Possible mixing of a diquark-antidiquark with a $p\bar{p}$ hadronic molecule

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We discuss the possibility that the two nearby resonances observed by BESIII partially below the $p\bar{p}$ threshold might be due to mixing between two metastable states with the same $J^{PC} = 0^{-+}$ quantum numbers, but rather different internal structure. One is a $p\bar{p}$ hadronic molecule and the other a bound state of a light-quark diquark and an antidiquark, both with spin 1 and isospin 0, a composite color antitriplet and triplet, respectively. The doubling of resonances, one of which may be interpreted as a hadronic molecule, while the other arises from $q\bar{q}$ annihilation in a state with vacuum quantum numbers, may be a more general feature than the specific case considered here.

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I. INTRODUCTION

Reference [1] reports an anomalous line shape of the $X(1840)$ resonance observed in the invariant mass recoiling against the photon in the decay $J/\psi \to \gamma 3(\pi^+\pi^-)$. A significant distortion at 1.84 GeV in the line-shape of the $3(\pi^+\pi^-)$ invariant mass spectrum is observed for the first time, which could be resolved by two overlapping resonant structures, $X(1840)$ and $X(1880)$. The new state $X(1880)$ is observed with a statistical significance larger than 10σ. The mass and width of $X(1880)$ are determined to be $1882.1 \pm 1.7 \pm 0.7$ MeV and $30.7 \pm 5.5 \pm 2.4$ MeV, respectively, while the mass and width of the $X(1840)$ are found to be $1842.2^{+7.1}_{-2.6}$ MeV and $83 \pm 14 \pm 11$ MeV, respectively. The region of ± 15 MeV around the central value of 1882 MeV extends down to 1867 MeV, i.e., about 10 MeV below the $p\bar{p}$ threshold at 1877 MeV. This might indicate the existence of a $p\bar{p}$ bound state.

The history of $p\bar{p}$ bound states goes back at least as far as the Fermi-Yang interpretation of the pion [2]. Attempts were made (superseded by the quark model and quantum chromodynamics) to describe the resonance spectrum [3] using hadron-antihadron interactions. Our more modest aim is to explain the two overlapping resonances, $X(1840)$ and $X(1880)$, in terms of dominant components of their wave functions.

Such doubling may have a broader context, applying to general cases of two-hadron molecules such as those discussed in Ref. [4]. In addition to the states in $3(\pi^+\pi^-)$ there is a broad state $X(1835)$ in the $\eta'\pi^+\pi^-$ channel [5] with mass $M(X) = 1826.5^{+15.0}_{-3.4}$ MeV, $\Gamma(X) = 242^{+14}_{-15}$ MeV, whose participation in the two-state interference we do not consider. We also restrict our attention to S-wave proton-antiproton states, ignoring states with relative angular momentum $\ell > 0$.

We first discuss behavior of wave function components under discrete symmetries (Sec. II). We then (Sec. III) interpret the observed spectrum in terms of two states: a proton-antiproton molecule and a diquark-antidiquark bound state, whose masses we calculate in Sec. IV. Other examples of similar two-state systems are noted in Sec. V, while we conclude in Sec. VI.

After the first version of this report had been issued, a parallel set of interpretations appeared in Ref. [6].

II. QUANTUM NUMBERS

We begin by identifying the likely J^{PC} quantum numbers of the two resonances. Both J/ψ and the photon have charge-conjugation eigenvalue *C-parity* $C = -1$. Therefore both $X(1840)$ and $X(1880)$ must have $C = +1$. The J^{PC} of $X(1840)$ is well-established to be 0^{-+} [5]. If one interprets $X(1880)$ as a $p\bar{p}$ resonance, its charge parity is given by

$$C_{p\bar{p}} = (-1)^{L+S} \tag{1}$$

where L and S are the orbital and spin angular momentum of the $p\bar{p}$ system, respectively. $X(1880)$ is very close to the $p\bar{p}$ threshold, so L must be zero, because orbital excitations cost

