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Formless Structure:

An Ontological Problem in Aristotle’s Theory of Number

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The literature on Aristotle's philosophy of arithmetic is still at a pre-paradigmatic stage. What progress has been made, though, reveals a potential gap in Aristotle's ontology. On Aristotle's standard account, number is an aggregate of discrete units, devoid of any further unifying form or structure. Given this, how can such an account explain how number has particular mathematical properties, such as being odd, prime, or square? The number 4, for instance, is simply four units taken together. Yet, this aggregation alone does not appear to explain why 4 is even or square. Indeed, Aristotle affirms that numbers have attributes apart from quantity (*An. Post*. 2.13 96a30-b5). Nevertheless, his reduction of number to a mere plurality of units threatens to render such attributes unintelligible within his framework. In what follows, I will reconstruct Aristotle's conception of number and argue that he is, in fact, committed to attributing properties to number beyond mere cardinality. I contend, however, that his account lacks the ontological resources to explain the basis of these properties. As a result, Aristotle's ontology of number fails to provide a satisfactory foundation for arithmetic.

In the first section, I reconstruct Aristotle's conception of number, emphasizing his commitment to its being a mere plurality of discrete units. In the second section, I articulate the central problem: this account cannot explain how numbers, so conceived, possess determinate mathematical properties that go beyond mere quantity. I argue that this failure reveals a structural gap in Aristotle's ontology of number. In the third section, I examine four interpretive strategies that aim to close this gap. I show, however, that each ultimately fails to reconcile Aristotle's metaphysical commitments with the explanatory demands of number.

Editorial note: All line references have been rounded to the nearest inclusive multiple of five to accommodate variations across translations of the Greek text. For example, 1324a7-b12 would be cited as 1324a5-b15.

I. An Aristotelian Account of Number

(I) IS NUMBER A UNITY? Aristotle defines number at several points in the *Metaphysics* as: “limited plurality,” “a plurality of units,” “plurality measurable by one,” and “a measured plurality and a plurality of measures” (*Met*. Δ.13 1020a10-15; I.1 1053a30-35; I.6 1057a1-5; N.1 1088a5-10).[2] Thus, the measurement of a plurality confers the status of “number” upon a unit or group of units. Although this much is undisputed, there is still disagreement about whether, for Aristotle, number is a heap or a tight unity (a whole). That is, there is no clear consensus as to the ontological status of number itself. Those who identify number as a unity point to Aristotle’s criticisms of his opponents.[3] Namely, he charges them with being unable to account for the unity of number (*Met*. K.2 1060b10-15; Λ.10 1075b30–40). Thus, it seems that a good account of number should be able to explain its unity (*Met*. H.6 1045a5-10). Proponents of the unity argument identify number as a hylomorphic unity, a solution that Aristotle himself offers to his interlocutors (*Met*. M.8 1084b1-10). A hylomorphic reading can be identified in his claims that the structure of a definition contains both a material and formal part (*Met*. H.6 1045a20-25). Thus, number—which Aristotle defines as having both material and formal features—seems to be aligned with this picture.

Nevertheless, there is a question as to whether or not we can draw specific ontological implications about Aristotle's own view from the structure of number's definition. Emily Katz argues that the unity of definition for number has no bearing on the ontological unity of number. That is, even when a definition contains both formal and material components, the object defined need not be a hylomorphic compound.[4] Aristotle’s definition of thunder as “extinction of fire in

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[2] Plurality is that which is potentially divisible into non-continuous parts (*Met*. Δ.13 1020a5-15).
[3] See Galluzzo (2018); Halper (1989); Mueller (1987)
[4] Katz, “What Numbers Could Not Be (For Aristotle),” 198-199

cloud" illustrates this point (*An. Post*. 2.8 93b5-15). Although this definition contains both a formal aspect (extinction of fire) and a material aspect (cloud), thunder itself is not a hylomorphic compound. Paradigmatic hylomorphic compounds—such as living organisms—are constituted by a form (soul) which actualizes the potential inherent in its matter (body) into a unified, enduring entity (*Met*. Z, H). Thunder, however, is an incidental, transient phenomenon dependent on external atmospheric conditions. It lacks both an intrinsic principle of unity and a persisting underlying substrate. The hylomorphic structure of thunder's definition does not automatically confer a hylomorphic structure upon thunder itself. The most favorable reading in support of number's unity appeals to the unity of its definition, which contains both form (measure) and matter (plurality). But as the case of thunder shows, definitional unity does not necessarily entail ontological unity. Thus, it is not evident that number is a unity.

As a further point, Katz asserts that Aristotle has no obligation to explain the unity of number.[5] This is only a problem for his opponents who identify number as a substance, which Aristotle does not (*Met*. Z.3 1029a15-25).[6] Number is a quantity (plurality) and not a "this" (substance) (*Met*. N.2.1089b30-1090a5). As such, he does not actually endorse the view that number is a hylomorphic unity (*Met*. M.9 1084b1-10). Aristotle, then, has no ultimate need to account for the tight unity of number.

