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ABSTRACT

Gains from trade and the sovereign bond market

The empirical literature shows that when sovereigns default trade volumes collapse, suggesting that trade openness of a country discourages defaults. I construct a simple two country model and show that this is not necessarily the case. Upon default, two opposing forces operate: an improvement in the terms of trade, which increases the sovereign's share of the gains from trade, and a reduction in the size of the total gains from trade. I show that under reasonable parameter values, the latter effect dominates: default costs are higher when the country is more open to trade. This leads to a lower interest rate in the sovereign bond market. By opening up to trade, the country enjoys not only the traditional gains from trade but also additional welfare gains, because the sovereign is now able to borrow more. Simple calibration implies that the additional gains are non-negligible in magnitude. This shows that a country may well achieve welfare gains in markets other than the goods trading market upon trade liberalization.

Imperfect Competition and the Transmission of Shocks: The Network Matters (joint with Emmanuel Dhyne and Glenn Magerman)

This paper studies the aggregate implications of the firm-to-firm production network structure. Using a dataset on all domestic transactions between Belgian firms, we establish two facts: firms charge higher markups if they have higher input shares within their customers, and firms experience larger churn of suppliers if they face a larger reduction in foreign goods' prices. Motivated by these two facts, we build a model where firms compete as oligopolies to supply inputs to each customer and where firms optimally choose their suppliers. The network structure becomes irrelevant in a benchmark case where we impose perfect competition and hold the network fixed. In this case, firm-level variables are sufficient to compute the welfare response to a large fall in import prices. Allowing for oligopolistic competition generates two counteracting forces within supplier-customer pairs. A supplier raises its markup

to a customer when its costs decline, but it reduces the markup if other firms supplying the same customer receive the shock. Further, allowing for endogenous networks amplifies the impact of the shock as firms begin importing and begin sourcing from other firms exposed to the import shock. Due to the omission of these dynamics, the aggregate response in the benchmark case is less than one quarter of those in the full estimated model.

Trade and Domestic Production Networks (joint with Felix Tintelnot, Magne Mogstad and Emmanuel Dhyne)

We use administrative data from Belgium with information on domestic firm-to-firm sales and foreign trade transactions to study how international trade affects firm efficiency and real wages. The data allow us to construct the buyer-supplier network of the Belgian economy. We document that most firms that do not directly import or export still have large indirect exposure to foreign trade, and that a firm's output is affected by idiosyncratic shocks to its buyers and suppliers. These empirical findings motivate and guide the development of a model with domestic production networks and international trade. We obtain new sufficient statistics results for the effects of trade in a model with fixed network structure, and we develop a tractable model of endogenous domestic production networks. Comparing our results to those we obtain using existing approaches highlights the importance of data on and modeling of domestic production networks in studies of international trade.

CHAPTER 1

GAINS FROM TRADE AND THE SOVEREIGN BOND MARKET

1.1 Introduction

What happens when a sovereign defaults on its bonds? The literature has found evidence that sovereign defaults result in trade disruption. Gopinath and Neiman (2014) document that imports of Argentina collapsed in the wake of its default from 2000 to 2002. Studying samples of defaults of over 150 countries, Rose (2005) finds that trade declines persistently when a sovereign defaults.

If the country experiences trade disruption upon default, then one may reasonably think that other things the same, a sovereign which economy is more open to trade is less likely to default. And if this is indeed the case, the more open sovereign may face a higher price in its bond market, since the bonds are safer for the investors. Therefore, by opening up to trade, the country may enjoy additional gains that come from its bond market in addition to the usual gains from trade. In this paper, I investigate this possible effect of trade on the sovereign bond market and answer the following questions: How does the sovereign's trade openness affect its cost of default? And how much is the additional welfare gain, if any?

To shed light on these questions, I construct a simple model. There are two endowment economies, where the sovereign government of the home country issues bonds in an incomplete market. The home country may also engage in goods trade with another country. One key assumption I make is that some fraction of home's endowment is destroyed when the sovereign decides to default.[1] With this assumption I compare the effect of the negative endowment shock on home's consumption in the two states of the world: the home country in autarky and the home country in free trade.

1. This common assumption can be interpreted in the following way. Government defaults disrupt the domestic financial sector, by reducing banks' wealth and by reducing banks' access to liquid assets (Perez (2015)). These mechanisms can trigger setbacks in domestic production.

When a country trades with another country, it's consumption is no longer affected linearly by its own endowment shock. Upon default, when a fraction of the endowment is destroyed, there are two opposing forces that come into play. First, there is a positive terms of trade effect. This effect mitigates the negative endowment shock, and decreases the cost of defaulting.[2] On the other hand, the same negative shock reduces the overall gains from trade that the two countries jointly enjoy. In other words, upon destruction of the endowment, the sovereign will obtain an increasing share of a pie that is decreasing in size.[3] If the latter effect is stronger meaning that consumption decreases more than one-to-one with the endowment shock, then the cost of default is larger in free trade than in autarky. If the terms of trade effect dominates and consumption decreases less than one-to-one, then the sovereign's cost of default is smaller than in autarky. Therefore, trade liberalization can either increase or decrease the cost of sovereign default.

Another key assumption I make is the bond market is incomplete. The one period sovereign bond is not state contingent, and in addition is subject to limited commitment: the government has an option to default and not pay back its debt at each period. Combining these with risk neutral foreign investors, the price of the sovereign bond reflects the probability of default. Therefore, a larger cost of default leads to a smaller probability of default, and this in turn results in a higher bond price. Facing a higher bond price, the risk averse sovereign will have a larger ability to smooth its consumption, and receives welfare gains in addition to the usual gains from trade.[4]

I demonstrate that under reasonable parameter values, the cost of default becomes larger when the country is open to trade. When the two goods in the two countries are not that

2. This straight forward positive terms of trade effects holds true because the model is a single sector Armington model.

3. As mentioned above, the assumption of proportional endowment loss is key in generating these two forces. In the sovereign default literature, this common assumption is needed to quantitatively make default more costly to match low default frequency. But here, in order to make the comparison between free trade and autarky, it is crucial that the endowment loss does not vary with the level of trade.

4. In this Armington model, the 'usual' gains from trade come from the love of variety.

easily substituted and when the home biases of the two countries are not too low, the positive terms of trade effect is dominated by the loss of gains from trade that was attached to the destroyed endowment. This higher cost of default leads to a higher bond price, which enables the sovereign to borrow more. A simple calibration in the case of Argentina increasing its trade with China in the early 2000s yields that the additional gains that the sovereign enjoys from a higher bond price is non-negligible - around one fifth of the usual gains from trade.

These results affect how we should think about gains from trade. The existing trade models - most notably summarized by Arkolakis, Costinot, and Rodríguez-Clare (2012) - produce surprisingly small gains from trade. Here, I show that the gains that a country enjoys from opening up to trade are not constrained to come from the goods market in which it trades. I provide a simple example that a country would enjoy non-negligible gains from the sovereign bond market, suggesting that welfare gains are achieved in dimensions that are beyond goods trading market.

The empirical evidence is broadly consistent with this theory. Rose and Spiegel (2004) investigated the link between bond prices and trade volumes, to determine whether countries repay their debt out of the fear that default might lead to trade disruption. They find a positive correlation between trade volumes and the claims of sovereign bonds by the creditor country, when they looked at bilateral trade and international banking claims from 20 creditor and 149 debtor countries in 1986-1999 period. Moreover, Manasse, Roubini, and Schimmelpfennig (2003) studied a panel of 47 countries for the 1970-2002 period and find low trade openness as one of the main predictors for sovereign debt crises. These evidence seem to suggest that sovereigns have more to lose upon default, when they become more open to trade.

In terms of the literature, this paper is positioned between two large bodies of work: a literature on the gains from trade; and one on sovereign default. On the trade side, the simple model I use is a two country Armington (1969) model, whose gains from trade are nested by the formula Arkolakis, Costinot, and Rodríguez-Clare (2012) offer. On the sovereign default side, I borrow a simplified version of the models that feature endogenous sovereign default. Developed by Eaton and Gersovitz (1981) and also subsequently by Aguiar and Gopinath (2006) and Arellano (2008), they construct sovereign defaults as optimal decisions of the sovereign. I combine the simplest versions of the models in these two literatures to generate an additional source of gains from trade.

Many others have looked at the intersection of trade and sovereign default models. Mendoza and Yue (2012) build a model of sovereign default of a sovereign that is trading with a foreign country. They assume that trade is disrupted upon default, and explain the dynamics of interest rates and debt levels. In this paper I do not assume exogenous trade disruption costs, but only the loss in endowments that the sovereign trades with. Asonuma (2014) also builds a model of sovereign default with trade but investigates the dynamics of real exchange rates upon defaults. Instead, in this paper I focus on the gains from trade that are generated in the sovereign bond market.

The remainder of this paper is organized as follows. I start out in Section 1.2 by presenting a simple two period model of a closed economy where the sovereigns issue bonds. In Section 1.3, I introduce trade to the model. Here I describe the two opposing effects that determine the sovereign's default cost. Then in Section 1.4, I build a fully dynamic model and present in Section 1.5 numerical results. Section 1.6 then concludes.

1.2 Baseline model

In this section I construct a two period model with a single endowment economy issuing one period defaultable bonds. The setup is almost identical to that of Arellano (2008) except that the model here is in two periods. Readers who are familiar with the model may skip to Section 1.3.

1.2.1 The sovereign

Consider a closed endowment economy with two periods, where benevolent sovereign maximizes the expected utility of the representative household. The representative household's expected utility in the first period is

$$\mathbb{E}_1\left[U\right] = \frac{C_1^{1-\epsilon}}{1-\epsilon} + \beta\mathbb{E}_1\left[\frac{C_2^{1-\epsilon}}{1-\epsilon}\right] \tag{1.1}$$

β is the discount factor, and ϵ governs intertemporal substitution. In the first period, the household has an endowment of Y_1 and the sovereign agrees with foreign investors on a contract (L, D). A contract (L, D) lets the sovereign borrow L units of consumption goods from risk neutral foreign investors in period one, by promising that it will repay D units of consumption goods in the second period. The implied price of the bond will be $q = \frac{L}{D}$ and gross interest rate $1/q$. In the second period, endowment Y_2 is realized: $Y_2 = \alpha + \varepsilon$, where $\varepsilon \in [-\alpha, \infty]$ is an i.i.d. random variable with CDF $F(\cdot)$.

Consumption in the first period is $C_1 = Y_1 + L$. In the second period, the sovereign decides either to repay or to default after observing the endowment shock. If it decides to repay, the sovereign will repay D as promised. If it decides to default, the sovereign does not have to pay back D, but incurs endowment loss of fraction θ. Thus the consumption at the second period becomes $C_2 = \max\{Y_2 - D,\ (1-\theta)Y_2\}$. The sovereign will default if the loss of default is smaller than the loss when repaying, $\theta Y_2 < D$. Thus given D at period 1, the probability of default becomes $F(\varepsilon^*)$ where

$$\varepsilon^* = \frac{D}{\theta} - \alpha. \tag{1.2}$$

I make a crucial assumption that the sovereign suffers a direct output loss upon default, and this holds throughout the paper. The rationale behind this assumption is that when the government defaults, it becomes harder for the domestic firms to finance their production.

This may happen through different channels: domestic financial markets may be disrupted upon default, or firms may lose access to international credit markets. The sovereign makes repayment decision by weighing this endowment loss against the cost of paying back the debt.

1.2.2 Foreign creditors

There are many atomistic risk neutral foreign investors making transactions with the sovereign. They face a world risk free interest rate of $1 + r$, and make zero profits. I focus on the representative investor, who takes price q as given, and maximize expected consumption by choosing how much to demand D. Thus their problem is

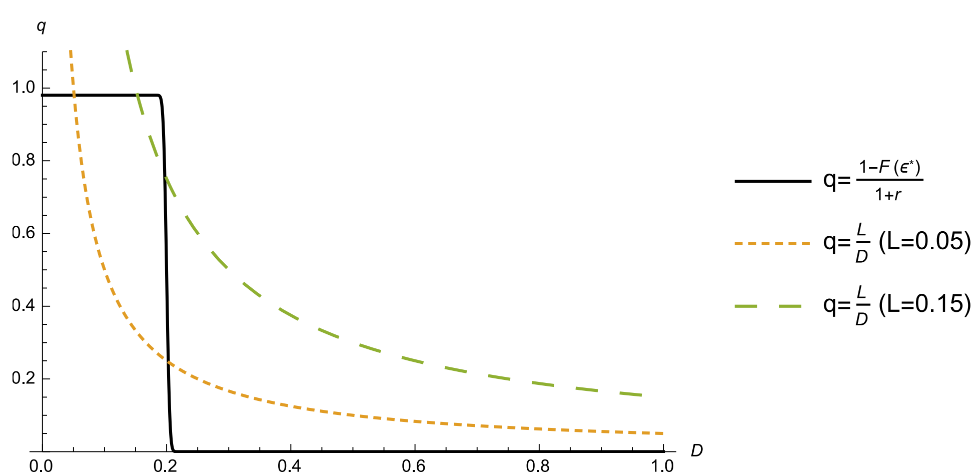
$$\max_D -L + \frac{1}{1+r} D(1 - F(\varepsilon^*)). \tag{1.3}$$

From the zero profit condition, the implied price for the sovereign bond becomes:

$$q = \frac{L}{D} = \frac{1 - F(\varepsilon^*)}{1+r}. \tag{1.4}$$

1.2.3 Equilibrium

Figure 1.1: Multiple solutions



Given $F(\cdot)$, equations (1.2) and (1.4) determine the demanded repayment amount D, price $q = \frac{L}{D}$, and the probability of default $F(\varepsilon^*)$ for each L that the government demands in the first period. Figure 1.1 plots the pricing schedule that is derived from equations (1.2) and (1.4).[5] The black line displays the price schedule $q = \frac{1-F(\varepsilon^*)}{1+r}$. When outstanding debt is small, the probability of default is close to zero so that the price will be almost equal to $q = \frac{1}{1+r}$. On the other hand, as outstanding debt becomes larger, the probability of default increases to 1, thus lowering the price eventually to 0. The price schedule is almost kinked near $D = 0.2$. This is where ε^* equals 0, which is the mean of the normal distribution. The slope around the kink is steep both because the value of θ and σ_ε^2 are set to be low. A small deviation from $D = 0.2$ will have propagating effect on ε^* due to equation (1.2), and the value of q would easily go to the two extremes.

In addition to the price schedule, there are two dashed lines. These are the price $q = L/D$ for different values of L, the amount that the sovereign borrowed in the first period. Given L, there are two solutions for bond price q and default probability $F(\varepsilon^*)$: one is the "good solution", where price is high and low probability of defaulting, and another, the "bad solution", is where price is low and high probability of defaulting. The first solution is where the creditors demand less repayment D as they believe that the sovereign is less likely to default in the next period. The second solution is where creditors demand large amount of repayment as compensation for high probability that they would not get repaid in the next period.

In this simple two period model, one cannot determine which solution is selected in the market. The foreign investors offer schedules of demanded repayment $\{D_{\text{good}}(L),\ D_{\text{bad}}(L)\}$. Additional assumption is needed to determine which will take place in the market. However, in the case of the model with infinite horizon which I will use later in this paper, Auclert and Rognlie (2014) prove that only the "good" solution is selected. For given L, the "bad

5. The specifications used for producing the figure are $\{(Y = 10, \alpha = 10, \theta = 0.02, r = 0.02, \sigma_\varepsilon = 0.2)\}$, where $F(\cdot)$ is a truncated normal distribution with support $[-\alpha, \infty]$ and variance σ_ε^2.

solution", the one with larger debt fails to be self-sustaining. Thus in infinite horizon, the sovereign takes only the schedule $D_{\text{good}}(L)$ as given, and chooses the L that maximizes its expected present value utility. Risk aversion of the sovereign makes the optimal L to be such that the sovereign smooths its consumption over the two periods.

1.3 Economies with trade

From this section I expand the model to investigate how having trade with another country affects sovereign bond prices. I first construct a simple Armington (1969) model where there are trade costs in both countries. Then I compare and analyze two states of the world, one with autarky where trade costs are infinite, and another with free trade where trade costs are zero.

The model is in two periods, where two countries trade goods in both periods. Each of the two countries is endowed with a single tradable intermediate good. Y_t and Y_t^* denote the endowments of Home and Foreign countries at time $t \in \{1, 2\}$. There are foreign investors who reside in a third country or the rest of the world. They lend to the Home country, which I call the sovereign.

The timings of the model are as follows. In the first period, endowments of both countries $\{Y_1, Y_1^*\}$ are realized. Trade takes place and each countries aggregate the intermediate goods from both countries into final consumption goods $\{c(Y_1, Y_1^*), c^*(Y_1, Y_1^*)\}$. Then the sovereign agrees on a contract of (L, D) with the foreign investors. The bonds are denominated in terms of final goods of the sovereign. Therefore final consumption of the two countries at the end of the first period are

$$\begin{cases} C_1 & = c(Y_1, Y_1^*) + L \\ C_1^* & = c^*(Y_1, Y_1^*). \end{cases} \tag{1.5}$$

I refer to the lower case c's as "pre-transfer" consumption as they are the amount of final goods that the countries have as a result of intermediate goods trade. Upper case C's denote consumption after the sovereign interacts with the investors.

$\{Y_2, Y_2^*\}$ are realized in the second period. Then the sovereign decides to default or not. If the sovereign repays, its final consumption will be the pre-transfer consumption minus the promised amount D. If the sovereign defaults, fraction θ of the country's endowment of intermediate goods are destroyed in exchange of not having to repay its debt. Therefore, if the sovereign repays, their final consumption are

$$\begin{cases} C_2 &= c(Y_2, Y_2^*) - D \\ C_2^* &= c^*(Y_2, Y_2^*) \end{cases} \tag{1.6}$$

and if the sovereign defaults, they are

$$\begin{cases} C_2 &= c((1-\theta)Y_2, Y_2^*) \\ C_2^* &= c^*((1-\theta)Y_2, Y_2^*). \end{cases} \tag{1.7}$$

1.3.1 Pre-transfer consumption

In this subsection I characterize the pre-transfer consumption of both countries under two regimes: autarky and free trade.

The preference of the two countries are specified as

$$\begin{cases} c_t &= \frac{1}{\Phi}(\lambda(x_{HHt})^{\frac{\sigma-1}{\sigma}} + (1-\lambda)(x_{FHt})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \\ c_t^* &= \frac{1}{\Psi}(\lambda^*(x_{FFt})^{\frac{\sigma-1}{\sigma}} + (1-\lambda^*)(x_{HFt})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \end{cases} \tag{1.8}$$

where $\Phi = \lambda^{\frac{\sigma}{\sigma-1}}$, $\Psi = (\lambda^*)^{\frac{\sigma}{\sigma-1}}$. x_{ijt}'s are the amount of intermediate goods shipped from i to j at time t. $\sigma \in (1, \infty]$ is the elasticity of substitution between the two goods, which gives trade elasticity of $1 - \sigma < 0$. λ, $\lambda^* \in (0, 1)$ represent the home bias for each country.

Goods market clearing conditions are

$$\begin{cases} Y_t & = x_{HHt} + \tau_t^* x_{HFt} \\ Y_t^* & = \tau_t x_{FHt} + x_{FFt} \end{cases} \tag{1.9}$$

where iceberg trade costs for goods that enter Home and Foreign are denoted as τ_t and τ_t^*.

Denoting ex-factory prices as p_t and p_t^*, and taking as given the endowments $\{Y_t, Y_t^*\}$ and trade costs $\{\tau_t, \tau_t^*\}$, I can write down 6 equations for 6 unknowns $\{x_{ijt}, p_t, p_t^*\}$:

$$\begin{cases} \frac{x_{FHt}}{x_{HHt}} & = (\frac{\lambda}{1-\lambda})^{-\sigma}(\frac{p_t}{\tau_t p_t^*})^{\sigma} \\ \frac{x_{HFt}}{x_{FFt}} & = (\frac{\lambda^*}{1-\lambda^*})^{-\sigma}(\frac{p_t^*}{\tau_t^* p_t})^{\sigma} \\ Y_t & = x_{HHt} + \tau_t^* x_{HFt} \\ Y_t^* & = \tau_t x_{FHt} + x_{FFt} \\ \tau_t^* p_t x_{HFt} & = \tau_t p_t^* x_{FHt} \\ p_t^* & = 1 \end{cases} \tag{1.10}$$

where I normalize the ex-factory price of the Foreign country.

The solution gives the amount of consumption in each country.

$$\begin{cases} c_t(Y_t, Y_t^*) & = Y_t(1 + \phi^{-1}\tau_t^{1-\sigma} p_t^{\sigma-1})^{\frac{1}{\sigma-1}} \\ c_t^*(Y_t, Y_t^*) & = Y_t^*(1 + \psi^{-1}(\tau_t^*)^{1-\sigma} p_t^{1-\sigma})^{\frac{1}{\sigma-1}} \end{cases} \tag{1.11}$$

where $\phi = (\frac{\lambda}{1-\lambda})^{\sigma}$ and $\psi = (\frac{\lambda^*}{1-\lambda^*})^{\sigma}$ and terms of trade p_t is determined by

$$\frac{Y_t}{Y_t^*} = \frac{1}{p_t} \frac{1 + \phi \tau_t^{\sigma-1} p_t^{1-\sigma}}{1 + \psi (\tau_t^*)^{\sigma-1} p_t^{\sigma-1}}. \tag{1.12}$$

Given this general solution with trade costs, I compare the pre-transfer consumption in two states of the world: autarky and free trade. In autarky, the trade costs are set as infinity, thus the pre-transfer consumption will simply be the endowment of the country.

$$\begin{cases} c_t^{\text{AUT}} & = Y_t \\ (c_t^*)^{\text{AUT}} & = Y_t^* \end{cases} \tag{1.13}$$

In free trade, trade costs are 0, meaning ($\tau = \tau^* = 1$).

$$\begin{cases} c_t^{\text{FT}}(Y_t, Y_t^*) & = Y_t(1 + \phi^{-1} p_t^{\sigma-1})^{\frac{1}{\sigma-1}} \\ (c_t^*)^{\text{FT}}(Y_t, Y_t^*) & = Y_t^*(1 + \psi^{-1} p_t^{1-\sigma})^{\frac{1}{\sigma-1}} \end{cases} \tag{1.14}$$

where p_t is determined by

$$\frac{Y_t}{Y_t^*} = \frac{1}{p_t} \frac{1 + \phi p_t^{1-\sigma}}{1 + \psi p_t^{\sigma-1}}. \tag{1.15}$$

Equation (1.14) says that consumption is the product of the endowment and the term that captures gains from trade. Also note that the terms of trade is determined solely by the available endowments of the two countries. The sovereign bond contracts (L, D) are denominated in terms of final goods, but they are considered non-tradables and do not affect the terms of trade p_t.

1.3.2 Default decisions

Consider the second period where the sovereign has outstanding debt D. As in the previous section, I assume that fraction θ of the country's endowment is destroyed upon default.

First in autarky, the sovereign defaults if the loss of consumption upon default, or the difference between the pre-transfer consumption with full endowment and one with partially destroyed endowment is smaller than the outstanding debt D.

$$\text{Default in autarky if} \qquad Y - D < (1-\theta)Y$$
$$\Leftrightarrow \theta Y < D \tag{1.16}$$

In free trade, the sovereign makes a similar default decision. The sovereign defaults when the loss of consumption upon default is smaller than outstanding debt D. But in free trade, both countries' pre-transfer consumption can be denoted as functions of the two countries' endowments.

$$\text{Default in free trade if} \qquad c(Y, Y^*) - D < c((1-\theta)Y, Y^*)$$
$$\Leftrightarrow c(Y, Y^*) - c((1-\theta)Y, Y^*) < D \tag{1.17}$$

The left hand sides of equations (1.16) and (1.17) express the cost of default. If θY is larger than $c(Y, Y^*) - c((1-\theta)Y, Y^*)$, then the cost of default is larger in autarky than in free trade, and vice versa.

Figure 1.2: Pre-transfer consumption schedules

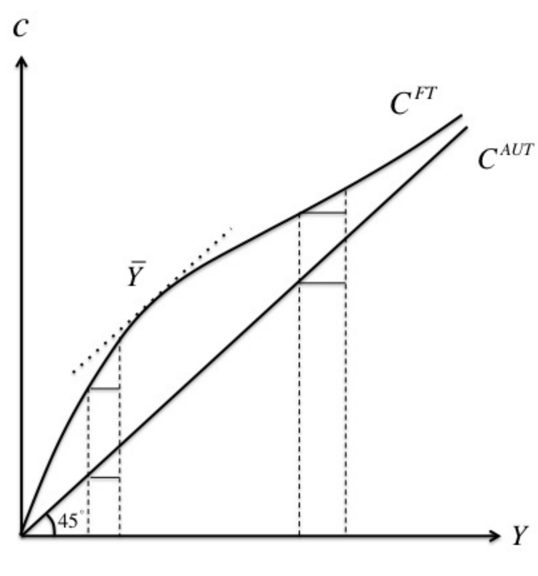


Figure 1.2 describes the pre-transfer consumption schedules in autarky and in free trade,

where Y^* is fixed. It shows that depending on the relative size of the endowments in the two countries, cost of default in autarky can either be larger or smaller than in free trade. When the country's endowment is smaller than $\bar{Y}$ where $\frac{\partial C^{FT}}{\partial Y}|_{Y=\bar{Y}} = 1$, the sovereign loses more of its consumption in free trade than in autarky, with the same reduction of endowment. On the other hand, when the country's endowment is larger than $\bar{Y}$, it is less costly to default under free trade than under autarky.

To understand the mechanism, it is essential to capture the two forces that come into play when the sovereign opens up to trade. In autarky, the destruction of endowment that occurs upon default linearly affects consumption, as can be seen from equation (1.13). But in free trade, the same loss of endowment will not have as simple effect on consumption.

$$\frac{\partial}{\partial Y} c^{\mathrm{FT}}(Y) = Y g'(p) \frac{\partial p}{\partial Y} + g(p) \tag{1.18}$$
$$\text{where} \quad g(p) = (1 + \phi^{-1} p^{\sigma-1})^{\frac{1}{\sigma-1}}.$$

Equation (1.18) shows that the slope of pre-transfer consumption is decomposed in two terms. $g(p) > 1$ captures the gains from trade, and p is determined by equation (1.15). One force that operates is the terms of trade effect, which is captured by the first term. Since $g'(p) > 0$ and $\frac{\partial p}{\partial Y} < 0$, endowment loss upon default would lead to a more favorable terms of trade for the sovereign. This positive terms of trade effect would push consumption up, and this is true for all range of Y. The second force is the loss of gains from trade, which is captured by the second term. The sovereign will not only lose consumption linearly, but also the positive gains from trade that was attached to the destroyed endowment. This negative effect on consumption is larger when initial endowment Y is relatively small, that is, when the gains from trade is large. The terms of trade effect is an effect that allocates larger fraction of the total gains from trade that the two countries enjoy. But the second effect reduces the size of total gains from trade. Therefore under free trade, destruction of endowment will give the sovereign an increasing share of a pie that is decreasing in size.

When home's relative endowment size is small, the loss of gains from trade dominates the terms of trade effect, and the sovereign loses more compared to the same destruction of endowment under autarky. In this region of endowment, the sovereign suffers a larger cost of default under free trade. On the other hand, if the initial amount of endowment is large enough, the gains from trade that disappears with the marginal reduction of endowment is negligible. Then terms of trade effect dominates. In this region of endowment the sovereign has smaller cost of default in free trade.

The appendix discusses how $\bar{Y}$ moves with different parameters of $(\sigma, \lambda, \lambda^*)$. I find that as substitutability increases, $\bar{Y}^1$ decreases. This is because substitutability across goods will diminish the gains from trade, thus the terms of trade effects tend to dominate more upon negative endowment shock. Under reasonable values of the above parameters where the elasticity of substitution is not too high and where the two home biases are not too low, $\bar{Y}$ becomes relatively high. That is, under realistic parameters, the cost of default is higher in free trade than in aurarky in most countries.

1.4 Infinite horizon model

Here I extend the model constructed in Section 1.3 into an infinite horizon model. Then I define a recursive equilibrium of the economy.

1.4.1 Sovereign government's problem

The key ingredients in the two period model presented before remain in this section. The two countries receive stream of intermediate goods endowments $\{Y_t, Y_t^*\}$ that they trade with each other. Also, the sovereign has access to contracts (L_t, D_{t+1}) with the foreign investors who reside in a third country. As before, the contracts are one period state non-contingent bonds that give L_t unit of consumption goods to home's representative household at period

t, with the promise of paying back D_{t+1} units in period $t+1$. The price of the contract is thus $q_{t+1} = \frac{L_t}{D_{t+1}}$.

Home's expected utility in the initial period is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\epsilon}}{1-\epsilon}. \tag{1.19}$$

The pre-transfer consumption of the two countries in each period are the same as in the previous section. In each period, given the endowments $\{Y_t, Y_t^*\}$, the two countries obtain pre-transfer consumption schedules denoted in equations (1.14) and (1.15).

In each period, the state of the economy is the outstanding government debt and the two countries' endowment $\{D_t, Y_t, Y_t^*\}$. Given the states, the sovereign has the option to default or to repay its debt in each period. Thus the sovereign's value at period t is

$$V(D_t, Y_t, Y_t^*) = \max\{V^R(D_t, Y_t, Y_t^*), V^D(Y_t, Y_t^*)\}. \tag{1.20}$$

$V^R(D_t, Y_t, Y_t^*)$ is the value of the sovereign if it repays the debt.

$$V^R(D_t, Y_t, Y_t^*) = \max_{D_{t+1}} [\frac{C_t^{1-\epsilon}}{1-\epsilon} + \beta \mathbb{E}_t[V(D_{t+1}, Y_{t+1}, Y_{t+1}^*)]] \tag{1.21}$$

$$\text{s.t.} \quad C_t = c_t(Y_t, Y_t^*) + q_{t+1} D_{t+1} - D_t \tag{1.22}$$

$V^D(Y_t, Y_t^*)$ is the value of the sovereign if it decides to default. In that case, θ fraction of Y_t is destroyed and the sovereign is excluded from the foreign bond market. From the next period, the sovereign has probability π to return to the bond market with zero outstanding debt.

$$V^D(Y_t, Y_t^*) = \frac{C_t^{1-\epsilon}}{1-\epsilon} + \beta\{\pi \mathbb{E}_t[V(0, Y_{t+1}, Y_{t+1}^*)] + (1-\pi)\mathbb{E}_t[V^D(Y_{t+1}, Y_{t+1}^*)]\} \tag{1.23}$$

$$\text{s.t.} \quad C_t = c_t((1-\theta) Y_t, Y_t^*) \tag{1.24}$$

Note that in this infinite horizon model, the sovereign suffers not only the direct output cost upon default, but also the cost of being excluded from the international capital market. Thus in each period, the sovereign weighs the present value cost of paying back the debt at that period against the present value cost of suffering an output loss and not being able to smooth consumption for a probabilistic length of time.

1.4.2 Foreign investors' problem

The foreign investors reside outside the two economies, and they are small and large in numbers. Taking as given the bond price q_{t+1}, the investors maximize profits, which is zero in equilibrium.

$$\max_{D_{t+1}} -q_{t+1}D_{t+1} + \frac{1}{1+r}D_{t+1}\mathbb{1}\{\text{Repay}\}.$$

Therefore the implied price for the sovereign bond is

$$q_{t+1} = \frac{L_t}{D_{t+1}} = \frac{\text{Prob(Repay)}}{1+r}. \tag{1.25}$$

1.4.3 Recursive equilibrium

I assume the endowment process of the two countries to follow a log-normal AR(1) process. The shocks to the two streams are uncorrelated.

$$\begin{cases} \log(Y_t) & = \log(\bar{Y}_t) + \rho(\log(Y_{t-1}) - \log(\bar{Y}_t)) + \varepsilon_t \\ \log(Y_t^*) & = \log(\bar{Y}_t^*) + \rho^*(\log(Y_{t-1}^*) - \log(\bar{Y}_t^*)) + \varepsilon_t^* \end{cases} \tag{1.26}$$

where

$$\begin{pmatrix} \varepsilon_t \\ \varepsilon_t^* \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma\right), \quad \Sigma = \begin{pmatrix} \sigma^2 & 0 \\ 0 & (\sigma^*)^2 \end{pmatrix} \tag{1.27}$$

and $(\bar{Y}_t, \bar{Y}_t^*)$ are mean income of the two economies.

Summing all up, the exogenous parameters of this model are $\{\beta, r, \epsilon, \lambda, \lambda^*, \sigma, \theta, \pi\, \rho, \rho^*, \sigma, \sigma^*\}$ and the state of the economy is $\{D_t, Y_t, Y_t^*\}$.

The recursive equilibrium of this economy is a set of policy functions for

1. Home and foreign's pre-transfer consumption.

2. Sovereign's default decision and bond issuance.

3. Bond price functions.

such that

1. Taking as given the price of bonds and goods' relative prices, both the sovereign and foreign pre-transfer consumption satisfy their optimization problem.

2. Goods market clear.

3. Taking as given the bond price, the sovereign's default decision satisfies its dynamic problem.

4. Bond price reflect sovereign's default probability and consistent with the investors maximization problem.

1.5 Numerical analysis

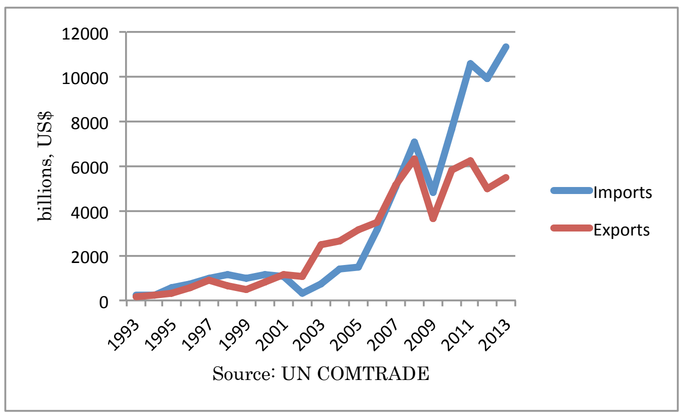
In this section I depart from the theoretical analysis and partially calibrate the model into data to answer how much gains does a sovereign bond issuing country enjoy by having trade.

1.5.1 Experiment

Figure 1.3 shows the commodity trade flows between Argentina and China from 1993 to 2013. We see from this figure that the trade between two countries shot up from around

2002. In this experiment, I compare the two states of the world as in the previous sections. The first is analogous to the autarky economy in previous theoretical analyses. I estimate the endowment shock process from the real GDP sequence of Argentina from 1993 to 2013, and compute the recursive equilibrium defined in the previous section, but with infinite trade costs. The second is analogous to the free trade economy with two countries. I additionally estimate the Chinese endowment process using the Chinese real GDP data from 2002 to 2013, and consider this as the trading partner in the model. Then I compute the recursive equilibrium where the two countries have trade with zero trade costs.

Figure 1.3: Argentina's trade flows with China



In the numerical analysis, I modify the endowment cost that the sovereign has to take upon default. Instead of assuming that uniform fraction of endowment is destroyed upon default, I assume the endowment upon default follows the rule below.

$$Y_t^{\text{default}} = \begin{cases} \gamma \mathbb{E}(Y_t) & \text{if } Y_t > \gamma \mathbb{E}(Y_t) \\ Y_t & \text{if else} \end{cases} \tag{1.28}$$

Equation (1.28) says that the fraction of destroyed endowment is weakly increasing in the endowment size. This construction is the same as done in Arellano (2008) and other papers in the literature. When output realization is low, the sovereign does not suffer any output destruction and is only deprived of its ability to smooth consumption. But when the endowment is larger than a certain threshold, the sovereign starts to suffer increasing fraction of direct output loss upon default. This assumption that default cost is larger in "good times" than in "bad times" is needed in order to produce the observed feature that the sovereign is more likely to default when faced with low endowment shock. This assumption is also in line with models that have endogenous production. Mendoza and Yue (2012) assumes that domestic firms lose access to world credit markets upon sovereign default, thus cannot finance working capital to import certain fraction of intermediate goods. With this setup, they find that the GDP loss upon default is increasing in TFP shock.

1.5.2 Data and parameters

Argentinian quarterly GDP data are taken from National Institute of Statistics and Censuses (INDEC).[6].Chinese quarterly GDP data are taken from National Bureau of Statistics of China (NBS).[7] I estimate the parameters that govern the endowment processes in equation (1.26) using maximum log-likelihood method.[8] The parameters estimated are $\{\rho = 0.91,\ \rho^* = 0.81,\ \sigma = 0.015,\ \sigma^* = 0.016\}$, where Argentina is the Home country and China is the Foreign country. Given the estimated endowment processes, I then discretize the process into a finite state Markov chain using the procedure described in Hussey and Tauchen (1991).

Table 1.1 summarizes the parameter specifications. The discount rate $\beta = 0.86$ is taken from Asonuma (2014), which is set to match the Argentina's average default frequency documented in Sturzenegger and Zettelmeyer (2007). Risk free rate r is set to be 0.017, which is taken from Arellano (2008). The endowment threshold γ is also taken from Arellano (2008) and set to be 0.969. The probability of reentry π is set as 0.282 which is consistent with the findings in Gelos, Sahay, and Sandleris (2011).

6. The data is then converted to constant prices in US dollars in 2005, using inflation and exchange rates from US Bureau of Economic Analysis (BEA) and Central Bank of Argentina.

7. The data is then converted to constant prices in US dollars in 2005, using inflation and exchange rates from World Development Indicators of the World Bank, and the People's Bank of China. It is also seasonally adjusted by X-12-ARIMA Seasonal Adjustment Program by the US Census Bureau.

8. I detrend the data with Hodrick-Prescott filter using smoothing parameter of 1600.

Parameter	Value	Comment
Discount rate	$\beta = 0.86$	Asonuma (2014)
Risk free rate	$r = 0.017$	Arellano (2008)
Risk aversion	$\epsilon = 2$	RBC literature
Weights of x_{HH} and x_{FF}	$\lambda = 0.5,\ \lambda^* = 0.5$	Asonuma (2014)
Elasticity of substitution	$\sigma = 5$	Simonovska and Waugh (2014)
Endowment threshold	$\gamma = 0.969$	Arellano (2008)
Probability of reentry	$\pi = 0.282$	Gelos, Sahay, and Sandleris (2011)
AR(1) persistence	$\rho = 0.91,\ \rho^* = 0.81$	Estimated
AR(1) standard deviation	$\sigma = 0.016,\ \sigma^* = 0.015$	Estimated

Table 1.1: Specifications of parameters

1.5.3 Results

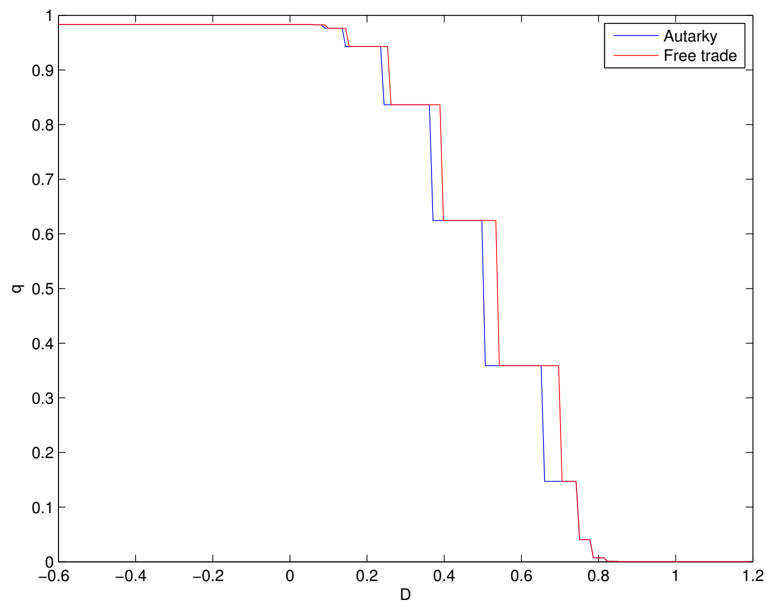
Note that the results do not exactly reveal the gains that Argentina enjoys by opening up commodity trade with China. Obviously, the results are limited given the stylized setup of the model. Nevertheless, they are informative in grasping the magnitude of how much a sovereign gains through the bond market by opening up to trade.

Figure 1.4 shows the bond price schedules in both autarky and in free trade. The price schedules show that the sovereign faces higher price for the same amount of debt in free trade, reflecting the lower probability of default.

Though the basic structure of the model is the same as in the two period model established in the last section, the magnitude of the result seems to be smaller in this infinite horizon model. This is because the assumption of what happens to the endowment upon default has changed. In the previous section, I assumed a uniform 2% reduction of endowment, but here the reduction only happens when the endowment is higher than 97% of its mean value. Thus for endowments that are in the lower half of the distribution, the default cost is zero. This diminishes the difference of default costs between free trade and autarky, resulting in smaller magnitude of bond price differences.

As briefly mentioned in the introduction, there are two sources that the sovereign gains from. One is the traditional gains from trade, that comes from the "love of variety" feature in the CES utility specification. Another is the ability to smooth out consumption by issuing

Figure 1.4: Price function in different regimes



bonds.

This ability to smooth out consumption is affected by two ingredients that create incompleteness of the bond market. The first is the fact that the sovereign bonds are state non-contingent. By issuing state non-contingent bonds, the sovereign tries to smooth consumption paths as in Mendoza (1991). But since the sovereign's discount rate β is smaller than the world discount rate $\frac{1}{1+r}$, the impatient sovereign will always borrow. Moreover, the endogenous bond price will make the sovereign easier to borrow in times when endowment is large, since the probability of defaulting is lower in those states. This results in the sovereign borrowing more in good times than in bad times. The second is that the bonds are defaultable. By defaulting when endowments are low and large outstanding debt, the government mitigates the negative effect of low endowment shock to consumption. Since it is harder for the sovereign to borrow when the endowment is low, the sovereign defaults with smaller outstanding debt in low endowment periods. Therefore this defaulting channel works as smoothing out consumption, similar to the notion explained in Zame (1993).

Given the above equilibrium features, I move on to the simulation results. To examine the effect of moving from autraky to free trade on bond price and on welfare, I simulate the model for 1000 times, given the obtained policy functions. Each trial of simulation is run for 2000 periods, and the last 200 periods are taken to analyze the steady state distribution. In analyzing the bond prices, I take out the periods when the sovereign is in default, since the bond price are not well defined. Following the literature, I consider the 74 periods prior to default events, when analyzing pre-default statistics. Table 1.2 reports the simulation results along with other statistics. The first column, which reports statistics where trade is not allowed, is comparable to those from Arellano (2008). By having trade with another country, the sovereign's default probability decreases and that is reflected in increase of its sovereign bond price. It is interesting to note that the average debt to output ratio is smaller in free trade. The sovereign's capacity to accumulate debt expands when it moves to free trade from autarky, but this does not necessarily mean that the sovereign has higher debt to output ratio along the steady state.

	Autarky	Free trade
Avg. interest rate spread $\frac{1}{q} - 1$ (%)	3.25	2.66
Pre-default interest rate spread (%)	3.48	2.85
Avg. default probability (%)	3.10	2.55
Avg. debt to output (%)	4.81	3.96

Table 1.2: Simulation result (in quarterly frequency)

Facing more favorable bond price, the sovereign enjoys not only the traditional gains from trade, but also an additional intertemporal gains from trade. Table 1.3 reports the increase in present value welfare at steady state, with and without sovereign bond market. As for testing robustness, I alter the values of σ, the substitutability across goods from the two countries. Instead of setting $\sigma = 5$ as in the baseline case, I also see how the numbers change when $\sigma = 3$ and $\sigma = 7$. The increase in welfare without bond issuance is calculated using the corresponding endowment sequences that are used in the simulation. This welfare gains from trade is around 16% in the baseline case, and it is consistent with the present value of the static gains from trade. On the other hand, the gains from opening up to trade when the sovereign is issuing bonds become even larger, to 19%. The difference of those two gains, which is around 3%, comes from the fact that the sovereign has more capacity to borrow, and faces higher bond price. Therefore, the sovereign enjoys additional gains from trade from the sovereign bond market, which is around one fifth of the traditional gains from trade.

The numbers change when substitutability across goods is different. As expected, higher substitutability reduces the amount of gains from trade that the sovereign enjoys. But it also diminishes the relative size of the additional gains that the sovereign enjoys from the sovereign bond market. Recall figure 1.2 in Section 1.3.2. In the static version of the model, higher substitutability reduces the region of the home country's endowment where default cost is higher in free trade than in autarky. In other words, higher σ decreases $\bar{Y}$. This means that for Y that is less than $\bar{Y}$, the slope of the consumption schedule in free trade diminishes and becomes closer to that of the consumption schedule in autarky. The cost of default is still higher than in autarky, but diminishes as substitutability increases. The difference between free trade and autarky becomes smaller, which leads to less additional gains by opening up to trade. Nevertheless, I find that the additional gains that come from the sovereign bond market is non-negligible in magnitude.

	$\sigma = 5$	$\sigma = 3$	$\sigma = 7$
Without bond issuance (%)	16.3	29.3	10.9
With bond issuance (%)	19.4	41.5	12.3
Additional gains relative to traditional gains (%)	19	42	13

Table 1.3: Gains from trade

1.6 Conclusion

This paper develops a simple model where the sovereign trades with another country and also issues bonds in an incomplete market. With this model, I answer two questions: Does opening up to trade increase or decrease the sovereign's cost of default? And how much is the additional welfare gains, if any?

For the first question, I demonstrate that the answer depends on which of the two opposing forces dominates upon default: the improvement in the terms of trade and the loss of gains from trade. I show that under reasonable parameter values, the latter dominates, which increases the cost of default in free trade.

For the second question, I show that in the case of Argentina trading with China, Argentina enjoys an additional gains from trade from the sovereign bond market, which is around one fifth of the traditional gains from trade. This demonstrates that a country may enjoy non-negligible amount of additional gains that comes from markets that are outside the goods trading market.

As a last note, I should mention that the model that I establish in this paper is very crude and thus the results should not be taken to represent the actual gains that Argentina enjoyed. Nevertheless this paper is the first to focus on the extra gains from trade that a sovereign will enjoy in the sovereign bond market, and the model is useful to capture the magnitude of how large those gains are. The numbers I find is not a huge amount, but also not too small that it can be ignored.

CHAPTER 2

IMPERFECT COMPETITION AND THE TRANSMISSION OF SHOCKS: THE NETWORK MATTERS

This chapter is based on a joint work with Emmanuel Dhyne and Glenn Magerman.

2.1 Introduction

Does the structure of the firm-to-firm production network affect how the economy responds to shocks in the aggregate? Results from Hulten (1978) imply that the network structure is irrelevant up to a first order approximation in an efficient and closed economy. Under perfect competition and fixed network structures, firm-level variables such as firms' total sales, are sufficient in evaluating how aggregate variables respond to firm-level shocks. Due to this theoretical result and the scarcity of data on firm-to-firm transactions, the literature has widely assumed away the possibility that the network structure matters, or assumed network structures in which firm-level information is sufficient to work with.

In this paper, we analyze a detailed administrative dataset on all domestic firm-to-firm transactions in Belgium and establish two novel facts. First, firms charge higher average markups when they have larger input shares within their customers. This holds even after controlling for the firms' sectoral market shares. In addition, the variations in our metric of average input share within the customer firms are more important in predicting firms' profitability than those of the commonly used metric of sectoral market share. These results suggest that in addition to the *firm-level* market share within the sector, the firm's pairwise input shares to each customer capture the *pair-level* pricing power that the firm has to each of its customers. Second, we evaluate how firms alter linkages in response to an exogenous reduction in import prices. We borrow insights from Autor, Dorn, and Hanson (2013) and Hummels, Jørgensen, Munch, and Xiang (2014) and take the increase in firms' imports from China in the 2000s as a trade shock. We find that the more exposed the firms are to the import supply shocks, the more churn they have in their suppliers.

Motivated by these facts, we build a model that has two key departures from perfect competition and fixed network structures. The first departure is oligopolistic competition in firm-to-firm trade where firms charge different markups to each customer firm. The relative size of the supplier in the total input sourcing of each customer becomes the relevant determinant of the supplier's markup on that transaction. This is in contrast with a setting in which firms' sectoral market shares determine their firm-level market power. The second departure is endogenous network formation, where firms face fixed costs and optimally choose which firms to supply from.

Our model presents a network irrelevance result in the benchmark case where we shut down both oligopolistic competition and endogenous networks. In addition to perfect competition and fixed networks, this benchmark case imposes strong restrictions: exports are in terms of composite final goods, and there is common substitutability across goods in technology and preference. In this case, shocks still transmit to other firms along the production chain. But firm-level variables such as firms' total sales and their direct exposure to the shock, become sufficient statistics for evaluating global changes in the aggregate variables.

We model oligopolistic competition with a nested CES structure as in Atkeson and Burstein (2008). Rather than the more conventional implementation where a firm's share in the sector's purchases determine its elasticity and markup, in our model a firm charges a higher markup to a customer when its share in that customer's purchases is larger. This departure leads to two counteracting effects on aggregate variables. First, variable markups imply there will be an incomplete pass-through from a supplier's input price reduction to its output price reduction since the supplier will increase its markup, what we call the "attenuation effect." Second, the other suppliers that sell goods to the same customer will reduce their markups in face of increased competition, what we call the "pro-competitive effect."

We model endogenous network formation as firms choosing which set of suppliers to source from. They additionally decide whether to import and/or export. Each linkage requires payment of a fixed cost and firms maximize their net profits given other firms' sourcing decisions and prices. Upon a reduction in foreign price, firms can start to directly import from abroad. They can additionally source from firms whose goods have become relatively cheaper. These will amplify the aggregate response to the foreign price reduction as the input costs of firms that changed suppliers discretely drop.

Guided by our model, we estimate the CES parameters so that the firm-level average markups – averages of the model implied markups on sales to other producers and to the final consumer – provide best fit of those implied from the data. We then study how the aggregate price index and welfare respond to the reduction in a foreign good's price. We start by analyzing the predictions from the benchmark case where the network structure is irrelevant. We then investigate how adding oligopolistic competition and endogenous networks alter aggregate predictions.

