Kaluza-Klein graviton freeze-in and big bang nucleosynthesis

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In models featuring extra spatial dimensions, particle collisions in the early Universe can produce Kaluza-Klein gravitons. Such particles will later decay, potentially impacting the process of big bang nucleosynthesis. In this paper, we consider scenarios in which gravity is free to propagate throughout n flat, compactified extra dimensions, while the fields of the Standard Model are confined to a $(3+1)$-dimensional brane. We calculate the production and decay rates of the states that make up the Kaluza-Klein graviton tower and determine the evolution of their abundances in the early Universe. We then go on to evaluate the impact of these decays on the resulting light element abundances. We identify significant regions of previously unexplored parameter space that are inconsistent with measurements of the primordial helium and deuterium abundances. In particular, we find that for the case of one extra dimension (two extra dimensions), the fundamental scale of gravity must be $M_\star \gtrsim 2\times 10^{13}$ GeV ($M_\star \gtrsim 10^{10}$ GeV) unless the temperature of the early Universe was never greater than $T\sim 2$ TeV ($T\sim 1$ GeV). For larger values of n, these constraints are less stringent. For the case of $n=6$, for example, our analysis excludes all values of $M_\star$ less than $\sim 10^6$ GeV, unless the temperature of the Universe was never greater than $T\sim 3$ TeV. The results presented here severely limit the possibility that black holes were efficiently produced through particle collisions in the early Universe's thermal bath.

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I. INTRODUCTION

Measurements of the primordial light element abundances provide us with our earliest probe of cosmic history, allowing us to constrain the expansion rate and overall composition of our Universe as early as ~ 1 s after the big bang. In particular, this information indicates that the early Universe was radiation dominated and at least as hot as a few MeV [1–5]. Little is known, however, about the thermal history of our Universe prior to the onset of big bang nucleosynthesis.

At extremely high temperatures, particles in the thermal plasma can scatter to produce gravitational excitations, leading to the production of a stochastic gravitational wave background [6,7]. Such considerations allow us to constrain the maximum temperature of the very early Universe, $T_{\rm max} \lesssim M_{\rm Pl} \sim 10^{19}$ GeV [8,9]. If those gravitational waves were later diluted, such as through inflation, this constraint would instead apply to the temperature of subsequent reheating, $T_{\rm RH} \lesssim M_{\rm Pl} \sim 10^{19}$ GeV.

If our Universe has extra spatial dimensions, gravitational excitations can be be produced more efficiently and at lower temperatures. As a result, we can potentially place much more stringent constraints on the maximum temperature of the Universe at early times in such scenarios. In this study, we will focus on the model proposed by Arkani-Hamed, Dimopoulos, and Dvali (ADD) [10–12], which features n extra dimensions that are flat and compactified on a torus of radius, R. Unlike gravity, all of the Standard Model fields are confined to a three-dimensional brane, the volume of which constitutes the $(3+1)$-dimensional spacetime that we experience.

This class of models was originally proposed as a possible solution to the electroweak hierarchy problem [10]. In particular, the effective four-dimensional reduced Planck scale, $\overline{M}_{\rm Pl} = M_{\rm Pl}/\sqrt{8\pi} \approx 2.4\times 10^{18}$ GeV, is related to the fundamental $(n+4)$-dimensional Planck scale, $M_\star$, according to the following [10,12]:

$$\overline{M}_{\rm Pl}^2 = R^n M_\star^{2+n}. \tag{1}$$

Thus, for the appropriate values of R and n, the fundamental scale of gravity could be similar to electroweak scale,

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$M_\star \sim$ TeV. In such a scenario, the apparent hierarchy between the Planck scale and the electroweak scale would be a consequence of the Standard Model's localization on the $(3+1)$-dimensional brane.

Since the ADD model was proposed more than two decades ago, stringent constraints have been placed on this class of scenarios. In particular, data from the Large Hadron Collider require that $M_\star$ must be greater than several TeV [13,14]. Tests of the gravitational force law at submillimeter distances further constrain $R \lesssim 30$ μm [15–17], corresponding to $M_\star \gtrsim 5.4 \times 10^8$ GeV for $n=1$ and $M_\star > 3.6 \times 10^3$ GeV for $n=2$. The requirement that neutron stars are not overly heated by Kaluza-Klein graviton decays further requires $M_\star \gtrsim 1.7 \times 10^5$ GeV for $n=2$ and $M_\star \gtrsim 7.6 \times 10^4$ GeV for $n=3$ [18] (for further discussion, see the entry on extra dimensions in the Particle Data Group's Review of Particle Physics [19]).

In light of the constraints from the Large Hadron Collider, we will not attempt to motivate this study by appealing to the electroweak hierarchy problem, but rather by the broader possibility of extra spatial dimensions, such as within the context of string theory [11,20]. With this in mind, we will consider values of $M_\star$ that range from several TeV up to the Planck scale.

In this class of models, particle collisions in the early Universe can result in the efficient production of Kaluza-Klein (KK) gravitons. For gravitons lighter than $m_{\rm KK} \sim 10^5$ GeV, such states will decay during or after the era of big bang nucleosynthesis, producing energetic Standard Model particles that can break apart helium nuclei through the processes of photodissassociation or hadrodissociation. Such decays can reduce of the primordial helium abundance and increase the abundance of primordial deuterium. We explore the impact of these decays and use measurements of the light element abundances to place constraints on this class of scenarios.

The remainder of this paper is organized as follows. In Sec. II, we evaluate the evolution of the Kaluza-Klein graviton abundance in the early Universe, including their production via freeze-in and their subsequent decays. In Secs. III and IV, we discuss the impact of these decays on the primordial light element abundances and use this information to place constraints on this class of models. In Sec. V, we consider the impact of these constraints on the possibility that black holes could be efficiently produced through particle collisions in the early Universe. We summarize our main results in Sec. VI.

II. KALUZA-KLEIN GRAVITON FREEZE IN

In the ADD scenario, all of the Standard Model fields are restricted to propagate within the $(3+1)$-dimensional brane. In contrast, gravitons are free to propagate throughout the $(n+4)$-dimensional bulk. To observers on the brane, these massless spin-2 gravitons appear as massive Kaluza-Klein states. More specifically, for each level of the Kaluza-Klein graviton tower, there exists one spin-2 state, $\tilde{h}_m$, $(n-1)$ spin-1 states, and $n(n-1)/2$ spin-0 states, $\tilde{\phi}_m$, all with masses given by $m_{\tilde{h}_m} = m_{\tilde{\phi}_m} = m/R$, where m is the level of the Kaluza-Klein tower. The spin-1 states are entirely decoupled and will play no role in the calculations performed here [21].

