

Supporting Information for ”Vertically resolved analysis of the Madden-Julian Oscillation highlights the role of convective transport of moist static energy”

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Text S1. Vertically integrated MSE analysis

We also perform the vertically integrated MSE analysis for our aquaplanet experiments (Figure S1). Below is the derivation. We have copied and pasted Equation (1) in the main text here for convenience.

$$\partial_t h' + \nabla_h \cdot (\vec{u}h)' + \partial_p(\omega h)' = Q'. \quad (\text{S1})$$

Instead of directly projecting Equation (S1) onto $h'(x, y, p)$, we first integrate Equation (S1) from the surface to the top of the atmosphere to get the integrated MSE budget:

$$\partial_t \langle h' \rangle + \langle \nabla_h \cdot (\vec{u}h)' \rangle = \langle Q' \rangle. \quad (\text{S2})$$

Here, $\langle \cdot \rangle$ represents vertical integral, and the vertical convergence term vanishes after integration. Then, we project Equation (S2) onto $\langle h' \rangle$ and get the contribution of each term to the maintenance of the MJO:

$$\frac{1}{2} \partial_t \overline{\langle h' \rangle^2} + \overline{\langle h' \rangle \langle \nabla_h \cdot (\vec{u}h)' \rangle} = \overline{\langle h' \rangle \langle Q' \rangle}. \quad (\text{S3})$$

We normalize Equation (S3) by the variance of vertically integrated MSE ($\mathcal{C} = \overline{\langle h' \rangle^2}$) and get the vertically integrated MSE variance budget:

$$\text{contribution to growth} = \frac{\text{Equation (S3)}}{\mathcal{C}}. \quad (\text{S4})$$

Similarly, we can assess the contribution of each term to the propagation of the MJO by projecting Equation (S2) onto $\partial_t \langle h' \rangle$:

$$\text{contribution to propagation} = \frac{\overline{\partial_t \langle h' \rangle \cdot \text{Equation (S2)}}}{\mathcal{D}}, \quad (\text{S5})$$

where $\mathcal{D} = (\partial_t \langle h' \rangle)^2$. The vertically integrated analysis for the maintenance and propagation of the MJO is shown in Figure S1.

Text S2. Hovmöller diagrams of OLR anomalies

Figure S2 shows the Hovmöller diagrams of the OLR anomalies for the entire simulation period of nine years in the three simulations, while Figure S3 shows those for the first 20 days to highlight the initiation stage.

Text S3. Horizontal Structures of the MJO composite

The MJO composites in the control and the mechanism-denial experiments share similar horizontal structures (Figure S4).

Text S4. Vertical Structures of the MJO composite

Figure S5 shows the vertical structures of the MJO composite in the control simulation. The results complement Figure 3 by showing the combined contribution of convection (CRM), boundary layer, and large-scale dynamics to the tendencies of DSE and moisture components.

Figure S6 shows the residual of the MSE budget in the three simulations (Equation (1)). The residual of the budget is negligibly small.

