured in quarters, and $\mathbf{Z}_{i,j,t}$ is the same set of control variables in Regression (3.1). I use the firm's MasterScore v2 at time $t-1$, the pair's initial balance, lender fixed-effects, borrower fixed effects, industry fixed effects and time fixed-effects to address the endogeneity of the relationship length with the pair's loan balance, as explained in Section 3.1. In this regression, I use observations in the baseline sample that are not selected for Regression (3.8). I also standardize the proxy for credit supply shocks for better economic interpretation.

Results from Regression (3.9) are displayed by Table 3.3. Column (1) demonstrates the regression result with no control variable; and column (2) shows the result with all control variables. As demonstrated, different specifications yield consistent results, with the R^2 improving slightly by the inclusion of more controlling factors. Column (1) shows a positive coefficient of the credit supply shock proxy, implying that an increase in the lender's credit supply would increase the growth rate of the balance between a lender-borrower pair. The relationship length coefficient and interaction coefficient are both negative, implying that longer relationships respond less to credit supply shocks. Column (2) shows that controlling for relationship length endoegneity does not affect the conclusion. In column (2), the proxy for credit supply shock has coefficient 0.24, suggesting that an one-standard deviation credit supply shock would increase a lender-borrower pair's balance growth by 0.24%. However, an one-quarter increase in the relationship length would decrease the balance growth rate's response by a level of 0.23%, plus more than 0.02% of the shock's magnitude. That is to say, one more quarter's relationship is associated with lessened responsiveness of the pair's balance growth to a credit supply

Table 3.3: Loan Balance Sensitivity to Credit Supply Shocks vs. Relationship Length

Dependent variable: $\Delta L_{i,j,t}$ (%)		
	(1)	(2)
$\hat{S}_{j,t}$	$1.52\times10^{-2***}$	0.24
	$(9.04\ \times10^{-2})$	(0.16)
$T_{i,j,t-1} \times \hat{S}_{j,t}$	$-1.49\times10^{-2**}$	$-1.68\times10^{-2***}$
	(6.92×10^{-3})	(3.64×10^{-3})
$T_{i,j,t-1}$	$-8.46\times10^{-2***}$	-0.21^{***}
	(1.98×10^{-2})	(2.77×10^{-2})
MasterScore		$-4.08\times10^{-2***}$
		(4.72×10^{-3})
Initial balance ($100,000)		$4.00\times10^{-2***}$
		(1.11×10^{-2})
Observations	414658	392492
R^2	0.002	0.259
Time FE	No	Yes
Firm FE	No	Yes
Lender FE	No	Yes
Other FE	No	SIC + CBSA

Notes: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. Results from Regression (3.9). The unit of the initial balance is 100,000 in 2012Q1 dollars. The standard errors are clustered by two ways.

shock by 7%. The reduction of balance sensitivity is even larger for firms of a lower default probability, as indicated by the MasterScore v2. The influence of a pair's initial balance is positive and significant, meaning that relationships with larger loan balances see faster credit growths.

I then study if the relationship length has the same implication for the transmission of positive and negative supply shocks. I therefore revise Regression (3.9) as follows:

$$\begin{aligned}\Delta L_{i,j,t} =&\beta_0 + \beta_1 \left(\hat{S}_{j,t} \times \mathbb{1}\{\hat{S}_{j,t} \geq 0\}\right) + \beta_2 \left(\hat{S}_{j,t} \times \mathbb{1}\{\hat{S}_{j,t} < 0\}\right) \\ &+ \beta_3 \left(T_{i,j,t-1} \times \hat{S}_{j,t} \times \mathbb{1}\{\hat{S}_{j,t} \geq 0\}\right) + \beta_4 \left(T_{i,j,t-1} \times \hat{S}_{j,t} \times \mathbb{1}\{\hat{S}_{j,t} < 0\}\right) \\ &+ \beta_5 T_{i,j,t-1} + \mathbf{\Gamma}'\mathbf{Z}_{i,j,t} + \varepsilon_{i,j,t}\end{aligned} \tag{3.10}$$

Table 3.4 displays results from Regression (3.10). The results suggest that, while a one-quarter increase in the relationship length has a significant and negative effect on the relationship's loan balance responsiveness to a positive credit supply shock, the relationship length's increase does not have a significant influence on the loan balance responsiveness to a negative credit supply shock. That is to say, the results from Regression (3.9) that longer relationships are less responsive to credit supply shocks are driven by positive credit supply shocks.