When evaluating the model with oligopolistic competition and fixed networks, we compute the changes in the aggregate price index using the observed input shares and the estimated CES parameters following a technique developed by Dekle, Eaton, and Kortum (2007). We find that oligopolistic competition in firm-to-firm trade slightly attenuates movements in the aggregate price index. The magnitudes of the net effects are small because the attenuation and pro-competitive effects largely cancel each other out in each customer firm's input market. Nevertheless, we analytically characterize the magnitudes of the two and their net effects. We demonstrate that a measure of the firm's exposure to the shock, either directly or indirectly through its suppliers, can help us understand whether the firm faces higher or lower markups on average. Moreover, we argue that the nature of the shock is also key in determining the magnitudes of the net effects. The shock of foreign price reduction hitting all the importers produces small net attenuation effect. But if the same price reduction hits only a single importer, the magnitudes of the net attenuation effects become much larger.

For the analysis of the model with endogenous networks, we rely on simulations of the estimated model. We use a model with a smaller set of firms, since simulating an endogenous network with the number of firms observed in the data is computationally infeasible. Even so, we find that endogenous networks significantly amplify aggregate responses.

Overall, the benchmark case can capture less than a quarter of aggregate responses that are implied by our estimated full model. The differences in aggregate responses are mostly driven by firms switching from non-importers to importers. In addition, we also find that oligopolistic competition in firm-to-firm trade makes a quantitative difference in aggregate responses through its interaction with endogenous networks. In our model, oligopolistic competition means that firms face a greater degree of double marginalization compared to a case where firms engage in monopolistic competition. Firms face higher markups in each transaction, and they accumulate throughout the firm-to-firm network. Higher input costs alter firms' decisions in choosing their suppliers.

This paper is closely related to the growing body of literature that studies aggregate outcomes beyond the network irrelevance result of Hulten (1978). Baqaee (2014a) theoretically shows that extensive margins of firm entry and exit can amplify idiosyncratic shocks. Baqaee and Farhi (2017) analyze the importance of second order effects of firm-level TFP shocks. They emphasize the roles of substitutability across inputs, returns to scale, factor reallocation, and structure of the network.[1] While they focus on second order effects in an economy without market frictions, we focus on how market frictions produce different aggregate outcomes in response to large shocks. We specifically focus on two deviations from the efficient economy, which we find to be relevant in the data: oligopolistic competition in firm-to-firm trade and endogenous networks.

We build on the literature that focuses on the aggregate implications of oligopolistic competition. Grassi (2016) develops a model in which firms engage in oligopolistic competition in an economy with sectoral input-output linkages and studies the contribution of firm-level

1. For other papers that investigate the effects beyond the network irrelevance result, see Altinoglu (2015), Liu (2016), and Bigio and La'o (2017) for models where firms face financial constraints, and Pasten, Schoenle, and Weber (2017) for models with price rigidities.

shocks on the aggregate dynamics.[2] Effects similar to our attenuation and pro-competitive effects are studied extensively in other contexts. For example, Feenstra, Gagnon, and Knetter (1996) study how the degree of price pass-through varies with the firm's export market share. Amiti, Itskhoki, and Konings (2017) study how firms prices respond to changes in the prices of their competitors. Atkeson and Burstein (2008) focus on incomplete price pass-through to explain deviations of international relative prices from relative PPP.[3] All these papers analyze oligopolistic competition where firms compete with others within the same sector, implying that the firm's market power is captured by its market share in its sector.[4] In contrast to these papers, we propose a novel view on competition between firms. Instead of the *firm-level* market share within the sector being the determinant of the firm's market power, we suggest that the *pair-level* input shares across its customers are the relevant metrics for the firm's ability to charge markups.

This paper is also related to papers that study the aggregate implications of firms changing suppliers. For example, Lim (2015) points out the importance of extensive margins in firm-to-firm relationships.[5] Tintelnot, Kikkawa, Mogstad, and Dhyne (2017) empirically show that shocks to a firm's actual suppliers and customers transmit to the firm itself even after controlling for shocks that affect the firm's potential suppliers and customers. This suggests that there are rigidities in firm-to-firm relationships, which also motivates our model where firms pay fixed costs when choosing suppliers.[6] They also build a tractable model of

2. As in Grassi (2016), we focus on strategic complementarities across suppliers in the style of Atkeson and Burstein (2008). See Krugman (1979), Ottaviano, Tabuchi, and Thisse (2002), Melitz and Ottaviano (2008) and Zhelobodko, Kokovin, Parenti, and Thisse (2012) for imperfect competition where complementarities arise from the demand side.

3. See Neiman (2011) for a similar model of variable markups that allows for arm's length and intra-firm transactions.

4. There are also cases in which aggregate volatilities can be captured by the distribution of market shares. See for example Gabaix (2011), where the Herfindahl-Hirschman Index (HHI) is the main metric that captures aggregate volatility.

5. Other papers that focus on the formation of domestic firm-to-firm relationships include Bernard, Moxnes, and Saito (2016b) and Oberfield (2017).

6. One of the findings of Tintelnot, Kikkawa, Mogstad, and Dhyne (2017) is that firms increase their scale in response to positive import shocks to their suppliers, as well as to themselves. Papers that study the

endogenous network formation. Unlike ours, their model relies on the assumption that firms do not obtain profits from firm-to-firm trade, and the resulting network is constrained to be acyclic.

This paper also contributes to a recently growing literature on how shocks transmit through the production network. Carvalho, Nirei, Saito, and Tahbaz-Salehi (2014) and Boehm, Pandalai-Nayar, and Flaaen (2016) have found that shocks to suppliers transmit to firms by looking at firms that sourced from Japanese firms impacted by the 2011 Tohoku earthquake. Barrot and Sauvagnat (2016) have also found shock transmission through production linkages by looking at firms sourcing from firms located in places hit by natural disasters in the US. In the context of sector-to-sector linkages, Acemoglu, Akcigit, and Kerr (2015a) study the propagation of demand and supply shocks.[7] In this paper, shocks on firms indeed transmit to other firms along the production chain. Our main result is that the structure of the production network matters in the aggregate because the magnitudes of these network effects cannot be solely captured by firm-level observables.

Finally, our paper is related to the considerable literature on micro shocks translating to aggregate movements. Firm- or sector-level shocks may not wash out when evaluating aggregate fluctuations if the firm- or sector-level size distributions are fat-tailed (Gabaix, 2011; Carvalho and Gabaix, 2013) or if the input-output structures are asymmetric (Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi, 2012).[8] In particular, Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) show that the economies with different input-output structures may produce different aggregate output volatilities in response to the same sector-

effects of import shocks on firms include Gopinath and Neiman (2014), Halpern, Koren, and Szeidl (2015), Magyari (2016), Antras, Fort, and Tintelnot (2017) and Furusawa, Inui, Ito, and Tang (2017).

7. These network effects are also studied in other contexts and have found to generate instabilities in the system as a whole. For example, Scheinkman and Woodford (1994) point out that small independent shocks to different firms can lead to instability of the system through their nonlinear interactions. Elliott, Golub, and Jackson (2014) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015b) study the stability of financial networks.

8. Di Giovanni, Levchenko, and Mejean (2014) and Magerman, De Bruyne, Dhyne, and Van Hove (2016) study the two potential sources of aggregate fluctuations together. Yeh (2016) points out that large firms tend to be less volatile, leading to mitigated effects of fat-tailed firm size distributions in the aggregate.

level shocks.[9] As aforementioned, we focus on exact changes in response to large shocks instead of the variance of the changes. In a Cobb-Douglas model that builds on Long and Plosser (1983), the irrelevance of the network structure still holds when one focuses on the changes in aggregate variables.

This paper proceeds as follows. Section 2.2 describes the data. We also provide two pieces of descriptive evidence. First, we show that firms' input shares across suppliers are skewed, and the variation in input shares are not entirely driven by firm-level components. Second, we show that there is large churn in supplier-customer relationships. Section 2.3 establishes the two empirical facts: suppliers charge higher markups if their input shares to customers are higher and firms alter suppliers in response to shocks. Section 2.4 outlines the model and presents the network irrelevance result in the benchmark case. Section 2.5 estimates the parameters of the model, and Section 2.6 conducts counterfactual analysis where a reduction in foreign price is taken as the shock. Finally, Section 2.7 concludes.

2.2 Data and descriptive evidence

In this section we start by introducing our main data sources. We then provide descriptive evidence that activities at the pair-level cannot be fully captured by firm-level components alone, and that there is a large churn in supplier-customer relationships.

2.2.1 NBB B2B dataset

Our main dataset is the National Bank of Belgium (NBB) Business-to-Business (B2B) transactions database, which is a panel of VAT-id to VAT-id transactions among the universe of Belgian VAT-ids over years 2002-2014. As explained in detail in Dhyne, Magerman, and Rubinova (2015), all enterprises in Belgium are assigned unique VAT-ids and are required to

9. Other papers that study the importance of micro shocks on aggregate volatility include Jovanovic (1987), Durlauf (1993), Bak, Chen, Scheinkman, and Woodford (1993), Horvath (1998), Horvath (2000), Carvalho (2010), Foerster, Sarte, and Watson (2011), Di Giovanni, Levchenko, and Mejean (2014), Stella (2015), Atalay (2017), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017).

report total yearly sales to other VAT-ids that are larger than 250 Euro. We also make use of the VAT declarations, in which we observe their total sales and total purchases. In addition, we merge the datasets with the annual account filings and the international trade dataset. From the annual accounts we observe the primary sector of each VAT-id (NACE Rev. 2, 4-digit), total sales, labor cost, ownership relations to other VAT-id's, location (ZIP code), and other variables that are standard in the annual accounts. In the international trade dataset we observe the values of imports and exports of goods at the VAT-country-product (CN 8-digit)-year level.

One firm can have multiple VAT-ids. In our paper, we focus on the effect of inter-firm pricing and inter-firm linkage formations on the aggregate variables. The nature of these pricing and linkage formation decisions may be different from those at the within-firm level. Thus we aggregate VAT-ids up to the firm-level using ownership filings in the annual accounts and foreign ownership filings in the Balance of Payments survey. In the Balance of Payments survey, we observe for each VAT-id the name and the country of the foreign firm that owns at least 10 percent of the shares, along with the associated ownership share. We group all VAT-ids into firms if they are linked with more than or equal to 50% of ownership, or if they share the same foreign parent firm that holds more than or equal to 50% of their shares. See Appendix B.1.1 for further details.

2.2.2 Sample selection

For our sample of the analysis, we select private and non-financial sector Belgian firms that report positive labor cost. Following De Loecker, Fuss, and Van Biesebroeck (2014), we select firms that report tangible assets of more than 100 Euro and positive total assets for at least one year throughout our sample period. Table 2.1 describes the coverage of our selected sample compared to the Belgian aggregate statistics.[10]

10. In Appendix B.1.2, we also report the coverage of the full sample constructed in Dhyne, Magerman, and Rubinova (2015). There we also provide aggregate statistics of the B2B dataset and some descriptive statistics of the production network.

In the table, one can see that our selected sample covers the aggregate statistics well. However, note that the total sales in our sample turn out to be larger than those in the aggregate statistics. The differences can be explained by the fact that the output values in the aggregate statistics sum up value added for trade intermediaries instead of their gross output, hence the smaller numbers in the aggregate statistics.

Table 2.1: Coverage of selected sample

Year	Private, non-financial		Imports	Exports	Selected sample				
	GDP	Output			Count	V.A.	Sales	Imports	Exports
2002	149	411	210	229	122,460	123	586	179	189
2007	192	546	300	314	136,370	157	757	280	269
2012	212	626	342	347	139,605	170	829	296	295

Notes: All numbers except for Count are in terms of billion Euro in current prices. Belgian GDP and output are for all private and non-financial sectors. Data for Belgian aggregate statistics are from Eurostat. Value added is the sum of value added reported in the annual accounts. Total sales in our selected sample are larger total output in the aggregate statistics because the output values in the aggregate statistics sum up value added for trade intermediaries instead of their gross output.

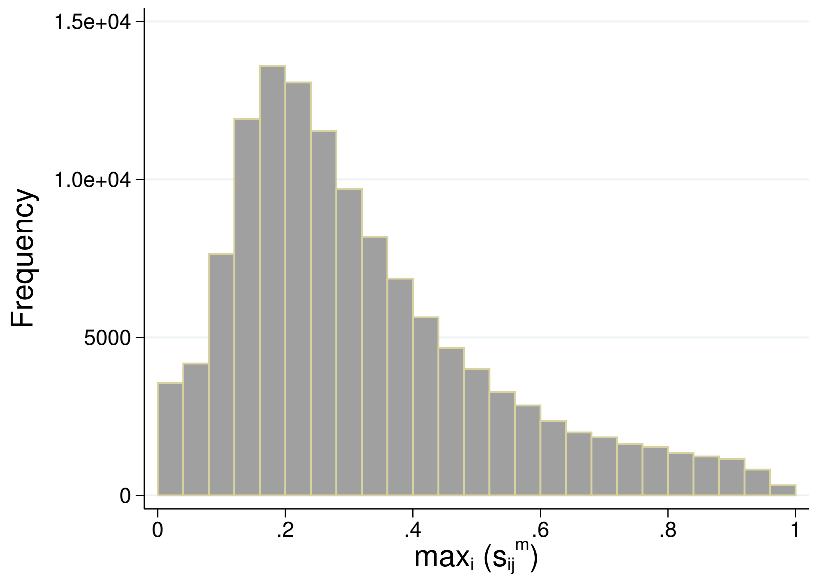
2.2.3 Descriptive evidence

In this section, we provide descriptive evidence that motivate our empirical analysis in Section 2.3. We first show that firms' input shares across suppliers are skewed, and then that the variation in pairwise input shares is not entirely driven by firm-level components. Finally, we show that there is large churn in supplier-customer relationships.

Skewed input shares across suppliers

Figure 2.1 plots a histogram for the input shares of the largest suppliers for all customer firms in 2012 that have more than 10 suppliers. The input share of the largest supplier for the median firm in this figure is 27%.

Figure 2.1: Input shares of the largest suppliers



Notes: s_{ij}^m is defined as firm i's goods share among firm j's input purchases from other Belgian firms and abroad. The above histogram shows the distribution of $\max_i \left(s_{ij}^m\right)$, which is the maximum value of s_{ij}^m for each customer firm j in 2012 that has more than 10 suppliers. The median value is 0.27.

Together with the fact that the median firm has 28 suppliers, it indicates that suppliers' input shares are highly skewed throughout the economy. For each customer, few suppliers tend to account for most of its input purchases. In Appendix B.1.3 we present a histogram of the Herfindahl-Hirschman Index (HHI) of s_{ij}^m for the same set of firms with at least 10 suppliers. We find that 50% of firms have a HHI above 0.15. 26% of firms have a HHI above 0.25%.

Pairwise components are driving the variations in input shares

However, high skewness in input shares may simply be caused by firm-level components. For example, one may argue that the skewness of input shares across suppliers is coming from the skewness in the suppliers' productivity distribution. If that is indeed the case, one would expect that a firm with a high input share on a particular customer would also be one with high total sales. To investigate this further, we compute for each firm the rank correlation between its suppliers' input shares and their total sales.

Consider the firm on the left of Figure 2.2. This firm is purchasing goods worth 10, 5, and 1 Euro from its three suppliers, a, b, and c, respectively. The three suppliers' total sales are 100, 50, and 10 Euro. The ordering of the firm's suppliers according to the input shares aligns with the ordering of their total sales. Thus, the rank correlation for the firm is 1. One the other hand, consider the firm on the right of the figure. The transaction values are identical to the firm on the left, but the three suppliers' total sales are 10, 50, and 100 Euro, respectively. Here the ordering of the two are opposite, so the rank correlation for the firm is -1.

Figure 2.2: Example for computing rank correlations

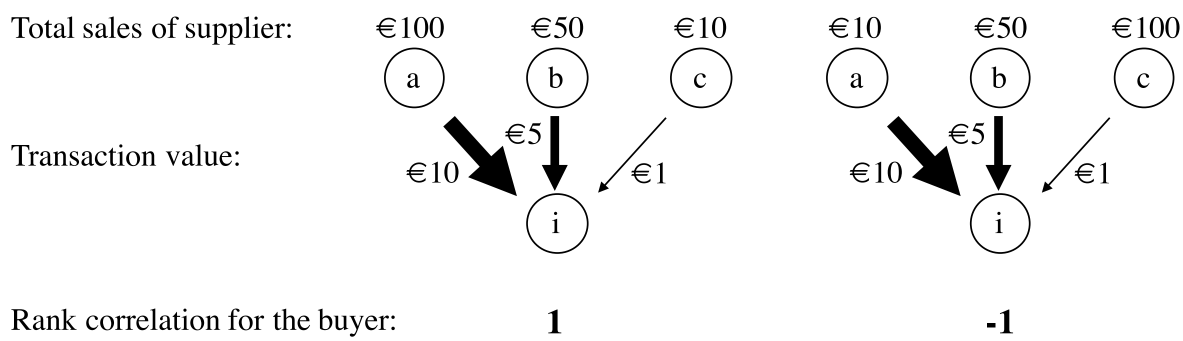
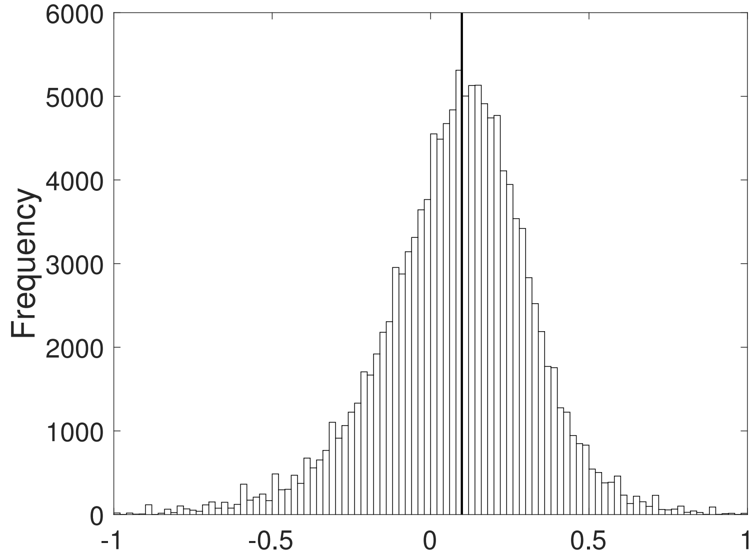


Figure 2.3 displays the histogram of the correlation coefficients. The median firm's coefficient is around 0.10. 35% of firms have correlation coefficients that are zero or negative. This result indicates that a firm with high input share on a particular customer is not necessarily large.[11] It illustrates that pairwise match components play a large role in firm-to-firm trade in addition to firm-level components.

11. This becomes the case if the distributions of firms' output shares to each customer are skewed. In Appendix B.1.4 we provide a figure analogous to Figure 2.1, but for output shares. The output shares are indeed skewed, where more than 20% of the output of a median firm goes to its largest customer.

Figure 2.3: Histogram of rank correlation of suppliers' input shares and total sales



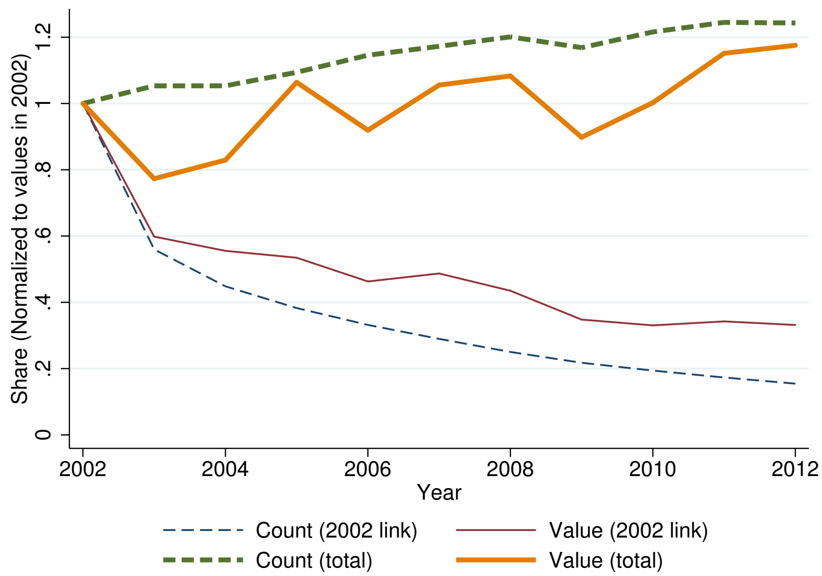
Notes: This figure shows a histogram of Spearman's rank correlation coefficients between s_{ij}^m and TotalSales_i for suppliers of j for all j with 5 or more suppliers. The vertical line depicts the median correlation coefficient of 0.10.

Indeed, in Figure 2.3 we plot the unconditional rank correlations which do not take into account the difference in the goods produced by suppliers. The low correlations in the figure may come from the fact that a supplier's good is heavily used in firms from one sector, but not from firms in others. In Appendix B.1.5 we take into account this heterogeneity of input compositions across sector-to-sector relationships. We calculate the rank correlations for each firm, but now for each group of suppliers in each sector. Even after conditioning the analysis within each of the sector-to-sector relationships, we see the same pattern as in Figure 2.3. We also show in Appendix B.1.5 that the results are qualitatively the same when we use the Pearson correlations instead of the rank correlations.

Large churn in supplier-customer relationships

We also see that there is a large churn in supplier-customer relationships. Figure 2.4 plots the share of firm-to-firm links present in 2002 that survived through 2012. It also depicts the evolution of total firm-to-firm links in terms of both numbers and values. One can see that there is substantial churn in the linkages: less than 40% of the links that were present in 2002 were still there in 2012 in terms of the values. In terms of pure numbers, linkage survival decreases to less than 20% in 10 years.

Figure 2.4: Evolution of firm-to-firm links



The evidence of this section confirms that pairwise patch components play a large role in firm-to-firm networks and that there is a large churn in linkages. In the next section, we establish the two facts that motivate our model.

2.3 Motivating empirical results

In this section we establish two empirical facts that will motivate our model in Section 2.4: firms charge higher average markups when they have higher input shares within their customers, and firms change their suppliers in response to an exogenous reduction in prices of imports.

Markups positively associated with input shares within customers

We start by exploring the relationship between firm-level markups and firms' average input shares within their customers. We ask if the two are positively associated with each other, even after controlling for firm-level sectoral market shares. A positive relationship suggests that firms' market power contains pair-level components that come from each individual customer in addition to firm-level components that are captured by sectoral market shares.

Firm-level markups, $\mu_{i,t}$, are measured as the ratios of firms' total sales over variable costs (sum of goods purchases and labor costs). Firm-level sectoral market shares, $\text{SctrMktShare}_{i,t}$, are computed at the NACE 4-digit level. This measure captures firms' market power in models that feature oligopolistic competition in which firms' output is aggregated at the sectoral level.

We also construct a measure that captures the input shares firms have within their customers. For each supplier-customer pair, we can compute the share of sales from the supplier firm i to the customer firm j out of j's total input purchases: $s^m_{ij} = \frac{\text{Sales}_{ij}}{\text{InputPurchases}_j}$. Using these pairwise input shares, we compute firm i's weighted average input shares to its customers at year t, as

$$\begin{aligned}\overline{s^m_{i\cdot,t}} &= \sum_{j \in W_{i,t}} \frac{\text{InputPurchases}_{j,t}}{\sum_{k \in W_{i,t}} \text{InputPurchases}_{k,t}} s^m_{ij,t} \\ &= \frac{\sum_{j \in W_{i,t}} \text{Sales}_{ij,t}}{\sum_{j \in W_{i,t}} \text{InputPurchases}_{j,t}},\end{aligned}$$

where $W_{i,t}$ is the set of i's customers at year t. Total input purchases are assigned as weights for each customer firm.

With these variables, we run the following regression:

$$\mu_{i,t} = \beta \, \text{SctrMktShare}_{i,t} + \gamma \, \overline{s^m_{i\cdot,t}} + \varphi \, X_{i,t} + \delta_t + \epsilon_{i,t}, \tag{2.1}$$

where firm-level controls and year fixed effects are included.

Table 2.2 reports the results. The specification of the first column includes sector fixed effects, and the specifications of the second and the third columns include firm fixed effects. First, in all specifications we see a positive relationship between markups and firm-level market shares. This is not surprising, as it may be because of the mechanical relationship between both variables. The numerators in both variables are firms' total sales. For example, the result on the third column indicates that within each firm, an increase of one standard deviation in the firm's market share is associated with an increase of around 6.9 percentage points in the firm's markup.

However, even after controlling for firms' market shares, the coefficients on the firms' average input shares to customers are positive. The third column indicates that within each firm a single standard deviation increase in average input shares to customers leads to around an increase of 17 percentage points in the firm's markup. This positive correlation indicates that firms have greater ability to charge markups if they have higher shares within their customers' inputs.

The relative size of the two coefficients is also worth discussing. Across all specifications, we see much larger coefficients on the average input shares compared to those on the firm-level market shares. In addition, we show in Table B.5 in Appendix B.2.1 that the R-squared increases more when adding the average input shares on the RHS, as opposed to adding the firm-level market shares. These results indicate that the variations in the average input shares within customers' inputs are more important for firms' ability to charge markups than the variations in the sectoral market shares.

Table 2.2: Firm-level markups and input shares

	Firm-level markups		
	(1)	(2)	(3)
$\text{SctrMktShare}_{i,t}$ (4-digit)	0.0929***	0.0430***	0.0686***
	(0.00928)	(0.00963)	(0.0129)
Average input share $\overline{s^m_{i\cdot,t}}$	0.298***	0.182***	0.173***
	(0.0130)	(0.00938)	(0.00925)
N	1099496	1089209	1070602
Year FE	Yes	Yes	Yes
Sector FE (4-digit)	Yes	No	No
Firm FE	No	Yes	Yes
Controls	Yes	No	Yes
R2	0.0994	0.619	0.625

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. Controls include firms' indegree, outdegree, employment, total assets, and age.

The result of the positive correlation between markups μ_i and average input shares $\overline{s^m_{i\cdot,t}}$ is robust under different average measures of $\overline{s^m_{i\cdot,t}}$: either taking simple averages or taking median values. It is also robust when using other measures of pairwise input shares. For example, instead of using s^m_{ij} we use s_{ij}, which is the firm i's sales share in j's total variable inputs (goods purchases plus labor costs). Another alternative share we use is the supplier's sales share among the customer's inputs that are classified as the same goods as the supplier's, either at the 2-digit or 4-digit level. We report the results of other robustness checks in Appendix B.2.1.[12]

12. The positive correlation is also robust to alternative markup measures. The measure of markups we use is consistent with the model we construct in Section 2.4, which is static and features CRS production technology. Firms might also use additional factors, such as capital inputs, and production technology may differ across sectors. Given these possibilities, we show positive correlation under alternative measures of

Larger churn in suppliers when exposed to larger reduction in input prices

As shown in Section 2.2.3, there is a large churn in supplier-customer relationships. The median firm has a churn of around 20% of its suppliers annually, in terms of values. Here we show that in addition to random changes, there is a systematic relationship between churn of suppliers and an exogenous shock to import opportunities from abroad. We take the reduction in prices of Chinese imported goods throughout the 2000s as the shock. Belgian imports from China more than doubled in the 2000s after its accession to the WTO. We interpret this as a decrease in the prices of Chinese goods available in the international market over the same period.[13]

We regress changes in firms' share of continuing and added suppliers on firms' increase in Chinese sourcing over the periods:

$$\Delta Y_i = \beta \, \Delta CS_i + \gamma \, X_{i,t_0} + \delta_{s(i)} + \epsilon_i, \tag{2.2}$$

where ΔY_i denotes the shares of continuing and added suppliers scaled by the values at the initial period.[14] ΔCS_i denotes the increase in Chinese sourcing scaled by the total input value of the firm at the initial period.

$$\Delta CS_i = \frac{\Delta V_{China,i}}{\text{TotalInput}_{i,t_0}}.$$

We add sector fixed effects and firm-level controls at the initial period.

The OLS regression of equation (2.2) is subject to an endogeneity issue. Increases in Chinese sourcing may be triggered by factors that also affect firm activities, including decisions

firm-level markups following De Loecker and Warzynski (2012). See Appendix B.2.2 for details.

13. In Appendix B.1.7 we compare Chinese imports to Belgium with imports from other countries to Belgium and show that the rapid increase in imports was not common with regard to other countries.

14. For example, consider a firm with 10 suppliers that dropped 3 and added 5, resulting in 12 suppliers. The share of continuing suppliers (in numbers) is calculated as 7/10, and the share of added suppliers (in numbers) is calculated as 5/10.

on which firms to source from. To capture the increase in firms' Chinese imports driven by factors exogenous to firms, we instrument firms' increase in Chinese sourcing using changes in Chinese exports to eight non-European developed countries.[15] The instrument for ΔCS_i becomes

$$\Delta IV_i = \sum_k \frac{V_{ALL,i,k,t_0}}{\text{TotalInput}_{i,t_0}} \Delta \frac{V_{China,Rich,k}}{V_{World,Rich,k}}, \tag{2.3}$$

where k represents products at the NACE 4-digit level. We first construct a sectoral measure of the increase in Chinese exports to the developed countries by taking the changes in Chinese goods' share in the developed countries' imports ($\Delta \frac{V_{China,Rich,k}}{V_{World,Rich,k}}$). We then convert product level measures into firm-level measures by using firm specific weights for each product ($\frac{V_{ALL,i,k,t_0}}{\text{TotalInput}_{i,t_0}}$). The weights measure firm i's exposure to sector k goods at the initial period by taking the ratio of the sum of product k inputs over the total inputs.

The idea of the instrument is similar to that of Autor, Dorn, and Hanson (2013). Differently, we use the change in Chinese goods' share in developed countries' imports as the product level measure that captures the increase in Chinese exports to developed countries. Instead of simply taking the growth rate of Chinese exports, this way we can remove the demand effects that increased developed countries' growth in imports from developing countries as a whole.

Our instrument is also at the firm-level, with variation coming from across firms that are within sectors. Similar to that of Bartik (1991), the instrument is valid if the variations in firms' initial exposure to each product are not correlated with unobservable firm-level characteristics that may affect firms' domestic sourcing decisions.

Table 2.3 reports both the first and second stage results when the changes in suppliers are computed in terms of values. The first and second columns of Panel A represent the second stage results where the LHS variables are the shares of continuing and added suppliers. The third and the fourth columns of Panel A decompose the effect of the added suppliers. Out

15. Australia, Canada, Chile, Japan, Korea, New Zealand, the UK, and the USA.

of the firms that were added, the third column shows the shares which were incumbent, meaning firms that existed in the initial period. The fourth column displays the shares which were newly born.[16] All variables are computed as average yearly changes over the sample period. Panel B reports the first stage results. We see that the first stage coefficient is positive and statistically significant.

The results first suggest that firms experience greater churn in suppliers when the price of imported goods' is further reduced. A one standard deviation increase in the change of Chinese sourcing leads to firms dropping around 13% of domestic suppliers on a yearly basis. The same shock also leads to firms adding around 11% of domestic suppliers. Given that the median firm loses around 19% and adds around 25% of suppliers in terms of value on a yearly basis, the magnitudes of churn induced by the import supply shock are significant. The results also show that the additions of links mostly come from the rewiring of links among existing firms, and not from firms that entered the market.[17]

We reported the results for the changes in suppliers in terms of values, but the same features remain robust when conducting the same analysis in terms of numbers.[18] In addition, we find qualitatively the same results when we analyze the changes in customers. We report these results in Appendix B.2.3 in addition to their OLS results and first stage results.

2.4 Model

In the previous section, we established two empirical facts: firms charge higher markups when they have higher input shares within their customers, and firms alter linkages in response

16. As the variables are computed as average yearly changes, the coefficients of the third and fourth columns need not exactly add up to the coefficients in the second columns.

17. The coefficients for the added suppliers which were entrants are small relative to the coefficients on the total added suppliers. However, the ratios are large compared to how much new entrants account for in the aggregate economy. In our sample period, new entrants account for around 3% of the total sales in the total economy, where our regression results imply that entrants account for around 12% of firms that were added as suppliers.

18. The results are also qualitatively robust when using variables in terms of yearly changes with additional year fixed effects, instead of average yearly changes over 2002 to 2012.

Table 2.3: First and second stage results

Panel A: Second stage result				
	Changes in suppliers (in terms of value)			
	(1)	(2)	(3)	(4)
	Continuing suppliers	Added suppliers	Added suppliers: Incumbent firms	Added suppliers: New firms
ΔCS	-0.128^{***}	0.110^{***}	0.0973^{***}	0.0128^{***}
	(0.0283)	(0.0334)	(0.0316)	(0.00366)
N	56146	56146	56146	56146
Controls	Yes	Yes	Yes	Yes

Panel B: First stage result	
	(1)
	ΔCS
ΔIV	0.00370^{***}
	(0.000649)
R2	0.0255
F Stat	32.48

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. The coefficients of the second stage results are X-standardized. Controls include firm age and employment size in 2002 with sector fixed effects (NACE 2-digit) and geographic fixed effects (NUTS 3). The same controls are used in the first stage results. ΔCS is the firm's average yearly increase of Chinese imports from 2002 to 2012 scaled by its total inputs in 2002. ΔCS is instrumented by the weighted sum of the sectoral change in Chinese goods' share in developed countries' total imports from 2002 to 2012. Standard errors are clustered at the NACE 2-digit-NUTS 3 level.

to shocks. These facts motivate a model of a small open economy where firms engage in oligopolistic competition within each customer's inputs, and where firms optimally choose suppliers. In this section, we set up the model and define the equilibrium. Then we turn to a special case of the model and present a network irrelevance result.

There are representative households inelastically supplying a fixed amount of labor. There is a homogeneous goods sector under perfect competition. These goods are also freely traded, and enables us to pin down wages. In the heterogeneous goods sector, there are a fixed number of domestic firms each producing a differentiated good. Labor, goods from other domestic firms, and/or imported goods are used for production. Firms sell their goods to final consumption, to other firms, and/or abroad.

We treat firms to be infinitesimal in the final demand market and assume monopolistic competition. On the other hand, we assume oligopolistic competition in firm-to-firm trade, which generates pairwise markups. Lastly, firms make decisions on their sourcing sets (including whether to import), and their exporting decisions.

2.4.1 Preference

There is a mass of representative households each providing one unit of labor. Households have Cobb-Douglas preference on the goods from the homogenous goods sector, Y, and on the goods from the heterogeneous goods sector. Within the heterogeneous goods sector, the representative household has a CES preference over all firms' goods with substitution parameter σ. We assume that goods are substitutes, thus $\sigma > 1$. We also assume that households do not directly consume foreign goods in the heterogeneous goods sector. The household's preference is denoted as

$$U = \left(\sum_{i \in \Omega} \beta_{iH} q_{iH}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\alpha\sigma}{\sigma-1}} Y^{1-\alpha}, \tag{2.4}$$

where Ω denotes the set of domestic firms in the heterogeneous goods sector and α is the Cobb-Douglas share on the heterogeneous goods sector. q_{iH} denotes the quantity of goods that firm i sells to the household. Given the price that i charges to the household, p_{iH} , q_{iH} can be written as

$$q_{iH} = \beta_{iH}^{\sigma} \frac{p_{iH}^{-\sigma}}{P^{1-\sigma}} \alpha E, \tag{2.5}$$

where E denotes the aggregate expenditure. P denotes the price index of the heterogeneous goods sector:

$$P = \left(\sum_{i \in \Omega} \beta_{iH}^{\sigma} p_{iH}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{2.6}$$

The price index of the aggregate economy, $\tilde{P}$, is a Cobb-Douglas aggregate of P and the price of the homogeneous good, p_y: $\tilde{P} = \left(\alpha^{\alpha} (1-\alpha)^{1-\alpha} \right)^{-1} P^{\alpha} p_y^{1-\alpha}$.

We model demand from abroad to have the same structure as the domestic household. Let I_{iF} be an indicator of whether firm i is an exporter or not. Given a price that i charges on exported goods, p_{iF}, export quantity, q_{iF}, can be written as

$$q_{iF} = p_{iF}^{-\sigma} D^{*}, \tag{2.7}$$

where D^{*} is the exogenous demand shifter from abroad.

2.4.2 Technology and market structure

Each firm in the heterogeneous goods sector produces a single differentiated good. In addition to labor inputs, they purchase goods from other firms and/or imported goods as intermediate goods. On the output side, they sell their goods to other domestic firms and/or export, at the same time selling directly to final demand. We treat firms to be infinitesimal in the final demand market and assume monopolistic competition. Thus firms charge constant markups on their goods when selling to the final consumer. We also assume that firms apply the same markups when exporting.

When firms sell goods to other domestic firms, the assumption of infinitesimal suppliers for each customer is not consistent with the data. Firms tend to have highly concentrated input share distributions, where a handful of top supplier firms account for the majority of firms' goods purchases. Moreover, in Section 2.3, we found that firms charge higher markups when they have higher input shares to customers. Thus we assume oligopolistic competition in firm-to-firm trade, where firms charge different markups to different customers depending on the shares they have in their customers' goods purchases. In doing so, we take the framework of Atkeson and Burstein (2008) and apply to firms' pricing decisions in firm-to-firm trade.

Motivated by the findings in Section 2.3, we also model firms to optimally make domestic sourcing decisions as well as importing and exporting decisions. We assume that firms pay fixed costs in order to supply from another domestic firm, and also for importing and exporting.[19]

We first lay out the firms' problem given the production network in Section 2.4.2. Then we describe the endogenous formation of the production network in Section 2.4.2.

Production given network

Let Z_i be firm i's set of domestic suppliers, and let I_{iF} and I_{Fi} be indicators for the exporting and importing status of firm i. In this subsection we take these as given.

Firms in the homogeneous goods sector produce goods with a linear technology with respect to labor:

$$y = l^Y. \tag{2.8}$$

Firms in the heterogeneous goods sector have a CES production function over the labor inputs and intermediate goods bundle. The intermediate goods bundle itself is a CES bundle

19. We take this approach because we see a positive relationship between firms' sales to domestic final demand and their number of domestic suppliers, as reported in Appendix B.1.8. These size advantages for firms with larger number of domestic suppliers are suggestive of fixed costs associated with domestic sourcing.

of goods from the firms' suppliers and foreign goods. We denote the elasticity of substitution across labor inputs and the intermediate goods bundle to be η, and the substitution parameter across firms' goods and imported goods to be ρ. We assume both parameters to be above one: $\rho, \eta > 1$.[20]

The implied unit cost of firm i becomes

$$c_i = \phi_i^{-1} \left(\omega_l^{\eta} w^{1-\eta} + \omega_m^{\eta} p_{mi}^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{2.9}$$

where ϕ_i is i's core productivity. ω_l and ω_m denote CES weights in the production function on labor and intermediate goods. w denotes wage, and p_{mi} is the firm specific price index of intermediate goods. p_{mi} varies with firms' sourcing strategy Z_i and I_{Fi}:

$$p_{mi} = \left(\sum_{j \in Z_i} \alpha_{ji}^{\rho} p_{ji}^{1-\rho} + I_{Fi} \alpha_{Fi}^{\rho} p_F^{1-\rho} \right)^{\frac{1}{1-\rho}}. \tag{2.10}$$

The term p_{ji} denotes the price that firm j charges for its goods when selling to firm i. p_F denotes the exogenous price of the foreign good. The terms α_{ji} and α_{Fi} reflect how salient goods from firm j and foreign are as inputs for firm i.

Before discussing the market structures of the final demand market and of the firm-to-firm markets, let us derive the firms' shares on inputs implied by the above CES structures. The share of firm i's variable costs spent on labor, s_{li}, is:

$$s_{li} = \frac{\omega_l^{\eta} w^{1-\eta}}{c_i^{1-\eta} \phi_i^{1-\eta}}. \tag{2.11}$$

20. When we estimate both ρ and η in Section 2.5.1, we do not impose any restrictions concerning the relative magnitudes of ρ and η. We find the point estimate of ρ to be larger than that of η, meaning that firms' goods are more substitutable with each other than with labor.

The intermediate goods' share, s_{mi}, becomes

$$
\begin{aligned}
s_{mi} &= 1 - s_{li} \\
&= \frac{\omega_m^{\eta} p_{mi}^{1-\eta}}{c_i^{1-\eta} \phi_i^{1-\eta}}.
\end{aligned} \tag{2.12}
$$

Among i's variable costs spent on intermediate goods, the share of firm j's good, s_{ji}^m, and the share of foreign goods, s_{Fi}^m, are:

$$
\begin{aligned}
s_{ji}^m &= \alpha_{ji}^{\rho} \frac{p_{ji}^{1-\rho}}{p_{mi}^{1-\rho}} \\
s_{Fi}^m &= I_{Fi} \alpha_{Fi}^{\rho} \frac{p_F^{1-\rho}}{p_{mi}^{1-\rho}}.
\end{aligned} \tag{2.13}
$$

Analogously, we can write s_{ji} and s_{Fi} as the shares of j's goods and foreign goods, out of i's total variable costs: $s_{ji} = s_{ji}^m s_{mi}$ and $s_{Fi} = s_{Fi}^m s_{mi}$.

We assume monopolistic competition for firms in the heterogeneous goods sector when they sell to final demand. Firms charge a constant markup over marginal cost. We assume the same when firms export:

$$
p_{iH} = p_{iF} = \frac{\sigma}{\sigma - 1} c_i. \tag{2.14}
$$

We introduce oligopolistic competition in firm-to-firm trade in the following way. When selling to firm j, firm i sets price p_{ij} that maximizes variable profits by taking as given prices of j's other suppliers and j's unit cost and output, c_j and q_j. Solving the firm's profit maximization problem yields the following price:

$$
\begin{aligned}
p_{ij} &= \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} c_i \\
\varepsilon_{ij} &= \rho \left(1 - s_{ij}^m\right) + \eta s_{ij}^m.
\end{aligned} \tag{2.15}
$$

The markup firm i charges on firm j depends on the input share that i's goods have in j's intermediate goods, s_{ij}^m. If a supplier has an infinitesimally small share in the customer's intermediate goods bundle ($s_{ij}^m \to 0$), then all the competition the supplier engages in is with the other suppliers that share the same customer. Then the price converges to what we obtain when assuming monopolistic competition: a constant markup of $\frac{\rho}{\rho-1}$. As the supplier's input share on the customer increases, then not only does the supplier engage in competition with the other suppliers, but also with the labor input that the customer firm employs. Thus, the elasticity of demand that the supplier faces, ε_{ij}, is a weighted average of ρ and η with the weight on η being s_{ij}^m. When the supplier is the only firm supplying the customer ($s_{ij}^m \to 1$), the markup converges to $\frac{\eta}{\eta-1}$. The intuition of how pairwise markups depend on pairwise shares are identical to what is described in Atkeson and Burstein (2008). The difference is that here the relevant shares and markups are defined for each supplier-customer pair.

As mentioned above, we assume that the supplier takes as given the customer's unit cost and output. A plausible alternative would be to assume that the supplier firm internalizes the change in demand for the customer's good when deciding on its price. In that case, the supplier needs to know the output composition of the customer firm to infer the elasticity of demand that it is facing. As firms are not likely to observe the flow of goods that are far from itself in the production chain, we find our assumption to be reasonable.[21]

We assume Bertrand competition as our baseline case. One can alternatively assume firms engage in Cournot competition, where firms set quantity q_{ij} to maximize variable profits. In that case, the demand elasticity that firm i faces, ε_{ij}, becomes a weighted harmonic mean of the two CES parameters ρ and η: $\varepsilon_{ij} = \left(\frac{1}{\rho}\left(1 - s_{ij}^m\right) + \frac{1}{\eta} s_{ij}^m\right)^{-1}$. As we show in

21. This assumption that firms have incomplete information about firms that are far from itself in the production chain is similar to that considered by Antràs and de Gortari (2017). In Appendix B.4.2 we discuss in detail the optimal prices that firms charge their customers under alternative assumptions. When a firm internalizes the effect of its price on the demand for the customer's goods, the markup it charges not only depends on s_{ij}^m but also on quantities that the customer sells to other firms and the quantities that it sells to final demand. One can also assume that firms take as given a constant demand elasticity that firms assume their customers face. In this case, if one assumes that the value of this demand elasticity is η, the pricing equation collapses to that of equation (2.15).

Section 2.5.1, the estimates of the CES parameters are not affected much by this alternate specification.

Finally, let us describe firms' output. A firm sells its goods to households, abroad (if the firm is an exporter), and also to other domestic firms. Therefore we have

$$q_i = q_{iH} + q_{iF} + \sum_{j \in W_i} \alpha_{ij}^{\rho} \frac{p_{ij}^{-\rho}}{p_{mj}^{1-\rho}} s_{mj} c_j q_j, \tag{2.16}$$

where W_i is the set of i's customers.

Formation of the production network

Let us now describe how firms make their decisions on sourcing and participation in international trade. In our model, customer firms pay fixed costs to form links with suppliers. Firm i pays a random firm-specific fixed cost, f_{Di}, when supplying from a domestic supplier. Analogously, when the firm decides to import or export, it has to pay random firm-specific fixed costs of f_{Fi} and f_{iF}, respectively. All fixed costs are in terms of labor.

The firm maximizes its variable profits net of these fixed costs by choosing the set of domestic suppliers, Z_i, and importing/exporting statuses, I_{Fi} and I_{iF}. The variable profits of firm i come from sales to final demand, exports, and sales to other domestic firms. Taking as given others' sourcing strategies and participation decisions in international trade, the variable profit of i is a function of its own sourcing strategies and importing/exporting statuses:

$$
\begin{aligned}
\pi_i^{var}\left(Z_i, I_{Fi}, I_{iF}\right) =& \frac{1}{\sigma} \underbrace{\beta_{iH}^{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i\left(Z_i, I_{Fi}\right)^{1-\sigma} \frac{\alpha E}{P^{1-\sigma}}}_{\text{Sales to HH}} \\
&+ I_{iF}\frac{1}{\sigma}\underbrace{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i\left(Z_i, I_{Fi}\right)^{1-\sigma} D^*}_{\text{Exports}} \\
&+ \sum_j \frac{1}{\varepsilon_{ij}} \underbrace{\alpha_{ij}^{\rho} p_{ij}\left(Z_i, I_{Fi}\right)^{1-\rho} \frac{s_{mj} c_j q_j}{p_{mj}^{1-\rho}}}_{\text{Sales to } j}.
\end{aligned} \tag{2.17}
$$

The total profit of the firm becomes variable profits net of fixed costs.

$$
\pi_i\left(Z_i, I_{Fi}, I_{iF}\right) = \pi_i^{var}\left(Z_i, I_{Fi}, I_{iF}\right) - \sum_{j \in Z_i} w f_{Di} - I_{Fi} w f_{Fi} - I_{iF} w f_{iF}. \tag{2.18}
$$

Thus the firm's problem becomes

$$
\max_{Z_i, I_{Fi}, I_{iF}} \pi_i\left(Z_i, I_{Fi}, I_{iF}\right). \tag{2.19}
$$

It is important to note that we do not assume firm pair-specific fixed costs for domestic sourcing. Our assumption of fixed costs for domestic sourcing, f_{Di}, is i specific, which implies that given its importing and exporting decisions, a firm only has to evaluate N different sourcing sets for its domestic suppliers: no sourcing, only from the firm with the lowest unit cost, from two firms with the lowest unit costs, and so on. This substantially reduces the number of evaluations, from 2^{N-1} to N.[22] At the same time, the model predicts a strict pecking order in the sourcing strategies. The set of customers of a firm with the most outdegree includes the set of customers of a firm with the second most outdegree, and so on.

22. This assumption is similar to that of Blaum, Lelarge, and Peters (2016), where they assume firms' importing fixed costs vary across firms but common across sourcing countries.

2.4.3 Equilibrium

Here we close the model and describe the equilibrium. We assume that the profits firms make are distributed back to the households. We also assume that labor is mobile across homogeneous and heterogeneous goods sectors, and that both sectors are active both at home and abroad. We take the homogeneous good's price as the numeraire, and since markets in the homogeneous goods sector are perfectly competitive, wages can be taken as given in that sector. We also assume balanced trade. The household's budget constraint becomes

$$E = wL + \sum_{i \in \Omega} \pi_i, \tag{2.20}$$

where L denotes the mass of households. Trade balance and labor market clearing conditions are the following:

$$[\text{TB}]: 0 = \underbrace{\sum_{i \in \Omega} I_{iF} p_{iH}^{1-\sigma} D^*}_{\text{Hetero. exports}} - \underbrace{\sum_{i \in \Omega} I_{Fi} s_{Fi} c_i q_i}_{\text{Hetero. imports}} + \underbrace{wl^Y - (1-\alpha) E}_{\text{Net exports of homog.}} \tag{2.21}$$

$$[\text{LMC}]: wL = \sum_{i \in \Omega} s_{li} c_i q_i + \sum_{i \in \Omega} \left(\sum_{j \in Z_i} wf_{Di} + I_{Fi} wf_{Fi} + I_{iF} wf_{iF} \right) + wl^Y, \tag{2.22}$$

where l^Y is the domestic labor allocated to the production of homogeneous goods.[23]

Let us first characterize the equilibrium under a fixed network structure.

1 (Equilibrium under a fixed network)**.** Take as given foreign demand D^* and foreign price p_F. Assume that the total amount of labor associated with the fixed costs is less than the total supply of labor L. An equilibrium for the model where the production network and firms' participation in international trade are exogenous and fixed is a set of variables $\left\{w, P, E, q_i, l^Y\right\}$ that satisfy equations (2.5)-(2.7), (2.9)-(2.16), (2.18), and (2.20)-(2.22).

23. The assumption of both sectors being active in both countries are crucial, as without it the trade balance condition would not hold.

Under a fixed network and given wages, one can find prices by solving for the fixed-point problem of firm-level unit costs, c_i, from equations (2.9), (2.10), (2.13) and (2.15). After backing out all the pairwise shares and prices, including the aggregate price index P, one can then solve for the fixed point of aggregate expenditure, E, from equations (2.5), (2.7), (2.16), (2.18), and (2.20).

Let us now turn to the equilibrium with endogenous network formation. We cannot rule out the potential multiplicity of the equilibrium that arises from firms' problem described in equation (2.19). Suppose that a firm guesses it will face high unit cost and thus face less demand for its good. Then it would expect less variable profits, and as a result it would not source from many suppliers. Then the firm will indeed end up having high unit costs. Conversely, if a firm guesses it will have low unit cost, then the guess will be realized by the firm sourcing from many firms.