The spin-2 Kaluza-Klein gravitons decay to Standard Model fields with the following partial widths [21]:

$$\begin{aligned}
\Gamma_{\tilde{h}_m \to \gamma\gamma} &= \frac{m^3_{\tilde{h}_m}}{80\pi \overline{M}^2_{\rm Pl}},\\
\Gamma_{\tilde{h}_m \to ZZ} &= \frac{13 m^3_{\tilde{h}_m}}{960\pi \overline{M}^2_{\rm Pl}} \left(1 - \frac{4m_Z^2}{m^2_{\tilde{h}_m}}\right)^{1/2} \left(1 + \frac{56 m_Z^2}{169 m^2_{\tilde{h}_m}} + \frac{48 m_Z^4}{169 m^4_{\tilde{h}_m}}\right),\\
\Gamma_{\tilde{h}_m \to WW} &= \frac{13 m^3_{\tilde{h}_m}}{480\pi \overline{M}^2_{\rm Pl}} \left(1 - \frac{4m_W^2}{m^2_{\tilde{h}_m}}\right)^{1/2} \left(1 + \frac{56 m_W^2}{169 m^2_{\tilde{h}_m}} + \frac{48 m_W^4}{169 m^4_{\tilde{h}_m}}\right),\\
\Gamma_{\tilde{h}_m \to gg} &= \frac{m^3_{\tilde{h}_m}}{10\pi \overline{M}^2_{\rm Pl}},\\
\Gamma_{\tilde{h}_m \to HH} &= \frac{m^3_{\tilde{h}_m}}{480\pi \overline{M}^2_{\rm Pl}} \left(1 - \frac{4m_H^2}{m^2_{\tilde{h}_m}}\right)^{5/2},\\
\Gamma_{\tilde{h}_m \to f\bar{f}} &= \frac{g_f m^3_{\tilde{h}_m}}{640\pi \overline{M}^2_{\rm Pl}} \left(1 - \frac{4m_f^2}{m^2_{\tilde{h}_m}}\right)^{3/2} \left(1 + \frac{8 m_f^2}{3 m^2_{\tilde{h}_m}}\right), \qquad (2)
\end{aligned}$$

where g_f is the number of internal degrees of freedom of the fermion species (4 for each charged lepton, 2 for each neutrino, and 12 for each quark).

For $m_{\tilde{h}_m} \gtrsim$ TeV, the sum of these partial widths is given by[1]

$$\Gamma_{\tilde{h}_m} \approx \frac{71 m^3_{\tilde{h}_m}}{240\pi \overline{M}^2_{\rm Pl}}. \qquad (3)$$

The partial widths of the spin-0 Kaluza-Klein graviton states are given by [21]

[1]Note that this quantity is sometimes calculated including decays into right-handed neutrinos, in which case the numerical prefactor is instead given by $293/960\pi$.

$$\Gamma_{\tilde{\phi}_m\to\gamma\gamma}=0,$$
$$\Gamma_{\tilde{\phi}_m\to ZZ}=\frac{m^3_{\tilde{\phi}_m}}{(n+2)48\pi\overline{M}^2_{\rm Pl}}\left(1-\frac{4m_Z^2}{m^2_{\tilde{\phi}_m}}\right)^{1/2}\left(1-\frac{4m_Z^2}{m^2_{\tilde{\phi}_m}}+\frac{12m_Z^4}{m^4_{\tilde{\phi}_m}}\right),$$
$$\Gamma_{\tilde{\phi}_m\to WW}=\frac{m^3_{\tilde{\phi}_m}}{(n+2)24\pi\overline{M}^2_{\rm Pl}}\left(1-\frac{4m_W^2}{m^2_{\tilde{\phi}_m}}\right)^{1/2}\left(1-\frac{4m_W^2}{m^2_{\tilde{\phi}_m}}+\frac{12m_W^4}{m^4_{\tilde{\phi}_m}}\right),$$
$$\Gamma_{\tilde{\phi}_m\to gg}=0,$$
$$\Gamma_{\tilde{\phi}_m\to HH}=\frac{m^3_{\tilde{\phi}_m}}{(n+2)48\pi\overline{M}^2_{\rm Pl}}\left(1-\frac{4m_H^2}{m^2_{\tilde{\phi}_m}}\right)^{1/2}\left(1+\frac{2m_H^2}{m^2_{\tilde{\phi}_m}}\right)^2,$$
$$\Gamma_{\tilde{\phi}_m\to f\bar{f}}=\frac{g_f m_f^2 m_{\tilde{\phi}_m}}{(n+2)48\pi\overline{M}^2_{\rm Pl}}\left(1-\frac{4m_f^2}{m^2_{\tilde{\phi}_m}}\right)^{1/2}\left(1-\frac{2m_f^2}{m^2_{\tilde{\phi}_m}}\right), \quad (4)$$

which for $m_{\tilde{\phi}_m} \gg$ TeV sums to

$$\Gamma_{\tilde{\phi}_m}\approx\frac{m^3_{\tilde{\phi}_m}}{(n+2)12\pi\overline{M}^2_{\rm Pl}}. \quad (5)$$

In Fig. 1, we plot of the lifetimes of the Kaluza-Klein gravitons as a function of their mass for the case of $M_\star = 10^6$ GeV and $n = 6$.

Kaluza-Klein gravitons can be produced in the early Universe through the inverse decays of Standard Model particles, such as $e^+e^- \to \tilde{h}_m$, for example. It follows from the principle of detailed balance that, in equilibrium [22], the production rate of a given particle species will be equal to the rate of its destruction. We can use this information to calculate the rates at which the various Kaluza-Klein graviton modes will be produced through the collisions of Standard Model particles in the thermal bath [22–25][2]:

$$P_{\tilde{h}_m}=n^{\rm Eq}_{\tilde{h}_m}\langle\Gamma_{\tilde{h}_m}\rangle,$$
$$\approx n^{\rm Eq}_{\tilde{h}_m}\Gamma_{\tilde{h}_m}\frac{K_1(m_{\tilde{h}_m}/T)}{K_2(m_{\tilde{h}_m}/T)},$$
$$P_{\tilde{\phi}_m}=n^{\rm Eq}_{\tilde{\phi}_m}\langle\Gamma_{\tilde{\phi}_m}\rangle,$$
$$\approx n^{\rm Eq}_{\tilde{\phi}_m}\Gamma_{\tilde{\phi}_m}\frac{K_1(m_{\tilde{\phi}_m}/T)}{K_2(m_{\tilde{\phi}_m}/T)}, \quad (6)$$

where $\langle\Gamma_{\tilde{h}_m}\rangle$ and $\langle\Gamma_{\tilde{\phi}_m}\rangle$ are the thermally averaged decay widths [26], T is the temperature of the decaying particle population, and K_1 and K_2 are modified Bessel functions of the second kind. The equilibrium number densities of Kaluza-Klein gravitons are given by

$$n^{\rm Eq}_{\tilde{h}_m}=\frac{5}{2\pi^2}\int_{m_{\tilde{h}_m}}^\infty\frac{(E^2-m^2_{\tilde{h}_m})^{1/2}}{e^{E/T}-1}EdE,$$
$$n^{\rm Eq}_{\tilde{\phi}_m}=\frac{n(n-1)}{4\pi^2}\int_{m_{\tilde{\phi}_m}}^\infty\frac{(E^2-m^2_{\tilde{\phi}_m})^{1/2}}{e^{E/T}-1}EdE, \quad (7)$$

where the factor of $n(n-1)/2$ in the second expression accounts for the multiplicity of spin-0 states at each level of the Kaluza-Klein tower.