Table 3.4: Loan Balance Sensitivity to Positive/Negative Shocks vs. Relationship Length

	Dependent variable: $\Delta L_{i,j,t}$ (%)	
	(1)	(2)
$\hat{S}_{j,t} \times \mathbb{1}\{\hat{S}_{j,t} \geq 0\}$	-9.02×10^{-2}	0.69^{***}
	(9.68×10^{-2})	(0.19)
$\hat{S}_{j,t} \times \mathbb{1}\{\hat{S}_{j,t} < 0\}$	2.29^{**}	-3.93^{***}
	(0.76)	(0.90)
$T_{i,j,t-1} \times \hat{S}_{j,t} \times \mathbb{1}\{\hat{S}_{j,t} \geq 0\}$	$-1.06\times10^{-2*}$	$-1.87\times10^{-2***}$
	(4.57×10^{-3})	(5.20×10^{-3})
$T_{i,j,t-1} \times \hat{S}_{j,t} \times \mathbb{1}\{\hat{S}_{j,t} < 0\}$	-9.80×10^{-2}	5.24×10^{-2}
	(4.93×10^{-2})	(3.23×10^{-2})
MasterScore		$-4.06\times10^{-2***}$
		(4.71×10^{-3})
Initial balance ($100,000)		$3.69\times10^{-2**}$
		(1.13×10^{-2})
Observations	414658	392492
R^2	0.002	0.259
Time FE	No	Yes
Firm FE	No	Yes
Lender FE	No	Yes
Other FE	No	SIC + CBSA

Notes: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. Results from regression (3.10). The unit of the initial balance is 100,000 in 2012Q1 dollars. The standard errors are clustered by two ways.

The standard errors in Table 3.9 and Table 3.4 do not need to be corrected as in a regular two-step estimation for two reasons. First, the estimation error of each lender-cell pair in the first Regression (3.8) is not magnified through the same observation in Regression (3.9) or (3.10), because these two regressions use different sub-samples of the data. Second, the first-step regression's estimation residuals are much smaller than the estimated credit supply shock proxies, implying that the estimation error is unlikely to matter for the second-step regression. To be specific, I compute a measure E for the magnitude of the first-step residuals relative to the magnitude of the estimated credit supply shock proxies:

$$E = \left(\mathrm{var}(\frac{\mathrm{var}(\hat{\epsilon}_{c,j,t}|j,t)}{N(j,t)})\right) / \left(\mathrm{var}(\hat{S}_{j,t})\right)$$

where $\mathrm{var}(\hat{\epsilon}_{c,j,t}|j,t)$ is the variance of residuals for lender j with each of the CBSA-SIC cells to which it lends at time t, $N(j,t)$ is the number of cells lender j lends to at time t, and $\hat{S}_{j,t}$ is the estimated proxy for the credit supply shock to lender j at the time. A small E indicates that the variance of the estimation error for each lender is small relative to the variance of the estimated credit supply shock proxies. The E measure for the first-step Regression (3.8) is only about 5.09%. These two reasons together suggests that the estimation error's influence on the standard errors in Table 3.3 and Table 3.4 is negligible.

Chapter 4

Model

In this chapter, I introduce a parsimonious competitive search model where firms search for loans and banks post optimal long-term lending contracts subject to firms' limited commitment, which is a version of Boualam (2018). Banks' funding cost is the aggregate state of the economy and contracts are state-contingent. I use this model to interpret the empirical results in Section 3 and draw implications on the aggregate economy.

4.1 Environment

The economy is populated by two types of infinitely-lived agents: a continuum of measure one of homogeneous firms and a large continuum of homogeneous banks. All agents have discount factor $\beta \in (0, 1)$.

Firms are risk-averse with utility function

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right],$$

where c_t is the firm's consumption in period t, and $u : \mathbb{R}^+ \to \mathbb{R}^+$ satisfies $u' > 0$, $u'' < 0$ and $\lim_{c \to 0} u'(c) = \infty$. Each firm is able to produce an undifferentiated good using capital k with a standard technology $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that $f' > 0$, $f'' < 0$, $f(0) = 0$ and $\lim_{k \to 0} f'(k) = \infty$. Firms do not own capital and can produce only if financed externally. Without funding, a firm can only consume b each period. Firms do not have any access to any storage technology. Here I model firms as risk-averse and completely reliant on external funding to focus on small businesses.

Banks are risk-neutral. In each period t, they have access to deposits at an exogenous funding cost r_t that follows a 2-state Markov process on $\{r^l, r^h\}$ with transition matrix

$$\Pi = \begin{bmatrix} p^l & 1-p^l \\ 1-p^h & p^h \end{bmatrix}$$

where $0 < r^l < r^h$ and $p_l, p_h \in (0,1)$. The funding cost is the only source of aggregate risk in this economy. I label the low funding cost state r^l the "normal" state, and the high funding cost state r^h the "recession" state.

