Resonant Self-Interacting Dark Matter from Dark QCD

Yu-Dai Tsai,[1,2,3,*] Robert McGehee,[4,5,†] and Hitoshi Murayama[4,6,5,‡]

[1]*Department of Physics and Astronomy, University of California, Irvine, California 92697-4575, USA*
[2]*Fermilab, Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA*
[3]*University of Chicago, Kavli Institute for Cosmological Physics, Chicago, Illinois 60637, USA*
[4]*Department of Physics, University of California, Berkeley, California 94720, USA*
[5]*Ernest Orlando Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[6]*Kavli Institute for the Physics and Mathematics of the Universe (WPI), University of Tokyo, Kashiwa 277-8583, Japan*

(Received 2 February 2021; revised 7 January 2022; accepted 10 March 2022; published 27 April 2022)

We present new models utilizing QCD-like dark sectors to resolve small-scale structure problems. These models of resonant self-interacting dark matter in a dark sector with QCD are based on analogies to the meson spectra in standard model QCD. We introduce a simple model that realizes resonant self-interaction (analogous to the ϕ-K-K system) and thermal freeze-out, in which dark mesons are made of two light quarks. We also consider asymmetric dark matter composed of heavy and light dark quarks to realize a resonant self-interaction (analogous to the $\Upsilon(4S)$-B-B system) and discuss the experimental probes of both setups. Finally, we comment on the possible resonant self-interactions already built into SIMP and ELDER mechanisms while using lattice results to determine feasibility.

DOI: 10.1103/PhysRevLett.128.172001

Introduction.—The study of dark matter (DM) has been one of the most important topics in particle physics, astrophysics, and cosmology. Although there is overwhelming evidence of DM, we know next to nothing about its nature. Observations involving halo or subhalo structures [1] may shed light on this mystery. Historically, core versus cusp [1–6], too-big-to-fail [7], and diversity problems [8] have indicated the potential existence of DM self-interaction (see, e.g., Ref. [9]), although baryonic feedback [10–13] provides an alternative explanation of these small-scale puzzles.

The Bullet cluster [14–16], along with halo shape observations [17,18], sets an upper bound on DM self-interactions around $\sim\mathrm{cm}^2/\mathrm{g}$. Given that a larger cross section could be preferable for smaller-scale halos [19], introducing a velocity dependent self-interaction to explain the small-scale structure issues is well motivated.

The preferred DM self-interaction strength is near that of nuclear interactions [9]. Thus, it is interesting to consider a QCD-like theory in which such strength of interaction emerges. Additionally, one of the simplest ways to achieve such velocity dependence solely in the dark sector is via resonant scattering [20], although one can also achieve that by exchanging a light mediator through t-channel processes [19]. Suppose there is a resonance in the DM self-interactions just above the threshold of twice its mass. Then this resonant self-interacting DM may miss this resonance in systems with large velocity dispersions, such as clusters of galaxies, while it may frequently hit the resonance in systems with small velocity dispersions, such as dwarf galaxies. This would lead to cross section enhancement at small velocities, yielding the desired velocity dependence. This solution typically requires that the resonance have a mass $(10^{-6} - 10^{-4})m_{\mathrm{DM}}$ above twice the DM mass.

In this Letter, we will consider multiple models with mediators just above the threshold which explains such resonances. To achieve these resonances, we need look no further than standard model (SM) QCD in which many cases of such resonances exist naturally. Perhaps the most famous example of near-threshold resonance is in the triple-α reaction in stellar burning, $\alpha\alpha \to {}^8\mathrm{Be}$, $\alpha{}^8\mathrm{Be} \to {}^{12}\mathrm{C}^*$ (7.66 MeV 0^+ excited state of ^{12}C),

$$\frac{m({}^8\mathrm{Be}) - 2m(\alpha)}{m({}^8\mathrm{Be})} = 0.000012, \tag{1}$$

$$\frac{m({}^{12}\mathrm{C}^*) - m({}^8\mathrm{Be}) - m(\alpha)}{m({}^{12}\mathrm{C}^*)} = 0.000026. \tag{2}$$

This example is often invoked as evidence for the anthropic principle [21,22]. Even though they are less pronounced, there are numerous examples of near-threshold resonances in QCD, such as

$$\frac{m(\phi) - 2m(K^0)}{m(\phi)} = 0.024, \tag{3}$$

$$\frac{m(D^{0*}) - m(D^0) - m(\pi^0)}{m(D^{0*})} = 0.0035, \tag{4}$$

$$\frac{m(B_{s1}) - m(B^*) - m(K^0)}{m(B_{s1})} = 0.0011, \tag{5}$$

$$\frac{m[\Upsilon(4S)] - 2m(B^0)}{m[\Upsilon(4S)]} = 0.0019. \tag{6}$$

Some of these illustrative near resonances are shown in Fig. 1. Most examples are not pure accidents: QCD dynamics require there to be such near-threshold resonances. In a heavy-light meson $(Q\bar{q})$, its mass is essentially the sum of the heavy quark mass m_Q and the effect of the strong interaction $\sim\Lambda_{\rm QCD}$. On the other hand, for the heavy-heavy meson $(Q\bar{Q})$, its mass is twice the heavy quark mass $2m_Q$ and the effect of binding. In the limit $m_Q \gg \Lambda_{\rm QCD}$, it is clear $m_{Q\bar{Q}} \approx 2m_{Q\bar{q}}$ is the zeroth-order approximation. To be more precise, we need to understand the quarkonium potential, discussed in the section about the heavy quark model. On the other hand, the mass splitting between D^* and D is due to the hyperfine interaction between magnetic moments and is approximately $\sim\Lambda_{\rm QCD}^2/m_Q$ which is not related to $m_\pi \approx (m_q\Lambda_{\rm QCD})^{1/2}$. We consider this example to be a pure accident.

In the following sections, we discuss three specific scenarios. First, we outline a model with two light quarks, with one much heavier than the other, in which dark "kaons" freeze out to the correct relic abundance and the resonance is analogous to $K^+K^- \to \phi$. We then discuss an asymmetric DM model in which DM particles are mesons with one heavy and one light quark, and the resonance is similar to $B^0\bar{B}^0 \to \Upsilon(4S)$. The closeness to threshold $\Delta \equiv 1–2m_{\rm PS}/m_V$ in both must be quite significant, where $m_{\rm PS}$ is the mass of the pseudoscalar meson and m_V is the mass of the vector meson. Finally, we describe a model directly based on the strongly interacting massive particle (SIMP) framework discussed in Ref. [23] and use lattice results to determine the parameters for the resonance. The dark QCD confinement scales in our models are above $\sim$10 MeV, and any excess entropy in the dark sector has time to safely transfer to the SM prior to the neutrino decoupling and the big bang nucleosynthesis (BBN). One other recent work connecting dark QCD and small-scale structure can be found in Ref. [24], but their dark QCD scale is significantly lower than that of SM and is drastically different from our setup.

