Risk-Sharing Externalities

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Financial crises typically occur because firms and financial institutions are highly exposed to aggregate shocks. We propose a theory to explain these exposures. We study a model where entrepreneurs can issue state-contingent claims to consumers. Even though entrepreneurs can use these instruments to hedge negative shocks, they do not necessarily do so because insuring against these shocks is expensive, as consumers are also harmed by them. This effect is self-reinforcing because riskier balance sheets for entrepreneurs imply higher income volatility for the consumers, making insurance more costly in equilibrium. We show that this feedback is quantitatively important and leads to inefficiently high risk exposure for entrepreneurs.

I. Introduction

The exposure of financial institutions to risks from the subprime mortgage market is widely seen as a root cause of the financial crisis of 2008–9. This exposure created the potential for shocks in the housing market

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to be heavily amplified, as recognized early on by Greenlaw et al. (2008). Why did banks not do more to protect their balance sheet—say, by shedding some of their riskier positions or by choosing a safer funding structure? More generally, why were these risks not better spread across the economy?

Spurred by the global financial crisis, economists have developed models in which balance sheet losses of financial institutions can negatively affect firms' hiring and investment decisions—for example, Gertler and Kiyotaki (2010), Jermann and Quadrini (2012), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014). These contributions provide the framework now commonly used to quantify the importance of financial factors over the business cycle and to design appropriate policy responses. However, these models sidestep the questions raised above, by assuming that the "specialists"—the agents who represent financial institutions—have limited risk-management tools. In particular, a common assumption in these models is that specialists hold only one risky asset and issue non-state-contingent debt, so that their risk exposure is mechanically linked to their leverage. In this paper, we break this tight link by allowing specialists to issue fully state-contingent debt and study why they choose to be exposed to aggregate risk, whether this exposure is socially efficient, and if not, what the appropriate policy response is.

Our paper makes two contributions. First, we offer an explanation of why specialists are exposed to aggregate risk. Our mechanism builds on a general equilibrium effect: when the net worth of specialists falls and the economy experiences a financial crisis, the income of all other agents contracts as well. Due to this feature, insuring these states of the world ex ante is costly, and this reduces the specialists' incentive to hedge. Second, we show that equilibrium risk management is suboptimal from the point of view of social welfare and study corrective policies. The optimal policy requires differentially taxing debt to be repaid in bad states, and the associated welfare gains cannot be achieved by a simpler, non-state-contingent tax on borrowing.

We develop these arguments in the context of a model with two groups of agents: consumers and entrepreneurs. Entrepreneurs are the specialists, and we can think of them as representing a sector that consolidates financial institutions and the nonfinancial firms that borrow from them. Entrepreneurs borrow from consumers to finance their purchases of factors of production, capital, and labor. The source of risk in the economy is a shock that affects the "quality" of capital held by the entrepreneurs, as in Gertler and Karadi (2011) and Brunnermeier and Sannikov (2014). Owing to limited enforcement, the entrepreneurs face an upper bound on their ability to raise funds from consumers. This implies that reductions in the aggregate net worth of the entrepreneurs can lead to a contraction in economic activity and the labor income of consumers. This is the general equilibrium effect (or, "macro spillover") at the core of our positive and normative results.

The entrepreneurs in our model can issue a full set of state-contingent claims. This assumption is meant to capture a variety of ways in which financial institutions can make their balance sheet less exposed to aggregate shocks—for example, by choosing between debt and equity financing, choosing debt of different maturities, choosing debt denominated in different currencies, taking derivative positions, and so on. By appropriately using state-contingent claims, the entrepreneurs can hedge their net worth against aggregate shocks. For example, they can promise smaller payments to consumers when the economy is hit by a negative shock. This would imply that consumers bear more aggregate risk and would stabilize entrepreneurs' net worth. A more stable net worth would dampen financial amplification in the economy.

We start by studying the positive implications of the model, focusing on the equilibrium allocation of risk between consumers and entrepreneurs. We show that the elasticity of entrepreneurs' net worth to aggregate shocks depends on two key model ingredients: the strength of the macro spillover described above and the risk aversion of consumers. The macro spillover implies that states of the world in which the entrepreneurs have low net worth are also states in which the consumers have low labor income. Risk aversion implies that consumers demand a premium for bearing risk in these states of the world. These two ingredients combined make it costly for entrepreneurs to hedge.

We first show this result theoretically, in a special case of our model that is analytically tractable. Next, we show that this mechanism can be quantitatively strong and produce a large exposure of entrepreneurs to aggregate risk. Specifically, under plausible calibrations our economy with state-contingent debt produces an elasticity of entrepreneurial net worth to aggregate shocks and a degree of financial amplification that is quantitatively comparable to those obtained in the corresponding economy where entrepreneurs can issue only non-state-contingent debt. These results are not driven by the type of aggregate shocks we consider, as we obtain very similar results when the aggregate shock affects the pledgeability of capital, as in Jermann and Quadrini (2012), rather than the capital stock.

The presence of the macro spillover not only hinders risk-sharing between consumers and entrepreneurs but also generates a pecuniary externality that makes the privately optimal portfolio choices of the agents socially inefficient. To understand the source of this externality, consider the problem of consumers. When choosing their financial assets, they do not understand that any payment received in a given state of the world reduces the net worth of entrepreneurs, and it negatively affects the current and future wages of consumers if the collateral constraint binds. Because consumers fail to internalize these negative spillovers, they overvalue payments received in these states relative to what a social planner would do. Thus, the interest rate for debt instruments indexed to these states is inefficiently low, and these low interest rates induce entrepreneurs to take on excessive risk.

In the last part of this paper, we study the optimal policy of a social planner that can impose Pigouvian taxes on the state-contingent claims issued by the entrepreneurs. We show that the optimal policy does not tax debt uniformly. Rather, it levies higher taxes on debt instruments indexed to states in which collateral constraints are tighter. These policies are successful in reducing the risk exposure of the entrepreneurs, and the resulting equilibrium features less financial amplification.

We finally contrast the optimal policy with a policy that taxes all debt instruments uniformly. These taxes could reduce the risk exposure of entrepreneurs because they reduce their incentives to issue debt. However, entrepreneurs respond by cutting mostly debt indexed to good states of the world, so their overall risk exposure changes little. That is, these tools are effective in reducing leverage, but they generate an incentive for entrepreneurs to substitute toward riskier types of debt. These substitution effects provide a cautionary tale for macroprudential tools that target leverage uniformly.

Related literature.—This paper is related to the large literature on the role of financial factors in the amplification and propagation of aggregate shocks. This literature goes back to the seminal contributions of Bernanke and Gertler (1986), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999) and has been very active following the global financial crisis. The logic of financial amplification in these models builds on two main assumptions: the presence of a financial constraint and incomplete financial markets. The first assumption implies that aggregate shocks affecting the net worth of specialists propagate to the rest of the economy, while the second assumption restricts the ability of the specialists to hedge aggregate shocks ex ante.

Important contributions in this literature show that the assumed incompleteness of financial markets is critical for financial amplification. Krishnamurthy (2003) introduces state-contingent claims in a three-period version of Kiyotaki and Moore (1997) and shows that the amplification mechanism disappears, as specialists perfectly hedge their net worth. Di Tella (2017) shows an analogous result in the context of a dynamic model similar to Brunnermeier and Sannikov (2014). Our incomplete hedging results may appear surprising in light of these contributions. However, as argued above, our results require two ingredients: risk-averse consumers and an active macro spillover. One or both of these ingredients are muted in these papers. Other papers that find limited amplification in more quantitative models are Carlstrom, Fuerst, and Paustian (2016), Cao and Nie (2017), and Dmitriev and Hoddenbagh (2017). The mechanism identified in our paper is potentially at work in those models, but—as we discuss in section IV—their calibrations make it quantitatively weak.

The literature has explored other mechanisms to explain why specialists are exposed to aggregate risk even if they can hedge it. Some papers explore different types of shocks. Di Tella (2017) obtains imperfect hedging in response to shocks to idiosyncratic volatility, while Dávila and Philippon (2017) obtain it in response to shocks to the degree of financial market completeness. Other papers look at alternative models of the financial friction. In particular, Rampini and Viswanathan's (2010) imperfect hedging result relies on the collateral constraint being always binding and on collateral values being insensitive to the shock, while Asriyan (2018) obtains imperfect hedging by combining information and trading frictions, which leads to distorted state prices. A large literature, including Schneider and Tornell (2004) and Farhi and Tirole (2012), emphasizes the possibility of collective moral hazard, whereby specialists choose to be exposed to aggregate risk, given the expectation of government bailouts when a large enough number of them is in trouble. Finally, a recent literature focuses on neglect of downside risks, coming from deviations from rational expectations—for example, Bordalo, Gennaioli, and Shleifer (2018) and Farhi and Werning (2020). Our approach emphasizes a simple general equilibrium spillover from specialists to the rest of the economy, and we see it as complementary to these other approaches.

Our welfare analysis is related to the large literature on inefficiencies and pecuniary externalities in models with financial market imperfections, going back to Geanakoplos and Polemarchakis (1986) and Kehoe and Levine (1993). The pecuniary externality that matters in our model is "distributive"—using the language introduced by Dávila and Korinek (2018)—and works through wages and labor income. This connects our paper to Caballero and Lorenzoni (2014), Bianchi (2016), and Itskhoki and Moll (2019), although we are the first to explore the implications that this type of pecuniary externality has on risk-sharing.

A number of papers study models in which constrained inefficiency takes the form of excessive leverage and derive implications for macroprudential policy (see Bianchi and Mendoza 2018 and references therein). Unlike many of these papers, we study an environment where the specialists can issue multiple types of debt rather than simply a noncontingent bond and study the optimal policy when the planner can tax these debt instruments differently. Lorenzoni (2008) and Korinek (2018) also allow for state-contingent claims and different Pigouvian taxation on these claims. Differently from them, we study optimal policy in an environment where risk premia are endogenous, and we compare it with a more restricted policy that taxes these instruments uniformly. A key insight from our analysis is that, in the presence of state contingency, some simple policies, such as a restriction on total leverage, may be ineffective in reducing risk-taking or can even backfire and lead to increased risk.

Our emphasis on inefficient risk-taking also connects our paper to work by Farhi and Werning (2016) that focuses on the effects of consumers' risk-taking on the demand for goods in environments with nominal rigidities. Their analysis shows the possibility of excessive risk-taking due to an aggregate demand externality. We instead emphasize the risk-taking by the financial and corporate side of the economy and its transmission through labor demand and the equilibrium volatility of labor income.[1]

Finally, the macro spillover that plays a central role in this paper was also present in our previous work on self-fulfilling currency crises (Bocola and Lorenzoni 2020). However, the analysis of how that spillover affects amplification and efficiency is novel to this paper.

The paper is organized as follows. Section II introduces the model. Section III studies a special case that is analytically tractable. Section IV presents numerical results for a calibrated version of the model. Section V presents the welfare analysis and its implications for macroprudential policies. Section VI concludes.

II. Model

We consider an economy populated by two groups of agents of equal size: consumers and entrepreneurs. Entrepreneurs accumulate capital, which is used together with labor to produce the final good, and they issue financial claims. Consumers earn labor income and buy financial claims from entrepreneurs. Financial claims are state-contingent promises to repay one unit of consumption in the next period. We now describe the details of the environment and define an equilibrium.

A. Environment

1. Technology and Shocks

Time is discrete and indexed by $t = 0, 1, 2 \ldots$. Uncertainty is described by a Markov process that takes finite values in the set $\mathcal{S}$. We denote by s_t the state of the process at time t and by $s^t = (s_0, s_1, \ldots, s_t)$ the history of states up to period t. The process for s_t is given by the transition matrix $\pi(s_{t+1}|s_t)$.