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significant energy. We have $L + S = S =$ even. For $p\bar{p}$ one can have $S = 0$ or $S = 1$. It follows that $S = 0$ and therefore $J = 0$. The parity of a $p\bar{p}$ system is $(-1)^{L+1} = -1$. So finally

$$J^{PC}[X(1840)] = J^{PC}[X(1880)] = 0^{-+}. \qquad (2)$$

III. TWO-STATE INTERPRETATION

If one interprets either $X(1840)$ or $X(1880)$ as a metastable $p\bar{p}$ bound state, a question arises immediately as to the physical nature of the other state. If one of the two states is a hadronic molecule, it is hard to come up with a plausible scenario in which the other state is also a hadronic molecule. With only light quarks and antiquarks in the game, it is hard to generate such a large splitting between below-threshold states. We are therefore led to search for some completely different configuration for the state which is not a hadronic molecule, yet has the same quantum numbers and a similar mass.

The transition to a final state near $\bar{p}p$ threshold is governed by the following regularity, illustrated in Fig. 1 [7]: If two hadrons have at least one $q\bar{q}$ pair in common, they should form at least one resonance via $q\bar{q}$ annihilation when the center-of-mass (c.o.m.) 3-momentum is less than some critical value p_0^*. For meson-meson scattering $p_0^{*MM} = 350$ MeV, while for meson-baryon scattering $p_0^{*MB} = 250$ MeV. Extrapolating to baryon-antibaryon scattering, one estimates $p_0^{*BB} = 200$ MeV [7]. Acceptable intermediate states include any below strong threshold.

Motivated by analogy to the system of the $\chi_{c1}(3872)$ meson [$X(3872)$], it has been noted (see, e.g., the discussion in Ref. [8]) that the unusual properties of this meson can be understood if one assumes that the physical $\chi_{c1}(3872)$ is a mixture of two very different objects which have the same $J^{PC} = 1^{++}$ quantum numbers and happen to be close in mass: a $\bar{D}D^*$ hadronic molecule and a P-wave

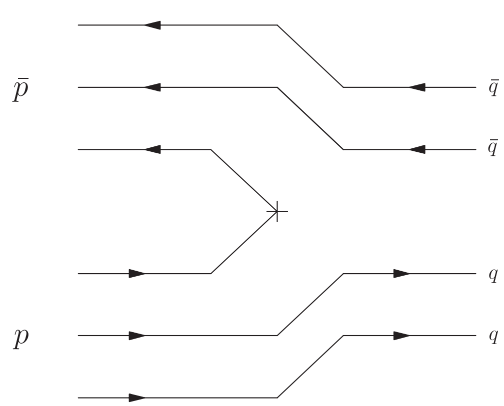

FIG. 1. Systems involving contribution of $^3P_0$ amplitude (the symbol +) to proton-antiproton annihilation into a diquark-antidiquark resonance.

charmonium, also known as $\chi_{c1}(2P)$. (One estimate of the $\chi_{c1}(2P)$ mass is 3920.5 MeV [9].)

We now argue that in the $X(1840)$-$X(1880)$ system the analogs are a $p\bar{p}$ hadronic molecule and a P-wave excitation of a narrow diquark-antidiquark system. The possibility of narrow high-mass exotic states has been raised in [10–12]. There it was pointed out that narrow high-mass states can arise despite large phase space when two nearly degenerate states are coupled to the same dominant decay mode. A necessary condition for such a proposal is that the P-wave diquark-antidiquark system must have the same quantum numbers and a mass close to the $p\bar{p}$ threshold. Conservation of angular momentum demands that the quark-antiquark pair which annihilate to leave the diquark-antidiquark system must have the quantum numbers $S = 1$, $L = 1$, and $J = 0$, or $^3P_0$ in a flavor singlet. The role of *production* of a flavor-singlet $^3P_0$ pair in quarkonium decay has been recognized for many years [13–15]. In the following we check the quantum numbers and estimate the mass of the diquark-antidiquark ground state using the standard toolbox of the nonrelativistic quark model.

Let us start from the quantum numbers. We have two spin-1 diquarks, which are each other's antiparticles: ud (or uu) $\bar{u}\,\bar{d}$ (or $\bar{u}\,\bar{u}$), obviously with isospin $= 1$, also known as "bad diquarks." Their color representations are $\bar{3}_c$ and 3_c, respectively. It is clear that they can be combined to a color and isospin singlet.