Given this potential multiplicity, we focus on a particular equilibrium following Atkeson and Burstein (2008) and Edmond, Midrigan, and Xu (2015). We focus on an equilibrium that results from firms sequentially making sourcing and international trade participation decisions. We order firms in terms of productivity, and let the most productive firm in the economy make domestic sourcing and importing/exporting decisions. Taking the first firm's decisions as given, the second most productive firm makes its own decisions, and so on.[24] We essentially solve a large fixed-point problem of the production network, where all firms choose the optimal domestic sourcing and international trade participation decisions, taking as given the decisions of other firms. The resulting equilibrium is a pairwise stable equilibrium, where no firm has an incentive to drop its existing supplier or an incentive to add new suppliers.[25]

24. We find that altering this ordering has little impact on the aggregate variables. Similar to what is discussed in Edmond, Midrigan, and Xu (2015), the differences in the networks across the orderings come from differences in decisions on sourcing from marginal suppliers, which have little impact in the aggregate variables.

25. We describe the computational algorithm for the network formation in Appendix B.3. Existence of such equilibrium is not theoretically guaranteed. However, we find that the network generally converges to a fixed point in the numerical analysis.

Note that in each evaluation of the network structure, we solve the equilibrium described by Definition 1. Firms set prices that maximize variable profits, taking as given the network structure. Consistent with the concept of "Nash-in-Nash" equilibrium (Collard-Wexler, Gowrisankaran, and Lee, 2016), we do not allow firms to consider alternations in linkages when setting prices.

Lastly, we highlight some differences in the approach we take for endogenous network formation compared to Tintelnot, Kikkawa, Mogstad, and Dhyne (2017). In their framework, firms are sorted so that they can only supply from the firms previous in the ordering. This results in an acyclic network, where there exists at least one ordering of firms so that all directed edges face one direction. Additionally, they assume that firms do not charge markups when selling to other domestic firms. This makes the network formation problem more tractable, as firms' profits are not affected by the sourcing decisions of the firms downstream in the ordering.

Our paper puts emphasis on imperfect competition in firm-to-firm trade, and one of our main focuses is on pairwise variable markups in firm-to-firm trade. Thus we employ another approach that focuses on an equilibrium arising from sequential decision making. The resulting networks we obtain are not confined to ones that are acyclic. Since the sourcing decision of a firm is affected by those of other firms (subsequent in the ordering) through the changes in its profit, the fixed-point problem of the network we solve remains computationally demanding.

2.4.4 Network irrelevance under the benchmark case

Let us now consider the network irrelevance results under special cases in the model. Consider the change in price index and welfare, given an exogenous change in foreign price. The following proposition and lemma demonstrate that under certain assumptions, firm-level variables are sufficient in computing aggregate responses. These results resemble that of Hulten (1978) and of Baqaee and Farhi (2017), but focus on global changes in a setup with international trade. Following Dekle, Eaton, and Kortum (2007), let the change in variable x from the pre-shock equilibrium x to the post-shock equilibrium $x^{'}$ be $\hat{x} = x^{'}/x$.

1. *Only composite final consumption goods are exported.*

2. *Preferences and technologies have common CES parameters, $\sigma = \eta = \rho$.*

3. *Goods are competitively priced, $p_i = c_i \; \forall i \in \Omega$.*

4. *The domestic firm-to-firm network is exogenous and fixed.*

1 (Network irrelevance with a common CES parameter)**.** *Suppose that Assumptions 1-4 hold. Denote $\tilde{\sigma}$ as the common CES parameter from Assumption 2. Then the change in aggregate price index in the heterogeneous goods sector, $\hat{P}$, can be expressed as*

$$\hat{P}^{1-\tilde{\sigma}} = \sum_{i \in \Omega} \frac{p_i q_i}{\alpha E + Exports} \left(s_{li} + s_{Fi} \hat{p}_F^{1-\tilde{\sigma}} \right), \tag{2.23}$$

and the change in aggregate welfare, $\hat{U}$, can be expressed as:

$$\hat{U} = \left(\sum_{i \in \Omega} \frac{p_i q_i}{\alpha E + Exports} \left(s_{li} + s_{Fi} \hat{p}_F^{1-\tilde{\sigma}} \right) \right)^{\frac{-\alpha}{1-\tilde{\sigma}}}. \tag{2.24}$$

Proof. See Appendix B.4.3. □

This result shows that under these assumptions, one does not need any information on how firms are linked with other firms in evaluating aggregate changes. Firms' direct exposure to the shock are captured by firms' foreign input shares, s_{Fi}. The importance of each firm in the production network is captured by the Domar (1961) weight, $\frac{p_i q_i}{\alpha E + \text{Exports}}$. These two firm-level variables are the sufficient statistics when one is interested in how the aggregate price index and welfare respond to a foreign price change.

However, in order to compute the changes in price index and welfare, one needs to know the value of $\tilde{\sigma}$ in addition to the firm-level observables. In the following lemma, we impose a stronger assumption in preference and technologies and obtain a network irrelevance result where aggregate changes can be computed solely by firm-level observables.

5. *Assume Cobb-Douglas functions in preferences and technologies, $\sigma = \eta = \rho = 1$.*

1 (Network irrelevance under the benchmark case)**.** *Suppose that Assumptions 1, 3, 4, and 5 hold. Then the change in aggregate price index in the heterogeneous goods sector, $\hat{P}$, can be expressed as*

$$\ln \hat{P} = \sum_{i \in \Omega} \frac{p_i q_i}{\alpha E + \mathit{Exports}} s_{Fi} \ln \hat{p}_F, \tag{2.25}$$

and the change in aggregate welfare, $\hat{U}$, can be expressed as:

$$\ln \hat{U} = -\alpha \sum_{i \in \Omega} \frac{p_i q_i}{\alpha E + \mathit{Exports}} s_{Fi} \ln \hat{p}_F. \tag{2.26}$$

Under the Cobb-Douglas assumption in both preference and technology, one obtains a log-linear expression where aggregate movements are essentially the weighted sum of shocks that hit each firm. As the necessary variables are all observables in standard datasets, we use this case in Lemma 1 as the benchmark case in the counterfactual analysis and characterize the produced differences in the predictions between the benchmark case and the full model. We will also discuss predictions from the case in Proposition 1 under various values of $\tilde{\sigma}$.

Let us now discuss the assumptions. First, it is worth noting that the four assumptions in both Proposition 1 and Lemma 1 work as sufficient conditions in obtaining the network irrelevance result. In both Proposition 1 and Lemma 1, instead of having firms export their differentiated goods separately abroad, we assume that goods from all firms are bundled up to a composite final good, and that they are either consumed by the domestic households or exported abroad (Assumption 1).[26] By treating the exports of firms in the same way as their sales to final demand, we can use aggregate consumption and aggregate exports as the denominator of the Domar weights.

26. One can alternatively interpret this assumption as all firms in the heterogeneous goods sector either export or do not export at all.

At first sight Assumptions 3 and 4 may not seem consistent with Assumption 2 in Proposition 1, where one usually assumes monopolistic competition in a CES demand framework. One can interpret this combination of assumptions in terms of the following. Consider an economy where firms are endowed with production technologies that also specify which other firms and countries to buy from, thus fixing the production network. And when there is another identical firm ready to enter the market and take over production, firms charge a competitive price.

In Proposition 1, one might conjecture that relaxing Assumption 3 and having constant and common markups in firm-to-firm trade would still produce the network irrelevance result. It turns out that it is not the case. As we show in detail in Appendix B.4.4, we obtain equation (2.23) because firms' Domar weights, which capture the importance of firms as suppliers of goods, coincides with a measure of firms' importance as consumers of goods. This is only possible when Assumption 3 holds.

2.5 Estimation

There are three sets of parameters that we estimate separately. First is the set of CES parameters in the preference and production functions: $\{\eta, \rho, \sigma\}$. The second set governs the distribution of productivities. The third is the set of parameters that determine fixed costs of forming domestic links and fixed costs of participating in international trade. We describe the estimation procedures for the three sets of parameters in the following subsections.

2.5.1 Estimating the CES parameters

We estimate the CES parameters $\{\eta, \rho, \sigma\}$ by exploiting the firm-to-firm shares that we observe in the data. Recall that in equation (2.15) pairwise markups $\mu_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij}-1}$ are functions of parameters $\{\eta, \rho\}$ and observables s_{ij}^m. We have also assumed that markups firms charge on goods to domestic households and on exported goods, μ_{iH}, are $\frac{\sigma}{\sigma-1}$.

In our static model, a firm's input cost equals its sum of sales, each deflated by the destination-wise markups:

$$c_i q_i = \sum_j \frac{V_{ij}}{\mu_{ij}} + \frac{V_{iH}}{\mu_{iH}} + \frac{V_{iF}}{\mu_{iH}}. \tag{2.27}$$

We observe the input costs $c_i q_i$ and firms' destination-wise sales: sales to firm j, V_{ij}, sales to households, V_{iH}, and exports, V_{iF}.[27] Using these observables, we estimate the CES parameters $\{\sigma, \rho, \eta\}$ by minimizing the Euclidian distance between both sides of equation (2.27):

$$\min_{\eta,\rho,\sigma} \sum_i \left[c_i q_i - \left(\sum_j \frac{V_{ij}}{\mu_{ij}} + \frac{V_{iH}}{\mu_{iH}} + \frac{V_{iF}}{\mu_{iH}} \right) \right]^2 . \tag{2.28}$$

Since firms' markups to final demand, μ_{iH}, are constants $\frac{\sigma}{\sigma-1}$, the variations in the ratio of firms' sales to final demand $(V_{iH} + V_{iF})$ over firms' total inputs $(c_i q_i)$ pins down the value of σ. Firm-to-firm markups, μ_{ij}, are functions of pair specific shares, s_{ij}^m, and two parameters, ρ and η. Thus the ratio of firm-to-firm sales $\left(V_{ij}\right)$ over suppliers' input costs $(c_i q_i)$ and the input shares $\left(s_{ij}^m\right)$ jointly determine the value of the two parameters.[28]

The underlying assumption of this estimation procedure is that there are measurement errors in firms' labor costs, which is a component of $c_i q_i$. We assume that these errors are not correlated with the RHS variables of equation (2.27). The parameters are identified under this assumption, since firms' labor costs only appear on the LHS as one component of supplier i's total inputs and not in the RHS variables. Table 2.4 reports the estimation results.[29]

27. We compute variable input costs $c_i q_i$ by summing up firms' labor costs, purchases from other domestic firms, and imports. We assume that labor costs in our data are variable costs, as distinguishing fixed costs from variable costs is impossible.

28. Edmond, Midrigan, and Xu (2015) use a similar procedure with sectoral market shares to infer one of the CES parameters in models with variable markups.

29. To illustrate the fit of the model under the estimated parameters, in Appendix B.5.1 we provide the distribution of errors at the firm level, i.e., the difference between the LHS and RHS of equation (2.27).

Table 2.4: Estimated values for $\{\eta, \rho, \sigma\}$

	η	ρ	$\frac{\sigma}{\sigma-1}$
Estimate	1.27	2.78	1.25
s.e.	1.07	0.31	0.05
	η	ρ	σ
	(Labor and goods)	(Firm's goods in production)	(Firms' goods in consumption)
Implied value	1.27	2.78	4.99

Notes: Standard errors are based on 100 bootstrap samples drawn with replacement.

We find that in the production function, the substitution parameter across labor and goods, is 1.27. Within intermediate goods, the substitution parameter across goods from different firms and imported goods is 2.78. In the preference function, we find that the substitution parameter across goods is 4.99. The estimated values fall in plausible ranges. With a sectoral layer in the production function, the survey of Anderson and van Wincoop (2004) finds that the elasticity of substitution across goods in the production function within sectors to be in the range of 5 to 10. As we do not have a sectoral layer, it is plausible that our estimate of ρ is smaller.[30]

Robustness

In our model, firms engage in Bertrand competition in firm-to-firm trade. In an alternate specification we assume that firms engage in Cournot competition, which leads to a different

30. Our approach of estimating CES parameters is different from that of other papers that estimate substitution parameters at higher frequencies. For example Boehm, Pandalai-Nayar, and Flaaen (2016), Barrot and Sauvagnat (2016), and Atalay (2017) find much lower estimates in the production function parameters. In contrast, we estimate CES parameters using implied markup levels.

formula for pairwise markups μ_{ij}:

$$p_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} c_i$$
$$\varepsilon_{ij} = \left(\frac{1}{\rho} \left(1 - s_{ij}^m \right) + \frac{1}{\eta} s_{ij}^m \right)^{-1}.$$

In Appendix B.5.2 we estimate the three parameters under this setup, and we find similar estimates.[31]

Our estimates for the three parameters are also not affected when one assumes oligopolistic competition in the final goods market. This is because for most firms, shares in the final goods consumption are infinitesimal, which validates our assumption of monopolistic competition.

Finally, it is worth pointing out that we do not have capital goods in our model. We sum firms' total labor costs, purchases from other domestic firms, and imported goods in our measurement of firms' total inputs, $c_i q_i$. Missing capital inputs will lower our measurement of $c_i q_i$. If the degree of capital intensity is correlated with the firm's sales, then it violates our assumption of uncorrelated errors. To accommodate this potential issue, we take into account firms' capital inputs in two alternative ways. First, we uniformly scale up labor costs of firms by assuming a common labor-to-capital share. Second, we compute firm-level capital costs from the annual accounts data. As the results in Appendix B.5.4 reveal, we find similar estimates in both cases.

31. In another alternative setup, we estimate ρ and σ by assuming constant markups in firm-to-firm trade, where firms charge $p_{ij} = \frac{\rho}{\rho-1} c_i$. Here we also obtain similar estimates, where the estimated value of ρ is slightly smaller than what we estimate here. The results are reported in Appendix B.5.3.

2.5.2 Estimating the productivity distribution

We then recover the productivity distribution from the identity equation implied by the model:

$$\ln \phi_i = \frac{1}{\sigma - 1} \ln V_{iH} + \frac{1}{\eta - 1} \ln s_{li} + \ln \left(\frac{\sigma}{\sigma - 1} \omega_l^{\frac{-\eta}{\eta-1}} P^{-1} \alpha^{\frac{-1}{\sigma-1}} E^{\frac{-1}{\sigma-1}} \right). \tag{2.29}$$

Equation (2.29) implies that the log productivity of a firm can be recovered up to a scale, from firms' sales to households, V_{iH}, and from firms' labor input shares, s_{li}. We assume that the productivity distribution is log-normal, and we estimate the dispersion parameter to be 2.44.

Note that the firm's sales to households and firm's labor share both determine the firm's productivity. Since we assume constant markups in firms' sales to final demand, the variation in firms' sales to households reflects the variation in firms' unit costs. However, the variation in unit costs is not driven by the variation in firms' productivity alone.

A firm may have low unit cost simply because its core productivity is high, but it might also be buying cheap goods from other firms. Therefore, we need to control for the effects that come from firms' sourcing strategies. Notice from equation (2.11) that the variation in firms' labor share comes only from firms' sourcing strategies, as we assume that wage is common across firms. In order to pin down the variation in firms' core productivity, equation (2.29) controls for firms' labor share in addition to firms' sales to final demand.

2.5.3 Estimating the fixed cost distributions

The remaining parameters in need of estimation govern the fixed cost distributions. We assume that firms' fixed costs for sourcing from a domestic supplier, f_{Di}, are drawn from a common distribution, $F_D(\cdot)$. Firms' fixed costs for importing and exporting, f_{Fi} and f_{iF}, are drawn from the common distributions $F_{IM}(\cdot)$ and $F_{EX}(\cdot)$. We additionally assume that the three distributions are log-normal, independent from each other, and that they have a common dispersion parameter Φ^{disp}. We estimate the three scale parameters Φ_D^{scale}, Φ_{IM}^{scale}, and Φ_{EX}^{scale}, along with the common dispersion parameter Φ^{disp} via simulated methods of moments.

When running model simulations under endogenous networks, we additionally assume that the saliency terms in preference and production functions, $\left\{\beta_{iH}, \alpha_{ij}, \alpha_{Fi}\right\}$, to be equal to 1. We also calibrate the rest of the parameters. We set the production weights on labor inputs and goods inputs, ω_l and ω_m, to be 0.3 and 0.7, respectively, to match the average labor input share of 0.34 in our sample. The Cobb-Douglas share in the preference function on the heterogeneous goods sector α is set to 0.55 to match the aggregate share of the private and non-financial sectors in Belgium. We set the foreign demand D^* to be 10^{14} so that it matches the average export share for exporting firms' output of 0.2. Analogously, we set foreign price p_F to be 5 so that it matches the average imported goods' share for importing firms' inputs of 0.31. Finally, we set L to be 10^{10}.

For determining the three scale parameters of domestic sourcing, importing and exporting, we use the aggregate shares of firms that are sourcing from at least one domestic firm, m_{dom}, shares of firms that are importing, m_{imp}, and shares of firms that are exporting, m_{exp}, as moments. We use the correlation between domestic indegrees and outdegrees of firms, m_{cor}, to infer the dispersion parameter in the fixed costs. The intuition is as follows. Suppose that the dispersion parameter is zero. Then all firms draw a common number for the domestic fixed costs. In that case, the most productive firm will be the firm that sources from the most firms, and it will also be the firm that sources to the most firms. Thus, the correlation between indegrees and outdegrees becomes one. As the dispersion parameter increases from zero, it will no longer be the case that the most productive firm is the firm that sources from the most firms, and the correlation decreases. In all, we have four moments to identify four parameters.

Let us denote δ as the set of four moments, $\left\{\Phi_D^{scale}, \Phi_{IM}^{scale}, \Phi_{EX}^{scale}, \Phi^{disp}\right\}$, and the vector of four moments generated by the model as $\hat{m}\left(\delta\right)$. Given δ, we can calculate the difference between the moments generated by the model and the moments from the data:

$$\hat{y}(\delta) = m - \hat{m}(\delta) = \begin{bmatrix} m_{dom} - \hat{m}_{dom} \\ m_{imp} - \hat{m}_{imp} \\ m_{exp} - \hat{m}_{exp} \\ m_{cor} - \hat{m}_{cor} \end{bmatrix}.$$

We assume that the following moment condition holds at the true parameter values δ_0:

$$E\left[\hat{y}(\delta_0)\right] = 0.$$

Therefore, we estimate δ by solving the following minimization problem

$$\min_{\delta} \left[\hat{y}(\delta)\right]' \boldsymbol{W} \left[\hat{y}(\delta)\right],$$

where $\boldsymbol{W}$ is a weighting matrix.[32] We report the estimated values in Table 2.5, as well as the standard errors from a bootstrap method. In the bootstrap method, we draw a different set of firms with different productivities each time.

Table 2.5: Estimated values for the fixed cost parameters

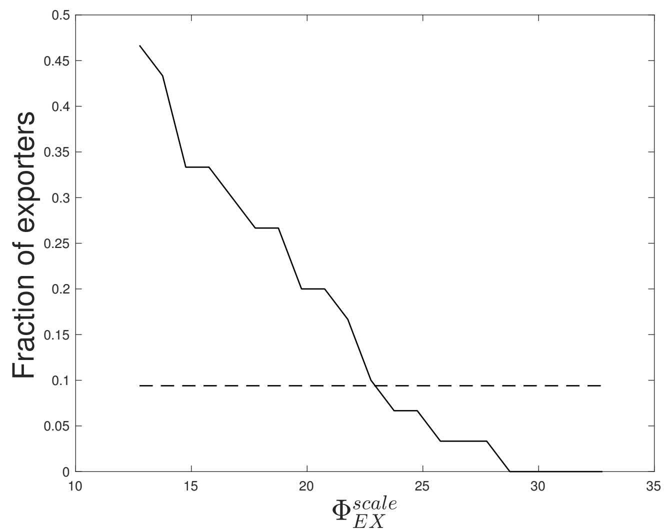
	Φ_D^{scale}	Φ_{IM}^{scale}	Φ_{EX}^{scale}	Φ^{disp}
Estimate	2.37	21.10	22.76	6.10
s.e.	0.38	0.28	0.33	0.56

We provide local identification for the four parameters, which we display for Φ_{EX}^{scale} in Figure 2.5 as an example. Fixing other parameters, one can see that as the scale parameter for exporting fixed costs increases, less firms become exporters. This monotonic relationship determines the value of the parameter. We display this local identification for the other

32. In practice, we weight the moments equally.

parameters in Appendix B.5.5.

Figure 2.5: Local identification of Φ_{EX}^{scale}



Notes: On the x-axis we plot Φ_{EX}^{scale}, the scale parameter for the distribution of fixed costs for exporting, which we vary while fixing all other parameters to their estimated values. On the y-axis we plot the share of exporters. The horizontal line indicates the observed value in the data. The monotonic relationship determines the value of Φ_{EX}^{scale}.

Finally, we note that due to computational limitations, we can only simulate the economy with a limited number of firms. Our problem of endogenous network formation is arguably complex. One has to solve a large fixed-point problem of a cyclic network where, within each network, one has to solve another fixed-point problem of pairwise prices and shares. In our estimation, we simulate the economy with 30 firms.

One may argue that 30 firms may not be able to entirely capture the effects of firm-to-firm trade on the aggregate movements. In Appendix B.7, we build a variant of our model adjusted to represent one single manufacturing sector where we focus on firm-to-firm trade within the sector. In this sector, the top 30 firms account for almost all the sales and firm-to-firm trade. We argue that the results from the counterfactual exercise under this partial equilibrium model are qualitatively the same as what we will show in Section 2.6.

2.5.4 Model fit

Table 2.6 reports the model fit for targeted and non-targeted moments under the estimated parameters. One can see that the model does well in fitting the targeted statistics. As for the non-targeted moments, the model succeeds in predicting positive correlations between firms' sales and indegrees/outdegrees. It also succeeds in generating a weak negative assortativity in the network: a negative correlation between suppliers' sales and customers' sales. Finally, the last three rows report the model fit in terms of matching the distributions of pairwise input shares, which dictate the level of markups. The model seems to suitably match the magnitudes of the median and the 25th percentile. The model fails to match the right tail of the input share distribution. However, the pairwise markups derived in equation (2.15) are increasing and convex in the pairwise input shares under the estimated parameters of ρ and η. Therefore, there is little difference in the level of markups in the region where input shares are close to zero.

Table 2.6: Targeted and non-targeted moments

Panel A: Targeted moments		
	Data	Model
Fraction of firms sourcing from domestic firms	0.98	0.97
Fraction of importers	0.15	0.17
Fraction of exporters	0.09	0.10
Corr(Indeg, Outdeg)	0.65	0.65

Panel B: Non-targeted moments		
	Data	Model
Corr(Sales, Indeg)	0.48	0.24
Corr(Sales, Outdeg)	0.51	0.33
Corr(Sales_i, Sales_j)	-0.02	-0.06
25th percentile s_{ij}^m	3.1×10^{-4}	3.0×10^{-4}
Median s_{ij}^m	1.8×10^{-3}	3.4×10^{-3}
75th percentile s_{ij}^m	8.2×10^{-3}	4.5×10^{-2}

2.6 Counterfactual analysis

We conduct counterfactual analysis with the model parameters that we have recovered in the previous section. In particular, we focus on an exogenous reduction in a foreign good's price, p_F, and analyze how it affects aggregate variables, such as price index and welfare. We start with the results from the benchmark case where the network structure is irrelevant. We then add market frictions one by one, and finally present the results from the full model.

In the first subsection, we do counterfactual analysis where the network is fixed. We analyze the predictions from the benchmark case then add monopolistic competition in firm-to-firm trade, and then finally add oligopolistic competition. In all three cases, we use the firm-to-firm network that we observe in the data, along with the estimated CES parameters from Section 2.5.1. In the next subsection, we add endogenous networks. Here we switch to the estimated full model and conduct model simulations. Finally, in the last subsection, we consider the model with a common CES parameter that features a network irrelevance result (described in Proposition 1). We do so by evaluating aggregate changes under different values of the common CES parameter.

2.6.1 Under fixed network

We characterize the differences in aggregate predictions made by models with a fixed network. We focus on the movements in the aggregate price index. First, we consider the predictions from the benchmark case. We have shown in equation (2.25) of Lemma 1 that given the change in the foreign good's price, $\hat{p}_F$, the change in the aggregate price index, $\hat{P}$, can be solely determined from firm-level variables. The aggregate price index falls more in response to a reduction in the foreign good's price if firms that have larger sales have higher exposure to foreign goods.

In the second case, we add constant markups in firm-to-firm trade and compare the movements in the aggregate price index with those from the benchmark case. In this second case we relax Assumptions 1, 3, and 5. Instead we assume CES structures in preference and technology, using the estimated parameters for σ, ρ, and η. In this case, the fall in the aggregate price index becomes even larger. Firms and households now face CES substitution parameters that are larger than one, and are able to shift their expenditures more towards goods that became relatively cheaper. This contributes to larger movements in the aggregate price.

For the third case, we consider firms charging variable markups in firm-to-firm trade. When one adds variable markups, there are two counteracting forces that push the aggregate price index in the opposite directions: the attenuation effect and the pro-competitive effect.

Consider a firm facing a price reduction in one of its inputs. In the constant markup case, the firm's output prices go down proportionally to the input's share in the firm's total inputs. However, in the variable markup case, the firm will increase its markups as it enjoys larger shares in its customers' inputs. This attenuation effect leads to incomplete price pass-through and reduces the aggregate movement in the price index. On the other hand, the other firms that sell goods to the same customer may also receive positive cost shocks. In that case, the firm reduces its markup in the face of increased competition. This pro-competitive effect leads to larger movements in the aggregate price index. Overall, the net effect on the movements in the aggregate price index can either be positive or negative.

In both the constant markup case and the variable markup case, following a technique developed by Dekle, Eaton, and Kortum (2007), we can compute the change in the aggregate price index using the observed input shares and the estimated CES parameters. In Appendix B.4.5 we present the system of equations for the changes in firms' unit costs and pairwise markups.

We display the results in Figure 2.6. The first line in the figure displays the change in price index computed using equation (2.25) in the benchmark case.[33] As the foreign good's price falls (as $\hat{p}_F$ moves from right to left), the aggregate price index falls. In the benchmark case, 40% reduction in the foreign good's price ($\hat{p}_F = 0.6$) leads to a drop in the aggregate price of about 25%.[34] In the second case, we indeed see a larger reduction in the aggregate price index. With higher substitutability across goods, the price index now falls by around 30% when $\hat{p}_F$ is 0.6.

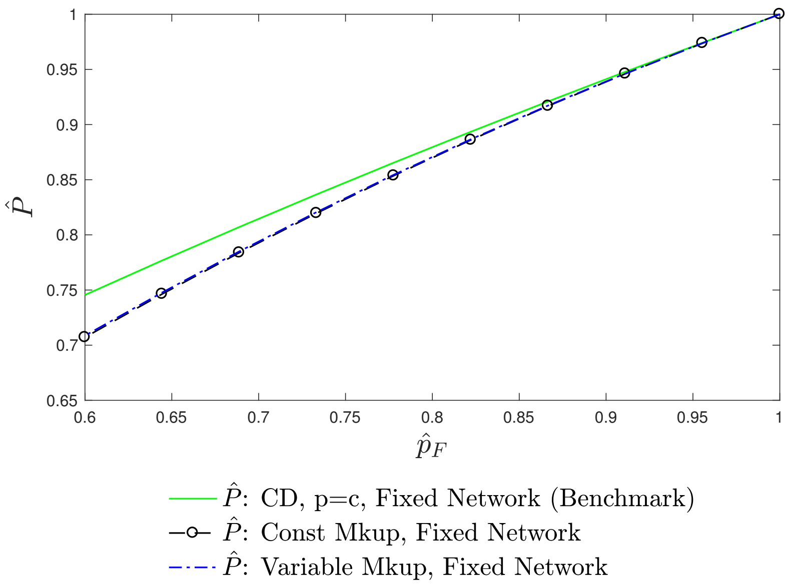
Adding pairwise variable markups does not seem to make significant changes in the movements of aggregate prices, as one cannot visually distinguish the third line in Figure 2.6 from the second line. As aforementioned, the two counteracting forces cancel each other

33. This prediction exhibits nonlinearity as the $\hat{P}$ is computed for global changes. In Appendix B.6.1 we also provide predictions for the first order approximated changes, as considered in Hulten (1978).

34. Under the estimated value of $\rho = 2.78$ and fixed demand, $\hat{p}_F = 0.6$ corresponds to an increase in import value of around 150%. Chinese imports to Belgium have also increased by around the same amount in the 2000s. Accordingly, we consider foreign price shocks between $\hat{p}_F = 1$ and $\hat{p}_F = 0.6$, treating $\hat{p}_F = 0.6$ as the largest possible shock to the economy.

out. But in the aggregate, the third line in Figure 2.6 is slightly above the second, indicating that the attenuation effect weakly dominates the pro-competitive effect.

Figure 2.6: $\hat{P}$ under the fixed network



Characterizing the attenuation and pro-competitive effects

Even though the net effects turn out to be small, it is worth exploring the underlying mechanisms. To characterize the two effects of adding variable markups, we work with the system of cost changes that are approximated at the first order. In the case where firms charge constant markups, the system of equations for the first order approximated changes in prices given $\frac{\mathrm{d}p_F}{p_F}$ and parameters are as follows:

$$\frac{\mathrm{d}c_i}{c_i} = \sum_{j \in Z_i} s_{ji} \frac{\mathrm{d}c_j}{c_j} + s_{Fi} \frac{\mathrm{d}p_F}{p_F}. \tag{2.30}$$

And in the case where firms charge pairwise variable markups in firm-to-firm trade where the network is fixed, the system of first order changes in prices is expressed in equations (2.31), (2.32), and (2.34). The changes in firms' unit costs are now affected by the changes in the unit costs of their suppliers and also the changes in the markups they charge:

$$\frac{\mathrm{d}c_i}{c_i} = \sum_{j \in Z_i} s_{ji} \left(\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}} + \frac{\mathrm{d}c_j}{c_j} \right) + s_{Fi} \frac{\mathrm{d}p_F}{p_F}, \tag{2.31}$$

where

$$\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}} = \underbrace{-\Gamma_{ji} \frac{\mathrm{d}c_j}{c_j}}_{\text{attenuation effect}} + \underbrace{\Gamma_{ji} \frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}}_{\text{pro-competitive effect}} . \tag{2.32}$$

The term Γ_{ji} represents the elasticity of the markup μ_{ji} with respect to the supplier's cost c_j:

$$\begin{aligned} \Gamma_{ji} &= -\frac{\partial \mu_{ji}}{\partial c_j} \frac{c_j}{\mu_{ji}} \\ &= \frac{\Upsilon_{ji} \left(1 - s_{ji}^m\right)}{1 - \Upsilon_{ji} s_{ji}^m}, \end{aligned} \tag{2.33}$$

where

$$\Upsilon_{ji} = \frac{\left(\rho - \varepsilon_{ji}\right)(\rho - 1)}{\left(\varepsilon_{ji} - 1\right)\varepsilon_{ji} + \left(\rho - \varepsilon_{ji}\right)(\rho - 1)}. \tag{2.34}$$

The term $\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}$ represents the average price change from suppliers other than j:

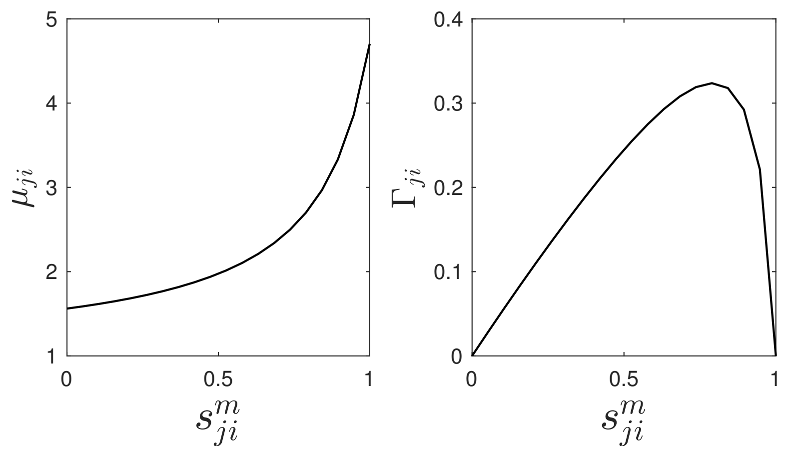
$$\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}} = \frac{\sum_{k \in Z_i, k \neq j} s_{ki}^m \left(\frac{\mathrm{d}\mu_{ki}}{\mu_{ki}} + \frac{\mathrm{d}c_k}{c_k} \right) + s_{Fi}^m \frac{\mathrm{d}p_F}{p_F}}{1 - s_{ji}^m}. \tag{2.35}$$

The term $\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}$ in equation (2.31) captures the additional effect that firm j has on firm i's unit cost by adding variable markups. $\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}$ can be decomposed into two. The first term in equation (2.32) captures the attenuation effect, since reduction in j's cost leads to an increase in markup μ_{ji}. On the other hand, the second term in equation (2.32) says that the markup μ_{ji} is also affected by i's other suppliers besides j. If the prices of other suppliers and imported goods decline on average, then μ_{ji} decreases.

The magnitudes of both effects are governed by two components. First, the term Γ_{ji}, which is the elasticity of markup with respect to the supplier's cost, governs the maximum possible magnitudes of the two. As one can see in Figure 2.7, markup μ_{ji} is increasing and convex with respect to the input share s^m_{ji}. And when the input share converges to either 0 or 1, the markup converges to a constant, making the elasticity Γ_{ji} converge to 0. The elasticity displays a hump shape with respect to s^m_{ji} and is largest when the share is around 0.8. This allows both attenuation and pro-competitive effects to be large. However, the magnitudes of the two effects are also affected by how much shock the supplier or the other suppliers received. For example, even if the input share for a specific pair is in the region where the elasticity Γ_{ji} is large, if the supplier's cost did not decrease at all, there will be no attenuation effect. The degrees of cost reductions by the suppliers govern the degree of attenuation effects within the same values of input shares. Likewise, the average degrees of price changes by other suppliers determine the degree of pro-competitive effects within the same value of input shares.[35]

35. In Appendix B.6.2 we plot these pairwise attenuation and pro-competitive effects with respect to input shares. We also show that the degree to which suppliers and the other suppliers received the shock are correlated with a measure capturing the exposure of firms to foreign goods, both directly and indirectly through their suppliers.

Figure 2.7: Markup μ_{ji} and elasticity Γ_{ji} with respect to input share s^m_{ji}



Notes: The figure plots the pairwise markup, μ_{ji}, and elasticity of μ_{ji} with respect to c_j, Γ_{ji}, in equation (2.32), as a function of s^m_{ji}. We use the parameter values of $\rho = 2.78$ and $\eta = 1.27$.

Characterizing the net effects

We now characterize the aggregate magnitudes of the two effects in a similar fashion, by computing the first order approximations of the changes in aggregate prices. We decompose the aggregate price change into three, as shown in equation (2.36):

$$\begin{aligned} \frac{\mathrm{d}P}{P} =& \sum_i s_{iH} \left(\sum_{j \in Z_i} s_{ji} \frac{\mathrm{d}c_j}{c_j} + s_{Fi} \frac{\mathrm{d}p_F}{p_F} \right) \\ &+ \sum_i s_{iH} s_{mi} \sum_{j \in Z_i} s^m_{ji} \frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}. \end{aligned} \tag{2.36}$$

The first line represents the effects that are present in the constant markup case: firms' cost changes come from their direct exposures to foreign price change, and their exposure to each supplier's cost change. The second line represents captures the aggregate effects of the net changes in individual markups. The term $\sum_{j \in Z_i} s^m_{ji} \frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}$ captures the average movements of markups that firm i faces, each weighted by the input share for each supplier.[36]

To help understand which firm faces higher markups from its suppliers and which firm faces reductions in markups, in Figure 2.8 we plot firms' average change in markups $\sum_{j\in Z_i} s^m_{ji}\left(\hat{\mu}_{ji}-1\right)$ against a measure that captures firms' *indirect* exposure to foreign goods, $s^{Indirect}_{Fi}$. We first construct the measure of "total foreign input share", s^{Total}_{Fi}, that captures firm i's exposure to foreign inputs by summing up its direct exposure, and its suppliers' exposure, and so on:[37]

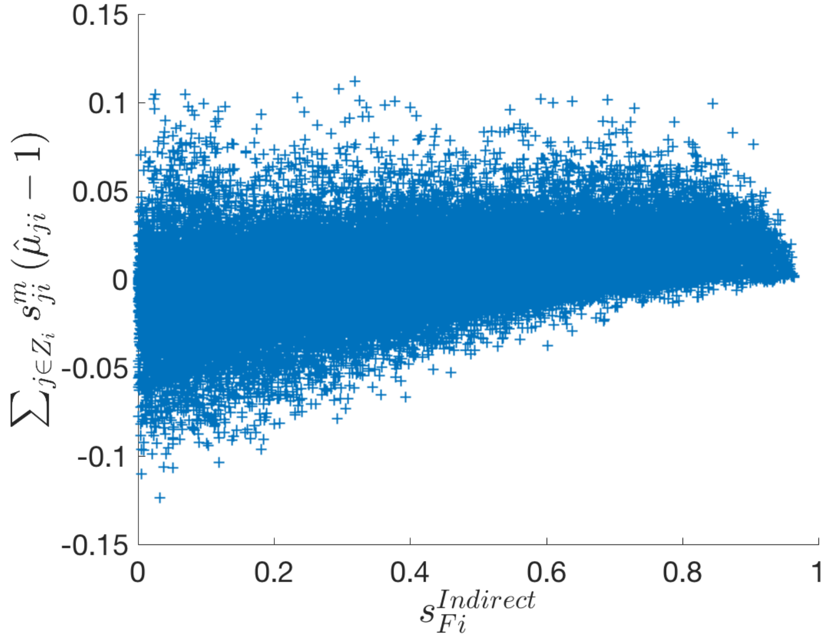
$$s^{Total}_{Fi} = s_{Fi} + \sum_{k\in Z_i} s_{ki} s^{Total}_{Fk}.$$

We then subtract firms' direct exposure to foreign inputs: $s^{Indirect}_{Fi} = s^{Total}_{Fi} - s_{Fi}$. One can see that there is a positive correlation between the two measures. Consider a firm with high value of $s^{Indirect}_{Fi}$, which supplier with high input share is highly exposed to foreign imports. In this case the attenuation effect dominates the pro-competitive effect, as the supplier with high input share raises its markup charged to the firm.

36. The sum of the change in markups, $\sum_i s_{iH} s_{mi} \sum_{j\in Z_i} s^m_{ji} \frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}$, can be decomposed into two components: the sum of the attenuation effects, $-\sum_i s_{iH} s_{mi} \sum_{j\in Z_i} s^m_{ji}\Gamma_{ji}\frac{\mathrm{d}c_j}{c_j}$, and the sum of the pro-competitive effects, $\sum_i s_{iH} s_{mi} \sum_{j\in Z_i} s^m_{ji}\Gamma_{ji}\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}$. In Appendix B.6.3 we plot the three components of the change in the aggregate price index. Though the net aggregate effect is small, the aggregate attenuation effect and pro-competitive effect are non-negligible in magnitude. In addition, in Appendix B.6.4 we characterize the magnitudes of average attenuation and pro-competitive effects at the firm-level using the HHI of input shares.

37. s^{Total}_{Fi} is defined by Tintelnot, Kikkawa, Mogstad, and Dhyne (2017), and there is a one-to-one mapping between $\frac{\mathrm{d}c_j}{c_j}$ and s^{Total}_{Fj} under the benchmark case of our model.

Figure 2.8: Average change in markups and $s_{Fi}^{Indirect}$



Notes: The figure plots $\sum_{j \in Z_i} s_{ji}^m (\hat{\mu}_{ji} - 1)$ upon $\hat{p}_F = 0.6$, against firms' indirect exposure to foreign goods, $s_{Fi}^{Indirect}$.

The nature of the shock also matters in explaining the correlation between $\sum_{j \in Z_i} s_{ji}^m (\hat{\mu}_{ji} - 1)$ and $s_{Fi}^{Indirect}$. The shock we are focusing on here affects all the importers (accounting for around 15% of all firms) directly, and many other firms at the same time. The median value of the total foreign input share, s_{Fi}^{Total}, is around 41%. The shock being large scale, it is plausible to imagine that many firms have multiple suppliers which experience roughly the same degree of cost reductions. In these cases, both attenuation and pro-competitive effects tend to cancel each other out.

To illustrate this point, in Appendix B.6.5 we study an alternative shock where we hit only one importer with the foreign price reduction. We demonstrate that the positive correlation between the average movements in markups and firms' indirect exposure to the shock is much stronger. Moreover, in this case the net aggregate effects of adding variable markups are much larger. The differences in the changes in aggregate price index under constant markups and under variable markups are around 0.5% when considering the shock that hit all importers. But when considering the shock that hit a single importer, then the differences in the $\hat{P}$ becomes around 3% to 5%.[38]

2.6.2 Under endogenous networks

In this section we additionally consider cases where firms are allowed to change their sourcing sets and status for importing/exporting. In doing so, we depart from the firm-to-firm trade data and contend with the estimated model.

There are several forces that move the aggregate price index in different directions. First, as the foreign good's price falls, some firms that were initially not importers may decide to become importers. This leads to discrete reductions in unit costs of such firms, hence discrete reductions in the input costs of their customers and so on. The aggregate price index falls more as a result. Changes in domestic firm-to-firm linkages also affect the movements in the aggregate price index.

A firm will likely drop a supplier when its price becomes relatively higher. This pushes up the price index, as there are less firm-to-firm links to transmit cost reductions. On the other hand, a firm's sourcing decision may be complementary across suppliers and the firm may decide to add a new supplier, which diminishes the price index.

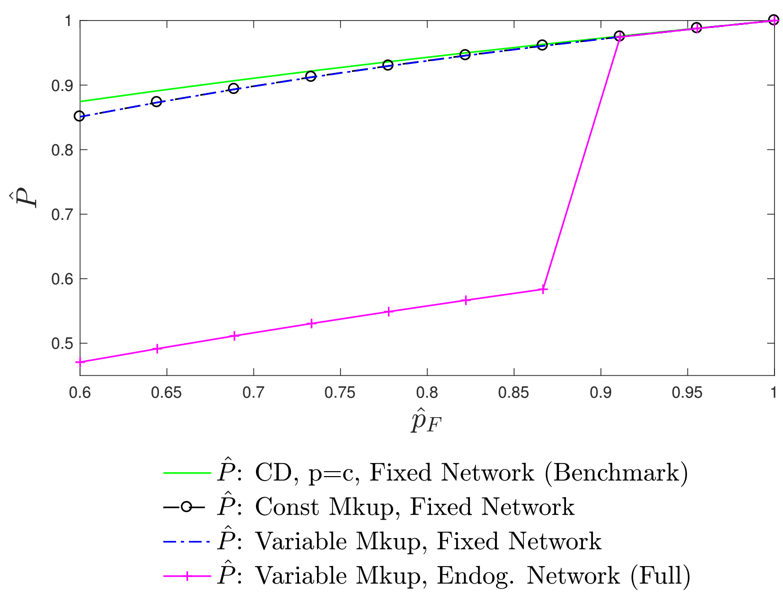
In Figure 2.9 we report the model's predictions on how aggregate price moves under four different cases. For the first three cases under fixed network - the baseline case, model with constant markups, and model with variable markups - the estimated model displays similar patterns as we have seen in Figure 2.6. Price index falls more under constant markups, but adding variable markups has small net effect.[39] The fourth line depicts the change in price

38. The net effect on the aggregate price index also depends on the underlying CES parameters. In Appendix B.6.6 we illustrate how different values of ρ and η affect the markup elasticities Γ_{ji}'s and, in turn, impact the aggregate effects on $\hat{P}$.

39. The estimated model predicts a smaller change in aggregate prices, compared to the predictions using the full firm-to-firm network data. This is because of the calibration strategy that we employ. We parametrize the model so that it matches the average imported goods' share for the importing firms, and not the aggregate

index under the full model, where firms charge variable markups and change linkages. One can see that allowing firms to change linkages amplifies changes in price index. The kink when $\hat{p}_F$ is around 0.9 indicates the start of importing for a firm that originally was not an importer. This result is expected, as the new importer will be capitalizing on cheaper foreign goods. The discrete drop of the firm's unit cost leads to a further reduction in aggregate price.

Figure 2.9: $\hat{P}$ from the estimated model



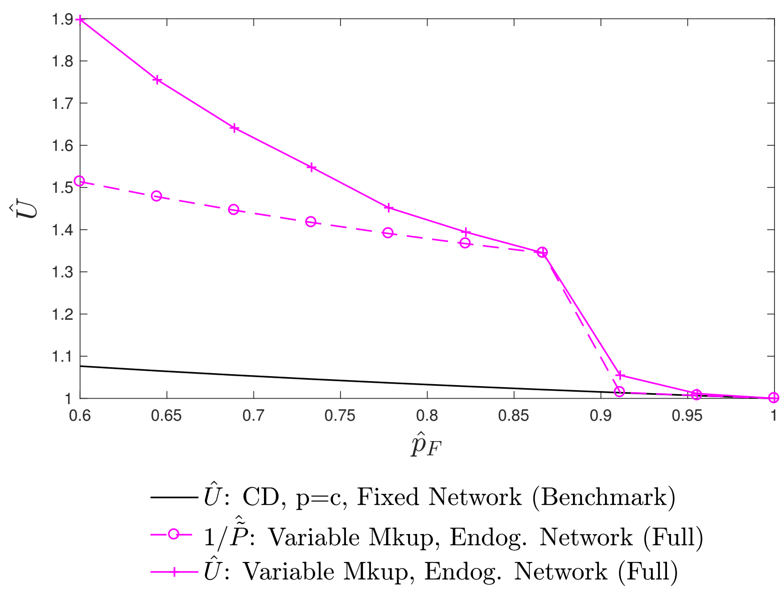
The movement in aggregate welfare is of natural interest. In the benchmark case, the change in aggregate welfare is simply the inverse of the change in the aggregate price index, as firms do not earn profits. But in the full model, it is also positively related to the change in aggregate net profits. In Figure 2.10 we compare the change in aggregate welfare implied by the benchmark case of the model with the change in aggregate welfare implied by the full model. We also plot the inverse of the change in the aggregate price index to illustrate the contributions of the change in aggregate net profits to the change in welfare in the full model.

We find that the changes in aggregate profits greatly magnify the changes in welfare,

import share over GDP.

further differentiating the implications from our full model from those of the benchmark case. We also find that when a firm switches from a non-importer to an importer when $\hat{p}_F$ is around 0.9, the difference between the $\hat{U}$ and $1/\hat{\tilde{P}}$ diminishes. This is because the firm starts to pay additional fixed costs of importing, which reduces its profits net of fixed costs.

Figure 2.10: $\hat{U}$ in the benchmark case and full model



Changes in domestic firm-to-firm linkages

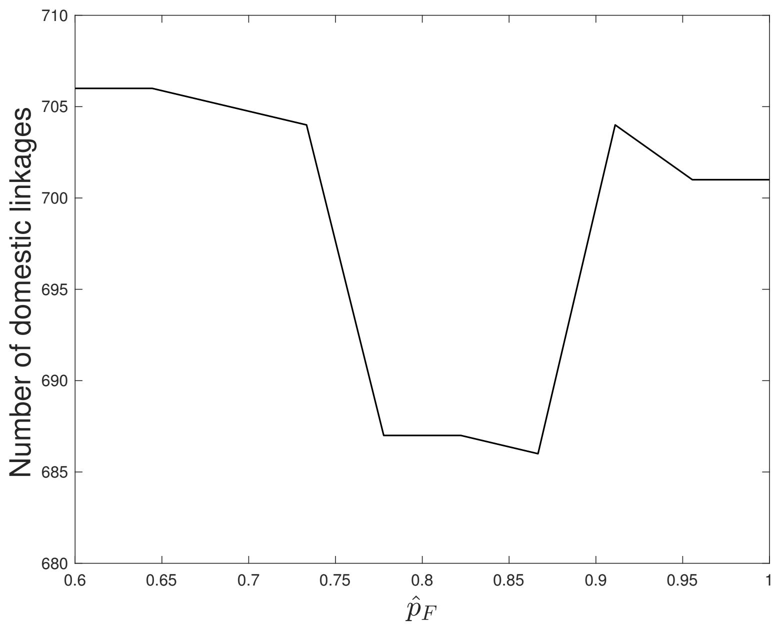
We also see changes in domestic firm-to-firm linkages, which we show in Figure 2.11. First note that as we have seen in Table B.2, the firm-to-firm network is extremely sparse in the Belgian data. However, in the model, we generate a dense firm-to-firm input-output matrix. In order to reduce the number of possible sourcing sets, we assume fixed costs for domestic sourcing to be at the firm-level, and not at the pair level. Nevertheless, analyzing how the network evolves in our model provides important insights.

One can see from Figure 2.11 that the number of domestic linkages generally increases as the foreign price falls, except when $\hat{p}_F$ is around 0.9, which is where the non-importing firm decides to become an importer. The increase in number of linkages come from firms adding new suppliers with reduced output prices. The large drop in linkages when $\hat{p}_F$ is around 0.9

comes from the customer firms of the firm which became an importer. Facing large reduction in one of their input prices, they drop some of their other domestic suppliers.[40]

With regard to the empirical evidence that we presented in Section 2.3, we cannot test whether the same results hold true in this model, as we have too few firms. However, in the model we see the effects that we found in the Belgian data: as firms experience positive cost shocks, they experience larger churn in their suppliers. In the model, we see firms adding new suppliers as their existing suppliers' prices become cheaper. Also, we see firms drop existing suppliers in response to a larger input cost reduction.

Figure 2.11: Number of domestic firm-to-firm linkages



40. The fact that firms may either drop or add suppliers in response to a reduction in one of its input prices may be confusing to readers familiar to Antras, Fort, and Tintelnot (2017), Tintelnot, Kikkawa, Mogstad, and Dhyne (2017), or Furusawa, Inui, Ito, and Tang (2017). In these papers the authors construct models where the firm's marginal benefit of adding a supplier is increasing in the set of other suppliers. If the same feature holds in our model, we would expect firms to only add new suppliers when hit by a positive input price shock. However these complementarities across sourcing sets do not necessarily hold in our model. As in Tintelnot, Kikkawa, Mogstad, and Dhyne (2017), in our model the marginal profit that a firm gains from sales to final demand or from exports is indeed increasing in the firm's sourcing set. However, the marginal profit that a firm gains from sales to other domestic firms is decreasing in the set of other suppliers. Thus whether a firm's sourcing decision is complementary across suppliers depends on the share of profits that come from firm-to-firm sales or from sales to final demand. We confirm this in the simulation results. We find that the firms which dropped other suppliers when one of its suppliers became an importer, had higher output shares to other domestic firms, than those which did not drop suppliers.