The capital stock is subject to random depreciation captured by the stochastic parameter u_t. Namely, k_{t-1} units of capital accumulated at the end of time $t-1$ yield $u_t k_{t-1}$ units of capital that can be used in production at time t and a residual stock of $(1-\delta)u_t k_{t-1}$ units of capital after production.

[1] See also Dávila and Korinek (2018) for a general analysis of the role of state-contingent wedges in models of pecuniary externalities.

The parameter u_t depends on the state of the Markov process according to the function $u_t = u(s_t)$ and is the only exogenous source of uncertainty in the model. The variable u_t is similar to the capital quality shock used in Gertler and Karadi (2011) and Brunnermeier and Sannikov (2014).

Entrepreneurs have exclusive access to the technology that allows capital accumulated in period $t - 1$ to be productive in period t, so all capital is held by entrepreneurs in equilibrium. The entrepreneurs use capital and labor services provided by consumers to produce final goods according to the Cobb-Douglas production function:

$$y_t = (u_t k_{t-1})^{\alpha} l_t^{1-\alpha}.$$

The labor market is perfectly competitive, and the wage rate is w_t. We assume that entrepreneurs need to pay a fraction γ of the wage bill before their revenues are realized. This assumption ensures that the financial conditions of entrepreneurs can have a contemporaneous effect on labor demand (see Jermann and Quadrini 2012). In general, all equilibrium variables are functions of the history s^t, but whenever no confusion is possible we leave this dependence implicit in the subscript t.

2. Preferences

Entrepreneurs have log preferences over consumption streams $\{c_{e,t}\}$, so they maximize

$$\mathbb{E}_t\left[\sum_{t=0}^{\infty}\beta_e^t \log(c_{e,t})\right].$$

Consumers have Epstein-Zin preferences, so their utility is defined recursively as

$$V_t = \left\{(1-\beta)x_t^{1-\rho} + \beta\left[\mathbb{E}_t(V_{t+1}^{1-\sigma})\right]^{(1-\rho)/(1-\sigma)}\right\}^{1/(1-\rho)},$$

where x_t is given by

$$x_t = c_t - \chi\frac{l_t^{1+\psi}}{1+\psi}.$$

This specification of the consumers' utility eliminates the wealth effect on labor supply, as in Greenwood, Hercowitz, and Huffman (1988).

3. Financial Markets and Limited Commitment

Each period, agents trade a full set of one-period state-contingent claims. Let $q(s_{t+1}|s^t)$ represent the price at time t of a claim that pays one unit of consumption at $t + 1$, conditional on history $s^{t+1} = (s^t, s_{t+1})$. We denote by $a(s^t)$ the claims held by consumers at the beginning of period t. Similarly, $b(s^t)$ denotes the claims owed by entrepreneurs at the beginning of the period. Market clearing requires that

$$a(s^t) = b(s^t)$$

for every history s^t.

Entrepreneurs enter period t with $u_t k_{t-1}$ units of capital (in efficiency units) and debt b_t. Each period t is divided into three stages. In the first stage, entrepreneurs hire workers and issue within-period debt to pay for a fraction γ of their wage bill $w_t l_t$. In the second stage, production takes place, goods are sold, and entrepreneurs pay the remaining fraction of the wage bill $(1-\gamma)w_t l_t$ and decide whether to repay their total liabilities $b_t + \gamma w_t l_t$ or default. If they default, entrepreneurs can hide the firms' profits and a fraction $1-\theta$ of the undepreciated capital stock and start anew with initial wealth

$$y_t - (1-\gamma)w_t l_t + (1-\theta)(1-\delta)u_t k_{t-1}.$$

In the third and last stage, entrepreneurs issue new liabilities $b(s^{t+1})$ and use these resources along with their net worth to buy capital goods.[2]

Notice that we assume that an entrepreneur who defaults is not excluded from financial markets.[3] It follows that the entrepreneur chooses repayment if and only if

$$y_t - w_t l_t - b_t + (1-\delta)u_t k_{t-1} \geq y_t - (1-\gamma)w_t l_t + (1-\theta)(1-\delta)u_t k_{t-1}.$$

Making explicit the dependence on the state of the world, this constraint is equivalent to the state-contingent collateral constraint

$$b(s^t) + \gamma w(s^t)l(s^t) \leq \theta(1-\delta)u(s_t)k(s^{t-1}). \tag{1}$$

B. *Competitive Equilibrium*

In a competitive equilibrium, consumers choose sequences for consumption, labor supply, and state-contingent claims to maximize their utility subject to the budget constraint

$$c(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)a(s^{t+1}) = w(s^t)l(s^t) + a(s^t)$$

[2] If entrepreneurs default, we assume that the fraction θ of capital not hidden by the entrepreneurs gets destroyed. Alternative assumptions are possible here, as default happens only off the equilibrium path.

[3] A similar assumption is made in Rampini and Viswanathan (2010) and Cao, Lorenzoni, and Walentin (2019).

for each history s^t and a no-Ponzi-game condition. The first-order condition for $a(s^{t+1})$ takes the form

$$q(s_{t+1}|s^t) = \beta\pi(s_{t+1}|s_t)\left(\frac{x(s^t)}{x(s^{t+1})}\right)^{\rho}\left(\frac{RW(s^t)}{V(s^{t+1})}\right)^{\sigma-\rho}, \tag{2}$$

where $RW(s^t) = \mathbb{E}_t[V(s^t, s_{t+1})^{1-\sigma}]^{1/(1-\sigma)}$. Optimal labor supply requires that

$$\chi l(s^t)^{\psi} = w(s^t). \tag{3}$$

Entrepreneurs choose sequences for consumption, capital, labor demand, and state-contingent claims to maximize their utility subject to the collateral constraints (1) and their budget constraint

$$c_e(s^t) + k(s^t) = n(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)b(s^{t+1}),$$

where $n(s^t)$ represents the net worth of the entrepreneurs

$$n(s^t) = y(s^t) - w(s^t)l(s^t) + (1-\delta)u(s_t)k(s^{t-1}) - b(s^t). \tag{4}$$

Denoting by $\mu(s^{t+1})$ the Lagrange multiplier on the collateral constraint in state s^{t+1}, we can write the entrepreneurs' first-order conditions for $b(s^{t+1})$ as

$$q(s_{t+1}|s^t)\frac{1}{c_e(s^t)} = \beta_e\pi(s_{t+1}|s_t)\left(\frac{1}{c_e(s^{t+1})} + \mu(s^{t+1})\right). \tag{5}$$

This condition is a standard intertemporal Euler equation with state-contingent debt and collateral constraints.

Combining equations (2) and (5), we obtain

$$\beta_e\frac{1/c_e(s^{t+1}) + \mu(s^{t+1})}{1/c_e(s^t)} = \beta\left(\frac{x(s^t)}{x(s^{t+1})}\right)^{\rho}\left(\frac{RW(s^t)}{V(s^{t+1})}\right)^{\sigma-\rho}. \tag{6}$$

This is the risk-sharing condition that determines the allocation of aggregate risk in this economy. On the right-hand side, there is the consumers' marginal rate of substitution between consumption at time t and consumption at $t+1$ in state s^{t+1}. On the left-hand side, there is a similar expression for entrepreneurs: the marginal value of a unit of resources in state s^{t+1} for entrepreneurs includes both the marginal utility of consumption $1/c_e(s^{t+1})$ and the shadow value of relaxing their collateral constraint $\mu(s^{t+1})$.

The optimality conditions for labor and capital take the following form:

$$\frac{1}{c_e(s^t)}\left[(1-\alpha)[u(s_t)k(s^{t-1})]^{\alpha}l(s^t)^{-\alpha} - w(s^t)\right] = \gamma w_t\mu(s^t), \tag{7}$$

$$\frac{1}{c_e(s^t)} = \mathbb{E}_t\left\{\beta_e \frac{1}{c_e(s^{t+1})}\left[\alpha u(s_{t+1})^{\alpha}\left(\frac{l(s^{t+1})}{k(s^t)}\right)^{1-\alpha} + (1-\delta)u(s_{t+1})\right]\right\} + \beta_e\theta(1-\delta)\mathbb{E}_t[u(s_{t+1})\mu(s^{t+1})]. \quad (8)$$

The first condition shows that there is a wedge between the marginal product of labor and the wage if the collateral constraint is binding, because hiring labor requires some capacity to borrow. The second condition is a standard intertemporal condition for capital accumulation. Relative to a frictionless economy, investing in capital has the additional benefit of relaxing the collateral constraints, which is captured by the last term on the right-hand side.[4]

The advantage of assuming log preferences for entrepreneurs is that their consumption function is linear in net worth, $c_e(s^t) = (1-\beta_e)n(s^t)$, irrespective of whether the collateral constraint is binding. This property, proved in section A of the online appendix, simplifies the analysis of the equilibrium.

An equilibrium is given by sequences of quantities $\{c(s^t), c_e(s^t), k(s^t), l(s^t), a(s^t), b(s^t)\}$ and prices $\{w(s^t), q(s_{t+1}|s^t)\}$, such that the quantities solve the individual optimization problems above and markets clear.

C. Discussion

Before moving on, let us discuss some of the simplifying assumptions we made. First, our model does not feature endogenous asset prices, as the price of capital is always one. This mutes a canonical feedback between asset prices and entrepreneurial net worth, which may lead to inefficiently high levels of risk-taking, as shown, for example, in Lorenzoni (2008). In the current paper, we abstract from this channel to isolate the novel mechanism that works via the endogeneity of labor income. We do not expect endogenous asset prices to substantially change the mechanism investigated here.

Second, the main driving force in the model is a shock to the quality of capital. In our framework, this shock substitutes for the missing volatility of asset prices and allows us to generate sizable movements in the value of assets held by entrepreneurs. As we discuss in section IV and in more detail in section C of the online appendix, our mechanism does not rely on this specific source of risk and is still present with different types of aggregate shocks.

[4] This does not mean that more capital is invested relative to an economy without the collateral constraints because, in equilibrium, the collateral constraints also affect the stochastic discount factor of entrepreneurs.

Finally, entrepreneurs and consumers are assumed to be distinct agents, a fairly common assumption in the literature. There are different ways to interpret this assumption. One is to view the entrepreneurs as the controlling shareholders of the financial firms they represent and to interpret all equity financing they raise as part of the state-contingent claims issued. The other one is to interpret the entrepreneurs as all the shareholders of these firms, with consumers being barred from holding shares. In the second interpretation, it would be interesting to allow for the possibility of issuing shares to all agents, subject to some friction (as in, e.g., Gertler and Kiyotaki 2010), something we leave to future work.

III. Equilibrium Risk-Sharing and Financial Amplification

In this section and the next, we characterize the risk-sharing problem of consumers and entrepreneurs and show how it affects the economy's response to aggregate shocks. In particular, we study to what extent the effects of the capital quality shocks are amplified due to the presence of the collateral constraint and how this "financial amplification" depends on the equilibrium allocation of aggregate risk between consumers and entrepreneurs. In this section, we consider a simplified version of the model and focus on analytical results. In the next section, we go back to the full model and derive numerical results.

We consider a special case of our economy in which all uncertainty is resolved in one period. The economy starts at date 0 with $u_0 = 1$. At $t = 1$, the capital quality u_1 is drawn from a continuous distribution on the interval $[\underline{u}, \bar{u}]$, with density $f(u_1)$. From $t = 2$ on, the capital quality is deterministic and equal to $u_t = 1$. We make some additional simplifying assumptions: entrepreneurs and consumers have the same discount factor, $\beta_e = \beta$; the elasticity of intertemporal substitution is infinite, $\rho = 0$; there is no working capital requirement, $\gamma = 0$; and labor supply is inelastic at $l_t = 1$.