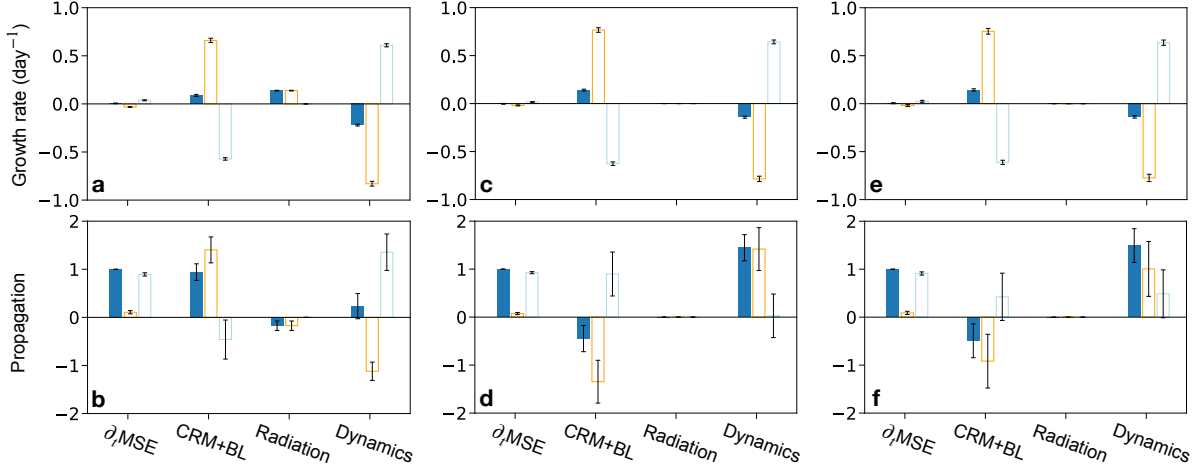


Figure S1. The vertically integrated MSE analysis for the control simulation (first column), HomoRad simulation (second column), and FixedRad simulation (third column). The first and second rows show individual terms' contribution to MJO's maintenance (Equation (S4)) and propagation (Equation (S5)), respectively. The solid blue bar represents MSE component; the open orange and blue bars represent the corresponding DSE and moisture components, respectively. Note: 'Dynamics' only corresponds to the second term in Equation (S1), because the vertical convergence term vanishes after integral.