(II) WHOLES AND TOTALS. Not only does Aristotle have no obligation to account for the tight unity of number, but he also denies that number possesses the internal unity characteristic of wholes. Number, he argues, is a total, not a whole. Quantities are *totals* when the position of their parts "does not make a difference" (*Met*. Δ.26 1024a1-10; *Cat*. 6 5a20-30). In contrast, for a

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[5] Katz, "What Numbers Could Not Be (For Aristotle)," 200-201

[6] Although Aristotle maintains only substances have definitions, it is not a problem that he defines number, which is not a substance. This is only a definition in a derivative sense (*Met.* Z.5 10311a1-15).

whole, the relative position of its parts is salient to its identity and existence. Consider the difference, for instance, between a bundle of wood (total) and a wooden house (whole). The precise arrangement of the pieces of wood is irrelevant to that of the bundle, but critical to that of the house. For Aristotle, "Water and all liquids and number are called totals, but 'the whole number' or 'the whole water' one does not speak of, except by an extension of meaning" (*Met*. Δ.26 1024a1-10). In other words, the predicate "whole" in "the whole number" (which might suggest unity) is only metaphorical. Even if we may consider the many units as *one* number, this unity is extrinsic—like calling a pile of wood "one pile." There is no internal principle of unity that binds the units together in the way the parts of a whole are unified. In short, number is a discrete quantity whose position of its parts (its units) is irrelevant to its identity. Note that even if the units are interchangeable, their specific arrangement has no bearing on the *what-it-is*, since number is nothing but an aggregation of units.[7]

(III) HEAPS. I follow Katz in holding that, because Aristotle classifies number as a *total* rather than a *whole*, it must be a kind of heap.[8] Specifically, a heap is an aggregation of parts that are not unified under a single form.[9] The view, then, is that number falls within the category of a total, of which heap is a subcategory.[10] This heap is not a *mere* (i.e., indeterminate) heap, but a *measured* heap—a plurality made determinate by a principle of measure. The main worry is that if number were a mere heap, it would seem to have no identity criteria. That is, there is nothing by which to differentiate number—it would be utterly indeterminate. To address this, Katz

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[7] Aristotle does seem to support the claim that the units are interchangeable (*Met*. M.7 1082b1-10). He resists differentiating between units or the indexing of particular units.

[8] See also Maher (2011).

[9] Koslicki "The Structure of Objects," 132n18

[10] Koslicki maintains that the category of totals is not synonymous with the category of heaps. Following this distinction, Katz holds that heaps are a subcategory of totals. For a more detailed discussion of heaps and totals, see Katz, "What Numbers Could Not Be (For Aristotle)," 210n67 and Koslicki, "The Structure of Objects," Chapter VI.

proposes that the identity of a number is derived from its measure. Although it remains a heap lacking in intrinsic unity, it is determinate: its quantity is its identity. [11]

To borrow an example from Katz, consider the difference between a mere sandheap, a measured sandheap, and a sandcastle (Figure 1).[12] A mere sandheap is an indeterminate plurality: it has no identity criteria and arbitrary addition or subtraction of grains does not alter its identity (Figure 1A). It remains a mere sandheap. Once we introduce measurement (e.g., grams), the sandheap becomes a determinate plurality (Figure 1B). [13] If sand is added or removed, a given measured sandheap becomes a different measured sandheap. As a measured plurality, this measured sandheap behaves exactly like number, whose measurement of units constitutes its identity criteria. Crucially, this sandheap is still a heap because it is a total, not a whole. The spatial or structural arrangement of the sand grains is irrelevant to its identity. A sandcastle, however, is a whole (Figure 1C). Its parts are arranged in a specific way such that it has a form (walls, towers, turrets, etc.). Katz observes that if one were to stomp on the sandcastle, it would no longer be a sandcastle but, rather, a sandheap. Yet, if one were to stomp on a sandheap, it would remain a sandheap. And if a measured sandheap were stomped on, it would still contain the same number of grains of sand.

[11] Note that "identity criteria" is distinct from an internal unifying principle. An identity criterion is used to identify an object for the sake of differentiation. Without this, we cannot say what a thing is (its identity). For example, the color of an apple can be used to differentiate between a red apple and a green apple. Likewise, the measure of a number can be used to differentiate between a heap of 3 units and a heap of 4 units. Assigning identity criteria to number does not necessarily import any claims of internal unity.
[12] Katz, "What Numbers Could Not Be (For Aristotle)," 213-214
[13] While the matter of the sandheap remains the same, it transforms into a determinate plurality. It is in this way that the ontological status changes. For example, consider the difference between a pile of sticks (unmeasured) and a pile of 41 sticks (measured). The latter has a clearly defined identity indexing its nature in a way that the former does not.