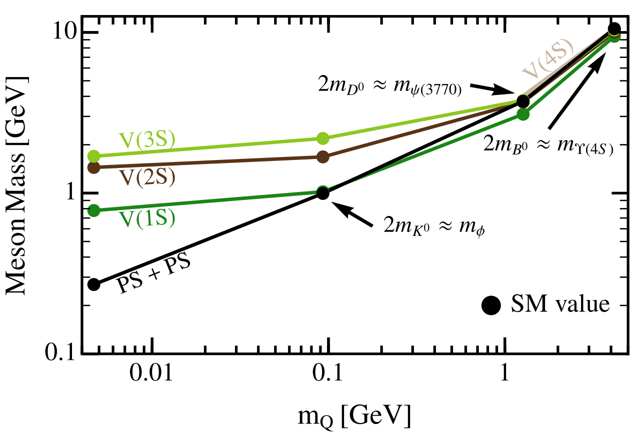

FIG. 1. A selection of the SM meson spectrum as a function of the larger quark mass in each meson, m_Q. Extrapolations of twice the pseudoscalar meson mass (PS + PS), of the first vector meson mass [V(1S)], of the second vector meson mass [V(2S)], of the third vector meson mass [V(3S)], and of the fourth vector meson mass [V(4S)] are shown. For $m_Q = m_d$, we show π^0 as well as the average masses of the first three ρ and ω states. For $m_Q = m_s$, we show K^0 and the first three ϕ's. For $m_Q = \{m_c, m_b\}$, we show D^0 and B^0 as well as the first four ψ and Υ states, respectively.

With our discussions of QCD mesons and resonances complete, future references to quarks (e.g., u) and mesons (e.g., K) in this Letter will refer to dark sector analogs to the SM states unless otherwise noted.

Light quark model.—We first assume a QCD-like gauge theory $SU(3)_D$ in the dark sector. DM is composed of dark "kaons" [25] composed of two dark quarks with masses much smaller than the dark QCD scale, labeled u and s, with $m_s \gg m_u$. The quarks are charged under a dark $U(1)_D$ as $u(+1)$ and $s(0)$ which is broken, resulting in a massive dark photon A_D. We also assume a kinetic mixing between $U(1)_D$ and $U(1)_{\rm EM}$ of the form $\mathcal{L} \supset 1/2 \cdot \epsilon F_{\mu\nu} F_D^{\mu\nu}$.

DM self-interactions: The desired resonant self interaction is provided by the dark ϕ exchange saturating the Breit-Wigner cross section in the P wave. We assume $\Delta \sim 10^{-7.8}$ for these dark mesons [20]. We also need $(\sigma_0/m_{\rm DM}) \sim 0.1\ ({\rm cm}^2/g)$ in order for the low-velocity limit of the self-interaction cross section to fit small-scale structure observations [20]. Thus, we calculate the 4-kaon interaction in the dark sector. The self-interaction mediated by A_D is negligible for the parameters we consider.

We define $U = e^{2i\Pi/f_K}$, $\Pi = K^a T^a = \frac{1}{2} K^a \tau^a$, $2{\rm Tr}(\Pi^2) = K^a K^a$; f_K is the dark kaon decay constant. First, consider the nonderivative couplings. The relevant chiral Lagrangian terms are

$$\mathcal{L} = \frac{1}{2}\frac{m_K^2 f_K^2}{m_u + m_s} {\rm Tr}\left[U^\dagger \begin{pmatrix} m_u & 0 \\ 0 & m_s \end{pmatrix} + \begin{pmatrix} m_u & 0 \\ 0 & m_s \end{pmatrix} U\right] \tag{7}$$

$$\supset -m_K^2 K^+K^- + \frac{1}{4}\left(\frac{2m_K^2}{3f_K^2}\right)(K^+K^-)^2. \tag{8}$$

The relevant derivative couplings are

$$\mathcal{L} = \frac{f_K^2}{4}\mathrm{Tr}\partial_\mu U^\dagger \partial^\mu U$$
$$= \partial_\mu K^+ \partial^\mu K^- - \frac{2m_K^2}{3f_K^2}(K^+K^-)^2$$
$$- \frac{1}{2f_K^2}(K^+K^-)\partial_\mu\partial^\mu(K^+K^-) + O(K^6). \qquad (9)$$

We assume K^0 is heavier than $K^\pm$ by $\sim$10% (which can be induced by the L_7 term in the chiral Lagrangian [27–29]), so that only the $K^\pm$ states make up DM. From here on, we define $m_K = m_{K^\pm}$ to be the masses of the dark charged kaons. The neutral kaon is unstable and cannot be a DM candidate because it can decay into, for example, four electrons, through an off shell dark photon. In halos today, there are only $K^\pm$ interactions. After taking into account the derivative terms, the self-interaction cross section for $K^+K^- \to K^+K^-$ is $\sigma_{K^+K^-} = (1/16\pi)(m_K^2/f_K^4)$.

To match the fitted low-velocity limit of the self-interaction cross section [20], we set $(\sigma_0/m_{\mathrm{DM}}) = \frac{1}{2}(\sigma_{K^+K^-}/m_K) \simeq 0.11^{+0.10}_{-0.05}$ cm^2/g. $m_{\mathrm{DM}} = m_K$ is the DM mass, and σ_0 is the low-velocity limit of the DM self-interaction cross section.

Requiring the correct σ_0/m_{DM} in our model fixes the relation between m_K and f_K, $\sim(0.07 \pm 0.01)$ GeV $(\mathrm{m_K}/\mathrm{GeV})^{1/4}$. If we match this to the SM ratio of $m_K/f_K \sim 0.32$ [30], we get $m_K \sim$ 100–160 MeV for the dark kaon (region I). On the other hand, if we consider the SM ϕ-K-K system, its $\gamma = g_V^2/(384\pi) \sim 0.02$ and $m_K \sim$ 0.9–1.5 GeV (region II). g_V is the coupling constant, and the definition of γ can be found in Ref. [20]. We delineate the ranges of m_K which correspond to each of these two assumptions in Fig. 2. Even though these regions do not overlap, one could consider a different gauge group or simply a different N_c (see the Supplemental Material [31] for more discussions, which also includes Ref. [32–49]). For example, the regions could move closer [50] for $N_c = 2$.