Matching Banks and firms match on a decentralized credit market. In each period, one bank can finance only one firm, and one firm can be financed by only one bank. The credit market is modeled through a competitive search framework. At the beginning of each period, banks choose whether to pay a positive cost s to post a lending contract and enter the market. Each firm observes all the contracts posted by active banks and chooses to direct their search to the one they prefer. Then bilateral matching takes place. If a bank and a firm match, the pair enter into a lending relationship described by the contract. Each contract delivers a value V_0 to the firm and can be characterized by V_0. Each period, all relationships face a constant probability $\sigma \in (0,1)$ of separation.

For each funding cost r and contract V_0, let $\mu(r, V_0)$ denote the mass of unfunded firms searching for contract V_0 in state r, and $\nu(r, V_0)$ the mass of banks offering V_0 in state r. The mass of matches created in each sub-market V_0 for each funding cost r is given by a constant-returns-to-scale (CRS) matching function. Let $\Theta(r, V_0) \equiv \frac{\nu(r,V_0)}{\mu(r,V_0)}$ denote the tightness of sub-market V_0 in state r. Then, the probability that an unfunded firm searching in sub-market V_0 when the state is r matches with a bank can be represented by a non-decreasing function $p(\Theta(r, V_0))$ with $p : [0, \infty) \to [0, 1]$. The probability that a bank posting V_0 when the state is r is matched a firm is represented by a non-increasing function $q(\Theta(r, V_0)) = \Theta(r, V_0)p(\Theta(r, V_0))$ with $q : [0, \infty) \to [0, 1]$. Furthermore, p and q

satisfy

$$p(0) = 0, \quad q(0) = 1, \quad \lim_{\theta \to \infty} p(\theta) = 1, \quad \lim_{\theta \to \infty} q(\theta) = 0.$$

Optimal Contracts Contracts are long-term and state-contingent. Let

$$r_t^\tau \equiv \{r_t, r_{t+1}, \dots, r_\tau\}$$

denote the history of the realizations of the funding cost from period t to period $\tau \geq t$. Then a contract V_0 signed at time t is defined by two functions $k_\tau(r_t^\tau)$ and $d_\tau(r_t^\tau)$, where $k_\tau(r_t^\tau) > 0$ is the amount of capital the bank lends to the firm and $d_\tau(r_t^\tau)$ is the payment from the firm in each period τ as functions of the history of the funding cost since the beginning of the relationship r_τ^t.

Banks fully commit to the contract. However, a funded firm can decide to walk away from the contract at any time and steal a proportion $\eta \in (0, 1]$ of the capital obtained from the bank that period. After a firm walks away from a contract, they will be able to return to the credit market and search for a new contract in any following period with probability $\gamma \in [0, 1]$. Banks' optimal contracting problem can be written in a recursive form. Let V denote the firm's expected value from the contract, which is

$$V = u(f(k) - d) + \beta \mathbb{E}\left[\sigma V^u(r') + (1 - \sigma)V'(r')|r\right] \tag{PK}$$

where k is the amount of capital the bank lends to the firm in that period, d is the payment from the firm in that period, and $V^u(r')$ is the value of an unfunded firm searching for a lending contract when the funding cost is r'.

The firm will stay in the contract relationship if and only if the value of the contract is not smaller than the value of consuming the stolen capital ηk and searching for a new lending contract whenever it becomes possible. The incentive compatibility constraint can then be written as

$$V \geq u(\eta k) + \beta D(r) \tag{IC}$$

where $D(r)$ is the continuation value of a firm who walks away from the contract when the state is r and takes the form

$$D(r) = \mathbb{E}\left[\gamma V^u(r') + (1-\gamma)\left(u(b) + D(r')\right)|r\right]. \tag{4.1}$$

The optimal contract for a given promised contract value V_0 solves the following problem

$$B(r,V) = \max_{k,d,V'(r^l),V'(r^h)} \quad d - rk + \beta(1-\sigma)\mathbb{E}[B(r', V'(r'))|r] \tag{P1}$$

subject to the promise keeping constraint (PK), the incentive compatibility (IC), the non-negativity constraint $k \geq 0$ and $d \geq 0$, and the initial value $V = V_0$.

4.2 Equilibrium

As equilibrium notion, I use a natural extension of a standard competitive search equilibrium in Wright et al. (2017). In equilibrium, banks post optimal lending contracts and make zero profit. Conditional on the contracts posted in a given state, firms optimally direct their search towards their preferred contract. it is important to define beliefs about the market tightness even for contracts that are not posted in the equilibrium.