We want to stress that the Kalzua-Klein graviton population never reaches its equilibrium abundance. We are merely relating the rate of Kaluza-Klein graviton production to the decay rate of those particles under the condition of equilibrium.

For $T \gtrsim$ TeV, these production rates reduce to

$$P_{\tilde{h}_m}\approx\frac{5\zeta(3)T^3}{\pi^2}\frac{71m^3_{\tilde{h}_m}}{240\pi\bar{M}^2_{\rm Pl}}\frac{K_1(m_{\tilde{h}_m}/T)}{K_2(m_{\tilde{h}_m}/T)},$$
$$P_{\tilde{\phi}_m}\approx\frac{n(n-1)\zeta(3)T^3}{2\pi^2}\frac{m^3_{\tilde{\phi}_m}}{(n+2)12\pi\overline{M}^2_{\rm Pl}}\frac{K_1(m_{\tilde{\phi}_m}/T)}{K_2(m_{\tilde{\phi}_m}/T)}. \quad (8)$$

Note that the production rate of spin-2 Kaluza-Klein gravitons exceeds that of spin-0 states by a factor of $\sim 283(n+2)/[16n(n-1)] \approx 4.7$–35 (for $n = 6-2$). The lifetimes of the scalar modes, however, are longer than those of the spin-2 modes by a factor of $\sim 283/70$,

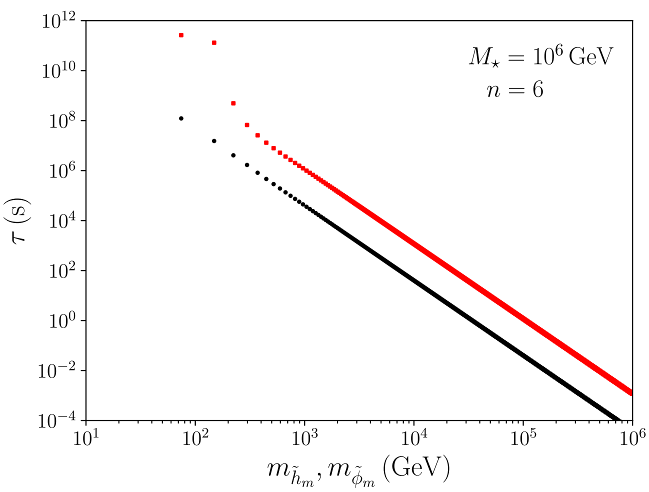

FIG. 1. The lifetimes of Kaluza-Klein gravitons as a function of their mass, for the case of $M_\star = 10^6$ GeV and $n = 6$ (for which the lightest Kaluza-Klein states have a mass of $1/R \approx 74$ GeV). The black dots represent the spin-2 states ($\tilde{h}_m$), while the red squares correspond to the spin-0 states ($\tilde{\phi}_m$). We expect those Kaluza-Klein modes with $\tau \gtrsim 1$ s to decay after the onset of big bang nucleosynthesis, potentially impacting the primordial light element abundances.

[2]We have not included the effects of Pauli blocking or Bose enhancement in our calculation. In the case of Kaluza-Klein graviton production, the phase space density of these particles, f, is always much less than the equilibrium density, ensuring that $1 + f \approx 1$. For the case of Kaluza-Klein graviton decays, we are primarily concerned with decays that occur during the era of big bang nucleosynthesis ($T \lesssim$ MeV), by which time the Kaluza-Klein gravitons will be highly nonrelativistic, again ensuring the validity of the approximation, $1 - f \approx 1$.

leading to complementary impacts on the light element abundances.

We are now in a position to calculate the evolution of the abundances of the Kaluza-Klein gravitons by solving the following coupled set of differential equations (for each level of the Kaluza-Klein tower, m):

$$\frac{dn_{\tilde{h}_m}}{dt} = -3Hn_{\tilde{h}_m} + P_{\tilde{h}_m},$$
$$\frac{dn_{\tilde{\phi}_m}}{dt} = -3Hn_{\tilde{\phi}_m} + P_{\tilde{\phi}_m}, \qquad (9)$$

where $H = (8\pi\rho/3M_{\rm Pl}^2)^{1/2}$ is the rate of Hubble expansion and $\rho = \rho_{\rm SM} + \sum_m \rho_{\tilde{h}_m} + \sum_m \rho_{\tilde{\phi}_m}$ is the total energy density of the Universe, $\rho_{\rm SM} = \pi^2 g_\star T^4/30$ is the energy density in Standard Model particles, and $g_\star$ is the number of relativistic degrees of freedom. We evolve the temperature of the Standard Model bath by applying entropy conservation, $T \propto a^{-1} g_{\star,S}^{-1/3}$, where $g_{\star,S}$ is the number of relativistic degrees of freedom in entropy.

In Fig. 2, we plot the production rate of Kaluza-Klein gravitons (divided by three powers of the temperature) for several selected levels of the Kaluza-Klein tower, and for the case of the $M_\star = 10^6$ GeV and $n = 6$. Since these production rates are many order of magnitude below the rate of Hubble expansion, the Kaluza-Klein graviton abundances never reach their equilibrium values, placing us safely within the regime of thermal freeze-in. Unlike more typical freeze-in scenarios, however, the large multiplicity of Kaluza-Klein

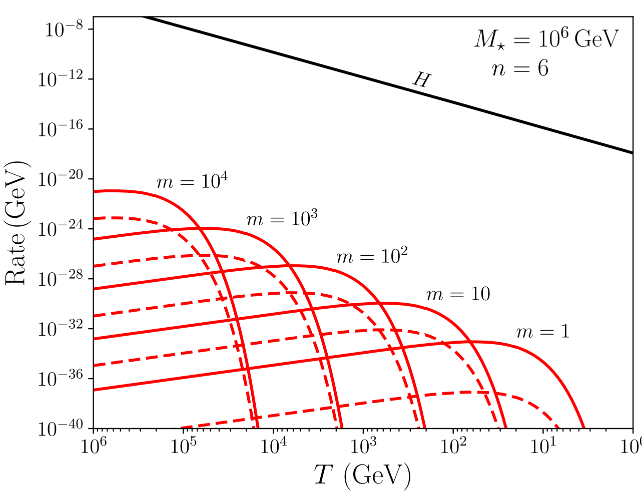

FIG. 2. The production rate of Kaluza-Klein gravitons divided by three powers of temperature, P/T^3, for the case of $M_\star = 10^6$ GeV and $n = 6$. We show results for Kaluza-Klein gravitons with levels of $m = 10^4, 10^3, 10^2, 10$, and 1. The solid red lines correspond to the spin-2 states, $\tilde{h}_m$, while the dashed red lines correspond to the scalar states, $\tilde{\phi}_m$. For comparison, we also show the value of the hubble rate, H, which is much larger than the production rate of Kaluza-Klein gravitons, ensuring that equilibrium is never achieved.

states can greatly enhance the total energy density of these particles that is produced. The case shown in Fig. 2, for example, is effectively that of the simultaneous freeze-in of $\sim 2 \times 10^4$ different particle species, each of which contributes to the total resulting abundance.