The interaction between variable markups and endogenous networks

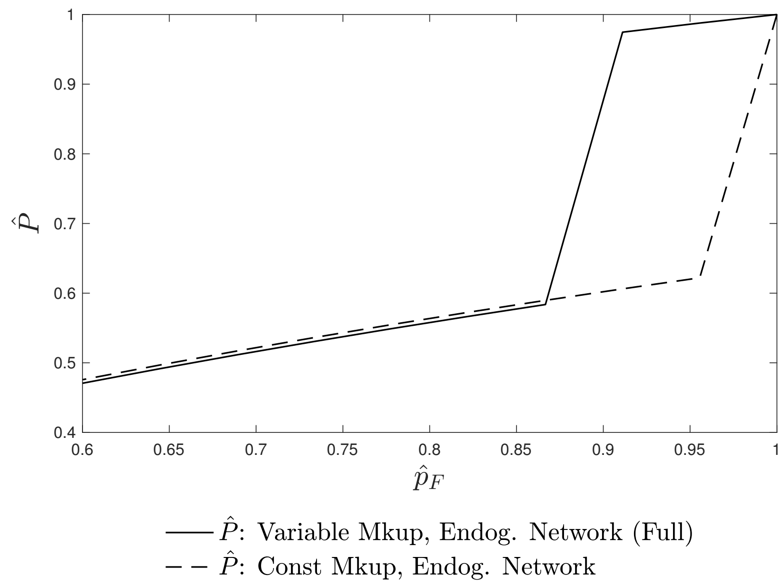
Lastly, we discuss the interaction between variable markups and firms' sourcing decisions. To investigate how much of the aggregate movements implied by our full model are coming from the interaction between variable markups and endogenous network formation, we construct an economy where firms charge constant markups in firm-to-firm trade and form optimal sourcing sets. We contrast the model's counterfactual implications with those of our full model, where firms charge variable markups and make optimal sourcing decisions.

Figure 2.12 plots the changes in the aggregate price index implied by the two models.[41] Though the predictions of $\hat{P}$ from the two models are not far apart when the change in foreign price is large, there is a stark difference when a large shift in aggregate price occurs. In the constant markup case, firms charge markups of $\frac{\rho}{\rho-1}$ when selling goods to other domestic firms. This markup is the lower bound of markups implied by the variable markup case, where firms charge $\frac{\rho}{\rho-1}$ only when their input shares to the customers are infinitesimally small. This smaller degree of double marginalization lets firms in the constant markup case face cheaper input prices and generate larger variable profits given firm-to-firm networks compared to those in the variable markup case. Therefore, in the constant markup case, firms are able to add domestic suppliers and switch to importing, even under a smaller reduction in foreign price. This explains why the large drop in aggregate price change - caused by a firm switching from a non-importer to an importer - occurs upon higher $\hat{p}_F$ in the constant markup case.

As we have seen in Section 2.6.1, adding variable markups in firm-to-firm trade alone contributes to a small change in aggregate variables when fixing firm-to-firm networks. We then documented that most of the changes in aggregate variables in the full model are due to firms changing their sourcing sets. But variable markups in firm-to-firm trade produce both qualitative and quantitative differences in aggregate changes, through the interaction with endogenous network formation. These results imply that the difference in the aggregate

41. We provide discussions for the changes in welfare in Appendix B.6.7.

Figure 2.12: $\hat{P}$ under endogenous networks: variable and constant markups

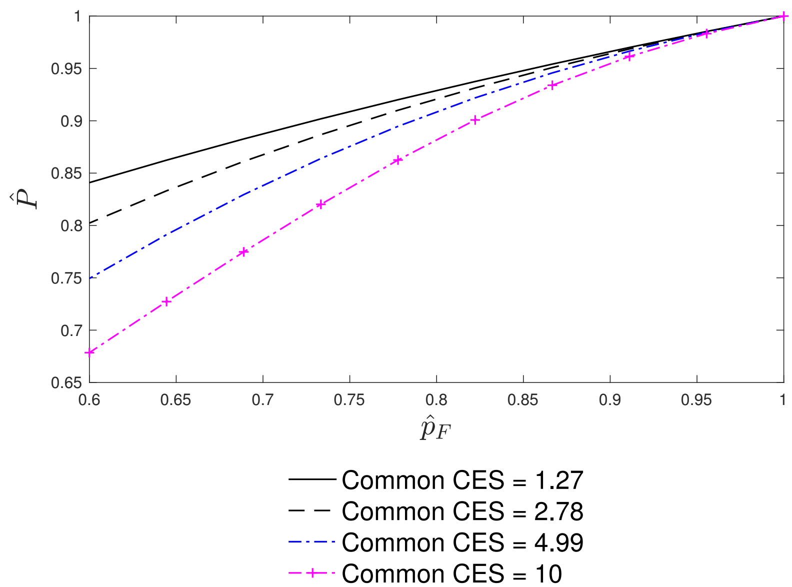


implications are driven by both margins.

2.6.3 Network irrelevance under common CES parameter

Finally, we consider the case of the model described in Proposition 1, where change in price index and welfare can also be written by firm-level variables, but with an assumption for the value of $\tilde{\sigma}$. Figure 2.13 plots the movement in aggregate price, under four different values of $\tilde{\sigma}$. Higher value of $\tilde{\sigma}$ translates to higher substitutability of inputs, thus resulting in larger reduction in aggregate price. By assigning value of $\tilde{\sigma}$ that is between the estimated values of η and ρ, one can match the prediction of $\hat{U}$ from the variable markup case. However, the predictions under the value of $\tilde{\sigma}$ as high as 10 do not match the predictions under the full model. The magnitudes of aggregate price changes caused by endogenous network formation are so large that this model of common CES parameter is unable to generate such large changes under the plausible range of $\tilde{\sigma}$.

Figure 2.13: $\hat{P}$ under Proposition 1



2.7 Conclusion

In this paper we studied how oligopolistic competition in firm-to-firm trade and firms' ability to choose suppliers affect aggregate response to shocks. With a model that incorporates the two elements, we analyze how aggregate variables, such as price index and welfare, respond to an exogenous reduction in the foreign good's price. We contrast them with the predictions from the benchmark case, in which the network is irrelevant. While we focus on the transmission of a foreign trade shock, all results and intuitions offer insight into the response other types of shocks: shocks at the industry- or firm-level, or domestic shocks.

Our model proposes a novel view on competition between firms. Instead of the market share within the sector being the determinant of the firm's market power, we suggest that the relative size of the firm in the total input sourcing of its customers is the relevant metric. The novel data on firm-to-firm transactions support this view: firms charge higher markups if they have higher average input shares within their customer firms, controlling for their sectoral market shares.

We find that the model produces both qualitatively and quantitatively different aggregate predictions compared to the benchmark case. The benchmark case of the model can only capture less than a quarter of movements in price index and welfare that are implied by the full model. In particular, allowing firms to optimally decide their sourcing set significantly alters aggregate predictions. When the foreign price goes down, firms that initially did not import switch and become importers. This amplifies the movement in the aggregate price index and welfare. We also find that oligopolistic competition in firm-to-firm trade makes a quantitative difference in the aggregate responses through the interaction, with firms making optimal sourcing decisions.

Our findings contrast that of Hulten (1978), where the network structure is irrelevant in the aggregate in an efficient economy, up to the first order. Our results imply that firm-level variables do not work as sufficient statistics when evaluating aggregate outcomes, and indicate the need for information on firm-level input-output structures.

This paper also adds depth to various policy questions, as it lays a framework analyzing how aggregate variables are affected by the two market frictions in firm-to-firm trade. Our counterfactual analysis considers the effect of a particular policy episode: exogenous reduction in the foreign good's price. Using this framework, one can explore other policy experiments and analyze their aggregate effects.

CHAPTER 3

TRADE AND DOMESTIC PRODUCTION NETWORKS

This chapter is based on a joint work with Felix Tintelnot, Magne Mogstad, and Emmanuel Dhyne.

3.1 Introduction

Over the past few decades, the focus of research on international trade has shifted from countries and industries towards firms. This shift is in no small part due to the increased availability of firm-level transaction data on trade. One important insight from this data is that few firms directly import or export goods (**?**). However, the concentration of imports and exports does not necessarily imply that few firms benefit from foreign trade. Even if firms themselves do not import or export, they may still buy from or sell to domestic firms that trade internationally. Capturing this channel, however, is challenging since domestic firm-to-firm transactions are rarely observed. In the absence of such data, quantification of the effects of foreign trade on all firms requires strong assumptions, such as a common intermediate good (**?**, and Blaum, Lelarge, and Peters, 2016) or the same import shares across importing firms within broad industries (Caliendo and Parro, 2015, and Costinot and Rodríguez-Clare, 2014).

The goal of this paper is to combine data on domestic firm-to-firm sales with information on foreign trade transactions to study how international trade affects real wages and efficiency of all firms, including those that do not directly export or import. Our analysis employs a panel dataset with detailed information on Belgian firms for the years 2002-2014. This dataset is based on several data sources that we have linked through identifiers. Annual accounts provide data on input factors and output, custom records and intra-EU declarations give information on exports and imports, and a value-added tax (VAT) registry provides information on domestic firm-to-firm transactions. Using this data, we empirically examine several new dimensions of firms in international trade before developing and estimating a model of trade and domestic production networks.

In Section 2, we describe the data, construct the domestic production (buyer-supplier) network of the Belgian economy, and provide two new empirical findings. The first is that most firms are exposed to foreign trade through their production network. While only 15% of firms import directly, 97% of firms obtain foreign inputs either directly or indirectly through domestic suppliers which use foreign inputs in their production process. Indeed, most firms are heavily dependent on foreign inputs, but only a small number of firms show that dependence through the direct imports observed in firm-level transaction data on trade. For example, in a majority of firms, at least 40% of input costs are spent on goods that are imported directly or indirectly.

The second empirical finding is that foreign trade shocks seem to propagate across firms within production networks. Following Hummels, Jørgensen, Munch, and Xiang (2014), we measure trade shocks as changes in world export supply and world import demand of country-product combinations in which the firm had a previous trade relationship. They argue that these shocks are plausibly exogenous and that their impact varies markedly across firms, because the firms — even within the same sector — do not have all inputs in common. Using our data, we find that positive export shocks to the firm's buyers and positive import shocks to the firm's suppliers both tend to increase the firm's output, even after controlling for direct shocks to the firm itself and for shocks to the set of potential buyers and suppliers of the firm.

Taken together, these two empirical findings highlight that information about domestic firm-to-firm transactions is key to understand the extent to which firms rely on foreign input and to analyze the propagation of trade shocks. Motivated and guided by this evidence, we develop and estimate a model of domestic production networks and international trade. In our model, firms combine imports, inputs produced by other domestic firms, and labor to produce differentiated products with a constant elasticity of substitution production function.

Firms are finite and monopolistically competitive.

In Section 3, we assume a fixed network structure (i.e., the buyer-supplier relationships do not change in response to trade shocks) and quantify how international trade affects firms' production costs and consumer prices. The cost reduction for an individual firm due to international trade depends on two quantities only: the share of input costs that is spent on goods that are imported directly or indirectly and the elasticity of substitution in the production function. We apply this sufficient statistics formula to our data, and find that international trade is important in reducing the cost of production for plausible values of the elasticity of substitution. For example, with an elasticity of substitution in the production function of 2, we calculate that shutting down international trade would increase the cost of the majority of Belgian firms by at least 70%. To compute the welfare gains from trade, we combine information on firms' sales to domestic households with an assumption about the elasticity of substitution in the utility function. Our baseline results imply that the consumer price index in Belgium would be 77% higher in the absence of international trade.

While assuming a fixed network structure is convenient to take the model to the data, it does not allow us to capture how buyer-supplier relationships may change in response to trade shocks. In Section 4, we therefore develop a model of trade with endogenous network formation. In particular, we let firms optimally choose their set of suppliers (i.e. the firm's sourcing strategy) subject to a buyer-supplier-specific fixed cost for adding a supplier. Allowing for endogenous network formation is challenging for two reasons. First, firms face a large discrete choice problem of which suppliers to include in their sourcing strategy. Second, firms' sourcing strategies are interdependent, creating a large fixed point problem: firms take into account the expected sourcing strategies of others in order to determine their own optimal sourcing strategy, all the while knowing that other firms are thinking in the same way.

Building on Jia (2008) and Antràs, Fort, and Tintelnot (2016), we overcome the first challenge by using lattice theory to solve firms' large combinatorial discrete choice problems.

To address the second challenge we consider the formation of an acyclic network, postulating an ordering of firms and restricting the eligible set of suppliers to firms that appear prior to the buyer.[1] While restrictive, this assumption allows us also to solve a model of firm trade with endogenous formation of domestic buyer-supplier relationships. We use method of simulated moments to estimate the model, and then perform counterfactuals to draw inference about the impact of shutting down international trade with and without endogenous network formation. Our findings suggest that allowing for endogenous formation of buyer-supplier relationships tend to attenuate the effects of banning trade on firms' cost. Under endogenous network formation, we find a price index increase around 15% lower than under a fixed network.

Our paper contributes to a growing literature on the economy-wide effects of *foreign sourcing*.[2] Many studies use aggregate data only, relying on the assumption that firms import intensities are equalized — which is at odds with the data. Using firm-level data on trade transactions, Blaum, Lelarge, and Peters (2016) show that accounting for heterogeneity in import exposure significantly affects the measurement of the gains from international trade. Their model assumes that firms can import directly and purchase a common intermediate good. Taking advantage of data on domestic firm-to-firm transactions, we relax the assumption of a common intermediate good, and derive a parsimonious sufficient statistics formula for a model with a fixed production network. We also go beyond the fixed network structure, solving a model of endogenous network formation with a finite number of firms and fixed costs for adding suppliers. This contribution builds on the global sourcing model of Antràs, Fort, and Tintelnot (2016). While they distinguish between final good and intermediate good sectors, we consider a more general input-output structure between firms. In addition, our model captures not only the firms' decisions with respect to foreign sourcing

1. See Spiegler (2016) for a recent contribution in economics studying belief formation in a directed acyclic network.

2. See for example **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, and **?**.

but also their choices of domestic sourcing strategies.[3]

Our paper also relates to a literature on the formation and consequences of *domestic production networks*.[4] Bernard, Moxnes, and Saito (2016b) adapt the model of Antràs, Fort, and Tintelnot (2016) to search for domestic suppliers in different locations, where each location has a continuum of intermediate-good-producing firms. They find significant improvements in firm performance from a reduction in internal search costs in Japan. Furusawa, Inui, Ito, and Tang (2017) develop a variant of the global sourcing model of Antràs, Fort, and Tintelnot (2016), and use Japanese buyer-supplier link data to test the model's predictions. Oberfield (2017) develops a theory in which the network structure of production forms endogenously among firms that each purchase a single input. Lim (2015) develops a dynamic model of network formation in which each firm has a continuum of domestic suppliers. With a continuum of suppliers and buyers, the sales from one firm to another are negligibly small and a link between two particular firms has no effect on aggregate outcomes. In contrast to these papers, we develop a model of endogenous network formation with a finite set of suppliers, and incorporate both firm exporting and importing decisions. While our theory assumes simple solutions for the pricing game between firms, **?** explore the segmentation of markets for different buyers, with supplier firms having heterogeneous bargaining power in the supplier-buyer relationships.

Finally, our paper relates to a literature that analyzes how production networks matter for *aggregate effects and transmission of idiosyncratic firm shocks*. Gabaix (2011) provides conditions under which granular shocks can affect aggregate fluctuations. Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) study the transmission of shocks along sectoral input-output networks. Magerman, De Bruyne, Dhyne, and Van Hove (2016) test both channels with the Belgium domestic firm-to-firm data. Barrot and Sauvagnat (2016), Boehm,

3. Our work is also related to the analysis in Caliendo and Parro (2015) and Ossa (2015). They find the gains from trade to be larger when taking sectoral input-output linkages into account.

4. A growing body of work studies how firms meet international trading partners. See for example Chaney (2014), Chaney (2016), Morales, Sheu, and Zahler (2015), and Eaton, Kinkins, Tybout, and Xu (2016).

Flaaen, and Pandalai-Nayar (2015), and Carvalho, Nirei, Saito, and Tahbaz-Salehi (2014) use natural disasters to study the propagation of shocks in production networks. Carvalho and Voigtländer (2015) analyze the adoption of inputs by innovators and the evolution of the domestic production network. Hulten (1978) provides conditions under which the underlying network structure is irrelevant for quantifying the propagation of shocks — up to a first-order approximation — as long as firms' initial size and the magnitudes of the idiosyncratic shocks are observed.[5] In recent work, Baqaee and Farhi (2017) illustrate that the second-order effects of shock propagation arising from networks can be large.[6] Our paper extends the analysis of shock propagation to foreign trade shocks, while allowing buyer-supplier relationships to change in response to these shocks.

3.2 Trade and production networks: Data and evidence

This section describes the data, documents firms' direct and indirect exposure to foreign trade, and shows how the output of a firm is affected by trade shocks to its buyers and suppliers.

3.2.1 Data sources and sample selection

Our analysis draws on three administrative data sources from Belgium, accessible only at the National Bank of Belgium, for the years 2002-2014. These data sources can be linked through unique identifiers, assigned and recorded by the government for the purpose of collecting value-added taxes (VAT). Below we briefly describe our data and sample selection, while additional details are given in Appendix C.3.

5. As shown by **?**, to apply the Hulten theorem to a small open economy setting one would need very strong conditions that are immediately violated in our data: in particular, all firms would need have a constant ratio of export sales to sales to domestic final consumers.

6. Other recent contributions to determining the effects of networks include Baqaee (2014b), **?**, as well as in the context of financial frictions, Bigio and La'o (2017) and Liu (2016). Atalay, Hortacsu, Roberts, and Syverson (2011) characterize the buyer-supplier network of the US economy.

The first data source is the Business-to-Business (B2B) transactions database (see also Dhyne, Magerman, and Rubinova (2015)). By law, all Belgian firms are required to file the annual sales to each buyer (provided the annual sales to a given buyer exceeds €250). Thus, the B2B dataset allows us to measure accurately the identity of the firms' suppliers and buyers. The second data source is the annual accounts filed by Belgian firms. These data contain detailed information from the firms balance sheets on output (such as revenues) and inputs (such as capital, labor, intermediates) as well as 4-digit (NACE) industry codes and geographical identifiers at the zip code level. In addition, the annual accounts include information about ownership shares in other enterprises. The third set of data source is the Belgian customs records and the intra-EU trade declarations. These data contain information about international trade transactions in each year and for every firm. Both imports and exports are disaggregated by product and origin or destination.

One challenge with using the Belgian data is that the information is recorded at the level of the VAT-identifier. The problem is that a given firm may have several VAT-identifiers (for accounting or tax reasons).[7] While organizational choices and transactions across units within a firm are of interest, our paper is centered on trade between firms. Thus, if a firm has multiple VAT-identifiers, we aggregate all data up to the firm level using information from the balance sheets about ownership structure. Details of the aggregation are outlined in Appendix C.3.1. In 2012, for example, the aggregation converts 896,000 unique VAT-identifiers into 860,000 unique firms. Of these firms, 842,000 had a single VAT-identifier. However, the 18,000 firms with multiple VAT-identifiers are important, accounting for around 60% of the total output in the dataset.

After constructing a firm-level dataset, we impose a few sample restrictions. We exclude firms in the government or financial sector. In addition, we restrict the sample to firms with positive labor costs and employment, tangible assets of more than €100, and positive

7. Existing papers tend to abstract from this issue, analyzing the data at the level of the VAT-identifier. See e.g. Amiti, Itskhoki, and Konings (2014), Magerman, De Bruyne, Dhyne, and Van Hove (2016) and Bernard, Blanchard, Van Beveren, and Vandenbussche (2016a)).

total assets in at least one year during our sample period. These criteria are similar to the ones used by De Loecker, Fuss, and Van Biesebroeck (2014). Applying these criteria reduces the number of firms significantly. In, 2012, for example, only 139,605 firms satisfy the above criteria. The large reduction in sample size is mostly driven by the exclusion of local firms without employees (self-employment) from the sample (687,700 firms in 2012). Lastly, we drop foreign firms with no local production activity in Belgium from the sample. These account for a sizable fraction of imports and exports, but have no domestic production activity in Belgium.

Table 3.1 illustrates that our selected estimation sample of firms provides relatively good coverage of aggregate value added, gross output, exports and imports. However, total sales in our sample is larger than what are reported in the national statistics. The reason is that the output of trade intermediaries in the national statistics is measured by their value added instead of their total sales. We refer to Appendix C.3.2 for the same statistics for all Belgian firms.

Table 3.1: Coverage of selected sample

Year	GDP	Output	Imports	Exports	Selected sample				
	(Excl. Gov. & Fin.)				Count	V.A.	Sales	Imports	Exports
2002	149	411	210	229	122,460	123	586	179	189
2007	192	546	300	314	136,370	157	757	280	269
2012	212	626	342	347	139,605	170	829	296	295

Notes: All numbers except for Count are denominated in billion Euro in current prices. Belgian GDP and output are for all sectors excluding public and financial sector. See Appendix C.3.2 for the same statistics for the total economy. Data for Belgian GDP, output, imports and exports are from Eurostat.

3.2.2 Direct and indirect exposure to foreign trade

The Belgian data allow us to construct the buyer-supplier relationships of the Belgian economy, and therefore document firms' direct and indirect exposure to foreign trade.

We define firm j's *total foreign input share* as the sum of firm j's *direct foreign input share*, s_{Fj}, and the direct foreign input share of firm j's suppliers, suppliers' suppliers, and so forth, each weighted by the firm-pair-specific input shares $(s_{ij}, s_{ki}, ...)$:

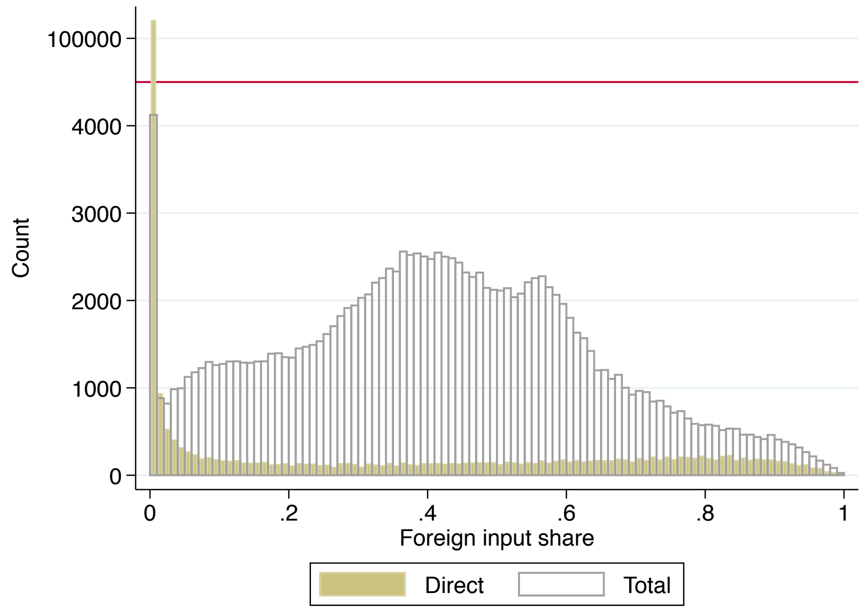
$$s_{Fj}^{Total} = s_{Fj} + \sum_{i \in Z_j^D} s_{ij} \underbrace{\left[s_{Fi} + \sum_{k \in Z_i^D} s_{ki} \left(s_{Fk} + \cdots \right) \right]}_{s_{Fi}^{Total}}, \tag{3.1}$$

where Z_j^D denotes the set of domestic suppliers of firm j, and the denominator of the input shares is the sum of labor costs, purchases from other firms, and imports. Note that the definition of the total foreign input share is recursive: a firm's total foreign input share is sum of its direct foreign input share and the share of its inputs from other firms multiplied by those firms' total foreign input shares. While many firm-level datasets contain information about the direct foreign input share s_{Fj}, our data also offer information about firm-pair-specific input shares, s_{ij}. As a result, we are able to calculate the total foreign input share for every firm. We note that there is one inherent assumption in our definition of the total foreign input share: When a firm sells its output to multiple firms or final consumers, the foreign input share in the costs of producing these goods is assumed to be the same for all buyers (i.e., independent of the identity of the buyer). This assumption is consistent with the model we develop in Sections 3 and 4, where each firm produces a single product.

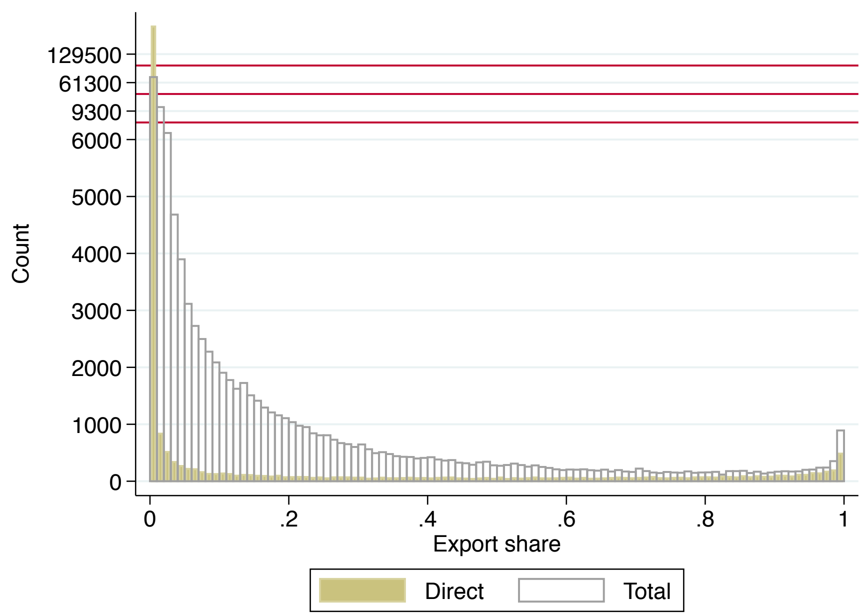
In Figure 3.1a, we display a histogram of the total and direct foreign input shares of the Belgian firms. While only 15% of firms import directly, 97% of firms obtain foreign inputs either directly or indirectly through domestic suppliers which use foreign inputs in their production process. Indeed, most firms are heavily dependent on foreign inputs, but only a small number of firms show that dependence through the direct foreign input shares observed in firm-level transaction data on trade. In the median firm, for example, the total foreign input share is 41%. By comparison, the total foreign input shares are 21% and 60% at at the 20th and 80th percentile. We present direct and total foreign input shares by sector in Appendix C.4.1. Even in the service sector, in which firms have a very low share of direct foreign inputs, the median firm's total foreign input share is as large as 28%.

Figure 3.1: Histogram of direct and indirect linkages to foreign trade

(a) Direct and total foreign input share



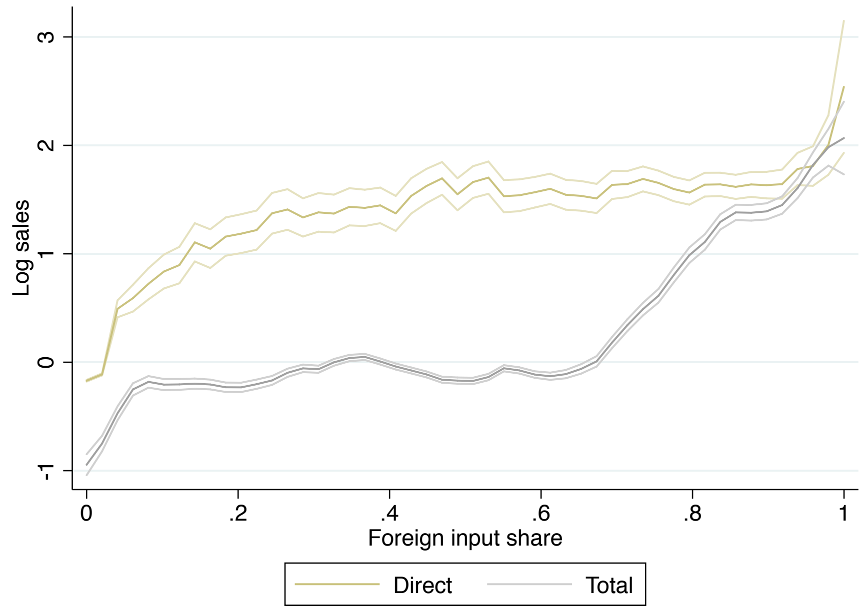
(b) Direct and total export share



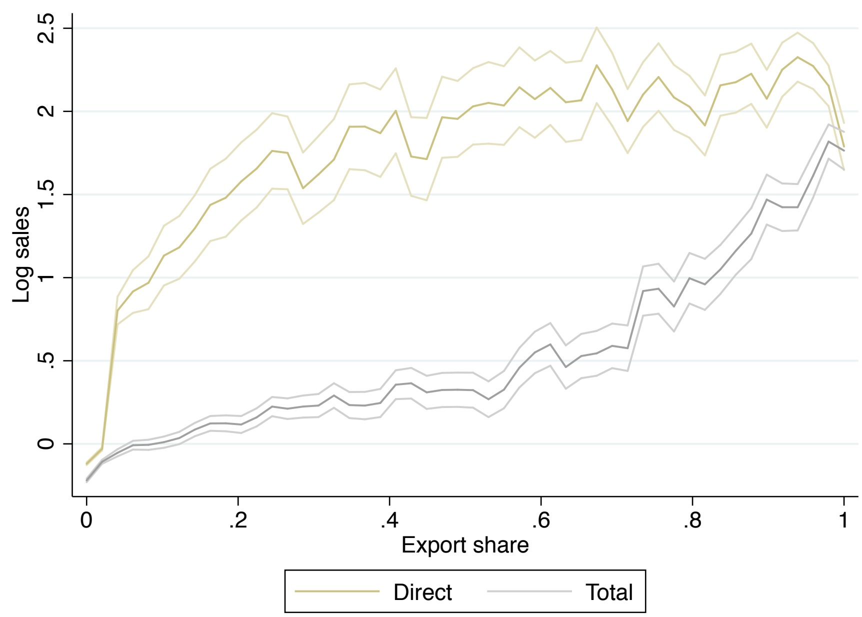
Notes: Total foreign input share of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct foreign input share, and s_{ji} is j's share among i's inputs. Total export share firm i, r_{iF}^{Total} is calculated by solving $r_{iF}^{Total} = r_{iF} + \sum_{j \in W_i} r_{ij} r_{jF}^{Total}$ where r_{iF} is i's share of exports in its revenue, and r_{ij} is share of i's revenue that arises from sales to firm j. The figures are based on the analysis of 139,605 private sector firms in Belgium in 2012. The horizontal lines represent scale breaks on the vertical axis.

Figure 3.2: Size premium of direct and indirect linkages to foreign trade

(a) Foreign inputs



(b) Exports



Notes: The two figures display the smoothed values with 95% confidence intervals of kernel-weighted local polynomial regression estimates of the relationship between firms' sales and their levels of participation in foreign trade. We use the Epanechnikov kernel function with kernel bandwidth of 0.01, pilot bandwidth of 0.02, degree of polynomial smooth at 0, and smooth obtained at 50 points. Log sales are demeaned with 4-digit industry fixed effects.

Figure 3.1b performs a similar exercise, but now looks at *total export shares* and *direct export shares*. We calculate the total export share of firm j, r_{jF}^{Total}, as the sum of the share of revenue of firm j coming from directly export goods, r_{jF}, and the share of revenue coming from goods sold to other domestic firms, multiplied by those firms' total export shares:

$$r_{jF}^{Total} = r_{jF} + \sum_{i \in W_j} r_{ji} r_{iF}^{Total}, \tag{3.2}$$

where W_j denotes the set of domestic buyers of firm j, and the denominator of the export shares is the total revenue of the firm. Direct export is even more concentrated than direct import, both on the intensive and extensive margin. While only 10% of firms export directly, 82% of firms export either directly or indirectly by selling to domestic buyers which subsequently trade internationally. In terms of trade volume, however, export remains relatively concentrated even after taking the indirect export into account. The total export share is only 2% in the median firm, whereas it is 19% at the 80th percentile. In contrast, Figure 3.1a showed that most firms are heavily dependent on foreign inputs. This difference is partly driven by the service sector. While many firms in this sector (e.g., restaurants) rely on foreign inputs — often obtained indirectly through domestic suppliers — relatively few export directly or sell to domestic firms that are exporting directly or indirectly (see Appendix C.4.2 for direct and total foreign input shares by sector).[8]

Across a wide range of countries and industries, firms that directly export or import have been shown to be larger than other firms. A natural question is whether the positive association between firm size and international trade also carries over to indirect export or import. Figure 3.2 investigates this, calculating the average size of firms by direct and total foreign input shares as well as by direct and total export shares. We demean the log of firm sales using the firm's four-digit industry average, so that a firm with log sales of zero

8. Another possible explanation is that it is difficult to measure all forms of export in the service sector. For example, when a foreigner eats at a Belgian restaurant, it is technically an export transaction. However, such transactions are not recorded in our data.

is the size of an average firm in its industry. Figure 3.2 illustrates that average firm size is increasing in both direct and total foreign input shares. However, firms that import directly tend to be much larger than firms that buy foreign inputs through domestic firms. Indeed, firms with less than 60% in total foreign input shares are, on average, of similar size as the average firm in their industry. A similar pattern is evident for size and export. Firms with very high total export shares tend to be large. However, over most of the total export share distribution, there is only a weak relationship between firm size and total export share. Taken together, the results in Figure 3.2 suggest that firms do not have to be large to rely heavily on foreign inputs or to have most of their sales going ultimately to a foreign country.

3.2.3 Trade shocks and the production network

The analysis in Section 3.2.2 showed that most firms are exposed to foreign trade through their production network. This finding raises the questions of whether, and to what extent, trade shocks propagate across firms within production networks. To investigate these questions, we build on the work by Hummels, Jørgensen, Munch, and Xiang (2014), who construct measures of trade shocks from changes in world export supply and world import demand of country-product combinations in which the firm had a previous trade relationship. They argue these shocks are plausibly exogenous, and show that their impact varies markedly across firms, because the firms do not have all inputs in common. Hummels, Jørgensen, Munch, and Xiang (2014) use the trade shocks to estimate wage effects of offshoring and exporting in Denmark. We apply the same identification strategy to the Belgian setting with the goal of examining whether trade shocks to the firm's actual buyers or suppliers have stronger impact on its output than trade shock to potential suppliers and customers.

To make the identification strategy precise, consider the following regression model in first-differences:

$$\begin{aligned}\Delta \log Y_{it} = &\beta_X^C \Delta \log X_{it}^C + \beta_M^S \Delta \log M_{it}^S \\ &+ \beta_X^{PC} \Delta \log X_{it}^{PC} + \beta_M^{PS} \Delta \log M_{it}^{PS} \\ &+ \beta_X \Delta \log X_{it} + \beta_M \Delta \log M_{it} + \varphi_t + \epsilon_{it}. \end{aligned} \tag{3.3}$$

where Y_{it} denotes the total sales of firm i in year t, and Δ denotes the change in the variable from year $t-1$ to t. In addition to calender time fixed effects φ_t, we include three sets of explanatory variables. The first is the measures of import shocks to firm i's suppliers $\Delta \log M_{it}^S$ and export shocks to firm i's' buyers $\Delta \log X_{it}^C$. Our goal is to consistently estimate the coefficients on these variables, β_X^C and β_M^S. However, there are several threats to identification. One is that $\Delta \log M_{it}^S$ and $\Delta \log X_{it}^C$ are likely to correlate with trade shocks to i's potential suppliers and buyers. We therefore include measures of import shocks to firm i's potential suppliers, $\Delta \log M_{it}^{PS}$, and export shocks to firm i's' potential buyers, $\Delta \log X_{it}^{PC}$. Another concern is that $\Delta \log M_{it}^S$ and $\Delta \log X_{it}^C$ could be correlated with trade shocks that affect the firm i directly (through it's direct import demand or supply). To address this concern, we control for export shocks, $\Delta \log X_{it}$, and import shocks, $\Delta \log M_{it}$, to firm i itself.

To take the regression model to the data, we need to construct the various measures of trade shocks. To this end, we follow the shift-share approach in Hummels, Jørgensen, Munch, and Xiang (2014). To construct an export shock for firm i, $\Delta \log X_{it}$, we use information about the firm's product-country-level exports in year $t-1$ (the share variable capturing firm-specific exposure), and the aggregate shift in world import demand for each country and product:

$$\Delta \log X_{it} = \log \underbrace{\sum_{k,c} r_{ic,t-1}^{k,X} \text{WID}_{k,c,t}}_{X_{it,t-1}} - \log \underbrace{\sum_{k,c} r_{ic,t-1}^{k,X} \text{WID}_{k,c,t-1}}_{X_{it-1,t-1}} .$$

The term $r^{k,X}_{ic,t-1}$ is the share of exports of firm i at year $t-1$ that falls on product k sold to country c, and $\text{WID}_{k,c,t}$ is the world import demand (excluding imports from Belgium) of country c for product k.[9] We measure the export shock for firms' buyers in a similar way. For firm i's buyers, we construct the weighted average of their export demand shocks, $\Delta \log X^C_{it}$, using i's output share to each buyer in the previous year as the weights:

$$\Delta \log X^C_{it} = \log \sum_j r_{ij,t-1} X_{jt,t-1} - \log \sum_j r_{ij,t-1} X_{jt-1,t-1}.$$

Finally, we measure the trade shocks to the firms' potential buyers, $\Delta \log X^{PC}_{it}$. The potential buyers of firm i include both the buyers of i's goods and other firms in the same (4-digit) sector as the actual buyers. We weight sectors for each firm according to the share of the firm's revenue that is sold to firms' from that sector, $r_{iu,t-1}$.[10] We then construct an export shock for each sector as a weighted aggregate of export shocks to all firms of that sector.[11] We combine these terms to construct an export shock to the potential buyers of firm i:

$$\Delta \log X^{PC}_{it} = \log \sum_u r_{iu,t-1} X^{-i}_{ut,t-1} - \log \sum_u r_{iu,t-1} X^{-i}_{ut-1,t-1}.$$

9. We use NACE 4 digit level to classify products k.

10. Let sectors, at the NACE 4-digit level, be denoted with u. $r_{iu,t} = \sum_{j \in W^u_{it}} \frac{\text{Sales}_{ijt}}{\text{TotalSales}_{it}}$, where W^u_{it} denotes the set of customers of i producing sector u goods at time t. We fix all weights at the previous year $t-1$.

11. For the weights that firm i assigns to each firm within a sector, we use the firms' sales to domestic final demand as corresponding weights. We have experimented with different weights and obtained similar results. These weights vary at the firm i - sector u level, as we exclude firm i's own exports and imports if firm i is producing sector u good.

$$X^{-i}_{ut,t-1} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} X_{jt,t-1}$$
$$X^{-i}_{ut-1,t-1} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} X_{jt-1,t-1},$$

where U_t is the set of firms producing sector u good at t, and V_{iHt} is firm i's sales to domestic final demand at t.

To construct the import shock variables to the firm itself, its suppliers, and its potential suppliers, we use a similar procedure. These variables use information about changes in aggregate export supply in foreign countries and the past sourcing of firms from these countries. We describe their construction in Appendix C.5.1.

Table 3.2: Reduced form results

	$\Delta \ln$ Total Sales	$\Delta \ln$ Total Sales	$\Delta \ln$ Total Sales
$\Delta \ln X_{it}$	0.106***	0.103***	0.089***
	(0.008)	(0.008)	(0.009)
$\Delta \ln M_{it}$	0.183***	0.178***	0.156***
	(0.010)	(0.010)	(0.012)
$\Delta \ln X_{it}^{PC}$		0.027***	0.025***
		(0.002)	(0.003)
$\Delta \ln M_{it}^{PS}$		0.040***	0.039***
		(0.004)	(0.005)
$\Delta \ln X_{it}^{C}$			0.122***
			(0.013)
$\Delta \ln M_{it}^{S}$			0.041***
			(0.018)
N	87100	87100	87100

Notes: Standard errors are clustered at the firm level. All variables are in terms of yearly log differences for the period 2002-2012. All specifications include year fixed effects. We truncate outliers of each variables at the top and bottom 1% level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3.2 shows the estimation results from the regression model with changes in the firm's total sales as the dependent variable (see Table C.9 in Appendix C.5.2 for results with changes in domestic sales and domestic inputs as dependent variables). The results in column 1 suggest that firms that experience positive trade shocks tend to increase their sales. The estimates in column 2 suggest that trade shocks to potential buyers and suppliers also affects the firm's sales. However, as shown in column 3, shocks to the firms' actual buyers and suppliers matter more for the firm's sales than shocks to the potential buyers and suppliers.[12] In terms of magnitudes, the estimates suggest that an 10 percent exogenous increase in the foreign demand of goods for firm i's buyers leads to a 1.2 percent increase in

12. Note that the actual buyers and suppliers are included in the set of potential buyers and suppliers. Thus, the coefficients β_X^C and β_M^S should be interpreted as the additional effect of a trade shocks to the firm's actual buyers and suppliers as compared to the effect of trade shocks to the firm's potential buyers and suppliers

the sales of firm i. The pass through of shocks to firm i's suppliers is smaller. A 10 percent exogenous increase in the foreign supply of goods to firm i's suppliers leads to a .4 percent increase in the sales of firm i.

Taken together, the results in Table 3.2 suggest that sectoral input-output tables are not sufficient to analyze the propagation of trade shocks. The output of a firm is significantly affected by idiosyncratic shocks to its buyers and suppliers. This finding is consistent with Carvalho, Nirei, Saito, and Tahbaz-Salehi (2014), who show that the disruption caused by a Japanese earthquake in 2011 propagated through upstream and downstream supply chains. Motivated by this evidence, we proceed by developing a model of international trade and domestic production networks.

3.3 A model of trade with fixed production networks

We now develop a model of trade and domestic production networks, and use it to quantify how international trade affects firms' production costs and consumer prices. While this section assumes a fixed network structure — which is convenient to take the model to the data — we allow, in Section 4, the buyer-supplier relationships to change in response to trade shocks.

3.3.1 Model

We describe a small open economy called Belgium. Before describing the model, we briefly discuss the notation. Since there exist many bilateral directed flows in our model, we will often have two subscripts. In such cases, the first subscript denotes the origin of the good and the second subscript denotes the destination of the good.

Preferences and Demand

Each consumer supplies one unit of labor inelastically. Consumers are assumed to have identical, homothetic CES preferences over consumption goods:

$$U = \left(\sum_{k \in \Omega} \left(\beta_{kH} q_{kH} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{3.4}$$

where Ω denotes the set of available products in the small open economy, k denotes a product, and H denotes domestic final demand from households. Since all consumers have the same, homothethic CES preferences for consumption, we can write the aggregate final consumer demand (in quantities) for product k, given price p_{kH}, as:

$$q_{kH} = \beta_{kH}^{\sigma-1} \frac{p_{kH}^{-\sigma}}{P^{1-\sigma}} E, \tag{3.5}$$

where E denotes the aggregate expenditure in Belgium and P denotes the domestic consumer price index:

$$P = \left(\sum_{j \in \Omega} \beta_{jH}^{\sigma-1} p_{jH}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{3.6}$$

We assume that final goods are substitutes and therefore $\sigma > 1$.

Demand from abroad for product k takes a similar functional form:

$$q_{kF} = \beta_{kF}^{\sigma-1} \frac{p_{kF}^{-\sigma}}{P_F^{1-\sigma}} E_F, \tag{3.7}$$

where β_{kF} is a product-k-specific foreign demand shifter, p_{kF} is the price of product k abroad, and P_F and E_F denote the foreign price index and expenditure, respectively .

Production and market structure

Firms produce single products. We will use i, j, k to index firms or products. The products are differentiated across firms. Firms sell the same product to final consumers and to other firms as an intermediate input, though not all firms sell to other firms, and not each pair of firms has a buyer-supplier relationship. Note that we allow Belgian firms to sell directly to foreign consumers, while all foreign products reach Belgian consumers indirectly through the importing of inputs by Belgian firms.[13]

We treat every firm as infinitesimal when selling to final consumers. Hence, when selling to domestic or foreign final consumers, we assume the market structure is monopolistic competition. When selling to other firms, the assumption of infinitesimal size is no longer reasonable, however, since most firms just have a few selected suppliers. We assume that in the Nash bargaining between buyer and supplier, the buyer has the full bargaining power. Given the assumptions on technology described below, this will imply that the supplier sells

13. The assumption that foreign goods reach Belgian consumers only through Belgian firms is reasonable because in the data nearly all imports are carried out by firms. We make the assumption that Belgian firms can reach foreign consumers directly to avoid modeling foreign firms in detail.

at marginal cost to the buyer firm. We note that in this section with exogenous networks, our main propositions and quantitative results are unchanged if firms charge positive and possibly heterogeneous mark-ups to customer firms as long as these are fixed. **?** analyze a network economy with variable firm-to-firm mark-ups. In our paper, the arguably strong assumption of the bargaining power in firm-to-firm transactions being on the buyer's side will be critical for modeling the network formation game in a tractable manner.[14]

This section assumes a fixed network structure: we take as given the set of firms, Z_j, from which each firm j is eligible to purchase inputs. For importing firms, Z_j contains also foreign, F, as an eligible supplier. Sometimes we will refer to the set of domestic suppliers of firm j, which we denote by Z_j^D.

Firms use a CES input bundle of workers and domestic and foreign inputs with elasticity of substitution $\rho > 1$ in the production function. We assume that $\sigma > \rho$, implying that consumers are more price-elastic than firms in their purchase of goods. Given the CES production function, we can write the cost function of firm j as:

$$c_j(Z_j) = \frac{1}{\phi_j} \left(\sum_{k \in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho} \right)^{1/(1-\rho)} . \tag{3.8}$$

The first term in the cost function, ϕ_j, denotes the exogenous total factor productivity of firm j. Following Antràs, Fort, and Tintelnot (2016), we will call $\Theta_j(Z_j) = \sum_{k \in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}$ the *sourcing capability* of firm j, and Z_j the *sourcing strategy* of firm j. The sourcing strategy may include both domestic and foreign sourcing. The price of labor is denoted by w_ℓ. The share of variable costs by firm j that is spent on intermediate inputs produced by firm $k \in Z_j$ is:

14. Also in the industrial organization literature, corner solutions in the bargaining game are sometimes assumed to obtain tractable solutions for network formation problems. For example, when studying the determinants of the hospital networks offered by health plans, Ho (2009) assumes that hospitals make take-it-or-leave-it offers to all health plans in the market.

$$s_{kj} = \frac{p_{kj}q_{kj}}{c_j q_j} = \frac{\alpha_{kj}^{\rho-1} p_{kj}^{1-\rho}}{\Theta_j(Z_j)}, \tag{3.9}$$

where α_{kj} reflects how salient the good produced by firm k is as an input for firm j. Analogously, the share of variable costs by firm j that is spent on labor is:

$$s_{\ell j} = \frac{w_\ell \ell_j}{c_j q_j} = \frac{\alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}}{\Theta_j(Z_j)}, \tag{3.10}$$

while the *direct* foreign input share of firm j (assuming $F \in Z_j$) is:

$$s_{Fj} = \frac{p_{Fj}q_{Fj}}{c_j q_j} = \frac{\alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho}}{\Theta_j(Z_j)}. \tag{3.11}$$

Before deriving an expression for the total sales of a firm, we discuss the pricing problem of the firm. Due to CES preferences and monopolistic competition, firms charge a constant mark-up over marginal costs, $\mu = \frac{\sigma}{\sigma-1}$, when selling to final consumers at home or abroad. When selling to other firms, firms engage in Nash bargaining with the full bargaining power on the side of the buying firm. The buyer will make the supplier just indifferent between selling to the firm or not, and therefore firms sell at marginal costs to other firms.

In order to sell abroad, firms incur iceberg transport costs, τ. In this section, we take export participation, I_{jF}, as given ($I_{jF} = 1$ for all exporting firms and $I_{jF} = 0$ otherwise) and endogenize it in Section 3.4. Firms' total sales consist of the sum of domestic sales to final consumers, foreign sales to final consumers, and domestic sales to other firms. Let firm j's total sales be denoted by:

$$x_j = \underbrace{\beta_{jH}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\frac{E}{P^{1-\sigma}}}_{\text{Domestic sales to final consumers}} + \underbrace{I_{jF}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}}}_{\text{Exports}}$$

$$+ \underbrace{\sum_k I(j \in Z_k)\mu^{1-\rho}\phi_j^{\rho-1}\alpha_{jk}^{\rho-1}\Theta(j)\frac{x_k/\mu_k}{\Theta_k(Z_k)}}_{\text{Domestic sales to firms}}, \tag{3.12}$$

where μ_k denotes the average mark-up of firm k. Recall that the firm charges a constant mark-up to final consumers and a zero mark-up to other firms. Hence, μ_k depends on the distribution of firm k's sales.

Given that firms make their profits only on sales to final consumers, we can write the *variable* profits of firm j given a sourcing strategy, Z_j, and export participation, I_{jF}, as

$$\begin{aligned}\pi_j^{var}(Z_j, I_{jF}) =& \frac{1}{\sigma}\beta_{jH}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\frac{E}{P^{1-\sigma}} \\ &+ I_{jF}\frac{1}{\sigma}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}}.\end{aligned} \tag{3.13}$$

Firms' dependence on foreign inputs

We now calculate the exposure of firms to foreign inputs, taking into account that the direct and total foreign input share can be substantially different. Let s_{Fj} denote the *total* foreign input share of firm j:

$$s_{Fj}^{Total} = s_{Fj} + \sum_{i \in Z_j^D} s_{ij}s_{Fi}^{Total}. \tag{3.14}$$

The definition of total foreign input share is intuitive in a model with single product firms in which each firm uses the same fraction of foreign inputs in the production sold to every buyer. Proposition 1 shows the link between the total foreign input shares and the cost reduction from international trade in our model.[15]. In the proposition, we also present results based on two alternative modeling assumptions that researchers often make when they do not have access to domestic firm-to-firm transaction information.

Proposition 1 (Cost increases from banning foreign inputs)**.** *Assume* $\rho > 1$.