The DM self-interaction mediated by the dark photon A_D is suppressed as $(m_K/m_{A_D})^4$, and the interaction strength is much smaller than that of four-meson interaction so that it can be neglected in this consideration.

Freeze-out: Here, we consider the process that sets the DM relic abundance. We assume A_D is heavier than $K^\pm$. Since A_D is heavier than $K^\pm$, before K^0 decays (suppressed by one loop, ε^4, and $m_{A_D}^{-8}$), it annihilates via $K^0K^0 \to K^+K^-$. The annihilation $K^+K^- \to A_D A_D \to e^+e^-e^+e^-$ can also happen, but it is suppressed by ε^4, much smaller than the freeze-out cross section. The primary freeze-out process we consider is thus $K^+K^- \to A_D \to$ SM.

The generic choice of m_{A_D} and m_K is mostly excluded in our parameter region of interest. However, one can invoke another resonance to open up the parameter space. In addition to the resonance in self-interactions induced by the vector meson, one can also arrange the dark photon mass so

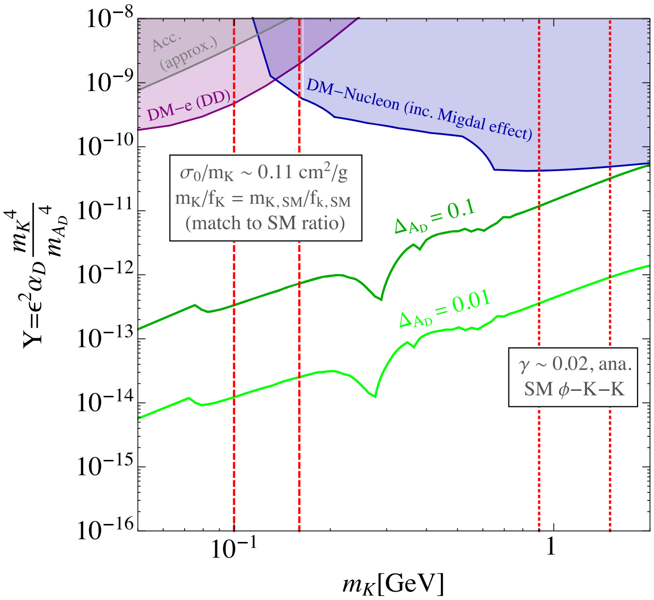

FIG. 2. The most motivated mass ranges for resonant self-interaction, analogous to the ϕ-K-K system discussed in the text, are enclosed by red dashed and dotted lines. The green curves give the correct relic abundance with $\Delta_{A_D} = (m_{A_D}^2 - 4m_{\mathrm{DM}}^2)/4m_{\mathrm{DM}}^2 = 0.1$ and 0.01, reproduced from Ref. [51]. The purple regime is constrained by DM-electron direct detection; the gray regime is the approximate accelerator bound (see text for discussions), and the blue region is constrained by the DM-nucleon scattering (including the Migdal effect).

that it goes on resonance for the freeze-out process, to allow smaller Y to produce the correct relic abundance and avoid accelerator as well as direct-detection constraints [51–55]. We define $\Delta_{A_D} \equiv [(m_{A_D}^2 - 4m_K^2)/4m_K^2]$. In Fig. 2, we show the A_D resonant cases with $\Delta_{A_D} = 0.1$ and 0.01, along with constraints from direct-detection [56–60] and accelerator experiments [61–66]. We assume $m_{A_D} = 2m_K$ for the direct-detection constraints and rescale the accelerator constraints accordingly [67]. We also checked that this model is safe from the cosmic microwave background (CMB) and halo constraints.

Heavy quark model.—We want the near-threshold resonance to emerge directly from the theory for the model discussed in this section. We consider one light quark u and two heavy quarks c and b and assume the c and b abundances are fixed by their asymmetries, $n_c = n_{\bar{b}}$. There are many ways to populate asymmetric DM (see, e.g., [68–70] and references therein) which will work for this GeV scale DM [71]. So, we remain agnostic about the origin of the asymmetry. We also assume the heavy quarks have a common mass, m_Q, and refer to either heavy quark as Q. This assumption is unnecessary for successful phenomenology, and is made only for the simplicity of discussions. The resonance is $D^0(c\bar{u})B^+(u\bar{b}) \to \Upsilon(c\bar{b})(nS)$ for some excited level n, and $m_D = m_B$ is the DM mass for these heavy-quark mesons. Despite being motivated by the presence of the heavy quarks, this resonance requires some level of accident which we proceed to estimate. The relic abundance of the DM particles, D^0 and B^+, are set by the asymmetry of n_c and $n_{\bar{b}}$.

We introduce a massive dark photon γ' corresponding to a broken $U(1)'$ dark gauge group which the lightest pseudoscalar dark meson, $\pi(\bar{u}u)$, decays through Ref. [73]. We assume a similar coupling as the SM π^0 to two photons and that the decay proceeds through a heavy-fermion loop. Note that γ' here is different from the dark photon A_D introduced in the section about the light quark model since γ' decays entirely to visible SM particles. We assume a kinetic mixing between $U(1)'$ and $U(1)_{EM}$ of the form $\mathcal{L} \supset 1/2 \cdot \epsilon F^{\mu\nu} F'_{\mu\nu}$.