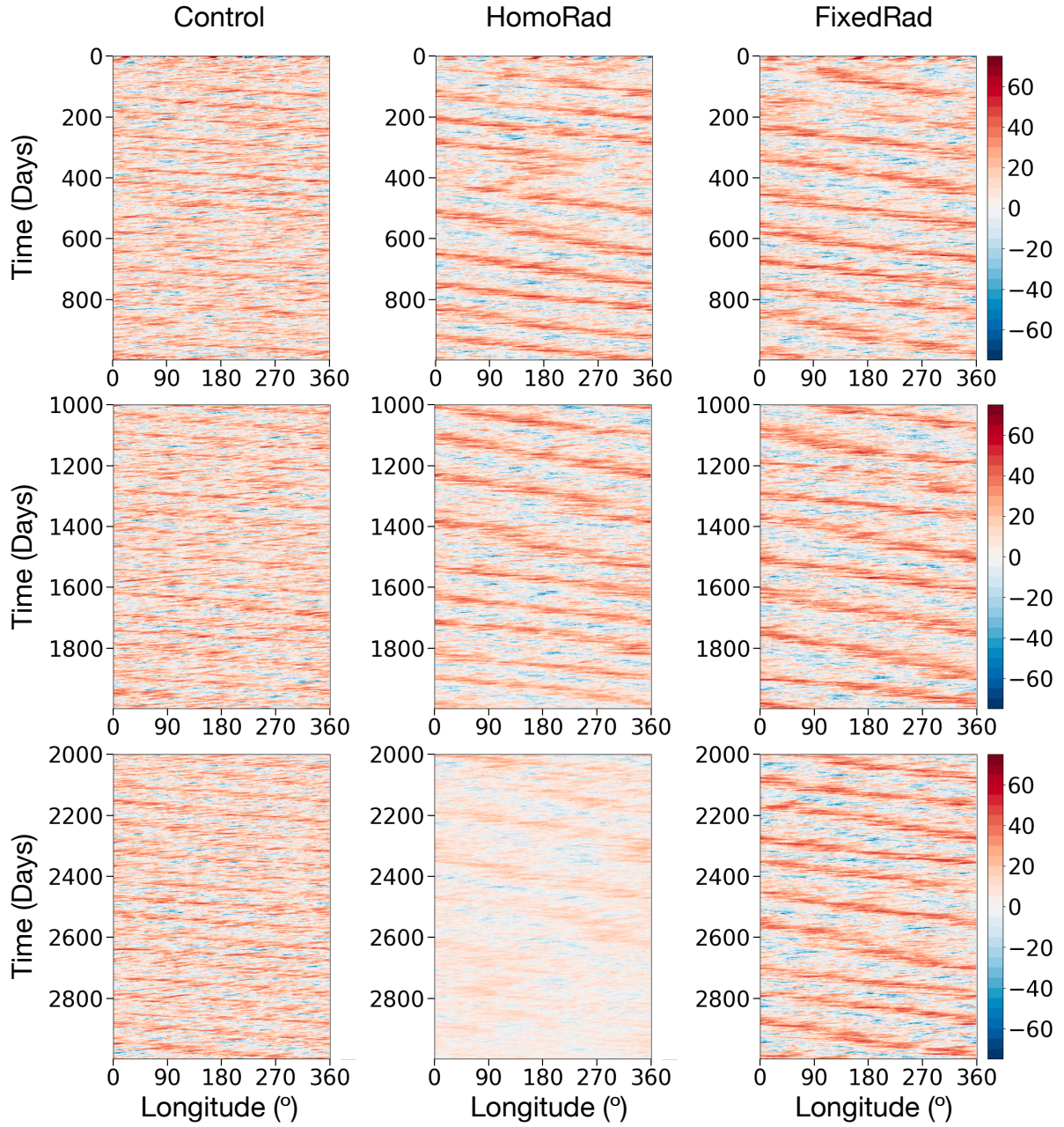


Figure S2. Hovmöller diagrams of OLR anomalies for the entire simulation period.

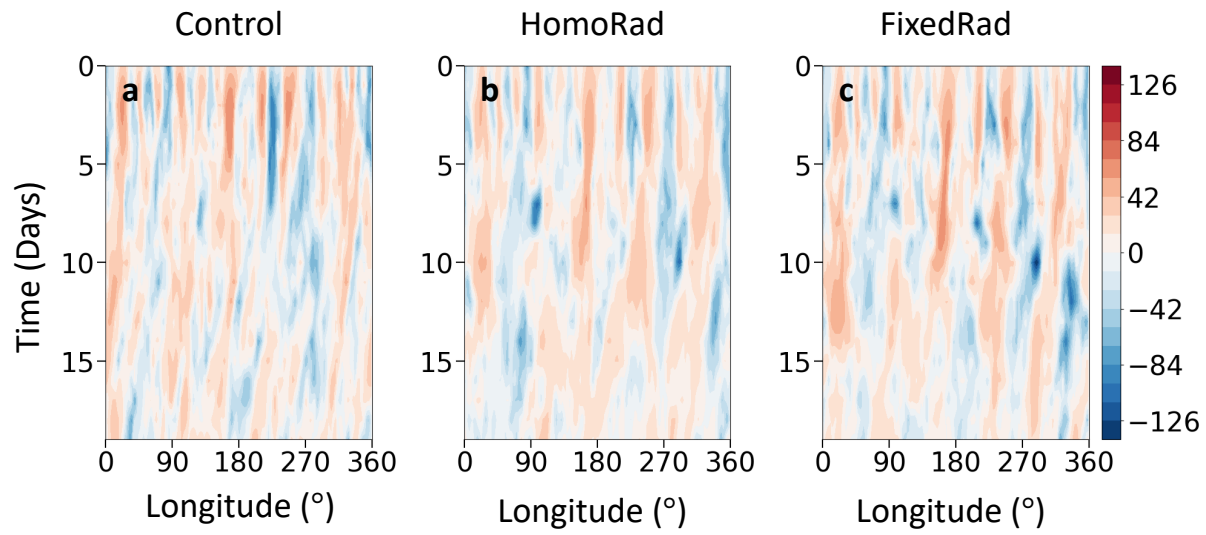


Figure S3. Hovmöller diagrams of OLR anomalies for the first 20 days.

