Randomized Benchmarking with Non-Markovian Noise and Realistic Finite-Time Gates

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We analyze the impact of non-Markovian classical noise on single-qubit randomized benchmarking experiments, in a manner that explicitly models the realization of each gate via realistic finite-duration pulses. Our new framework exploits the random nature of each gate sequence to derive expressions for the full survival probability decay curve which are nonperturbative in the noise strength. In the presence of non-Markovian noise, our approach shows that the decay curve can exhibit a strong dependence on the gate implementation method, with regimes of both exponential and power law decays. We discuss how these effects can complicate the interpretation of a randomized benchmarking experiment, but also how to leverage them to probe non-Markovianity.

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Introduction—Randomized benchmarking (RB) protocols are a powerful tool for characterizing errors in quantum processors. They rely on the application of random gate sequences to robustly extract average properties of the noise without having to perform full quantum process tomography. For gate-independent Markovian error models, standard RB protocols predict an exponential decay of the survival probability [1–5]. Under standard assumptions, the measured decay rate can be directly used to extract the average gate infidelity. Unfortunately, in many relevant settings the dominant noise is non-Markovian (i.e., correlated in time) [6–8]. While RB protocols could still be useful in this context, it is not clear what they measure in the presence of non-Markovian noise. RB protocols have been used in many situations where the Markovian assumption is not valid [9,10].

The above concerns have motivated many recent works studying RB and non-Markovian noise [11]. With such noise, it has been shown that the decay of the survival probability can be nonexponential [12–14], can converge more slowly to its mean [15,16] and can even be used to learn the noise spectrum [17]. However, typical approaches make approximations that can miss important physics. In particular, the assumption of instantaneous gates can miss the potentially rich interplay between finite-duration gates and temporally correlated noise.

Here, we address these concerns by modeling single-qubit RB in a physically motivated manner. We consider a qubit that is driven by both finite-duration pulses (used to implement the chosen sequence of random gates) as well as classical non-Markovian, Gaussian noise [see, e.g., Fig. 1(b)]. As we will show, this approach naturally captures the effects of noise correlations between adjacent gates which are present due to the finite-duration gate implementation. This effect would be hard to capture using other approaches [see, e.g., Fig. 1(a)]. We develop a novel method that builds on Refs. [18–20] to understand the average dynamics of the qubit, averaged both over noise realizations

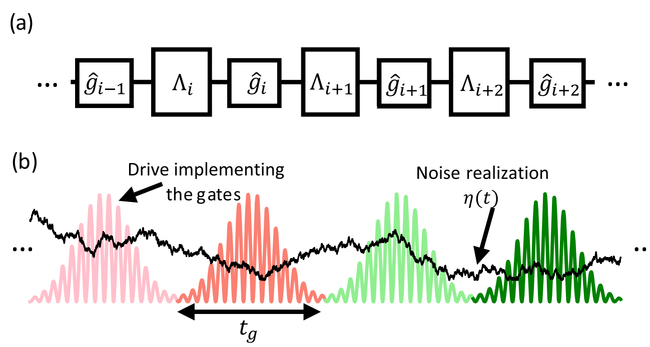

FIG. 1. (a) Common approach to model RB: each gate $\hat{g}_i$ is applied instantaneously, followed by a noisy map Λ_i. Errors can be temporally correlated but accurately describing their dependence on the drives implementing the gates is challenging. (b) Schematic of our model: noise is modeled as a classical stochastic process $\eta(t)$, gates are implemented via finite-duration pulses.

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and over gate sequences. By exploiting the randomness of the control pulses, we are able to obtain expressions for the full survival probability decay curve that are nonperturbative in the noise strength. We show that the fully averaged qubit dynamics is described by a time-dependent depolarizing channel, with the form of the time-dependent rate encoding the complex interplay of non-Markovian noise and the specific finite-time gate implementation.