Definition 1. A competitive search equilibrium is a pair of unfunded firm values $\{V^u(r^l), V^u(r^h)\}$, a pair of optimal contract values $\{V_0(r^l), V_0(r^h)\}$ with $\mathbb{V}$ denoting the set of all posted contracts, a set of market tightness functions $\Theta(r^s, V) : \mathbb{R}_+ \to [0, \infty)$ for $s = l, h$, and a set of optimal policies $k(r, V) : \{r^l, r^h\} \times \mathbb{R}_+ \to [0, \infty)$, $d(r, V) : \{r^l, r^h\} \times \mathbb{R}_+ \to [0, \infty)$, $V'(r', r, V) : \{r^l, r^h\} \times \{r^l, r^h\} \times \mathbb{R}_+ \to (0, \infty)$, such that:

(i) Banks' profit maximization and free entry: $\forall\ V_0 \in \mathbb{R}_+,\ r \in \{r^l, r^h\}$,

$$q(\Theta(r, V_0))\mathbb{E}\left[B(r', V'(r', r, V_0))|r\right] - s \leq 0 \tag{4.2}$$

with equality if $V_0 \in \mathbb{V}$.

(ii) Optimal policies $k(\cdot)$, $d(\cdot)$, $V'(\cdot)$ solve the banks' optimal contracting problem (P1), $\forall\ r, r' \in \{r^l, r^h\}, V \in \mathbb{R}_+$.

(iii) Unfunded firms' optimal search: $\forall\, V_0 \in \mathbb{R}_+,\ r \in \{r^l, r^h\}$,

$$V^u(r) \geq u(b) + \beta \left[p(\Theta(r, V_0)) + (1 - p(\Theta(r, V_{)})))\mathbb{E}[V^u(r')|r] \right] \tag{4.3}$$

with equality if $\Theta(r, V_0) < \infty$, where

$$V^u(r) = \max_{V_0 \in \mathbb{R}_+} \; u(b) + \beta \left[p(\Theta(r, V_0)) + (1 - p(\Theta(r, V_0)))\mathbb{E}[V^u(r')|r] \right]. \tag{4.4}$$

Boualam (2018) shows that a unique competitive search equilibrium exists with a sufficiently small posting cost s.

4.3 Contract Characterization

In this section, I derive some properties of the optimal contract between the bank and the firm. First, I characterize the relationship between the bank's and the firm's value flows.

Lemma 1. The Pareto frontier $B(r, V)$ is continuously differentiable, strictly decreasing in r and V, and concave with respect to V.

The lemma shows the relationship between the firm value and the lender profit. Intuitively, the bank expects less profit when the funding cost is higher. Also, a higher firm value implies that the bank needs to compensate the firm more by increasing the loan amount or reducing the requested payment in the current period or some future period. The implication is that the bank has to sacrifice some profit in some period during the relationship. Therefore a higher firm value is associated with a smaller bank value. Concavity of $B(r, V)$ comes from the fact that, due to decreasing marginal utility of the firm, the bank has to offer more compensation to the firm to increase the firm value by a certain extent if the current firm value is higher. So the bank's marginal cost of increasing the firm value is higher with a larger V.

The following contract properties focus on how the bank chooses compensation for the firm over time and between the two ways of increasing the firm value – increasing the loan amount to the firm in a particular state and providing better insurance for the firm against funding cost stochasticity.

Proposition 1. The optimal policy for the bank's lending to the firm is:

$$k(r,V) = \begin{cases} \tilde{k}(r), & \text{if } V \geq u(\eta\tilde{k}(r)) + \beta D(r), \\ \frac{u^{-1}(V-\beta D(r))}{\eta} < \tilde{k}(r), & \text{if } V < u(\eta\tilde{k}(r)) + \beta D(r) \end{cases} \tag{4.5}$$

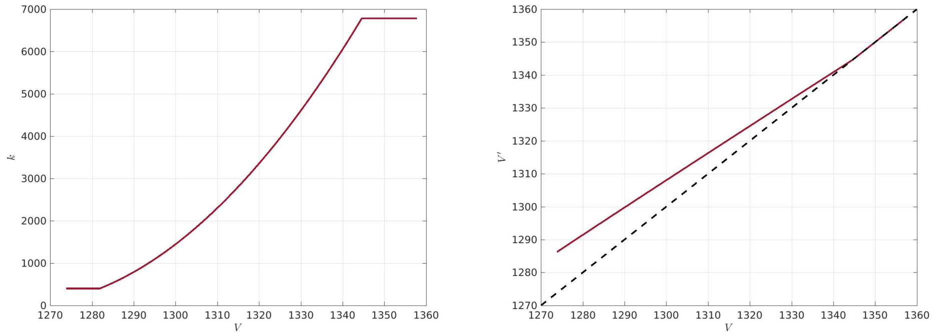
where $\tilde{k}(r) = \text{argmax}_k\, f(k) - rk$ is the first-best amount of financing to the firm in absence of the limited commitment when the state is r.