Next, we have to make sure that we can obtain a bound state with $J^{PC} = 0^{-+}$. The intrinsic parity of the two-diquark system is $+1$. To get a negative parity, we need to put them in a P-wave, i.e., $L = 1$. Then to get the total angular momentum $J = 0$, we combine the two vectors to $S = 1$, so that $\vec{J} = \vec{L} + \vec{S} = 0$. The consistency check is whether the charge parity comes out right. For two spin-1 bosons with $S = 1$ and $L = 1$ we indeed have $C = 1$, as illustrated by, e.g., $\eta(1270)$ with $J^{PC} = 0^{-+}$ which has a $\rho^+\rho^-$ decay mode [5].

IV. MASSES

The calculation of the mass of the diquark-antidiquark system is straightforward. The mass of each "bad" diquark is just the constituent mass of the two light quarks (we ignore isospin breaking), plus color-hyperfine repulsion in spin-1 state

$$M_{\text{diq},S=1} = 2m_q + a_{HF} = 2\times 363 + 50 = 776\ \text{MeV}. \qquad (3)$$

The mass of the diquark-antidiquark state is just the sum of the masses of the two diquarks, plus the cost of the P-wave excitation. The latter is estimated using a method previously employed in Ref. [16]. The residual energy difference ΔE_R after accounting for quark masses is found to be approximately linear in reduced mass μ_R of the two-body system

$$\Delta E_R = 417.37 - 0.2141\mu_R \tag{4}$$

The reduced mass of the diquark-antidiquark system is $\mu_R = (1/2)(776) = 388$ MeV, so $\Delta E_R = 334$ MeV and we estimate the mass of the diquark-antidiquark configuration with $J^{PC} = 0^{-+}$

$$M_{\mathrm{diq-diq}} = 2 \times 776 + 334 = 1886 \text{ MeV}. \tag{5}$$

In order for the molecular proton-antiproton state to be near the threshold of $2m_p = 2(938.27) = 1876.5$ MeV, we must identify it with the $X(1880)$, leaving the $X(1840)$ to be the state predicted in Eq. (5). This is probably acceptable, given the crude nature of the approximation (4). The opposite order, with $X(1840)$ identified as the $p\bar{p}$ molecule and $X(1880)$ taken as the diquark-antidiquark candidate, implies an unexpectedly large molecular binding energy.

V. OTHER SYSTEMS

The doubling of resonances, one of which may be interpreted as a hadronic molecule while the other arises from $q\bar{q}$ annihilation in a state with vacuum quantum numbers, i.e., $^3P_0$, may be a more general feature than the case considered here. One would then expect such resonances as $f_0(980)$, $\Lambda(1405)$, and $D_s(2317)$ (see Table I of Ref. [4]) to be accompanied by a partner at a nearby mass.

A. $f_0(980)$

The $f_0(980)$ is a spin-zero isospin-zero resonance with couplings to $\pi\pi$, $K\bar{K}$, $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $\eta\pi\pi$, nucleon-antinucleon, $\gamma\gamma$, and tetraquark configurations. These final states are sorted out in an extensive section of the Particle Data Group's Review of Particle Physics [17]. Analogy with the doubled-resonance picture would regard it as a combination of a $K\bar{K}$ molecule with small binding energy and a quark-antiquark $^3P_0$ state with a mixture of $^3P_0$ configurations $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$. The doubled-resonance structure of the $f_0(980)$ is consistent with the analysis of Ref. [18]. However, recent reviews of the status of the $f_0(980)$ region ([19,20]) need only one pole. The former summarizes: "…the $f_0(980)$ is close to the $\bar{K}^0K^0$ threshold (995 MeV) and the width is near 40 MeV. There is always only one pole in … the $\pi\pi \to KK$ coupled channels and it is most likely to be a KK molecule." The latter finds a $f_0(980)$ pole at $996 \pm 7 - i25^{+10}_{-6}$ MeV.