We find that depending on noise parameters and gate implementation, non-Markovian noise can yield RB survival probabilities that decay either exponentially or as a power law. Perhaps not surprisingly, noise with a short correlation time (e.g., comparable to the gate time) yields exponential decay. More surprisingly, the corresponding decay rate does not correspond to the average gate infidelity and can vary by a factor of almost 2 by changing the gate implementation. This raises questions about the interpretation of RB decay curves, even in the seemingly simple case where one obtains a purely exponential decay. Similar questions were raised in Ref. [21] due to the gauge freedom present in the definition of the average gate infidelity. In contrast, for our problem we show that this difference is instead linked to the non-Markovian nature of the noise. While we apply our new technique to single-qubit RB, it can be generalized to analyze other protocols such as two-qubit RB and randomized compiling (RC) [22] (see [23]).

Model—We consider a standard single-qubit randomized benchmarking experiment [1] in which each gate sequence consists of $L+1$ gates sampled uniformly from the Clifford group. The qubit is subject to classical, Gaussian, stationary noise $\eta(t)$ [autocorrelation function $S(t)$], which couples (without loss of generality) to the qubit operator $\hat{\sigma}_z$. While our approach applies to any noise spectrum, in the main text we take as a paradigmatic example noise with an exponentially decaying $S(t)$ (i.e., an Ornstein-Uhlenbeck process), with strength σ and correlation time τ_c. Using a bar to denote noise averages:

$$S(t-t') \equiv \overline{\eta(t)\eta(t')} = \sigma^2 \exp(-|t-t'|/\tau_c). \qquad (1)$$

This form interpolates between Markovian and strongly non-Markovian limits, and is also directly relevant to many experiments (see e.g., [28]). More general noise spectra are considered in Supplemental Material (SM) [23]. Each gate in the RB sequence is implemented by a pulse of duration t_g. Working in a rotating frame at the qubit frequency, the Hamiltonian for a single realization of the noise $\eta(t)$ and a specific gate sequence $\vec{\beta}$ (an $L+1$ length vector) is

$$\tilde{H}_\eta(t,\vec{\beta}) = \vec{\Omega}(t,\vec{\beta}) \cdot \vec{\hat{\sigma}} + \eta(t)\hat{\sigma}_z, \qquad (2)$$

where $\vec{\Omega}(t,\vec{\beta})$ encodes the pulse envelope implementing the sequence and $\vec{\hat{\sigma}}$ is the Pauli vector. Importantly, because the noise is non-Markovian, the effect of the noise on the qubit depends on $\vec{\Omega}(t,\vec{\beta})$ even though the control Hamiltonian is independent of $\eta(t)$. As the goal of this Letter is to understand the global decay features of the survival probability, we neglect state preparation and measurement errors (which ideally do not impact the decay rate). We also assume that the zeroth gate of the sequence β_0 is implemented instantaneously. As shown in SM [23], this has a negligible influence on our findings, but greatly simplifies our analysis.

We work in an interaction picture with respect to the noise-free Hamiltonian $\hat{H}_0(t,\vec{\beta}) = \vec{\Omega}(t,\vec{\beta}) \cdot \vec{\hat{\sigma}}$, where the dynamics is described by the stochastic Hamiltonian $\hat{H}_\eta(t,\vec{\beta}) = \eta(t)\hat{\sigma}_z(t,\vec{\beta})$ with $\hat{\sigma}_z(t,\vec{\beta}) = \hat{U}_0^\dagger(t,\vec{\beta})\hat{\sigma}_z\hat{U}_0(t,\vec{\beta})$ and $\hat{U}_0(t,\vec{\beta}) = \mathcal{T}\ \exp\left(-i\int_0^t dt' \hat{H}_0(t',\vec{\beta})\right)$. This is a useful frame as there is no noise-free dynamics, letting us isolate the impact of noise. Qubit evolution here is $\check{\mathcal{U}}_\eta(t,\vec{\beta})\hat{\rho} \equiv \hat{U}_\eta(t,\vec{\beta})\hat{\rho}\hat{U}_\eta^\dagger(t,\vec{\beta})$ where $\hat{\rho}$ is the initial qubit density matrix and $\hat{U}_\eta(t,\vec{\beta}) = \mathcal{T}\ \exp\left(-i\int_0^t dt' \hat{H}_\eta(t,\vec{\beta})\right)$. The quantity of interest in an RB experiment is the average survival probability:

$$P_0(t) = \langle \overline{tr[\hat{\rho}_0\hat{\rho}_\eta(t,\vec{\beta})]} \rangle_{\vec{\beta}}, \qquad \hat{\rho}_\eta(t,\vec{\beta}) \equiv \check{\mathcal{U}}_\eta(t,\vec{\beta})\hat{\rho}_0, \qquad (3)$$

where $\langle\cdot\rangle_{\vec{\beta}} = (1/24^{L+1})\sum_{\vec{\beta}}[\cdot]$ is the average over the gate sequences. We assume that the initial state $\hat{\rho}_0$ is uncorrelated with the noise. Our task then reduces to computing the noise and sequenced averaged evolution superoperator:

$$\check{\mathcal{U}}_{\mathrm{avg}}(t) = \langle \overline{\check{\mathcal{U}}_\eta(t,\vec{\beta})} \rangle_{\vec{\beta}}. \qquad (4)$$

This yields the survival probability via $P_0(t) = \mathrm{tr}[\hat{\rho}_0\check{\mathcal{U}}_{\mathrm{avg}}(t)\hat{\rho}_0]$.

The most obvious next step is to compute the average over the noise in Eq. (4). This is nontrivial: even though $\eta(t)$ is Gaussian, the noncommuting structure of Eq. (2) gives rise to an infinite set of cumulants. In some cases this hierarchy can be truncated to yield useful descriptions [18], but this is an approach that is perturbative in the noise strength and fails for noise with long correlation times. We instead follow a different route, and first average over the random variable $\vec{\beta}$ (i.e., over different random gate sequences) for a fixed noise realization $\eta(t)$. This yields an alternate kind of cumulant expansion, which we truncate to second order to get a *nonperturbative* approximation in the noise strength. As we show in SM [23], this cumulant approximation is similar to the one introduced in Refs. [24,25], but is not restricted to noise with limited temporal correlations. Instead, it is able to capture the impact of temporal correlations between nonadjacent gates to any order in the noise strength. It is motivated by the decoupling properties of random gate sequences [26,29], which generate an effective correlation-time of t_g for the dynamics, making higher cumulants vanish on that timescale.

After averaging over gate sequences within this approximation, we obtain a propagator $\check{\mathcal{U}}_\eta(t)$ that still depends on the specific noise realization. After m full gates are applied, it is given by

$$\check{\mathcal{U}}_\eta(mt_g) = \check{\mathcal{I}} + \Lambda_\eta(mt_g)\sum_\alpha \check{\mathcal{D}}[\hat{\sigma}_\alpha], \tag{5}$$

$$\Lambda_\eta(mt_g) = \frac{1}{4} - \frac{1}{4}\exp\left(-4\int_0^{mt_g} dt'\Gamma_\eta(t')\right), \tag{6}$$

where $\check{\mathcal{I}}$ is the identity superoperator and $\check{\mathcal{D}}$ is the Lindblad dissipator: $\check{\mathcal{D}}[\hat{O}]\hat{\rho} = \hat{O}\,\hat{\rho}\,\hat{O}^\dagger - \frac{1}{2}(\hat{O}^\dagger\hat{O}\,\hat{\rho} + \hat{\rho}\hat{O}^\dagger\hat{O})$. This is a depolarizing channel whose strength has a nontrivial dependence on m as determined by the effective stochastic rate $\Gamma_\eta(t)$. This in turn is given by (see SM [23]):

$$\Gamma_\eta(nt_g+\tau) = \frac{1}{3}\int_{(n-1)t_g}^{nt_g+\tau} dt'\eta(nt_g+\tau)\eta(t')f(nt_g+\tau,t'), \tag{7}$$