Given the assumptions above, we can characterize an equilibrium in two steps. First, we study the equilibrium from date 1 on, taking as given the equilibrium level of capital and the contingent bonds chosen by entrepreneurs and consumers at date 0, $\{k_0, b_1(u_1), a_1(u_1)\}$. This part of the analysis is standard. Second, we go back to date 0 and study how these variables are determined in equilibrium. This is the novel part of our analysis.

A. *Continuation Equilibrium*

From date 1 on, the economy follows a deterministic path. Since there is no uncertainty and $\rho = 0$, the interest rate is constant and equal to $1/\beta - 1$. In addition, the absence of working capital requirements means that firms are unconstrained in hiring labor, so wages are equal to the marginal product of labor

$$w_t = (1 - \alpha)(u_t k_{t-1})^\alpha.$$

The dynamics of k_t and n_t are characterized as follows. For a finite number of periods J, the collateral constraint binds and the dynamics of capital and net worth are determined by the recursion

$$k_t = \frac{\beta n_t}{1 - \beta\theta(1 - \delta)}, \qquad n_{t+1} = \alpha k_t^\alpha + (1 - \delta)(1 - \theta)k_t, \tag{9}$$

given an initial condition for net worth at date $t = 1$. The first expression in (9) comes from the fact that entrepreneurs save a fraction β of their wealth and they can lever it at most by the factor $1/[1 - \beta\theta(1 - \delta)]$. The second expression is obtained by combining the definition of net worth in equation (4), the wage derived above, and the binding collateral constraint (1). After J periods, the collateral constraint is slack, n_t is constant in all following periods, and the capital stock reaches the unconstrained level

$$k^* = \left(\frac{\alpha\beta}{1 - (1 - \delta)\beta}\right)^{1/(1-\alpha)}. \tag{10}$$

The number of periods J that the economy spends in the constrained region depends on the value of net worth at date 1,

$$n_1(u_1) = \alpha(u_1 k_0)^\alpha + (1 - \delta)u_1 k_0 - b_1(u_1). \tag{11}$$

In the above expression, we use $n_1(u_1)$ to denote the equilibrium relation between net worth and the capital quality shock u_1. If $n_1(u_1)$ is above the threshold $n^* = k^*[1 - \beta\theta(1 - \delta)]/\beta$, then the entrepreneur has enough resources to finance the unconstrained level of capital k^*. In this case, $J = 0$ and the economy reaches the first-best allocation in period 1. Otherwise, $J > 0$, and the economy evolves according to (9).

Because $\rho = 0$, we can use the consumers' intertemporal budget constraint to compute consumers' utility at $t = 1$:

$$V_1 = (1 - \beta)\left[a_1(u_1) + \sum_{t=0}^{\infty}\beta^t w_{t+1}\right]. \tag{12}$$

For future reference, it is useful to split the present value of labor income into two parts, $w_1 = (1 - \alpha)(u_1 k_0)^\alpha$ and $W \equiv \sum_{t=1}^{\infty} \beta^t(1 - \alpha)k_t^\alpha$. In equilibrium, W is a function only of the entrepreneurs' net worth n_1. We denote this relation by $W(n_1)$. A higher value of n_1 implies a (weakly) higher path of capital accumulation and therefore a (weakly) higher path of wages.

The next lemma summarizes the properties of the continuation equilibrium.

LEMMA 1 (Continuation equilibrium). There is a unique continuation equilibrium that depends only on the state variables k_0, u_1, and $b_1(u_1)$ and does not depend on the parameter σ. In the continuation equilibrium, the collateral constraint is binding for a finite number of periods J, with $J = 0$ if and only if $n_1(u_1) \geq n^*$. The present value of future wages at $t = 1$, $W(n_1(u_1))$, is strictly increasing for $n_1 < n^*$ and constant for $n_1 \geq n^*$.

B. *Risk-Sharing at Date 0 and Financial Amplification*

Equation (11) shows that the shape of the function $n_1(u_1)$ depends on the portfolio choices of entrepreneurs at $t = 0$—that is, on $b_1(u_1)$. We now study how $b_1(u_1)$ is determined in equilibrium.

To simplify our discussion, we focus on the special case in which the collateral constraints at date 0, $b_1(u_1) \leq \theta(1 - \delta)u_1 k_0$, do not bind in equilibrium, so $\mu_1(u_1) = 0$ for all u_1.[5] The risk-sharing condition (6) then takes the form

$$\left(\frac{RW_0}{V_1(u_1)}\right)^{\sigma} = \frac{n_0}{n_1(u_1)} \qquad \text{for all } u_1. \tag{13}$$

From equation (13) we can derive the equilibrium sensitivity of entrepreneurs' bond issuance and net worth to the capital quality shock, as shown in the next proposition. Let $\omega(u_1)$ denote the period 1 ratio of entrepreneurs' wealth to total wealth in the economy, including the human wealth of consumers—that is,

$$\omega(u_1) \equiv \frac{n_1(u_1)}{n_1(u_1) + a_1(u_1) + w_1(u_1) + W(n_1(u_1))}.$$

PROPOSITION 1. There exists a level of the consumers' coefficient of relative risk aversion $\hat{\sigma} > 0$ such that if $\sigma \in [0, \hat{\sigma}]$ and

$$n_0 \geq \alpha\left(\bar{u}\hat{k}\right)^{\alpha} + (1 - \theta)(1 - \delta)\bar{u}\hat{k},$$

where $\hat{k} \equiv \alpha\beta\mathbb{E}(u_1^{\alpha})/\{1 - (1 - \delta)\beta\mathbb{E}(u_1^{\alpha})\}$, then, in equilibrium, the date 0 collateral constraint does not bind, $\mu_1(u_1) = 0$ for all u_1, and the sensitivities of debt payments and entrepreneurs' net worth to the u_1 shock are

[5] This restriction does not imply that the collateral constraint does not bind at $t = 0, 1, 2, \ldots$. Indeed, as we have seen in the analysis of continuation equilibrium, $\mu_t > 0$ for $t = 2, 3, \ldots, J$ when $n_1(u_1) < n^*$.

$$\begin{aligned} b_1'(u_1) &= \alpha^2 u_1^{\alpha-1} k_0^\alpha + (1-\delta)k_0 \\ &\quad - \frac{\omega}{\omega + (1-\omega)(1/\sigma) - \omega W'(n_1)} \left(\alpha u_1^{\alpha-1} k_0^\alpha + (1-\delta)k_0\right), \end{aligned} \tag{14}$$

$$n_1'(u_1) = \frac{\omega}{\omega + (1-\omega)(1/\sigma) - \omega W'(n_1)} \left(\alpha u_1^{\alpha-1} k_0^\alpha + (1-\delta)k_0\right). \tag{15}$$

The proof of this proposition is presented in the print appendix. Equations (14) and (15) provide an expression for the sensitivities of b_1 and n_1 to the shock u_1 in terms of the endogenous quantities k_0, $\omega(u_1)$, and $n_1(u_1)$ (the dependence on u_1 is omitted for readability).

We can use equations (14) and (15) and the results in lemma 1 to identify the forces that determine financial amplification in this model. As a benchmark, in the first-best case with no collateral constraints we have $k_t = k^*$ for all $t \geq 1$, implying that the shocks to capital quality do not affect the choice of capital by entrepreneurs.[6] Therefore, any positive response of k_1 to the u_1 shock is a form of financial amplification. Figure 1 illustrates the debt payments of entrepreneurs, their net worth, and choice of capital as a function of the capital quality shock when consumers are risk neutral (solid lines) and when they are risk averse (dashed lines).

Let us first study the case when consumers are risk neutral: $\sigma = 0$. In this case, we can see from proposition 1 that

$$b_1'(u_1) = \alpha^2 k_0^\alpha u_1^{\alpha-1} + (1-\delta)k_0, \qquad n_1'(u_1) = 0.$$

When consumers are risk neutral, debt payments in equilibrium are structured so that entrepreneurs pay more to consumers when the realization of the capital quality shock is good. This state contingency in debt payments allows the entrepreneurs to perfectly hedge against the aggregate shock—$n_1(u_1)$ is independent of u_1. Because $n_1'(u_1) = 0$, we also know from the characterization of the continuation equilibrium that k_1 is independent of u_1. Therefore, when consumers are risk neutral, there is no financial amplification in the model, in the sense that $k_1'(u_1) = 0$ as in the economy without the collateral constraint.[7] This echoes the baseline result in Krishnamurthy (2003).

When consumers are risk averse ($\sigma > 0$), they demand insurance against low capital quality states because those are states with a low present value of labor income. In equilibrium, this reduces the sensitivity of debt payments to the capital quality shock, and the net worth of entrepreneurs becomes positively related to u_1. If $n_1(u_1) \geq n^*$, k_1 is still independent from

[6] This is due to the assumption that consumers have linear preferences after $t = 1$ and the capital quality shock is independent and identically distributed.

[7] The level of k_1 can be different from the first-best, because entrepreneurs may still be constrained if $\mathbb{E}[u_1^\alpha] < 1$ and their initial level of net worth n_0 is small enough.

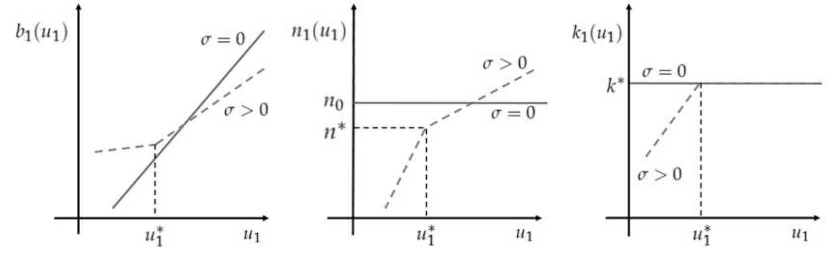

FIG. 1.—Debt payments, net worth, and capital as functions of the capital quality shock u_1.

u_1. However, a sufficiently negative capital quality shock at $t = 1$ can lead net worth to fall below the threshold n^*, in which case entrepreneurs are constrained and the level of capital falls below its first-best. In figure 1, this occurs for realizations of the capital quality shock below u_1^* (see dashed line).

This discussion emphasizes that the degree of financial amplification depends on the equilibrium sensitivity of net worth to u_1. Equation (15) identifies two key determinants of this elasticity: σ and W'. We now discuss the role of these two elements in detail.

The expression in parentheses on the right-hand side of (15) represents the effect of u_1 on the economy's resources. How much of that effect is borne by the entrepreneurs depends on the ratio

$$\frac{\omega}{\omega + (1 - \omega)(1/\sigma) - \omega W'(n_1)}.$$

To interpret this ratio, let us separately consider the cases $n_1(u_1) \geq n^*$ and $n_1(u_1) < n^*$.

If $n_1(u_1) \geq n^*$, then $W'(n_1)$ is zero, and the ratio above is simply

$$\frac{\omega}{\omega + (1 - \omega)(1/\sigma)}.$$

Define the risk tolerance as the inverse of the coefficient of relative risk aversion. Then the risk tolerance of the entrepreneurs is one—due to log preferences—and the average risk tolerance in the economy, weighted by the agents' wealth shares, is $\omega + (1 - \omega)1/\sigma$. Therefore, we obtain the standard result that agents share aggregate risk in proportion to their risk tolerance: the less risk tolerant consumers are, the higher the sensitivity of entrepreneurial net worth to the aggregate shock in equilibrium. See Gârleanu and Panageas (2015) for an example.