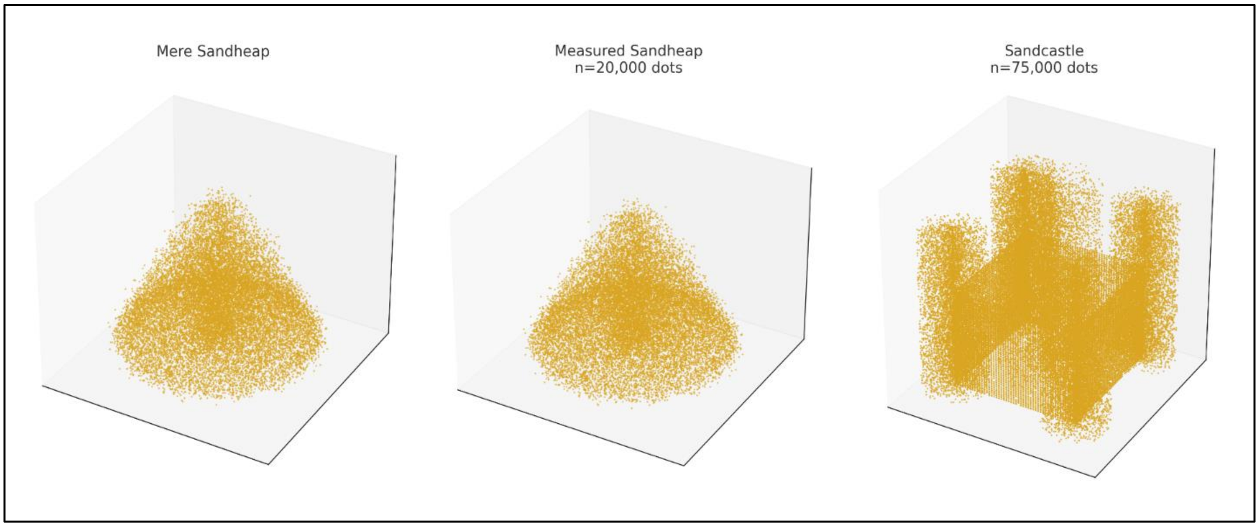

Figure 1A **Figure 1B** **Figure 1C**

Figure 1[14]

I endorse Katz's reading of Aristotle's conception of number as the most textually and philosophically plausible. The view that number is a tight unity becomes untenable given Aristotle's affirmation of number as a total and his denial that it is a substance. Katz's notion of a measured heap also avoids the pitfalls of number as a mere heap. Number—even if not a unity—still has meaningful ontological status consistent with Aristotle's definition of "limited plurality." That is, it is not just a *mere* heap but a *measured* heap. A major worry with this view, though, is that it cannot account for the attributes that Aristotle explicitly ascribes to number beyond its quantity. Aristotle is unequivocal in maintaining that numbers have special qualities, but it is unclear if Katz's account—which insists that numbers are nothing other than the quantity of their units—can make room for this.[15] On Katz's view, number is merely the plurality of its units, reducible to the measurement of that plurality or "what it is once" (*Met*. Δ.14 1020b5-10). On this view, number has neither substance nor form—it is pure matter. But if number is nothing

[14] The code used to generate this Matplotlib output was supplied by ChatGPT-4o (November 2024).
[15] Katz, "What Numbers Could Not Be (For Aristotle)," 210n62

more than quantified plurality, then there appears to be no internal basis for attributing to it any qualitative properties. Qualities such as oddness, evenness, or plane and solid dimensionality, which Aristotle treats as essential to numbers, become unintelligible. Thus, if Katz's interpretation of number as a measured heap is correct (as I maintain), then Aristotle faces a serious explanatory difficulty: his account must explain how numbers can possess non-quantitative features if their essence is defined solely in terms of quantity.

(IV) INTELLIGIBLE MATTER. If Katz's interpretation is correct, and number is nothing but its matter (its units), then this matter must also serve as the ontological basis for the qualities that Aristotle attributes to number. Aristotle identifies this matter as *intelligible matter*, which he distinguishes from the *perceptible matter* of sensible objects. While perceptible matter pertains to things *qua* changeable, intelligible matter pertains to things *qua* unchangeable (*Met.* Z.10 1036a5-15). For example, the bronze of a bronze sphere is perceptible matter because it possesses sensible qualities like color and weight, apprehended by the senses. However, the sphere also has mathematical features, such as magnitude and continuity, which are not apprehended *qua* sensible. These are unchangeable, intelligible properties which are independent of any material instantiation. Regardless of the color or weight of a sphere, it still has universal properties (such as three-dimensional continuity). This is precisely the intelligible matter that is present in "perceptible things but not insofar as they are perceptible" (*Met.* Z.10 1036a5-15). The geometer, for instance, must mentally subtract all changeable, perceptible qualities to isolate the bronze sphere's dimensional continuity.[16] Thus, intelligible matter is not separable in existence but is separable in thought: it is inherent in sensibles but isolated by the intellect through

[16] Katz, "Why Aristotle Can't Do without Intelligible Matter," 136

abstraction. The question, then, is whether the intelligible matter of number (its units) can similarly ground the non-quantitative properties Aristotle attributes to number.

On Katz's reading, the intelligible matter for number is "a sensible plurality just insofar as it consists of indivisibles that measure it."[17] Put differently, the intelligible matter is sensible plurality *qua* its units (the source of number's measure). The key upshot here is that number is divisible into its units. For example, the intelligible matter of the number six is that it consists of six units, which provide the measurement of six. In short, the number six is equivalent to six units. In doing so, we can "take" these six units together to represent the number six.[18] This view is consistent with number being a total, since the particular arrangement of its units is irrelevant to the ontology of the structure.

But this is also precisely why intelligible matter cannot account for number's qualities. While intelligible matter can ground number's cardinality, (the fact that it is *six* rather than *five* or *seven*) it cannot account for properties such as evenness, oddness, or dimensionality. Number, existing just as intelligible matter, cannot yield these qualities. As such, intelligible matter cannot serve as the ontological ground for the qualitative features that Aristotle himself attributes to numbers. Hence, appeal to intelligible matter does not resolve the explanatory gap but only clarifies its location: if number is nothing but a measured plurality of units, then there remains no evident basis for its non-quantitative properties.

II. Beyond Quantity

(I) Composite Qualities. The preceding analysis has reconstructed Aristotle's ontology of number as a determinate quantity—specifically, as a measured plurality of units.

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[17] Katz, "Why Aristotle Can't Do without Intelligible Matter," 124
[18] Katz, "What Numbers Could Not Be (For Aristotle)," 208n58

Yet, this view encounters a serious challenge. Namely, Aristotle himself takes number to be more than just mere quantity. This challenge is not merely interpretive (that is, for Katz), but arises from Aristotle's own metaphysical commitments. If number is nothing but an aggregation of its units, then it is unclear how it can also exhibit distinct qualities.