$$f(t_1,t_2) = \langle tr[\hat{\sigma}_z(t_1,\vec{\beta})\hat{\sigma}_z(t_2,\vec{\beta})]\rangle_{\vec{\beta}}, \tag{8}$$

where $n\in\mathbb{Z}$ and $\tau\in[0,t_g)$ are defined by $t=\tau+nt_g$. Equation (7) is only valid for $n>0$; when $n=0$, the lower bound of the integral becomes 0 [30]. The function $f(t_1,t_2)$ is the average overlap of the evolved noise operator at different times and is the only quantity that depends on the gate implementation. Note that for each noise realization, the sequence-averaged evolution is a time-dependent depolarizing channel. Further, the instantaneous rate $\Gamma_\eta(t)$ depends both on the behavior of the noise during the "current" gate period, and during the previous gate, as reflected by the integration bounds of Eq. (8). The noise at even earlier times does not contribute directly to the instantaneous stochastic rate (as in this case, the earlier time and t are separated by one or more complete random gates and hence a full twirl). However, longer-range temporal correlations will ultimately contribute once we perform an average over $\eta(t)$.

While Eq. (5) still has to be averaged over the noise $\eta(t)$, this step is greatly simplified: we now just need to average a single scalar quantity $\Lambda_\eta(t)$. Since $\Lambda_\eta(t)$ is the exponential of a squared Gaussian random variable $\eta(t)$, its average reduces to a functional determinant, which can be computed using various methods (e.g., [31]). In the remainder of this Letter, we present two approximate methods to gain analytic insight. The first relies on a weak noise approximation and is valid for small correlation times. The second, assumes that the noise is constant on the timescale of t_g, which is valid for long correlation times. While one or the other of these approximations is needed to get simple analytic expressions, we show in SM [23] that at least one of them holds for almost any correlation times, making them jointly sufficient to describe the decay in nearly all regimes.

Before diving into these approximations, we can compute the noise average exactly in two opposite limits [23]: Markovian and quasistatic noise. In the first case, $S(t)=\gamma\delta(t)$ and, as expected, the survival probability decays exponentially:

$$P_0^{\mathrm{M}}(mt_g) = \frac{1}{2} + \frac{1}{2}\exp\left(-\frac{4}{3}\gamma mt_g\right). \tag{9}$$

In the second case, $S(t)=\sigma^2$ and the survival probability decays like a power law:

$$P_0^{\mathrm{qs}}(mt_g) = \frac{1}{2} + \frac{1}{2\sqrt{1+\frac{8}{3}\sigma^2 t_g^2[mF_{\mathrm{curr}}+(m-1)F_{\mathrm{prev}}]}}, \tag{10}$$

where $F_{\mathrm{prev,curr}}$ are the integrals of $f(t_1,t_2)$ corresponding to the average overlap of the noise operator between times belonging to the same gate or adjacent gates, respectively:

$$F_{\mathrm{curr}} = \frac{1}{t_g^2}\int_0^{t_g} dt_1\int_0^{t_1} dt_2 f(t_1,t_2), \tag{11}$$

$$F_{\mathrm{prev}} = \frac{1}{t_g^2}\int_{t_g}^{2t_g} dt_1\int_0^{t_g} dt_2 f(t_1,t_2). \tag{12}$$

These factors depend on how the gates are implemented, and will be useful going forward.

Master equation description—To average Eq. (5) over the noise, the simplest approximation is to perform a weak noise expansion and derive a time-local master equation [18]. This can be justified even in the long-time limit provided that the noise has a small correlation time, such that $\sigma^2\tau_c t_g \ll 1$.