Given fixed linkages between firms, and leaving domestic nominal wages, w_ℓ*, unchanged, the total cost increase from banning foreign inputs is:*

$$\hat{c}_j \mid_{total}^{p_{F\cdot}\to\infty} = \left(1 - s_{Fj}^{Total}\right)^{1/(1-\rho)}. \tag{3.15}$$

Ignoring linkages and indirect effects (i.e., assuming there is no pass-through of cost changes from domestic suppliers) and leaving domestic nominal wages, w_ℓ*, unchanged, the direct cost increase from banning foreign inputs is:*

$$\hat{c}_j \mid_{direct}^{p_{F\cdot}\to\infty} = (1 - s_{Fj})^{1/(1-\rho)}. \tag{3.16}$$

Finally, if one assumes an economy with roundabout production in which firms' outputs are aggregated to a composite intermediate input according to equation (3.4) *and the composite intermediate input is the only firm-to-firm input in equation* (3.8)*, the cost increase from banning foreign inputs,* $\hat{c}_j \mid_{roundabout}^{p_{F\cdot}\to\infty}$*, is implicitly defined as:*

$$\left(\hat{c}_j \mid_{roundabout}^{p_{F\cdot}\to\infty}\right)^{1-\rho} = s_{\ell j} + s_{Dj}\left(\sum_k s_{kD}\left(\hat{c}_k \mid_{roundabout}^{p_{F\cdot}\to\infty}\right)^{1-\sigma}\right)^{\frac{1-\rho}{1-\sigma}}, \tag{3.17}$$

where s_{Dj} *is the share of firm j's domestic intermediate good purchases and* s_{kD} *is the share of firm k in the intermediate good bundle (measured by firm k's share in total domestic sales).*

The result on the firm-level cost increase from banning imports, $\hat{c}_j \mid_{total}^{p_{F\cdot}\to\infty}$, reflects that a firm's cost will not only rise according to it's own direct foreign input share, but also

15. Note that the assumption made in the proposition that nominal wages are unchanged is not that restrictive, since nominal wages can be normalized to any value under autarky

according to its suppliers' foreign input shares, suppliers' suppliers' foreign input share, and so forth. This firm-level exposure is summarized in the total foreign input share expression. The elasticity of substitution in the production function, ρ, indicates how easy it is to switch to alternative inputs, including labor. Given the observed total foreign input share, a lower value of ρ leads to larger cost increases from banning foreign trade.

In contrast, the expression $\hat{c}_j \mid_{direct}^{p_{F\cdot}\to\infty}$, yields only the direct effect of on firm-level cost from banning foreign trade (mirroring the results in earlier work by Arkolakis, Costinot, and Rodríguez-Clare (2012) and Blaum, Lelarge, and Peters (2016)). It can be rationalized from our model under a network in which importers sell all their output to final consumers, and hence the total foreign input share of domestic suppliers of other firms in equation (3.14) is zero. Since the observed production networks differ from this extreme case, using the formula from equation (3.16) leads to cost increases from banning foreign inputs that are too low compared to the full effect summarized in equation (3.15).

In the absence of firm-to-firm transaction data, it is possible to approximate the indirect effect by assuming a roundabout production structure (as for example in Blaum, Lelarge, and Peters (2016)).[16] Under the roundabout production assumption, every firm with the same intermediate input share will have the same indirect exposure to foreign goods. However, this assumption is at odds with our data, and could create bias in the calculation of the firm-level cost effects from trade.

Aggregation and equilibrium

We now describe the aggregation of our model, discuss how firm profits are redistributed to consumers and define the equilibrium. In the model with a fixed production network we abstract from fixed costs of linkage formation, and hence $\pi_j = \pi_j^{var}$.

16. Blaum, Lelarge, and Peters (2016) obtain a total cost reduction result from foreign inputs in which each firm buys the same CES bundle of intermediate inputs. The production function in their paper is Cobb-Douglas in intermediate inputs and own labor input. Our roundabout cost reduction result in equation (3.17) is similar to their result, but derived for a production function which is CES between labor input and intermediates.

We assume that the set of Belgian firms is fixed and that firm profits are distributed to workers in Belgium. We consider Belgium as a small-open economy and assume that there are no foreign asset holdings and that trade is balanced. Hence aggregate expenditure in Belgium is given by

$$E = w_\ell L + \sum_k \pi_k. \tag{3.18}$$

Balanced trade implies that aggregate exports are equal to aggregate imports:

$$\sum_j I_{jF} \beta_{jF}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j (Z_j)^{(\sigma-1)/(\rho-1)} \tau^{1-\sigma} \frac{E_F}{P_F^{1-\sigma}} = \sum_j \frac{1}{\mu_j} s_{Fj} x_j. \tag{3.19}$$

Labor market clearing implies that labor income is equal to firms' labor costs:

$$w_\ell L = \sum_j \frac{1}{\mu_j} s_{\ell j} x_j. \tag{3.20}$$

We next define the equilibrium for the small open economy.

Definition 1 (Equilibrium given a fixed network structure)**.** *Given foreign expenditure, E_F, foreign price index, P_F, and a set of prices by foreign suppliers, $\{p_{Fj}\}_j$, an equilibrium for the model with a fixed network structure and fixed export participation is a wage level, w_ℓ, price index for the consumer, P, and aggregate expenditure, E, such that equations* (3.6), (3.8), (3.9), (3.10), (3.12), (3.13), (3.18), (3.19), *and* (3.20) *hold.*

We establish uniqueness of the equilibrium in the closed economy in the following lemma.

Lemma 1 (Uniqueness of equilibrium under closed economy)**.** *Define a $K \times K$ matrix A where where the (i, j) element is $\phi_j^{\rho-1} \alpha_{ij}^{\rho-1}$ if $i \in Z_j$ and 0 otherwise, and K denotes the number of Belgium firms. Assume the matrix $\left(I - A'\right)$ is invertible, where I is the identity matrix. Then under a closed economy, for a given nominal wage, there exists a unique equilibrium defined in Definition 1.*

Lemma (1) is useful because it implies that the counterfactual equilibrium without trade is unique. We next proceed to discuss the change in the aggregate price index arising from banning international trade.

Proposition 2 (Change in aggregate price index from banning international trade)**.** *Let* s_{iH} *denote firm i's share in household demand in in the initial equilibrium prior to raising the barriers to trade.*

Given a fixed set of firms, network structure and nominal wage, the price index change from banning international trade can be summarized as follows:

$$\hat{P} \mid_{total}^{p_{F.}\to\infty} = \left(\sum_i s_{iH} \left(\hat{c}_i \mid_{total}^{p_{F.}\to\infty} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{3.21}$$

If the price of intermediate goods is assumed to be unchanged, the price index change can be expressed as

$$\hat{P} \mid_{direct}^{p_{F.}\to\infty} = \left(\sum_i s_{iH} \left(\hat{c}_i \mid_{direct}^{p_{F.}\to\infty} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{3.22}$$

Finally, if one assumes roundabout production as defined in proposition 1, then the expression becomes

$$\hat{P} \mid_{roundabout}^{p_{F.}\to\infty} = \left(\sum_i s_{iH} \left(\hat{c}_i \mid_{roundabout}^{p_{F.}\to\infty} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{3.23}$$

The change in the price index from banning international trade is a weighted aggregate of each firm's cost increase with the weight equal to the firm's share in domestic household demand, s_{iH}, in the initial economy with international trade. Propositions 1 and 2 imply that the change in the aggregate price index depends on the underlying network structure. To see this, consider a production network in which all the imports are made by firms that had no sales to other domestic firms or to the domestic final consumers. In that case, all the cost increases from banning foreign goods would accrue to firms for which the share in domestic household demand is zero, and therefore the price index effect would be zero as well. Suppose instead that importers of foreign goods had no sales to other domestic firms, but sold all their output to domestic final consumers. Then, the price index increase is given by equations (3.16) and (3.22). However, with positive sales to other domestic firms by the importers, the price index effect is given by equations (3.15) and (3.21).

A corollary of the results from propositions 1 and 2 is that two economies with the same elasticities of substitutions in production and in the utility functions, and the same levels of aggregate imports and exports, GDP, *and* gross production, can have different gains from trade. We illustrate this in a simple numerical example in Appendix C.2.[17]

The above results illustrates that knowing the underlying micro-structure of the economy is relevant for the quantitative analysis of the gains from trade. In the following subsection, we make use of the detailed information about domestic firm-to-firm transactions in our data when we calculate the welfare gains from trade for the Belgian economy.

3.3.2 Empirical results

In this section, we provide a quantitative analysis of how international trade affects firms' production costs and the consumer price index. As shown above, this analysis requires information on the observed firm-to-firm transactions, firm-level output, international trade flows, and labor input, in combination with estimates or assumptions for the elasticity of substitution in the production function, ρ, and the utility function, σ. Throughout the paper, the baseline specification assumes σ is equal to 4 and ρ is equal to 2.[18] We perform sensitivity analysis to examine how the results vary with the choice of these parameter values.

To assess the implications of banning foreign inputs, we compute the firm level cost

17. This example is related to the discussion in **?** that the gains from trade can be arbitrarily large in a model with sequential production, as the number of stages of production increases. However, note that the ratio of gross production to GDP also rises in their example as the number of stages gets larger. Here we hold the level of gross production and GDP fixed, and illustrate that the gains from trade can still differ.

18. Using data for the US, Antràs, Fort, and Tintelnot (2016) estimate $\sigma = 3.85$ and **?** estimate a level of σ between 3 and 5 among various manufacturing industries.

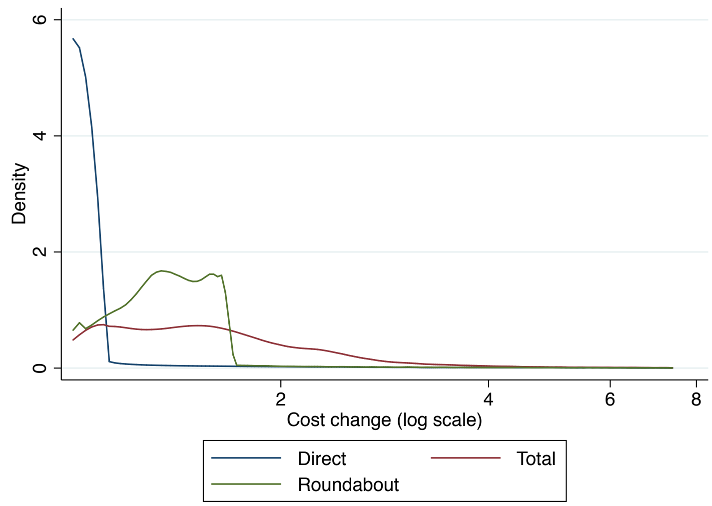
increases by making use of proposition 1. We use the firm-to-firm network structure as observed in 2012. Proposition 1 tells us that the total shares of foreign inputs for each firm, s_{Fj}^{Total}, translates to the cost changes that firms face from banning foreign inputs. Figure 3.3 displays the cost increase of firms from banning international trade (in the red line). This figure also reports the cost increase under the assumptions of the direct effect (blue line) and roundabout economy (green line). In these two cases, it is not necessary to observe domestic firm-to-firm transactions to calculate the cost increases.

As evident from Figure 3.3, international trade matters much more for firms' production costs if we use our model with fixed domestic production networks than in the direct effect and roundabout economy. For the median firm (in the distribution of cost changes), the cost increase from banning foreign inputs is 70% in our model with domestic production networks, 41% in the model with the roundabout economy assumption, and zero when considering only the direct effect. As expected, the cost increases are the lowest under the direct effect assumption. In the roundabout production economy, the non-importing firm's cost increase is bounded above by the price increase of the composite intermediate good. By comparison, when taking the actual production network into consideration, many firms have a cost increase above 65%, while the roundabout model suggest that very few importing firms have cost increases of that magnitude.

In Appendix C.9, we plot the same distributions for different parameter values of σ and ρ. Even for alternative parameters, the distribution of cost changes under the actual production network has a much thicker right tail than the distribution under the roundabout production assumption. While the distributions become closer to each other if $\sigma = \rho = 2$, even in that case, the 90th percentile of firm-level cost increases under roundabout is 152%, whereas it is 245% when the actual firm-to-firm transactions are used in the calculation (see Table C.12 in the Appendix).[19]

19. To read Table C.12, recall that in the roundabout economy, σ is the parameter used in the aggregation to the intermediate input bundle. Note that altering σ does not affect the calculation of $\hat{c}_j \mid_{total}^{p_{F\cdot}\to\infty}$, but it affects the calculation of $\hat{c}_j \mid_{roundabout}^{p_{F\cdot}\to\infty}$.

Figure 3.3: Distributions of $\hat{c}$ from banning imports



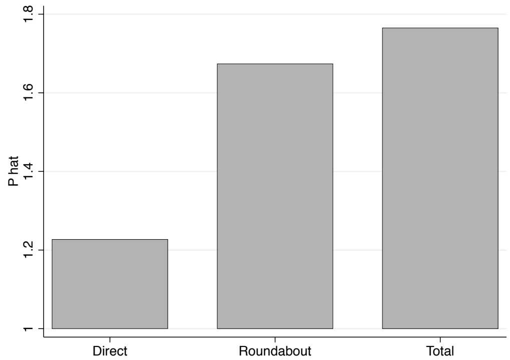
Notes: The parameters used are $\rho = 2, \sigma = 4$. See the appendix for derivations and plots for different parameter values.

In Figure 3.4, we make use of proposition 2 to analyze the implications for the aggregate consumer price index from banning foreign inputs. This figure reports the changes in aggregate price indices, $\hat{P}$, for the three different cases.[20] The increase in the price index is much larger when one takes into account the domestic production network than when one only takes into account firms' direct exposure to foreign inputs. While the price index increases by 23% if only the direct effect of banning imports is considered, it rises by as much as 77% if the full network structure of domestic production is taken into account. Compared to the model of roundabout production, we also find a larger change in the price index when using the information contained in the firm-to-firm transaction data. Under roundabout production, the domestic price index increases by 67%.

In Appendix C.9, we report the analogous numbers for $\hat{P}$ under different parameters of σ and ρ. As one would expect, the gains from trade get larger as σ and ρ get lower. We robustly find that the price index increase from banning international trade is larger

20. Note that s_{iH} is not observable in standard datasets. However, firm i's share of total domestic sales, s_{iD}, is usually observed. As shows in Appendix section C.1.1, s_{iD} and s_{iH} are identical in the roundabout case. We use $s_{iD}$when calculating the price index changes for the direct effect economy and the roundabout economy.

Figure 3.4: Change in aggregate price index, $\hat{P}$, from banning imports



Notes: The parameters used are $\rho = 2, \sigma = 4$. See the appendix for derivations and plots for different parameter values.

in our model than under the roundabout economy assumption, as long as σ is sufficiently large relative to ρ. On the one hand, the assumption in the roundabout economy that each firm obtains the same exposure to foreign goods through its domestic intermediate input purchases leads to larger gains from trade in the roundabout economy, as it is harder for consumers to substitute away from firms that face higher costs. On the other hand, the common assumption in roundabout production economy models (**?** and Blaum, Lelarge, and Peters, 2016) is that aggregation of the intermediate input bundle occurs analogous to the aggregation for final consumption goods (equation (3.4)), using σ as the elasticity of substitution in that aggregation. That makes it easier in the intermediate input bundle to substitute from firms exposed to large imports to non-importing firms, leading to lower cost increases and lower gains from trade. The second effect dominates when σ is sufficiently large as compared to ρ, leading to larger gains from trade when using the actual firm-to-firm production network data.

Our analysis so far has taken the network structure of firms as fixed. This has allowed us to derive analytic solutions for the firm-level cost and aggregate price index changes in the absence of international trade that could then be calculated easily with the firm-to-firm transaction and international trade data available to us. In the following section, we analyze how the network forms endogenously and the implications of endogenous networks for the quantification of the gains from trade.

3.4 A model of trade with endogenous production networks

This section develops a model of trade with endogenous network formation, allowing buyer-supplier relationships to change in response to trade shocks. The model builds on the theoretical framework presented in Section 3.3.1. We assume the same preferences, demand functions, production technology, and market structure as in the model with fixed production networks. What is new is that firms now optimally choose their set of suppliers (i.e. the firm's sourcing strategy) and decide whether to import and export. We first describe the model with endogenous network formation and discuss how it can be solved. Then we estimate the model, and use it to quantify how international trade affects firms' production costs and consumer prices with and without endogenous network formation.

3.4.1 Model

Determination of firm sourcing strategy, import and export participation

We assume that only buyers initiate linkages with other domestic firms. Forming linkages to suppliers is costly, and firm j incurs a random, firm-pair-specific, fixed cost f_{kj} to add supplier k. The realization of fixed costs is known to the firm at the time it selects suppliers. Firms in our model make profits due to positive mark-ups in sales to domestic and foreign final consumers. Since the buyer is assumed to have all the bargaining power, firms do not make profits from sales to other firms. Hence, variable profits are proportional to firm-level sales to final consumers.

Given a sourcing strategy, Z_j, and export participation choice, I_{jF}, the profits of firm j are equal to variable profits minus the fixed costs of domestic and foreign sourcing,

$\sum_{k \in Z_j} f_{kj} w_\ell$, and the fixed costs of exporting, $I_{jF} f_{jF} w_\ell$:

$$\pi_j(Z_j, I_{jF}) = \pi_j^{var}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj} w_\ell - I_{jF} f_{jF} w_\ell. \tag{3.24}$$

We assume that firm j exogenously meets a set of eligible suppliers, $\mathbf{Z}_j$. Firm j then endogenously decides on the set of suppliers and whether or not to export:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j\ ,\ I_{jF} \in \{0, 1\}. \tag{3.25}$$

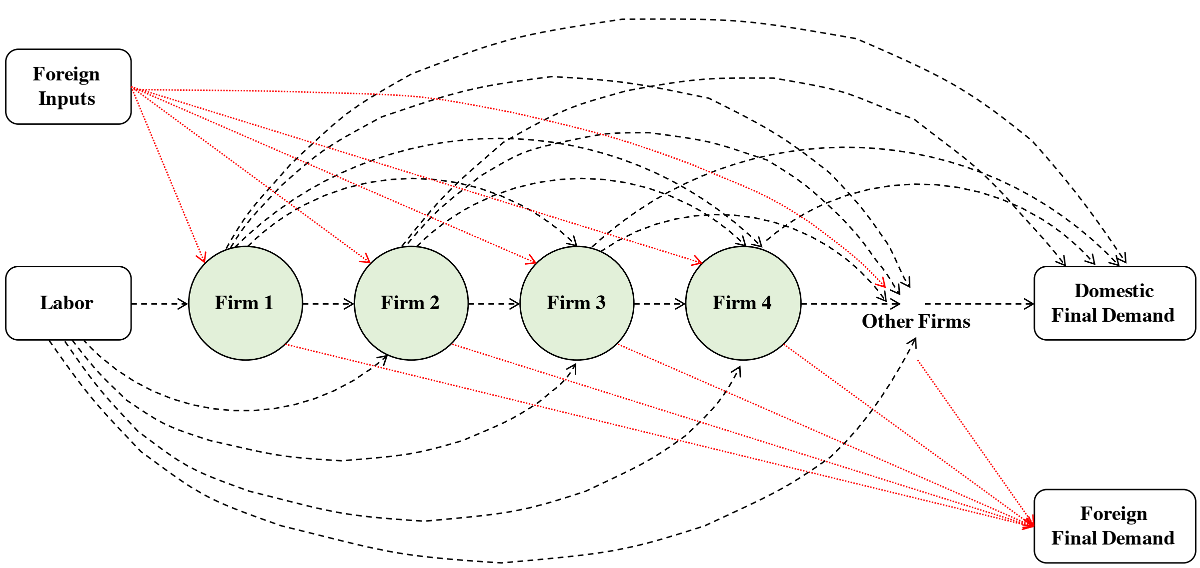
Under the assumption made earlier that final demand is more elastic than the elasticity of substitution between inputs in the production function, $\sigma > \rho$, the sourcing decision is complementary across suppliers as well as complementary to the exporting decision. In other words, the marginal benefit of adding a supplier is increasing in the set of existing suppliers and it is higher if the firm exports. We can therefore follow the same approach as Antràs, Fort, and Tintelnot (2016), namely the adaption of the Jia (2008) algorithm, to solve the problem described in (3.25), given knowledge about the costs of the eligible suppliers in $\mathbf{Z}_j$.

To solve the model with endogenous network formation, we therefore need to determine the set of eligible suppliers, $\mathbf{Z}_j$. When solving the problem for every firm described in (3.25), a key issue that arises is that each firm needs to guess a set of costs for its eligible suppliers, where the costs themselves are equilibrium objects and depend on everyone's sourcing decision. Not only is it extremely challenging computationally to find a fixed point in the set of costs for all firms so that these costs are consistent with everyone's optimal sourcing decision, but also the uniqueness of such a fixed point is rather unlikely. On the one hand, if firms guess that suppliers have very high unit costs, this could result in the formation of very few linkages and lead to high unit costs overall. On the other hand, if firms guess that suppliers have very low unit costs, this could result in the formation of many linkages and lead to low unit costs overall. To get around these problems, we consider the formation of an acyclic network, postulating an ordering of firms and restricting the eligible set of suppliers to firms that appear prior to the buyer.

To be concrete, all firms can choose to import foreign inputs and to export their output abroad. However, the set of eligible domestic suppliers varies across firms. Specifically, we order firms in a sequence $S = \{1, 2, 3, ..., N\}$ that restricts the set of eligible suppliers, as illustrated in Figure 3.5. Because we assumed buyers has the full bargaining power in any firm-to-firm transactions, firms only need to know the choices of the firms prior in the sequence. Taken together, these assumptions make the network formation tractable, as we describe below.

Firm 1 is first in the sequence and can only hire labor inputs. To make its decision of how much labor to hire, firm 1 only needs to know the wage level, w_ℓ, and domestic market demand, $\frac{E}{P^{1-\sigma}}$. Firm 2 is second in the sequence and can hire both labor inputs as well as purchase the input produced by firm 1. To make these decisions, firm 2 needs to know the wage and market demand level as well as the cost of its eligible supplier (firm 1). Firm 3 can hire labor and purchase the inputs from firm 1, firm 2, or both. And so on. Given a guess for equilibrium wages, w_ℓ, and domestic market demand, $\frac{E}{P^{1-\sigma}}$, one can solve the problems of the firms sequentially.

Figure 3.5: Endogenous network formation – eligible connections



Although the above ordering of firms simplifies the problem, we are still limited in the number of possible sourcing strategies we can feasibly evaluate. We therefore restrict the set of eligible suppliers for firm j, $\mathbf{Z}_j$, to be a random subset from the set of firms prior to firm j in the sequence. The suppliers for firm j are then optimally chosen as the solution to the problem in (3.25). In practice, we choose the cardinality of $\mathbf{Z}_j$ to be at most 200, so the firm still chooses among 2^{200} possible supplier sets. By following Antràs, Fort, and Tintelnot (2016)'s adaption of Jia (2008)'s algorithm, we are then able to tractably solve the discrete choice problem.[21] The firm's order in the sequence of supplier choices and its set of eligible suppliers are becoming attributes of the firm and therefore primitives of the model.

Imposing a tie-breaking rule that in the case of indifference a supplier is included ensures a unique solution to the problem in (3.25). As a consequence, the network formation will also be unique given a set of wages and a guess for the price index. We can then alter wages, price index, and expenditure to achieve labor market clearing, trade balance, and a fixed point for the price index and expenditure. Importantly, we are searching here only for a fixed point in wages and price index (only 2 scalars) as opposed to searching for a large fixed point vector in every firm's costs and searching strategies. In other words, the ordering approach implies that even with a rich micro structure and firm-level heterogeneity, knowing only two equilibrium variables is sufficient to solve sequentially the firms' problems. We discuss the aggregation and the equilibrium with endogenous network structure more formally below.

Aggregation and equilibrium

The model aggregation and equilibrium are broadly similar to the case with fixed networks. However, there are a few notable differences. The first is that firms incur fixed costs – paid in units of labor – to add a domestic buyer, import, or export. Therefore, the labor market

21. We describe the computational algorithm to solve for the firm's problem in Appendix C.7. The procedure is very similar to the one described in Antràs, Fort, and Tintelnot (2016). Here, we also develop a greedy algorithm in the case the the differences in the lower and upper bounds for the optimal solution are too wide to evaluate the profits of all feasible combinations in between. As in Antràs, Fort, and Tintelnot (2016), we find that in about 99% of the cases the lower and upper bounds are perfectly overlapping (see Table C.10 in the Appendix).

clearing condition becomes:

$$w_\ell L = \sum_j \frac{\mu_j - 1}{\mu_j} s_{\ell j} x_j + w_\ell \sum_j \left(\sum_{k \in Z_j} f_{kj} + I_{jF} f_{jF} \right). \tag{3.26}$$

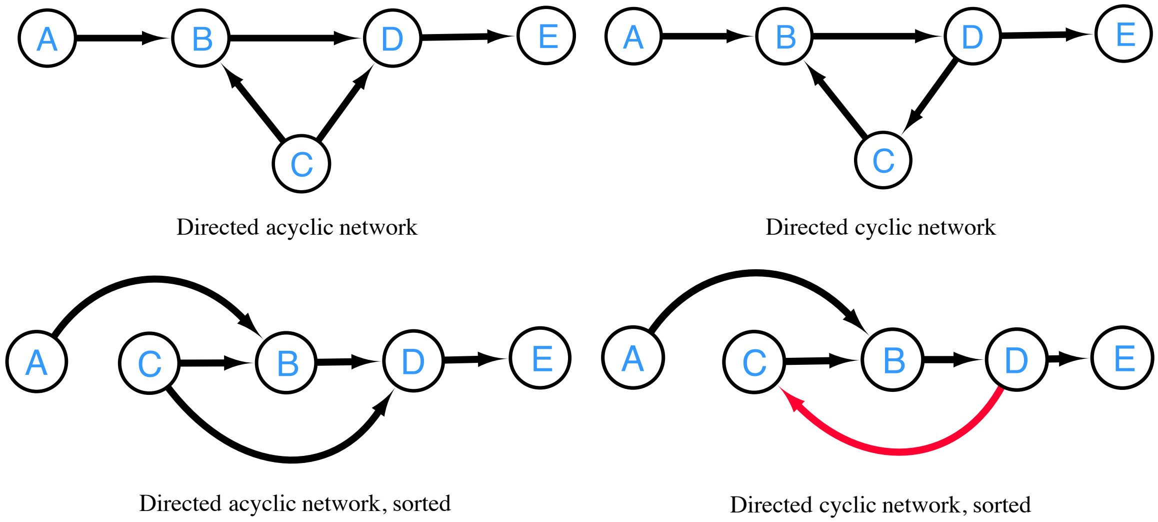
Additionally, a firm's profit function now subtracts the incurred fixed costs, and a firm's sourcing strategy and export participation are now endogenous choices. However, the trade balance condition remains unchanged. The following definition formally describes equilibrium with an endogenous network structure.

Definition 2 (Equilibrium with endogenous network structure)**.** *Given foreign expenditure, E_F, foreign price index, P_F, and a set of prices by foreign suppliers, $\{p_{Fj}\}_j$, as well as set of eligible suppliers, $\mathbf{Z}_j$ that satisfies acyclicity of the network, an equilibrium for the model with endogenous network structure and endogenous export participation is a wage level, w_ℓ, price index for the consumer, P, and aggregate expenditure, E, as well as a set of sourcing strategies and export participation choices, such that the firm's optimization problem in* (3.25)*, and equations* (3.6)*,* (3.8)*,* (3.9)*,* (3.10)*,* (3.12)*,* (3.13)*,* (3.18)*,* (3.19)*, and* (3.26) *hold.*

3.4.2 Assessing the assumptions about the shape of the network

Given the assumptions we invoked to solve the endogenous network formation, the resulting network will be acyclic. As shown in Figure 3.6, in an acyclic firm network, there exists at least one way to sort firms so that all directed edges face one direction. In contrast, in a cyclical network at least one edge will face the opposite direction. This feature of our network formation mechanism is admittedly restrictive. We now perform two checks to assess how well the Belgian data can be approximated by an acyclical network.

Figure 3.6: Examples of acyclic and cyclic networks



How cyclic is the production network?

Let $\nu(i)$ be an ordering of firms that maps firms $\{i, j, k, \cdots\} \in \Theta$ into numbers from $\{1, \cdots N\}$. To describe how cyclical the Belgian production network is, we want to find the optimal $\nu(k)$ that minimizes the following objective function:

$$\min_{\{\nu(k)\}} \sum_{i,j} \mathbf{1}\left\{i \in Z_j\right\} \mathbf{1}\left\{\nu(i) > \nu(j)\right\},$$

where Z_j is the supplier set of firm j. Solving this problem corresponds to minimizing the number of directed edges that are facing the direction opposite to that of the sorting order. In other words, we try to find an ordering that minimizes the number of arrows facing to the left in the cyclic network in Figure 3.6.

To solve this problem, which is also known as the feedback arc set problem, we adopt an algorithm proposed by Eades, Lin, and Smyth (1993). The details of the computational algorithm and implementation are presented in Appendix C.6. The algorithm offers a local minimum, showing that at most 17% of edges in the whole firm-to-firm network in 2012 violate acyclicity.[22] We also search for an ordering that minimizes the value of firm-to-firm

22. While there is no perfect reference point for this figure, we can compare it to the structure of the

sales in violation of acyclicity.[23] We find that no more than 22% of firm-to-firm sales are in violation of acyclicity. We will refer to the former as the unweighted ordering algorithm and the latter as the weighted ordering algorithm.

A natural question that arises is how different the structure of an economy with an acyclic network is in comparison to the economy observed in the data. One way to make this comparison is to calculate input-output tables with and without the firms in buyer-supplier relationships that violate acyclicity. We find that when calculating input-output tables with 72 sectors, the correlation between the input-output table coefficients from the full data and the data without links in violation of the ordering is 0.92 when using the the unweighted ordering algorithm output. The correlation is even higher, 0.97, when using the weighted ordering algorithm output.[24]

Gains from trade under fixed networks: cyclic versus acyclic production networks

Another way to assess the assumption of an acyclic network is to examine how the results based on the exogenous network model change if we exclude transactions that violate acyclicity. It is reassuring to find that estimated effect of international trade on consumer prices and firms' cost of production are very similar if we only uses firm-to-firm sales that are consistent with the acyclic network obtained by the ordering algorithm described in Section 3.4.2. Specifically, we keep the direct import share of each firm the same as in the data, set

directed social network Twitter. Simpson, Srinivasan, and Thomo (2016) calculate that 23% of edges are in violation of acyclicity in the Twitter network in the year 2010.

23. Specifically, we solve the following problem: $\min_{\{\nu(k)\}} \sum_{i,j} x_{ij} \mathbf{1}\left\{\nu(i) > \nu(j)\right\}$, where x_{ij} is the value of the sales from firm i to firm j.

24. To construct the input-output table coefficients, we aggregate firm-to-firm transactions within the supplying and buying sector. We note that this procedure differs from the national account definition of an input-output table. First, the rows and columns of our aggregated tables are referring to the main sectors of the buyers and suppliers, but these firms can also have a significant share of their production in other sectors. Second, in national account tables, the contribution of the retail and wholesale sectors to the production of the other goods only refer to the trade margin of retailers and wholesalers. In our data, the retail and wholesale sectors are accounted based on their total sales and total input consumption and not on their trade margin.

all transactions in violation of the ordering to zero, and adjust all other domestic firm-to-firm input shares such that share of each firm j's input purchases, $\sum_{i \in Z_j} s_{ij}$, is unchanged.[25] The results presented in Table C.14 in the Appendix show that the gains from trade under an exogenous network are virtually identical if we only use the subset of transactions for which the domestic production network is acyclic.

3.4.3 Estimation of model parameters given endogenous network

When allowing for endogenous network formation, we are not able to analytically solve the model. Instead, we will structurally estimate the model and provide numerical results for the counterfactual analysis.

In the estimation of our model, we simulate 100,000 Belgian firms, which is close to the 139,605 firms in our sample in 2012. A firm is characterized by a core productivity level, a set of eligible suppliers that satisfies the ordering, a vector of fixed cost draws for all eligible suppliers, a vector of firm-pair-specific cost shifters, a foreign input cost shifter, fixed costs of importing and exporting, and a foreign demand shifter. We normalize firms' labor productivity shifters, $\alpha_{\ell j} = 1$, and firms' domestic final demand shifters, $\beta_{jD} = 1$.

As a first step of the estimation, we recover the productivity distribution of firms (scaled by some general equilibrium objects) from the identity

$$\frac{x_{iH}^{1/(\sigma-1)}}{s_{\ell i}^{1/(1-\rho)}} = \phi_i \frac{PE^{1/(\sigma-1)}}{\mu w_\ell}. \tag{3.27}$$

Observing all the terms on the left hand side enables us to estimate the distribution $\phi_i \frac{PE^{1/(\sigma-1)}}{\mu w_\ell}$. After visually inspecting the distribution, we assume it is log-normal, and estimate the scale parameter to be -2.12 and the dispersion parameter to be 1.37.

We next turn to the estimation of the parameters for the distribution of the firm-pair-

25. The only exception is when there is no other domestic supplier of a firm, in which case in the data with only acyclic transactions the domestic firm-to-firm input share is set to zero.

specific shifter in the production function, α_{kj}, the foreign input cost shifter, α_{Fj}, the foreign demand shifter, β_{jF}, as well as the distribution of fixed cost parameters for domestic firm-to-firm purchases, $w_\ell f_{kj}$, fixed cost of importing, $w_\ell f_{Fj}$, and fixed cost of exporting $w_\ell f_{jF}$. We again impose log-normality of the distributions and estimate the scale and dispersion parameters of those distributions. We assume that α_{Fj}, α_{kj}, and β_{jF} are independent draws from three log-normal distributions which share a common dispersion parameter, $\Phi^{\alpha,\beta}_{disp}$, and have different scale parameters, $\Phi^{\alpha_F}_{scale}$, $\Phi^{\alpha_{\text{dom}}}_{scale}$, $\Phi^{\beta_F}_{scale}$, respectively. Similarly, the fixed cost draws for domestic purchases from other firms, imports, and exports, are drawn independently from three log-normal distributions with scale parameters $\Phi^{f_{\text{dom}}}_{scale}$, $\Phi^{f_{\text{imp}}}_{scale}$, $\Phi^{f_{\text{exp}}}_{scale}$, and a common dispersion parameter, Φ^{f}_{disp}. The parametric restrictions on the common dispersion parameters imply we need to estimate only 2 instead of 6 dispersion parameters. Overall, there are 8 parameters to be estimated.

We use method of simulated moments to estimate our parameters. We target three sets of moments to match. The first set of moments is helpful in estimating the parameters affecting domestic-firm-to firm purchases. The draws of the fixed costs govern the extensive margins of firm-to-firm trade. Thus, we use information about firms' numbers of suppliers to identify $\Phi^{f_{\text{dom}}}_{scale}$ and Φ^{f}_{disp}. To do this, we match the model to have the same quartile distribution of number of suppliers. Following the procedure used by **?**, we include in the first vector of moments generated by the model, $\hat{m}_1(\Phi)$, the proportion of firms that has a number of suppliers equal to the first, second, third, and fourth quartile in the data. The draws of α_{kj} govern the distribution of both the intensive margin and the extensive margin of firm-to-firm transactions. To identify the parameters Φ^{α}_{scale} and $\Phi^{\alpha,\beta}_{disp}$, we target statistics on the labor share of firms. Again, we aim to match the fraction of firms in the data that have labor shares in the first, second, third, and fourth quartile of the actual labor share in the data. Relatedly, we also aim to match the distribution of the actual firm-to-firm input shares (conditional on observing trade between firms). Using the same procedure as above, we include the fraction of firms in the four quartile bins of that distribution (using as thresholds the quartiles observed in the data). This generates 12 elements in the vector $\hat{m}_1(\Phi)$.

The second set of moments is helpful in estimating the parameters affecting imports and exports. We include in $\hat{m}_2(\Phi)$ the share of firms that import and export, respectively. We also include the fraction of firms falling into the bins of the first, second, third, and fourth quartile of imports in firm-level inputs in the data. Similarly, the fraction of firms that have the ratio of exports to firm-level domestic sales as in the quartiles in the data. There are 10 elements in the vector $\hat{m}_2(\Phi)$. Finally, as a third set of moments we include aggregate targets such as the ratio of aggregate exports to aggregate final demand, and the weighted aggregate of firm-level sales to households and foreign input shares, which correspond to the sufficient statistics for the price index increase under fixed networks. There are 3 elements in the vector $\hat{m}_3(\Phi)$.

We describe the difference between the moments in the data and in the simulated model by $\hat{y}(\Phi)$:

$$\hat{y}(\Phi) = m - \hat{m}(\Phi) = \begin{bmatrix} m_1 - \hat{m}_1(\Phi) \\ m_2 - \hat{m}_2(\Phi) \\ m_3 - \hat{m}_3(\Phi) \end{bmatrix},$$

and the following moment condition is assumed to hold at the true parameter value Φ_0:

$$E\left[\hat{y}(\Phi_0)\right] = 0. \tag{3.28}$$

The method of simulated moments selects the model parameters that minimize the following objective function:

$$\hat{\Phi} = \arg\min_{\Phi} \left[\hat{y}(\Phi)\right]^{\top} \mathbf{W} \left[\hat{y}(\Phi)\right], \tag{3.29}$$

where $\mathbf{W}$ is a weighting matrix.[26]

26. We weight the moments equally, hence the weighting matrix is the identity matrix.

Estimation results

Table 3.3 shows the values of the estimated parameters.

Table 3.3: Estimated parameters

Preference and production				Fixed costs			
$\hat{\Phi}^{\alpha_{\text{dom}}}_{scale}$	$\hat{\Phi}^{\alpha_F}_{scale}$	$\hat{\Phi}^{\beta_F}_{scale}$	$\hat{\Phi}^{\alpha,\beta}_{disp}$	$\hat{\Phi}^{f_{\text{dom}}}_{scale}$	$\hat{\Phi}^{f_{\text{imp}}}_{scale}$	$\hat{\Phi}^{f_{\text{exp}}}_{scale}$	$\hat{\Phi}^{f}_{disp}$
-6.69	-2.36	-1.94	2.40	-2.64	8.01	9.47	8.65

The magnitude of the estimated parameters are difficult to interpret, since their scale is affected by the choice of normalizations for the foreign market size and the price of the foreign input. We therefore focus on the model fit given these parameter estimates.

Table 3.4 shows how well we fit the moments that we target. Note that instead of showing the moments directly (i.e., the fraction of firms falling into each quartile bin), we show the values of the 25th, 50th, and 75th percentiles in both the data and model. The model does a pretty good job at fitting the targeted statistics of firm-to-firm transactions. Most statistics are very close between the model and the data. With respect to fitting firm-level statistics of trade, the model under-predicts the fraction of firms directly involved into exporting and importing (9.4% and 15.4% in the data and 6.7% and 7.9% in the model, respectively). However, the model does a decent job of fitting the aggregate importance of trade, illustrated by a similar ratio of aggregate exports to aggregate sales to domestic final demand in both model and data. The statistics summarizing the gains from trade under fixed networks are also similar in model and data.

We also examine how well the model fits moments that were not directly targeted in the estimation. Specifically, we have not targeted directly the association of size between buyers and suppliers that trade with each other. Consistent with the data, the model predicts a weak negative correlation between the number of suppliers of the buying firms (indegree buyer) and the number of buyer firms of suppliers (outdegree supplier). Similarly, the correlation between sales of the buying and selling firm is close to zero both in the data and in the model.

Table 3.4: Model fit: targeted moments

	Data	Model
Number of suppliers 25th percentile	15	15
Number of suppliers 50th percentile	28	30
Number of suppliers 75th percentile	49	53
Share of labor costs 25th percentile	0.12	0.21
Share of labor costs 50th percentile	0.28	0.33
Share of labor costs 75th percentile	0.50	0.66
Firm-to-Firm share 25th percentile	0.0002	0.0001
Firm-to-Firm share 50th percentile	0.0015	0.0004
Firm-to-Firm share 75th percentile	0.0069	0.0035
Share of firms that export	0.094	0.067
Share of exports in total firm sales 25th percentile	0.008	0.001
Share of exports in total firm sales 50th percentile	0.120	0.020
Share of exports in total firm sales 75th percentile	0.649	0.185
Share of firms that import	0.154	0.079
Share of imports in firm inputs 25th percentile	0.008	0.012
Share of imports in firm inputs 50th percentile	0.239	0.066
Share of imports in firm inputs 75th percentile	0.653	0.317
Ratio of aggregate exports to aggregate sales to domestic final demand	1.02	0.87
$\hat{P}\mid_{total}^{p_{F\cdot}\to\infty}=\left(\sum_i s_{iH}\left(1-s_{Fj}^{Total}\right)^{\frac{1-\sigma}{1-\rho}}\right)^{\frac{1}{1-\sigma}}$	1.77	1.93
$\hat{P}\mid_{direct}^{p_{F\cdot}\to\infty}=\left(\sum_i s_{iH}\left(1-s_{Fj}\right)^{\frac{1-\sigma}{1-\rho}}\right)^{\frac{1}{1-\sigma}}$	1.21	1.13

Notes: Percentiles are calculated based on all firms in the sample; Share of labor costs refers to the fraction of labor costs in costs (labor costs + domestic purchases + imports) and the percentiles are calculated based on all firms the sample; Firm-to-Firm share refers to the fraction of costs a firm spends on one particular supplier and the percentiles are calculated for all firm-to-firm transactions; The percentiles for share of exports in total firm sales are calculated for all firms with positive export sales. The percentiles for share of imports in firm inputs are calculated for all firms with positive import purchases.

Table 3.5: Model fit: non-targeted moments

	Data	Model
Corr (Indegree Buyer, Outdegree Supplier)	-0.05	-0.06
Corr (Sales Buyer, Sales Supplier)	-0.02	0.01

3.4.4 Counterfactual with endogenous network formation

Equipped with the parameter estimates of our model, we next turn to the quantitative analysis of the gains from trade for the Belgian economy under endogenous production networks. We repeat the same counterfactual experiment as under fixed networks, and ask how the economic outcomes in the Belgian economy would change if the barriers to trade were infinite. To shut down trade, we make the costs to import and export prohibitively large, so that no firm will engage in international trade. We keep the size of the domestic labor force and all other parameters at their estimated level. We then solve the problem of a closed economy and normalize the nominal wage to the same level it took in the open economy. To solve for the counterfactual equilibrium, we find a fixed point in the market demand, $\frac{E}{P^{1-\sigma}}$. After finding the fixed point in market demand, the labor market clears as well due to Walras' law.

We are interested in how international trade affects the firms' production costs, the structure of the domestic production network, and the aggregate price index. We start by discussing the change in the overall price index, and then look into the micro-changes that lead to these price index effects. In our simulated economy, the price index increase by 93 percent with a fixed network structure and 80 percent when we allow for endogenous network formation. Hence, we find that in our simulated economy with endogenous networks, the price index rises around 15 percent less than under a fixed network.[27]

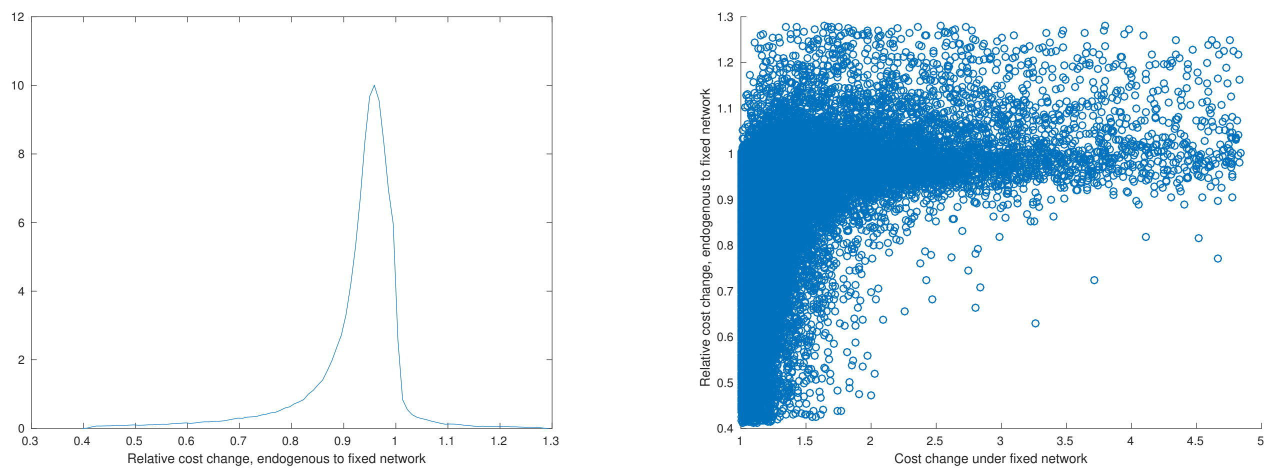
We next look at the distribution of firms' cost changes. Under fixed networks, we have seen that every firm's cost weakly increases when banning international trade. Interestingly, this is no longer the case under endogenous networks. Firms with little direct or indirect engagement in foreign trade actually benefit from banning international trade. This is because they can expand relatively to the firms' that are engaged in international trade. Our simulated model suggests that in the absence of international trade, 24 percent of firms actually would have weakly *lower* costs than with international trade.

27. Specifically, $\frac{80}{93} - 1 = -.14$.

Compared to the counterfactual analysis with a fixed network structure, most firms have lower cost increases under endogenous network structure. We show the distribution of relative cost increases under endogenous and exogenous networks in Figure 3.7a. Note that a small fraction of firms does have higher cost increases under endogenous networks. This occurs if a firm not only stops importing and exporting but also drops domestic suppliers from its sourcing strategy. Figure 3.7b shows that firms that have relatively large cost increases under fixed networks tend to have even larger cost increases under endogenous networks.

Figure 3.7: Cost changes: Endogenous vs. exogenous network

(a) Density (b) Scatter



Notes: The left panel shows the density of the ratio of cost changes from banning international trade under endogenous and fixed networks. For most firms this ratio is smaller than one, implying lower cost increases under endogenous networks for those firms. The right panel is a scatter plot of the ratio of cost increases under endogenous and fixed networks (vertical axis) against the cost increases from banning international trade under fixed networks (horizontal axis). Firms that have high cost increases under fixed networks sometimes have even larger cost increases under endogenous networks. Observations below the 1st and above the 99th percentile of cost increases are excluded from the figure.

These heterogeneous cost changes are driven by changes in the domestic linkages between firms. We find that in the simulated economy, the number of domestic firm-to-firm linkages increases from 3.70 million to 4.16 million when trade is shut down, an increase of around 12 percent.[28] Underlying this net increase, there is churn in firm-to-firm linkages: Around 2 percent of the domestic linkages that exist under free trade are no longer active in the

28. In the actual Belgian firm-to-firm data, there are around 6 million domestic firm-to-firm transactions, hence our simulated economy under-predicts the number of linkages between firms.

absence of trade.

3.5 Conclusion

In this paper, we used administrative data from Belgium with information on domestic firm-to-firm sales and foreign trade transactions to study how international trade affects firm efficiency and real wages. Our paper offered three sets of results. First, we documented that most firms that do not directly import or export still have large indirect exposure to foreign trade, and that a firm's output is affected by idiosyncratic shocks to its buyers and suppliers. These empirical insights motivated the development of a model with domestic production networks and trade.

Second, we derived new sufficient statistics results for how international trade affects firms' production costs. Assuming a fixed network structure, the cost reduction for an individual firm due to international trade depends only on the share of input costs that is spent on goods that are imported directly or indirectly and the elasticity of substitution in the production function. We applied this sufficient statistics formula to our data, and compared the results to those we obtain using existing approaches. This comparison highlights the importance of data on and modeling of domestic production networks in studies of international trade.

Lastly, we developed a novel framework for analyzing the endogenous formation of the production network. We make the model tractable by focusing on the formation of an acyclic rather than a cyclic production network. While restrictive, this allowed us to solve a model of firm trade with endogenous formation of domestic buyer-supplier relationships. Reassuringly, we found that the vast majority of buyer-supplier relationships in Belgium can be described by an acyclic production network. Moreover, both sectoral input-output tables and the gains from trade under a fixed network structure do not change materially if we restrict attention to transactions for which the domestic production network is acyclic. Our approach to endogenous network formation may prove useful in contexts other than trade where researchers are increasingly interested in the formation and consequences of domestic production networks.

APPENDIX A

APPENDIX TO GAINS FROM TRADE AND THE SOVEREIGN BOND MARKET

A.1 Pre-transfer consumption schedules in autarky and free trade

In this section I will show that there is a threshold value of Y such that if the endowment is below that amount, the cost of defaulting would be higher in free trade than in autarky. I will also show numerically that the threshold is very large under reasonable parameters.

Figure 1.2 illustrates the pre-transfer consumption schedules in autarky and free trade, as functions of Y. It is assumed that Y^* is fixed and treated as a parameter. The two schedules are as defined in equations (1.13) and (1.14).

$$\begin{cases} c^{\text{AUT}}(Y) & = Y \\ c^{\text{FT}}(Y) & = Y(1+\phi^{-1}p^{\sigma-1})^{\frac{1}{\sigma-1}} \\ & = Y \cdot g(p) > Y. \end{cases}$$

Given these schedules, the cost of defaulting is represented by the slope of the consumption schedules. If the slope is larger, then the loss of consumption by losing a fraction of endowment Y by declaring default would be larger.

We know that both consumptions will be 0 when the endowment Y is 0.

$$\begin{cases} c^{\text{AUT}}(0) & = 0 \\ \lim_{Y \xrightarrow{+} 0} c^{\text{FT}}(Y) & = 0. \end{cases}$$

Also, as Y goes to infinity, the gains from trade will disappear ($g(p) \to 1$), and the two consumption schedules will converge.

$$\lim_{Y\to\infty}(c^{\text{FT}}(Y) - c^{\text{AUT}}(Y)) = 0.$$

In addition, positive gains from trade, or $g(p) > 1$ assures that consumption in free trade is larger than consumption in autarky.

$$c^{\text{FT}}(Y) > c^{\text{AUT}}(Y) \quad \forall\, Y \in (0,\infty).$$

Therefore, using the mean value theorem, there exists at least one $Y \in (0,\infty)$ such that the slope of the free trade consumption will be equal to that of autarky.