B. $\Lambda(1405)$

The $\Lambda(1405)$ is one of the first hadron resonances, making its appearance before 1960 [21]. It couples to the open channel $\pi\Sigma$ (threshold $\sim 137 + 1193 = 1330$ MeV) and the closed channel $\bar{K}N$ (threshold $\sim 495 + 939 = 1434$ MeV). It is represented by a two-pole structure [22–25], with pole 1 very close to the $\bar{K}N$ threshold and imaginary part corresponding to a rather narrow resonance. There is less agreement about the position of pole 2 but its real part is below 1400 MeV and its imaginary part corresponds to a broader resonance than pole 1.

Proceeding by analogy with the system $X(1840–1880)$, we would identify pole 1 (narrow, near threshold) as an analog of $X(1880)$, and pole 2 (broader, some distance below threshold) as an analog of $X(1840)$.

TABLE I. Predictions of $M(D_{s0}(^3P_0))$ ([27])(MeV).

2409	2484	2509	2380	2344	2409
[27]	[28]	[26]	[29]	[30]	[31]

C. $D_{s0}(2317)$

A relativistic quark model of charmed nonstrange and strange meson masses [26] predicts all S-wave and P-wave masses to a satisfactory extent with the exception of the spin-zero $D_{s0}(2317)$ and spin-1 $D_{s1}(2460)$. Its prediction of $M(D_{s0}(^3P_0))$ is compared with others in Table III of Ref. [27], summarized in Table 1.

Reference [26] finds the relativistic D_{s0} mass higher than any other calculation, and farther from the observed value of D_{s0}. We propose that $D_{s0}(2317)$ be accepted as the analog of $X(1840)$ while there should exist a distinct molecular state near D^0K^+ threshold ($1864.8 + 493.7 = 2359$ MeV) which would be the analog of $X(1880)$.

VI. CONCLUSIONS

Our treatment of the doubled resonance around 1840 and 1880 MeV has several features:

(a) The J^{PC} quantum numbers work out for both proton-antiproton (molecule, narrow, X(1880), near threshold) and diquark-antidiquark [broader, X(1840)].

(b) The masses work out almost by construction for proton-antiproton and rather nontrivially for diquark-antidiquark.

(c) We have identified a simple and nontrivial mechanism allowing the transition between the two configurations.

A consequence of our conjecture is that the spin-parity of the $3(\pi^+\pi^-)$ system must be determined to be 0^{-+} without interference from the broad $\eta'\pi^+\pi^-$ state or others around 1835 MeV.

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[1] M. Ablikim *et al.* (BESIII Collaboration), Observation of the anomalous shape of $X(1840)$ in $J/\psi \to \gamma 3(\pi^+\pi^-)$ indicating a second resonance near $p\bar{p}$ threshold, Phys. Rev. Lett. **132**, 151901 (2024).

[2] E. Fermi and C. N. Yang, Are mesons elementary particles?, Phys. Rev. **76**, 1739 (1949).

[3] O. D. Dalkarov, V. B. Mandelzweig, and I. S. Shapiro, On possible quasinuclear nature of heavy meson resonances, Nucl. Phys. **A21**, 88 (1970); O. D. Dalkarov, Comment on Chan Hong-Mos Talk: Quasinuclear States in Baryon-Antibaryon Systems, in *Proceedings of 19th Int. Conf. on High Energy Physics, Tokyo, 1978*, https://cds.cern.ch/record/870622/files/c78-08-23-p152.pdf.

[4] F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao, and B. S. Zou, Hadronic molecules, Rev. Mod. Phys. **90**, 015004 (2018); **94**, 029901(E) (2022).

[5] R. L. Workman *et al.* (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).

[6] B. Q. Ma, Protonium: Discovery and prediction, arXiv:2406.19180.

[7] J. L. Rosner, Compulsory resonance formation, Phys. Rev. D **6**, 2717 (1972).

[8] M. Karliner and J. L. Rosner, $X(3872)$, X_b, and the $\chi_{b1}(3P)$ state, Phys. Rev. D **91**, 014014 (2015).

