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FORWARD-LOOKING LOAN LOSS PROVISIONING UNDER IMPERFECT FORECASTS

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To Stefan, who taught me about forecasting and life.

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ABSTRACT

The current expected credit losses (CECL) model stipulates that loan loss provisions should be forward-looking. I document that banks increasingly rely on macroeconomic forecasts following the implementation of CECL, yet most forecasts are not rational. I build a theoretical model to study the implications of CECL for bank provisions, lending, aggregate output in the economy, and bank stability. Additionally, I derive the optimal minimum capital requirement within the CECL framework. I first explore the implications under rational expectations and then consider bank expectations influenced by Kahneman and Tversky's (1972) representativeness heuristic to capture the empirical properties of macroeconomic forecasts. My model demonstrates that the representativeness heuristic results in overreaction to news, leading to underprovisioning, excessive lending and risk-taking in reaction to good news, and the opposite in reaction to bad news. Due to timely loan loss provisioning under CECL, the optimal capital constraint is time-varying in response to the macroeconomic conditions and the underlying risk in the economy. In contrast to rational expectations, the representativeness heuristic necessitates a binding capital constraint even when bank equity is high and the social cost of bank failure is low.

CHAPTER 1

INTRODUCTION

Banks in the US are now subject to the Current Expected Credit Loss (CECL) provisioning framework, introduced to enhance the timely recognition of credit losses and aleviate procyclicality of the banking system.[1] CECL represents a fundamental change in credit loss accounting, requiring the provisions of financial institutions to reflect future expected credit losses. Specifically, banks should consider not only past due and current information, but also reasonable and supportable forecasts of future economic conditions when determining provisions. These provisions should be accounted for as early as the time of loan origination, relying on forecasts of future losses, even prior to the availability of information regarding the borrower's repayment behavior. Consequently, macroeconomic forecasts have become a key component of the provisioning framework, given their significant influence on a borrower's ability to repay loans. Surprisingly, existing academic literature has paid limited attention to the influence of macroeconomic forecasts on provisions under CECL.[2] Additionally, there has been a gap of exploration into how the properties of these forecasts impact provisions and regulatory capital, despite the wealth of macroeconomic literature documenting systematic errors in macroeconomic forecasts, which constitutes the central focus of this paper.

I study the implications of CECL's mandate that bank provisions be forward-looking while considering the possibility that the underlying macroeconomic forecasts may not be rational and prone to behavioral biases. In particular, I present a theoretical model to examine how the characteristics of macroeconomic forecasts affect bank lending, aggregate output, and bank stability over the economic cycle. Subsequently, by considering the attributes of bank provisions under CECL, I derive the optimal regulatory policy in setting minimum

1. Accounting Standards Update (ASU) 2016-13 (ASC 326) and background information in Accounting Standards Update (ASU) 2016-13 (ASC 326) and https://www.federalreserve.gov/supervisionreg/topics/faq-new-accounting-standards-on-financial-instruments-credit-losses.htm

2. One exception is Lu and Nikolaev (2022).

capital requirements.

I begin by presenting stylized facts about macroeconomic forecasts that aid the motivation of my theoretical model. First, I document a noticeable increase in a bank's discussions of macroeconomic expectations in their financial statements after the implementation of CECL. For example, CECL-adopting banks have additional disclosures to note the macroeconomic factors used to estimate expected credit losses and to discuss how changes in the forecasts affected changes in the bank's loan loss allowances. These disclosures indicate that macroeconomic forecasts have assumed a heightened role within the provisioning framework.

Next, I explore the properties of a bank's macroeconomic expectations. Forecasting macroeconomic conditions is fundamental under CECL, as credit losses depend on the state of the economy. Despite the predominant paradigm that expectations are rational, a substantial body of empirical macroeconomic and behavioral finance literature strongly rejects the hypothesis of rational expectations (Coibion and Gorodnichenko, 2015; Bordalo et al., 2018; Bordalo et al., 2020; Beckmann and Reitz, 2020; Afrouzi et al., 2023; Barrero, 2022). The evidence against rational expectations is consistent across various forecasting agents, macroeconomic variables, and forecast horizons. To illustrate the issue and demonstrate that it is relevant in this setting, I test the properties of macroeconomic forecasts provided by banks and professional forecasting institutions using the Blue Chip Financial Forecasts survey. Following the methodology of Coibion and Goridnichenko (2015) and Bordalo et al. (2020), I document that the majority of forecasters overreact to news. This finding of overreaction is important because it contradicts the models previously used to explain flaws in macroeconomic forecasts related to the signal extraction process (Lucas, 1972; Kydland and Prescott, 1982; Woodford, 2003), infrequent information updating (Mankiw and Reis, 2002), and rational inattention (Sims, 2003).

Having provided evidence that a bank's forecasts overreact to news, my model explicitly considers such overreaction an important characteristic of macroeconomic forecasts. I follow

Bordalo et al. (2020), who reconcile the finding of overreaction to news by Kahneman and Tversky's (1972) representativeness heuristic. I use the formalization of the heuristic by Gennaioli and Shleifer (2010) and Bordalo et al. (2018). The representativeness heuristic implies that agents overweigh the future states whose likelihood increases when new information arrives. The source of deviation from rational updating arises when the forecaster rather than properly evaluating all possible states, as a shortcut, focuses on the more likely ones when forming expectations.

I present a stylized model of bank lending in which a representative bank is subject to a capital adequacy constraint and accounts for provisions using CECL. I consider macroeconomic expectations, which are subject to the representativeness heuristic, and compare the results to a rational expectations benchmark. There are two dates of interest in this model. On the first date, the bank raises deposits and originates two categories of loans: a risk-free loan and a risky loan, where the risk arises from the borrower's susceptibility to macroeconomic conditions. That is, the occurrence of default on the risky loan depends on the future macroeconomic state and, therefore, macroeconomic expectations play a crucial role in the bank's lending decision. The bank observes the current macroeconomic state, forms expectations about the future, and chooses the interest rates for the two types of loans. The bank accounts for provisions in accordance with CECL, which subsequently impacts its regulatory capital. The bank's decision determines the total amount of lending, aggregate output, and expected surplus in the economy; it also determines the bank's exposure to macroeconomic risk, and hence, the probability of bank failure. On the next date, the bank observes if the risky borrower repays or defaults on the loan and accounts for the corresponding profit or loss. At that time, the bank also receives the return from the safe loan. If it has sufficient funds, the bank repays its depositors; otherwise, it fails. I assume that bank failure incurs a social cost that reduces the overall welfare in the economy.

My analysis begins by assuming an exogenous minimum capital adequacy ratio. In

Section 5, I relax this assumption and derive the optimal regulatory capital, taking into account the properties of bank lending and provisions under CECL. Consistent with the macroprudential approach to regulation, in which the bank does not fully internalize the cost of its own failure, the regulator chooses the minimum capital constraint to maximize the welfare in the economy, already taking into account the social cost of bank failure.

The model highlights the following mechanisms: Macroeconomic expectations affect bank lending, provisioning, and minimal capital requirements through two primary channels. Firstly, the bank's forecast of future states informs its estimation of the risky borrower's probability of default. Consequently, the bank adjusts the interest rate offered to the risky borrower to reflect this risk, leading to variations in the originated loan amount and corresponding risk exposure. This channel, distinct from CECL, is inherent to the bank's lending decision-making process, wherein banks screen borrowers and decide on loan issuance based on expected defaults. This relationship is well-documented in the finance literature, particularly in studies exploring the interplay between expectations and credit cycles.