Equation (15) highlights a second determinant of the equilibrium risk-taking behavior of entrepreneurs, which operates only when the collateral constraint in the continuation equilibrium binds, $n_1(u_1) < n^*$. Because $W'(n_1) > 0$ in this constrained region, we can see from equation (15) that the share of the shock borne by entrepreneurs is larger.[8] The intuition for the last result is that a reduction in $n_1(u_1)$ in the constrained region reduces consumers' lifetime labor income, making them more willing to purchase state-contingent claims that pay off in that contingency. In equilibrium, this makes it harder for the entrepreneurs to smooth their net worth in those states of the world, increasing the sensitivity of $n_1(u_1)$ to u_1. In other words, the response of $n_1(u_1)$ increases the background risk perceived by consumers endogenously, making it costlier for the entrepreneurs to insure against the aggregate shock.

The importance of endogenous labor income in the results above can also be seen by comparing our model with a different environment with an "AK" technology. With this production function, consumers do not earn labor income and their consumption is financed only by holdings of financial assets. In section D of the online appendix, we show that such model features no financial amplification relative to the first-best economy even when consumers are risk averse, as long as they have the same constant relative risk aversion preferences as entrepreneurs. This case is closely related to the no-amplification result in Di Tella (2017), who also considers an economy with an AK technology.

IV. Quantitative Analysis

In this section, we go back to the fully fledged stochastic model and use numerical simulations to evaluate the strength of financial amplification under plausible calibrations of the model parameters. We compare our baseline economy with complete markets with two other economies: a first-best economy, equivalent in all respects to the benchmark with the exception that entrepreneurs do not face the collateral constraints (1), and an incomplete markets economy, in which entrepreneurs can issue only non-state-contingent bonds, so the following additional constraint is present:

$$b(s^t, s_{t+1}) = \bar{b}(s^t) \qquad \forall\ (s^t, s_{t+1}).$$

In the incomplete markets economy, the limited enforcement friction implies the financial constraints

[8] This is the reason why in fig. 1 the relation between $n_1(u_1)$ and u_1 is steeper when $n_1(u_1) < n^*$.

$$\bar{b}(s^t) + \gamma\omega(s^{t+1})l(s^{t+1}) \leq \theta(1-\delta)u(s_{t+1})k(s^t) \quad (16)$$

for all (s^t, s_{t+1}).

A. *Calibration*

Table 1 reports the model parameters used in our simulations. A period in the model corresponds to a quarter. We set the following parameters to standard values: the capital income share α is 0.33, the depreciation of capital is 2.5%, the discount factor of consumers β is 0.99, and the Frisch elasticity of labor supply ψ is 1. In addition, we choose χ so that worked hours are equal to 1 in the deterministic steady state of the model. We further set ρ to 1, so that consumers have a unitary elasticity of intertemporal substitution as entrepreneurs. The parameter γ represents the fraction of the wage bills that needs to be paid in advance by entrepreneurs. We set it to 0.50, in the midrange of values considered in the literature.[9] Conditional on the above parameters, β_e and θ control the steady-state level of the capital-to-net-worth ratio (k_{ss}/n_{ss}) and the return to capital. We choose β_e and θ so that the former equals 4 and the latter is 50 annualized basis points above the risk-free rate.[10] This gives us β_e equal to 0.984 and θ equal to 0.818. For the consumers' risk aversion, σ, we do not pick a single value but present numerical results for different values ranging in the interval [1, 10].

We assume that the capital quality shock takes two possible values, $u_t = \{u_H, u_L\}$ with $u_H = 1$. Thus, the calibration of this process consists of choosing values for u_L and for transition probabilities. In line with Gertler and Karadi (2011), we set $u_L = 0.925$ and $P(u_{t+1} = u_L | u_t = u_L) = 0.66$. We further set $P(u_{t+1} = u_H | u_t = u_H) = 0.99$, so that financial crises in the model are rare events. In section C of the online appendix, we perform two robustness checks. First, we consider smaller and less persistent capital quality shocks. Second, we study a version of our model where the exogenous shock moves the pledgeability parameter θ rather than capital quality. In both cases, we find results comparable to those presented in this section.

[9] For example, Jermann and Quadrini (2012) set this parameter to 1 in their sensitivity analysis, while Bianchi and Mendoza (2018) set it to 0.16. The key results presented in this section survive when using smaller or larger values for γ within this range.

[10] The entrepreneurs in our model consolidate financial and nonfinancial firms. Using US data, Gertler and Karadi (2011) target an average leverage ratio of 4 for the consolidated financial and nonfinancial corporate sector. The excess returns to capital that arise in the deterministic steady state reflect deviations from arbitrage induced by the presence of binding collateral constraints. Gârleanu and Pedersen (2011) and Bocola (2016) document that these arbitrage rents were sizable during the global financial crisis of 2008–9, but they typically average few basis points in advanced economies in normal times. We chose 50 basis points to be consistent with this evidence.

TABLE 1
Model Parameters

Parameter	Concept	Value
α	Capital income share	.330
δ	Capital depreciation	.025
β	Discount factor, consumers	.990
ψ	Frisch elasticity	1.000
χ	Disutility of labor	1.980
ρ	Inverse elasticity of intertemporal substitution, consumers	1.000
γ	Fraction of wages paid in advance	.500
β_e	Discount factor, entrepreneurs	.984
θ	Fraction of pledgeable assets	.818
u_L	Capital quality in low state	.925
$\Pr(u' = u_L \mid u = u_L)$	Transition probability	.660
$\Pr(u' = u_H \mid u = u_H)$	Transition probability	.990

B. *Results*

In table 2, we report statistics computed on model-simulated data using three different values of the consumers' coefficient of relative risk aversion: $\sigma = 1$, $\sigma = 5$, and $\sigma = 10$. In each case, we report results for the first-best economy (FB), the economy with incomplete financial markets (IM), and the baseline economy with state-contingent claims (CM).

For each specification, we simulate the model for $T = 200{,}000$ periods and select the periods in which the capital quality shock switches from H to L between $t - 1$ and t. Panel A reports the average percentage change

TABLE 2
Entrepreneurs' Balance Sheet and Financial Amplification

	$\sigma = 1$			$\sigma = 5$			$\sigma = 10$		
	FB	IM	CM	FB	IM	CM	FB	IM	CM
	A. Quantities								
$\Delta(\log n_t)$		−25.21	−2.96		−25.24	−6.75		−25.30	−16.46
$\Delta(\log \tilde{n}_t)$		−97.92	16.94		−97.47	−14.48		−96.58	−72.29
$\Delta(\log l_t)$	−1.89	−4.80	−1.67	−1.89	−4.57	−6.41	−1.89	−4.11	−11.16
$\Delta(\log i_t)$	−6.45	−16.66	−6.37	−7.60	−17.58	−9.94	−8.99	−19.00	−17.69
$\Delta(\log y_t)$	−3.77	−5.72	−3.63	−3.77	−5.57	−6.80	−3.77	−5.26	−9.99
	B. Entrepreneurs' Balance Sheet								
n_{t-1}		7.89	6.23		7.86	6.20		7.79	6.67
$\tilde{n}_{t-1}$		2.29	.95		2.29	.96		2.28	1.56
k_{t-1}/n_{t-1}		3.09	3.94		3.09	3.93		3.08	3.56
$b_{L,t}/b_{H,t}$		1.00	.91		1.00	.93		1.00	.97

Note.—Each economy is simulated for $T = 200{,}000$ periods. For each simulation, we select every j such that $u_{j-1} = u_H$ and $u_j = u_L$. We then compute a given statistic x_j and average across j. In panel A, the changes in the variables are multiplied by 100 to obtain percentage changes.

in entrepreneurial net worth when the switch occurs and the average percentage change in

$$\tilde{n}_t = \theta(1 - \delta)u_t k_{t-1} - b_t(u_t),$$

a variable that measures the entrepreneurs' maximum capacity to issue intraperiod loans to finance working capital. Both variables are relevant to understanding how financial factors affect the demand for capital and labor by entrepreneurs. Panel A also reports the average percentage change in labor, investment, and output. Panel B reports indicators for the entrepreneurs' balance sheet in the period immediately preceding the L shock: the average net worth, the average value of $\tilde{n}_{t-1}$, the average leverage ratio, and the average ratio between bonds issued in period $t - 1$ contingent on the L state realizing at time t and those contingent on the H state, respectively denoted $b_{L,t}$ and $b_{H,t}$. In the incomplete market economy, this ratio is always equal to one by construction.

Let us start with the case $\sigma = 1$ and look at the differences between the three economies. In the first-best economy, a negative capital quality shock lowers the marginal product of labor, leading to a reduction in labor demand and a fall in hours worked. The direct effect of the shock, coupled with the reduction in labor input, leads to a fall in output. Investment falls because the u_t shock is persistent, so a reduction in u_t reduces the incentives to accumulate capital.

In the incomplete market economy, the shock has larger effects on labor, investment, and output. The differences are due to the financial amplification mechanism. In the incomplete market economy, entrepreneurs issue non-state-contingent claims and face the collateral constraint (16). The first ingredient implies that their balance sheet is exposed to aggregate risk: a negative capital quality shock reduces the value of the capital held but not the value of entrepreneurs' liabilities. So, both n_t and $\tilde{n}_t$ fall (on average by 25% and 98%, respectively). The second ingredient implies that these balance sheet effects depress the demand for capital and labor by entrepreneurs. The combination of these two forces leads to a deeper recession relative to the first-best.

When entrepreneurs can issue state-contingent claims, the fall in labor, investment, and output are comparable to those of the first-best economy. That is, the financial amplification mechanism is muted. Unlike in the incomplete market case, entrepreneurs can now insure against the capital quality shock by reducing their contingent liabilities in state L. Panel B of table 2 shows that this is precisely what they do in equilibrium: the ratio $b_{L,t}/b_{H,t}$ is on average 0.91, meaning that entrepreneurs promise to pay less in the L state. This liability structure implies that both n_t and $\tilde{n}_t$ are less affected by the negative capital quality shock, eliminating the first step of the amplification mechanism described above. These results mirror the findings in Carlstrom, Fuerst, and Paustian (2016), Cao and Nie

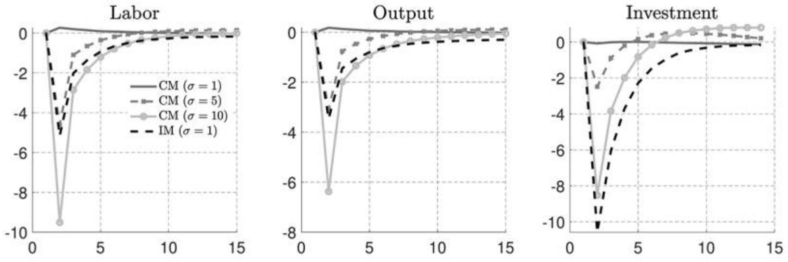

FIG. 2.—IRFs. We compute $2 \times M$ simulations of length T. We initialize the simulations at $t = 0$, setting each state variable at the mean of the ergodic distribution. In the first M simulations, we set $u_1 = u_L$; in the others, we set $u_1 = u_H$. The IRFs are computed taking the difference in logs between the first and second set of simulations, averaging across M. We use $M = 5{,}000$ and $T = 15$. The plots show the differences between the IRFs of the model considered and the first-best IRFs.

(2017), and Dmitriev and Hoddenbagh (2017). They study financial accelerator models with endogenous labor income and log utility for consumers and show that in their economies financial amplification is muted when debt contracts can be indexed to aggregate shocks.