In Fig. 3, we plot the evolution of the nonrelativistic densities of Kaluza-Klein gravitons, $\rho_i = n_i m_i$, compared to the total energy density in Standard Model particles, for the case of $M_\star = 10^6$ GeV and $n = 6$, and for two values of the initial temperature, $T_{\rm max}$. Results are shown several selected values of the Kaluza-Klein levels, m, as well as for the sum of all modes. In the left frame, one can notice that the $m = 10^3$ states begin to appreciably decay prior to the onset of big bang nucleosynthesis.

III. KALUZA-KLEIN GRAVITON DECAYS DURING BIG BANG NUCLEOSYNTHESIS

Measurements of the primordial deuterium [27,28] and helium [29–31] abundances provide us with the earliest probe of our Universe's thermal history, confirming that our Universe was radiation dominated and generally well described by ΛCDM cosmology throughout the era of big bang nucleosynthesis, which began $t \sim 1$ s after the big bang [32–37]. Such measurements allow us to place stringent constraints on the expansion history of our Universe, as well as on any energy injection that may have taken place during or after this era [38–61].

The presence of Kaluza-Klein gravitons in the early Universe could have potentially impacted the light element abundances in a number of ways. In particular, their decay products could have broken up helium nuclei through the processes of photodissociation and hadrodissociation, reducing the abundance of primordial helium while enhancing that of deuterium. These and other such processes have been modeled in detail in a number of publicly available codes [62–64]. In this study, we make use of the results of Kawasaki *et al.*, who evaluated the impact of decaying particles on the resulting light element abundances [48] (see also Refs. [45,47,49–57,59,60,65]). In particular, the authors of Ref. [48] derived constraints on the lifetime and abundance of a decaying particle species for various values of the particle's mass and dominant decay modes. These constraints were presented in terms of the mass of the decaying particles multiplied by the number of such particles per unit entropy, MY, as evaluated prior to their decays, $t \ll \tau$. Whereas that study considered the impact of only one decaying particle species at a time, we are concerned here with the decays of the entire tower of Kaluza-Klein gravitons. To recast the results of Kawasaki *et al.* for the case at hand, we treat all Kaluza-Klein states with lifetimes within a given decade (for example, $\tau = 10^2$–10^3 s) as a single particle species, with a lifetime and mass equal to that of the median Kaluza-Klein state within that group. We then compare this to Fig. 12 of Kawasaki *et al.* in order to determine whether a given

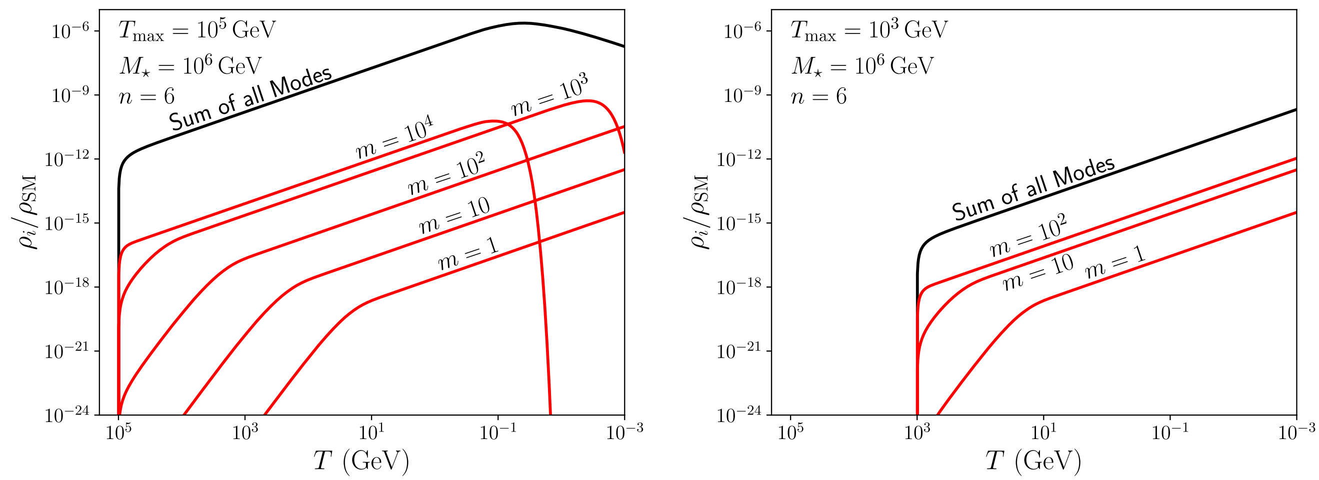

FIG. 3. The accumulated nonrelativistic densities of Kaluza-Klein gravitons, $\rho_i = n_i m_i$, divided by the total energy density in Standard Model particles for the case of $M_\star = 10^6$ GeV and $n = 6$, and for two values of the initial temperature, $T_{\rm max}$. We show results for the Kaluza-Klein modes with levels of $m = 10^4$, 10^3, 10^2, 10, and 1, as well as for the sum of all modes. The red lines represent the abundances of the spin-2 states, while the black lines include both spin-2 and scalar states. Note that the $m = 10^4$ and $m = 10^3$ states begin to appreciably decay prior to the onset of big bang nucleosynthesis.

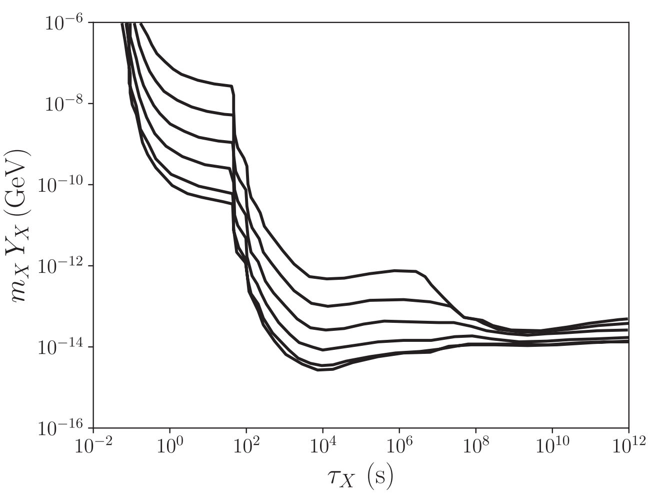

FIG. 4. Constraints on a particle species decaying to quarks from measurements of the primordial element abundances, as presented in Ref. [48]. From top to bottom, the curves correspond to the upper limits on the abundance of particles of mass 10^6, 10^5, 10^4, 10^3, 10^2, and 30 GeV. These constraints are presented in terms of the number density of decaying particles per unit entropy (prior to their decays), $Y_X = n_X/s$.

scenario is consistent with the measured helium and deuterium abundances. We repeat this procedure for each decade of lifetime, allowing us to produce conservative constraints on the values of $M_\star$ and n, as a function of the initial temperature of the Universe, $T_{\rm max}$. We include in our calculations only those graviton decays that proceed to quarks or gluons, and apply Kawasaki's constraints on the $u\bar{u}$ channel (which are almost entirely indistinguishable from those found in the cases of other hadronic final states). The constraints are presented by Kawasaki *et al.* are shown in Fig. 4.