Making this approximation to second order in the noise, the equation of motion for the density matrix averaged over the noise and gate sequences $\hat{\rho}_{\mathrm{avg}}(t) = \overline{\langle\hat{\rho}_\eta(t,\vec{\beta})\rangle_{\vec{\beta}}}$ is

$$\partial_t\hat{\rho}_{\mathrm{avg}}^{(2)}(t) = \overline{\Gamma_\eta(t)}\sum_{\alpha\in\{x,y,z\}} D[\hat{\sigma}_\alpha]\hat{\rho}_{\mathrm{avg}}(t), \tag{13}$$

where $\overline{\Gamma_\eta(t)}$ is the noise-averaged decay rate which is t_g periodic after the first full gate. We can then compute the survival probability:

$$P_0^{(2)}(mt_g) = \frac{1}{2} + \frac{1}{2}\exp(-4\epsilon')\exp[-4\epsilon(m-1)], \tag{14}$$

$$\epsilon = \int_{jt_g}^{(j+1)t_g} dt'\overline{\Gamma_\eta(t')} \quad \text{for } j>0, \tag{15}$$

where Eq. (15) does not depend on j due to the periodicity of $\overline{\Gamma_\eta(t)}$ and where $\epsilon' = \int_0^{t_g} dt'\overline{\Gamma_\eta(t')}$. We see that to second

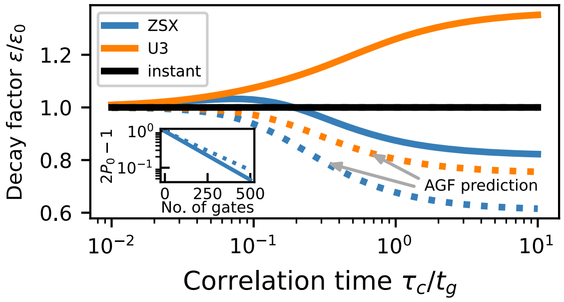

FIG. 2. Survival probability decay factors ϵ/ϵ_0 from Eq. (15) for three gate implementations with $\epsilon_0 = 8\gamma_0/3$ (the limit $\tau_c/t_g \to 0$ of ϵ). Solid lines show the decay factor that would be observed in a RB experiment. Dotted lines show the decay factor ϵ' predicted by the AGF $\bar{F}$. The difference arises from noise correlations between adjacent gates. Inset: survival probability decay from RB (solid) vs AGF prediction (dotted) with $\tau_c = 0.5t_g$ and $\gamma_0 = 2.5 \times 10^{-3}$. All shown τ_c/t_g values fall within the regime of validity of Eq. (14).

order in the noise strength, the survival probability decays exponentially. While an exponential decay is reminiscent of Markovian RB, we also see that the corresponding decay rate depends both on the gate implementation via $f(t_1, t_2)$ and on the noise correlation function $S(t_1 - t_2)$, an effect that would be absent for truly Markovian noise.

To study the dependence on choice of gate implementation, we consider three possible strategies for constructing pulses. The first, *ZSX*, uses finite duration X pulses interleaved by instantaneous Z pulses. A given Clifford gate is implemented as $R_Z(\phi+\pi)\sqrt{X}R_Z(\theta+\pi)\sqrt{X}R_Z(\lambda)$ for appropriate ϕ, θ, λ. This implementation is common in RB experiments, as all Clifford gates take the same time (each $\sqrt{X}$ takes a time $t_g/2$). The second, *U*3, corresponds to the application of the constant pulse of duration t_g that applies the shortest rotation that implements the gate on the Bloch sphere. The third, *instant*, corresponds to an instantaneous application of the gate followed by an idle time t_g. While this is unphysical, we include it as it is often used when modeling RB experiments, see, e.g., [11–13,15,32].