The most explicit statement of this tension appears in *Met*. Δ.14, where Aristotle maintains that mathematical entities, including numbers, possess *qualities* in addition to *quantities*. Specifically, he writes that numbers can be represented in multiple dimensions (*Met*. Δ.14 1020b1-10). For example, the number 12 is not only an aggregation of twelve units, but also manifests in different configurations, such as 3x4 in the plane (Figure 2) or 3x2x2 in the solid (Figure 3).

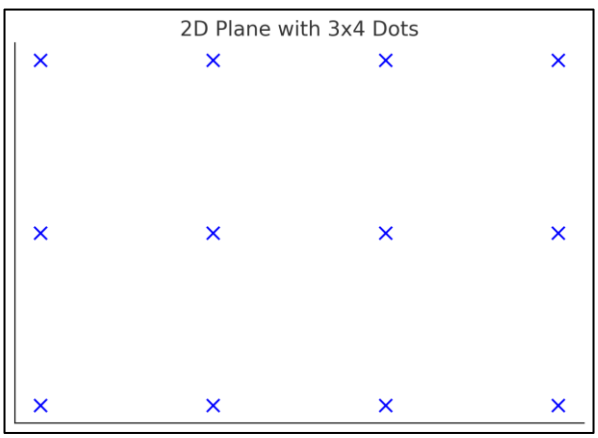

Figure 2[19]

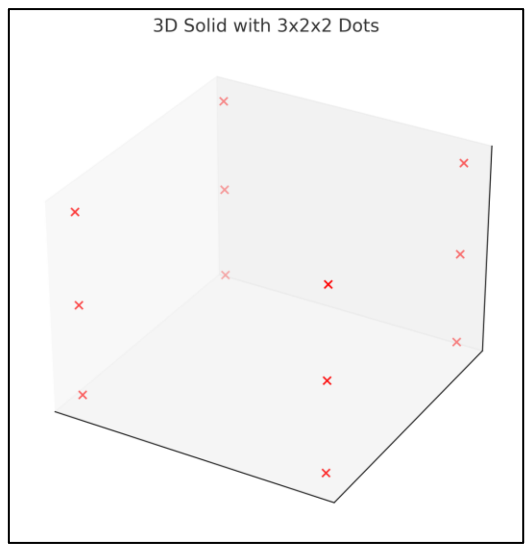

Figure 3[20]

These dimensional structures are not reducible to a mere plurality of units, since they involve relational and spatial organization—features that require more than aggregation. Aristotle makes this point explicit when he states that "in general, that which exists in the essence of numbers

[19] The code used to generate this Matplotlib output was supplied by ChatGPT-4o (November 2024).
[20] The code used to generate this Matplotlib output was supplied by ChatGPT-4o (November 2024).

besides quantity is quality" (*Met.* Δ.14 1020b5–10). This statement commits Aristotle to the view that the number 12 is more than a mere aggregation of 12 units. There is an additional aspect—quality—that characterizes how these units relate to one another. As such, number is not reducible to cardinality alone; it is both a specific quantity (12) and an object with distinct qualities (e.g., being representable into the 3x4 or 3x2x2). Aristotle reaffirms this in *Met*. M.8—for numbers, "quality is said to belong after quantity" (*Met*. M.8 1083a10-15). In other words, while quantity may be logically or ontologically prior, quality nonetheless belongs essentially to number. Thus, number is not merely a measure or count of discrete units but an object with both quantity and quality.

This yields a significant metaphysical tension in Aristotle's account. As I have shown, he is committed to the view that number is a total rather than a whole. That is, number is a quantity for which the position of its units does not matter (*Met*. Δ.26 1024a1-10; *Cat*. 6 5a20-30). For instance, the number 12 remains 12 regardless of how its units are grouped or scattered. In this sense, the totality of number is indifferent to the spatial or relational arrangement of its constituent units. However, Aristotle also asserts that number possesses qualities, such as being plane and solid, which necessarily depend on structural relations among its units. This assertion contradicts the very notion of a total, which is indifferent to arrangement (even if the units are interchangeable). Having plane or solid dimensionality implies that the arrangement of parts—how units are grouped or organized—*does* matter. Indeed, such relational significance is characteristic of wholes, which Aristotle explicitly denies number is, except perhaps metaphorically (*Met*. Δ.26 1024a5-10). This produces a puzzle: on the one hand, Aristotle insists that number is a total, reducible to pure quantity and indifferent to arrangement. On the other hand, he attributes qualities to number (e.g., composite in several dimensions) that seem to demand an internal structure, making it more like a whole. Thus, in virtue of what can number, as a total, also possess qualities? If number's essence is defined by its status as a total, then any introduction of essential qualitative aspects appears to undermine this definition. Yet, Aristotle consistently affirms both claims: that number is a total and that it has qualities.