Secondly, macroeconomic forecasts impact bank-expected credit losses, which are provisioned at the time of loan origination under CECL. These provisions reduce the bank's regulatory capital. When the bank's equity is relatively low so that the bank is constrained by minimum capital requirements, the obligation to provision for expected credit losses curtails the bank's ability to originate loans. This aspect represents a novel aspect of the provisioning framework and aligns with empirical findings, such as those observed by Granja and Nagel (2023), which demonstrate that CECL affects bank lending through regulatory capital. More generally, macroeconomic expectations generated by the bank influence its compliance with minimum capital requirements. In favorable macroeconomic conditions, the bank chooses to reduce interest rates and increase loan origination. Simultaneously, it anticipates lower credit losses and accounts for a smaller amount of provisions, which boosts its regulatory capital and makes it easier for the bank to comply with minimum capital requirements. In other words, under CECL there is an endogenous relationship between bank expectations and regulatory capital as early as the time of loan origination, which results in time-varying capital adequacy ratio of the bank.

Next, I document that if macroeconomic forecasts are subject to overreaction to news, this is a source of excessive procyclicality compared to the rational expectations benchmark. Under the representativeness heuristic, good macroeconomic news leads to excessive optimism about the future macroeconomic state compared to the rational expectations benchmark. When the bank is overly optimistic, it lowers interest rates to expand lending, underprovisions, and originates a riskier portfolio. Bad news, in contrast, exacerbates the expectations for a downturn. When the bank holds an excessively pessimistic view of future economic conditions, it responds by raising the interest rate of the risky borrower. This adjustment reflects a higher anticipated probability of default, and provisions being higher that these under rational expectations. Consequently, the bank originates fewer risky loans, reducing the overall risk exposure of its portfolio. This constrained loan origination, in turn, leads to a dampened economic output.

Having established the link between expectations, provisions, and lending, I now derive the optimal regulatory capital. The need for bank regulation stems from the social cost of bank failure because the bank does not internalize this cost in its lending decision. Under CECL, the macroeconomic conditions affect not only the risky lending exposure of the bank and the probability of bank failure, but also bank provisions. Hence, the optimal regulatory capital is time-varying. Under rational expectations, there is a role for bank regulation when the bank is highly leveraged and the expected social cost of bank failure is high. In all other cases, it is optimal for the regulator to leave the bank unconstrained by a minimum capital requirement. When expectations exhibit overreaction to news, it is still optimal to set a binding capital constraint when the bank is substantially leveraged and the social cost of bank failure is high, but more interestingly, there is a need for constraining bank lending even when bank equity is high or bank failure cost is low. These results underscore the interaction between the provisioning framework and bank regulation under CECL and provide evidence for the increased need for bank regulation to counter the effects of overreaction to news.

My paper contributes to several strands of literature, including the literature on the effects of banking regulation, accounting standards, and behavioral macroeconomics. I provide a theoretical framework of the link between macroeconomic forecasts, bank provisioning, and lending under CECL, contributing to the growing theoretical literature in this area (Bouvatier and Lepetit, 2012; Mahieux et al., 2023; Bertomeu et al., 2023; Huber, 2021). A novel feature of my framework is explicitly modeling the properties of macroeconomic forecasts and linking bank lending decisions to economic activity and welfare. To the best of my knowledge, my paper is the first to study how the properties of macroeconomic forecasts affect loan loss provisions, and my findings are relevant to users of accounting numbers, such as investors and regulators.

I also contribute to the ongoing debate on whether bank capital requirements should change in response to CECL (see also Mahieux et al., 2023). I derive the optimal minimum capital requirements under CECL by considering the link between loan loss provisions and regulatory capital. I find that the level of the minimum capital requirements should change due to CECL, and that it should vary over time. Moreover, due to the inherent overreaction to news in macroeconomic forecasts, there is a need for bank regulation, even in cases that are not considered problematic, if banks use rational expectations. From a methodological point of view, I explicitly show how the occurrence of bank failure is linked to the bank's lending decision, provisioning framework, and the evolution of its balance sheet under CECL.

My paper also contributes to the growing empirical literature studying the potential procyclical effects of forward-looking provisioning by highlighting behavioral biases in macroeconomic forecasts as a potential source of procyclicality (Beatty and Liao, 2011; Abad and Suarez, 2018; Krüger et al., 2018; Covas and Nelson, 2018; Cohen and Edwards, 2017; Chae et al., 2018; Chen et al., 2022; Kim et al., 2023). This paper also relates to the empirical literature on the effect of International Financial Reporting Standard 9 (IFRS 9) and CECL on the accuracy and predictive power of provisions by providing a theoretical framework for how forward-looking provisioning impacts the properties of provisions (López-Espinosa et al., 2021; Gee et al., 2022; Bonaldi et al., 2023). This paper supports the concern that future credit losses are difficult to predict accurately by drawing attention to systematic errors in macroeconomic expectations (Harris et al., 2018; Lu and Nikolaev, 2022).

Although deviations from rationality have gained attention in macroeconomic and finance research, this is less common in accounting literature.[3] Behavioral studies in accounting are mostly limited to the study of judgement and decision-making by managers and auditors, although heuristics are likely applied to many other settings within accounting. By combining macroeconomic signals and behavioral biases in expectation formation, I present novel theoretical predictions about the effects of CECL.

Finally, my paper relates to finance literature by connecting macroeconomic expectations, lending behavior, and credit cycles (e.g. Ma et al., 2021; Bordalo et al. (2018)). In contrast to these papers, I look at the role of accounting standards and capital requirements in establishing the mechanism for this relationship.

3. Some exceptions are Chan et al. (2004), Koch and Wüstemann (2009), and Kochetova and Salterio (2003) who provide a review of the older literature.

CHAPTER 2

USE AND PROPERTIES OF MACROECONOMIC EXPECTATIONS

Expectations play a central role in economics. Empirically, numerous studies have demonstrated that the forecasts of various economic agents serve as insightful indicators of these agents' underlying beliefs and, importantly, stand as a good predictor of their actual behavior (Greenwood and Shleifer, 2014; Gennaioli et al., 2016; Tanaka et al., 2020).

2.1 Use of macroeconomic forecasts under CECL

CECS stipulates that bank provisions should reflect forward-looking information. In particular, banks should recognize all loan losses at their expected value based on reasonable and supportable forecasts of future economic conditions. Macroeconomic conditions are an important factor in loan performance and, therefore, macroeconomic forecasts are a key ingredient for estimating expected credit losses under the new accounting standard.