IV. RESULTS

The main results of this study are present in Fig. 5, where we plot the maximum temperature of the Universe that is consistent with the measured light element abundances, as a function of $M_\star$ and n. Where the curves are solid in this figure, the light element abundances provide the strongest constraint on this class of models. In contrast, whereas the curves are dashed, the constraints derived here are less restrictive than others that have been presented in the literature (as summarized in Sec. I).

To understand the results that appear in Fig. 5, note that we are applying constraints that apply to particles with lifetimes ranging from $\tau \sim 0.1$ to 10^{12} s. From Eq. (3), we find that this range corresponds to Kaluza-Klein gravitons with masses in the range of $m_{\tilde{h}_m} \sim 3$ to $\sim 7 \times 10^4$ GeV. Thus, if the Universe was never at a temperature greater than $\mathcal{O}$(GeV), the only Kaluza-Klein gravitons that could be produced would be too long-lived to be constrained by the present analysis (although other constraints, such as those from the cosmic microwave background could still be potentially restrictive). On the other hand, in order for the Kaluza-Klein gravitons to not decay prior to 0.1 s (or equivalently, for those states to be lighter than $m_{\tilde{h}_m} \sim 7 \times 10^4$ GeV), the value of $M_\star$ must not be too high. For the $n = 1$ case, for example, the first Kaluza-Klein mode has a mass of $m_{\tilde{h}_1} \sim 7 \times 10^4$ GeV for $M_\star \sim 7 \times 10^{13}$ GeV, explaining why our constraints do not extend to larger values of the fundamental Planck

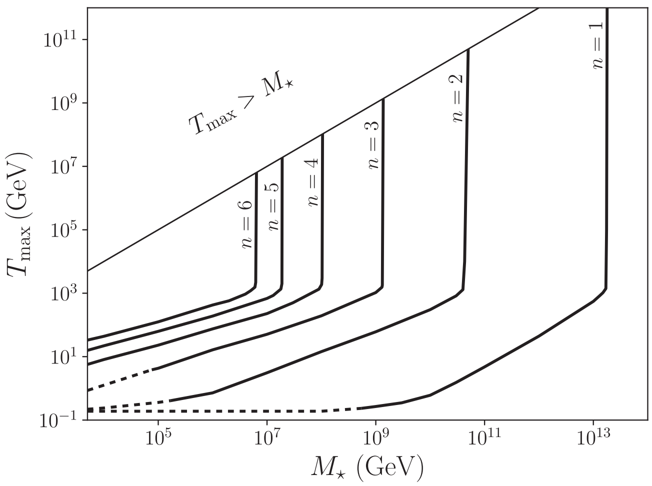

FIG. 5. The upper limits derived in this study on the maximum temperature of the Universe, $T_{\max}$, as a function of $M_\star$ and n. The dashed portions of the lines represent values of $M_\star$ that are ruled out by other considerations (see Sec. I). We only consider values of the temperatures that are below the fundamental Planck scale, $T < M_\star$.

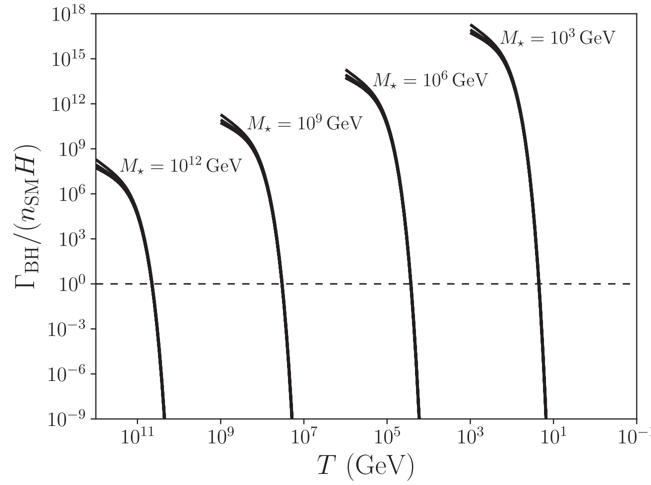

FIG. 6. The integrated production rate of black holes per Standard Model particle per Hubble time, for several values of $M_\star$ and for $n = 1$, 2, 4 and 6 (this result depends only very weakly on the value of n). For comparison, we plot as a dashed curve the value at which each Standard Model particles produces an average of one black hole per Hubble time, $\Gamma_{\rm BH}/(n_{\rm SM}H) = 1$.

scale. Alternatively, for $n = 6$, $m_{\tilde{h}_1} \sim 7 \times 10^4$ GeV corresponds to a value of $M_\star \sim 2 \times 10^8$ GeV. In this later case, you may notice that our constraints only extend up to $M_\star \sim 6 \times 10^6$ GeV. The reason that they do not extend to higher values of $M_\star$ is that the number of decaying particles per unit entropy is not particular large in this case and the decays occur around $\tau \lesssim 0.1$ s, where the constraints are rather weak (see Fig. 4). If we instead consider $M_\star \sim 6 \times 10^6$ GeV, then the lightest Kaluza-Klein mode has a mass of $m_{\tilde{h}_1} \sim 800$ GeV and a lifetime of $\tau \sim 8 \times 10^4$ s. Constraints in this lifetime range are much more stringent, allowing us to exclude values of $T_{\max}$ that are comparable to or larger than the mass of the lightest Kaluza-Klein mode.

V. IMPLICATIONS FOR BLACK HOLE PRODUCTION

It has long been appreciated that small-scale inhomogeneties in the early Universe may have led to the formation of primordial black holes [66,67]. Alternatively, in models with extra spatial dimensions, black holes could have been produced through the collisions of particles in the thermal bath, in particular if the total energy of a collision exceeds the fundamental Planck scale, $M_\star$ [68] (see also Refs. [69,70]).