(II) QUALITIES AS IDENTITY CRITERIA. This tension introduced by the qualities of composite numbers is pervasive. In multiple passages, Aristotle explicitly ascribes additional properties to numbers beyond mere quantity (*An. Post*. 1.4 73a35-40; *Met*. Γ.2 1004b10–15). The cleanest formulation of this comes in an example. In *An. Post*. 2.13 96a30-b10, Aristotle defines the triad as: "a number, odd, and prime in both senses."[21, 22] Individually, these attributes could apply to many different numbers but, collectively, they uniquely designate the number three.[23] The significance of this presentation, for my purposes, rests in its ability to pick out the number three without reference to a specific measure. Rather, Aristotle identifies the triad using a combination of essential qualities to distinguish its essence. He explicitly claims that this demonstration reveals "the substance of triad" (*An. Post.* 2.13 96a30–b10). This is clear affirmation that, for Aristotle, the number three is not a mere collection of units (a total), but something with intrinsic qualities that distinguish it. Yet, if the sole identity criterion of number is nothing but its measure (its units), Aristotle should not be able to identify number in this way.

[21] A major question is whether or not this formulation suffices as a definition of the triad. A definition must be more than just a mere listing of connected attributes (such as odd and prime in both senses) (*Met*. H.6 1045a10-15). If this formula does not constitute a proper definition, then it might seem to undermine the claim that number has an essence extending beyond quantity. Yet even if this formulation does not meet Aristotle's criteria for definition, his inclusion of additional qualities within number reveals an undeniable commitment to number as more than mere quantity. Aristotle acknowledges that qualities such as oddness "cannot be defined apart from number," suggesting an inseparable connection between numbers and their qualities (*Met*. Z.5 1031a1–5). This tight linkage implies that if number is to have any qualities at all, then its ontology must account for this possibility.

[22] "…prime in both senses" i.e., neither the product nor sum of two integers. Aristotle does not ordinarily classify one as a number. As such, the only possible numbers that are "prime in both senses" are 2 and 3. Of these two options, only 3 is odd. This is why "number, odd, and prime in both senses" must refer to the triad.

[23] One might ask if "triad" is distinct from the number three and, therefore, inapplicable in this case. However, Aristotle uses sets and numbers interchangeably (*An. Post.* 2.13 96a30; *Physics* 4.14 224a1–5), suggesting that the triad can stand in for the number three.

Moreover, these qualities—being odd and prime—are relational and structural with respect to its units. There is no conception of these qualities without the understanding that the individual units of the number are arranged a certain way. Put simply, these qualities necessarily entail a relation among units, but this is incoherent given number is a measured heap (a total) and not a unity (a whole).

In an odd number, for example, the spatial positioning of the remaining unit is significant. It is, in short, in the essence of oddness that a unit be leftover. For example, when we say that three is an odd number, what we mean is that when all possible units are paired together, there is a remainder of one unit. While all the units are the same and interchangeable, the representation of an odd number (say, in a 2D plane) demands a particular kind of arrangement of units. Put differently, we no longer take all the units together to have a view of the number's measure. Rather, in determining such a quality, we must take *subsets* of units together. In doing so, we impose an artificial structure. Notice that the same principle applies when we represent the number 4 as a 2x2 square in a two-dimensional plane. To represent an essential property of the number, there is only one possible structure. This means that the property of being square inherently demands a specific arrangement (namely, a 2x2 structure). It is precisely this structure that precludes a categorization of "total." Thus, in order to see certain essential features of number, we must treat it as if it were not a total. Aristotle, however, insists that number must be a total (*Met*. Δ.26 1024a1-10; *Cat*. 6 5a20-30). In order to explain number's qualities, number cannot be a total. Yet, number *is* a total.

(III) ESSENTIAL QUALITIES. Aristotle's broader treatment of number's qualities only reinforces this tension. He explicitly identifies several mathematical properties (odd and even, prime and compound, square and oblong) as "essential attributes" that would be "impossible for them not to inhere in their subject" (*An*. *Post*. 1.4 73a35-40; 73b15-25). And in the *Metaphysics*, Aristotle further reaffirms that number *qua* number has special qualities such as "oddness and evenness, commensurability and quality, excess and defect" (*Met*. Γ.2 1004b10–15). However, the acknowledgment of these properties clashes directly with his characterization of number as a total. If number is to be defined solely by its quantity and not its arrangement, how can it also bear attributes that depend on structure, such as being square or solid? This contradiction is further underscored by Aristotle's claim that qualities like "oddness" cannot be defined apart from number itself (*Met.* Z.5 1031a1–5). If these qualities are essential and inseparable, then the essence of number cannot consist of quantity alone. That is, we would be missing something deeply important about number if we could not account for its essential qualities (e.g., being odd). Not only are numbers in general (the genus) even or odd, so are *particular* numbers. It is not an accidental or contingent feature that the number three is odd. Thus, the insistence that number is a total (pure quantity) is fundamentally incompatible with his attribution of qualities that rely on relational structure. If number is truly a total, then its essence cannot accommodate these essential properties. Yet, if these properties are intrinsic to number, then number cannot remain a total but must instead be understood as a whole—a possibility Aristotle denies except through metaphor (*Met.* Δ.26 1024a5–10). Thus, the evidence is overwhelming that there is an Aristotelian commitment to (and failure to account for) mathematical properties within numbers.

III. Is There a Way Out?

Having presented the problem, I will, in the following section, offer four potential resolutions. I show that each fails to resolve the puzzle.

(I) POTENTIALITY AND ACTUALITY. One plausible avenue to explain how number can possess qualities despite being a total is to appeal to Aristotelian conceptions of potentiality and actuality. Aristotle often distinguishes between what a thing is in actuality (its realized state) and what it is in potentiality (the capacities inherent in its being) (*Met*. Θ). Applying this distinction to number, we can see how its qualities might emerge not as intrinsic features of its actual being but rather as a manifestation of its inherent potentialities.