Kim et al. (2023) provide evidence that banks rely more heavily on forward-looking information following the adoption of CECL, as intended by the new standard. However, their analysis potentially captures both macroeconomic and other forward-looking information. As I am primarily interested in reliance on macroeconomic forecasts alone, I conducted a textual analysis of bank financial statements; this analysis confirmed that banks have increasingly focused on macroeconomic forecasts after the adoption of CECL. I used filings of 10Ks and 10Qs of banking corporations from the SEC EDGAR database, which I matched to bank Call Reports from the Federal Deposit Insurance Corporation. I searched for any references to macroeconomic forecasts in both annual and quarterly reports.[1] Figure 1 plots

1. Specifically, I searched for forward-looking expressions related to future macroeconomic conditions, such as mentions of "macroeconomic forecast," "macroeconomic outlook," and "macroeconomic scenarios."

the references to macroeconomic expectations for CECL-adopting banks and non-adopting banks over time. The group of CECL-adopters only contains banks that adopted the new provisioning method in Q1 2020, while the other group consists of banks that used the incurred loss framework during the entire reporting period.[2] We see a sharp increase in the share of banks that discuss macroeconomic expectations in their financial statements post-implementation of CECL. Banks typically include a section on allowances for credit losses, which explains the key drivers of their model of expected credit losses as well as the key macroeconomic variables upon which it depends. This section often presents the forecast of these variables, in qualitative or quantitative terms, as well as how changes in the macroeconomic forecasts affect the allowance for credit losses. Appendix A contains examples of these sections. This evidence highlights the growing significance of macroeconomic forecasts within the provisioning framework and underscores the need to study how the attributes of these forecasts influence provisions.

2.2 Properties of macroeconomic expectations

I now explore the properties of macroeconomic forecasts using data from Blue Chip Financial Forecasts. Blue Chip is an organization that surveys institutional forecasts, covering professional forecasters, financial institutions, insurance companies, and wealth management companies. The surveys are conducted monthly, and the panelists are asked to provide forecasts of real GDP growth, inflation, and interest rates for the current quarter and all five future quarters. The sample contains forecasts by 230 institutions, 99 of which are financial institutions, and runs from January 1983 through February 2023.

Previous literature provides evidence that the forecasting institutions in the Blue Chip

2. ASU 2016–13 was initially set to take effect in January 2020 for all SEC filers, except for smaller reporting companies. Due to the COVID-19 pandemic, the CARES Act provided firms with an option to delay CECL adoption until the earlier of the first date of an eligible financial institution's fiscal year that begins after the date when the COVID-19 national emergency is terminated or January 1, 2022.

Financial Forecasts sample act in line with their disclosed forecasts. Wang (2021) matched a subset of banks from the Blue Chip data to their Call Report's balance sheet information and found that a bank's allocations to Treasuries vary positively and significantly with their bond returns forecasts. Ma et al. (2022) document that a bank's GDP growth forecasts from the Blue Chip sample are in line with their baseline projections in the Federal Reserve's FR Y-14A form, which are used by banks for capital assessments and stress testing.

I test the hypothesis of rational expectations following the methodology of Coibion and Goridnichenko (2015), which studies the relationship between forecast revisions and forecast errors. If the rational expectations hypothesis holds, forecast errors should not be predictable from the information available at the time of making the forecast. Under the hypothesis of rational expectations, the forecaster should be taking into account all the available information at the time of forecasting and process it optimally using Bayesian updating. Therefore, future forecast errors should not be systematically related to prior forecast revisions. Empirically, the predictability of forecast errors can be assessed using the following regression:

$$x_{t+h} - F_t x_{t+h,i} = \alpha + \beta(F_t x_{t+h,i} - F_{t-1} x_{t+h,i}) + \delta_h + +\delta_i + \delta_x + e_{t,h,i}$$

where i is an index of the forecasting institutions, $h \in [1, 5]$ is the forecast horizon in quarters, and $x \in \{inflation, GDPgrowth\}$ is the forecast variable. Furthermore, δ_i, δ_h, and δ_x capture fixed effects for the forecasting institution, forecasting horizon, and forecast macroeconomic variable. Let $x_{t+h,i}$ denote the realized value of the variable at time $t + h$, while $F_t x_{t+h,i}$ is the forecast of the variable h quarters ahead that is produced by forecaster i at time t, and $F_{t-1} x_{t+h,i}$ is the first lag of this variable, i.e., forecast produced at time $t-1$. In essence, the equation regresses the forecast error, $x_{t+h} - F_t x_{t+h,i}$, on the magnitude of the revision of the forecast, $F_t x_{t+h,i} - F_{t-1} x_{t+h,i}$. The main coefficient of interest is β. The hypothesis of rational expectations is consistent with $\beta = 0$. If $\beta \neq 0$, the full information rational expectations hypothesis can be rejected. In particular, finding $\beta < 0$ is consistent with overreaction to news, while $\beta > 0$ is consistent with underreaction.

Table 1 shows the results for the full sample in columns (1)-(2) and for the sample of banks in columns (3)-(4). In all cases, the negative and statistically significant coefficient between forecast errors and forecast revisions indicates that we can reject the hypothesis of rational expectations, in favor of evidence of overreaction to news. My results showing that bank macroeconomic forecasts exhibit systematic errors is in line with the long-standing literature on biases in survey expectations (Pesaran, 1987; Zarnowitz, 1985). Even though traditional economic analysis has been dominated by the Rational Expectations Hypothesis, the growing availability of survey-based microeconomic data on expectations has unveiled notable and quantitatively significant departures from rational expectations.[3] Moreover, my results are consistent with recent evidence about overreaction to news in the field of behavioral finance (Bordalo et al., 2020, Afrouzi et al., 2023). Overall, the analysis in this section implies that assuming bank forecasts are rational ignores the widespread deviations from such behavior in the data. Therefore, in the subsequent sections of this paper, I take the possibility of expectation biases seriously and investigate their implications for bank provisions, lending practices, and overall economic outcomes.

2.3 Representativeness heuristic

In this paper, I reconcile the empirical finding that macroeconomic forecasts exhibit overreaction to news using Kahneman and Tversky's (1972) representativeness heuristic. Kahneman and Tversky's groundbreaking research has documented numerous cognitive biases that impact the way humans assess probabilities (Kahneman and Tversky, 1972, 1974, 1983). They have proposed the representativeness heuristic, rooted in the human tendency to judge the probability of an event by how closely it resembles a preconceived example or stereotype, as a source of departure from rational evaluation of probabilities. Kahneman and Tversky's

3. For review, see Pesaran and Weale(2006) and Coibion, Goridnichenko and Kamdar (2018).

representativeness heuristic has survived substantial experimental scrutiny over the years, with numerous researchers highlighting its importance (Gilovich et al., 2002). Even sophisticated statistical forecasting models can be susceptible to the influence of heuristics through the input of exogenous assumptions and managerial overlays.

Previously, the literature has tried to account for the predictability of forecast errors by emphasizing possible deviations from full information due to information rigidities, while maintaining the assumption of rational expectations. Examples within this domain are the sticky-information model of Mankiew and Reis (2002), the noisy signal extraction models of Lucas (1973), Kydland and Prescott (1982) and Woodford (2003), as well as the rational inattention model of Sims (2003). However, the recent findings that forecasts overreact to news holds particular significance because it contradicts the predictions of these models.