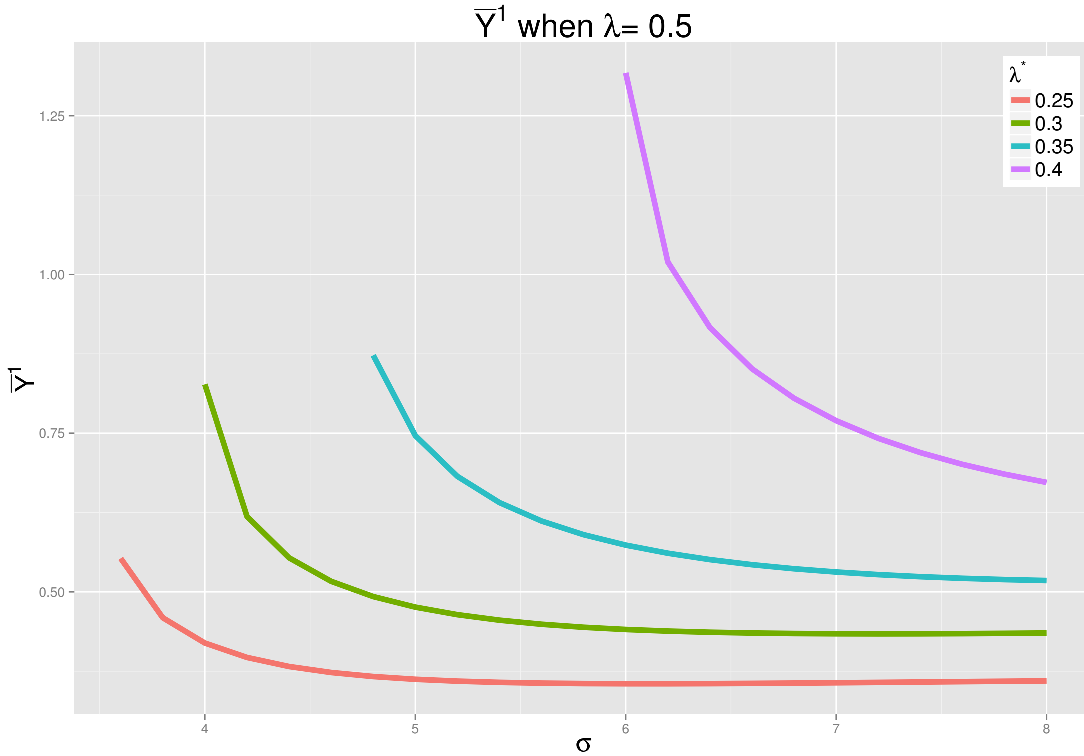
$$\exists\, \bar{Y} \in (0,\infty) \text{ s.t. } \frac{\partial c^{\text{FT}}(Y)}{\partial Y} = \frac{\partial c^{\text{AUT}}(Y)}{\partial Y} = 1. \tag{A.1}$$

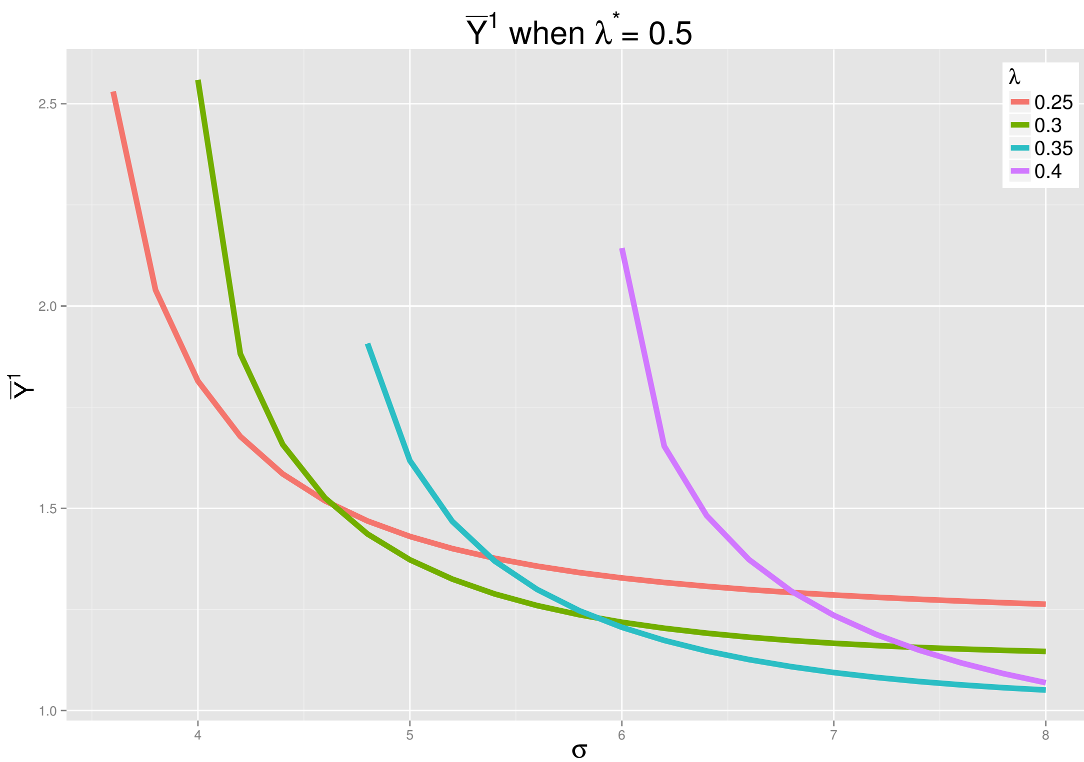
Take the sequence of $\{\bar{Y}\}$ that satisfies equation (A.1), and order them as $\{\bar{Y}^i\}, i = \{1, 2, \cdots\}$ so that $\bar{Y}^i < \bar{Y}^{i+1}\, \forall i$. For all values of Y such that $Y < \bar{Y}^1$, the slope of the consumption schedule is higher in free trade than in autarky, meaning the default cost is also higher.

Figure A.1 plots the numerically determined $\bar{Y}^1$, relative to Y^*. As substitutability increases, the gains from trade diminishes thus the terms of trade effects dominates. This lowers the value of $\bar{Y}^1$. Also, as foreign bias increases, the terms of trade deteriorates since the foreign country values less of home's good. This pushes up the value of $\bar{Y}^1$. Lastly, the change in the sovereign's home bias has an ambiguous effect on $\bar{Y}^1$. An increase in home bias will improve the terms of trade, but at the same time the sovereign values less of the cheaper imported goods.

Nevertheless, one can see from these plots that under reasonable parameters, which is σ around 5, and home biases at least as big as 0.5, the value of $\bar{Y}^1$ is very large. In the numerical exercises that I conduct in section 1.5, the variable space comfortably fits into the region where $Y < \bar{Y}^1$.

Figure A.1: $\bar{Y}^1$ with different $\sigma, \lambda, \lambda^*$





APPENDIX B

APPENDIX TO IMPERFECT COMPETITION AND THE TRANSMISSION OF SHOCKS: THE NETWORK MATTERS

B.1 Data and other statistics

B.1.1 Aggregating VAT-ids into firms

Our datasets are all at the VAT-id level. Using the same procedure as in Tintelnot, Kikkawa, Mogstad, and Dhyne (2017), we aggregate the VAT-ids into firms. As mentioned in the main text, we group all VAT-ids into firms if they are linked with more than or equal to 50% of ownership, or if they share the same foreign parent firm that holds more than or equal to 50% of their shares. To determine if the two VAT-ids share the same foreign parent firm, we use a "fuzzy string matching" method and compare the all possible pairs of the foreign parent firms' names. In order to correct for misreporting, we also make the following correction. If the two separate VAT-ids were paired as one firm in the year before and the year after, we pair the two into one firm in that year.

We then identify one VAT-id as the "head VAT-id" for each group of multiple VAT-ids. This "head VAT-id" will work as the identifier of the firm. We also make corrections on which VAT-id becomes the "head VAT-id" of the firm, so that the identifiers of the firms become consistent over time. For the procedure to choose the "head VAT-id" and the corrections, see Appendix C.1 of Tintelnot, Kikkawa, Mogstad, and Dhyne (2017).

In converting the VAT-id level variables into firm level variables, we simply sum up the variables if the variables are numeric. For variables such as total sales and inputs, we correct for double counting that arises from VAT-id-to-VAT-id trade that occur within firms. For non-numeric variables, we take the values of its "head VAT-id".

B.1.2 Coverage and descriptive statistics

Table B.1 reports the coverage of the full sample constructed in Dhyne, Magerman, and Rubinova (2015).

Table B.1: Coverage of all Belgian firms

Year	GDP	Output	Imports	Exports	All Belgian firms				
					Count	V.A.	Sales	Imports	Exports
2002	275	556	210	229	714,469	210	812	204	217
2007	345	715	300	314	782,006	274	1080	294	282
2012	387	823	342	347	860,373	300	1244	320	317

Notes: All numbers except for Count are in terms of billion Euro in current prices. Data for Belgian aggregate statistics are from Eurostat. Value added is the sum of value added reported in the annual accounts. Total sales in our selected sample are larger total output in the aggregate statistics because the output values in the aggregate statistics sum up value added for trade intermediaries instead of their gross output.

Table B.2 shows the aggregate statistics of the dataset. The number of firm-to-firm links in the economy is much smaller than the number of all possible links among all firms. This indicates that the production network is extremely sparse. We also note that the amount of total firm-to-firm sales sums up to an amount larger than the total value added.

Table B.2: Aggregate statistics of the B2B dataset

Year	Num. links	Num. links / Possible links	Total B2B sales	Total B2B sales / V.A.
2002	4,905	0.03%	208	170%
2007	5,752	0.03%	220	140%
2012	6,097	0.03%	245	144%

Notes: Number of links are in the thousands and the total B2B sales are in terms of billions of Euro in current prices.

Table B.3 shows the distribution of the pairwise input shares s_{ij}^m, defined as the share of goods from firm i, among j's input purchases. We also report the distributions for the number of suppliers and customers. Though the median firm has as many as 28 suppliers, the median value of the pairwise input share s_{ij}^m is very small. In addition, one can see that the distribution of the number of customers is much more skewed than the number of suppliers.

Table B.3: Descriptive statistics of the production network

	Mean	Percentiles				
		10%	25%	50%	75%	90%
$s_{ij}^m = \text{Sales}_{ij}/\text{InputPurchases}_j$	1.62%	0.00%	0.00%	0.18%	0.82%	3.15%
Num. suppliers	45	8	15	28	49	86
Num. customers	45	0	1	7	27	86

B.1.3 HHI of input shares across suppliers

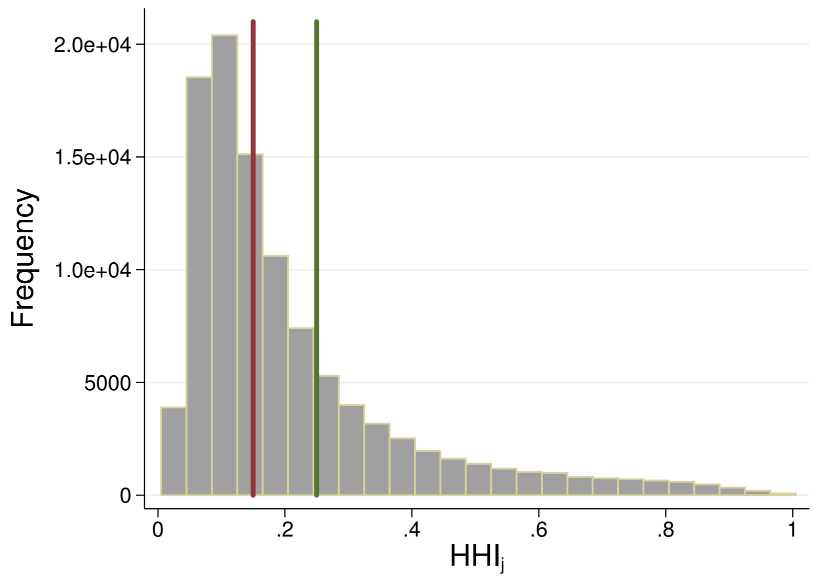
In this section we compute the HHI of s_{ij}^m for all customer firms j, across suppliers i. Figure B.1 displays the histogram of these firm-level HHI. We find that 50% of firms have a HHI above 0.15. 26% of firms have a HHI above 0.25%.

While there is no perfect reference for the HHI for s_{ij}^m for each customer firm j, the US Department of Justice and FTC consider markets in which the HHI is between 0.15 and 0.25 to be moderately concentrated. Markets in which the HHI is above 0.25 are considered highly concentrated (U.S. Department of Justice and Federal Trade Commission, 2010).

B.1.4 Distribution of firms' output shares

Figure B.2 plots a histogram for the output shares of the largest customers for all supplier firms in 2012 that have more than 10 customers. The output share of the largest customer for the median firm in this figure is 22%.

Figure B.1: HHI of suppliers' input shares



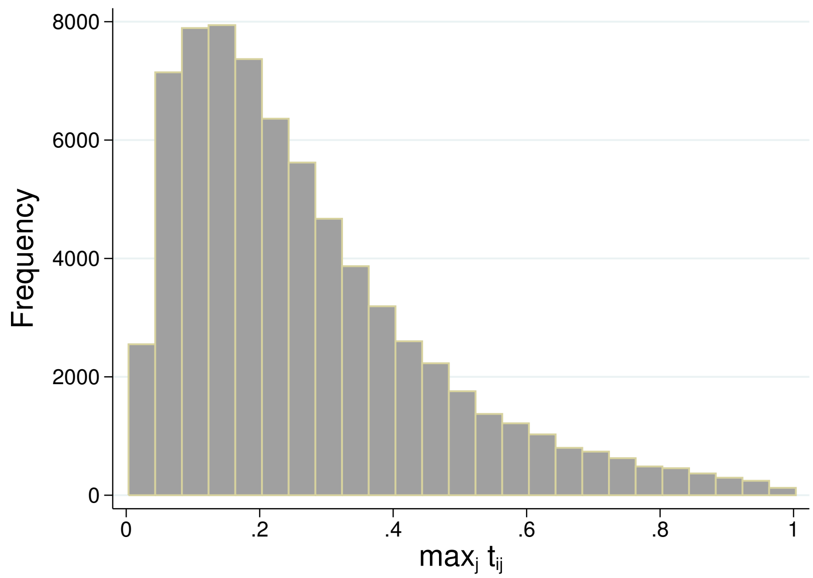
Notes: s_{ij}^m is defined as firm i's goods share among firm j's input purchases from other Belgian firms and abroad. The above histogram shows the HHI of s_{ij}^m for all customer firms j in 2012 that have more than 10 suppliers. The median value is 0.15. The two vertical lines indicates HHI being 0.15 and 0.25.

B.1.5 Disconnect between pairwise input shares and market shares

In this section we show that firms that have high input shares on a particular customer are not necessarily the ones that are large, even after looking at supplier-customer relationships within each sector-to-sector pair. For each firm, we compute the rank correlations of suppliers' input shares and their total sales. But unlike what was done in Section 2.2.3, we do so for each group of suppliers in each sector at the NACE 2-digit level. We compute the rank correlation for suppliers in a sector, if there are 5 or more suppliers in that sector supplying to the firm.

We obtain distributions of rank correlations, for each sector-to-sector pair. Figure B.3 plots the histogram of the median rank correlations, for each distribution. The median value of these median rank correlations is 0.20, which is higher than the unconditional median value from Figure 2.3. However, we still see a large role that pairwise match components play, even within the same sector-to-sector relationships.

Figure B.2: Output shares of the largest customers



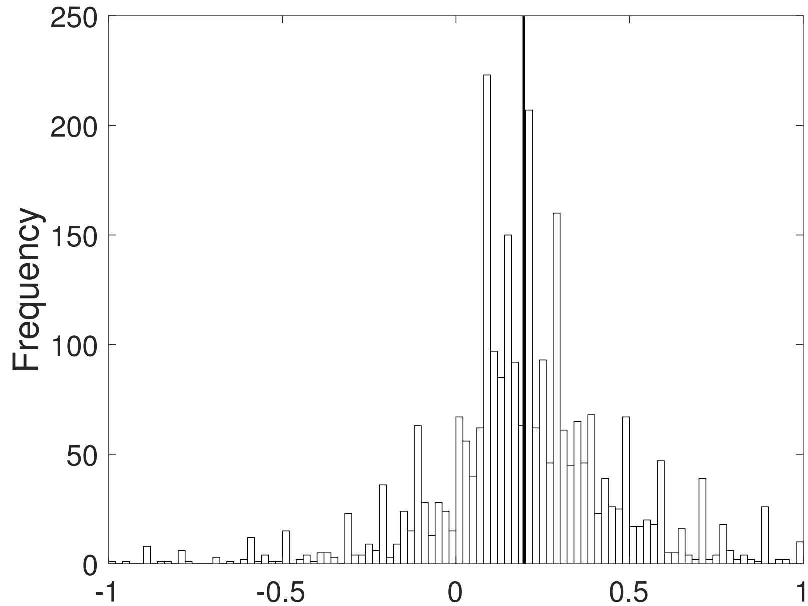
Notes: t_{ij} is defined as the share of firm i's goods that were sold to firm j, out of firm i's total sales to other domestic firms. The above histogram shows the distribution of $\max_j(t_{ij})$, which is the maximum value of t_{ij} for each supplier firm i in 2012 that have more than 10 customers. The median value is 0.22.

Instead of computing the rank correlations, we find that the results when we compute the Pearson correlations are qualitatively the same. Figure B.4 shows the histogram of the correlation coefficients, not taking into account the sectoral heterogeneity. Figure B.5 shows the histogram of the median correlations coefficients, for each sector-to-sector pair. Compared to the rank correlation distributions, both figures have fatter right tails. However, the median values are lower than those from the rank correlations.

B.1.6 Changes in suppliers and customers

Table B.4 shows the median values for firms' supplier and customer churning. This shows that is a significantly high rate of churn in both suppliers and customers. A median firm loses around 19% of suppliers and 26% of customers in terms of value at a yearly basis. They also add around 25% and 34% of suppliers and customers, relative to the previous years' values.

Figure B.3: Median rank correlations



Notes: For each customer firm j, we compute the rank correlations of suppliers' input shares s_{ij}^{m} and their total sales, for each sector in which 5 or more of j's suppliers are in. This figure shows a histogram of the median correlation coefficients, across each sector-to-sector pairs. The vertical line depicts the median value of 0.20.

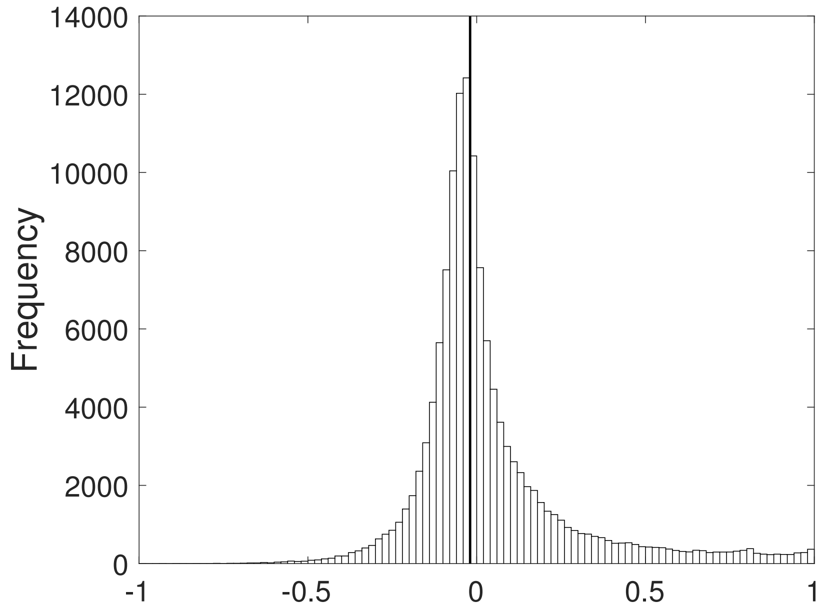
Table B.4: Median values for changes in suppliers and customers

	Yearly avg. (02-12)		10-year (02-12)	
Median	Cont. Share	Added Share	Cont. Share	Added Share
Supplier (Number)	0.60	0.43	0.22	0.92
Supplier (Value)	0.81	0.25	0.32	0.92
Customer (Number)	0.51	0.55	0.13	0.86
Customer (Value)	0.74	0.34	0.19	0.88

B.1.7 Chinese imports

The following figures compare the change in Chinese imports with those from other countries. Figure B.6 shows the evolution of Chinese imports compared with imports from the top five exporters to Belgium. Figure B.7 shows the same series, now compared with imports from

Figure B.4: Histogram of Pearson correlation of suppliers' input shares and total sales



Notes: This figure shows a histogram of Pearson correlation coefficients between s_{ij}^m and TotalSales_i, for suppliers of j for all j with 5 or more suppliers. The vertical line depicts the median correlation coefficient of -0.02.

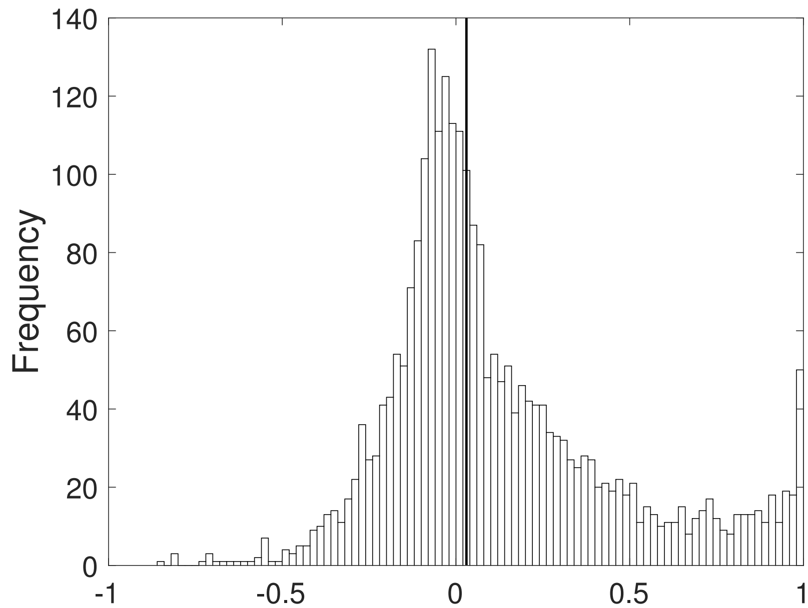
countries that are classified as in the same income category with China.[1] These figures indicate that the increase in imports as rapid as that of China did not occur to other comparable countries.

B.1.8 Sales and number of domestic suppliers

Figure B.8 shows the relationship between firms' sales to domestic final demand and their number of domestic suppliers. The positive relationship remain robust when taking firms' total sales instead. It is also robust after demeaning the sales variable with sector fixed effects, or when only considering firms that are not importing from abroad. These size advantages for firms with larger number of domestic suppliers are suggestive of fixed costs associated with domestic sourcing.

1. See World Bank classifications by income.

Figure B.5: Median Pearson correlations



Notes: For each customer firm j, we compute the Pearson correlations of suppliers' input shares s_{ij}^{m} and their total sales, for each sector in which 5 or more of j's suppliers are in. This figure shows a histogram of the median correlation coefficients, across each sector-to-sector pairs. The vertical line depicts the median value of 0.03.

Figure B.6: Chinese imports compared with other top exporters

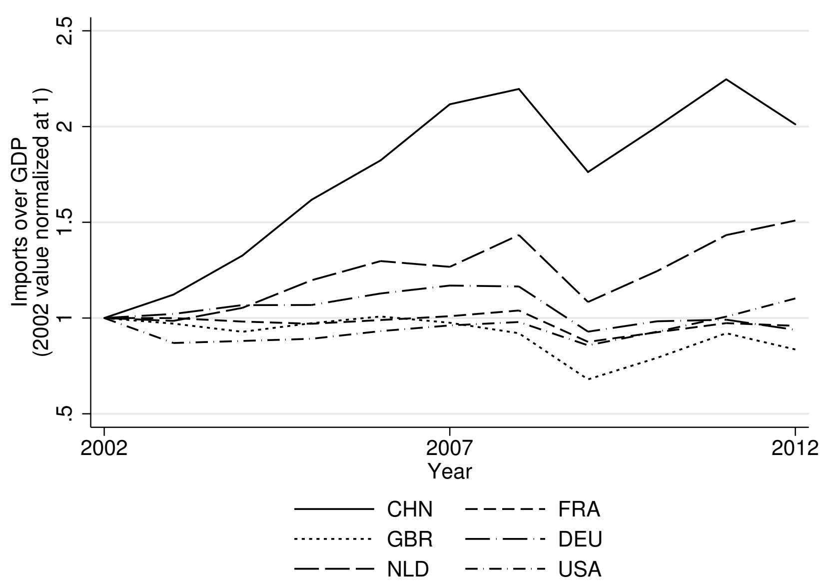


Figure B.7: Chinese imports compared with other middle income countries

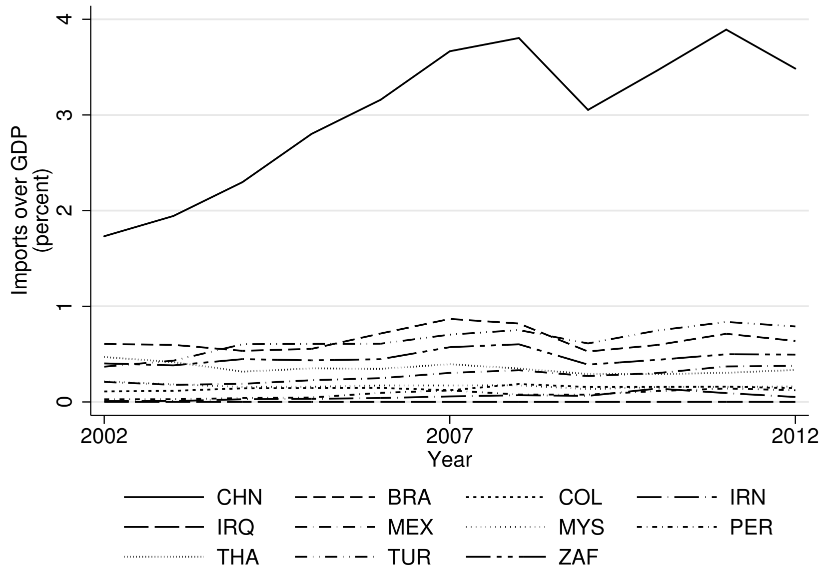
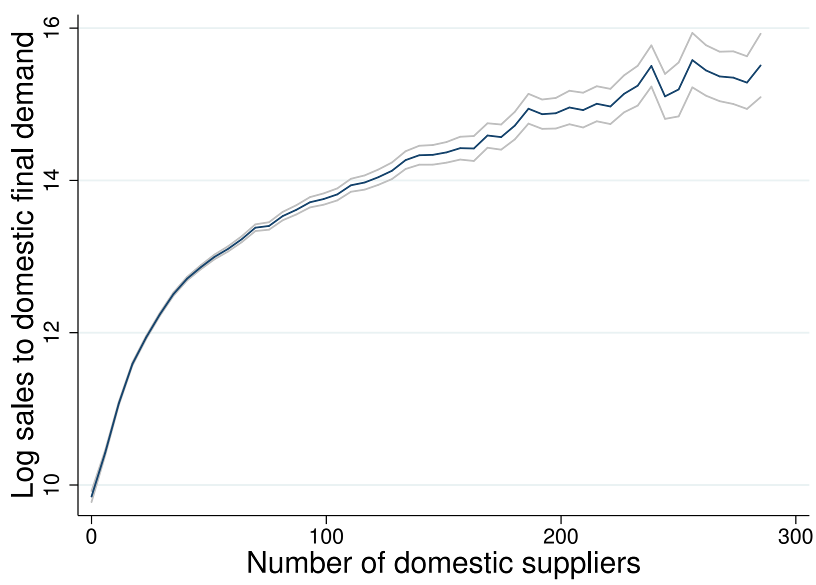


Figure B.8: Sales to domestic final demand and number of domestic suppliers



Notes: This figure shows the local polynomial regression plots of firms' log sales to final demand on the number of domestic suppliers, along with the 95% confidence intervals.

B.2 Additional empirical results

B.2.1 Additional results on markups and input shares

First, we show that firms' average input shares on customers have greater power in explaining the variation of firms' average markups, compared to firm-level market shares. In Table B.5 we report the regression results when we add the two RHS variables one by one, for each of the three specifications in Table 2.2. The 4th, 8th and 12th columns are identical to the three columns in Table 2.2. For each specification reported in the main text, we add three additional specifications. One with neither average input shares nor firm-level market shares on the RHS, and ones with each variable without the other. In all three sets of specifications, the increase in R-squared by adding average input shares alone on the RHS is larger than the increase in R-squared by adding sectoral market shares alone.

We then show that the positive relationship between markups and firms' average input shares are robust in other specifications. Table B.6 shows additional results when firm-level fixed effects are included. The second and the third columns are identical to the second and the third columns in Table 2.2.

Table B.7 shows additional results when sector-level fixed effects are included. The second column is identical to the first column in Table 2.2.

In our main specification, we drop firms that have no sales to other Belgian firms. Table B.8 shows the results when we include such firms in the regression, by treating their average input shares to other firms as zero.

B.2.2 Alternative markup estimates

In the main text, we recover firm-level average markups using the equation implied from the static model with CRS production function: $\mu_i = \frac{p_i q_i}{c_i q_i}$. To account for additional heterogeneity such as usage in capital inputs, here we recover firm-level markups following De Loecker and Warzynski (2012) and show that the positive correlation between firms' markups and their average input shares within their customers are still present even under these alternative markup estimates.

Let us first briefly describe the estimation procedure. When a firm is engaging in cost minimization under the existence of at least one flexible input X, the markup of firm i at time t can be expressed as

$$\mu_{it} = \theta_{it}^{X} \frac{p_{it}q_{it}}{p_{it}^{X} X_{it}},$$

where θ_{it}^{X} is firm i's output elasticity with respect to X, and $p_{it}^{X} X_{it}$ is the input value of X. As the input value share of the flexible input X is directly observed, it remains for us to estimate the value of θ_{it}^{X} to recover firm-level markups. In order to estimate the output elasticity, we assume a translog production function. We also assume that the technology parameters do not vary within sectors, thus we estimate the production function sector by sector at the NACE 2-digit level. We also allow for measurement errors in the output. Therefore, the production function to estimate becomes

$$\begin{aligned} y_{it} =& \alpha_l l_{it} + \alpha_k k_{it} + \alpha_m m_{it} + \alpha_{ll} l_{it}^2 + \alpha_{kk} k_{it}^2 + \alpha_{mm} m_{it}^2 \\ &+ \alpha_{lk} l_{it} k_{it} + \alpha_{km} k_{it} m_{it} + \alpha_{lm} l_{it} m_{it} + \omega_{it} + \varepsilon_{it}, \end{aligned}$$

where y_{it}, l_{it}, k_{it}, and m_{it} denote gross output, labor, capital, and material inputs, all in logs. The estimates from a least squares model would be biased as firm productivity ω_{it} is unobserved, and is potentially correlated with the inputs of the firm, which results in biased estimates of the technology parameters $\boldsymbol{\alpha}$. To overcome this issue, we follow Levinsohn and Petrin (2003) and use a “proxy” method. We assume that the innovation process of the firm-level productivities follow:

$$\omega_{it} = g_t\left(\omega_{it-1}\right) + \xi_{it}.$$

We identify $\boldsymbol{\alpha}$ via the following moment conditions:

$$E\left[\xi_{it}\left(\boldsymbol{\alpha}\right)\boldsymbol{z}_{it}\right]=\mathbf{0},$$

where $\boldsymbol{z}_{it}$ is a vector of lagged input variables:

$$\begin{aligned}\boldsymbol{z}_{it}=&[l_{it-1},k_{it},m_{it-1},\\&l_{it-1}^{2},k_{it}^{2},m_{it-1}^{2},\\&l_{it-1}k_{it},k_{it}m_{it-1},l_{it-1}m_{it-1}].\end{aligned}$$

The underlying assumption is that capital inputs are chosen a period ahead, and should be orthogonal to the future innovations of productivity. For other inputs, it is assumed that lagged variables are orthogonal to productivity innovations, as they are already chosen by the firm.

We estimate $\boldsymbol{\alpha}$ via GMM, and recover θ_{it}^{X} by assuming that material inputs are flexible. Once we recover firm-level markups μ_{it}, we run the regression of equation (2.1) in the main text. Table B.9 reports the results. Also in these alternative estimates of firm-level markups, there is a positive relationship between markups and firms' average input shares within their customers even after controlling for firm size variables.

Table B.5: Firm-level markups and input shares, R-squared across specifications

	Firm-level markups											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\text{SctrMktShare}_{i,t}$ (4-digit)		0.0960***		0.0929***		0.0431***		0.0430***		0.0691***		0.0686***
		(0.00954)		(0.00928)		(0.00965)		(0.00963)		(0.0130)		(0.0129)
Average input share $\overline{s^m_{i\cdot,t}}$			0.301***	0.298***			0.182***	0.182***			0.174***	0.173***
			(0.0132)	(0.0130)			(0.00939)	(0.00938)			(0.00929)	(0.00925)
N	1099496	1099496	1099496	1099496	1089209	1089209	1089209	1089209	1070602	1070602	1070602	1070602
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector FE (4-digit)	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	No
Firm FE	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes
R2	0.09200	0.09379	0.09772	0.09940	0.6177	0.6179	0.6186	0.6188	0.6232	0.6237	0.6240	0.6246

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. Controls include firms' indegree, outdegree, employment, total assets, and age.

Table B.6: Firm-level markups and input shares, with firm fixed effects

	Firm-level markups					
	(1)	(2)	(3)	(4)	(5)	(6)
$\text{SctrMktShare}_{i,t}$ (4-digit)	0.0431***	0.0430***	0.0686***			
	(0.00965)	(0.00963)	(0.0129)			
$\text{SctrMktShare}_{i,t}$ (2-digit)				0.0348***	0.0347***	0.0755***
				(0.00902)	(0.00895)	(0.0110)
Average input share $\overline{s^m_{i\cdot,t}}$		0.182***	0.173***		0.182***	0.172***
		(0.00938)	(0.00925)		(0.00937)	(0.00924)
N	1089209	1089209	1070602	1089694	1089694	1071051
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Controls	No	No	Yes	No	No	Yes
R2	0.618	0.619	0.625	0.618	0.619	0.624

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. Controls include firms' indegree, outdegree, employment, total assets, and age.

Table B.7: Firm-level markups and input shares, with sector fixed effects

	Firm-level markups			
	(1)	(2)	(3)	(4)
$\text{SctrMktShare}_{i,t}$ (4-digit)	0.0960^{***}	0.0929^{***}		
	(0.00954)	(0.00928)		
$\text{SctrMktShare}_{i,t}$ (2-digit)			0.0696^{***}	0.0670^{***}
			(0.00886)	(0.00848)
Average input share $\overline{s^m_{i\cdot,t}}$		0.298^{***}		0.314^{***}
		(0.0130)		(0.0140)
N	1099496	1099496	1099987	1099987
Year FE	Yes	Yes	Yes	Yes
Sector FE	4-digit	4-digit	2-digit	2-digit
Controls	Yes	Yes	Yes	Yes
R2	0.0938	0.0994	0.0665	0.0728

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. Controls include firms' indegree, outdegree, employment, total assets, and age.

Table B.8: Firm-level markups and input shares, including firms without firm-to-firm sales

	Firm-level markups		
	(1)	(2)	(3)
$\text{SctrMktShare}_{i,t}$ (4-digit)	0.101***	0.0449***	0.0746***
	(0.00998)	(0.00967)	(0.0136)
Average input share $\overline{s^m_{i\cdot,t}}$	0.248***	0.146***	0.146***
	(0.0124)	(0.00805)	(0.00801)
N	1285251	1293120	1259087
Year FE	Yes	Yes	Yes
Sector FE (4-digit)	Yes	No	No
Firm FE	No	Yes	Yes
Controls	Yes	No	Yes
R2	0.100	0.622	0.620

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. Controls include firms' indegree, outdegree, employment, total assets, and age.

Table B.9: Firm-level markups and input shares, using alternative markup estimates

	(1)	(2)	(3)
$\text{SctrMktShare}_{i,t}$ (4-digit)	0.00395***	-0.00179**	-0.000488
	(0.00122)	(0.000830)	(0.00103)
Average input share $\overline{s^m_{i\cdot,t}}$	0.0690***	0.0117***	0.0112***
	(0.00375)	(0.00139)	(0.00136)
N	602903	584131	584131
Year FE	Yes	Yes	Yes
Sector FE (4-digit)	Yes	No	No
Firm FE	No	Yes	Yes
Controls	Yes	No	Yes
R2	0.629	0.917	0.917

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. We use firm-level markups recovered using methods from De Loecker and Warzynski (2012) as the LHS variables. The coefficients are X-standardized. Standard errors are clustered at NACE 2-digit-year level.

B.2.3 Additional results for Section 2.3

We start by reporting the results when the LHS share variables are calculated in terms of numbers. Table B.10 shows the results analogous to those of Table 2.3, but now shares are computed in terms of numbers. The results are qualitatively similar compared to the results in the main text where shares are calculated in terms of values: as firms experience exogenous reductions in Chinese goods' price they also experience larger churn in both suppliers and customers.

Next, we report in Table B.11 the results analogous to Table 2.3, but taking the changes in customers on the LHS. The results are qualitatively the same as when we take the changes in suppliers on the LHS. The coefficients on the customer changes are larger, but as reported in Appendix B.1.6 there are larger churn in customers than in suppliers.

Finally, we report in Table B.12 the OLS results for the specification shown in Table 2.3. We find that the OLS coefficients are smaller in magnitude than the IV estimates, and the differences in magnitudes are broadly similar to those of Antras, Fort, and Tintelnot (2017) and Hummels, Jørgensen, Munch, and Xiang (2014).

Table B.10: First and second stage results for changes in suppliers (in terms of number)

Panel A: Second stage result					Panel B: First stage result	
	(1) Continuing suppliers	(2) Added suppliers	(3) Added suppliers: Incumbent firms	(4) Added suppliers: New firms		(1) ΔCS
ΔCS	-0.149^{***}	0.122^{***}	0.119^{***}	0.00275^{***}	ΔIV	0.00370^{***}
	(0.0275)	(0.0236)	(0.0238)	(0.00134)		(0.000649)
N	56146	56146	56146	56146	R2	0.0255
Controls	Yes	Yes	Yes	Yes	F Stat	32.48

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. The coefficients of the second stage results are X-standardized. Controls include firm age and employment size in 2002 with sector fixed effects (NACE 2-digit) and geographic fixed effects (NUTS 3). The same controls are used in the first stage results. ΔCS is the firm's average yearly increase of Chinese imports from 2002 to 2012 scaled by its total inputs in 2002. ΔCS is instrumented by the weighted sum of the sectoral change in Chinese goods' share in developed countries' total imports from 2002 to 2012. Standard errors are clustered at the NACE 2-digit-NUTS 3 level.

Table B.11: First and second stage results for changes in customers (in terms of value)

Panel A: Second stage result					Panel B: First stage result	
	(1) Continuing customers	(2) Added customers	(3) Added customers: Incumbent firms	(4) Added customers: New firms		(1) ΔCS
ΔCS	-0.325^{***}	0.314^{***}	0.285^{***}	0.0395^{***}	ΔIV	0.00377^{***}
	(0.0686)	(0.0890)	(0.0815)	(0.00832)		(0.000660)
N	55280	55280	55280	55280	R2	0.0256
Controls	Yes	Yes	Yes	Yes	F Stat	32.74

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. The coefficients of the second stage results are X-standardized. Controls include firm age and employment size in 2002 with sector fixed effects (NACE 2-digit) and geographic fixed effects (NUTS 3). The same controls are used in the first stage results. ΔCS is the firm's average yearly increase of Chinese imports from 2002 to 2012 scaled by its total inputs in 2002. ΔCS is instrumented by the weighted sum of the sectoral change in Chinese goods' share in developed countries' total imports from 2002 to 2012. Standard errors are clustered at the NACE 2-digit-NUTS 3 level.

Table B.12: OLS results for changes in suppliers (in terms of value)

	(1) Continuing suppliers	(2) Added suppliers	(3) Added suppliers: Incumbent firms	(4) Added suppliers: New firms
ΔCS	-0.00121^{***}	0.0104^{***}	0.00919^{***}	0.00114^{***}
	(0.000390)	(0.000948)	(0.000898)	(0.000112)
N	56146	56146	56146	56146
R2	0.140	0.108	0.100	0.0753
Controls	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. $*p < 0.10, **p < 0.05, ***p < 0.01$. The coefficients are X-standardized. Controls include firm age and employment size in 2002, with sector fixed effects (NACE 2-digit) and geographic fixed effects (NUTS 3). ΔCS is the firm's average yearly increase of Chinese imports from 2002 to 2012 scaled by its total inputs in 2002. Standard errors are clustered at the NACE 2-digit-NUTS 3 level.

B.3 Algorithm for network formation

Given firms' productivities ϕ_i and parameters, we follow the steps below to simulate the network formation game. As mentioned in the main text, we focus on an equilibrium that results from firms sequentially making sourcing and international trade participation decisions. This sequential sourcing decisions serve as an equilibrium selection rule.

1. Initialize the economy where no firm is sourcing from any other domestic firm, and no firm is participating in international trade. Solve for all prices and aggregate expenditure E, from equations (2.5), (2.7), (2.9), (2.10), (2.13), (2.15), (2.16), (2.18), and (2.20).

2. Order firms in terms of productivity, from the most productive to the least productive.

3. Start with the most productive firm, $i = 1$. Taken as given all other firms' decisions $\{Z_i, I_{Fi}, I_{iF}\}_{i \neq 1}$, evaluate $4N$ possible sets of decisions and choose the set $\left\{\hat{Z}_1, \hat{I}_{F1}, \hat{I}_{1F}\right\}$ that yields the largest variable profit net of fixed costs.[2] To compute the net profits for each possible set of $\{Z_1, I_{F1}, I_{1F}\}$, solve the system of equations (2.5), (2.7), (2.9), (2.10), (2.13), (2.15), (2.16), (2.18), and (2.20). Update the firm's decision to its optimal set $\left\{\hat{Z}_1, \hat{I}_{F1}, \hat{I}_{1F}\right\}$.

4. Repeat the previous step for firms $i = 2, 3, \cdots N$ in sequence. After the last firm makes its decision, record the economy's network structure $\{Z_i, I_{Fi}, I_{iF}\}_{i \in \Omega}$.

5. Repeat steps 3 and 4, until the resulting network structure $\{Z_i, I_{Fi}, I_{iF}\}_{i \in \Omega}$ converges to a fixed point.

2. There are N possible sets of Z_1: {no sourcing, source from a firm with lowest unit cost, source from two firms with lowest unit costs, $\cdots$, source from all}. Interacting with two possible choices for both importing and exporting decisions, $I_{F1} \in \{0, 1\}$ and $I_{1F} \in \{0, 1\}$, the firm has $4N$ possible sets of $\{Z_1, I_{F1}, I_{1F}\}$ to evaluate.

B.4 Theoretical results

B.4.1 Derivation of equation (2.15)

Consider firm i selling its goods to j. Firm i chooses p_{ij} to maximize profits, taking into account the effect of p_{ij} on j's price index for its intermediate goods, p_{mj}. It takes as given j's unit cost and production, c_j, and q_j, as well as j's sourcing set, Z_j and I_{Fj}. The firm's problem is as follows:

$$\max_{p_{ij}} \left(p_{ij} - c_i\right) q_{ij}$$
$$s.t. p_{ij} q_{ij} = \alpha_{ij}^{\rho} p_{ij}^{1-\rho} p_{mj}^{\rho} m_j$$
$$p_{mj} m_j = \omega_m^{\eta} p_{mj}^{1-\eta} \phi_j^{\eta-1} c_j^{\eta} q_j.$$

Solving the above problem while taking into account that $\frac{\partial p_{mj}}{\partial p_{ij}} \neq 0$ yields

$$p_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} c_i$$
$$\varepsilon_{ij} = \rho\left(1 - s_{ij}^{m}\right) + \eta s_{ij}^{m}.$$

B.4.2 Alternative market structures

In our model we assume the following when firms participate in firm-to-firm trade. When selling to firm j, firm i sets price p_{ij} by internalizing the effect of p_{ij} on j's price index for its intermediate goods, p_{mj}. However, it takes as given j's unit cost and total production, c_j and q_j. This yields our pricing equation of

$$p_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} c_i$$
$$\varepsilon_{ij} = \rho\left(1 - s_{ij}^{m}\right) + \eta s_{ij}^{m}.$$

In this section we discuss alternative market structures in firm-to-firm trade. We discuss the pricing equations that result in firms internalizing their prices' effect on the customers' unit costs and total production.

Fixed demand shifters

First we consider a case where firm i takes as given the two demand shifters that firm j faces - one from sales to other firms $\left(D_{jB}\right)$ and another from sales to final demand $\left(D_{jH}\right)$:

$$q_j = c_j^{-\rho} D_{jB} + c_j^{-\sigma} D_{jH}.$$

When one solves this problem the pricing equation becomes

$$\begin{aligned} p_{ij} &= \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} c_i \\ \varepsilon_{ij} &= \left(1 - s_{ij}^m\right)\rho + s_{ij}^m \left(\left(1 - s_{mj}\right)\eta + s_{mj}\left(s_{jB}^q \rho + s_{jH}^q \sigma\right)\right). \end{aligned}$$

The term s_{jB}^q is the quantity output share of firm j's goods that were shipped to other firms, and the term s_{jH}^q is the quantity output share of firm j's goods that were shipped to final demand:

$$\begin{aligned} s_{jB}^q &= \frac{c_j^{-\rho} D_{jB}}{q_j} \\ s_{jH}^q &= \frac{c_j^{-\sigma} D_{jH}}{q_j} = 1 - s_{jB}^q. \end{aligned}$$

This implies that the firm needs to know the quantity output shares of its customers.

Constant demand elasticity for customers' goods

We also consider a case where firm i does not know the output compositions of its customer j, but assumes that j is facing a common demand elasticity of ν. In this case q_j can be written as

$$q_j = c_j^{-\nu} D_j,$$

in which firm i takes as given the demand shifter, D_j. When one solves the problem of firm i under this setup, the pricing equation becomes

$$p_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} c_i$$
$$\varepsilon_{ij} = \left(1 - s_{ij}^m\right) \rho + s_{ij}^m \left(\left(1 - s_{mj}\right) \eta + s_{mj} \nu\right).$$

Notice that if we additionally assume that $\nu = \eta$, the above equation collapses to equation (2.15).

B.4.3 Proof of Proposition 1

From Assumption 3, no firm generates profits. Hence, the change in welfare, $\hat{U}$, is the inverse of the change in the aggregate price index:

$$\hat{U} = \hat{P}^{-\alpha}. \tag{B.1}$$

From Assumptions 2, 3 and equation (2.6), we have

$$\hat{P}^{1-\tilde{\sigma}} = \sum_{i \in \Omega} s_{iH} \hat{c}_i^{1-\tilde{\sigma}}, \tag{B.2}$$

where $\tilde{\sigma}$ is the common CES parameter and s_{iH} is firm i's share in the final demand market for the heterogeneous goods sector: $\frac{p_{iH}^{1-\tilde{\sigma}}}{P^{1-\tilde{\sigma}}}$. From Assumptions 2, 3, and equation (2.9), we obtain the change in unit costs: $\hat{c}_i^{1-\tilde{\sigma}} = \sum_k s_{ki} \hat{c}_k^{1-\tilde{\sigma}} + s_{li} + s_{Fi} \hat{p}_F^{1-\tilde{\sigma}}$. Rearranging this into matrix form yields

$$\hat{\boldsymbol{c}}^{1-\tilde{\sigma}} = \left(I - S^{'}\right)^{-1} \left(\boldsymbol{s}_{l\cdot} + \boldsymbol{s}_{F\cdot}\hat{p}_F^{1-\tilde{\sigma}}\right), \tag{B.3}$$

where the (i, j) element of matrix S is s_{ij}, and $\boldsymbol{s}_{F\cdot}$, $\boldsymbol{s}_{l\cdot}$ are vectors where their i'th elements are s_{Fi} and s_{li}.

On the output side, the revenue of firm i, $p_i q_i$, is the sum of sales to households, exports, and sales to other firms. From Assumption 1, the share of each firm among exports are equal to that among sales to households, s_{iH}. Thus from Assumptions 1 and 3, we obtain

$$p_i q_i = s_{iH}\alpha E + s_{iH}\text{Exports} + \sum_j s_{ij} p_j q_j. \tag{B.4}$$

Rearrange this into matrix form and obtain

$$\frac{\boldsymbol{p} \circ \boldsymbol{q}}{\alpha E + \text{Exports}} = (I - S)^{-1} \boldsymbol{s}_{\cdot H}, \tag{B.5}$$

where $\boldsymbol{s}_{\cdot H}$ is a vector where its i'th element is s_{iH}. Equation (B.5) implies that the firm-level measure $\frac{p_i q_i}{\alpha E + \text{Exports}}$ captures the centrality of each firm as a supplier of goods to final demand (including exports). This is analogous to the "supplier centrality" defined in Baqaee (2014a).

Finally, combine equations (B.2), (B.3) and (B.5) to yield

$$\hat{P}^{1-\tilde{\sigma}} = \sum_{i \in \Omega} \frac{p_i q_i}{\alpha E + \text{Exports}} \left(s_{li} + s_{Fi}\hat{p}_F^{1-\tilde{\sigma}}\right).$$

Then from equation (B.1), we have

$$\hat{U} = \left(\sum_{i \in \Omega} \frac{p_i q_i}{\alpha E + \text{Exports}} \left(s_{li} + s_{Fi}\hat{p}_F^{1-\tilde{\sigma}}\right)\right)^{\frac{-\alpha}{1-\tilde{\sigma}}}. \quad \square$$

B.4.4 Case of constant markups in firm-to-firm trade

Consider a case where firms charge constant and common markups $\tilde{\mu}$ in firm-to-firm trade. Then Proposition 1 no longer holds because of the following. Equations (B.2) and (B.3) remain the same, but equation (B.4) no longer holds. Since there are markups in firm-to-firm trade, we instead have

$$\begin{aligned} p_i q_i &= s_{iH}\alpha E + s_{iH}\text{Exports} + \sum_j s_{ij} c_j q_j \\ &= s_{iH}\alpha E + s_{iH}\text{Exports} + \sum_j s_{ij}\tilde{\mu}^{-1} p_j q_j. \end{aligned}$$

Rearranging to matrix form, we find

$$\frac{\boldsymbol{p}\circ\boldsymbol{q}}{\alpha E + \text{Exports}} = \left(I - \tilde{S}\right)^{-1}\boldsymbol{s}_{\cdot H},$$

where the (i, j) element of matrix $\tilde{S}$ is now $s_{ij}\tilde{\mu}^{-1}$. In this case, the matrix used in capturing firms' centrality as consumers of foreign goods does not match with the one used in capturing firms' centrality as suppliers of goods to final demand.