The comparison of the three cases (FB, IM, and CM) is very different once we move to the next columns on the right, which correspond to economies with higher consumer risk aversion ($\sigma = 5, 10$). Table 2 shows that the behavior of the FB and IM economies does not change much once we increase σ, a result related to the findings in Tallarini (2000). However, the CM economy behaves very differently: the average ratio $b_{L,t}/b_{H,t}$ increases to 0.93 when $\sigma = 5$ and to 0.97 when $\sigma = 10$. Entrepreneurs use state-contingent debt less to protect their net worth against a negative shock, and as a result the sensitivity of n_t and $\tilde{n}_t$ to the shock increases. The larger fall in these two variables constrains entrepreneurs' demand of labor and capital, leading to a deeper recession. With $\sigma = 5$, the fall in labor and output in the economy with complete markets is comparable to that of the economy with incomplete markets.[11] Increasing σ further leads to more risk-taking by entrepreneurs and stronger financial amplification.

Figure 2 gives a more complete representation of the dynamics following the shock, plotting impulse response functions (IRFs) for labor, output, and investment for different values of consumers' risk aversion. Because for the IM economy the IRFs are virtually identical for the different values of σ, the figure reports only the $\sigma = 1$ case. In addition, to better

[11] This happens despite the fact that with complete markets the fall in net worth is smaller than with incomplete markets. The reason is that the economy with incomplete markets starts from a higher level of net worth in equilibrium, so the postshock levels of net worth in the two economies are quantitatively similar.

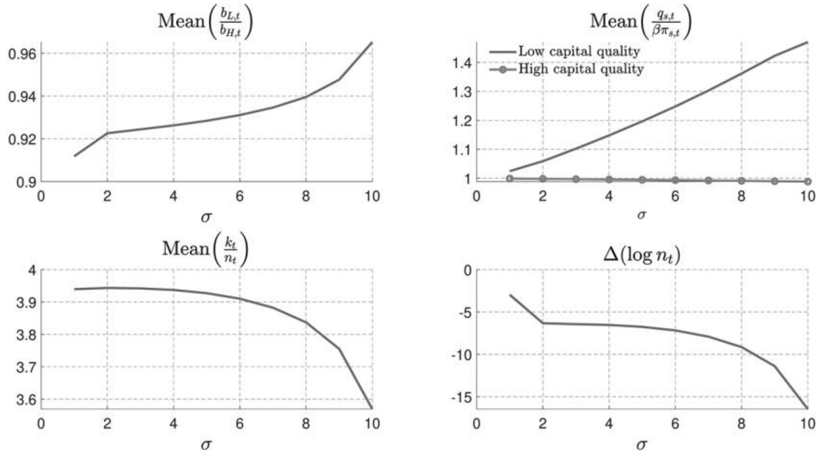

FIG. 3.—Asset prices and entrepreneurs' balance sheet. For each value of σ, we simulate the complete market economy for $T = 200{,}000$ periods, and we compute average values of $q(s_{t+1}|s^t)/(\beta\pi(s_{t+1}|s^t))$ (*top right*), $b_{L,t}/b_{H,t}$ (*top left*), k_t/n_t (*bottom left*), and the percentage change in net worth after a negative capital quality shock (*bottom right*).

visualize financial amplification, we plot the difference between the IRFs in the model considered and the IRFs in the first-best economy. The CM economy features essentially no financial amplification when $\sigma = 1$. As we increase σ, labor, output, and investment respond by more than in the first-best. Quantitatively, the responses are comparable to those of the economy with incomplete markets for plausible levels of σ.[12]

Behind the aggregate outcomes plotted in figure 2, there is the fact that entrepreneurs in the CM economy choose riskier balance sheets when σ is higher, as shown in the top left panel of figure 3. To provide an interpretation of this result, in figure 3 we plot three other variables. In the top right panel, we plot the average value of $q(s_{t+1}|s^t)/(\beta\pi(s_{t+1}|s^t))$, for $s_{t+1} = H$ and for $s_{t+1} = L$ in economies with different levels of consumers' risk aversion σ. This ratio measures the price of buying insurance against state s_{t+1} relative to the risk-neutral price—a measure of the insurance premium for each state. The remaining panels report the average entrepreneurs' leverage and the average percentage change in net worth after a low capital quality shock.

[12] In the CM economy with $\sigma = 1$, the impact responses of labor and output are slightly weaker than in the first-best case. The reason for this apparently odd behavior is that, in this calibration of the CM economy, entrepreneurs are on average more constrained in choosing the labor input after the H shock (when they would like to hire more) than after the L shock. So the fall in labor and output when the economy switches from the H to the L state is smaller than in the first-best.

After a low capital quality shock, consumers' current and future labor incomes decline. For low values of σ, this has a small effect on the insurance premium $q(L|s^t)/(\beta\pi(L|s^t))$, which remains close to one. However, as we increase σ, consumers are less willing to sell insurance against the L state and the premium increases. This incentivizes entrepreneurs to sell more L-contingent debt, so the average $b_{L,t}/b_{H,t}$ ratio increases with σ. As this ratio increases, the entrepreneurs' net worth becomes more sensitive to the capital quality shock and, in general equilibrium, makes consumers' incomes even more procyclical, reinforcing the process.

The figure shows that there is also a countervailing force at work: as entrepreneurs take on more aggregate risk, they partly adjust by reducing their investment in capital, thus reducing their leverage k_t/n_t, as seen in the bottom left panel. This force, however, only partly offsets the mechanism described above.

C. *Isolating the General Equilibrium Spillover on Labor Income*

The mechanism just described contains two steps: first, risk-averse consumers are willing to pay high premia for insuring a bad realization of the capital quality shock; second, high insurance premia endogenously make consumers' incomes more sensitive to the shock, reinforcing the first step. The results in table 2 and figure 2 show that the combined effect of these two steps can be quantitatively relevant. We now attempt a decomposition to evaluate the importance of the second step—that is, to evaluate how much the macro spillover in our model reinforces the direct effect of consumers' risk aversion.

We consider an economy that is identical to that of section II except that consumers earn the counterfactual wage that would arise in the first-best economy.[13] Wages still respond to the capital quality shock—as they do in the first-best—but they are not affected by the changes in investment and labor demand that are due to the presence of the collateral constraint. By construction, in this economy there is no spillover from entrepreneurs' net worth to consumers' labor income. For brevity, we call it the "no spillover" economy.

Table 3 reports the average response of key variables to the low capital quality shock in the first-best economy, in the benchmark economy with state-contingent claims, and in the economy with no spillover. In all cases, we set $\sigma = 5$. Columns 1 and 2 reproduce results in table 2. Column 3 shows that the amplification mechanism is substantially reduced if we shut down the macro spillover. Net worth falls by 3.2% instead of 6.8%,

[13] See sec. E of the online appendix for a detailed description of this version of the model.

TABLE 3
QUANTIFYING THE GENERAL EQUILIBRIUM SPILLOVER

	First-Best (1)	Benchmark (2)	No Spillover (3)
	A. Quantities		
$\Delta(\log n_t)$		−6.75	−3.24
$\Delta(\log l_t)$	−1.89	−6.41	−1.65
$\Delta(\log i_t)$	−7.44	−9.94	−9.89
$\Delta(\log y_t)$	−3.77	−6.80	−3.61
	B. Prices and Entrepreneurs' Balance Sheet		
$\Delta(\log LI_t)$	−3.77	−12.82	−3.29
$q_{L,t}/\pi_{L,t}$		1.20	1.03
$q_{H,t}/\pi_{H,t}$		1.00	1.00
k_{t-1}/n_{t-1}		3.93	3.96
$b_{L,t}/b_{H,t}$		.93	.91

NOTE.—See table 2's note.

and the responses of labor, investment, and output are comparable to those of the first-best economy.

Panel B helps us understand this result. Absent the spillover, labor income falls by 3.3% after the negative capital quality shock, substantially less than the 12.8% of the benchmark model. Thus, even if consumers are more risk averse than entrepreneurs, they do not bid the price up as much for insuring a low realization of the capital quality shock: $q_{L,t}/(\beta\pi_{L,t})$ is 1.03 in the no spillover economy, compared with 1.20 in our benchmark economy. Given these state prices, entrepreneurs have a better incentive to stabilize their net worth by reducing their contingent debt in the L state. In summary, to generate quantitatively meaningful financial amplification in our model, we need both consumers to be more risk averse than entrepreneurs and labor income to be sufficiently responsive to entrepreneurs' net worth.

V. Welfare Analysis

We now turn to the welfare implications of the model. In section V.A, we set up the policy problem of a planner that can tax entrepreneurs' assets and liabilities. We then study the solution to this problem in two steps. In section V.B, we analytically characterize the solution to the planner's problem in the special case of section III. We show that the laissez-faire competitive equilibrium is inefficient, with entrepreneurs hedging less than what is socially efficient because they do not internalize the stabilizing effects of their risk-mitigation strategies on consumers' labor income. In section V.C, we go back to the general model—calibrated as in section IV—and numerically study the optimal policy aimed at correcting this externality. In section V.D, we study the relation between the policy prescriptions described here and policy interventions routinely used in practice to deal with financial instability.

A. The Planner's Problem

We start from the laissez-faire equilibrium studied in section IV and consider a planner who intervenes for one period only: the planner sets proportional taxes or subsidies on capital purchases and on state-contingent claims issued by entrepreneurs at time t. In addition, the planner can make a lump-sum transfer at date t to redistribute the efficiency gains between consumers and entrepreneurs.

The timing of events within a period is as in the general model, and we assume that the planner intervenes in the third stage of period t—after production has taken place and after entrepreneurs have chosen whether to default, at the moment in which they choose their capital investment and trade state-contingent claims with consumers. Given this timing, the planner cannot relax the collateral constraint in period t, because employment and production have already occurred. The collateral constraint in future periods is also unaffected, because the planner intervenes for only one period. Therefore, all welfare gains are solely due to the planner inducing different choices of capital and state-contingent debt at time t.[14]

Let $\mathbf{s} = [u, K, B]$ be the vector of aggregate state variables in period t. Using the recursive notation of section A of the online appendix, we write the entrepreneur's problem as follows:

$$\max_{c_e, l, b'(\mathbf{s}), k'} \log(c_e) + \beta_e \mathbb{E}_{\mathbf{s}}[V^e(b'(\mathbf{s}'), k'; \mathbf{s}')],$$

$$n = (uk)^\alpha l^{1-\alpha} - w(\mathbf{s})l + (1-\delta)uk - b,$$

$$c_e + [1 + \tau_k(\mathbf{s})]k' \leq n + \sum_{\mathbf{s}'}[1 - \tau_b(\mathbf{s}'|\mathbf{s})]q(\mathbf{s}'|\mathbf{s})b'(\mathbf{s}') + T_e(\mathbf{s}),$$

$$b + \gamma w(\mathbf{s})l \leq \theta(1-\delta)uk,$$

where $\tau_k(\mathbf{s})$ represents a proportional tax on capital, $\tau_b(\mathbf{s}'|\mathbf{s})$ represents a tax on the sales of state-contingent claims that pay in state $\mathbf{s}'$, $T_e(\mathbf{s})$

[14] The main advantage of limiting our analysis to one-period interventions is simplicity. In the current formulation, the planner cannot circumvent the collateral constraint, even though Pigouvian taxes at time t are fully enforceable. In a model with multiperiod interventions, if taxes are fully enforceable it would be easy for the planner to circumvent the limited enforcement problem—by transferring resources to entrepreneurs when the constraint is binding and redistributing them back to consumers in future periods. Given that, we would need to introduce some form of limited enforcement of tax payments, which would substantially complicate the analysis.

represents a lump-sum transfer, and $V^e(.)$ is the value function of entrepreneurs, expressed as a function of the individual state variables (b, k) and of the aggregate state $\mathbf{s}$. Because the planner intervenes for only one period, V^e is the laissez-faire equilibrium value function.