Another important stream of literature explores whether the deviations in forecasts from rational expectations are driven by agency-related factors or behavioral biases. A growing body of evidence lends support to behavioral explanations for the observed data patterns. For example, Ehrbeck and Waldmann (1996) examine various models of strategic considerations as potential sources of bias in professional forecasts, but the data rejects the proposed models. In the domain of research exploring the factors behind credit cycles, Fahlenbrach and Stulz (2011) and Cheng, Raina, and Xiong (2014) offer evidence suggesting that distorted incentives alone may not be the sole explanations for the buildup of excessive risk-taking leading up to the 2007-2008 financial crisis. Instead, biased expectations have emerged as a potential explanation for the 2007-2008 financial crisis and a driver of credit cycles, more generally (Shleifer, 2011; Gennaioli and Shleifer, 2018; De Stefani and Zimmermann, 2022; Ma, 2022). Early work of Minsky (1977) and Kindleberger (1978) postulate that credit cycles originate from over-optimism leading to credit and investment booms. Consistent with this, a growing body of research, across different contexts, is finding evidence that agents' expectations are biased in the direction of overreaction to news, that is, agents tend to be overly optimistic amid favorable economic conditions, and vice versa (Greenwood and Shleifer, 2014; Piazzesi, Salomao, and Schneider, 2015; Bordalo, Gennaioli, and Shleifer, 2018; Bordalo, Gennaioli, Ma, and Shleifer, 2019; Gennaioli, Ma, and Shleifer, 2016; Richter and Zimmermann, 2019; De Stefani, 2021; Afrouzi et al., 2023).

Afrouzi et al. (2023) ran a large-scale randomized experiment where participants were asked to forecast an AR(1) process. In their setting, the process was fixed and stable, the participants were familiar with the structure of the process and observed its realizations without any information frictions, and their payoff only depended on the accuracy of the forecast. Given these experimental conditions, the researchers in this study found that forecasts display significant overreaction to the most recent observation.

In summary, unlike the existing models of sticky information, noisy information, rational inattention, and strategic behavior, the representativeness heuristic allows for reconciling the observed biases in macroeconomic forecasts. Moreover, the representativeness heuristic finds longstanding support in experimental research, which underscores its widespread influence. Given these reasons, I consider expectations based on the representativeness heuristic as detailed in the following section.

CHAPTER 3

THE MODEL

I present a model of bank lending and loan loss provisioning, in which macroeconomic expectations play a key role. I consider rational expectations as a benchmark and I also study expectations based on the representativeness heuristic, which are formalized below. The model comprises two sectors and a regulator: an entrepreneurial sector seeking loans and a representative bank extending these loans. The bank is subject to a minimum capital requirement set by the regulator. In this section, I assume that the capital requirement is a fixed exogenous parameter, which does not depend on the macroeconomic state. This setting resembles the current regulatory landscape in the United States, where the capital requirement is fixed over time. Once I have derived the link between the properties of macroeconomic expectations, provisions, and bank lending, I study the bank regulator's problem in Section 5 in order to determine the optimal level of the capital constraint.

The timing of events is as follows. At time t, the bank raises insured deposits and originates two categories of loans corresponding to the two types of entrepreneurs: one risk-free and the second risky, where the risk arises from the entrepreneurs' susceptibility to macroeconomic conditions. That is, the occurrence of default on the risky loan depends on the future macroeconomic state. At time t, the bank observes the current macroeconomic state, forms expectations about the future, and chooses the interest rates for the two types of loans, adhering to the minimum capital requirements. The bank accounts for provisions in accordance with CECL. At time $t+1$, the risky borrower repays or defaults on the loan and the bank also receives the return from the safe loan. If it has sufficient funds, the bank repays its depositors; otherwise, it fails. Deposits are fully insured by the regulator, but bank failure comes at a social cost that reduces the overall welfare in the economy.

3.1 Expectation formation

As detailed in the subsequent sections, the bank's loan origination and provisioning depend on macroeconomic expectations. Therefore, I start this section by outlining the assumed process governing the evolution of the macroeconomic state as well as the expectation formation process.

The macroeconomic state at time t, denoted by x_t, follows an AR(1) process:

$$x_t = \rho x_{t-1} + u_t$$

where $x_0 = 0$, $\rho \in (0, 1)$ is a known constant and the error terms u_t are i.i.d., $u_t \sim N(0, \sigma_u^2)$. Throughout the paper, I refer to the error terms in this process as macroeconomic shocks. Note that the coefficient ρ governs the persistence of the macroeconomic shocks over time.

3.1.1 Rational expectations

As a benchmark, I first consider expectation formation under full information and rational expectations. In this case, on each date t the bank observes the current and past macroeconomic states, $X_t = \{x_t, x_{t-1}, ..., x_0\}$. Let $f^{RE}(x_{t+T}|X_t)$ denote the distribution of $x_{t+T}|X_t$. Let $\mathbb{E}^{RE}[x_{t+T}|X_t]$ and $Var^{RE}(x_{t+T}|X_t)$ denote the first two moments of the distribution. Considering that the evolution of the macroeconomic state is governed by the process $x_{t+T} = \rho^T x_t + \sum_{i=0}^{T-1} \rho^i u_{t+T-i}$, for $T \geq 1$, we can see that the distribution of $x_{t+T}|X_t$ is a sum of normally distributed variables and, therefore, also normal. Proposition 1 characterizes this distribution.

Proposition 1. *Under the information set $X_t = \{x_t, x_{t-1}, ..., x_0\}$ and rational expectations,* *the distribution $f^{RE}(x_{t+T}|X_t)$ is normal and characterized by the following moments:*

$$\mathbb{E}^{RE}[x_{t+T}|X_t] = \rho^T x_t$$
$$Var^{RE}(x_{t+T}|X_t) = \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2$$

3.1.2 Representativeness heuristic

As discussed in section 2.3, I allow expectations to rely on Kahneman and Tversky's representativeness heuristic. I am using the formalization of the representativeness heuristic by Gennaioli and Shleifer (2010) and Bordalo et al. (2020). The representativeness heuristic implies that agents overweigh the future state whose likelihood has increased the most as a result of observing new information. Equivalently, this implies underestimating states that are less likely given the new information. In the setting of this paper, this implies that if the latest macroeconomic news is good, agents will overestimate the possibility for a continued expansion in the future (as good news is more representative of expansions) and underestimate the possibility of a recession, compared to what Bayesian updating would imply. The source of the deviation from rational updating is that the human mind does not retrieve all possible states but focuses on the more representative ones when forming expectations.

To illustrate this heuristic, consider the following example. Suppose there are three possible macroeconomic states in the future period $(t+1)$: (1) recession in which GDP falls by 5%, (2) the GDP stays unchanged, and (3) an economic boom in which GDP grows by 5%. Once the macroeconomic signal at time t is observed, suppose there are three possible scenarios about the information in the signal: (1) No news, i.e., the realized signal fully corresponds to the expectation from the previous period $t-1$, (2) Good news, i.e., the realized signal is better than the expectation from the previous period, and (3) Bad news, i.e., the signal falls short of the expectation. Table 2 presents the conditional distributions of the macroeconomic states under each of these scenarios.

Under rational expectations, the forecaster considers all possible states whose probabilities are presented in the table. In the case of no news: $\mathbb{E}^{RE}[\text{GDP growth}|\text{No news}] = 1.5\%$, in case of good news, $\mathbb{E}^{RE}[\text{GDP growth}|\text{Good news}] = 2.25\%$ and in case of bad news, $\mathbb{E}^{RE}[\text{GDP growth}|\text{Bad news}] = 0\%$.