The intuition behind Proposition 1 is that, the bank can safely lend the first-best amount of capital to the firm if and only if the firm's value from the contract is larger than the value from stealing the capital; otherwise, the amount of capital the bank can safely lend to the firm is bounded by the firm's limited commitment.

Proposition 2. With the funding cost fixed at r, the optimal contract features $V'(r) \geq V$ and $k'(r) \geq k$ with strict inequality if and only if $V < u(\eta\tilde{k}(r)) + \beta D(r)$.

Proposition 1 and Proposition 2 together imply that, for a fixed r and $r' = r$, the optimal policies $k(\cdot)$ and $V'(\cdot)$ follow the patterns in Figure 4.1. The left panel plots k against V, and the right panel plots V' against V. The combination of these two panels suggest how the contract evolves over time. With a constant bank funding cost, if the contract begins with a small firm so that the incentive constraint is binding, the firm value would increase in the next period, leading to a higher amount of capital in the next period. The firm value eventually converges to the level at which the incentive constraint just becomes slack. Consequently, the amount of capital lent by the bank eventually converges to the first-best amount $\tilde{k}$. Therefore, the bank-firm relationship length observed in the data can be associated with the firm value V which, given the funding cost, characterizes the evolution of the contract. As a result, in the model, limited commitment on the firm side results in an optimal contract with a gradually increasing amount of bank lending in a relationship. This optimal contract property is consistent with the first empirical finding of this paper, as presented by Table 3.1. Note that the increase in relationship capital takes only one period without the risk-averseness of firms.

Figure 4.1: Optimal Policies



(a) k Against V (b) V' Against V

Notes: The left panel shows the optimal capital policy k against the firm value V with a constant funding cost. In right panel, the solid line shows the optimal continuation firm value V' against the firm value V with a constant funding cost, and the dashed line represents the 45-degree line.

Proposition 3. For a given funding cost r and a given firm value V, the continuation firm value $V'(r')$ is decreasing in r'.

Proposition 3 implies that, conditional on the current state r, the bank promises the firm more continuation value if the funding cost in the next period is lower. A lower next-period funding cost implies a higher match surplus, so it is optimal for the bank to compensate the firm more in this case. This proposition also implies that in the next period, the loan amount is higher if the funding cost is lower, following the optimal capital policy (4.5). This proposition suggests that the bank provides only partial insurance for the firm against the funding cost stochasticity.

Chapter 5

Quantitative Analysis

In Section 5.1, I calibrate the model with features in the PayNet data, including the estimated slope of relationship lending with respect to the relationship length shown in Table 3.1. In Section 5.2, I validate the model with the empirical finding on how the relationship affects the responsiveness of loan balances to credit supply shocks, as shown in Section 3.2. Then, in the same section, I interpret the model validation using properties of the optimal contract derived in Section 4.3. Finally, in Section 5.3, I perform a counterfactual exercise and analyze how the distribution of relationship lengths in the economy matters for the aggregate effects of credit supply shocks.

5.1 Calibration

Function Forms I specify a CRRA utility function for the firms $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$ with ρ being the risk-aversion parameter. I model firms as risk-averse to describe small businesses that do not have diversified ownership. I assume the production function is $f(k) = k^{\alpha}$, where α is the elasticity of capital with respect to total output. The matching probabilities for unfunded firms and banks follow

$$p(\Theta) = \Theta(1 + \Theta^{\psi})^{-1/\psi}$$

$$q(\Theta) = (1 + \Theta^{\psi})^{-1/\psi}$$

where ψ denotes the matching function elasticity.

Fixed Parameters Table 5.1 lists all parameters I exogenously fix. The model period is one quarter, so I set the discount factor β to be 0.99. I choose the value of the funding cost in the normal state, r^l, and the recession state r^h, to match the average the 3-month Treasury Bill rates in NBER expansions and recessions from 1999Q1 to 2019Q1. I set the persistence of the normal state and the recession state to be respectively 0.9941 and 0.9929, following the calibration of Khan and Thomas (2013). In expectation, the economy spends about 7% of the time in recessions with an average recession lasting 12.8 quarters. I set the risk-aversion coefficient to be 0.5, following Boualam (2018). I choose the capital elasticity to total output to be 0.6. The reason is that, while the model abstracts from labor input in production, I assume firms optimally choose labor based on the amount of external financing. With the implied adjustment of labor, the elasticity of capital with respect to total production is about 0.6, as estimated by Cooper and Haltiwanger (2006).

Table 5.1: Fixed Parameters

Parameters	Description	Value
r^l	Bank funding cost in the normal state	1.76%
r^h	Bank funding cost in the recession state	2.12%
p_l	Persistence of the normal state	99.41%
p_h	Persistence of the crisis state	92.19%
β	Discount factor	0.99
ρ	Risk-aversion parameter	0.50
α	Capital share in total output	0.60

Notes: Parameters fixed exogenously in model calibration.