Consumers solve the problem

$$\max_{c,l,a'(\mathbf{s}')} (1-\beta)\left(c - \chi\frac{l^{1+\psi}}{1+\psi}\right)^{1-\rho} + \beta\left[\mathbb{E}_{\mathbf{s}}\left(V(a'(\mathbf{s}');\mathbf{s}')\right)^{1-\sigma}\right]^{(1-\rho)/(1-\sigma)},$$

$$\sum_{\mathbf{s}'} q(\mathbf{s}'|\mathbf{s})a'(\mathbf{s}') + c \le w(\mathbf{s})l + a + T_c(\mathbf{s}),$$

where $T_c(\mathbf{s})$ represents a lump-sum transfer and $V(.)$ is the laissez-faire equilibrium value function.

A competitive equilibrium with one-period government intervention is given by taxes and transfers, prices, and allocations such that consumers and entrepreneurs solve the optimization problems above, the bond market and capital market clear, $a'(\mathbf{s}') = b'(\mathbf{s}') = B'(\mathbf{s}')$, $k' = K'$, the labor market clears, and the government budget constraint holds.

We consider a planner who chooses the policies $\tau_b(\mathbf{s}'|\mathbf{s})$, $\tau_k(\mathbf{s})$, $T_c(\mathbf{s})$, $T_e(\mathbf{s})$ to maximize the utility of the consumers subject to giving entrepreneurs the same utility as in the laissez-faire equilibrium. Because the planner can always implement the laissez-faire allocation by setting zero taxes and transfers, any deviation from such benchmark is, by construction, a Pareto improvement. In section F of the online appendix, we show that the planner's optimum can be characterized by solving the primal problem

$$\max_{X,C_e,K',B'(\mathbf{s}')} \left\{(1-\beta)X^{1-\rho} + \beta\left[\mathbb{E}_{\mathbf{s}}[V(B'(\mathbf{s}');u',B'(\mathbf{s}'),K')]^{1-\sigma}\right]^{(1-\rho)/(1-\sigma)}\right\}^{1/(1-\rho)}$$

$$\text{subject to } X + C_e + K' \le (uK)^{\alpha}L(\mathbf{s})^{1-\alpha} + (1-\delta)uK - \chi\frac{L(\mathbf{s})^{1+\psi}}{1+\psi}, \tag{SP}$$

$$\log C_e + \beta_e\mathbb{E}_{\mathbf{s}}[V^e(B'(\mathbf{s}'),K';u',B'(\mathbf{s}'),K')] \ge V^e(B,K;\mathbf{s}),$$

where $L(\mathbf{s})$ represents the labor allocation of the laissez-faire equilibrium.[15] The first constraint is the resource constraint, and the second constraint ensures that the entrepreneurs are as well off as in the laissez-faire equilibrium.

To understand the planner's rationale for intervening, consider the first-order condition with respect to $B'(\mathbf{s}')$. After some manipulations, we obtain

[15] Because the planner cannot relax the entrepreneurs' collateral constraint, and due to the absence of a wealth effect on consumers' labor supply, the labor allocation in the planner's solution at date t is equivalent to that of the laissez-faire equilibrium.

$$\beta_e \frac{(1/C_e(\mathbf{s}')) + \mu(\mathbf{s}')}{1/C_e} - \beta\left[\frac{X}{X(\mathbf{s}')}\right]^{\rho}\left[\frac{RW(\mathbf{s})}{V(\mathbf{s}')}\right]^{\sigma-\rho}$$
$$= \beta_e C_e \frac{\partial V^e(\mathbf{s}')}{\partial B'(\mathbf{s}')} + \beta \frac{X^{\rho}[RW(\mathbf{s})]^{\sigma-\rho}V(\mathbf{s}')^{-\sigma}}{1-\beta}\frac{\partial V(\mathbf{s}')}{\partial B'(\mathbf{s}')}, \tag{17}$$

where $X(\mathbf{s}')$ and $C_e(\mathbf{s}')$ are the individual policy functions at the laissez-faire equilibrium and $\partial V(\mathbf{s}')/\partial B'(\mathbf{s}')$ is a short notation for the partial derivative of $V(B'(\mathbf{s}'); u', B'(\mathbf{s}'), K')$ with respect to its third argument (and similarly for $\partial V^e(\mathbf{s}')/\partial B'(\mathbf{s}')$).

The two terms on the left-hand side of (17) are equivalent to the terms in our baseline risk-sharing condition (6), which are equalized in every state of nature at the laissez-faire equilibrium. In the planner solution, however, there is a wedge between the two, represented by the terms on the right-hand side of equation (17). Differently from atomistic agents, the planner takes into account that by changing $B'(\mathbf{s}')$ it affects the net worth of entrepreneurs and thus the price of state-contingent claims and wages in equilibrium. The impact of these pecuniary externalities on consumers' and entrepreneurs' welfare are represented by the partial derivatives of V and V^e with respect to the aggregate state variable $B'(\mathbf{s}')$. As long as the terms on the right-hand side do not cancel out, the planner has incentives to impose taxes or subsidies on state-contingent debt to modify the allocation of risk between consumers and entrepreneurs.

B. *Optimal Policy in the Simple Model*

To shed light on how the pecuniary externalities discussed above affect the optimal policy, consider the special case of section III. Since the value function of consumers at date $t = 1$ is given by (12), the effect of increasing $B_1(u_1)$ on consumers' welfare is

$$\frac{\partial V_1}{\partial B_1(u_1)} = -(1-\beta)\sum_{t=1}^{\infty}\beta^t \frac{\partial w_{t+1}}{\partial n_1(u_1)} \leq 0. \tag{18}$$

A change in $B_1(u_1)$ affects consumers' welfare through its impact on their lifetime labor income. If the collateral constraint does not bind at u_1, then capital equals k^* in every period after $t = 1$, wages are independent of $B_1(u_1)$, and $\partial V_1/\partial B_1(u_1) = 0$. If the collateral constraint binds at u_1, however, we know from lemma 1 that capital accumulation depends on entrepreneurial net worth at date $t = 1$. A higher $B_1(u_1)$, by reducing net worth, leads to lower capital and lower wages for a finite number of periods, so $\partial V_1/\partial B_1(u_1) < 0$.

We can follow similar steps and study the impact of an increase in $B_1(u_1)$ on entrepreneurs' welfare. Using the envelope theorem and the fact that wages affect net worth one for one, we have that

$$\frac{\partial V_1^e}{\partial B_1(u_1)} = \sum_{t=1}^{\infty} \beta^t \frac{1}{c_{e,t+1}} \frac{\partial w_{t+1}}{\partial n_1(u_1)} \geq 0. \tag{19}$$

Similarly to consumers, a change in $B_1(u_1)$ affects entrepreneurs only through its impact on wages. Differently from consumers, however, an increase in $B_1(u_1)$ has a (weakly) positive spillover for entrepreneurs because it lowers their cost of labor.

The above discussion shows that the pecuniary externalities triggered by an increase in $B_1(u_1)$ hurt consumers and help entrepreneurs, so their overall effects on the optimal policy are in principle ambiguous. However, we can show that the negative effect on consumers dominates in the states in which the constraint is binding.

Substituting (18) and (19) on the right-hand side of equation (17) and using the fact that the consumption of entrepreneurs is proportional to their net worth, we have

$$\beta \frac{n_0}{n_1(u_1)} \sum_{t=1}^{\infty} \beta^t \frac{n_1(u_1)}{n_{t+1}} \frac{\partial w_{t+1}}{\partial n_1(u_1)} - \beta \left(\frac{RW_0}{V_1(u_1)} \right)^{\sigma} \sum_{t=1}^{\infty} \beta^t \frac{\partial w_{t+1}}{\partial n_1(u_1)}. \tag{20}$$

Let us evaluate this expression at the laissez-faire allocation studied in section III. Using the risk-sharing condition (13), the sign of (20) is equal to the sign of

$$\sum_{t=1}^{\infty} \beta^t \left[\frac{n_1(u_1)}{n_{t+1}} - 1 \right] \frac{\partial w_{t+1}}{\partial n_1(u_1)}.$$

If the collateral constraint binds at u_1, we know from section III.A that entrepreneurs' net worth increases over time, $n_t < n_{t+1}$, for a finite number of periods. So, in those states the expression in (20) is negative: the reduction in consumers' welfare is larger than the increase in entrepreneurs' welfare.

The derivations above suggest that, starting at the laissez-faire allocation, the planner has a motive to reduce entrepreneurs' debt payments in states where the constraint binds. The intuition is that reducing debt payments causes two reallocations in resources: the first, internalized by private agents, is a direct reallocation from consumers to entrepreneurs at $t = 1$; the second, not internalized, is a reallocation from entrepreneurs to consumers, caused by the general equilibrium increase in wages in periods $t = 2, 3, 4, \ldots$. Because the entrepreneurs are constrained at date 1, they value resources relatively more at $t = 1$ than in future periods, so the combined effects of these reallocations is to increase social welfare.

A planner who internalizes the general equilibrium effects above can achieve the social optimum using the taxes characterized in the following proposition.

Proposition 2. In the special case of section III, the taxes on state-contingent claims and capital that implement the planner's optimum are

$$\tau_b(u_1) = \frac{\sum_{t=1}^{\infty}\beta^t[1-(n_1(u_1)/n_{t+1})(1/(1+\mu_1(u_1)(1-\beta)n_1(u_1)))](\partial w_{t+1}/\partial n_1(u_1))}{1-(1/(1+\mu_1(u_1)(1-\beta)n_1(u_1)))\sum_{t=1}^{\infty}\beta^t(n_1(u_1)/n_{t+1})(\partial w_{t+1}/\partial n_1(u_1))} \geq 0, \quad (21)$$

$$\tau_k = \beta\mathbb{E}\left\{\frac{n_0}{n_1(u_1)}\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{n_1(u_1)}{n_{t+1}} - \frac{1+\mu_1(u_1)(1-\beta)n_1(u_1)}{1-\tau_b(u_1)}\right]\frac{\partial w_{t+1}}{\partial k_0}\right]\right\} \leq 0. \quad (22)$$

Proposition 2 provides expressions for the optimal taxes as a function of the planner's allocation and of the continuation equilibrium characterized in lemma 1.[16] Given the properties of the continuation equilibrium, we can characterize key properties of these taxes. The optimal tax on state-contingent claims, given by equation (21), is zero if the collateral constraint does not bind in state u_1 and is positive otherwise. This reflects the planner's motive, discussed above, to reduce entrepreneurs' debt payments when the collateral constraint binds. To the extent that $n_1(u_1)$ is increasing in u_1 in the planner's allocation, proposition 2 also implies that the planner levies taxes toward state-contingent claims that pay in low capital quality states.

The proposition also derives the optimal tax on capital, given by (22). In the proof of the proposition, presented in the print appendix, we show that τ_k is strictly negative when the collateral constraint at date $t = 1$ binds with positive probability. The planner's motive for subsidizing capital is closely related to that of taxing debt, as higher capital at date $t = 1$ triggers the same pecuniary externality of a reduction in entrepreneurs' debt payments that we studied earlier.

C. *Numerical Analysis*

We now go back to the full model, calibrated as in section IV, to give a quantitative assessment of the optimal taxes and their effects on the equilibrium allocation. In addition, we compare the optimal policy with a blunter policy that taxes borrowing equally in all states of the world. Specifically, we impose an additional constraint on the planner's problem, requiring $\tau_b(\mathbf{s}'|\mathbf{s})$ to be constant in $\mathbf{s}'$. The latter policy is equivalent to a simple leverage constraint of the type usually studied in existing models of macroprudential policy.