Using the representativeness heuristic, the forecaster is affected by the so-called "representativeness" of the state once new information has become available, which is defined as the ratio between the likelihood of the state given new information and the likelihood given no new information. The representativeness factor attempts to capture to what extent the likelihood of a state has changed in light of the latest information. More formally, for any macroeconomic state $x \in \{\text{Recession}, \text{No change}, \text{Boom}\}$ and any scenario of $News \in \{\text{No news}, \text{Good news}, \text{Bad news}\}$, the representativeness factor is defined as:

$$R(x) = \frac{f(x|\text{News})}{f(x|\text{No news})}$$

The forecast using the representativeness heuristic weighs the distribution of the states by their representativeness using the distorted posterior:

$$f^{RH}(x_t|X_t) = f^{RE}(x_t|X_t)R(x_t)^{\theta}\frac{1}{Z_t}$$

where $f^{RE}(x_t|X_t)$ is the underlying Bayesian conditional distribution, $R(x_t)$ is the representativeness factor of each state, and Z_t is a normalization factor ensuring that $f^{RH}(x_t|X_t)$ integrates to 1. The parameter $\theta \geq 0$ denotes the extent to which the forecast relies on the representativeness heuristic. In particular, when $\theta = 0$, the expectation formation process coincides with the rational one. However, when $\theta > 0$, forecasts that are based on the representativeness heuristic overestimate highly representative states and underestimate unrepresentative states. In the example above, forecasts based on the representativeness heuristic under good news overestimate the possibility for an economic boom compared to the rational forecast because good news makes this state more likely. For example, for $\theta = 0.5$, E^{RH}[GDP growth|Good news] $= 2.6\%$, which exceeds the rational forecast and demonstrates that there is an overreaction to good news. Similarly, under bad news, using the heuristic leads to overweighing the recession and neutral state and underweighing the booming state versus the rational forecast. For example, for $\theta = 0.5$, E^{RH}[GDP growth|Bad news] $= -0.7\%$, which is much lower that the rational forecast and again demonstrates overreaction to news.

Moving beyond the simplified example above, the representativeness heuristic is applied to the model in the following way. The distribution of $x_{t+T}|x_t$, perturbed by the representativeness heuristic is:

$$f^{RH}(x_{t+T}|X_t) = f^{RE}(x_{t+T}|X_t)\left[\frac{f(x_{t+T}|X_t)}{f(x_{t+T}|X_t = \rho x_{t-1})}\right]^{\theta}\frac{1}{Z}$$

Proposition 2. *Under the information set $X_t = \{x_t, x_{t-1}, \ldots, x_0\}$, the distribution of $x_{t+T}|X_t$, perturbed by the representativeness heuristic, is normal. Let $E^{RH}(x_{t+T}|X_t)$ and $Var^{RH}(x_{t+T}|X_t)$ denote the mean and variance of this perturbed distribution. The moments are presented by the following expressions:*

$$\mathbb{E}^{RH}(x_{t+T}|X_t) = (1+\theta)\mathbb{E}^{RE}(x_{t+T}|X_t) - \theta\mathbb{E}^{RE}(x_{t+T}|X_{t-1}) = \rho^T\left(x_t + \theta u_t\right)$$
$$Var^{RH}(x_{t+T}|X_t) = \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2$$

Note that when $\theta = 0$, i.e., when the agent puts no weight on the representativeness factor, this distribution coincides with the one under rational expectations.

3.2 Entrepreneurial sector

There are two types of entrepreneurs, each differing from each other by their project's sensitivity to the macroeconomic state. Apart from this sensitivity, entrepreneurs are identical.[1] Both entrepreneurs have no wealth and seek funds from a bank to finance their projects. At time t, each entrepreneur i seeks funding to finance the setup costs, K_{it}, and invests the raised funds into production. At time $t+1$ the return from the project is realized. The production technology of the entrepreneurs is given by:

$$Y_{it} = AK_{it}^{\alpha}$$

The term AK_{it}^{α}, where $\alpha \in (0, 1)$, represents a diminishing returns to scale technology, which can be interpreted as an investment technology with adjustment costs. The productivity parameter $A > 1$ is a fixed constant, which is large enough so that the net present value of the entrepreneurs' projects is positive.

The project of the first entrepreneur is risk-free, in the sense that its payoff is unaffected by macroeconomic conditions. In contrast, the second entrepreneur's project is risky, as its payoff is contingent upon macroeconomic conditions.

The risk-level of the second entrepreneur is indexed by the parameter γ, which is exogenous. If the macroeconomic state at time $t+1$ falls below this parameter, then the entrepreneur is unable to sell its production at time $t+1$, generates no payoff, and is unable to repay its loan at that time.[2] Lower levels of γ indicate lower levels of risk, in the sense

1. The assumption that the production function, and hence demand for credit, is identical for both types of entrepreneurs is not essential to this analysis. What is key is that one of the projects is not subject to macroeconomic risk, which is the only risk in the model. The role of the risk-free sector is to allow for the bank to invest in a risk-free asset, and not necessarily collapse in case the risky loan defaults. Moreover, the ratio of risky to risk-free loans provides a measure of the risk exposure of the bank.

2. To provide a more concrete example, imagine that at time $t+1$ consumers (not explicitly modeled here) are hit by a shock, such as unemployment or a shock to their wealth. The shock prevents the consumers from buying the entrepreneur's product, hence his or her production perishes.

that only more adverse macroeconomic states can impede payoff. Therefore, the probability that the second type of entrepreneur defaults is $h_t \equiv P(x_{t+1} \leq \gamma | X_t)$. The entrepreneur repays the loan if $x_{t+1} > \gamma$ and defaults if $x_{t+1} \leq \gamma$.

Note that, whenever I refer to the theoretical, rational probability of default, I denote it by h_t without any superscripts. When I refer to the probability of default evaluated by the bank, I denote it by h_t^ι, and I study the possibility for this evaluation to be done using by both rational expectations and the representativeness heuristic, i.e. $\iota \in \{RE, RH\}$.

Suppose the entrepreneurs are risk neutral. Taking the interest rate offered by the bank as given, each entrepreneur chooses the amount of investment (which is the same as the amount of loan) in order to maximize his or her expected payoff:

Entrepreneur 1 (risk-free project): $$\max_{K_{1t}} AK_{1t}^\alpha - K_{1t} - r_{1t}K_{1t}$$

Entrepreneur 2 (risky project): $$\max_{K_{2t}} (AK_{2t}^\alpha - K_{2t} - r_{2t}K_{2t})(1 - h_t)$$

The payoff of entrepreneur i consists of the production output AK_{it}^α, net of the investment amount K_{it} and the interest repayment to the bank $r_{it}K_{it}$. The risk-free entrepreneur attains this level of payoff regardless of the macroeconomic state, while the risky entrepreneur attains it only if the macroeconomic state is favorable (i.e. $x_{t+1} > \gamma$), which happens with probability $1 - h_t$.

The solution to the entrepreneurs' problem is the following. The optimal investment for both types of entrepreneurs, $i = 1, 2$, is:

$$K_{it} = \left(\frac{1 + r_{it}}{\alpha A} \right)^{-\frac{1}{1-\alpha}}$$

This equation implies a downward sloping demand for loans, which is the same for both types of entrepreneurs regardless of their different exposures to risk. This result stems from the assumption of limited liability: the risky entrepreneur derives utility only in the upside case, when they sell production and realize profits. Therefore, his or her optimal investment decision is not directly influenced by the macroeconomic state. As we will see below, the risky entrepreneur is indirectly affected by the macroeconomic conditions because of the bank prices in the state-dependent default risk through the interest rate charged for the loan.[3]

3.3 Banking sector

The banking sector consists of a representative bank. The bank's balance sheet at any time t is as follows:

The assets include the amount of the risk-free loan (K_{1t}) and the risky loan (K_{2t}). They are funded by equity (E_t) and insured deposits (D_t). When the bank accounts for provisions, the loan loss provision (LLP_t) lowers the net value of the loan K_t, which translates to a loss that lowers the amount of equity. The bank balance sheet identity $K_{1t} + K_{2t} - LLP_t = D_t + E_t - LLP_t$ holds at any time t.