For instance, a number like 12 exists merely as a total—a plurality of 12 units without any arrangement-dependent structure. However, 12 also has the *potential* to be represented in various configurations (such as 3x4 or 3x2x2). These representations depend on the context of mathematical inquiry but do not conflict with 12 being a total. Rather, one could argue, such representations reveal the inherent potentialities within the intelligible matter of 12. It is a plurality measured by its units, which can themselves be divided, combined, or otherwise arranged. The units have such-and-such potentialities but remain, in actuality, indifferently aggregated as a total. Thus, qualities like oddness, evenness, or dimensionality arise from these potentialities, actualized under specific conditions, rather than being intrinsic to the actual being of 12.

Although this framework may appear to be an elegant way to reconcile the qualities of number with its ontology as a total, this resolution faces insurmountable obstacles. While potentiality usually has a clear connection to its actuality rooted in its *telos* (e.g., the potential for a seed to become a tree), the connection is much less clear for number. The sorts of things that have a *telos* are natural, hylomorphic substances, which number is not. Number's actuality is nothing but measured plurality: an aggregation of units whose arrangement does not matter. It is unclear, then, how its potentiality could come to explain arrangement-dependent qualities like oddness or dimensionality. Put differently, if number's qualities can be explained by its potentiality, where does this potentiality come from? We have already ruled out (by definition) the possibility that number has a form, which is the primary candidate for supplying a *telos* (*Met.* Θ.8 1050a-b). More fundamentally, this is another indictment on the explanatory power of intelligible matter. Its actuality is characterized in terms of its arrangement-independent units, but this fails to ground the qualitative properties of number, which require an arrangement-dependent structure.

Even granting that it may be within the essence of number to have potential qualities, this view is an uncomfortable position to hold. In what sense is the number 3 only potentially odd? There is no understanding of 3 as odd without it being actually odd. The same is true for composite qualities. While the squareness of 4 might appear to be more removed from the essence of number than the oddness of 3, it is just as actual. There is no understanding of 4 being square separate from the actuality of 4. If someone were to say, “4 is potentially square,” we would think them confused. 4 *is* square. I grant that any physical or mental representation of this quality (e.g., in the plane or solid) may be construed as a potential representation of the quality. It may even be necessary that we need such a representation to understand squareness. But this does not import potentiality onto the quality itself—4 is never *potentially* square. I may not understand what a metaphor is until you give me an example, but nothing changes in the activity of “metaphor” in the process of your explanation. In this way, the characteristic features of a metaphor are not merely potentially so.

(II) AN EPISTEMOLOGICAL VIEW. A second possible option to account for number’s qualitative properties is to consider them as a reflection of number’s epistemology rather than its ontology. Specifically, it is important to draw a distinction between the epistemic role of these qualities and their ontological status. Under such a view, these properties are cognitive or heuristic tools employed by mathematicians to grasp, manipulate, and study numbers.

Representing 12 as 3x4 might merely serve as a convenient way to illustrate its factors or visualize it in a rectangular arrangement. Such a representation need not impose ontological weight on the number itself. Namely, this representation is not necessarily intrinsic in the being of 12 but rather reflects the scientific approach of mathematicians. On this view, the arrangement-based qualities of numbers are artifacts of mathematical practice, not inherent features of the number's being. This is not to deny the essential features of numbers (oddness, primeness, etc.) but only to clarify their status as epistemic heuristics.

This epistemological interpretation of number's qualities, treating them as heuristic tools rather than ontological attributes, while promising, ultimately misrepresents Aristotle's commitments. Aristotle explicitly identifies these mathematical properties as *essential attributes*, which belong to what a thing is in its essence (*An. Post*. 1.4 73a35-40). Such qualities are not external but inhere in numbers necessarily: "it is impossible for them not to inhere in their subject" (*An. Post*. 1.4 73b15-25). Aristotle is explicit about the ontological significance of these qualities, noting "that which exists in the essence of numbers besides quantity is quality" (*Met*. Δ.14 1020b5-10). He reaffirms this elsewhere, observing that "number *qua* number has peculiar attributes" which belong "to numbers in themselves or in relation to another" (*Met*. Γ.2 1004b10–15). Essential attributes, by definition, are not tools for analysis or representation, but pertain to what a thing is in itself. It is significant that Aristotle identifies the triad through its essential qualities (odd and prime in both senses) and not, say, "the pentad less the dyad" (*An. Post*. 2.13 96a30-b5). The triad's essence is not *just* a mere plurality of units; it is also a collection of essential qualities. This approach risks relegating mathematical properties to a kind of second-order reality, dependent on our engagement with the mathematical object. Such a move fatally undermines Aristotle's commitment to the reality of these qualities, which inhere essentially. These qualities are not imposed by the mathematician but are embedded in the very being of number itself. They "belong to all numbers" *qua* number (*Met*. Γ.2 1004b10–15). It is evident, then, that such qualities belong to the number and not to the mathematician.[24]

(III) ABSTRACTION. So, what is the mathematician doing when she considers the squareness of 4 or the oddness of 3? In ordinary language, we might refer to such an act as "abstraction." I might say, for instance, that I am abstracting the mathematical properties from the number when I consider it in terms of its dimensionality or its parity. However, this precise process of abstraction—and what it would mean for the ontology of number—is indeterminate. Aristotle gives us little to work with in the way of abstraction. The furthest his account takes us (as far as abstraction goes) is intelligible matter (*Met.* E.1 1026a5-15). I have shown, however, that intelligible matter is insufficient. If mathematical properties were to be actualized through intelligible matter, such an actualization necessarily demands a conception of an arrangement-dependent structure, which is precluded by Aristotle's characterization of number as a total. [25] Perhaps it is the case, then, that intelligible matter merely represents number in a derivative sense. Put differently, number exists actually as a total with qualities, but we can only grasp such qualities through intelligible matter.