Heavy-light meson and quarkonium spectrum: Following the discussion of Refs. [78–80], interactions of heavy quarks can be described by the nonrelativistic Schrödinger equation. The $c\bar{b}$ bound states have the logarithmic potential $V(r) = C \ln(r/r_0)$, where C is a parameter that can be calculated in lattice QCD and r_0 is the distance at which the log potential is equal to the threshold necessary for $\Upsilon(c\bar{b})$ to decay into $D^0(c\bar{u})+$ $B^+(u\bar{b})$. The level spacing of these quarkonium excited states is independent of m_Q [see Eq. (8) of Ref. [78]]:

$$m_{\Upsilon(nS)} - m_{\Upsilon(1S)} \approx C \ln\left(\frac{4n}{3}\right) \tag{10}$$

in the large n limit. The mass splitting is

$$\Delta_n \equiv m_{\Upsilon(nS)} - m_{\Upsilon[(n-1)S]} = C\left[\frac{1}{n} + \mathcal{O}\left(\frac{1}{n^2}\right)\right]. \tag{11}$$

The summed mass of the mesons with one heavy quark is [see also Eq. (6) of Ref. [78]]

$$m_D + m_B - m_{\Upsilon(1S)} = A + \frac{1}{2} C \ln\left(\frac{m_Q}{\Lambda}\right), \tag{12}$$

assuming $m_Q \gg \Lambda$, where Λ is the dark confinement scale [81]. The intersection of the summed scalar meson masses (black) with the different heavy quarkonium excited states (purple) is where resonance occurs as shown in Fig. 3.

The tuning to be on resonance can be reduced to $\Delta \times (m_Q/\Delta_n)$, where Δ is at the level of $10^{-7.8}$ [20]. In the large n limit, assuming the dimensionful parameters $A \sim C \sim \Lambda$ for simplicity, the m_Q which allows the sum of the pseudoscalar mesons' masses to fall between the $n-1$ and n levels is $m_Q \approx n^2 (4/3e)^2 \Lambda$ [by solving Eqs. (10)–(12)]; e is the exponential. The requisite level of accident (F.T.) to achieve the desired resonant self-interaction,

$$\text{F.T.} \equiv \Delta \times \frac{m_Q}{\Delta_n} \approx \Delta \times \left(\frac{4}{3e}\right)^2 n^3, \tag{13}$$

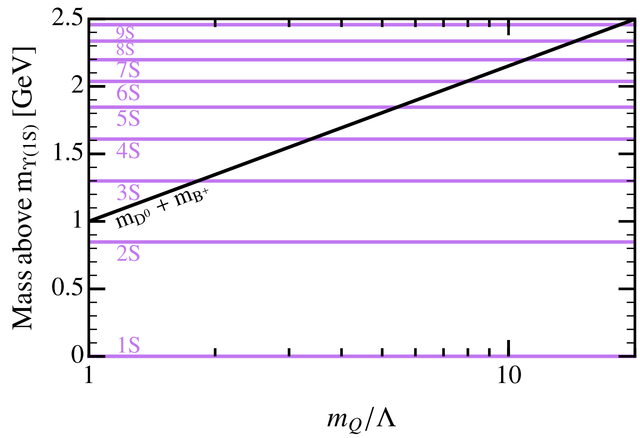

FIG. 3. The crossings of the sum of heavy quark pseudoscalar meson masses and heavy quarkonium excited states for different heavy quark masses, m_Q.

can then be reduced (getting closer to an order one number). When $n > 10$, the level of accident is reduced by as much as 10^5.

Log potential region: When m_Q is significantly larger than Λ, the quark potential is Coulombic for small n. The quark potential only becomes logarithmic, as assumed above, for large enough n, which can be estimated as follows. The Bohr radius of the system is $a = 1/(\alpha_s m_Q)$, where α_s is the dark gauge fine structure constant. The energy levels are roughly $E_n \sim (\alpha_s m_Q/n^2)$ in the Coulombic region, so $(\alpha_s m_Q/n^2) > \Lambda$ corresponds to the log potential region. Thus, for $m_Q \gtrsim 10\Lambda$ (assuming $\alpha_s \sim 1$), n needs to be larger than at least 4 for the quark system to have a logarithmic potential. This is consistent with our analysis above.

Experimental signature: We assume the dark π and the dark photon have the same couplings as their SM counterparts so that the former decays to the latter quickly after confinement. The dark photon must further decay to the SM to successfully transfer the excess, symmetric entropy from the dark sector prior to SM neutrino decoupling.

As mentioned previously, we assume $n_c = n_{\bar{b}}$ for simplicity. Now, let us further assume that $n_c + n_{\bar{b}} = n_{B,\text{SM}}$, where the latter is the asymmetric SM baryon number density. This could easily occur in a full model which includes a mechanism for all three asymmetries to be generated simultaneously. Requiring the asymmetric heavy-light mesons to reproduce the observed DM relic abundance yields $m_{\text{DM}} = m_p(\Omega_{\text{DM}} h^2/\Omega_{B,\text{SM}} h^2)$, where m_p is the proton mass. The DM mass $m_{\text{DM}} = m_D = m_B \sim m_Q$ in the heavy-quark limit. With n_Q and Eq. (13), we can write the required dark confinement scale as $\Lambda \approx m_Q(3e\Delta/4\ \text{F.T.})^{2/3} \sim m_p(\Omega_{\text{DM}}/\Omega_{B,\text{SM}})(3e\Delta/4\ \text{F.T.})^{2/3}$.

To enable the dark π to decay to a pair of dark photons, we require $2m_{\gamma'} \leq m_\pi \approx \Lambda$. Thus, the upper bound on the dark photon mass is set by the level of accident we permit

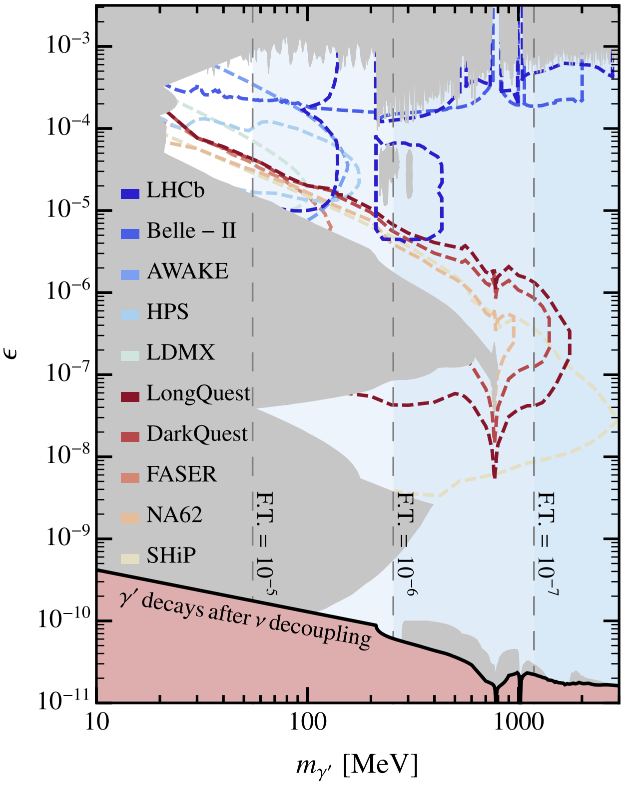

FIG. 4. The parameter space in the heavy quark model in which the dark photon decays quickly enough to transfer the dark π's entropy before neutrino decoupling. Existing constraints [82–91] are dark gray while projected sensitivities of future experiments are shown as dashed lines [83,91–104]. The dashed vertical lines show different levels of accident (F.T.) required to achieve the necessary resonant self interaction, as defined in Eq. (13).

associated to Λ. In Fig. 4, we show the dark photon parameter space in which the dark pions decay to dark photons which in turn decay to SM particles fast enough. We also show the relevant current and future experimental probes.