At time t, the bank observes the information set X_t, which contains the current macroeconomic state x_t, as well as all history of the previous states back to date 0: $X_t = \{x_t, x_{t-1}. \dots, x_0\}$. The bank does not forget the history, so as new macroeconomic information becomes available over time, their information set expands.

I now specify the actions the bank can take. At t, the bank starts with an exogenously given equity, E_t. The bank cannot issue equity, and its amount is only affected by profits and losses over the two dates of interest, t and $t+1$. At time t, the bank can originate deposits. I assume that deposits are fully insured and inelastically supplied; hence, I abstract from

3. Note that the assumption the entrepreneurs have no initial wealth is not crucial. Suppose the entrepreneurs have initial wealth W_0. The risky entrepreneur's investment decision is: $\max_{K_{2t}}(W_0 + AK_{2t}^{\alpha} - K_{2t} - r_{2t}(K_{2t} - W_0))(1 - h_t)$. The risky entrepreneur's investment decisions will still be driven by the upside case, yielding the same downward-sloping demand for loans for the two types of entrepreneurs. Due to limited liability, the disciplining role in considering future macroeconomic conditions will still come from the bank's lending decision.

studying the threat of bank runs. I do so because, in this paper, I focus on studying the regulator's role in monitoring the bank's risk-taking behavior rather than that of depositors. For simplicity, the interest rate is normalized to zero.[4] At time t, the bank also chooses the interest rates on loans, which can be differentiated across the different types of entrepreneurs: r_{it}. The bank indirectly chooses the amount of loans by setting their interest rates, and the amount of deposits that it raises is a residual term: $D_t = K_{1t} + K_{2t} - E_t$. In line with CECL, the bank also accounts for loan loss provisions.

At time $t+1$, the bank realizes payoff from the risk-free loan at the amount $r_{1t}K_{1t}$. Furthermore, the risky entrepreneur either repays or defaults on the loan. Formally, at date $t+1$, the loan generates a return of the amount $r_{2t}K_{2t}$ if $x_{t+1} > \gamma$, or a loss of K_{2t} otherwise. If the bank funds are not sufficient to repay its depositors, the bank fails, and the regulator makes a frictionless transfer to the depositors to fully cover their deposits.

The bank is subject to an exogenous capital adequacy requirement set by the regulator. The capital requirement resembles the Basel Tier 1 capital ratio, which postulates that the ratio of equity to risk-weighted assets should exceed a threshold: $\frac{E_t}{RWA_t} \geq \xi$, where E_t denotes the bank's equity, and RWA_t denotes the amount of the bank's risk-weighted assets at any time t. The capital requirement should always hold. According to the Basel Accord, risk weighted assets are calculated by assigning assets to different risk categories, with weights between 0 and 1 depending on the risk. The risk weight of the risk-free loan is

4. If the deposit interest rate is non-zero, the optimal interest rate on loans increases one-to-one by the deposit interest rate. Therefore, we can think of the current version of the loan interest rates offered by the bank as rates, net of the interest paid on deposits. The deposit interest rate will be priced in the interest paid by the entrepreneurs, limiting the demand for loans, output and entrepreneurial surplus, while some surplus is transferred to the depositors. However, the direction in which lending and total output vary with the macroeconomic state remains unchanged. I have also considered a version of the model in which the supply of deposits is elastic, and the investment in the risk-free asset is residual ($K_1 = D + E - K_2$). This version of the model is equivalent, but redistributes some surplus from the bank to the depositors, which I do not explicitly study now.

0 while that of the risky loan is 1; hence, the capital requirement can be expressed as:

$$\frac{E_t - LLP_t}{K_{2t} - LLP_t} \geq \xi$$

3.4 The bank's problem

Given the information at time t, the bank maximizes its expected cumulative profit by choosing the interest rates on loans, making sure the capital requirement is fulfilled.

Formally, the bank's problem is:

$$\max_{r_{1t}, r_{2t}} (r_{1t}K_{1t} + (1 - h_t^{\iota})r_{2t}K_{2t} - h_t^{\iota}K_{2t})$$

subject to:

$$\begin{aligned}
&\text{Loan demand:} && K_{it} = \left(\frac{1 + r_{it}}{\alpha A}\right)^{-\frac{1}{1-\alpha}} && i = 1, 2 \\
&\text{Capital adequacy:} && \frac{E_t - LLP_t}{K_{2t} - LLP_t} \geq \xi && \\
&\text{CECL:} && LLP_t = h_t^{\iota}K_{2t} &&
\end{aligned}$$

defined for bank expectations $\iota \in \{RE, RH\}$.

The bank chooses the interest rates on the two types of loans in order to maximize its expected profit. Bank profit consists of the payoff on the safe loan as well as the payoff on the risky loan, which is collected only in a favorable macroeconomic state (with probability h_t^{ι}) or a loss at the amount of the risky loan in case the macroeconomic state is adverse (with probability h_t^{ι}). By choosing the interest rates, the bank indirectly chooses the amount of loan origination, following the loan demand scheme, already derived in the previous subsection.

Under the expected credit loss provisioning method, the bank accounts for a loan loss provision at the amount of the expected credit loss. The latter depends on the macroeconomic information at time t through its effect on the expected future macroeconomic state. The bank accounts for the expected loss as a loan loss provision at time t: $LLP_t = h_t^\iota K_{2t}$. Therefore, the capital adequacy requirement is equivalent to $E_t - h_t^\iota K_{2t} \geq \xi(1 - h_t^\iota)K_{2t}$.[5]

In the bank optimization problem, the bank uses its own evaluation of default probability, h_t^ι. I consider the problem under both rational expectations and the representativeness heuristic, denoted by $\iota \in \{RE, RH\}$. Under rational expectations the bank estimates the default probability in the following way: $h_t^{RE} \equiv \mathbb{E}^{RE}[\mathbb{1}\{x_{t+1} \leq \gamma\}|X_t] = \int_\infty^\gamma f_{\tilde{x}_{t+1}|X_t}^{RE}(x_{t+1})dx_{t+1}$. If the bank bases its forecast based on the representativeness heuristic, $h_t^{RH} \equiv \mathbb{E}^{RH}[\mathbb{1}\{x_{t+1} \leq \gamma\}|X_t] = \int_\infty^\gamma f_{\tilde{x}_{t+1}|X_t}^{RH}(x_{t+1})dx_{t+1}$.

Proposition 3 shows the solution to the bank problem and the corresponding lending and output levels. The solution depends on whether the minimum capital requirement is binding. This, on the other hand, depend on the amount of equity, E_t, that the bank has.