We solve the planner's problem numerically and report the response of the economy to a negative capital quality shock in table 4. Specifically, we simulate the economy for many periods, select all the periods in

[16] Note that here we are not restricting the entrepreneurs to be unconstrained at date $t = 0$, so the Lagrange multiplier $\mu_1(u_1)$ can be positive in some states.

TABLE 4
OPTIMAL POLICY

	FB (1)	LF (2)	PL (3)	PL-c (4)
	A. Quantities			
$\Delta(\log n_t)$		−16.46	−16.11	−16.38
$\Delta(\log \tilde{n}_t)$		−72.29	−54.12	−72.33
$\Delta(\log l_t)$	−1.89	−11.16	−2.36	−11.16
$\Delta(\log i_t)$	−8.99	−17.69	−19.40	−21.78
$\Delta(\log y_t)$	−3.77	−9.99	−4.06	−9.95
	B. Taxes			
$1 - \tau_b(u_L)$		1.00	.80	1.00
$1 - \tau_b(u_H)$		1.00	1.00	1.00
$1 + \tau_k$		1.00	.98	.98

NOTE.—See table 2's note.

which the shock switches from u_H to u_L between $t-1$ and t, and report statistics regarding the entrepreneurs' balance sheet and the behavior of macroeconomic variables, assuming that the planner intervened at $t-1$. We compare four different cases: the first-best (FB), the laissez-faire equilibrium (LF), the equilibrium under optimal policy (PL), and the equilibrium under the constrained policy (PL-c). For this illustration, we set σ to 10.

First, let us consider the behavior of quantities in panel A. Under the optimal policy, financial amplification is substantially reduced: the falls in labor and output in column 3 are smaller than in column 2 and closer to column 1. In addition, comparing columns 3 and 4 shows that different tax rates on different state-contingent claims are critical for this result: a planner restricted to impose a uniform tax on state-contingent claims does not dampen financial amplification.

Panel B reports the average taxes set by the planner. The results for the PL economy are consistent with the analytical derivations of the simple model: the planner subsidizes capital accumulation and imposes a tax on bonds that pay in low capital quality states.[17] Quantitatively, the subsidy on capital is 2% on average, while the planner levies a tax of 19% on sales of L-contingent bonds and a zero tax on H-contingent bonds. These taxes induce the entrepreneurs to reduce their reliance on the L-contingent debt, which explains why their balance sheet is less exposed to the negative capital quality shock at date t and why financial amplification is muted.

[17] Incidentally, the subsidy on capital explains why investment falls more in the planner solution than in the laissez-faire equilibrium in panel A. Since the planner can intervene only at $t-1$, the investment subsidy is present only at date $t-1$, driving down investment between $t-1$ and t. In the laissez-faire equilibrium, this policy effect is absent.

Turning to the PL-c economy, we can see that the restricted planner chooses an optimal tax on debt close to zero on average. Consistently, balance sheets and aggregate effects in the PL-c economy are similar to those in the LF economy.

The result of a near-zero tax in the PL-c economy may appear surprising in light of several papers in the literature that report sizable optimal debt taxes in similar models with non-state-contingent debt. To better understand this result, figure 4 reports entrepreneurs' debt in the PL-c economy when the planner varies τ_b. The left panel shows that as τ_b increases, entrepreneurs reduce their contingent debt in both states of the world, but much more so in state H, so the ratio of L-contingent to H-contingent debt increases (right panel). Thus, a uniform tax on borrowing is not particularly effective in reducing the risk-taking of entrepreneurs, because entrepreneurs respond by reducing the degree of state contingency in future debt payments. Because of this feature, the planner chooses essentially not to engage in macroprudential policy. In models where debt is not state contingent, the private sector cannot respond to the tax by altering the degree of state contingency of debt payments, so a uniform tax on debt is more effective in curbing risk-taking incentives.

We summarize the discussion above in two observations. First, when borrowers have means to adjust the state contingency of their liabilities, the welfare benefits of a uniform tax on leverage may be overstated. Second,

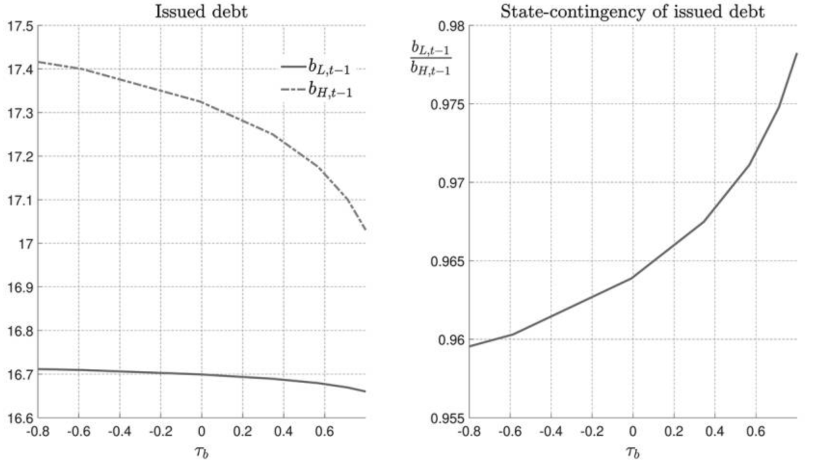

FIG. 4.—Tax on debt, leverage, and risk-taking. The left panel reports the equilibrium levels of $b_{L,t-1}$ and $b_{H,t-1}$ when varying τ_b. When constructing the figure, we set $u_{t-1} = 1$ and set the other state variables at $t - 1$ at the ergodic mean. In addition, the tax on capital and the transfers are set so that the level of capital remains at its optimal level in the constrained planner problem and the entrepreneur achieves the same utility as in the laissez-faire competitive equilibrium. The right panel is constructed in a similar fashion.

the ability of regulation to reduce financial amplification and improve welfare rests crucially on the ability to discourage the riskier forms of borrowing with targeted instruments.

D. *Bailouts and Financial Regulation*

We now discuss the connection between the welfare analysis above and policy interventions routinely used to deal with financial instability. In particular, we discuss bailouts and capital adequacy ratios.

Suppose we start at the laissez-faire equilibrium and, at date *t*, consumers and entrepreneurs have exchanged the state-contingent claims

$$a'(\mathbf{s}') = A'(\mathbf{s}') = b'(\mathbf{s}') = B'(\mathbf{s}').$$

Suppose that the government unexpectedly introduces state-contingent transfers $T'(\mathbf{s}')$ at date $t+1$, so the consumers receive $A'(\mathbf{s}') - T'(\mathbf{s}')$ and the entrepreneurs' net worth increases by $T'(\mathbf{s}')$, and suppose that these transfers are positive after low capital quality shocks and negative after high ones. We can interpret these transfers as bailouts to entrepreneurs in states of the world in which they are distressed, compensated by a levy in good states. It is possible to proceed as in the analysis above and construct examples in which the transfers $T'(\mathbf{s}')$ lead to a Pareto improvement.[18]

What is the problem with the policy above? If consumers and entrepreneurs anticipate that the policy will be in place, the contingent bailout turns out to be completely neutral. To be precise, suppose that the expected present value of the transfer $\Sigma_{\mathbf{s}'}\, q(\mathbf{s}'|\mathbf{s})T'(\mathbf{s}')$ is zero at the laissez-faire state prices. Then there exists an equilibrium in which the values of $b'(\mathbf{s}') - T'(\mathbf{s}')$ are identical to the values of $b'(\mathbf{s}')$ at the original laissez-faire equilibrium. In other words, the entrepreneurs completely undo the transfers, by taking additional risky debt in states of the world in which they expect to receive a bailout.[19] The fact that agents have access to perfect state-contingent markets means that they are more flexible in taking advantage of ex post government help. In this framework, this leads to an extreme form of moral hazard, as anticipated bailouts are essentially useless.

Turning to capital requirements, an alternative to the Pigouvian taxes introduced in section V.A is to introduce, at $t = 0$, restrictions to the issuance

[18] This does not require the government to have superior capacity to enforce payments, as we can build examples in which the transfers always respect the entrepreneurs' no-default constraint

$$b'(\mathbf{s}') - T'(\mathbf{s}') + \gamma w(\mathbf{s}')l' \leq \theta(1-\delta)u'k'.$$

[19] If the present value of the transfer is not zero, the effect of the policy is not neutral but is equivalent to a single ex ante, non-state-contingent transfer.

of debt in proportion to the assets held by the entrepreneurs, imposing the constraint

$$\sum_{\mathbf{s}'} \omega(\mathbf{s}'|\mathbf{s}) q(\mathbf{s}'|\mathbf{s}) b'(\mathbf{s}') \leq k'. \tag{23}$$

In the expression above, $\omega(\mathbf{s}'|\mathbf{s})$ represent risk weights applied to each state-contingent claim traded. In section F of the online appendix, we show that the optimal policy can be equivalently implemented by imposing constraint (23) on entrepreneurs, with the appropriate set of risk weights $\omega(\mathbf{s}'|\mathbf{s})$, and using a tax on capital.

It is important to notice that the presence of different risk weights $\omega(\mathbf{s}'|\mathbf{s})$ plays an essential role. If $\omega(\mathbf{s}'|\mathbf{s})$ was constant across states, the intervention would be analogous to a uniform tax on debt, which, as we saw in the previous subsection, is a poor substitute for a state-contingent tax. In practice, risk weights are more usually applied on the asset side of the balance sheet. Our framework provides a macroprudential argument for using risk weights on the liability side.[20]

VI. Conclusion

In this paper, we have asked why financial institutions tend to be exposed to aggregate risk despite the availability of several instruments to hedge this exposure. To answer this question, we have used a canonical financial accelerator model in which agents trade fully state-contingent claims. We have obtained two main results.

First, we showed that entrepreneurs may not hedge negative aggregate shocks in equilibrium because insuring these states can be too costly for them. We have isolated the importance of two factors for this result: the general equilibrium spillover of entrepreneurs' net worth on consumers' labor income and the risk aversion of consumers. Under plausible calibrations of our model, these two effects are strong enough to make the productive sector as exposed to aggregate risk as it would be in a corresponding economy where only a non-state-contingent bond can be used for risk management. These results show that it is feasible to introduce risk-management considerations in this class of models without compromising their ability to generate financial amplification.

Second, we showed that the resulting competitive equilibrium is constrained inefficient and it features too much exposure of entrepreneurs to aggregate risk. In the optimal policy, a planner reduces this exposure by taxing only certain debt instruments—specifically, those whose payments are indexed to the negative aggregate shocks. On the contrary,

[20] This connects the analysis here to papers that suggest imposing regulatory constraints based on the sensitivity of balance sheets to correlated shocks, as in Adrian and Brunnermeier (2016).

uniform taxes on all debt instruments, despite reducing overall leverage, are not effective in limiting the entrepreneurs' risk exposure because they incentivize a substitution toward riskier debt instruments. More generally, our results emphasize that macroprudential policies targeted toward certain debt instruments can be substantially more effective than policies that discourage leverage tout court—a common prescription of the incomplete market models used in the literature.

These policy prescriptions are obtained in an environment where a full set of state-contingent claims is available. In future research on macroprudential policy, it may be useful to consider models in between the two extremes of no state contingency and full state contingency, to more realistically capture the set of risk-management tools available to financial institutions.

Appendix

A. *Proof of Proposition 1*

We divide the proof of this proposition into two parts. First, we establish that if the collateral constraint does not bind at date 0, then the equilibrium sensitivities of entrepreneurial debt payments and net worth to the capital quality shocks are given by (14) and (15). Second, we show that the restrictions on the primitives in the statement of the proposition guarantee that the collateral constraint does not bind at date 0.