Fitted Parameters Table 5.2 shows parameters I choose to match moments from the data listed in Table 5.3. The objective of the calibration is to match the distribution of relationship lengths observed in the PayNet data, and the speed of the increase in the loan balance of a lender-firm relationship estimated in Section 3.1.

I compute empirical moments from observations from 2005Q1 to 2019Q1 of all lenders that enter the PayNet data before 2005Q1 to focus on a fixed pool of lenders for a long period of time. Then I evaluate the model fit using a simulated sample of 10000 firms for 57 quarters in the normal state. I use the method of simulated annealing to find fitted parameters.

First, I set the exogenous separation rate σ to be 6.58% to match the cross-sectional average of the likelihood that a lender-borrower relationship continues into the next quarter in the subset data described above.

The second set of parameters are the matching function elasticity ψ, the bank posting cost s and the unfunded firm consumption b. They govern the credit market tightness and therefore the distribution of relationship lengths in the modeled economy. I choose these parameters to target the empirical mean and standard deviation of the relationship length, as well as the average proportion of relationships in their first quarter.

The last set of parameters are the proportion of capital a funded entrepreneur can steal η and the probability of returning to the credit after capital stealing γ. These parameters govern the extent to which the firm's commitment is limited, and therefore how the loan amount increases over time in a lending relationship. I target the average increase in the loan balance as a percentage of the initial balance due to a one-quarter increase in the relationship length, as shown by Table 3.1, and the average rate of credit origination in the data defined as:

$$\text{credit origination rate} = \frac{\text{total new relationship balances} + \text{total increase in old relationship balances}}{\text{total balances in all relationships}} \tag{5.1}$$

Table 5.3 summarizes the targeted moments and the fit of the model.

Table 5.2: Fitted Parameters

Parameters	Description	Value
σ	Separation probability	6.58%
ψ	Matching function elasticity	1.6102
s	Bank posting cost	64.9283
b	Unfunded firm consumption	26.7730
η	Proportion of capital a funded firm can steal	0.2530
γ	Probability that a firm returns to the credit market after stealing	0.5632

Notes: Parameters chosen to match moments in Table 5.3.

As demonstrated, the model well predicts the distribution of relationship lengths in the data. Also, the model fits closely the estimated average loan balance growth rate and is thus very consistent with the first empirical finding in Section 3.1.

Table 5.3: Calibration Targets and Model Fit

Moments	Data	Model
Mean of relationship lengths	11.45	11.36
SD of relationship lengths	10.98	9.93
Average new relationship proportion	9.44%	9.33%
Credit origination rate	11.25 %	11.62%
Average increase in the loan balance per quarter as a percentage of the initial balance	9.33%	9.32%

Notes: Empirical moments targeted for model calibration and the model fit.

5.2 Model Validation

I validate the model calibration by showing that the model also predicts smaller loan balance responsiveness to credit supply shocks by longer relationships, as I empirically document in Section 3.2. In specific, I simulate a panel of 5000 firms for 81 periods to be consistent with the period 2000Q1 to 2019Q1 covered by PayNet. I consider changes of the funding cost as credit supply shocks, and set the path of the funding costs such that the funding cost is high during NBER recessions and low during NBER expansions. Then I run the following regression with the simulated panel:

$$\Delta L_{l,t} = \beta_0 + \beta_1 S_t + \beta_2 T_{l,t-1} \times S_t + \beta_3 T_{l,t-1} + \lambda_t + \varepsilon_{l,t} \tag{5.2}$$

where $\Delta L_{l,t}$ is the loan balance growth from period $t-1$ to t of a relationship in the simulation indexed by l, S_t is the aggregate capital growth, $T_{l,t-1}$ is the relationship's length in the previous period, and λ_t denotes the time fixed-effect. In this regression, I use the aggregate capital growth as a measure of the credit supply shock. I do not control for lender, borrower, or pair-specific variables as in the data, since the model abstracts from lender, borrower, and match heterogeneity.

Table 5.4 summarizes the result of Regression (5.2) and shows that the model is consistent with the data. This regression also yields a positive coefficient to the credit supply shock measure, and a negative coefficient to both the relationship length and the interaction term of the shock with the relationship length. I do not compare the exact magnitudes of these coefficient estimates with those in the empirical exercise because the model abstracts from the concept of individual banks. That is to say, the measure of credit supply shocks in Regression (5.2) are not comparable with the proxy of credit supply shocks in Regression (3.9), though they both demonstrate the extent to which the economy is subject to changes of the credit supply conditions.