5. Note that at $t+1$ equity evolves in the following way: $E_{t+1} = E_t - LLP_t + \pi_{t+1}$, where π_{t+1} is the profit of the bank at $t+1$, i.e., bank equity changes only with the amount of profit or loss, which is generated at time $t+1$. Therefore, if $(E_t - LLP_t)/(K_{2t} - LLP_t) \geq \xi$, in expectation, the capital constraint will also be fulfilled at time $t+1$, i.e., $\mathbb{E}[E_{t+1}/K_{2t+1}|X_t] \geq \xi$, because the bank would set its expected profits above zero. More precisely, if the bank sets high enough interest rates, it effectively rejects all loan applications and originates no loans, $K_{1t} = K_{2t} = 0$, in which case $\mathbb{E}[\pi_{t+1}|X_t] = 0$. Therefore, we can be sure that if the capital requirement is fulfilled at time t, it is expected to be fulfilled at time $t+1$ as well: $\mathbb{E}[E_{t+1}/(K_{2t} - LLP_t)|X_t] = \mathbb{E}[(E_t - LLP_t + \pi_{t+1})/(K_{2t} - LLP_t)|X_t] \geq \mathbb{E}[(E_t - LLP_t)/(K_{2t} - LLP_t)|X_t]$. Therefore, only the capital constraint for time t appears in the optimization problem.

Proposition 3. ***Case 1 (Bank - unconstrained by the capital requirement)***

If $E_t \geq \bar{E}$, the capital constraint is not binding. Then:

$$r_{1t} = \frac{1}{\alpha} - 1 \qquad r_{2t} = \frac{1}{\alpha(1-h_t^\iota)} - 1$$

$$K_{1t} = \left(\frac{1}{\alpha^2 A}\right)^{\frac{1}{\alpha-1}} \qquad K_{2t} = \left(\frac{1}{\alpha^2 A}\frac{1}{1-h_t^\iota}\right)^{\frac{1}{\alpha-1}}$$

$$Y_{1t} = A\left(\frac{1}{\alpha^2 A}\right)^{\frac{\alpha}{\alpha-1}} \qquad Y_{2t} = A\left(\frac{1}{\alpha^2 A}\frac{1}{1-h_t^\iota}\right)^{\frac{\alpha}{\alpha-1}}$$

Case 2 (Bank - constrained by the capital requirement)

If $E < \bar{E}$, the capital constraint is binding. Then:

$$r_{1t} = \frac{1}{\alpha} - 1 \qquad r_{2t} = \alpha A\left(\frac{h_t^\iota + \xi(1-h_t^\iota)}{E_t}\right)^{1-\alpha} - 1$$

$$K_{1t} = \left(\frac{1}{\alpha^2 A}\right)^{\frac{1}{\alpha-1}} \qquad K_{2t} = \frac{E_t}{h_t^\iota + \xi(1-h_t^\iota)}$$

$$Y_{1t} = A\left(\frac{1}{\alpha^2 A}\right)^{\frac{\alpha}{\alpha-1}} \qquad Y_{2t} = A\left(\frac{E_t}{h_t^\iota + \xi(1-h_t^\iota)}\right)^{\alpha}$$

$$\lambda = \frac{1}{\xi + (1-\xi)h_t^\iota}\left(\alpha^2 A(1-h_t^\iota)\left(\frac{\xi + (1-\xi)h_t^\iota}{E_t}\right)^{1-\alpha} - 1\right),$$

where λ denotes the Lagrange multiplier which corresponds to the capital constraint, and $\bar{E}$ denotes the following threshold:[6]

$$\bar{E} = (\xi + (1-\xi)h_t^\iota)\left(\alpha^2 A(1-h_t^\iota)\right)^{\frac{1}{1-\alpha}}$$

6. See Appendix B for details.

3.4.1 Unconstrained case

For the safe loan, the interest rate only depends on α, which is related to the loan demand elasticity. When loan demand is less elastic, i.e., α is small, a higher interest rate yields the maximized profit for the bank. In the case of the risky loan, the interest rate is not only tied to the elasticity of loan demand but also related to the entrepreneur's default risk. As risk increases, the bank requires a higher interest rate as compensation for expected losses, which, in turn, decreases the demand for the loan. Due to the higher default probability of the risky entrepreneur, the bank sets a higher interest rate compared to the one for the safe entrepreneur, resulting in a lower volume of risky loan origination. In this manner, the bank plays a disciplining role in loan origination by considering the impact of future macroeconomic conditions on the borrower's default probability.

3.4.2 Constrained case

When the bank is constrained by the minimum capital requirement, the optimal interest rate for the safe entrepreneur remains unchanged because regulatory capital only depends on the risky loan. In contrast, the interest rate for the risky entrepreneur is now higher compared to the unconstrained case, restricting the amount of risky loan origination. The interest rate for the risky entrepreneur is positively related to her default probability and the minimum capital requirement, and inversely related to the level of bank equity. Higher equity allows the bank to originate more loans without violating the capital adequacy requirement. Increasing the capital adequacy threshold (ξ) has the opposite effect. When the expectation about the probability of default on the risky loan (h_t^ι) improves, the bank requires a smaller amount of loan loss provisions, which relaxes the capital adequacy constraint and allows the bank to originate more loans. On the other hand, as the default risk increases, the required provisions also increase. This limits the amount of risky loan origination, but only up to the point where lending falls so much that the capital constraint becomes no longer binding.

Last, note that the problem is constrained when $E_t < \bar{E}$, i.e., when:

$$E_t < (\xi + (1-\xi)h_t^\iota)\left(\alpha^2 A(1-h_t^\iota)\right)^{\frac{1}{1-\alpha}}$$

In general, as the default risk rises, the capital constraint gains more slack. The reason for this is when the risky project has a higher default risk, the bank sets higher interest rates, which lowers risky loan origination that, in turn, results in a higher capital adequacy ratio. As the capital adequacy threshold increases, i.e., as ξ increases, the capital adequacy constraint has less slack, in the sense that a wider range of values of the parameters satisfy $E_t < \left(\xi + (1-\xi)h_t^\iota\right)\left(\alpha^2 A(1-h_t^\iota)\right)^{\frac{1}{1-\alpha}}$. In contrast, when the initial equity (E_t) increases, the constraint is relaxed.

3.5 Risk taking and aggregate output

Having shown the level of loan origination, I next characterize the corresponding expressions for loan loss provisions, the bank's risk exposure, the risk of bank failure and aggregate outcomes, which directly depend on the loan amounts K_{1t} and K_{2t}.

Loan loss provisions, LLP_t, reflect the bank's expected credit losses, which only stem from the risky loan:

$$LLP_t = h_t^\iota K_{2t}$$

The share of the bank's risky lending out of its total lending, R_t, which I use as a measure of the bank's risk exposure, is:

$$R_t = \frac{K_{2t}}{K_{1t} + K_{2t}}$$

Aggregate lending:

$$K_t = K_{1t} + K_{2t}$$

Aggregate output:

$$Y_t = Y_{1t} + Y_{2t} = AK_{1t}^{\alpha} + AK_{2t}^{\alpha}$$

Overall, increasing the origination amount of the risky loan results in elevated risk exposure, greater aggregate lending, and increased aggregate output.

CHAPTER 4

THE ROLE OF MACROECONOMIC EXPECTATIONS

In this section, I study the role of macroeconomic expectations within the model in greater depth. I begin by examining comparative statics on lending, risk-taking, and aggregate economic outcomes as expectations change. Subsequently, I explore how the results change when expectations are grounded in the representativeness heuristic as opposed to the rational expectations benchmark. At the end of the section, I also study the interaction between expectations and the minimum capital requirement.

4.1 Comparative statics regarding macroeconomic expectations

As shown in Section 3.4, lending depends on the borrower's probability of default. I now investigate how the expectation formation process affects the risky borrower's default probability, which is a key driver in the model.