Based on this specific dark-photon setup, the reduction of the level of an accident is at best $\sim 10^3$, not the value of 10^5 discussed below Eq. (13). However, one can consider other similar models to achieve a better reduction. For example, the dark pion can decay to completely secluded dark-sector particles (thus allowing a smaller Λ), and a better reduction of accident can be achieved. The secluded scenarios could, for example, affect the effective number of relativistic species and produce interesting signatures in cosmological observations (including BBN and CMB measurements), which are beyond the scope of this paper.

SIMP & ELDER DM as resonant SIDM.—Another natural place to expect resonances is in dark sectors with confining gauge groups. Two classes of such dark sectors that have their own strong motivations are strongly interacting massive particles (SIMPs) [105] and elastically decoupling relics (ELDERs) [106]. It is possible that the dark vector resonance we require to achieve the desired self-interacting dark matter (SIDM) behavior is already realized in SIMP or ELDER scenarios. For concreteness, we consider one of the simplest SIMP realizations where the $3 \to 2$ process is realized by a Wess-Zumino-Witten term in a dark chiral Lagrangian where the dark pions compose DM [23]. Motivated by specific realizations [48], we further restrict our consideration to an Sp(4) gauge group with $N_f = 2$ fermions in the fundamental, so that the flavor symmetry is $SU(4)/Sp(4)$ and there are five equal-mass pions comprising DM.

For this gauge and flavor structure, there exist lattice results for the corresponding spectra and decay constants after confinement in the dark sector [107,108]. In particular, there exists a single point at which the lightest pseudoscalar mass, i.e., the dark pion, is exactly half the mass of the lightest vector resonance. At this point, the ratio of the pseudoscalar mass to its decay constant is $m_\pi/f_\pi = 1.9$ [109].

At first glance for this ratio, we find that the SIMP mechanism does not quite work as the necessary m_K [23] causes the DM self-interaction to be too large and excluded by the Bullet Cluster bound [14–16]. However, to see whether this parameter set simultaneously explains both the abundance and the self-interaction cross section requires detailed modeling of pion scattering, including the vector meson exchanges, which is beyond the scope of this Letter and will be discussed elsewhere. Given a variety of QCD-like gauge theories, we believe a significant fraction of them lead to the correct phenomenology.

We have presented three new models to realize the resonant self-interacting dark matter, using pseudoscalar and vector meson states arising from a dark QCD. These models can motivate future small-scale studies, and lead to new experimental searches and lattice QCD studies in the dark sector.

We thank Asher Berlin, Xiaoyong Chu, Camilo Garcia-Cely, Manoj Kaplinghat, Gordan Krnjaic, Chris Quigg, Yuhsin Tsai, and Sean Tulin for useful discussions. The work of R. M. was supported by NSF Grant No. PHY-1915314 and the U.S. DOE Contract No. DE-AC02-05CH11231. The work of H. M. was supported by the NSF Grant No. PHY-1915314, by the U.S. DOE Contract No. DE-AC02-05CH11231, by the JSPS Grant-in-Aid for Scientific Research No. JP17K05409, MEXT Grants-in-Aid for Scientific Research on Innovative Areas No. JP15H05887 and No. JP15K21733, by WPI, MEXT, Japan, and Hamamatsu Photonics, K. K. Part of this document was prepared by Y.-D. T. using the resources of the Fermi National Accelerator Laboratory (Fermilab), a U.S. Department of Energy, Office of Science, HEP User Facility. Fermilab is managed by Fermi Research Alliance, LLC (FRA), acting under Contract No. DE-AC02-07CH11359. Part of this work was performed by Y.-D. T. at the Aspen Center for Physics, which is supported by the National Science Foundation Grant No. PHY-1607611.

The work of Y.-D. T. is supported in part by U.S. National Science Foundation Grant No. PHY-1915005. This research was supported in part by the National Science Foundation under Grant No. NSF PHY-1748958. Y.-D. T. is grateful for the hospitality of Kavli Institute for Theoretical Physics, University of California, Santa Barbara, and the Neutrinos as a Portal to New Physics and Astrophysics program.

*yt444@cornell.edu
†robertmcgehee@berkeley.edu
‡hitoshi@berkeley.edu; hitoshi.murayama@ipmu.jp