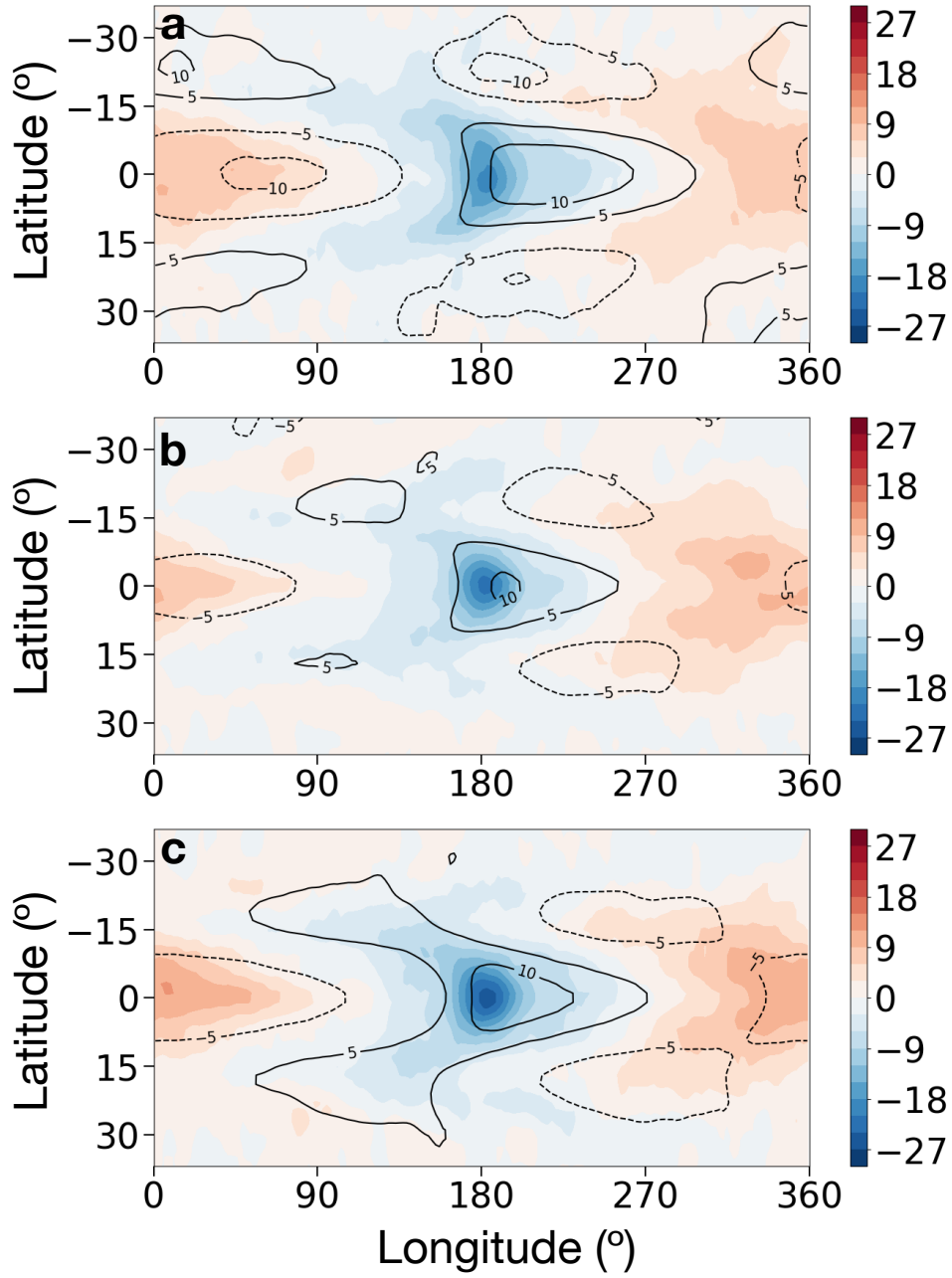


Figure S4. Horizontal structures of the MJO composite. (a) The control simulation. (b) The HomoRad simulation. (c) The FixedRad simulation. Color shading represents OLR anomalies, and contours represent geopotential anomalies at 200 hPa associated with the MJO.