Starting with the first part, we can use the expression for V_1 in equation (12) and the market clearing condition $a_1(u_1) = b_1(u_1)$ to write the equilibrium risk-sharing condition as

$$\left[\frac{RW_0}{(1-\beta)[b_1(u_1) + w(u_1) + W(n_1(u_1))]}\right]^{\sigma} = \frac{n_0}{n_1(u_1)} \qquad \forall\ u_1.$$

From the definition of net worth in (11), we have that $b_0(u_1) + w(u_1) = (u_1 k_0)^{\alpha} + (1-\delta)u_1 k_0 - n_1(u_1)$. Substituting this into the above expression and rearranging terms, we obtain

$$n_1(u_1) = \xi[(u_1 k_0)^{\alpha} + (1-\delta)u_1 k_0 - n_1(u_1) + W(n_1(u_1))]^{\sigma},$$

where $\xi = n_0/RW_0^{\sigma} > 0$ is a constant, independent of u_1. Differentiating with respect to u_1, we obtain

$$n_1'(u_1) = \sigma\frac{[\alpha u_1^{\alpha-1}k_0^{\alpha} + (1-\delta)k_0 - n_1'(u_1) + W'(n_1(u_1))n_1'(u_1)]n_1(u_1)}{(u_1 k_0)^{\alpha} + (1-\delta)u_1 k_0 - n_1(u_1) + W(n_1(u_1))}. \tag{A1}$$

Again using the definition of net worth in (11) and the market clearing condition for bonds, we have $(u_1 k_0)^{\alpha} + (1-\delta)u_1 k_0 - n_1(u_1) + W(n_1(u_1)) = a_1(u_1) + w(u_1) + W(n_1(u_1))$. Substituting this expression in the denominator of (A1) and using the definition of ω in the text, we can rewrite equation (A1) as

$$n_1'(u_1) = \sigma[\alpha u_1^{\alpha-1}k_0^{\alpha} + (1-\delta)k_0 - n_1'(u_1) + W'(n_1(u_1))n_1'(u_1)]\frac{\omega}{1-\omega}.$$

Collecting on the left-hand side the $n_1'(u_1)$ terms, we have

$$n_1'(u_1)\left[\frac{1}{\sigma}\frac{(1-\omega)}{\omega} + 1 - W'(n_1(u_1))\right] = \alpha u_1^{\alpha-1}k_0^{\alpha} + (1-\delta)k_0,$$

and solving for $n_1'(u_1)$ gives equation (15). The expression for $b_1'(u_1)$ is obtained by differentiating equation (11) with respect to u_1 and using equation (15) to substitute for $n_1'(u_1)$.

The second part of the proof shows that the conditions of the proposition are sufficient to guarantee that the collateral constraint does not bind at date 0. Let us assume first that $\sigma = 0$. In that case, the unconstrained level of capital at date 0 equals

$$k_0 = \hat{k} \equiv \frac{\alpha\beta\mathbb{E}(u_1^{\alpha})}{1-(1-\delta)\beta\mathbb{E}(u_1^{\alpha})}.$$

In addition, from the risk-sharing condition (13) we know that $n_1(u_1) = n_0$ for all u_1 when $\sigma = 0$. From the definition of $n_1(u_1)$ in equation (11), we then have

$$b_1(u_1) = \alpha(u_1\hat{k})^{\alpha} + (1-\delta)u_1\hat{k} - n_0.$$

The collateral constraint does not bind at date 0 if $b_1(u_1) < \theta(1-\delta)u_1\hat{k}$ for all $u_1 \in [\underline{u}, \bar{u}]$. Using the above expression for $b_1(u_1)$, we can rewrite these conditions as

$$n_0 > \alpha(u_1\hat{k})^{\alpha} + (1-\theta)(1-\delta)u_1\hat{k} \qquad \forall\ u_1 \in [\underline{u}, \bar{u}].$$

Because the right-hand side of the above expression increases in u_1, the condition on n_0 in the statement of the proposition guarantees that the above is satisfied for all $u_1 \in [\underline{u}, \bar{u}]$.

Let us now consider the case with $\sigma > 0$, and let k_0 be the unconstrained choice of capital by entrepreneurs. If the collateral constraint does not bind at date 0, $b'(u_1)$ is given by equation (14). Using that expression, we have

$$\frac{\partial}{\partial u_1}[b_1(u_1) - \theta(1-\delta)u_1k_0] = \alpha^2u_1^{\alpha-1}k_0^{\alpha} + (1-\theta)(1-\delta)k_0$$
$$- \frac{\sigma\omega}{\sigma\omega(1-W'(n_1)) + (1-\omega)}\left(\alpha u_1^{\alpha-1}k_0^{\alpha} + (1-\delta)k_0\right),$$

which, for a σ small enough, is positive for every u_1. So, for σ small enough, we have that

$$b_1(\bar{u}) < \theta(1-\delta)\bar{u}k_0 \tag{A2}$$

is a sufficient condition for $b_1(u_1) < \theta(1-\delta)u_1k_0$ for all $\in [\underline{u}, \bar{u}]$.

We now show that the condition on n_0 in the statement of the proposition guarantees that the inequality (A2) is satisfied. Because $V_1(u_1)$ increases in u_1, we have that $V_1(\bar{u}) \geq RW_0$. From the risk-sharing condition (13), it follows that $n_1(\bar{u}) \geq n_0$. So, from the definition of $n_1(u_1)$ we have that

$$b_1(\bar{u}) \leq [\alpha(\bar{u}k_0)^{\alpha} + (1-\delta)\bar{u}k_0] - n_0.$$

Because $k_0 \leq \hat{k}$ when $\sigma > 0$, we have that

$$n_0 > \alpha(\bar{u}\hat{k})^\alpha + (1-\theta)(1-\delta)\bar{u}\hat{k}$$

guarantees that the inequality (A2) is satisfied. So, for σ small enough, the condition on n_0 in the statement of the proposition guarantees that $b_1(u_1) < \theta(1-\delta)u_1k_0$ for all $u_1 \in [\underline{u}, \bar{u}]$. QED

B. *Proof of Proposition 2*

Let us start by solving for the optimal tax on state-contingent claims. Substituting equations (18) and (19) into (17), we have that the planner's allocation needs to satisfy the following condition for every u_1:

$$\left[\frac{RW_0}{V_1(u_1)}\right]^\sigma - c_{e,0}\left[\frac{1}{c_{e,1}(u_1)} + \mu_1(u_1)\right] = \left[\frac{RW_0}{V_1(u_1)}\right]^\sigma \sum_{t=1}^{\infty}\beta^t \frac{\partial w_{t+1}}{\partial n_1(u_1)} - c_{e,0}\sum_{t=1}^{\infty}\beta^t \frac{1}{c_{e,t+1}}\frac{\partial w_{t+1}}{\partial n_1(u_1)}. \tag{A3}$$

From the consumers' and entrepreneurs' problem, we also know that in any competitive equilibrium with taxes the following condition must hold for every u_1:

$$\left[\frac{RW_0}{V_1(u_1)}\right]^\sigma [1-\tau_b(u_1)] = c_{e,0}\left[\frac{1}{c_{e,1}(u_1)} + \mu_1(u_1)\right]. \tag{A4}$$

Substituting equation (A4) into the left- and right-hand sides of (A3) and simplifying, we have that

$$\tau_b(u_1) = \sum_{t=1}^{\infty}\beta^t\frac{\partial w_{t+1}}{\partial n_1(u_1)} - \frac{[1-\tau_b(u_1)]}{(1/c_{e,1}(u_1)) + \mu_1(u_1)}\sum_{t=1}^{\infty}\beta^t\frac{1}{c_{e,t+1}}\frac{\partial w_{t+1}}{\partial n_1(u_1)}. \tag{A5}$$

Given that $c_{e,1}(u_1) = (1-\beta)n_1(u_1)$ in the continuation equilibrium, we can use equation (A5) to obtain an expression for $\tau_b(u_1)$,

$$\tau_b(u_1) = \frac{\sum_{t=1}^{\infty}\beta^t[1-(n_1(u_1)/n_{t+1})(1/(1+\mu_1(u_1)(1-\beta)n_1(u_1)))](\partial w_{t+1}/\partial n_1(u_1))}{1-(1/(1+\mu_1(u_1)(1-\beta)n_1(u_1)))\sum_{t=1}^{\infty}\beta^t(n_1(u_1)/n_{t+1})(\partial w_{t+1}/\partial n_1(u_1))}. \tag{A6}$$

We use the properties of the continuation equilibrium in lemma 1 to sign $\tau_b(u_1)$. Specifically, we know that $\partial w_{t+1}/\partial n_1(u_1) \geq 0$, with strict inequality if the collateral constraint binds at u_1. In addition, we know that in the continuation equilibrium $n_1 \leq n_j$ for all $j > 1$, with strict inequality if the collateral constraint binds at $j-1$. Given that $\mu_1(u_1) \geq 0$, these properties guarantee that $\tau_b(u_1) \geq 0$, with strict inequality if the collateral constraint binds at u_1.

We follow a similar approach to solve for the tax on capital. From the primal problem we know that the planner's allocation must satisfy the following condition:

$$\beta c_{e,0}\mathbb{E}\left\{\frac{1}{c_{e,1}(u_1)}\left[\alpha u_1^\alpha k_0^{\alpha-1} + (1-\delta)u_1\right] + \theta\delta\mu_1(u_1)u_1\right\} = 1 + \beta c_{e,0}\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{c_{e,t+1}}\frac{\partial w_{t+1}}{\partial k_0}\right] - \beta\mathbb{E}\left[\left[\frac{RW_0}{V_1(u_1)}\right]^\sigma\sum_{t=0}^{\infty}\beta^t\frac{\partial w_{t+1}}{\partial k_0}\right]. \tag{A7}$$

In addition, the entrepreneurs' optimality condition for capital implies that in any competitive equilibrium with taxes, the following condition holds:

$$\beta c_{e,0}\mathbb{E}\left\{\frac{1}{c_{e,1}(u_1)}\left[\alpha u_1^{\alpha}k_0^{\alpha-1}+(1-\delta)u_1\right]+\theta\delta\mu_1(u_1)u_1\right\}=1+\tau_k. \tag{A8}$$

Inspecting equations (A7) and (A8), we can see that the optimal tax on capital must be

$$\tau_k=\beta c_{e,0}\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{c_{e,t+1}}\frac{\partial w_{t+1}}{\partial k_0}\right]-\beta\mathbb{E}\left[\left[\frac{RW_0}{V_1(u_1)}\right]^{\sigma}\sum_{t=0}^{\infty}\beta^t\frac{\partial w_{t+1}}{\partial k_0}\right]. \tag{A9}$$

Substituting for $[RW_0/V_1(u_1)]^{\sigma}$ in the above expression using equation (A4) and rearranging terms, we obtain the expression in the main text

$$\tau_k=\beta\mathbb{E}\left\{\frac{n_0}{n_1(u_1)}\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{n_1(u_1)}{n_{t+1}}-\frac{1+\mu_1(u_1)(1-\beta)n_1(u_1)}{1-\tau_b(u_1)}\right]\frac{\partial w_{t+1}}{\partial k_0}\right]\right\}. \tag{A10}$$

Again, we can use the properties of the continuation equilibrium to sign τ_k. First, we have that $\partial w_{t+1}/\partial k_0\geq 0$. Second, the term

$$\left[\frac{n_1(u_1)}{n_{t+1}}-\frac{1+\mu_1(u_1)(1-\beta)n_1(u_1)}{1-\tau_b(u_1)}\right]$$

is necessarily nonnegative, and it is strictly negative if the collateral constraint binds at u_1. It follows that $\tau_k\leq 0$, with strict inequality if the collateral constraint binds with positive probability at date 1. QED

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