Figure 2 plots survival probability decay rates ϵ predicted by Eq. (15) as a function of noise correlation time τ_c. To meaningfully compare the impact of varying τ_c, we also vary σ with τ_c so that the variance $\gamma_0 = \int_0^{t_g} dt_1 \int_0^{t_1} dt_2 S(t_1 - t_2)$ of the random phase acquired during a time t_g remains fixed. As shown in SM [23], all the values of τ_c/t_g shown on the x axis of Fig. 2 are within the regime of validity of Eq. (14) for values of $\gamma_0 = 2.5 \times 10^{-3}$. We see that ϵ varies by a factor of almost 2 between the *ZSX* and *U*3 gate implementations when $\tau_c \sim 10t_g$. This dependence on choice of gate implementation can be interpreted as arising from the ability of finite-duration gate pulses to perform a kind of dynamical decoupling cancellation or enhancement of correlated noise (as seen with *ZSX* and *U*3 implementations, respectively). As such, the implementation dependence is absent in the extreme Markovian limit. Note, however, that even for small but nonzero correlation times (e.g., $\tau_c \approx 0.1t_g$), the error rate per gate is higher than in the true Markovian limit $\tau_c/t_g \to 0$. We see that even a small departure from the Markovian limit can have an impact. Finally, the "instant" implementation is independent of τ_c as there is no possible dynamical decoupling between the pulses implementing the gates. The decay factor is instead set solely by the fixed random phase γ_0.

It is natural to ask whether the RB decay rate ϵ that we find is related to the average gate fidelity (AGF), defined as $\bar{F} = \int d\psi \langle\psi|\check{\mathcal{U}}_{\mathrm{avg}}(t_g)[|\psi\rangle\langle\psi|]|\psi\rangle$. This definition is equivalent to the survival probability after one gate $P_0(t_g)$, meaning that $\bar{F} = [\exp(-4\epsilon') + 1]/2$. Using standard RB theory for Markovian noise [2], the estimated AGF extracted from the decay curve $\check{\bar{F}}$ would be determined from $\check{\bar{F}} = [\exp(-4\epsilon) + 1]/2$. As shown in Fig. 2, ϵ (plain lines) and ϵ' (dotted lines) significantly differ from each other meaning that the AGF does not accurately predict the RB decay curve, even if it is exponential. Since $\bar{F} = P_0(t_g)$ is a physical quantity, the difference between $\check{\bar{F}}$ and $\bar{F}$ is not due to our choice of gauge for the AGF (as was studied in Ref. [21]). Instead, it reflects the fact that with non-Markovian noise, errors at time t will depend on what happened during earlier gates, something that cannot be captured by characterizing gates in isolation.

Coarse grained noise approximation—For noise with longer correlation times, the straightforward perturbative approach to Eq. (5) is no longer valid, as terms that are higher order in the noise cannot be neglected. Instead, we make use of the long correlation time $\tau_c \gg t_g$ and coarse grain the noise with negligible induced error, i.e., replace $\eta(t)$ in Eq. (7) by a set of stochastic random variables $\theta_i/t_g \equiv (1/t_g)\int_{it_g}^{(i+1)t_g} dt'\eta(t')$. This replacement reduces the noise averaging to the evaluation of a finite matrix determinant. Using this approximation (see SM [23] for details) we can express the survival probability after the application of m gates in terms of two $m \times m$ matrices: the correlation matrix of the coarse-grained noise $\mathbf{\Sigma}$ and a tridiagonal matrix $\mathbf{F}$ which encodes the effect of the finite-time gate implementation. Letting $\mathbb{1}$ denote the identity matrix, we have

$$P_0(mt_g) = \frac{1}{2} + \frac{1}{2\sqrt{\det(\mathbb{1} + \frac{8}{3}\mathbf{\Sigma}\mathbf{F})}}, \tag{16}$$

$$\Sigma_{i,j} = \overline{\theta_i\theta_j}, \tag{17}$$

$$\mathrm{F}_{i,j} = F_{\mathrm{curr}}\delta_{i,j} + \frac{1}{2}F_{\mathrm{prev}}(\delta_{i,j+1} + \delta_{i,j-1}). \tag{18}$$

Note that in the simple limit where gates are implemented instantaneously, Eq. (16) reproduces the expression