B.4.5 System of price changes under fixed networks

In this section we present the system of price changes for the variable markup case and derive its first order approximations. When firms charge pairwise variable markups in firm-to-firm trade and the network is fixed, we have the following system of equations for the changes in prices, given $\hat{p}_F$ and parameters.

$$\begin{aligned}
\hat{c}_i^{1-\eta} &= s_{li} + s_{mi}\hat{p}_{mi}^{1-\eta} \\
\hat{p}_{mi}^{1-\rho} &= \sum_{j\in Z_i} s_{ji}^m \hat{\mu}_{ji}^{1-\rho}\hat{c}_j^{1-\rho} + s_{Fi}^m \hat{p}_F^{1-\rho} \\
\hat{\mu}_{ji} &= \hat{\varepsilon}_{ji}\frac{\varepsilon_{ji}-1}{\hat{\varepsilon}_{ji}\varepsilon_{ji}-1} \\
\varepsilon_{ij} &= \rho\left(1-s_{ij}^m\right)+\eta s_{ij}^m \\
\hat{\varepsilon}_{ji} &= \frac{1}{\varepsilon_{ji}}\left(\rho\left(1-s_{ji}^m\hat{s}_{ji}^m\right)+\eta s_{ji}^m\hat{s}_{ji}^m\right) \\
\hat{s}_{ji}^m &= \hat{\mu}_{ji}^{1-\rho}\hat{c}_j^{1-\rho}\hat{p}_{mi}^{\rho-1}.
\end{aligned} \tag{B.6}$$

Taking first order approximations, we obtain

$$\begin{aligned}
\frac{\mathrm{d}c_i}{c_i} &= s_{mi}\frac{\mathrm{d}p_{mi}}{p_{mi}} \\
\frac{\mathrm{d}p_{mi}}{p_{mi}} &= \sum_{j\in Z_i} s_{ji}^m\left(\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}+\frac{\mathrm{d}c_j}{c_j}\right)+s_{Fi}^m\frac{\mathrm{d}p_F}{p_F} \\
\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}} &= -\frac{\left(\rho-\varepsilon_{ji}\right)(\rho-1)}{\left(\varepsilon_{ji}-1\right)\varepsilon_{ji}+\left(\rho-\varepsilon_{ji}\right)(\rho-1)}\left(\frac{\mathrm{d}c_j}{c_j}-\frac{\mathrm{d}p_{mi}}{p_{mi}}\right).
\end{aligned}$$

Further manipulating the above equation:

$$\begin{aligned}
\frac{\mathrm{d}c_i}{c_i} &= \sum_{j\in Z_i} s_{ji}\left(\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}+\frac{\mathrm{d}c_j}{c_j}\right)+s_{Fi}\frac{\mathrm{d}p_F}{p_F} \\
\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}} &= -\Gamma_{ji}\frac{\mathrm{d}c_j}{c_j}+\Gamma_{ji}\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}},
\end{aligned} \tag{B.7}$$

where Γ_{ji} equals the elasticity of markup μ_{ji} with respect to the supplier's cost c_j:

$$\begin{aligned}
\Gamma_{ji} &= -\frac{\partial \mu_{ji}}{\partial c_j}\frac{c_j}{\mu_{ji}} \\
&= \frac{\Upsilon_{ji}\left(1 - s_{ji}^m\right)}{1 - \Upsilon_{ji} s_{ji}^m} \\
\Upsilon_{ji} &= \frac{\left(\rho - \varepsilon_{ji}\right)\left(\rho - 1\right)}{\left(\varepsilon_{ji} - 1\right)\varepsilon_{ji} + \left(\rho - \varepsilon_{ji}\right)\left(\rho - 1\right)},
\end{aligned}$$

and $\hat{p}_{\tilde{j}i}$ represents the average price change from suppliers other than j:

$$\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}} = \frac{\sum_{k \in Z_i, k \neq j} s_{ki}^m \left(\frac{\mathrm{d}\mu_{ki}}{\mu_{ki}} + \frac{\mathrm{d}c_k}{c_k}\right) + s_{Fi}^m \frac{\mathrm{d}p_F}{p_F}}{1 - s_{ji}^m}.$$

The first order approximation of the change in aggregate price is

$$\frac{\mathrm{d}P}{P} = \sum_i s_{iH} \frac{\mathrm{d}c_i}{c_i}.$$

Combining with equation (B.7) yields

$$\begin{aligned}
\frac{\mathrm{d}P}{P} =& \sum_i s_{iH} \left(\sum_{j \in Z_i} s_{ji} \frac{\mathrm{d}c_j}{c_j} + s_{Fi} \frac{\mathrm{d}p_F}{p_F}\right) \\
&+ \sum_i s_{iH} s_{mi} \left(-\sum_{j \in Z_i} s_{ji}^m \Gamma_{ji} \frac{\mathrm{d}c_j}{c_j} + \sum_{j \in Z_i} s_{ji}^m \Gamma_{ji} \frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}\right).
\end{aligned} \tag{B.8}$$

B.5 Additional estimation results

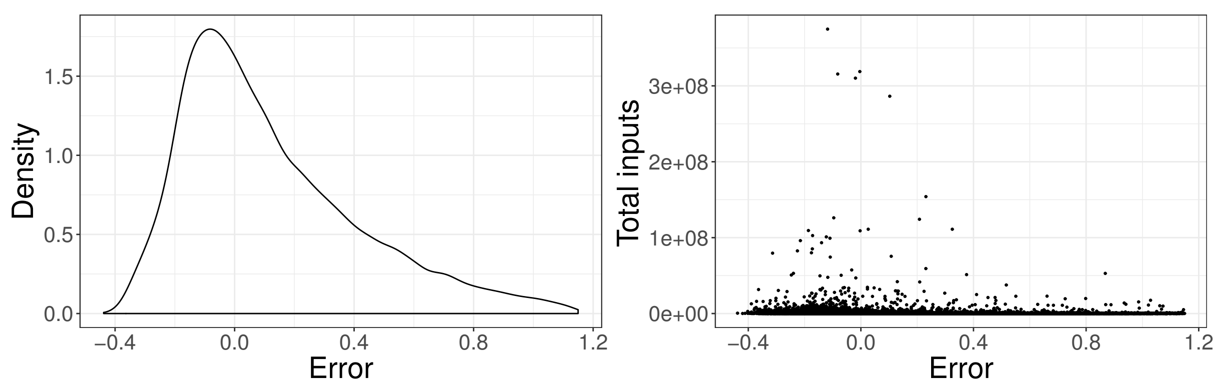
B.5.1 Distribution of errors

Here we provide the distribution of firm-level errors under the estimated CES parameters. We compute the size of the relative error for each firm i, where $\hat{\mu}$'s are the implied markups from estimated $\hat{\eta}, \hat{\rho}, \hat{\sigma}$:

$$\text{Error}_i = \frac{c_i q_i - \left(\sum_j \frac{V_{ij}}{\hat{\mu}_{ij}} + \frac{V_{iH}}{\hat{\mu}_{iH}} + \frac{V_{iF}}{\hat{\mu}_{iF}}\right)}{c_i q_i}.$$

We plot the distribution of these firm-level errors on the left of Figure B.9, and on the right we plot these errors against firms' size of total inputs, $c_i q_i$. One can see that the distribution of errors is concentrated around zero, and firms with large errors tend to be small firms.

Figure B.9: Distribution of firm level errors



Notes: The left figure displays the distribution of firm-level errors. The errors are defined as the difference between the LHS and the RHS of equation (2.27), relative to the size of the firm's total inputs. The right figure plots these errors against firms' total inputs.

B.5.2 Assuming Cournot competition in estimating CES parameters

When assuming Cournot competition in firm-to-firm trade instead, equation (2.15) becomes

$$p_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} c_i$$
$$\varepsilon_{ij} = \left(\frac{1}{\rho} \left(1 - s_{ij}^m \right) + \frac{1}{\eta} s_{ij}^m \right)^{-1}.$$

We follow the same procedure described in Section 2.5.1 and obtain the estimates shown in Table B.13.

Table B.13: Estimated values for $\{\eta, \rho, \sigma\}$ under Cournot competition

	$\frac{1}{\eta}$	$\frac{1}{\rho}$	$\frac{\sigma}{\sigma-1}$
Estimate	0.62	0.36	1.25
s.e.	0.18	0.04	0.05
	η	ρ	σ
	(Labor and goods)	(Firms' goods in production)	(Firms' goods in consumption)
Implied value	1.63	2.79	5.00

B.5.3 Assuming constant markups in estimating CES parameters

When assuming that firms charge constant markup $\frac{\rho}{\rho-1}$ when selling goods to other domestic firms, equation (2.15) becomes

$$p_{ij} = \frac{\rho}{\rho - 1} c_i.$$

We follow the same procedure described in Section 2.5.1 and obtain the estimates of ρ and σ shown in Table B.14.

Table B.14: Estimated values for $\{\rho, \sigma\}$ under constant markups

	$\frac{\rho}{\rho-1}$	$\frac{\sigma}{\sigma-1}$
Estimate	1.57	1.25
s.e.	0.11	0.04
	ρ	σ
	(Firms' goods in production)	(Firms' goods in consumption)
Implied value	2.74	4.99

B.5.4 Accounting for capital inputs in estimating CES parameters

In the model, total input $c_i q_i$ is an aggregate of labor costs and goods purchases. Here we account for capital inputs by interpreting labor as the composite input of labor and capital. As we do not directly observe capital rental costs for each firm, we take two alternate approaches.

First, we assume that firms have common labor shares, and uniformly scale up labor cost. We use the aggregate labor share of 0.6 that we compute as the total labor cost divided by the total value added. We report the estimation results in Table B.15.

Table B.15: Estimated values for $\{\eta, \rho, \sigma\}$ assuming common labor share

	η	ρ	$\frac{\sigma}{\sigma-1}$
Estimate	1.00	3.03	1.25
s.e.	0.66	0.47	0.05
	η	ρ	σ
	(Labor/capital and goods)	(Firms' goods in production)	(Firms' goods in consumption)
Implied value	1.00	3.03	4.96

Second, we assume that the user cost of capital consists of capital depreciation rate and the interest rate. Following Dhyne, Petrin, Smeets, and Warzynski (2017), we set the yearly depreciation rate as 8% and set the interest rate as the long-term interest rate in Belgium. We compute the capital rental costs using fixed tangible assets reported in the annual accounts. We report the estimation results in Table B.16.

Table B.16: Estimated values for $\{\eta, \rho, \sigma\}$ using capital from annual accounts

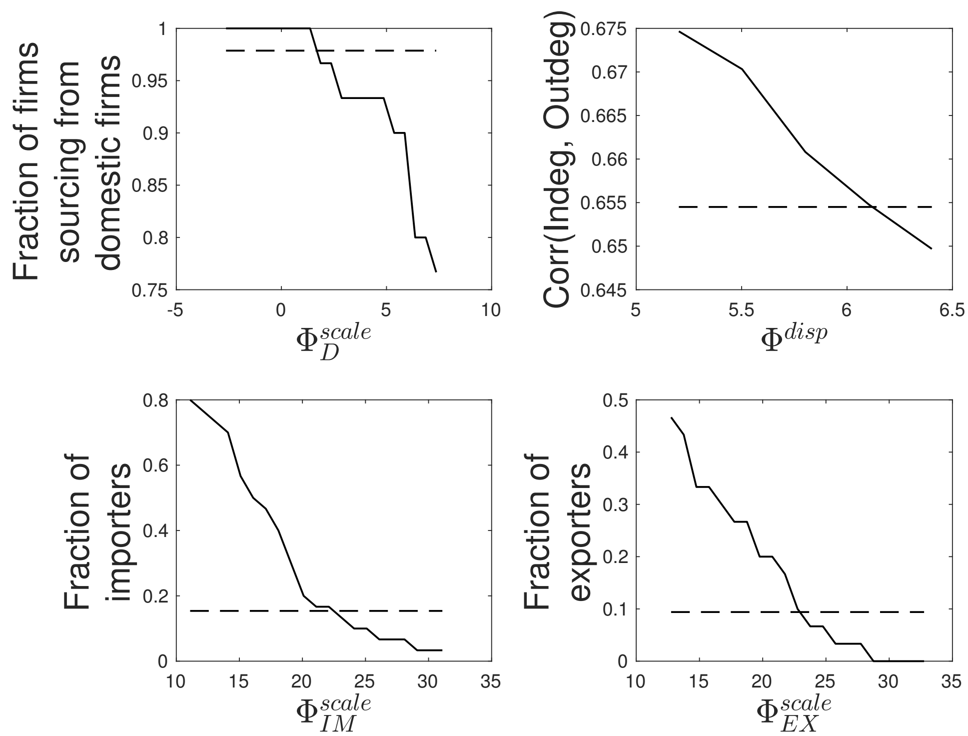
	η	ρ	$\frac{\sigma}{\sigma-1}$
Estimate	1.00	3.59	1.27
s.e.	0.93	0.65	0.04
	η	ρ	σ
	(Labor/capital and goods)	(Firms' goods in production)	(Firms' goods in consumption)
Implied value	1.00	3.59	4.77

In the two cases above, the estimates of η are both one. If a firm is a sole supplier to a customer, then its implied markup will be $\frac{\eta}{\eta-1}$, which is not well defined if $\eta = 1$. In our selected sample, there are only around 1000 firms that have single supplier, and we drop them from our estimation sample.

B.5.5 Local identification for the fixed cost parameters

Figure B.10 shows the local identification for all parameters that we estimate via SMM.

Figure B.10: Local identification of the fixed cost parameters



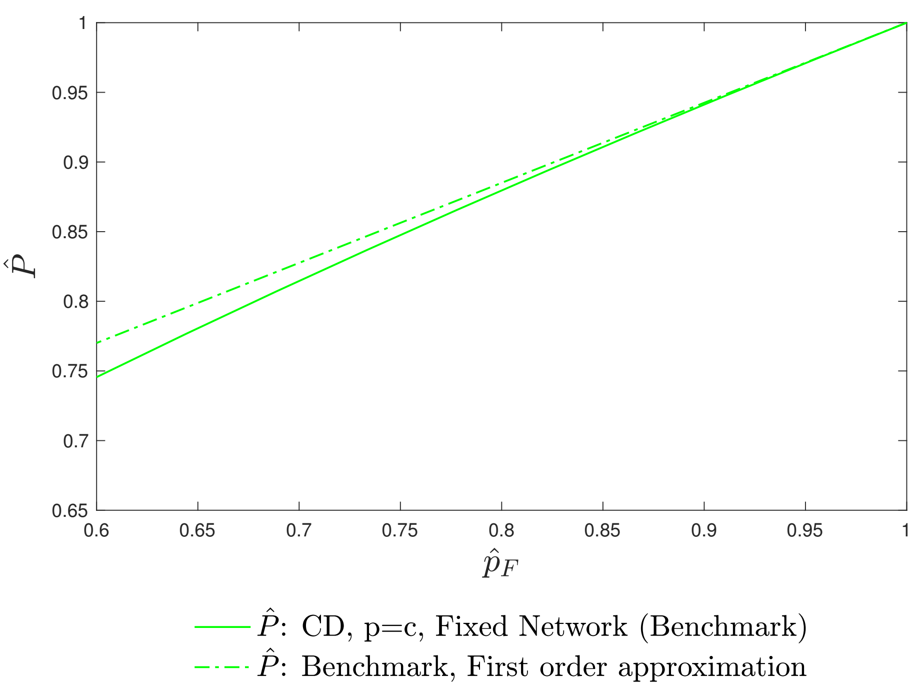
Notes: These figures illustrate local identification of the four fixed cost parameters. In each figure, on the x-axis we plot the parameter to identify, which we vary while fixing all other parameters to their estimated values. On the y-axes we plot the moments we use to identify the parameters. The horizontal lines indicate the observed value of the moment in the data.

B.6 Additional results from the counterfactual analysis

B.6.1 First order approximation of the benchmark case

Here we provide the first order approximated change in the aggregate price index under the benchmark case. The solid line in Figure B.11 displays the global change in price index under the benchmark case, and is identical to the first line in Figure 2.6 in the main text. The dotted line displays the first order approximation. The first order approximation requires smaller set of assumptions. While we assume Assumptions 1, 3, 4, and 5 for our benchmark case, we only need Assumptions 1 and 3 for the first order approximation.

Figure B.11: $\hat{P}$ under the benchmark case and its first order approximation



B.6.2 Pairwise attenuation and pro-competitive effects

We decomposed the change in pairwise markups into attenuation and pro-competitive effects in equation (2.32) of the main text:

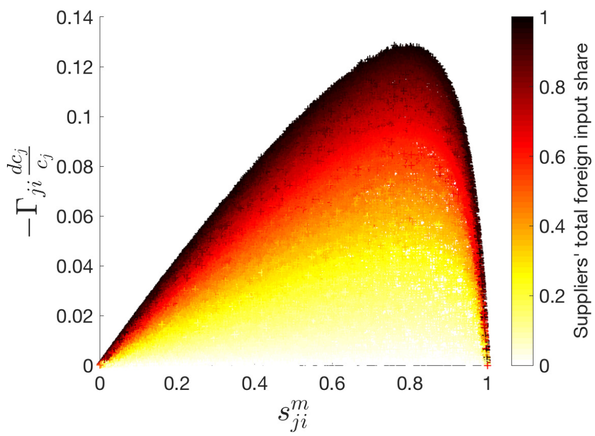
$$\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}} = \underbrace{-\Gamma_{ji}\frac{\mathrm{d}c_j}{c_j}}_{\text{attenuation effect}} + \underbrace{\Gamma_{ji}\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}}_{\text{pro-competitive effect}}.$$

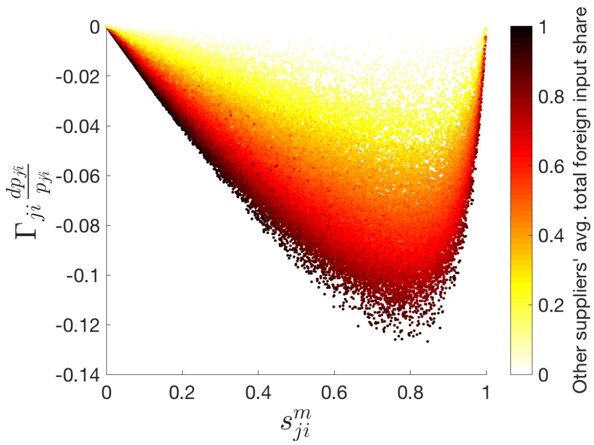
Here we plot these two effects, along with the net effects of $\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}$. First, in Figure B.12 we plot the pairwise attenuation and pro-competitive effects against the input shares s_{ji}^m. As we have argued in the main text, the upper bounds of the both effects display a hump shape with respect to s_{ji}^m. Within the same values of s_{ji}^m, there are variations in the two effects, depending on how much shock the supplier or other suppliers received, $\frac{\mathrm{d}c_j}{c_j}$ and $\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}$. The cost changes at the firm-level can be approximated by firm-level measures of total foreign input share, s_{Fj}^{Total}:

$$s_{Fj}^{Total} = s_{Fj} + \sum_k s_{kj} s_{Fk}^{Total}.$$

From Figure B.12, one can indeed see that within the same value of s_{ji}^m, pairs in which suppliers have a higher total foreign input share experience greater attenuation effect. Likewise, within the same value of s_{ji}^m, pairs in which the other suppliers have higher total foreign input share on average experience a greater pro-competitive effect.

Figure B.12: Attenuation and pro-competitive effects

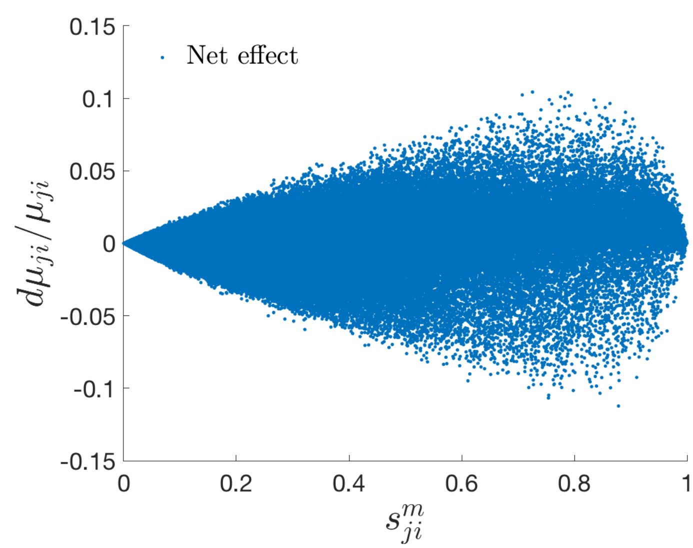




Notes: The left figure plots the pairwise attenuation effect, $-\Gamma_{ji}\frac{\mathrm{d}c_j}{c_j}$, against the input shares, s^m_{ji}. The right figure plots the pairwise pro-competitive effect, $\Gamma_{ji}\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}$, against the input shares. In both figures, we add colors that represents suppliers' total foreign input share, s^{Total}_{Fj}. In the left figure, the darker color indicates the higher value of s^{Total}_{Fj}. In the right figure, the darker color indicates the higher value of other suppliers' total foreign input share: $\frac{\sum_{k\in Z_i, k\neq j} s^m_{ki}s^{Total}_{Fk}+s^m_{Fi}}{1-s^m_{ji}}$.

Finally we plot the net effects of the two, $\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}$, in Figure B.13.

Figure B.13: Change in markups



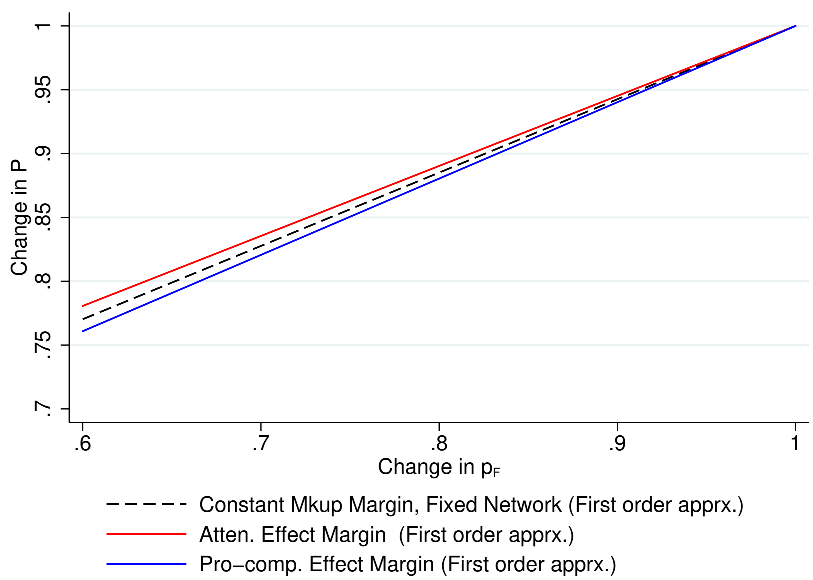
B.6.3 First order approximated changes in price index

Further manipulating equation (2.36), we can decompose the first order approximated change in price index into three components:

$$\begin{aligned}\frac{\mathrm{d}P}{P} = &\sum_i s_{iH}\left(\sum_{j\in Z_i} s_{ji}\frac{\mathrm{d}c_j}{c_j} + s_{Fi}\frac{\mathrm{d}p_F}{p_F}\right)\\ &-\sum_i s_{iH}s_{mi}\sum_{j\in Z_i} s^m_{ji}\Gamma_{ji}\frac{\mathrm{d}c_j}{c_j}\\ &+\sum_i s_{iH}s_{mi}\sum_{j\in Z_i} s^m_{ji}\Gamma_{ji}\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}. \end{aligned} \tag{B.9}$$

The first line of equation (B.9) describes the channels that are present in the constant markup case. The second line represents the aggregate attenuation effect, and the third line represents the aggregate pro-competitive effects. We plot in Figure B.14 the three components of the change in aggregate price index, computed from the firm-to-firm trade data in 2012.

Figure B.14: Attenuation and pro-competitive effects



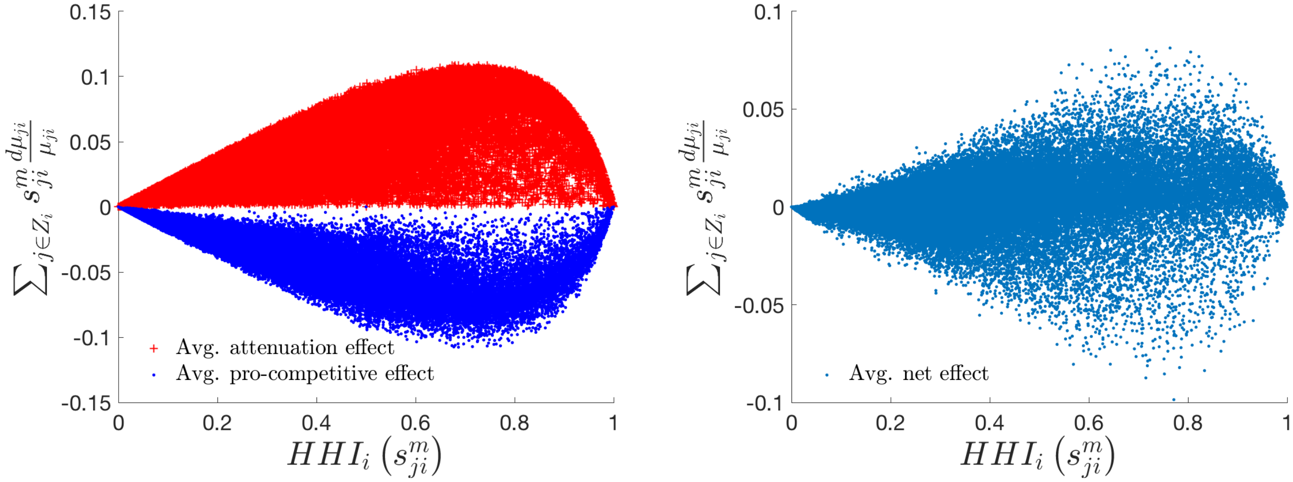
B.6.4 Average attenuation and pro-competitive effects

The average change in markups that firm i faces, $\sum_{j \in Z_i} s^m_{ji} \frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}$, can be decomposed into to: the average attenuation effect that the firm faces, and the average pro-competitive effect that the firm faces:

$$\sum_{j \in Z_i} s^m_{ji} \frac{\mathrm{d}\mu_{ji}}{\mu_{ji}} = \underbrace{-\sum_{j \in Z_i} s^m_{ji} \Gamma_{ji} \frac{\mathrm{d}c_j}{c_j}}_{\text{avg. attenuation effect}} + \underbrace{\sum_{j \in Z_i} s^m_{ji} \Gamma_{ji} \frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}}_{\text{avg. pro-competitive effect}} .$$

Here we plot these two effects at the firm-level, along with the net effects of $\sum_{j \in Z_i} s^m_{ji} \frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}$. We plot these against the HHI of input shares s^m_{ji} across suppliers j. Maximum magnitudes of both average attenuation and pro-competitive effects display a hump shape with respect to the HHI of input shares. If the input shares are completely diversified (HHI close to 0), then their markups do not change as all suppliers have infinitesimal input shares. Also, if the input shares are very skewed (HHI close to 1), suppliers' markups do not change, since one supplier has an input share close to 1 and all others have shares close to 0. The variation within the same value of HHI comes from different combinations of suppliers' input shares s^m_{ji} and their exposure to the shock $\left(\frac{\mathrm{d}c_j}{c_j}, \frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}\right)$.

Figure B.15: Average attenuation/pro-competitive effects



Notes: In the positive region of the left figure, we plot the average attenuation effect for each firm i, $-\sum_{j\in Z_i} s^m_{ji}\Gamma_{ji}\frac{\mathrm{d}c_j}{c_j}$, against the HHI of input shares, s^m_{ji}, across supplier firm j. In the negative region of the left figure, we plot the average pro-competitive effect for each firm i, $\sum_{j\in Z_i} s^m_{ji}\Gamma_{ji}\frac{\mathrm{d}p_{\not{j}i}}{p_{\not{j}i}}$, against the HHI of input shares, s^m_{ji}, across supplier firm j. In the right figure, we plot the net effect, $\sum_{j\in Z_i} s^m_{ji}\frac{\mathrm{d}\mu_{ji}}{\mu_{ji}}$, against the HHI of input shares.

B.6.5 Shock to one importer

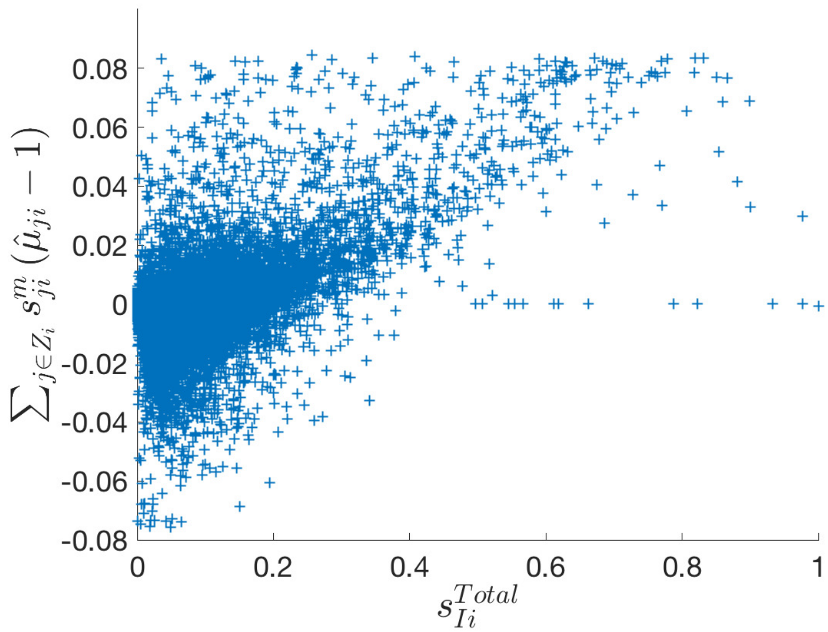
Here we consider a shock of foreign price reduction that hits a single importer, firm I. Analogous to Figure 2.8 in the main text, in Figure B.16 we plot the firms' average change in markups, $\sum_{j\in Z_i} s^m_{ji}\left(\hat{\mu}_{ji}-1\right)$, against the measure capturing firms' closeness to the shock. Instead of firms' indirect exposure to imports $s^{Indirect}_{Fi}$, we construct a measure s^{Total}_{Ii} that captures firms' exposure to firm I's goods:

$$
\begin{aligned}
s^{Total}_{Ii} &= \sum_{k\in Z_i} s_{ki}s^{Total}_{Ik} && \text{if } i\neq I \\
s^{Total}_{Ii} &= 1 && \text{if } i = I.
\end{aligned}
$$

One can see that the positive correlation between the two measures are stronger than in Figure 2.8.

In Table B.17, we report the changes in aggregate price index in response to the two different shocks. The magnitude of the change is smaller when the shock hits a single importer, but the magnitude of the net effects of adding oligopolistic competition becomes larger.

Figure B.16: Average change in markups and s_{Ii}^{Total}



Notes: The figure plots $\sum_{j\in Z_i} s_{ji}^m (\hat{\mu}_{ji}-1)$ upon $\hat{p}_F = 0.6$, against s_{Ii}^{Total}.

Table B.17: Changes in price index under two different shocks

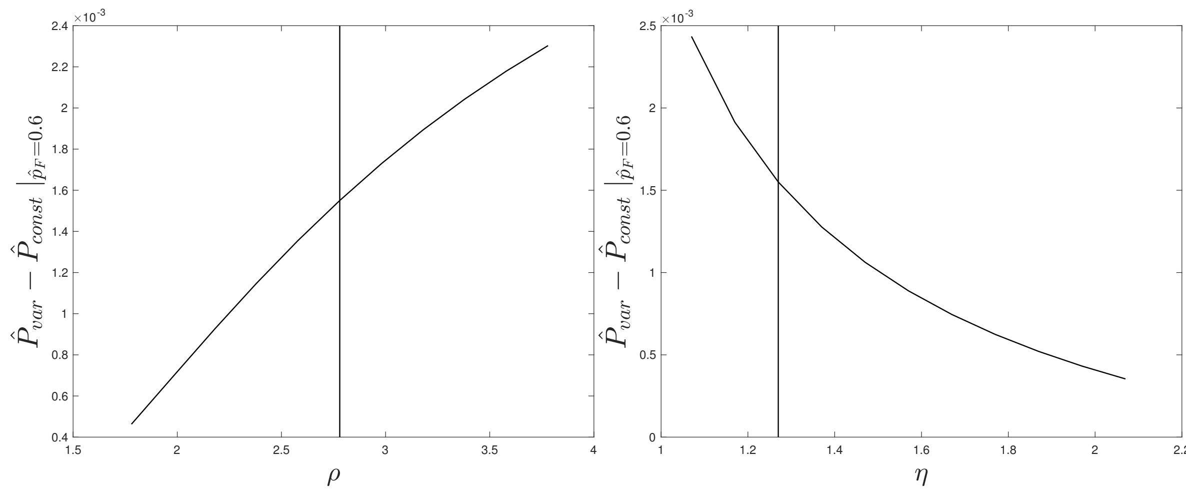
	Shock to all importers	Shock to one importer
$\hat{P}_{const}$	0.707	0.976
$\hat{P}_{var}$	0.709	0.977
$\left(\hat{P}_{var}-\hat{P}_{const}\right)/\left(1-\hat{P}_{const}\right)$	0.005	0.053

Notes: The changes in price index are evaluated when $\hat{p}_F = 0.6$.

B.6.6 Changes in price index under different CES parameters

Here we show that the net effect of adding variable markups on the aggregate price index depends on the values of the underlying CES parameters. Figure B.17 plots the net effect of variable markups on price index, $\hat{P}_{var} - \hat{P}_{const}$, against different parameters of ρ and η. The changes in price index are evaluated at $\hat{p}_F = 0.6$. One can see that larger values of ρ and smaller values of η lead to larger net effects, where the attenuation effect dominates the pro-competitive effect.

Figure B.17: The net effect of variable markups on $\hat{P}$ under different CES parameters



Notes: These figures plot the net effects of variable markups on price index, $\hat{P}_{var} - \hat{P}_{const}$, against different parameters of ρ and η. The changes in price index are evaluated at $\hat{p}_F = 0.6$. The vertical lines depict the estimated values of the CES parameters.

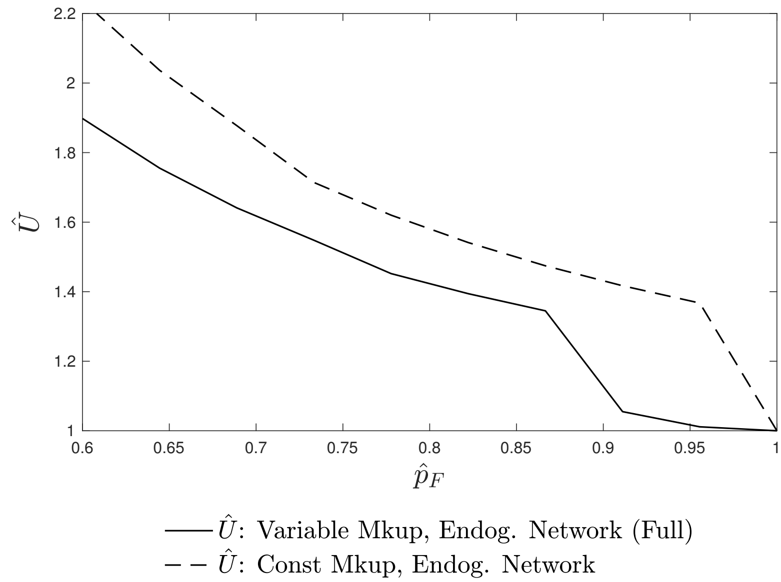
The effects of different ρ and η on the net effects can be explained by their effects on markup elasticities Γ_{ji}. As ρ gets larger and as η gets smaller, Γ_{ji} increases for all regions of s^m_{ji}. As explained in the main text, higher Γ_{ji} means larger maximum magnitudes of the pairwise attenuation and pro-competitive effects.

B.6.7 Changes in welfare under variable and constant markups

Here we explore the implications of the interaction between variable markups and endogenous networks on the changes in aggregate welfare, $\hat{U}$. Figure B.18 shows the analogous result for

Figure 2.12, but now for $\hat{U}$. In addition to the change in price index, the change in welfare also reflects the change in aggregate welfare as well as its initial level. Facing cheaper costs and smaller degree of double marginalization, both the level of initial aggregate profits and the increase in aggregate profits are much larger in the constant markup case. These contribute to larger predicted increase in aggregate welfare.

Figure B.18: $\hat{U}$ under endogenous networks: variable and constant markups



B.7 One sector model of firm-to-firm trade

In this section we outline a model of firm-to-firm trade within a single sector. We focus on firm-to-firm trade that occurs within the sector, and assume firms take demand and prices of goods outside the sector as exogenous. Estimating the parameters in this partial equilibrium model, we conduct the same counterfactual exercise as in main text, and show that it yields qualitatively the same results.

B.7.1 Model outline

We assume that firms take demand and prices outside the sector as exogenous. Let demand for firms' goods from domestic households be D, and those from firms outside the sector to be D_o. All firms in the sector produce goods by using labor input and outside sector goods. They all sell to households and to firms outside the sector. Their network decisions involve which firms in the same sector to source from, and whether to import/export.

Firms in the sector have the same production technologies as the heterogeneous goods sector in the main text, but treat the price of goods outside the sector, p_o, as exogenous. The implied unit cost of firm i in the sector is

$$c_i = \phi_i^{-1} \left(\omega_l^{\eta} w^{1-\eta} + \omega_m^{\eta} p_{mi}^{1-\eta} \right)^{\frac{1}{1-\eta}},$$

where p_{mi} is the firm specific price index for i's goods input. p_{mi} varies with the firm's sourcing strategy Z_i and I_{Fi}. Z_i is now a set of i's suppliers that belong to the same sector. We assume that all firms in the sector buy intermediate inputs from the outside sector.

$$p_{mi} = \left(\sum_{j \in Z_i} \alpha_{ji}^{\rho} p_{ji}^{1-\rho} + \alpha_{oi}^{\rho} p_o^{1-\rho} + I_{Fi} \alpha_{Fi}^{\rho} p_F^{1-\rho} \right)^{\frac{1}{1-\rho}}.$$

Now let us describe the market structure. As in the main text, we assume monopolistic competition when firms sell to final demand and export:

$$p_{iH} = p_{iH} = \frac{\sigma}{\sigma - 1} c_i.$$

We also assume that when firms sell their output to firms outside the sector, they engage in monopolistic competition. Given our assumption that all firms in the sector sell at least part of their output to firms outside the sector, it is reasonable to assume that firms act as if they were infinitesimally small. We posit that firms in the outside sector have the same production function, which leads to the price that firm i charges when selling goods to an outside sector firm being

$$p_{iO} = \frac{\rho}{\rho - 1} c_i.$$

As for the intra sector firm-to-firm trade, we maintain oligopolistic competition as in the main text. Firm i charges higher markup to j if i has larger input share in j's goods bundle:

$$p_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} c_i$$
$$\varepsilon_{ij} = \rho \left(1 - s_{ij}^m\right) + \eta s_{ij}^m.$$

Firms make their linkage formation decisions by maximizing their variable profits net of fixed costs. Their decisions involve choosing the set of suppliers in the same sector, Z_i, and importing/exporting statuses, I_{Fi} and I_{iF}. The variable profit of i is:

$$\pi_i^{var}(Z_i, I_{Fi}, I_{iF}) = \frac{1}{\sigma} \underbrace{\beta_{iH}^{\sigma} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i(Z_i, I_{Fi})^{1-\sigma} D}_{\text{Sales to HH}} + I_{iF} \frac{1}{\sigma} \underbrace{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i(Z_i, I_{Fi})^{1-\sigma} D^*}_{\text{Exports}}$$
$$+ \frac{1}{\rho} \underbrace{\left(\frac{\rho}{\rho-1}\right)^{1-\rho} c_i(Z_i, I_{Fi})^{1-\rho} D_o}_{\text{Sales to outside sector}}$$
$$+ \sum_{j \in W_i} \frac{1}{\varepsilon_{ij}} \underbrace{\alpha_{ij}^{\rho} p_{ij}(Z_i, I_{Fi})^{1-\rho} \frac{s_{mj} c_j q_j}{p_{mj}^{1-\rho}}}_{\text{Sales to } j},$$

where W_i is now the set of customers of i in the same sector. Taking as given other firms' decisions, the exogenous demand parameters D, D^* and D_o, and exogenous prices w, p_o and p_F, the firm maximizes the total net profit:

$$\pi_i(Z_i, I_{Fi}, I_{iF}) = \pi_i^{var}(Z_i, I_{Fi}, I_{iF}) - \sum_{j \in Z_i} w f_{Di} - I_{Fi} w f_{Fi} - I_{iF} w f_{iF}.$$

B.7.2 Estimation

We then apply this partial equilibrium model to the data. For the sector in which we focus on firm-to-firm trade, we use the NACE 2-digit sector 10, which is the "Manufacture of food products". In 2012, there were 3481 firms in that sector in our sample. Out of all the B2B inputs that firms in the sector purchased in 2012, around 25% were from firms in the same sector. Out of all the goods that firms in the sector sold, around 23% were to firms in the same sector.

Analyzing the firm-to-firm network with only 30 firms in this sector is reasonable, as the largest 30 firms accounted for around 99% of total sales in the sector. Moreover, around 99% of all the B2B sales values that occurred within that sector were sales where both the supplier and the customer firm were among the largest 30 firms.

For the three CES parameters $\{\eta, \rho, \sigma\}$ and the dispersion parameter of the productivity distribution, we use the values estimated in Section 2.5.1 and Section 2.5.2. We re-estimate the four fixed costs parameters in the same way as in Section 2.5.3, using the same set of moments.

For the rest of the parameters, we do the following calibration. First, we set the saliency terms $\{\beta_{iH}, \alpha_{ij}, \alpha_{oi}, \alpha_{Fi}\}$ to be equal to 1. We set the production weights on labor inputs and goods input, ω_l and ω_m to be 0.3 and 0.7 to match the average labor input share of 0.27 for the sector. We set the demand parameters D and D_o to be 5×10^{12} and 2×10^{10}, to match firms' average output shares to domestic final demand (0.58) and to firms outside the sector (0.28). We set D^* to 2×10^{12} to match exporting firms' average export share, 0.31. We normalize wage w to one, and set foreign price p_F and price of outside good p_o to be 1 and 2 so that they match importers' average import share (0.20) and firms' average input share of the outside goods (0.53), respectively.

Table B.18 reports the estimated fixed costs parameters, estimated via simulated methods of moments.

Table B.18: Estimated values for the fixed cost parameters

	Φ_D^{scale}	Φ_{IM}^{scale}	Φ_{EX}^{scale}	Φ^{disp}
Estimate	6.07	27.01	27.17	10.05

And in Table B.19 we report the fit of the model for the targeted moments under the estimated parameters.

Table B.19: Targeted moments

	Data	Model
Fraction of firms sourcing from domestic firms	0.90	0.90
Fraction of importers	0.19	0.20
Fraction of exporters	0.17	0.17
Corr(Indeg, Outdeg)	0.47	0.47

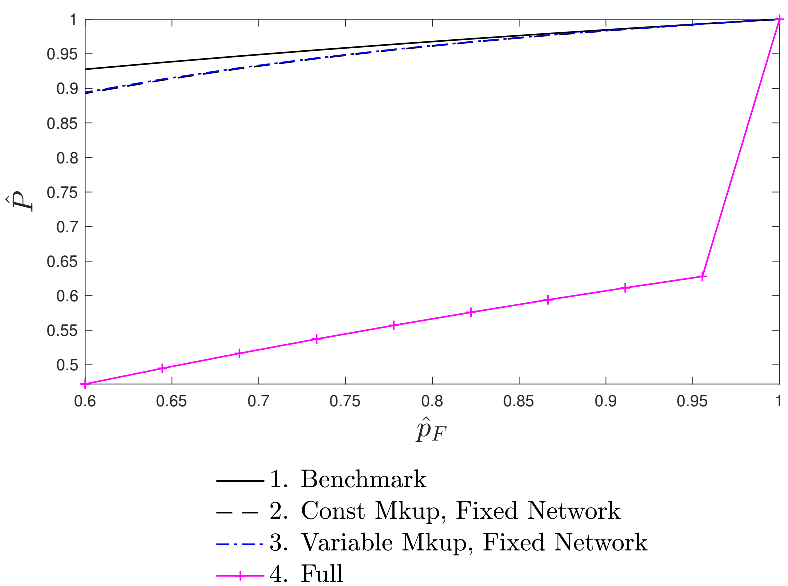
B.7.3 Counterfactual analysis

Here we conduct counterfactual analysis, analogous to Section 2.6.1 in the main text. We take an exogenous reduction in the foreign good's price, p_F, as the shock and focus on how the price index of the sector, P, changes. We define P as

$$P = \left(\sum_{i \in \Omega} \beta_{iH}^{\sigma} p_{iH}^{1-\sigma} \right)^{\frac{1}{1-\sigma}},$$

where Ω is the set of firms in the sector. Analogous to Figure 2.9 in the main text, we plot in Figure B.19 the changes in P under four cases. One can see that the qualitative results still hold in our one sector partial equilibrium model: firms' endogenous network formation can significantly alter aggregate implications.

Figure B.19: $\hat{P}$ in four cases



APPENDIX C

APPENDIX TO TRADE AND DOMESTIC PRODUCTION NETWORKS

C.1 Theoretical Results

C.1.1 Proof of Proposition 1

Total cost increase

We have

$$
\begin{aligned}
s_{Fj}^{Total} &= s_{Fj} + \sum_i s_{ij} s_{Fi}^{Total} \\
&= s_{Fj} + \sum_i s_{ij} \left[s_{Fi} + \sum_k s_{ki} \left(s_{Fk} + \cdots \right) \right]
\end{aligned}
$$

and

$$
c_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} c_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho} + \alpha_{Fj}^{\rho-1} \phi_j^{\rho-1} p_{Fj}^{1-\rho}.
$$

Consider the indirect effects (i.e., assuming that suppliers' cost increases will translate into price increases for their customers). The unit cost after $p_{Fj} \to \infty$ for all j (assuming that the nominal wage, w_ℓ, does not change) is

$$
\tilde{c}_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} \tilde{c}_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho},
$$

thus

$$
\begin{aligned}
\hat{c}_j^{1-\rho} \mid_{total}^{p_{F\cdot}\to\infty} &= \frac{\tilde{c}_j^{1-\rho}}{c_j^{1-\rho}} \\
&= \frac{\sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} \tilde{c}_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho}}{c_j^{1-\rho}} \\
&= s_{\ell j} + \sum_k s_{kj} \hat{c}_k^{1-\rho} \mid_{total}^{p_{F\cdot}\to\infty} \\
&= s_{\ell j} + \sum_k s_{kj} \left[s_{\ell k} + \sum_i s_{ik} \left(s_{\ell i} + \cdots \right) \right] \\
&= 1 - s_{Fj} - \sum_k s_{kj} + \sum_k s_{kj} \left[\left(1 - s_{Fk} - \sum_i s_{ik} \right) \right. \\
&\qquad \left. + \sum_i s_{ik} \left(\left(1 - s_{Fi} + \sum_l s_{li} \right) + \cdots \right) \right] \\
&= 1 - \left(s_{Fj} + \sum_k s_{kj} \left[s_{Fk} + \sum_i s_{ik} - \sum_i s_{ik} \left(1 - s_{Fi} + \sum_l s_{li} + \cdots \right) \right] \right) \\
&= 1 - \left(s_{Fj} + \sum_k s_{kj} \left[s_{Fk} + \sum_i s_{ik} \left(s_{Fi} + \cdots \right) \right] \right) \\
&= 1 - s_{Fj}^{Total}.
\end{aligned}
$$

Therefore, firms' change in unit costs upon $p_f \to \infty$ when considering the full network effects are as follows:

$$
\hat{c}_j \mid_{total}^{p_{F\cdot}\to\infty} = \left(1 - s_{Fj}^{Total} \right)^{\frac{1}{1-\rho}}.
$$

Direct cost increase

If only considering the direct effect (i.e., assuming that suppliers' cost increases will not translate into price increases for their customers), with $\rho > 1$ and $p_{Fj} \to \infty$ for all j (i.e., autarky), the cost for firm j becomes

$$\bar{c}_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} c_k^{1-\rho} + \alpha_{\ell j}^{\rho-1} \phi_j^{\rho-1} w_\ell^{1-\rho}.$$

Therefore,

$$\hat{c}_j^{1-\rho} \mid_{direct}^{p_{F\cdot} \to \infty} = \frac{\bar{c}_j^{1-\rho}}{c_j^{1-\rho}} = 1 - s_{Fj}.$$

Re-arranging yields the change in unit cost when considering only the direct effect:

$$\hat{c}_j \mid_{direct}^{p_{F\cdot} \to \infty} = \left(1 - s_{Fj}\right)^{\frac{1}{1-\rho}}.$$

Cost increase under roundabout production

In this roundabout production economy, firm j produces its goods with a CES production technology, using domestic intermediate goods, foreign imports, and labor. The implied unit cost of firm j becomes

$$c_j = \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} P_D^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho} \right)^{\frac{1}{1-\rho}},$$

where P_D is a price index of domestic intermediate goods. Associated input shares are $s_{Dj} = \frac{\phi_j^{\rho-1} \alpha_{Dj}^{\rho-1} P_D^{1-\rho}}{c_j^{1-\rho}}$, $s_{Fj} = \frac{\phi_j^{\rho-1} \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho}}{c_j^{1-\rho}}$, and $s_{\ell j} = \frac{\phi_j^{\rho-1} \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}}{c_j^{1-\rho}}$.

As in Blaum, Lelarge, and Peters (2016), we let domestic intermediate goods be produced via roundabout production, with CES substitution parameter σ. The price of an intermediate good is therefore equal to the CES price index,

$$P_D = \left(\sum_j \alpha_{jD}^{\sigma-1} p_{jD}^{1-\sigma} \right)^{\frac{1}{1-\sigma}},$$

where p_{jD} is the price that firm j charges in the aggregation process. Let $p_{jD} = c_j$. We can additionally define $s_{jD} = \frac{\alpha_{jD}^{\sigma-1} p_{jD}^{1-\sigma}}{P_D^{1-\sigma}}$, which is the firm j's contribution to the intermediate good. We use the firm's share of domestic sales:

$$s_{jD} = \frac{\text{B2B sales}_j + \text{Sales to HH}_j}{\sum_i (\text{B2B sales}_i + \text{Sales to HH}_i)}.$$

We assume $\alpha_{jD} = \beta_{jD}$, so that the two shares s_{jD} and s_{jH} are the same in the model.