[9] E. J. Eichten, K. Lane, and C. Quigg, New states above charm threshold, Phys. Rev. D **73**, 014014 (2006); **73**, 079903(E) (2006).

[10] M. Karliner and H. J. Lipkin, Possibility of narrow high-mass exotic states, arXiv:0710.4055.

[11] M. Karliner and H. J. Lipkin, Possibility of exotic states in the upsilon system, arXiv:0802.0649.

[12] M. Karliner and J. L. Rosner, Configuration mixing in strange tetraquarks Z_{cs}, Phys. Rev. D **104**, 034033 (2021).

[13] L. Micu, Decay rates of meson resonances in a quark model, Nucl. Phys. **B10**, 521 (1969).

[14] E. W. Colglazier and J. L. Rosner, Quark graphs and angular distributions in positive parity meson decays, Nucl. Phys. **B27**, 349 (1971).

[15] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Naive quark pair creation model of strong interaction vertices, Phys. Rev. D **8**, 2223 (1973).

[16] M. Karliner and J. L. Rosner, Scaling of *P*-wave excitation energies in heavy-quark systems, Phys. Rev. D **98**, 074026 (2018).

[17] T. Gutsche, C. Hanhart, R. E. Mitchell, and S. Spanier, Scalar Mesons below 1 GeV, Section 64 in [5].

[18] K. L. Au, M. R. Pennington, and D. Morgan, Experimental evidence for dynamics beyond the quark model: An extra $I = 0$ scalar meson near 1-GeV, Phys. Lett. **167B**, 229 (1986).

[19] D. L. Yao, L. Y. Dai, H. Q. Zheng, and Z. Y. Zhou, A review on partial-wave dynamics with chiral effective field theory and dispersion relation, Rep. Prog. Phys. **84**, 076201 (2021).

[20] R. Garcia-Martin, R. Kaminski, J. R. Pelaez, and J. Ruiz de Elvira, Precise determination of the $f_0(600)$ and $f_0(980)$ pole parameters from a dispersive data analysis, Phys. Rev. Lett. **107**, 072001 (2011).

[21] R. H. Dalitz and S. F. Tuan, A possible resonant state in pion-hyperon scattering, Phys. Rev. Lett. **2**, 425 (1959); The phenomenological description of *K*-nucleon reaction processes, Ann. Phys. (N.Y.) **10**, 307 (1960).

[22] J. X. Lu, L. S. Geng, M. Doering, and M. Mai, Cross-channel constraints on resonant antikaon-nucleon scattering, Phys. Rev. Lett. **130**, 071902 (2023).

[23] J. Bulova *et al.* (Baryon Scattering (BaSc) Collaboration), Two-pole nature of the $\Lambda(1405)$ resonance from lattice QCD, Phys. Rev. Lett. **132**, 051901 (2024).

[24] J. M. Xie, J. X. Lu, L. S. Geng, and B. S. Zou, Two-pole structures as a universal phenomenon dictated by coupled-channel chiral dynamics, Phys. Rev. D **108**, L111502 (2023).

[25] S. Navas *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **110**, 030001 (2024).

[26] D. Ebert, R. N. Faustov, and V. O. Galkin, Heavy-light meson spectroscopy and Regge trajectories in the relativistic quark model, Eur. Phys. J. C **66**, 197 (2010).

[27] R. H. Ni, Q. Li, and X. H. Zhong, Mass spectra and strong decays of charmed and charmed-strange mesons, Phys. Rev. D **105**, 056006 (2022).

[28] S. Godfrey and K. Moats, Properties of excited charm and charm-strange mesons, Phys. Rev. D **93**, 034035 (2016).

[29] J. Zeng, J. W. Van Orden, and W. Roberts, Heavy mesons in a relativistic model, Phys. Rev. D **52**, 5229 (1995).

[30] D. M. Li, P. F. Ji, and B. Ma, The newly observed open-charm states in quark model, Eur. Phys. J. C **71**, 1582 (2011).

[31] T. A. Lahde, C. J. Nyfalt, and D. O. Riska, Spectra and M1 decay widths of heavy light mesons, Nucl. Phys. **A674**, 141 (2000).