Table 5.4: Model Validation

Dependent variable: $\Delta L_{i,j,t}$ (%)

	Model	Data
S_t	9.40^{***}	0.24
	(6.10×10^{-2})	(0.16)
$T_{l,t-1} \times S_t$	$3.28\times10^{-2***}$	$-1.68\times10^{-2***}$
	(8.62×10^{-3})	(3.64×10^{-3})
$T_{l,t-1}$	-1.11^{***}	-0.21^{***}
	(3.46×10^{-3})	(2.77×10^{-2})

Notes: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. Column "Model" shows the result of Regression (5.2) using the simulated model; column "Data" shows the result of Regression (3.9) with all control variables using the baseline sample of the PayNet data.

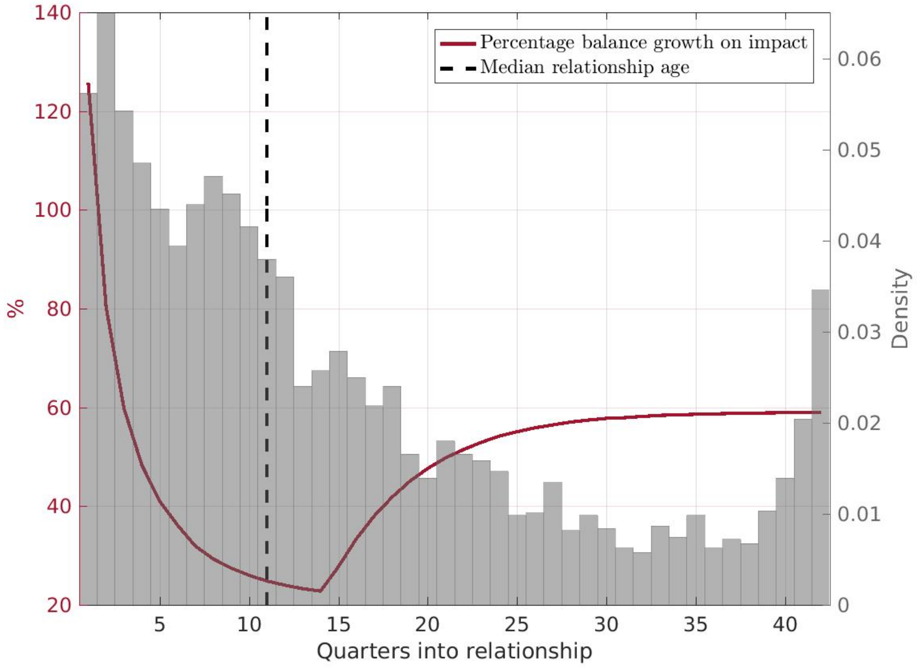
I explain the intuition behind Table 5.4 with the economy's response to the end of the recession in 2009Q3. Figure 5.1 plots the loan balance responsiveness, measured by the percentage growth of the loan balance, to a decrease in the funding cost against the relationship length, as well as the distribution of relationship lengths in 2009Q2 simulated from the calibrated model. With a decrease in the bank funding cost, loan balances of relationships of all lengths increase. The loan balance responsiveness is first decreasing in the relationship length up to the length of 14 quarters. These are the relationships that have not yet reached the first-best loan balance level in the recession state. Recall the firm incentive compatibility constraint (IC):

$$V \geq \underbrace{u(\eta k) + \beta \overbrace{D(r)}^{\text{continuation value of walking away}}}_{\text{value of walking away}}$$

where $D(r) = \mathbb{E}\left[\gamma V^u(r') + (1-\gamma)\left(u(b) + D(r')\right)|r\right]$. With a decrease in r, the credit market's match surplus in the next period goes up, implying a higher promised value to the firm by posted contracts $V_0(r)$ and a higher unfunded firm value $V^u(r)$. Since both expansions and recessions are persistent, the continuation value for a firm who walks away from the relationship, $D(r)$, goes up with $V^u(r)$. Consider two firms that are both credit-constrained in the recession state, one in a shorter relationship and the other in a longer relationship. As explained in Section 4.3, the firm in a shorter relationship has a smaller value from the contract and obtains less capital from the bank in each period. While the increase in $D(r)$ changes increases both firms' values of walking away by the same magnitude, the one in a shorter relationship sees a higher percentage increase in their value of walking away, as the value of consuming the stolen capital, $u(\eta k)$, is lower. In response to the positive credit supply shock, all banks increase their lender firm's value V from the relationship by extending the amount of the loan to satisfy the firm's incentive compatibility. Since the percentage increase of the value of walking away is higher for the firm in a shorter relationship, this firm is compensated by a higher percentage increase in capital lent by the bank, in comparison with the other firm who is in a longer relationship. As a result, for credit-constrained firms in the recession state, the growth of their loan balance after a positive credit supply shock is decreasing in the length of their relationship with the bank.