Consider a change in c_j, upon $p_{Fj} \to \infty$ for all j.

$$\begin{aligned} c_j &= \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} P_D^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho} \right)^{\frac{1}{1-\rho}} \\ \tilde{c}_j &= \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} \tilde{P}_D^{1-\rho} + \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho} \right)^{\frac{1}{1-\rho}} \\ \tilde{P}_D^{1-\sigma} &= \sum_j \alpha_{jD}^{\sigma-1} \tilde{c}_j^{1-\sigma} \end{aligned}$$

Combining these,

$$\tilde{c}_j^{1-\rho} = \phi_j^{\rho-1} \alpha_{Dj}^{\rho-1} \left(\sum_j \alpha_{jD}^{\sigma-1} \tilde{c}_j^{1-\sigma} \right)^{\frac{1-\rho}{1-\sigma}} + \phi_j^{\rho-1} \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}.$$

Thus,

$$\begin{aligned} \hat{c}_j^{1-\rho} \mid_{roundabout}^{p_{F\cdot} \to \infty} &= \frac{\phi_j^{\rho-1} \alpha_{Dj}^{\rho-1} \left(\sum_j \alpha_{jD}^{\sigma-1} \tilde{c}_j^{1-\sigma} \right)^{\frac{1-\rho}{1-\sigma}} + \phi_j^{\rho-1} \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}}{c_j^{1-\rho}} \\ &= s_{\ell j} + s_{Dj} \frac{\left(\sum_j \alpha_{jD}^{\sigma-1} \tilde{c}_j^{1-\sigma} \right)^{\frac{1-\rho}{1-\sigma}}}{P_D^{1-\rho}} \\ &= s_{\ell j} + s_{Dj} \left(\frac{\sum_j \alpha_{jD}^{\sigma-1} c_j^{1-\sigma} \hat{c}_j^{1-\sigma} \mid_{roundabout}^{p_{F\cdot} \to \infty}}{P_D^{1-\sigma}} \right)^{\frac{1-\rho}{1-\sigma}} \\ &= s_{\ell j} + s_{Dj} \left(\sum_j s_{jD} \hat{c}_j^{1-\sigma} \mid_{roundabout}^{p_{F\cdot} \to \infty} \right)^{\frac{1-\rho}{1-\sigma}}. \end{aligned}$$

The solution to this system of equations $\hat{c}_j^{1-\rho}\, |_{roundabout}^{p_{F}.\to\infty}$ is the change in unit costs of each firm, upon autarky.

C.1.2 Proof of Lemma 1

Using the result that firms sell to other firms are marginal cost and rearranging equation (3.8), we obtain

$$c_j^{1-\rho} = \sum_k \phi_j^{\rho-1} \alpha_{kj}^{\rho-1} c_k^{1-\rho} + \phi_j^{\rho-1} \alpha_{\ell j}^{\rho-1} w_\ell^{1-\rho}.$$

In matrix form, this equation becomes

$$\boldsymbol{c}^{1-\rho} = \left(I - A^{'}\right)^{-1} \boldsymbol{\phi}^{\rho-1} \circ \boldsymbol{\alpha}_\ell^{\rho-1} \circ w_\ell^{1-\rho},$$

where the (i, j) element of A is $\phi_j^{\rho-1}\alpha_{ij}^{\rho-1}$ if $i \in Z_j$ and 0 otherwise. The assumption that the matrix $\left(I - A^{'}\right)$ is invertible and the fact that under closed economy one can normalize wage w_ℓ, guarantee that there is a unique vector $\boldsymbol{c}$ that solves the equation above. With the cost vector $\boldsymbol{c}$ and constant mark-ups in sales to final consumers, one can compute a unique aggregate price index P according to equation (3.6). Given the cost vector and aggregate price index, one can then compute a unique aggregate expenditure E from equations (3.13) and (3.18).

C.1.3 Proof of Proposition 2

Denote post-shock equilibrium variable x with $\tilde{x}$. From equation (3.6), we have the expression for the price index after the shock,

$$\tilde{P} = \left(\sum_i \beta_i^{\sigma-1} \mu^{1-\sigma} \tilde{c}_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

Combining this expression with the pre-shock price index P, we have

$$\begin{aligned}\hat{P} &= \frac{\tilde{P}}{P} \\ &= \left(\frac{\sum_i \beta_i^{\sigma-1}\mu^{1-\sigma}\tilde{c}_i^{1-\sigma}}{P^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} \\ &= \left(\sum_i \frac{\beta_i^{\sigma-1}\mu^{1-\sigma}c_i^{1-\sigma}}{P^{1-\sigma}}\hat{c}_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\ &= \left(\sum_i s_{iH}\hat{c}_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\end{aligned}$$

where s_{iH} denotes firm i's share in final consumption. This equation says that the change in the aggregate price index depends on each firm's change in cost and its share in final consumption before the shock.

C.2 Numerical example: gains from trade

Here we demonstrate that economies with identical sets of aggregate exports, aggregate imports, aggregate gross production and GDP but with different firm-to-firm network structures can potentially generate different gains from trade. We will work with a simple example comparing two economies, where in both economies aggregate exports and imports are €100, aggregate gross production is €300, and GDP is €100.

Both economies consist of two firms. Table C.1 lays out the details of the two economies. In economy 1, both firms are identical, and the firm-to-firm network is symmetric in the sense that firms sell the same amount of goods to each other. In economy 2, firm 1 sells €100 to firm 2, while firm 2 does not sell any of its goods to firm 1. There is also asymmetry in sales to household and in exports, where firm 1 sells €10 to households and exports €40, while firm 2 sells €90 to households and exports €60. In the two economies, the aggregate values of imports, exports, gross production and value added are identical.

Table C.1: Two economies

	Economy 1		Economy 2	
	Firm 1	Firm 2	Firm 1	Firm 2
Imports	50	50	50	50
Exports	50	50	0	100
Gross production	150	150	150	150
Firm-to-firm sales	x_{12}	x_{21}	x_{12}	x_{21}
	50	50	100	0
Labor costs / Value added	50	50	100	0
Sales to households	50	50	50	50

Now let us compute the direct and total shares of foreign inputs for the two firms in two economies. Table C.2 summarizes the foreign input shares. The direct shares of foreign inputs for both firms in both economies are 1/3, as they all import €50 while their total inputs are €150. We can also compute the total shares of foreign inputs by solving the system of equations, from equation (3.14). In economy 2, firm 2 has higher exposure to foreign inputs because not only does the firm import directly, it also relies heavily on goods from firm 1, which also imports directly.

Table C.2: Direct and total shares of foreign inputs

	Economy 1		Economy 2	
	Firm 1	Firm 2	Firm 1	Firm 2
s_{Fi}	1/3	1/3	1/3	1/3
s_{Fi}^{Total}	1/2	1/2	1/3	5/9

Finally, let us compute the change in aggregate price index from banning imports. Applying equations (3.15) and (3.21), we obtain the changes in price index for economy 1 and economy 2:

$$\hat{P}_1 = \left(\frac{1}{2}\left(1-\frac{1}{2}\right)^{\frac{1-\sigma}{1-\rho}} + \frac{1}{2}\left(1-\frac{1}{2}\right)^{\frac{1-\sigma}{1-\rho}}\right)^{\frac{1}{1-\sigma}}$$

$$\hat{P}_2 = \left(\frac{1}{2}\left(1-\frac{1}{3}\right)^{\frac{1-\sigma}{1-\rho}} + \frac{1}{2}\left(1-\frac{5}{9}\right)^{\frac{1-\sigma}{1-\rho}}\right)^{\frac{1}{1-\sigma}}.$$

If we apply $\rho = 2$ and $\sigma = 4$, then we obtain $\hat{P}_1 = 2$ and $\hat{P}_2 = 1.73$. Even though the aggregate statistics (GDP, gross production, export and import) are identical across the two economies, the gains from trade in economy 1 turns out to be larger. This is because of the asymmetry in firm-to-firm trade where it creates different s_{Fi}^{Total} for the two firms. While in economy 1 the two firms have identical total exposure to foreign inputs, in economy 2, firm 2 has larger exposure. Upon ban of foreign inputs, in economy 2, households will be able to substitute away toward firm 1's goods, resulting in a smaller increase in aggregate price index.

C.3 Data Appendix

C.3.1 Grouping VAT-identifiers into firms

As mentioned in the main text, all our datasets are recorded at the VAT-identifier level. We utilize ownership filings in the Annual Accounts and information from the Balance of Payments survey in order to aggregate multiple VAT-identifiers into firms. In the ownership filings, each enterprise reports a list of all other enterprises of which it has an ownership share of at least 10% and the value of the share. In the Balance of Payments survey, Belgian enterprises with international financial linkages have to report their stock and flows of financial links. They have to report both the international participation they own and the foreign owners of financial participation in their capital if the participation represents at least 10% of the capital. The survey is designed to cover the population of Belgian enterprises involved in international financial transactions.

We group all VAT-identifiers into firms if they are linked with more than or equal to 50% of ownership. In addition, we group all VAT-identifiers into firms if they share the same foreign parent firm that holds more than or equal to 50% of their shares. We use a "fuzzy string matching" method to determine whether they share the same foreign parent firm, by obtaining similarity measures of all possible pairs of foreign firms' names. Lastly, in order to correct for misreportings, we also add links to the VAT-identifier pairs if the two were linked one year before and one year after. We define a firm as the group of VAT-identifiers that are directly and indirectly linked.

Given these groupings of VAT-identifiers, we then choose the "most representative" VAT-identifier for each firm. We use this "head VAT-identifier" as the identifier of the firm.[1] Then,

1. The criteria for determining the head VAT-identifier is as follows: (i) If there is only one VAT-identifier in the firm that filed all the full annual accounts, the VAT declarations, and the B2B filings, then this VAT-identifier is chosen as the head. (ii) If there are no such VAT-identifiers or multiple of them, then we choose the VAT-identifier that has the largest total assets reported. (iii) If there are no VAT-identifier that filed the annual accounts, then we choose the VAT-identifier that has the largest amount of total inputs, which is the sum of labor costs, B2B inputs, and imports.

Table C.3: Number of VAT-identifier in firms with multiple VAT-identifiers

	Mean	10%	25%	50%	75%	90%	max
Num. VAT-identifier	3	2	2	2	3	4	372

in order to make the identifiers consistent over time, we make the following adjustment: We take firms whose head VAT-identifier was not an identifier of any firm in the previous year. For such firms, if there exists a VAT-identifier within the firm which was a head VAT-identifier in the previous year, then we switch the firm identifier to that former head VAT-identifier.[2]

Having determined the head VAT-identifier for each firm with multiple VAT-identifiers, we aggregate all the variables up to the firm level. For variables such as total sales and inputs, we adjust the aggregated variables with the amount of B2B trade that occurred within the firm, correcting for double counting. For other non-numeric variables such as firms' primary sector, we take the value of its head VAT-identifier.

The number of VAT-identifiers for firms with multiple VAT-identifiers are shown in Table C.3.

C.3.2 Firm selection

Table C.4 displays the same numbers for Table 3.1, with statistics for all Belgian firms added.

Table C.4: Coverage of all Belgian firms and selected sample

Year	All Belgian Firms					Selected sample				
	Count	V.A.	Sales	Imports	Exports	Count	V.A.	Sales	Imports	Exports
2002	714,469	210	812	204	217	122,460	123	586	179	189
2007	782,006	274	1080	294	282	136,370	157	757	280	269
2012	860,373	300	1244	320	317	139,605	170	829	296	295

Notes: All numbers except for Count are denominated in billion Euro in current prices. Data for Belgian GDP, output, imports and exports are from Eurostat.

2. If there are multiple such VAT-identifier, then we choose the "most representative" VAT-identifier, using the same criteria as above.

C.3.3 Reporting thresholds of the international trade dataset

There are different reporting thresholds for the international trade dataset, depending on if the trade occurred with an extra-EU country or within the EU. The dataset covers all extra-EU exports and imports by firms with values higher than €1,000 or with weights bigger than 1,000kg. Nevertheless, we also observe values less than €1,000 as more firms use electronic reporting procedures. For intra-EU trade prior to 2006, the dataset covers all exports and imports by firms whose combined imports from intra-EU countries that are more than €250,000 a year. For intra-EU trade from 2006 onward, the thresholds for exports and imports changed to €1,000,000 and €400,000, respectively. Import reporting thresholds became €700,000 per year in 2010. While these reporting threshold for intra-EU trade imply we miss some trade transaction, they are set to capture at least 93% of aggregate Belgian trade in the micro-data, hence our data still contains the overwhelming majority of the value of Belgian trade.

C.3.4 Mapping CN codes into NACE codes

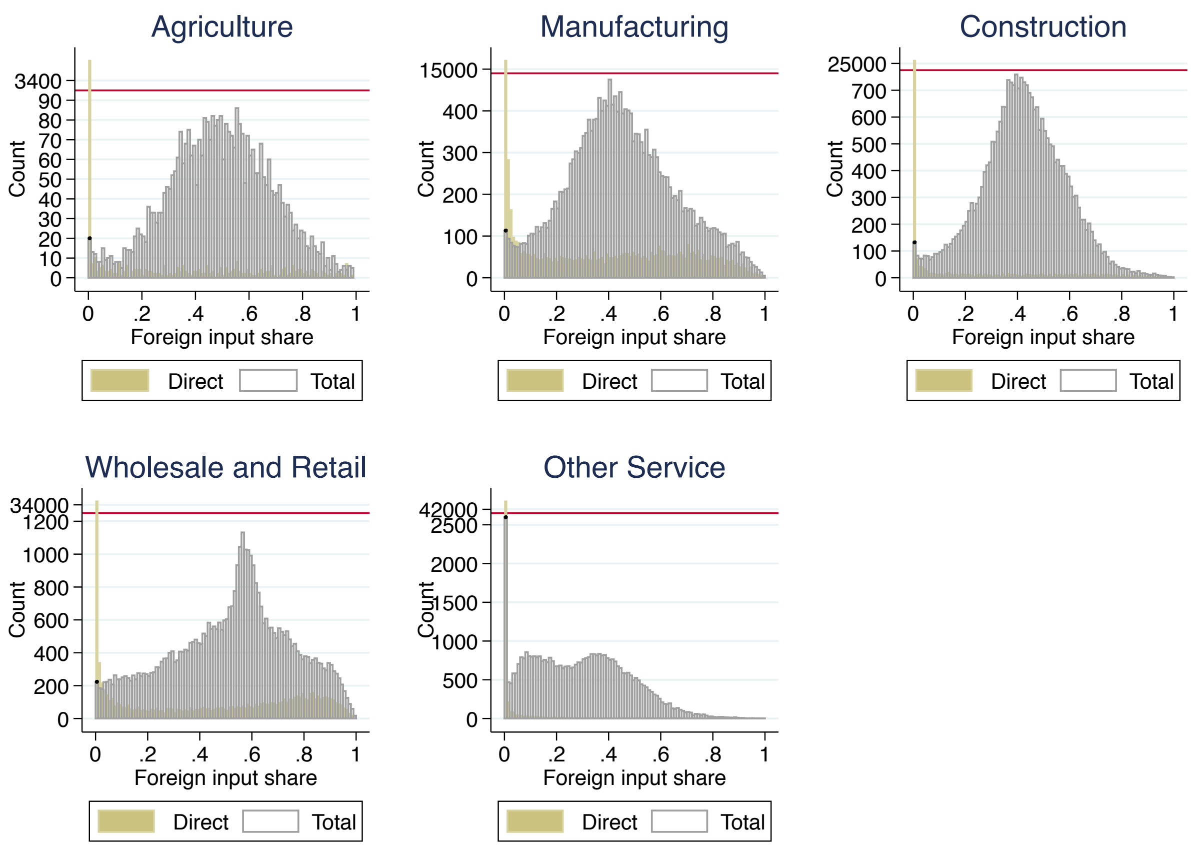
Our international trade dataset records products in Combined Nomenclature (CN) codes, up to 8 digits. On the other hand, all other datasets that we use record the enterprise's primary sector in NACE Rev.2 code. To concord the two classifications, we convert the CN 8 digit codes into NACE Rev.2 codes. As the first 6 digits of CN codes are identical to the contemporary Harmonized System (HS) codes, we first convert those HS 6-digit codes to Classification of Products by Activity (CPA) codes. We then convert CPA codes to NACE codes, using the fact that CPA 2008 codes are identical to NACE Rev.2 codes up to 4 digits. This conversion allows us to convert more than 98% of all international trade recorded in our dataset, in terms of values (in 2012).

C.4 Descriptive statistics

C.4.1 Direct and Total foreign input shares

In Figures 3.1a and C.1 we present both the direct and total foreign input shares first for the entire sample of private sector firms in Belgium and then differentiated by major sector.

Figure C.1: Histogram of direct and total foreign input share by firms' sector



Notes: The black dot indicates the ending of the bar for the total foreign input share. Total foreign input share of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct foreign input share, and s_{ji} is j's share among i's inputs. The figure is based on the analysis of 139,605 private sector firms in Belgium in 2012. The horizontal lines represent scale breaks on the vertical axis.

We summarize statistics on the distribution of the the direct and total foreign input share by firm's sector in Table C.5.

Table C.5: Distribution of direct and total foreign input share by firms' sector

Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture	0.03	0.27	0	0.49	0.68	0.49
Construction	0.01	0.11	0	0.42	0.48	0.42
Manufacturing	0.11	0.59	0	0.45	0.75	0.44
Wholesale and Retail	0.10	0.42	0	0.52	0.75	0.55
Other Services	0.01	0.18	0	0.29	0.41	0.28
Total	0.05	0.42	0	0.42	0.67	0.41

Notes: The numbers for the weighted mean are calculated using total input purchases of firms as the weight.

C.4.2 Direct and Total export share

In Figures 3.1b and C.2 we present both the direct and total export share for the entire sample of private sector firms in Belgium and then differentiated by major sector.

We summarize statistics on the distribution of the the direct and total export share by firm's sector in Table C.6.

Table C.6: Distribution of direct and total export share by firms' sector

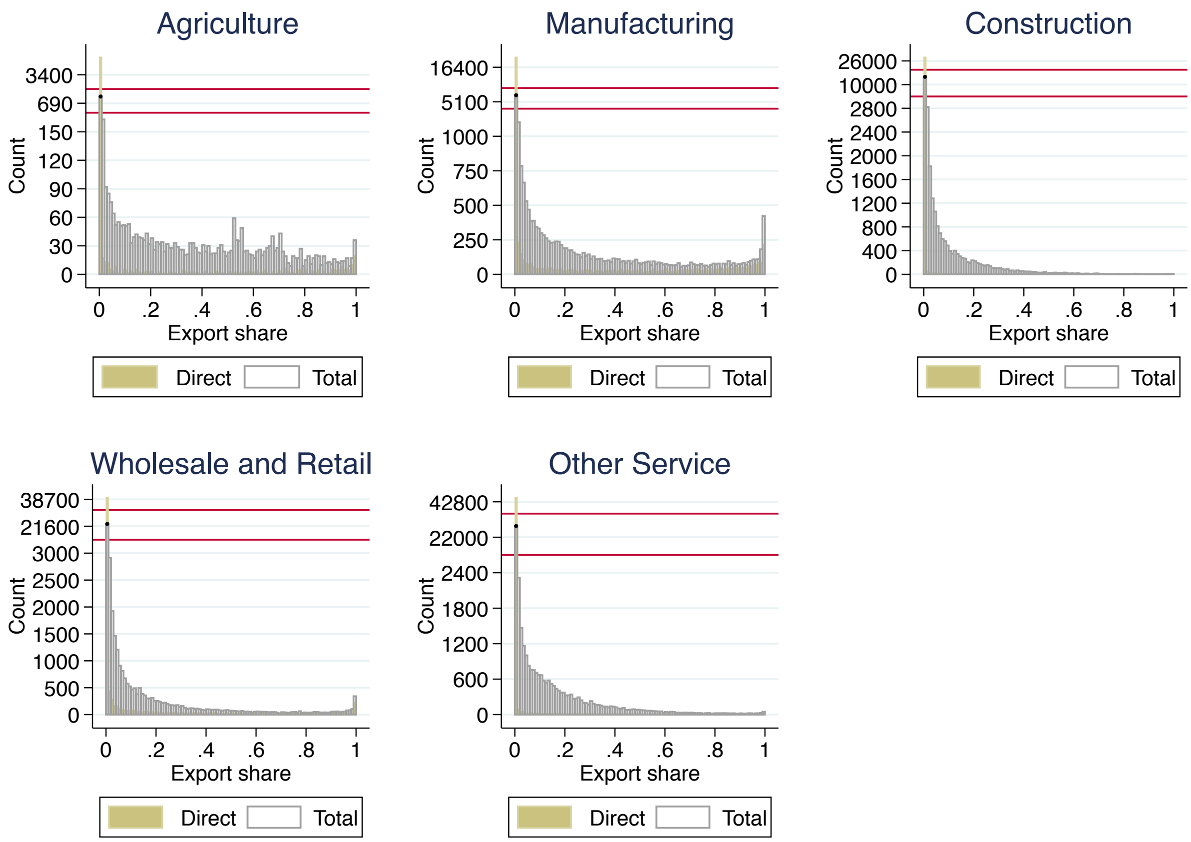
Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture	0.04	0.23	0	0.31	0.43	0.22
Construction	0.00	0.08	0	0.08	0.19	0.02
Manufacturing	0.10	0.60	0	0.25	0.68	0.10
Wholesale and Retail	0.04	0.22	0	0.10	0.30	0.01
Other Services	0.00	0.14	0	0.09	0.26	0.01
Total	0.03	0.36	0	0.12	0.45	0.02

Notes: The numbers for the weighted mean are calculated using total sales of firms as the weight.

C.4.3 Sectoral composition

Table C.7 shows the sectoral composition of our selected sample. Values for value added and output are in billion Euro.

Figure C.2: Histogram of direct and total export share by firms' sector



Notes: The black dot indicates the ending of the bar for the total export share. Total export share of firm i, s_{iF}^{Total} is calculated by solving $s_{iF}^{Total} = \tilde{s}_{iF} + \sum_{j \in W_i} \tilde{s}_{ij} s_{jF}^{Total}$ where $\tilde{s}_{iF}$ is i's share of exports out of its output, and $\tilde{s}_{ij}$ is share of i's output that went to firm j. W_i is the set of customers of i. The figure is based on the analysis of 139,605 private sector firms in Belgium in 2012. The horizontal lines represent scale breaks on the vertical axis.

Table C.7: Sectoral composition in 2012

Sector	Count	V.A.	Output	Imports	Exports
Agriculture	3,704	1.49	9.97	1.71	2.26
Construction	26,364	18.3	46.5	5.00	3.65
Manufacturing	20,385	55.5	322	147	194
Wholesale and Retail	42,999	31.8	245	85.3	54.5
Other Services	43,495	50.3	125	17.6	17.0
Other	2,658	12.7	80.5	39.8	24.3
Total	139,605	170	829	296	295

C.4.4 Link survival

Table C.8: 2002 Link Survival

Occurred In...	Count	Col %	Cum %
2002	3,570,077	65.5	65.5
2002 & 2007	912,028	16.7	82.2
2002 & 2012	191,566	3.5	85.7
2002 & 2007 & 2012	778,734	14.3	100.0

C.5 Transmission of shocks along production chain

C.5.1 Constructing the exogenous trade shocks

Below, we explain the construction of the variables $\Delta \log M_{it}$, $\Delta \log M_{it}^{S}$, and $\Delta \log M_{it}^{PS}$.

$\Delta \log M_{it}$, an exogenous shock affecting the imports of firm i, is constructed as follows:

$$\Delta \log M_{it} = \underbrace{\log \sum_{k,c} s_{ic,t-1}^{k,M} \text{WES}_{k,c,t}}_{M_{it,t-1}} - \underbrace{\log \sum_{k,c} s_{ic,t-1}^{k,M} \text{WES}_{k,c,t-1}}_{M_{it-1,t-1}},$$

where $s_{ic,t-1}^{k,M}$ is the share of imports of firm i at the initial year $t-1$ that falls on product k from country c, and $\text{WES}_{k,c,t}$ is the world export supply (excluding sales to Belgium) of country c for product k.

The import supply shock that i's suppliers received, $\Delta \log M_{it}^{S}$, is constructed as:

$$\Delta \log M_{it}^{S} = \log \sum_{k} s_{ki,t-1} M_{kt,t-1} - \log \sum_{k} s_{ki,t-1} M_{kt-1,t-1}.$$

Finally, the import supply shock to potential customers of firm i is:

$$\Delta \log M_{it}^{PS} = \log \sum_u s_{ui,t} M_{ut,t-1}^{-i} - \log \sum_u s_{ui,t-1} M_{ut-1,t-1}^{-i}.$$

Where the variables on the RHS are constructed analogous to footnotes 10 and 11.[3] [4]

C.5.2 Other reduced form results

Here we report the reduced form results where firms' changes in domestic sales and domestic inputs are on the LHS variable. The first column shows the results for firms' changes in totals are on the LHS, and is identical to the third column in Table 3.2.

Controlling for own shocks and shocks that potential customers and suppliers have received, both positive demand shock on a firm's actual customers and actual suppliers lead to an increase of the firm's domestic sales and domestic inputs.

---

3. The input share from sector u for firm i, $s_{iu,t}$, is defined as the share of inputs of i that came from firms producing sector u goods:

$$s_{uit} = \sum_{j \in Z_{it}^u} \frac{\text{Sales}_{jit}}{\text{TotalInputs}_{it}},$$

where Z_{it}^u denotes the set of suppliers of i producing sector u goods at time t.

4.

$$M_{ut,t-1}^{-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} M_{jt,t-1}$$

$$M_{ut-1,t-1}^{-i} = \sum_{j \in U_{t-1}, j \neq i} \frac{V_{jHt-1}}{\sum_{k \in U_{t-1}, k \neq i} V_{kHt-1}} M_{jt-1,t-1},$$

where U_t is the set of firms producing sector u good at t, and V_{iHt} is firm i's sales to domestic final demand at t.

Table C.9: Additional reduced form results

	$\Delta \ln$ Total Sales	$\Delta \ln$ Domestic Sales	$\Delta \ln$ Domestic Inputs
$\Delta \ln X_{it}$	0.089***	0.021	0.073***
	(0.009)	(0.013)	(0.010)
$\Delta \ln M_{it}$	0.156***	0.105***	0.093***
	(0.012)	(0.018)	(0.014)
$\Delta \ln X_{it}^{PC}$	0.025***	0.037***	0.024***
	(0.003)	(0.005)	(0.004)
$\Delta \ln M_{it}^{PS}$	0.039***	0.015**	0.085***
	(0.005)	(0.007)	(0.006)
$\Delta \ln X_{it}^{C}$	0.122***	0.127***	0.086***
	(0.013)	(0.022)	(0.016)
$\Delta \ln M_{it}^{S}$	0.041***	0.106***	0.078***
	(0.018)	(0.027)	(0.022)
N	87100	85795	87363

Notes: Standard errors are clustered at the firm level. All variables are in terms of yearly log differences from 2002 to 2012. All specifications include year fixed effects. We truncate outliers of each variables at the top and bottom 1% level. Firms' domestic sales are the sum of their sales to other domestic firms, and sales to domestic final demand. Firms' domestic inputs are the sum of their labor costs and input purchases from other domestic firms.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

C.6 Ordering algorithm

In this section we describe the implementation of the ordering algorithm to solve the feedback arc set problem. We begin by defining some terms and notation.

C.6.1 Terms and notation

- *graph / network*, $G = (V, E)$ - A collection of a set of edges E and set of vertices V. Edges describe the relationship between vertices. Two basic classifications of graphs are based on whether the edges are *directed* or *undirected* and whether they are *weighted* or *unweighted*
- $n = |V|$, $m = |E|$
- *cycle* - A path within a graph where a vertex is reachable from itself
- $d^+(u)$ - For a vertex $u \in V$ in a directed graph, number of outgoing edges
- $d^-(u)$ - For a vertex $u \in V$ in a directed graph, number of incoming edges

- $w^+(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of outgoing edges

- $w^-(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of incoming edges

- *sink* - A vertex $u \in V$ in a directed graph with $d^+(u) = 0$

- *source* - A vertex $u \in V$ in a directed graph with $d^-(u) = 0$

- *feedback arc set* - A set of edges from a directed cyclic graph that when removed make the graph acyclic

- $s = s_{left}s_{right}$ - Given 2 finite sequences s_{left} and s_{right} with the indicated notation we symbolize the *concatenation* operation. For example, if $s_{left} = (A, B, C)$ and $s_{right} = (X, Y, Z)$, then $s = s_{left}s_{right} = (A, B, C, X, Y, Z)$

- $\lfloor x \rfloor$ is the greatest integer less than or equal to x

C.6.2 Overview

The Belgian B2B data describes a weighted directed graph $G = (V, E)$. Vertices are firms and edges are sales between firms. The goal of the ordering algorithm is to order firms in a way such that a given firm only sells to firms further along in the ordering and only buys from firms that precede it. The condition desired by this ordering is known in graph theory as a *topological ordering* (**?**). A topological ordering exists if and only if a graph is directed and acyclic. The B2B data is cyclic. For the *unweighted* case our motivation is to find a feedback arc set of minimal cardinality, that is, what is the minimum number of transactions that we need to drop (i.e., the "violators") from our network to satisfy our ordering condition? For the *weighted case*, we seek to find a feedback arc set such that the cumulative weight of the violating transactions is minimized. Finding a minimum feedback arc set is computationally difficult but approximation algorithms exist.

C.6.3 Unweighted case

The algorithm we use for the paper was first presented by Eades, Lin, and Smyth (1993). This algorithm was chosen because it has a linear run time complexity, $O(m+n)$, and because of its relative implementation simplicity. The algorithm uses a greedy heuristic through which it builds the proposed ordering $s = s_{left}s_{right}$.[5] Vertices are initialized into several buckets: sinks, sources, and δ buckets, where for a vertex $u \in V$, $\delta(u) = d^-(u) - d^+(u)$.[6] At each iteration, the algorithm removes all sinks from the network and prepends them to a sequence s_{right}, removes all sources and appends them to a sequence s_{left}, and then removes the vertex with the lowest δ score (the most "source"-like vertex) and appends it to s_{left}.[7] Each removal requires updating the buckets to reflect the modified graph. The algorithm stops when the graph is empty. There will be $2n-1$ buckets, which can be formalized as follows:[8]

$$V_{-n+1} = V_{sources} = \{u \in V \mid d^-(u) = 0;\ d^+(u) > 0\}$$
$$V_{n-1} = V_{sinks} = \{u \in V \mid d^-(u) > 0;\ d^+(u) = 0\}$$
$$V_d = \{u \in V \mid d = \delta(u);\ d^-(u) > 0;\ d^+(u) > 0\}$$

The bucket V_{-n+1} contains all the vertices that are only the sources of edges. The bucket V_{n-1} contains all the vertices that are only the sinks of edges (in other words, vertices that

5. According to **?**, a greedy algorithm is "an algorithm that always takes the best immediate, or local, solution while finding an answer. Greedy algorithms find the overall, or globally, optimal solution for some optimization problems, but may find less-than-optimal solutions for some instances of other problems."

6. We have flipped the sign here compared to Eades, Lin, and Smyth (1993) to be consistent with the diagrams elsewhere in our paper.

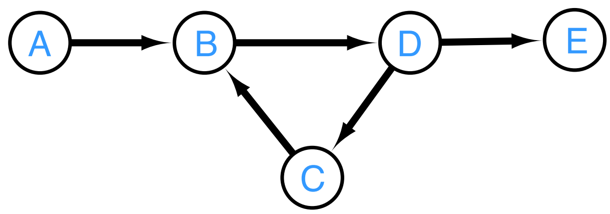
7. Eades, Lin, and Smyth (1993) take the vertex with the maximum δ score.

8. Eades, Lin, and Smyth (1993) assume that the graph G is simple (no bidirectional edges), and hence their original algorithm only requires $2n-3$ buckets.

are only receiving edges). Each V_d bucket contains vertices with d net incoming edges (conditional on these vertices having both outgoing and incoming edges).

C.6.4 Example execution on unweighted network

Consider the following network:



Let's trace the execution of the algorithm described by Eades, Lin, and Smyth (1993).

Initialization

Buckets:

A			D	C	B			E
sources	-3	-2	-1	0	1	2	3	*sinks*

$Ordering: \ s = s_{left} = s_{right} = ()$

First iteration:

Remove sinks

Updated buckets:

A				C, D	B			
sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = ()$, $s_{right} = (E)$, $s = s_{left}s_{right} = (E)$

Remove sources

Updated buckets:

				C, D, B				
sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = (A)$, $s_{right} = (E)$, $s = s_{left}s_{right} = (A, E)$

Remove vertex with lowest delta score

Updated buckets:

B								D
sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = (A, C)$, $s_{right} = (E)$, $s = s_{left}s_{right} = (A, C, E)$

Second iteration

Remove sinks

Updated buckets:

B								
sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = (A, C)$, $s_{right} = (D, E)$, $s = s_{left}s_{right} = (A, C, D, E)$

Remove sources

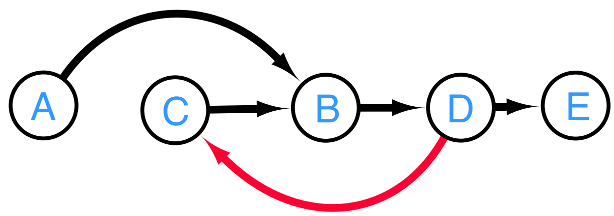
Updated buckets:

sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = (A, C, B)$, $s_{right} = (D, E)$, $s = s_{left}s_{right} = (A, C, B, D, E)$

Final output

Ordering : $s = s_{left}s_{right} = (A, C, B, D, E)$, *Violator edge set*: $\{(D, C)\}$



C.6.5 Weighted case

Simpson, Srinivasan, and Thomo (2016) have proposed a modification to adapt the Eades, Lin, and Smyth (1993) algorithm to solve the weighted problem. The required changes are:

1. In the initialization step, all edge weights need to be normalized to be between 0 and 1.

2. $\delta(u)$ is redefined as $\delta(u) = \lfloor w^-(u) - w^+(u) \rfloor$.

The key motivation behind these steps is to reformat the network so that the unweighted version of the algorithm could be used in an identical fashion as before, specifically without increasing the number of buckets. Without the floor in step 2, for any given network the number of buckets could become large.

C.7 Algorithm to solve for the firm's sourcing strategy and export participation

A firm is solving the problem described in (3.25), where profits are defined in equation (3.24) and variable profits are defined in equation (3.13). For convenience, we re-state the problem of firm j here:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j \ , \ I_{jF} \in \{0,1\},$$

where

$$\begin{aligned}
\pi_j(Z_j, I_{jF}) =& \frac{1}{\sigma}\beta_{jH}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\frac{E}{P^{1-\sigma}} \\
&+ I_{jF}\frac{1}{\sigma}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}} \\
&- \sum_{k\in Z_j} f_{kj}w_\ell - I_{jF}f_{jF}w_\ell. \\
=& \pi_j^{var}(Z_j, I_{jF}) - \sum_{k\in Z_j} f_{kj}w_\ell - I_{jF}f_{jF}w_\ell
\end{aligned}$$

In words, the firm is choosing its sourcing strategy, Z_j, and export participation, I_{jF}. We solve the firm's problem of choosing its sourcing strategy separately for $I_{jF} = 0$ and $I_{jF} = 1$. We then calculate the profits for these two cases and determine the firm is an exporter if and only if the profits under exporting are higher than under non-exporting.

Below we describe how we solve for the firm's optimal sourcing strategy for a given export participation choice.

C.7.1 Lower and upper bounds for the optimal sourcing strategy

We determine the lower and upper bound for the firm's sourcing strategy following the procedure in Jia (2008) and Antràs, Fort, and Tintelnot (2016).

Lower bound

We start from a guess of no sourcing from any other domestic supplier and no importing, $S_l^{(0)}$. We then check supplier by supplier whether the benefit of adding a supplier (given the current guess of not purchasing from any supplier) is positive. At iteration t, starting from $S_l^{(t)}$, we calculate the marginal benefit of adding supplier $k \notin S_l^{(t)}$, $k \in \mathbf{Z}_j$:

$$\pi_j^{var}(S_l^{(t)} \cup k, I_{jF}) - \pi_j^{var}(S_l^{(t)}, I_{jF}) - f_{kj} w_\ell.$$

If the marginal benefit to include supplier k is positive, in the next iteration we include supplier k in the guess for the sourcing strategy of firm j. Note that given $\sigma > \rho$, one is the least likely to determine the benefit of a supplier is positive when the current guess is no supplier. Hence if it is possible to include a supplier in this iteration, in all the next iterations the marginal benefit from this supplier will be positive as well.

Starting from $S_l^{(t)}$, we consider firm-by-firm if trading with a firm not contained in $S_l^{(t)}$ brings positive marginal benefit (i.e., the additional variable profits under this sourcing strategy exceed the additional fixed cost) or not. Then, we add all those firms which bring positive benefit to form $S_l^{(t+1)}$.

The process ends when $S_l^{(t)} = S_l^{(t+1)}$ or all eligible suppliers are in $S_l^{(t)}$ already. When the process ends (i.e., $S_l^{(t)} = S_l^{(t+1)}$), we denote the lower bound of the sourcing strategy for firm j as $S_l^* = S_l^{(t)} = S_l^{(t+1)}$.

Upper bound

To determine the upper bound we start from a guess of purchasing from every supplier (incl. foreign), $S_u^{(0)}$. We then check supplier-by-supplier whether the marginal benefit from dropping the supplier is positive. At iteration t, starting from $S_u^{(t)}$, we calculate the marginal benefit of dropping supplier $k \in S_u^{(t)}$ as:

$$\pi_j^{var}(S_l^{(t)} \setminus k, I_{jF}) - \pi_j^{var}(S_l^{(t)}, I_{jF}) + f_{kj} w_\ell.$$

The remainder of the procedure is very similar to the iteration for the lower bound but starting from the opposite direction (i.e., we drop from the next iteration $S_u^{(t+1)}$ all those suppliers for which the marginal benefit of dropping is positive). The ending criteria is the same. We denote the upper bound for the sourcing strategy as S_u^*.

C.7.2 From lower and upper bounds to optimal sourcing strategy

Once we obtain S_u^* and S_l^*, we consider 3 alternative cases. Let $D = \{x \in S_u^* \mid x \notin S_l^*\}$ denote the set with the elements that are in the upper bound but not in the lower bound for the sourcing strategy.

$S_u^* = S_l^*$

If the upper and lower bounds for the sourcing strategy are the same, then we have obtained the optimal sourcing strategy for the firm (for a given exporting choice).

S_u^* is close to S_l^*

When the cardinality of set D is less than or equal to 15, we consider S_u^* to be close to S_l^*.

In that case we evaluate the profits at all possible combinations of sourcing strategies in between S_u^* and S_l^*, including S_u^* and S_l^* themselves. We choose the combination that yields the highest total profit as the optimal sourcing strategy for the firm.

S_u^* is far from S_l^*

When the cardinality of set D is larger than 15, then evaluating the profits at all combinations of feasible sourcing strategies in between the two bounds would be too computationally intensive. For that case, we have developed the following greedy algorithm to determine the firm's sourcing strategy:

Starting from S_l^*, we calculate the marginal benefit from adding separately each supplier in D to the sourcing strategy S_l^*. Note that by construction the marginal benefit from adding each of these suppliers individually to S_l^* is negative (otherwise the algorithm in Section C.7.1 would have already added those suppliers to the lower bound). We order the suppliers in D by their marginal benefit of being added to S_l^*. If the cardinality of D is K, we consider $K-1$ alternative sourcing strategies. We first add the top 2 suppliers in D (those with the highest marginal benefit of being added evaluated at S_l^*) to S_l^*, then add the top3 suppliers to S_l^*, and so forth. Hence, we calculate the profits for $K-1$ alternative sourcing strategies.

In addition, we also follow a similar process starting from S_u^* and calculate the marginal benefit from dropping separately each supplier in D from the sourcing strategy S_u^*. Again, by construction, the benefit from dropping each of the suppliers individually is negative. We order the suppliers in D by their marginal benefit of being dropped from S_u^*. We then consider $K-1$ alternative sourcing strategies, in which $2, 3, ..., K$ suppliers are removed from S_u^* (following the ranking of their marginal benefit of dropping individually at S_u^*).

Then, out of these $2K-2$ sourcing strategies we choose the one with the highest total profit for the firm.

Note that, using the approach in Section C.7.2, the number of sourcing strategies we would need to calculate profits for would be 2^K (growing exponentially in K). The greedy algorithm developed here, requires evaluations of alternative sourcing strategies that grow linearly in K, making it feasible even in the rare case that the difference between S_u^* and S_l^* is large.

We present statistics on the cardinality of the differences in the bounds in Table C.10.

C.7.3 Statistics on the algorithm

Table C.10: Cardinality of differences in the upper and lower bounds

Number of firm draws × parameter iterations	Percent of cases in which Bounds are perfectly overlapping	Differences in bounds ≤ 15	Differences in bounds > 15
51,029,600,000	98.95	0.91	0.14

Notes: During the estimation we have to solve for each firm and parameter guess the firm's optimal sourcing strategy and exporting choice. This table presents aggregate statistics on the cardinality of the differences in the upper and lower bounds for the sourcing strategy summing over the outcomes for each firm, parameter guess, and exporting choice.

C.8 Algorithm for network formation

Below we describe the algorithm to solve for the network formation in three contexts: In Section C.8.1, we describe the algorithm to solve for the network formation and equilibrium for a given set of parameters. In Section C.8.2, we describe the algorithm to estimate the parameters of the model. In Section C.8.3, we describe the algorithm to solve for network formation and equilibrium in a closed economy.

C.8.1 Network formation given parameters

Given a set of parameters, size of the labor force, price of foreign goods, and foreign demand, we follow the steps below to simulate the network formation.

1. Firms with productivities ϕ_i are randomly sorted, and indexed with $i = 1, 2, 3, \cdots$. A firm's index determines the firm's set of eligible suppliers, $\mathbf{Z}_i$. The set of eligible suppliers is such that all feasible networks will be acyclic. Firms' draws of firm-pair-specific fixed cost of sourcing, fixed cost of importing and exporting, export demand, and benefits of importing, and firm-pair-specific cost shifters are also known at this point.

2a. All firms make a common guess of the wage level w_ℓ.

2b. All firms make a common guess of aggregate demand term: $A = EP^{\sigma-1}$.

3. We assume that firms decide on their sourcing strategies in sequence of i. Firm 1 decides its sourcing strategy and determines c_1, then firm 2 decides its sourcing strategy and determines c_2, and so on. When firms make their sourcing decisions, we assume that all firms are able to use labor and foreign inputs, but firm i is only able to choose its suppliers from its eligible supplier set $\mathbf{Z}_i$. We determine which suppliers among $\mathbf{Z}_i$ firm i sources from, using the algorithm described in Section C.7, and compute c_i. After the final firm $i = N$ decides its sourcing strategy, the whole vector $\boldsymbol{c}$ and the supplier sets of all firms are determined. At this point we have also solved for the firm's optimal export participation choice and export sales.

4. Given the network, the guesses for A and w, we are able to compute the equilibrium variables.

   (a) Sales to domestic final demand of firm i is computed as $X_{iH} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} A$ and to foreign final demand is computed as $X_{iF} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} \beta_{kF}^{\sigma-1} \frac{E_F}{P_F^{1-\sigma}}$.

   (b) The cost of inputs used for firm i's sales to domestic final demand is thus $C_{iH} = \frac{\sigma-1}{\sigma} X_{iH}$ and to foreign final demand is $C_{iF} = \frac{\sigma-1}{\sigma} X_{iF}$.

   (c) The total input costs of firms, C_i are calculated by solving the system of linear equations below:

$$C_i = C_{iH} + C_{iF} + \sum_j s_{ij} C_j,$$
$$\rightarrow \mathbf{C} = (I - S)^{-1} (\mathbf{C}_H + \mathbf{C}_F)$$

   where $\mathbf{C}$, $\mathbf{C}_H$, and $\mathbf{C}_F$ are vectors of C_i, C_{iH}, and C_{iF}, respectively, and the i, j element of matrix S is s_{ij}.

   (d) The total sales of firm i is then $X_i = X_{iH} + X_{iF} + C_i - C_{iH}$.

   (e) Firm profits and total expenditure on fixed costs.

5. We solve for equilibrium variables of A and w_ℓ in the following way: In the outer loop, we solve for wages such that the labor market clearing condition (3.26) is solved. In the inner loop, we iterate over steps 2b-4, such that a fixed point for the market demand level, A, is found.

C.8.2 Parameter estimation and network formation

One possible approach to estimating the parameters of the model is to simulate the model for each parameter guess according to the algorithm outlined in Section C.8.1, calculate the objective function in equation (3.29), and vary the parameter guesses to maximize the objective function. However, this requires for each parameter guess finding a fixed point in both the market demand, A, and a wage level, w_ℓ. Below, we describe a more computationally attractive algorithm to estimate the model.

Throughout the estimation, we set the domestic wage, $w_\ell = 1$, as well as the domestic market demand, $A = 1$. We ensure labor market clearing condition and the fixed point in market demand in the following way:

1. Of the 8 parameters to estimate, the mean foreign demand parameter is implicitly pinned down to take the value that satisfies the trade balance condition.

2. Instead of treating the size of the labor force as data (note that the units are arbitrary), we choose its level by setting: $L = \frac{AP^{1-\sigma} - \sum_i \pi_i}{w_\ell}$.

Note that $A = w_\ell = 1$. Under this level of the size of the labor force, $L = \frac{AP^{1-\sigma} - \sum_i \pi_i}{w_\ell}$, the fixed point for the market demand, A is automatically satisfied. Also, since the trade balance holds, the domestic labor market clears as well.

Given a parameter guess for $\hat{\Phi}^{\alpha_{\text{dom}}}_{scale}$, $\hat{\Phi}^{\alpha_F}_{scale}$, $\hat{\Phi}^{\alpha,\beta}_{disp}$, $\hat{\Phi}^{f_{\text{dom}}}_{scale}$, $\hat{\Phi}^{f_{\text{imp}}}_{scale}$, $\hat{\Phi}^{f_{\text{exp}}}_{scale}$, and $\hat{\Phi}^{f}_{disp}$, we vary $\hat{\Phi}^{\beta_F}_{scale}$ and go through steps 3 and 4 in Section C.8.1 to calculate the aggregate trade balance. Given the other parameters, the level of $\hat{\Phi}^{\beta_F}_{scale}$ is pinned down implicitly such that aggregate trade balance is equal to zero. Hence, instead of search for a fixed point in both

A and w_ℓ, we hold those fixed throughout the estimation and only need to find one fixed point in $\hat{\Phi}^{\beta_F}_{scale}$, that satisfies trade balance, for each guess of the other seven parameters.

C.8.3 Network formation in the closed economy

In the closed economy, we can normalize the domestic wage $w_\ell = 1$. We therefore only have to follow steps 2b-4 in Section C.8.1 to pin down the level of domestic market demand, A, in the closed economy.

C.9 Sensitivity results under exogenous network

Figure C.3: Distributions of $\log \hat{c}$ from banning imports, different parameter values

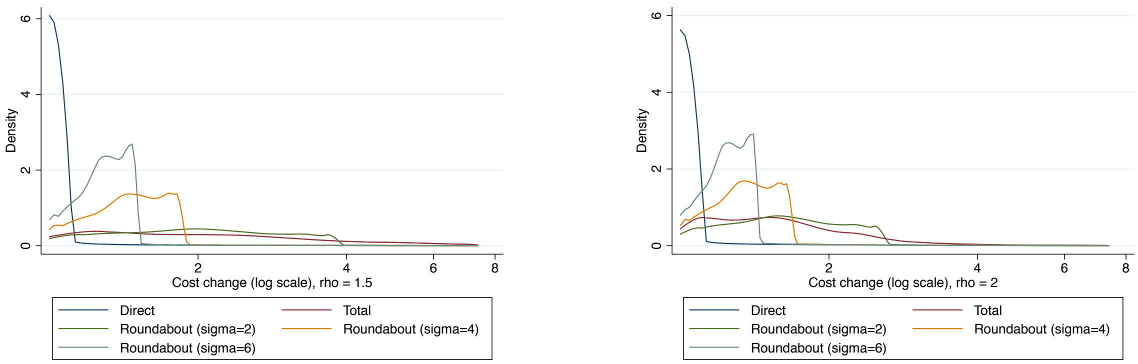


Table C.11 shows the median value of cost change $\hat{c}_i$ under different parameter values.

Table C.11: Median $\hat{c}_i$ under different values of σ and ρ

	Direct	Total	Roundabout		
			$\sigma = 2$	$\sigma = 4$	$\sigma = 6$
$\rho = 1.5$	1	2.89	2.37	1.53	1.33
$\rho = 2$	1	1.70	1.78	1.41	1.28

Table C.12: 90th percentile $\hat{c}_i$ under different values of σ and ρ

	Direct	Total	Roundabout		
			$\sigma = 2$	$\sigma = 4$	$\sigma = 6$
$\rho = 1.5$	1.08	11.93	3.76	1.85	1.50
$\rho = 2$	1.04	3.45	2.52	1.67	1.42

Table C.13: Change in price index $\hat{P}$ under different values of σ and ρ

ρ	σ	$\hat{P}\mid_{direct}$	$\hat{P}\mid_{total}$	$\hat{P}\mid_{roundabout}$
1.5	2	1.71	3.84	3.80
1.5	4	1.28	2.25	1.85
1.5	6	1.17	1.83	1.50
2	2	1.48	2.35	2.56
2	4	1.23	1.77	1.67
2	6	1.15	1.56	1.43

Table C.14: Change in price index $\hat{P}$ under acyclic network

ρ	σ	$\hat{P}\mid_{total}$	$\hat{P}\mid_{total}$, acyclic
1.5	2	3.84	3.93
1.5	4	2.25	2.28
1.5	6	1.83	1.84
2	2	2.35	2.37
2	4	1.77	1.78
2	6	1.56	1.57

Notes: The fourth column shows the change in price index from banning imports when taking into account the acyclic network structure. We obtain the acyclic network from the algorithm explained in Appendix C.6, for the weighted case where we minimize the value of violating transactions.

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