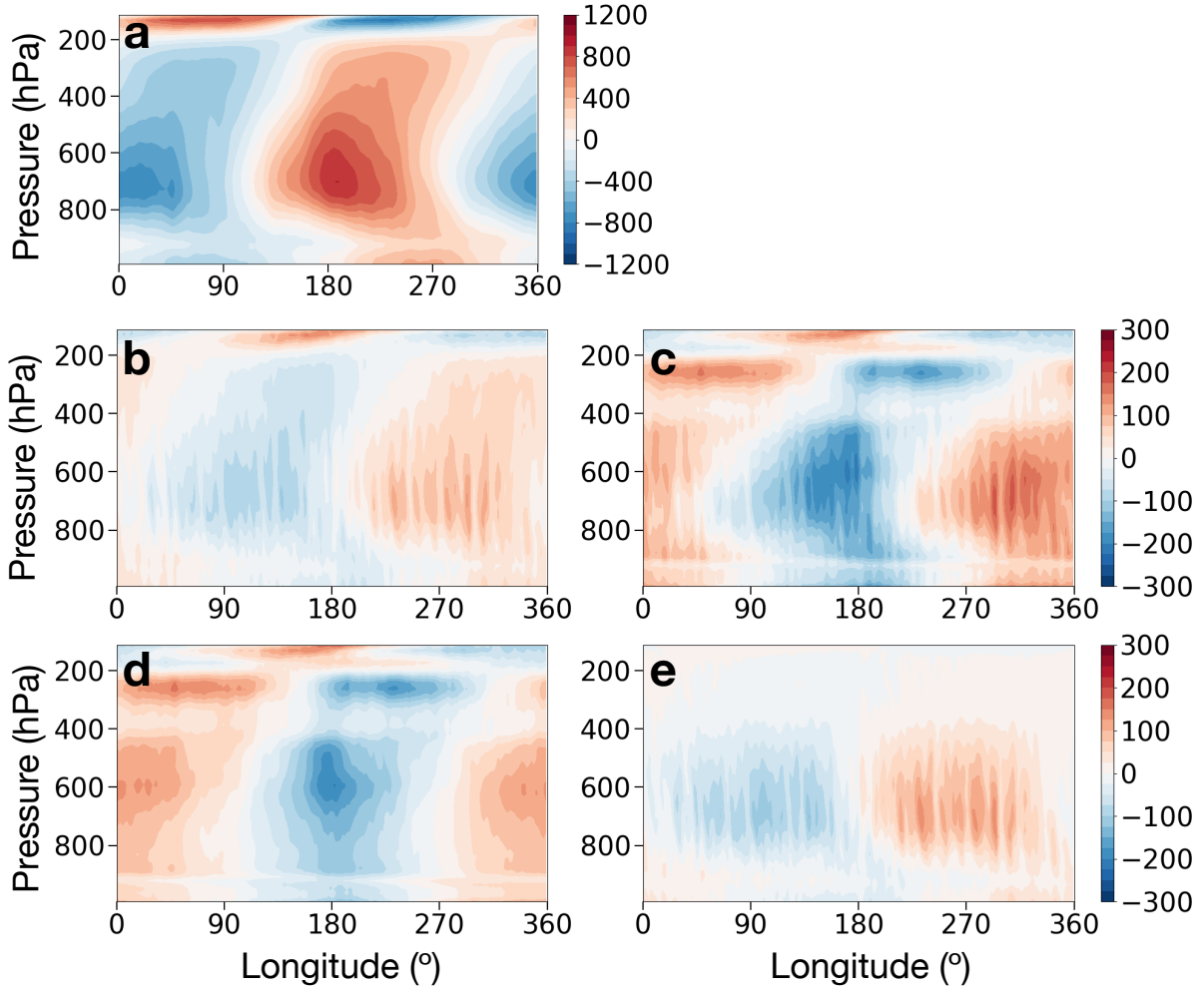


Figure S5. Vertical structures of the MSE budget of the MJO composite. (a) The MJO's MSE anomaly (J/kg). (b) Total MSE tendency $\partial_t \text{MSE}$ (J/kg/day). (c) Total MSE tendency due to convection (CRM), boundary layer, and large-scale dynamics (J/kg/day). (d) and (e) show the DSE and moisture components of (c), respectively (J/kg/day).

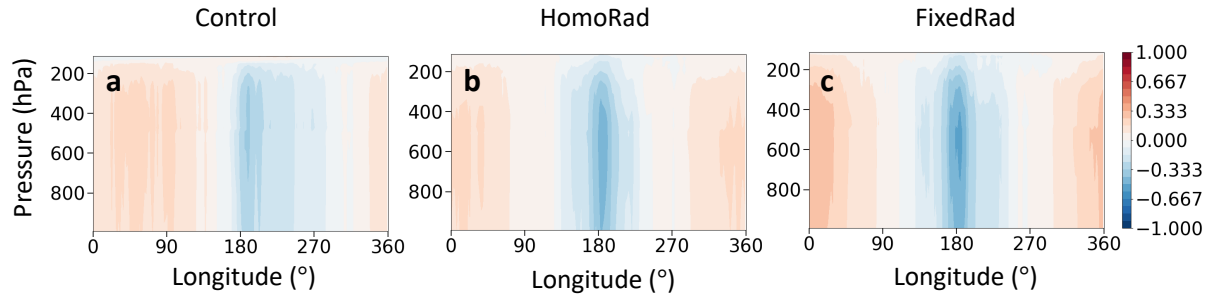


Figure S6. The residual of the MSE budget of the MJO composite (Equation (1)). The magnitude of the residual is two or more orders of magnitude smaller than the terms in Equation (1) (Figures 3 and S5).