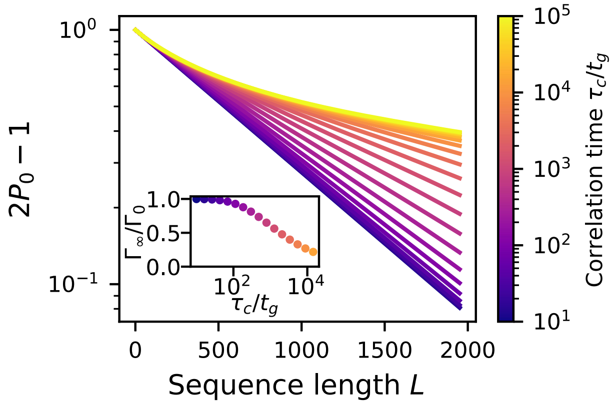

FIG. 3. Survival probability decay vs number of gates L for various τ_c (color bar), using the ZSX gate decomposition. Decay follows a power law for $Lt_g \ll \tau_c$ and becomes exponential with rate Γ_∞ for $Lt_g \gg \tau_c$. Inset: Γ_∞/Γ_0 vs τ_c, where Γ_0 is the initial decay rate, related to ϵ from Eq. (15). Γ_∞ is extracted by fitting longtime behavior to $A\ \exp(-\Gamma_\infty L t_g) + B$. Noise strength σ is fixed in the same way as in Fig. 2 with $\gamma_0 = 6.125 \times 10^{-4}$.

derived in Ref. [12]. We can in fact see Eq. (16) as a generalization of Ref. [12] as it also lets us understand the impact different choices of gate implementation. Indeed, because the matrix $\mathbf{F}$ is Toeplitz (except for the first time step), it can be seen as a scalar renormalizing the strength of the correlations in $\mathbf{\Sigma}$. In SM [23], we show that for noise with long correlation times, an additional coarse-grained approximation makes this statement rigorous. The renormalization factor is then simply given by $F = F_{\text{curr}} + F_{\text{prev}}$.

In Fig. 3 we used Eqs. (16)–(18) to study the survival probability for noise with long, but finite correlation times. We find that as a function of τ_c, the decay curves interpolate between exponential [Eq. (9)] and power law [Eq. (10)]. Furthermore as a function of the sequence length m, the curve initially decays as a power law, before transitioning to exponential decay with rate Γ_∞ for large m [33]. We also see in the inset that Γ_∞ decreases with τ_c. This is expected since here $\tau_c \gg t_g$, the noise can remain correlated among many gates leading to randomized dynamical decoupling effects [29]. This is in contrast with changing the gate implementation, which introduces noise averaging on the scale of t_g and which could either increase or decrease the decay rate.

We can therefore separate the roles of gate implementation and noise correlation function. The gate implementation changes the effective noise strength by introducing averaging effects on the timescale of t_g, while the noise correlation function affects the functional form of the decay by allowing averaging effects on the scale of multiple gate times.

Figure 3 also reveals that for correlation times around $10t_g$, the survival probability decay appears nearly exponential, making it hard to resolve the correlation time from the decay curve alone. Nevertheless, an RB experiment involving different gate implementations could be used as a flag for non-Markovianity.

Conclusion—Using a generalized cumulant approximation that goes beyond simple perturbative approaches, we studied the interplay between finite-duration gates and non-Markovian classical noise in RB experiments. We find that the decay remains exponential in the presence of noise with correlation times on the order of t_g, but with a decay rate that is highly dependent on the physical gate implementation and that does not correspond to the average gate infidelity. This complicates the interpretation of RB experiments under non-Markovian noise. Our work enables further research avenues: our generalized cumulant expansion applies to other protocols involving random gate sequences (e.g., RC and cycle benchmarking [22,26,34]). In [23] we demonstrate such calculations for both RC and two-qubit RB. Our results also suggest an approach for optimizing gate implementations when non-Markovian noise dominates (see [23]).

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Data availability—DOE will provide public access to these results of federally sponsored research in accordance with the DOE Public Access Plan [35].

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