In the context of the flat and compactified extra dimensions that we have considered in this study, the differential production rate of black holes in a thermal bath of temperature, T, is given by [71]

$$\frac{d\Gamma_{\rm BH}}{dM_{\rm BH}} = \frac{g_\star(T)^2}{8\pi^4}\left(\frac{8\Gamma(\frac{n+3}{2})}{n+2}\right)^{\frac{2}{n+1}} M_{\rm BH} T^2 \left(\frac{M_{\rm BH}}{M_\star}\right)^{\frac{2n+4}{n+1}} \times \left[\frac{M_{\rm BH}}{T} K_1(M_{\rm BH}/T) + 2K_2(M_{\rm BH}/T)\right] \times \Theta(M_{\rm BH} - M_\star), \tag{10}$$

where Γ is the gamma function, K_1 and K_2 are again the modified Bessel functions of the second kind, and Θ is the Heavyside step function. For $T \sim M_\star$, black holes can be produced at a very high rate. In particular, after dropping order one factors, this expression integrates to $\Gamma_{\rm BH} \sim M_\star^4$ for temperatures near the fundamental Planck scale. This indicates that the black hole production rate per particle could potentially be larger than the Hubble rate by a huge factor, roughly $\sim(\Gamma_{\rm BH}/n_{\rm SM})/H \sim 4(M_\star^4/T^3)/(T^2/M_{\rm Pl}) \sim 4M_{\rm Pl}/M_\star$. In contrast, the black hole production rate is dramatically suppressed at temperatures lower than $M_\star$. This behavior is confirmed in Fig 6, where we plot the integrated production rate of black holes per Standard Model particle per Hubble time, for several values of $M_\star$ and for $n = 1$, 2, 4, and 6 (this result depends only very weakly on the value of n).

If a large number of microscopic black holes had been generated in the early Universe, these objects could have had a number of potentially observable impacts. In particular, the products of their Hawking evaporation could have included particles that would act as dark radiation (and contribute to the effective number of neutrino species, $N_{\rm eff}$) or contribute to the dark matter density [72] (see also Refs. [73–75]). Alternatively, in some scenarios microscopic black holes could grow rapidly through accretion, significantly delaying their evaporation [68,76,77].

The constraints obtained in this study severely limit the rate at which black holes could have been produced through particle collisions in the early Universe. This can be seen by comparing the black hole production rates shown in Fig. 6 to the constraints we have presented in Fig. 5. For the case of $M_\star = 10^6$ GeV, for example, our constraints derived from the primordial light element abundances require $T_{\rm max} \lesssim 400$ GeV for any value of $n \leq 6$. For such temperatures, the black hole production rate is vanishingly small. Thus, for this value of $M_\star$, particle collisions will not produce any significant abundance of black holes in the early Universe. If we consider larger values of $M_\star$, then black hole production could still be potentially important. For $M_\star = 10^9$ GeV (10^{12} GeV), for example, large black hole production rates are still possible, provided that $n \geq 3$ ($n \geq 2$).

VI. SUMMARY AND DISCUSSION

In this study, we have considered the impact of Kaluza-Klein gravitons on the primordial light element abundances, as established during the era of big bang nucleosynthesis. In particular, we have focused on the ADD scenario, which features $n = 1$–6 extra spatial dimensions which are flat and compactified around a torus of radius, R. We have taken the fields of the Standard Model to be confined to a $(3+1)$-dimensional brane, allowing the fundamental scale of gravity to be much lower than the effective four-dimensional Planck scale, $M_\star \ll M_{\rm Pl}$ [10,10,12].

In this model, massless spin-2 gravitons propagating in the $n+4$ dimensional bulk appear to observers on the brane as a tower of massive Kaluza-Klein states, with an evenly spaced series of masses, $m_{\rm KK} = m/R$, where m is the level of the Kaluza-Klein tower. These Kaluza-Klein gravitons can be produced in the early Universe and, for $m_{\rm KK} \lesssim 10^5$ GeV, will subsequently decay during or after the era of big bang nucleosynthesis. Such decays can produce energetic Standard Model particles which can break up helium nuclei, significantly altering the predicted abundances of primoridal helium and deuterium.

We have calculated the production and decay rates of Kaluza-Klein gravitons in the early Universe, and evaluated the impact of these decays on the primordial light element abundances. These abundances, in turn, allow us to place constraints on Kalzua-Klein gravitons with masses in the range of $m_{\rm KK} \sim 3$ GeV to $\sim 7 \times 10^4$ GeV, as summarized in Fig. 5. For one extra dimension, $n = 1$, we exclude all values of $M_\star \lesssim 10^{13}$ GeV ($\lesssim 10^9$ GeV) unless the maximum temperature of the Universe was less than $\sim$500 GeV ($\sim$0.3 GeV). For larger values of n, our constraints are somewhat less stringent. For the case of $n = 6$, for example, our analysis excludes all values of $M_\star \lesssim 10^6$ GeV unless the maximum temperature of the Universe was less than $\sim$400 GeV.

The results presented here severely limit the possibility that black holes may have been produced in significant numbers through particle collisions in the early Universe's thermal bath [71]. For the case of $M_\star \lesssim 10^6$ GeV, we can rule out the possibility that any appreciable abundance of black holes was formed through such collisions, for any value of $n \leq 6$. For larger values of $M_\star$, thermal black hole production may still have been potentially important. For $M_\star = 10^9$ GeV, for example, efficient black hole production could have occurred in the early Universe, provided that $n \geq 3$.

Before closing, we note that the models we are considering here are similar to, but not the same as, the "dark dimension" scenario that has been proposed within the context of the Swampland program of string theory [78,79]. In particular, whereas Kaluza-Klein gravitons can decay into lighter KK modes in the dark dimension picture, extra-dimensional momentum conservation prevents such decays in the ADD model (decays into Standard Model particles are allowed, as the brane is a dynamical object which can recoil to take on the momentum of the decaying graviton). We leave the consideration of this class of models to future work.