This approach, however, is inconsistent with Aristotle's framework. The root of the problem is such: how can qualities that require structural arrangement exist within an entity whose ontology explicitly denies the importance of arrangement? To claim that such qualities are mediated through intelligible matter does not resolve this issue, but merely shifts the problem. If

[24] Aristotle is also committed to the reality of numbers ("the objects of mathematics exist") (*Met*. M.3 1078a30-35).
[25] I have also shown that we cannot leave mathematical properties as potentials. Aristotle is deeply committed to their essential character as features existing in actuality.

intelligible matter is supposed to account for our understanding of number as a total, then it also cannot simultaneously account for qualities that are arrangement-dependent.

Arithmetical abstraction runs up against an even deeper problem. Rather than isolating a numerical property, this abstraction ultimately produces a geometrical object. Katz's account of geometrical abstraction identifies a geometrical object by separating geometrical properties from the sensible object.[26] It is, in essence, a kind of "logical isolation" of the geometrical object from the sensible.[27] For example, a physical triangle is considered as a geometrical object only when it is considered as a three-sided figure with straight lines, not as a model with color and texture.

An arithmetical account of abstraction appears worryingly close to geometrical abstraction. For example, when the mathematician abstracts "squareness" from the 4 units, she inadvertently constructs something spatial. Indeed, this is an object whose units demand a specific position which number, as a measured heap, does not. If abstraction generates structural properties like squareness, then the mathematician has not merely identified an arithmetical property but has also imposed a geometrical structure onto number. The same is true of odd and even numbers, which are represented through either parity or imparity. That is, units standing in relation to each other. Thus, if the process of abstraction necessarily results in a structured whole with internal spatial relationships, then numbers are no longer being treated as pure quantitative units but as geometrical constructs. Yet, presumably, Aristotle wants to hold numbers and geometricals apart. Geometrical objects are themselves an abstraction from sensibles and they retain unity as wholes. Numbers, on the other hand, are mere totals—measured plurality. Abstraction of numerical properties from number fundamentally undercuts this view of number by transforming

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[26] Katz, Geometrical Objects as Properties of Sensibles," 483
[27] Katz, Geometrical Objects as Properties of Sensibles," 484n60

it into a geometrical. Consequently, the distinction between a numerical unit and a geometrical magnitude becomes indeterminate.

Therefore, abstraction, too, fails to produce number's qualities in a cogent way. The consequences of abstraction (the production of a whole) are not only opposed to Aristotle's view of number (a total), but they also introduce further questions about the status of number. Beyond failing to reconcile quality with quantity, it also introduces a geometrical puzzle about arithmetic.

(IV) FURTHER DISTINCTIONS. I will, finally, consider a series of further distinctions that might be invoked to dissolve the tension within Aristotle's account of number. [28] These distinctions attempt to reframe the qualitative properties as consistent with number being a total composed of positionless units. Although these distinctions may initially appear to undercut the force of the tension I raise, I argue that accepting these distinctions is unpalatable.

The first distinction proposes that arithmetical numbers themselves can be separated from their spatial representations. The distinction holds that this *spatial* view of units is not Aristotle's considered view. Rather, his position is that number is composed of *pure* units: these are indivisibles that have *no* spatial relationship (*Met.* M.8, 1084b15–20). It is true, for example, that I may *represent* the number 3 as a group of spatially arranged dots to illustrate its oddness. But such representations are not constitutive of number itself. Number is a plurality of positionless units, not a collection of spatially locatable objects. Hence, the oddness of 3, cannot, on this view, depend on its being arranged in a particular way. Oddness and evenness, though *representable* spatially, are not *in themselves* spatial properties. To consider 3's oddness on its own terms requires an appeal to its cardinality—to its being "just this many" units.

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[28] Many thanks to Emily Katz for raising these comments in correspondence.

How might we explain 3's oddness by appealing to its cardinality? A plausible proposal is a set-theoretic definition:[29]

- A number *n* is even if and only if its elements can be placed into two equinumerous subsets. That is, there exists a bijection between the subsets.
- A number *n* is odd if and only if no such partition exists. That is, every partition leaves one subset with exactly one more element.

This formulation treats number solely in terms of its cardinality. In other words, the only consideration is the *count* of elements, not their *arrangement*. However, the problem arises when we attempt to recognize (through demonstration or intuition) whether such a partition (the bijection condition) exists. To "place" units into subsets seems to require at least a conceptual ordering, grouping, or visual distinction. For example, if *n*=6, we can partition {●●●●●●} into {●●●} and {●●●}, which have the same cardinality, making 6 even. If *n* = 5, any partition (e.g., {●●●} and {●●}) results in different cardinalities, so 5 is odd. Thus, the most compelling definitions of evenness and oddness lean on the very spatiality that Aristotle's view denies. To place an element (unit) into a subset is to represent it in a certain kind of way.[30] While it may be true that number itself is nothing but a plurality of units, explaining its arithmetical properties requires further analytical tools (in this case, spatial arrangement).