[1] B. Moore, Evidence against dissipationless dark matter from observations of galaxy haloes, Nature (London) **370**, 629 (1994).
[2] J. Dubinski and R. Carlberg, The structure of cold dark matter halos, Astrophys. J. **378**, 496 (1991).
[3] J. F. Navarro, C. S. Frenk, and S. D. White, The structure of cold dark matter halos, Astrophys. J. **462**, 563 (1996).
[4] R. A. Flores and J. R. Primack, Observational and theoretical constraints on singular dark matter halos, Astrophys. J. **427**, L1 (1994).
[5] G. Gentile, P. Salucci, U. Klein, D. Vergani, and P. Kalberla, The cored distribution of dark matter in spiral galaxies, Mon. Not. R. Astron. Soc. **351**, 903 (2004).
[6] P. Salucci, The distribution of dark matter in galaxies, Astron. Astrophys. Rev. **27**, 2 (2019).
[7] M. Boylan-Kolchin, J. S. Bullock, and M. Kaplinghat, Too big to fail? The puzzling darkness of massive Milky Way subhaloes, Mon. Not. R. Astron. Soc. **415**, L40 (2011).
[8] K. A. Oman, J. F. Navarro, A. Fattahi, C. S. Frenk, T. Sawala, S. D. M. White, R. Bower, R. A. Crain, M. Furlong, M. Schaller *et al.*, The unexpected diversity of dwarf galaxy rotation curves, Mon. Not. R. Astron. Soc. **452**, 3650 (2015).
[9] S. Tulin and H.-B. Yu, Dark matter self-interactions and small scale structure, Phys. Rep. **730**, 1 (2018).
[10] J. F. Navarro, V. R. Eke, and C. S. Frenk, The cores of dwarf galaxy haloes, Mon. Not. R. Astron. Soc. **283**, L72 (1996).
[11] S. Gelato and J. Sommer-Larsen, On DDO 154 and cold dark matter halo profiles, Mon. Not. R. Astron. Soc. **303**, 321 (1999).
[12] J. Binney, O. Gerhard, and J. Silk, The dark matter problem in disc galaxies, Mon. Not. R. Astron. Soc. **321**, 471 (2001).
[13] O. Y. Gnedin and H. Zhao, Maximum feedback and dark matter profiles of dwarf galaxies, Mon. Not. R. Astron. Soc. **333**, 299 (2002).
[14] D. Clowe, A. Gonzalez, and M. Markevitch, Weak lensing mass reconstruction of the interacting cluster 1E0657-558: Direct evidence for the existence of dark matter, Astrophys. J. **604**, 596 (2004).
[15] M. Markevitch, A. Gonzalez, D. Clowe, A. Vikhlinin, L. David, W. Forman, C. Jones, S. Murray, and W. Tucker, Direct constraints on the dark matter self-interaction cross-section from the merging galaxy cluster 1E0657-56, Astrophys. J. **606**, 819 (2004).
[16] S. W. Randall, M. Markevitch, D. Clowe, A. H. Gonzalez, and M. Bradac, Constraints on the self-interaction cross-section of dark matter from numerical simulations of the merging galaxy cluster 1E 0657-56, Astrophys. J. **679**, 1173 (2008).
[17] M. Rocha, A. H. Peter, J. S. Bullock, M. Kaplinghat, S. Garrison-Kimmel, J. Onorbe, and L. A. Moustakas, Cosmological simulations with self-interacting dark matter I: Constant density cores and substructure, Mon. Not. R. Astron. Soc. **430**, 81 (2013).
[18] A. H. Peter, M. Rocha, J. S. Bullock, and M. Kaplinghat, Cosmological simulations with self-interacting dark matter II: Halo shapes vs. observations, Mon. Not. R. Astron. Soc. **430**, 105 (2013).
[19] M. Kaplinghat, S. Tulin, and H.-B. Yu, Dark Matter Halos as Particle Colliders: Unified Solution to Small-Scale Structure Puzzles from Dwarfs to Clusters, Phys. Rev. Lett. **116**, 041302 (2016).
[20] X. Chu, C. Garcia-Cely, and H. Murayama, Velocity Dependence from Resonant Self-Interacting Dark Matter, Phys. Rev. Lett. **122**, 071103 (2019).
[21] J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle* (Oxford University Press, Oxford, 1988).
[22] *Proceedings of the 63rd International Astronomical Union (IAU) Symposium, Copernicus Symposium II, held in Cracow, Poland, September 10-12, 1973*, edited by Malcolm S. Longair (Reidel, Dordrecht, 1974).
[23] Y. Hochberg, E. Kuflik, H. Murayama, T. Volansky, and J. G. Wacker, Model for Thermal Relic Dark Matter of Strongly Interacting Massive Particles, Phys. Rev. Lett. **115**, 021301 (2015).
[24] S. Alexander, E. McDonough, and D. N. Spergel, Strongly-interacting ultralight millicharged particles, Phys. Lett. B **822**, 136653 (2021).
[25] Even though we call the DM mesons kaons, they are the lightest $SU(3)_D$ states. This name was chosen since $m_s \gg m_u$ and we are motivated by the near-threshold resonance in the SM process $K^+K^- \to \phi$. Many other models have considered composite DM in a confined dark-sector gauge theory, e.g., [26].
[26] O. Antipin, M. Redi, A. Strumia, and E. Vigiani, Accidental composite dark matter, J. High Energy Phys. 07 (2015) 039.
[27] A. Pich, Chiral perturbation theory, Rep. Prog. Phys. **58**, 563 (1995).
[28] S. Scherer, Introduction to chiral perturbation theory, Adv. Nucl. Phys. **27**, 277 (2003).
[29] B. Kubis, An Introduction to chiral perturbation theory, in *Workshop on Physics and Astrophysics of Hadrons and Hadronic Matter* (Visva Bharati University, Shantiniketan, 2007).
[30] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **98**, 030001 (2018).
[31] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.172001 for details of dark mason models and astrophysical/cosmological constraints.
[32] R. Essig, E. Kuflik, S. D. McDermott, T. Volansky, and K. M. Zurek, Constraining light dark matter with diffuse