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[1] T. Hasegawa, N. Hiroshima, K. Kohri, R. S. L. Hansen, T. Tram, and S. Hannestad, MeV-scale reheating temperature and thermalization of oscillating neutrinos by radiative and hadronic decays of massive particles, J. Cosmol. Astropart. Phys. 12 (2019) 012.
[2] S. Hannestad, What is the lowest possible reheating temperature?, Phys. Rev. D **70**, 043506 (2004).
[3] G. F. Giudice, E. W. Kolb, and A. Riotto, Largest temperature of the radiation era and its cosmological implications, Phys. Rev. D **64**, 023508 (2001).
[4] M. Kawasaki, K. Kohri, and N. Sugiyama, MeV scale reheating temperature and thermalization of neutrino background, Phys. Rev. D **62**, 023506 (2000).
[5] M. Kawasaki, K. Kohri, and N. Sugiyama, Cosmological constraints on late time entropy production, Phys. Rev. Lett. **82**, 4168 (1999).
[6] J. Ghiglieri, G. Jackson, M. Laine, and Y. Zhu, Gravitational wave background from standard model physics: Complete leading order, J. High Energy Phys. 07 (2020) 092.
[7] J. Ghiglieri and M. Laine, Gravitational wave background from standard model physics: Qualitative features, J. Cosmol. Astropart. Phys. 07 (2015) 022.
[8] B. X. Hu and A. Loeb, An upper limit on the initial temperature of the radiation-dominated universe, J. Cosmol. Astropart. Phys. 01 (2021) 041.
[9] A. Ringwald, J. Schütte-Engel, and C. Tamarit, Gravitational waves as a big bang thermometer, J. Cosmol. Astropart. Phys. 03 (2021) 054.
[10] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, The hierarchy problem and new dimensions at a millimeter, Phys. Lett. B **429**, 263 (1998).
[11] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, New dimensions at a millimeter to a Fermi and superstrings at a TeV, Phys. Lett. B **436**, 257 (1998).
[12] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phenomenology, astrophysics and cosmology of theories with submillimeter dimensions and TeV scale quantum gravity, Phys. Rev. D **59**, 086004 (1999).
[13] A. M. Sirunyan *et al.* (CMS Collaboration), Search for new physics in final states with an energetic jet or a hadronically decaying W or Z boson and transverse momentum imbalance at $\sqrt{s} = 13$ TeV, Phys. Rev. D **97**, 092005 (2018).
[14] M. Aaboud *et al.* (ATLAS Collaboration), Search for dark matter and other new phenomena in events with an energetic jet and large missing transverse momentum using the ATLAS detector, J. High Energy Phys. 01 (2018) 126.
[15] E. G. Adelberger, J. H. Gundlach, B. R. Heckel, S. Hoedl, and S. Schlamminger, Torsion balance experiments: A low-energy frontier of particle physics, Prog. Part. Nucl. Phys. **62**, 102 (2009).
[16] J. Murata and S. Tanaka, A review of short-range gravity experiments in the LHC era, Classical Quantum Gravity **32**, 033001 (2015).
[17] W.-H. Tan, S.-Q. Yang, C.-G. Shao, J. Li, A.-B. Du, B.-F. Zhan, Q.-L. Wang, P.-S. Luo, L.-C. Tu, and J. Luo, New test of the gravitational inverse-square law at the submillimeter range with dual modulation and compensation, Phys. Rev. Lett. **116**, 131101 (2016).
[18] S. Hannestad and G. G. Raffelt, Supernova and neutron star limits on large extra dimensions reexamined, Phys. Rev. D **67**, 125008 (2003).
[19] R. L. Workman *et al.* (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[20] G. Shiu and S. H. H. Tye, TeV scale superstring and extra dimensions, Phys. Rev. D **58**, 106007 (1998).
[21] T. Han, J. D. Lykken, and R.-J. Zhang, On Kaluza-Klein states from large extra dimensions, Phys. Rev. D **59**, 105006 (1999).
[22] A. de Giorgi and S. Vogl, Warm dark matter from a gravitational freeze-in in extra dimensions, J. High Energy Phys. 04 (2023) 032.
[23] L. J. Hall and D. Tucker-Smith, Cosmological constraints on theories with large extra dimensions, Phys. Rev. D **60**, 085008 (1999).
[24] L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, Freeze-in production of FIMP dark matter, J. High Energy Phys. 03 (2010) 080.
[25] A. de Giorgi and S. Vogl, Dark matter interacting via a massive spin-2 mediator in warped extra-dimensions, J. High Energy Phys. 11 (2021) 036.
[26] M. Escudero, D. Hooper, G. Krnjaic, and M. Pierre, Cosmology with a very light $L_\mu - L_\tau$ gauge boson, J. High Energy Phys. 03 (2019) 071.
[27] R. Cooke, M. Pettini, R. A. Jorgenson, M. T. Murphy, and C. C. Steidel, Precision measures of the primordial abundance of deuterium, Astrophys. J. **781**, 31 (2014).
[28] S. Riemer-Sørensen and E. S. Jenssen, Nucleosynthesis predictions and high-precision deuterium measurements, Universe **3**, 44 (2017).
[29] Y. I. Izotov, T. X. Thuan, and N. G. Guseva, A new determination of the primordial He abundance using the HeI λ10830 Å emission line: Cosmological implications, Mon. Not. R. Astron. Soc. **445**, 778 (2014).
[30] E. Aver, K. A. Olive, and E. D. Skillman, The effects of He I λ10830 on helium abundance determinations, J. Cosmol. Astropart. Phys. 07 (2015) 011.
[31] A. Peimbert, M. Peimbert, and V. Luridiana, The primordial helium abundance and the number of neutrino families, Rev. Mex. Astron. Astrofis. **52**, 419 (2016).
[32] D. N. Schramm and M. S. Turner, Big bang nucleosynthesis enters the precision era, Rev. Mod. Phys. **70**, 303 (1998).
[33] G. Steigman, Primordial nucleosynthesis in the precision cosmology era, Annu. Rev. Nucl. Part. Sci. **57**, 463 (2007).
[34] F. Iocco, G. Mangano, G. Miele, O. Pisanti, and P. D. Serpico, Primordial nucleosynthesis: From precision cosmology to fundamental physics, Phys. Rep. **472**, 1 (2009).
[35] R. H. Cyburt, B. D. Fields, K. A. Olive, and T.-H. Yeh, Big bang nucleosynthesis: 2015, Rev. Mod. Phys. **88**, 015004 (2016).
[36] C. Pitrou, A. Coc, J.-P. Uzan, and E. Vangioni, Precision big bang nucleosynthesis with improved Helium-4 predictions, Phys. Rep. **754**, 1 (2018).
[37] B. D. Fields, K. A. Olive, T.-H. Yeh, and C. Young, Big-bang nucleosynthesis after Planck, J. Cosmol. Astropart. Phys. 03 (2020) 010; 11 (2020) 002(E).