My point, ultimately, is that spatial representation is *necessary* for any understanding of number's arithmetical properties, such as even or odd. My conception of 5 being odd is derived from an understanding that its units *are arranged a certain way*. This quality imputes a necessary structure, which is incompatible with the view that number is a measured heap consisting of a

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[29] Adapted from Euclid, *The Elements*, Book VII, defs. 6–7

[30] Crucially, while units are interchangeable, they must be *differentiated*. In this way, we can track distinct units. See *Met.* M.8 1081b-1082a.

mere plurality. To resolve this, one might suggest a separation: holding number (e.g., 5) and its qualities (e.g., being odd) as distinct categories. In this way, the incompatible structures of each representation—number as a total and quality as a whole—do not conflict with each other. But this is an improbable conclusion. Oddness is not an accidental feature of 5 but part of its essence. This is confirmed by Aristotle himself, who asserts that oddness “cannot be defined apart from number” (*Met*. Z.5 1031a1–5). To treat oddness as separable from 5 would be to treat it as a contingent property—precisely what Aristotle denies.

A second strategy for preserving Aristotle’s account appeals to an analogical distinction: geometrical terms like “square,” “oblong,” or “solid,” though originating in spatial contexts, are said to be applied to numbers not literally but analogically. On this view, these terms do not imply that numbers possess spatial features; rather, they refer to numerical relationships that mirror geometrical ones. For instance, a number is “square” if it results from multiplying a number by itself, just as the area of a geometric square results from multiplying equal lengths. These terms are borrowed from geometry to describe arithmetic patterns, not to impute any spatial character on the numbers themselves.

This analogical strategy seeks to preserve Aristotle’s view that number consists of positionless, indivisible units by denying that the geometrical properties attributed to numbers are literal. A number is square if and only if it is the result of multiplying some natural number by itself. What makes 4 square, on this account, is simply that 4=2*2—not that 4’s units are or must be spatially arranged. The squareness is taken to be algebraic, not spatial. Yet even this algebraic squareness presupposes a structure among units: it depends on how the units of 4 can be grouped, not just how many there are.

To get a grip on this algebraic formulation (that is, to actually recognize the "analogical" squareness of 4), we must appeal to how its units can be organized. Namely, the units comprise a set that can be partitioned into two equal-sized rows and two equal-sized columns. The key point is this: the explanation of 4's squareness depends not merely on how many units there are, but on how those units can be *organized relative to one another*.[31] Such a grouping, however, relies on the imposition of a structural relation between otherwise undifferentiated units. The squareness of 4, then, is grounded in its being the cardinality of a set that admits this kind of symmetric arrangement.

This is the crux of the issue: the mere cardinality of 4 *underdetermines* its squareness. The explanatory force lies not in the mere count of units, but in the possibility of subdivision that mirrors geometrical form.[32] It is only through a certain kind of arrangement—an internal symmetry—that the property becomes intelligible. Being square, then, is not merely an analogical feature of number. Its content lies in how units are configured, not simply in how many there are. Thus, the cardinality-based explanation reintroduces structure in a problematic way that mirrors the earlier case of oddness and evenness.

Oblong numbers illustrate the same dependence on structural arrangement.[33] A number is *oblong* if and only if it is the product of two consecutive natural numbers. That is, an oblong number is the result of $n*(n+1)$ where n is a natural number. For example, 6 is an oblong number because 6=2*3. But again, any demonstration or intuition that can account for this property must appeal to how six units can be grouped into 2 rows and 3 columns. The number of units in each column is one more than the number of units in each row (or vice versa). This spatial

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[31] Put differently, to explain what it is for 4 to be square requires more than stating a multiplication fact.
[32] Aristotle is clear that the units alone cannot produce quality. See *Met.* M.8 1083a10-15.
[33] The same logic applies for plane and solid numbers.

representation is not a feature of 6 as mere plurality, but of 6 as admitting a particular structural composition. Even if we are to deny that such a composition is spatial in any literal sense, it remains a kind of structure imposed upon the units. The content of the explanatory force comes from structure, not pure multiplicity.

In short, the analogical strategy fails: it reintroduces structure to explain number, and thus contradicts the very conception it aims to preserve. If number is truly nothing but a heap of positionless indivisibles, then it lacks the internal relations necessary to support predicates like "square" or "oblong." But if such properties are essential to number, as Aristotle affirms, then number must possess salient internal structure.

IV. Conclusion

Ultimately, I have argued that there is a fundamental and intractable tension within Aristotle's conception of number. Under the most promising interpretation, number is a measured heap—a discrete quantity whose units lack relative position, with no form organizing its parts. Yet, this account cannot make explanatory room for number's essential qualities, which Aristotle is deeply committed to. The identification of these qualities introduces structure and a particular arrangement of units, which the view denies is true of number. Thus, Aristotle's account features two independently coherent halves that are, in the end, incompatible. Barring a re-interpretation, Aristotle's commitments wind up undermining the integrity of his arithmetical project.

Lastly, it should be noted that this is not merely a localized difficulty within Aristotelian metaphysics but rather for any non-Platonist theory of number. To account for the unity of number is necessarily to account for the unity of its essential features. The conception of number as a measured plurality may suffice to ground cardinal identity, but it is ontologically insufficient to explain the presence of arithmetical and geometrical structure. If number possesses irreducible qualities, then its underlying ontology must be robust enough to support them. Any adequate account of number must, therefore, not only include the aggregation of units, but also a structure sufficient to explain the existence of its essential qualities.[34]

[34] I am grateful to Ermioni Prokopaki for her excellent supervision and feedback over numerous drafts. My thanks also to Agnes Callard and Emily Katz for their helpful questions and comments.

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