X-ray and gamma-ray observations, J. High Energy Phys. 11 (2013) 193.
[33] H. M. Lee and M.-S. Seo, Models for SIMP dark matter and dark photon, AIP Conf. Proc. **1743**, 060003 (2016).
[34] N. Bernal and X. Chu, $\mathbb{Z}_2$ SIMP dark matter, J. Cosmol. Astropart. Phys. 01 (2016) 006.
[35] Y. Hochberg, E. Kuflik, and H. Murayama, SIMP spectroscopy, J. High Energy Phys. 05 (2016) 090.
[36] H. M. Lee and M.-S. Seo, Communication with SIMP dark mesons via Z' -portal, Phys. Lett. B **748**, 316 (2015).
[37] S.-M. Choi and H. M. Lee, Resonant SIMP dark matter, Phys. Lett. B **758**, 47 (2016).
[38] N. Bernal, X. Chu, and J. Pradler, Simply split strongly interacting massive particles, Phys. Rev. D **95**, 115023 (2017).
[39] S.-M. Choi, H. M. Lee, and M.-S. Seo, Cosmic abundances of SIMP dark matter, J. High Energy Phys. 04 (2017) 154.
[40] Y. Hochberg, E. Kuflik, and H. Murayama, Dark spectroscopy at lepton colliders, Phys. Rev. D **97**, 055030 (2018).
[41] E. Kuflik, M. Perelstein, N. R.-L. Lorier, and Y.-D. Tsai, Phenomenology of ELDER dark matter, J. High Energy Phys. 08 (2017) 078.
[42] S.-M. Choi, Y. Hochberg, E. Kuflik, H. M. Lee, Y. Mambrini, H. Murayama, and M. Pierre, Vector SIMP dark matter, J. High Energy Phys. 10 (2017) 162.
[43] A. Berlin, N. Blinov, S. Gori, P. Schuster, and N. Toro, Cosmology and accelerator tests of strongly interacting dark matter, Phys. Rev. D **97**, 055033 (2018).
[44] Y.-D. Tsai, Elastically decoupling relics, multi-messenger searches for dark matter, and ν probes of new physics, Ph.D. thesis, Cornell University, 2018.
[45] R. K. Leane, T. R. Slatyer, J. F. Beacom, and K. C. Ng, GeV-scale thermal WIMPs: Not even slightly ruled out, Phys. Rev. D **98**, 023016 (2018).
[46] S.-M. Choi, H. M. Lee, P. Ko, and A. Natale, Resolving phenomenological problems with strongly-interacting-massive-particle models with dark vector resonances, Phys. Rev. D **98**, 015034 (2018).
[47] Y. Hochberg, E. Kuflik, and H. Murayama, Twin Higgs model with strongly interacting massive particle dark matter, Phys. Rev. D **99**, 015005 (2019).
[48] Y. Hochberg, E. Kuflik, R. McGehee, H. Murayama, and K. Schutz, Strongly interacting massive particles through the axion portal, Phys. Rev. D **98**, 115031 (2018).
[49] R. Laha, J. B. Muñoz, and T. R. Slatyer, INTEGRAL constraints on primordial black holes and particle dark matter, Phys. Rev. D **101**, 123514 (2020).
[50] Even though the large-N_c limit cannot be trusted, we sketch how these regions I and II come closer for $N_c = 2$ in the Supplemental Material.
[51] J. L. Feng and J. Smolinsky, Impact of a resonance on thermal targets for invisible dark photon searches, Phys. Rev. D **96**, 095022 (2017).
[52] E. Izaguirre, G. Krnjaic, P. Schuster, and N. Toro, Analyzing the Discovery Potential for Light Dark Matter, Phys. Rev. Lett. **115**, 251301 (2015).
[53] M. Ibe, H. Murayama, and T. T. Yanagida, Breit-Wigner enhancement of dark matter annihilation, Phys. Rev. D **79**, 095009 (2009).
[54] Patrick J. Fitzpatrick, Hongwan Liu, Tracy R. Slatyer, and Yu-Dai Tsai, New Pathways to the Relic Abundance of Vector-Portal Dark Matter, arXiv:2011.01240.
[55] Patrick J. Fitzpatrick, Hongwan Liu, Tracy R. Slatyer, and Yu-Dai Tsai, New Thermal Relic Targets for Inelastic Vector-Portal Dark Matter, arXiv:2105.05255.
[56] E. Aprile *et al.* (XENON Collaboration), Search for Light Dark Matter Interactions Enhanced by the Migdal Effect or Bremsstrahlung in XENON1T, Phys. Rev. Lett. **123**, 241803 (2019).
[57] D. Baxter, Y. Kahn, and G. Krnjaic, Electron ionization via dark matter-electron scattering and the migdal effect, Phys. Rev. D **101**, 076014 (2020).
[58] E. Aprile *et al.* (XENON Collaboration), Light Dark Matter Search with Ionization Signals in XENON1T, Phys. Rev. Lett. **123**, 251801 (2019).
[59] L. Barak *et al.* (SENSEI Collaboration), SENSEI: Direct-Detection Results on sub-GeV Dark Matter from a New Skipper-CCD, Phys. Rev. Lett. **125**, 171802 (2020).
[60] D. Amaral *et al.* (SuperCDMS Collaboration), Constraints on low-mass, relic dark matter candidates from a surface-operated SuperCDMS single-charge sensitive detector, Phys. Rev. D **102**, 091101 (2020).
[61] A. Aguilar-Arevalo *et al.* (MiniBooNE DM Collaboration), Dark matter search in nucleon, pion, and electron channels from a proton beam dump with MiniBooNE, Phys. Rev. D **98**, 112004 (2018).
[62] D. Banerjee *et al.*, Dark Matter Search in Missing Energy Events with NA64, Phys. Rev. Lett. **123**, 121801 (2019).
[63] J. Lees *et al.* (*BABAR*), Search for Invisible Decays of a Dark Photon Produced in e^+e^- Collisions at *BABAR*, Phys. Rev. Lett. **119**, 131804 (2017).
[64] M. Fabbrichesi, E. Gabrielli, and G. Lanfranchi, The dark photon, 10.1007/978-3-030-62519-1.
[65] A. Berlin, P. deNiverville, A. Ritz, P. Schuster, and N. Toro, On sub-GeV dark matter production at fixed-target experiments, Phys. Rev. D **102**, 095011 (2020).
[66] G. Krnjaic and S. D. McDermott, Implications of BBN bounds for cosmic ray upscattered dark matter, Phys. Rev. D **101**, 123022 (2020).
[67] However, the accelerator constraints should be modified accordingly for different values of Δ_{A_D}. As discussed in [65], both the visible searches for DM and invisible searches for A_D would be modified in the resonance regime because A_D would have nonnegligible branching ratios to SM particles for small Δ_{A_D}. This would involve a reanalysis of experimental data beyond the scope of this work.
[68] H. Davoudiasl and R. N. Mohapatra, On relating the genesis of cosmic baryons and dark matter, New J. Phys. **14**, 095011 (2012).
[69] K. Petraki and R. R. Volkas, Review of asymmetric dark matter, Int. J. Mod. Phys. A **28**, 1330028 (2013).
[70] K. M. Zurek, Asymmetric dark matter: Theories, signatures, and constraints, Phys. Rep. **537**, 91 (2014).
[71] It is possible to generate both the DM and baryon asymmetries in models with a dark QCD, e.g., [72].
[72] E. Hall, T. Konstandin, R. McGehee, and H. Murayama, Asymmetric matters from a dark first-order phase transition, arXiv:1911.12342.

[73] Massive dark photons can efficiently transfer entropy between the standard model and dark sectors before neutrino decoupling and are especially useful when there are many degrees of freedom from a dark QCD (e.g., [72,74–77]).

[74] K. Harigaya, R. McGehee, H. Murayama, and K. Schutz, A predictive mirror twin Higgs with small Z_2 breaking, J. High Energy Phys. 05 (2020) 155.