For relationships longer then 14 quarters, however, the growth in the loan balance actually increases in the relationship length. These relationships are already unconstrained in the recession state, meaning that they all receive the same loan amount $\tilde{k}(r^h)$ in the recession regardless of the relationship length. However, in the low funding cost state, their loan balances can still increase until they reach the maximum loan amount in this state, $\tilde{k}(r^l)$. As a result, the percentage growth of the loan balance after the positive

Figure 5.1: Loan Balance Responsiveness to a Positive Credit Supply Shock by Relationship Length



Notes: The solid red line shows the percentage increase in a relationship's loan balance upon a decrease in the bank funding cost in 2009Q3 by the relationship length in quarters. The grey bars show the distribution of relationship lengths in 2009Q2 in the simulated economy with the model calibration. The dashed black line shows the median length of lending relationships in 2009Q2.

credit supply shock for these firms is increasing in the relationship length.

The median relationship is at 11-quarter old, and the majority of relationships are in the region where the loan balance responsiveness decreases in the relationship length. As the result, the overall effect as shown by Table 5.4 is that the loan balance responsiveness to the interest rate cut is decreasing in the relationship length.

5.3 Aggregate Implications of Relationship Length Distribution

I now use the model to perform a counterfactual exercise and study how the aggregate lending's response to a positive credit supply shock differs in an economy with a higher firm ext rate. The economy has seen the firm exit rate decreasing in the past decades. Data from the Business Dynamics Statistics shows that the firm exit rate was 42.86% higher in

1980 than in 2016. I match this change of the firm exit rate in the counterfactual exercise by increasing the exogenous separation probability η by 42.86%.

Table 5.5: Aggregate Response to a Positive Shock: Baseline vs. Counterfactual

	Median $T_{l,t-1}$	Aggregate Credit Growth
Benchmark	11(Q)	39.13%
Benchmark policy + counterfactual distribution	8(Q)	45.15%
Counterfactual	8(Q)	35.32%

Notes: Aggregate lending responsiveness to a decrease in bank funding cost in 2009Q3 for the benchmark economy, the economy with the benchmark contract policies and the counterfactual relationship length distribution and the counterfactual economy. The relationship separation rate in the counterfactual case is 42.86% higher than in the benchmark case.

I compare the aggregate effect of the funding cost decrease across three cases: the benchmark case, the "decomposition" case in which the relationship length distribution is as in the counterfactual economy but the contract policies are held fixed as the benchmark economy, and the counterfactual case. Table 5.5 summarizes this comparison. As the counterfactual case features a higher relationship separation rate, the median relationship length is lower than in the benchmark case. If the contract policies remain as in the benchmark case, the aggregate lending growth due to a decrease in the bank funding cost at 45.15%, higher than the 39.13% aggregate lending growth in the benchmark case. This result is consistent with the model prediction that balances from longer relationships respond less to credit supply shocks. However, the shift in the relationship distribution also endogenously changes the optimal contract. In the counterfactual case, the higher separation rate of relationships makes relationships less attractive and thus decreases firms motives to stay in relationships. As a result, when there is a positive credit supply shock, the optimal contracts in the counterfactual economy features less loan balance growth compared to the benchmark contract for any relationship that has not reached the first-best loan balance amount. So, looking at the overall effect of the relationship length distribution change, I find that the counterfactual economy actually experiences only a 35.32% growth in aggregate lending in response to the same bank funding cost decrease, which is lower than in the benchmark case. Therefore, the endogenous change in the optimal contract due to the shift in the distribution of relationship lengths also plays an important role in affecting the aggregate effects of credit supply shocks.

This counterfactual exercise implies that the optimal contracting model is vital to fully evaluating the distributional effect of the relationship length on the transmission of credit supply shocks to aggregate lending.

Chapter 6

Conclusion

In this paper, I argue that the distribution of relationship lengths in the economy is important for the aggregate impact of credit supply shocks. In the first part of this study, I use novel micro data to show two important facts: (1) the loan balance between a lender-firm pair increases in the length of the lending relationship; (2) loan balances of longer relationships are less sensitive to credit supply shocks, in particular to positive credit supply shocks. In the second part of the paper, I use a parsimonious, micro-founded model of optimal loan contracts between banks and firms in a competitive search environment to interpret these two facts and discipline the model with the data. Finally, I consider a counterfactual economy to match with the higher firm exit rate in 1980, and find that in response to the same bank funding cost decrease, the counterfactual economy experiences a smaller aggregate lending growth compared to the benchmark economy, albeit featuring more short relationships in the relationship length distribution. This result suggests that the distribution of relationship lengths matters for the aggregate lending's responsiveness to credit supply shocks, not only by changing the composition of relationship lengths in the economy but also by endogenously influencing the optimal contracts in the equilibrium.

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