[38] S. Sarkar, Big bang nucleosynthesis and physics beyond the standard model, Rep. Prog. Phys. **59**, 1493 (1996).
[39] M. Pospelov and J. Pradler, Big bang nucleosynthesis as a probe of new physics, Annu. Rev. Nucl. Part. Sci. **60**, 539 (2010).
[40] K. Jedamzik and M. Pospelov, Big bang nucleosynthesis and particle dark matter, New J. Phys. **11**, 105028 (2009).
[41] M. Hufnagel, K. Schmidt-Hoberg, and S. Wild, BBN constraints on MeV-scale dark sectors. Part I. Sterile decays, J. Cosmol. Astropart. Phys. 02 (2018) 044.
[42] C. Keith, D. Hooper, N. Blinov, and S. D. McDermott, Constraints on primordial black holes from big bang nucleosynthesis revisited, Phys. Rev. D **102**, 103512 (2020).
[43] B. J. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, New cosmological constraints on primordial black holes, Phys. Rev. D **81**, 104019 (2010).
[44] G.-y. Huang, T. Ohlsson, and S. Zhou, Observational constraints on secret neutrino interactions from big bang nucleosynthesis, Phys. Rev. D **97**, 075009 (2018).
[45] L. Forestell, D. E. Morrissey, and G. White, Limits from BBN on light electromagnetic decays, J. High Energy Phys. 01 (2019) 074.
[46] M. Hufnagel, K. Schmidt-Hoberg, and S. Wild, BBN constraints on MeV-scale dark sectors. Part II. Electromagnetic decays, J. Cosmol. Astropart. Phys. 11 (2018) 032.
[47] P. F. Depta, M. Hufnagel, K. Schmidt-Hoberg, and S. Wild, BBN constraints on the annihilation of MeV-scale dark matter, J. Cosmol. Astropart. Phys. 04 (2019) 029.
[48] M. Kawasaki, K. Kohri, T. Moroi, and Y. Takaesu, Revisiting big-bang nucleosynthesis constraints on long-lived decaying particles, Phys. Rev. D **97**, 023502 (2018).
[49] V. Poulin and P. D. Serpico, Nonuniversal BBN bounds on electromagnetically decaying particles, Phys. Rev. D **91**, 103007 (2015).
[50] R. H. Cyburt, J. Ellis, B. D. Fields, F. Luo, K. A. Olive, and V. C. Spanos, Nucleosynthesis constraints on a massive gravitino in neutralino dark matter scenarios, J. Cosmol. Astropart. Phys. 10 (2009) 021.
[51] M. Kusakabe, T. Kajino, T. Yoshida, T. Shima, Y. Nagai, and T. Kii, New constraints on radiative decay of long-lived X particles in big bang nucleosynthesis with new rates of photodisintegration reactions of ^{4}He, Phys. Rev. D **79**, 123513 (2009).
[52] K. Jedamzik, Big bang nucleosynthesis constraints on hadronically and electromagnetically decaying relic neutral particles, Phys. Rev. D **74**, 103509 (2006).
[53] K. Kohri, T. Moroi, and A. Yotsuyanagi, Big-bang nucleosynthesis with unstable gravitino and upper bound on the reheating temperature, Phys. Rev. D **73**, 123511 (2006).
[54] K. Jedamzik, Did something decay, evaporate, or annihilate during big bang nucleosynthesis?, Phys. Rev. D **70**, 063524 (2004).
[55] M. Kawasaki, K. Kohri, and T. Moroi, Big-bang nucleosynthesis and hadronic decay of long-lived massive particles, Phys. Rev. D **71**, 083502 (2005).
[56] M. Kawasaki, K. Kohri, and T. Moroi, Hadronic decay of late—Decaying particles and big-bang nucleosynthesis, Phys. Lett. B **625**, 7 (2005).
[57] R. H. Cyburt, J. R. Ellis, B. D. Fields, and K. A. Olive, Updated nucleosynthesis constraints on unstable relic particles, Phys. Rev. D **67**, 103521 (2003).
[58] K. Kohri, Primordial nucleosynthesis and hadronic decay of a massive particle with a relatively short lifetime, Phys. Rev. D **64**, 043515 (2001).
[59] M. Kawasaki, K. Kohri, and T. Moroi, Radiative decay of a massive particle and the nonthermal process in primordial nucleosynthesis, Phys. Rev. D **63**, 103502 (2001).
[60] E. Holtmann, M. Kawasaki, K. Kohri, and T. Moroi, Radiative decay of a longlived particle and big bang nucleosynthesis, Phys. Rev. D **60**, 023506 (1999).
[61] M. Kawasaki and T. Moroi, Electromagnetic cascade in the early universe and its application to the big bang nucleosynthesis, Astrophys. J. **452**, 506 (1995).
[62] A. Arbey, J. Auffinger, K. P. Hickerson, and E. S. Jenssen, AlterBBN v2: A public code for calculating Big-Bang nucleosynthesis constraints in alternative cosmologies, Comput. Phys. Commun. **248**, 106982 (2020).
[63] C. Pitrou, A. Coc, J.-P. Uzan, and E. Vangioni, Precision big bang nucleosynthesis with the new code PRIMAT, J. Phys. Soc. Jpn. Conf. Proc. **31**, 011034 (2020).
[64] O. Pisanti, A. Cirillo, S. Esposito, F. Iocco, G. Mangano, G. Miele, and P. D. Serpico, PArthENoPE: Public algorithm evaluating the nucleosynthesis of primordial elements, Comput. Phys. Commun. **178**, 956 (2008).
[65] M. Kawasaki and T. Moroi, Gravitino production in the inflationary universe and the effects on big bang nucleosynthesis, Prog. Theor. Phys. **93**, 879 (1995).
[66] B. J. Carr and S. W. Hawking, Black holes in the early Universe, Mon. Not. R. Astron. Soc. **168**, 399 (1974).
[67] B. J. Carr, The primordial black hole mass spectrum, Astrophys. J. **201**, 1 (1975).
[68] J. A. Conley and T. Wizansky, Microscopic primordial black holes and extra dimensions, Phys. Rev. D **75**, 044006 (2007).
[69] P. C. Argyres, S. Dimopoulos, and J. March-Russell, Black holes and submillimeter dimensions, Phys. Lett. B **441**, 96 (1998).
[70] R. C. Myers and M. J. Perry, Black holes in higher dimensional space-times, Ann. Phys. (N.Y.) **172**, 304 (1986).
[71] A. Friedlander, K. J. Mack, S. Schon, N. Song, and A. C. Vincent, Primordial black hole dark matter in the context of extra dimensions, Phys. Rev. D **105**, 103508 (2022).
[72] A. Friedlander, N. Song, and A. C. Vincent, Dark matter from higher-dimensional primordial black holes, Phys. Rev. D **108**, 043523 (2023).
[73] D. Hooper, G. Krnjaic, and S. D. McDermott, Dark radiation and superheavy dark matter from black hole domination, J. High Energy Phys. 08 (2019) 001.
[74] D. Hooper, G. Krnjaic, J. March-Russell, S. D. McDermott, and R. Petrossian-Byrne, Hot gravitons and gravitational waves from Kerr black holes in the early universe, arXiv:2004.00618.

[75] A. Friedlander, N. Song, and A. C. Vincent, Dark matter from hot big bang black holes, Phys. Rev. D **108**, L081301 (2023).
[76] R. Guedens, D. Clancy, and A. R. Liddle, Primordial black holes in brane world cosmologies: Accretion after formation, Phys. Rev. D **66**, 083509 (2002).
[77] A. S. Majumdar, Domination of black hole accretion in brane cosmology, Phys. Rev. Lett. **90**, 031303 (2003).
[78] E. Gonzalo, M. Montero, G. Obied, and C. Vafa, Dark dimension gravitons as dark matter, J. High Energy Phys. 11 (2023) 109.
[79] M. Montero, C. Vafa, and I. Valenzuela, The dark dimension and the swampland, J. High Energy Phys. 02 (2023) 022.