[75] S. Koren and R. McGehee, Freezing-in twin dark matter, Phys. Rev. D **101**, 055024 (2020).

[76] I. Garcia Garcia, R. Lasenby, and J. March-Russell, Twin Higgs Asymmetric Dark Matter, Phys. Rev. Lett. **115**, 121801 (2015).

[77] I. Garcia Garcia, R. Lasenby, and J. March-Russell, Twin Higgs WIMP dark matter, Phys. Rev. D **92**, 055034 (2015).

[78] C. Quigg and J. L. Rosner, Counting narrow levels of quarkonium, Phys. Lett. **72B**, 462 (1978).

[79] C. Quigg and J. L. Rosner, Quarkonium level spacings, Phys. Lett. **71B**, 153 (1977).

[80] C. Quigg, Realizing the potential of quarkonium, AIP Conf. Proc. **424**, 173 (1998).

[81] If $m_c \neq m_b$, the expression in Eq. (12) would be more complicated, but the behavior in Fig. 3 would remain.

[82] A. Fradette, M. Pospelov, J. Pradler, and A. Ritz, Cosmological constraints on very dark photons, Phys. Rev. D **90**, 035022 (2014).

[83] J. Alexander *et al.*, Dark sectors 2016 workshop: Community report, arXiv:1608.08632.

[84] J. H. Chang, R. Essig, and S. D. McDermott, Revisiting supernova 1987A constraints on dark photons, J. High Energy Phys. 01 (2017) 107.

[85] E. Hardy and R. Lasenby, Stellar cooling bounds on new light particles: plasma mixing effects, J. High Energy Phys. 02 (2017) 033.

[86] M. Pospelov and Y.-D. Tsai, Light scalars and dark photons in Borexino and LSND experiments, Phys. Lett. B **785**, 288 (2018).

[87] D. Banerjee *et al.* (NA64 Collaboration), Search for a Hypothetical 16.7 MeV Gauge Boson and Dark Photons in the NA64 Experiment at CERN, Phys. Rev. Lett. **120**, 231802 (2018).

[88] R. Aaij *et al.* (LHCb Collaboration), Search for Dark Photons Produced in 13 TeV pp Collisions, Phys. Rev. Lett. **120**, 061801 (2018).

[89] R. Aaij *et al.* (LHCb Collaboration), Search for $A' \to \mu^+\mu^-$ Decays, Phys. Rev. Lett. **124**, 041801 (2020).

[90] R. H. Parker, C. Yu, W. Zhong, B. Estey, and H. Müller, Measurement of the fine-structure constant as a test of the Standard Model, Science **360**, 191 (2018).

[91] Y.-D. Tsai, P. deNiverville, and M. X. Liu, The High-Energy Frontier of the Intensity Frontier: Closing the Dark Photon, Inelastic Dark Matter, and Muon g-2 Windows, Phys. Rev. Lett. **126**, 181801 (2021).

[92] A. Celentano (HPS Collaboration), The heavy photon search experiment at jefferson laboratory, J. Phys. Conf. Ser. **556**, 012064 (2014).

[93] I. Jaegle (Belle Collaboration), Search for the Dark Photon and the Dark Higgs Boson at Belle, Phys. Rev. Lett. **114**, 211801 (2015).

[94] P. Ilten, J. Thaler, M. Williams, and W. Xue, Dark photons from charm mesons at LHCb, Phys. Rev. D **92**, 115017 (2015).

[95] S. Alekhin *et al.*, A facility to search for hidden particles at the CERN SPS: The SHiP physics case, Rep. Prog. Phys. **79**, 124201 (2016).

[96] P. Ilten, Y. Soreq, J. Thaler, M. Williams, and W. Xue, Proposed Inclusive Dark Photon Search at LHCb, Phys. Rev. Lett. **116**, 251803 (2016).

[97] A. Caldwell *et al.*, Particle physics applications of the AWAKE acceleration scheme, arXiv:1812.11164.

[98] A. Berlin, S. Gori, P. Schuster, and N. Toro, Dark sectors at the Fermilab SeaQuest experiment, Phys. Rev. D **98**, 035011 (2018).

[99] A. Berlin, N. Blinov, G. Krnjaic, P. Schuster, and N. Toro, Dark Matter, Millicharges, Axion and scalar particles, gauge bosons, and other new physics with LDMX, Phys. Rev. D **99**, 075001 (2019).

[100] A. Ariga *et al.* (FASER Collaboration), FASER's physics reach for long-lived particles, Phys. Rev. D **99**, 095011 (2019).

[101] NA62 Collaboration, 2018 NA62 status report to the CERN SPSC, Technical Reports No. CERN-SPSC-2018-010, SPSC-SR-229, CERN, Geneva, 2018.

[102] A. Apyan *et al.*, DarkQuest: A dark sector upgrade to SpinQuest at the 120 GeV Fermilab Main Injector, arXiv:2203.08322.

[103] J. L. Feng *et al.*, The Forward Physics Facility at the High-Luminosity LHC, arXiv:2203.05090.

[104] L. A. Anchordoqui *et al.*, The Forward Physics Facility: Sites, Experiments, and Physics Potential, arXiv:2109.10905.

[105] Y. Hochberg, E. Kuflik, T. Volansky, and J. G. Wacker, Mechanism for Thermal Relic Dark Matter of Strongly Interacting Massive Particles, Phys. Rev. Lett. **113**, 171301 (2014).

[106] E. Kuflik, M. Perelstein, N. R.-L. Lorier, and Y.-D. Tsai, Elastically Decoupling Dark Matter, Phys. Rev. Lett. **116**, 221302 (2016).

[107] E. Bennett, D. K. Hong, J.-W. Lee, C.-J. D. Lin, B. Lucini, M. Piai, and D. Vadacchino, Sp(4) gauge theories on the lattice: $N_f = 2$ dynamical fundamental fermions, J. High Energy Phys. 12 (2019) 053.

[108] E. Bennett, D. K. Hong, J.-W. Lee, C.-J. D. Lin, B. Lucini, M. Mesiti, M. Piai, J. Rantaharju, and D. Vadacchino, $Sp(4)$ gauge theories on the lattice: quenched fundamental and antisymmetric fermions, Phys. Rev. D **101**, 074516 (2020).

[109] Our convention for f_K matches that of Ref. [23], which is a factor of 2 larger than the convention used in Refs. [107,108].