Proposition 4. *For the probability of default, defined as* $h_t^\iota \equiv \mathbb{E}^\iota[\mathbb{1}\{x_{t+1} \leq \gamma\}|X_t] = \int_\infty^\gamma f^\iota_{\tilde{x}_{t+1}|X_t}(x_{t+1})dx_{t+1}$, *the following holds:*

$$(1) \qquad \frac{\partial h_t^\iota}{\partial \mathbb{E}^\iota[x_{t+1}|X_t]} \leq 0$$

$$(2) \qquad \frac{\partial h_t^\iota}{\partial Var^\iota[x_{t+1}|X_t]} \geq 0 \Leftrightarrow \gamma \leq \mathbb{E}^\iota[x_{t+1}|X_t]$$

where $\iota \in \{RE, RH\}$

Part (1) of the proposition states that improvement in macroeconomic expectations leads to a decrease in the probability of default. While I formally prove the proposition in the Appendix B, Figure 2 provides the intuition behind it. As the mean of the distribution of $x_{t+1}|X_t$ shifts to the right (Panel (a)), a lower mass of the distribution is below the risk parameter γ. As a result, the adverse states in which the risky entrepreneur would default become less likely. Panel (b) illustrates that the default probability falls below the one under the initial distribution, lowering the default hazard.

Part (2) of the proposition states that an increase in the variance of the distribution leads to higher default probability if and only if the risk parameter γ is below the mean. This is the case when the default probability is below 50%, which is the more relevant case banks operate in. The intuition is that when the variance of the macroeconomic shock σ_u^2 increases, adverse states (such that $x_{t+1} < \gamma$) will occur more often. As we can see in Panel (c), as the variance of the distribution increases, the default probability is weakly above the initial one whenever the threshold γ is below the mean of the distribution, and weakly below the initial default probability whenever γ is above the mean.

Having established the link between macroeconomic expectations and the probability of default, I now present the impact of changes in macroeconomic expectations on lending, output, and bank stability.

Proposition 5. *Both in the constrained and unconstrained case, for $\iota \in \{RE, RH\}$ the following conditions hold:*

$$\begin{array}{ll} \textit{Total lending:} & \dfrac{\partial K_t}{\partial \mathbb{E}^\iota[x_{t+1}|X_t]} \geq 0 \\ \textit{Total output:} & \dfrac{\partial Y_t}{\partial \mathbb{E}^\iota[x_{t+1}|X_t]} \geq 0 \\ \textit{Risk exposure:} & \dfrac{\partial R_t}{\partial \mathbb{E}^\iota[x_{t+1}|X_t]} \geq 0 \end{array}$$

Economic activity generated by the risk-free project does not depend on the macroeconomic state, and hence does not depend on macroeconomic expectations. Expectations shape the activity of the risky entrepreneur, and through that effect shape overall lending, output, and surplus in the economy. As expectations improve, the expected default probability of the risky entrepreneur falls, which allows for the bank to offer a lower interest rate for the project and originate a higher loan. This translates into a larger investment and larger output.

Proposition 5 also shows that, under rational expectations, optimal bank lending and economic output co-vary with the macroeconomic state. This stems from the fact that the macroeconomic conditions are persistent, making the current macroeconomic state informative about the expected future bank losses. Specifically, optimal lending behavior and risk-taking exhibits a cyclical pattern: expanding when expectations improve and contracting when expectations deteriorate.

4.2 Deviation from rational expectations

The mean and variance of the distribution of $x_{t+1}|X_t$ under the different expectation formation processes are summarized in Table 4. When the bank uses the representativeness heuristic in the expectation formation process, the distribution mean can either exceed or fall short of the rational expectations mean, depending on the sign of the macroeconomic news u_t. In case of good news, due to overreaction, the distribution of the future state $x_{t+1}|X_t$ will shift to the right of the one distribution under rational expectations. As we know from Proposition 4, this leads to an underestimation of the default probability of the risky entrepreneur. Conversely, if news is negative, the bank overestimates the borrower's probability of default compared to the rational expectations benchmark.

When agents are less capable of retrieving the probabilities of unrepresentative states, the reliance on the representativeness heuristic is higher (θ is higher). In this case, the overreaction to macroeconomic shocks is stronger, and the mean of the distribution $x_{t+1}|X_t$ under the representativeness heuristic lies further away from the rational benchmark.

Figure 3 illustrates how expectations vary with changes to the macroeconomic state based on a simulation of the model. In particular, I simulate the bank problem for different points in time s, as if the bank is born with E_s equity and makes lending decisions at time s, realizes payoffs and repays its deposits at time $s+1$, and dies. A new identical bank is born at time $s+1$ and the problem repeats, where the only difference is the macroeconomic state has changed based on the assumed AR(1) process: $x_{t+1} = \rho x_t + u_t$. For the simulation, I assume a persistence coefficient $\rho = 0.7$ and error term $u_t \sim N(0,1)$. Figure 3 illustrates the forecast's overreaction to news; namely, when $u_t > 0$ ($u_t < 0$), the expected macroeconomic state based on the representativeness heuristic is higher (lower) compared to the one based on rational expectations.

I now explore how differences in the expectation formation process affect provisions, lending, aggregate output, and risk taking. In particular, I study how deviating from rational expectations to expectations based on the representativeness heuristic affects lending and all other variables of interest. In addition to the variables already presented in the previous section, I study loss overhang based on the definition of unreported expected losses by Bushman and Williams (2015) and denote it by $H_t^{RH} = (h_t^{RE} - h_t^{RH})K_t^{RH}$. Suppose the bank uses the representativeness heuristic in forming expectations. In this case, the bank originates K_t^{RH} loans and provisions an amount of $h_t^{RH} K_{2t}^{RH}$. However, the rational estimate of expected losses on this loan is $h_t^{RE} K_{2t}^{RH}$. Therefore, if $H_t^{RH} > 0$, it indicates a loss overhang, meaning the bank has accounted for less than the rational evaluation of the expected loss. Conversely, if $H_t^{RH} < 0$, it indicates that the bank has overprovisioned for the loan.

Proposition 6. *Let $\mathbb{E}^{RH}[x_{t+1}|X_t]$ and $\mathbb{E}^{RE}[x_{t+1}|X_t]$ denote the mean of the distribution of the future state under rational expectations and the representativeness heuristic, respectively.*
(a) Excessive optimism. If $\mathbb{E}^{RH}[x_{t+1}|X_t] > \mathbb{E}^{RE}[x_{t+1}|X_t]$, then:

$$h_t^{RH} < h_t^{RE}, \qquad H_t^{RH} < 0, \qquad K_t^{RH} > K_t^{RE}, \qquad Y_t^{RH} > Y_t^{RE}, \qquad R_t^{RH} > R_t^{RE}$$

(b) Excessive pessimism. If $\mathbb{E}^{RH}[x_{t+1}|X_t] < \mathbb{E}^{RE}[x_{t+1}|X_t]$*, then:*

$$h_t^{RH} > h_t^{RE}, \qquad H_t^{RH} > 0, \qquad K_t^{RH} < K_t^{RE}, \qquad Y_t^{RH} < Y_t^{RE} \qquad R_t^{RH} < R_t^{RE}$$

Proposition 6 demonstrates that the representativeness heuristic is a source of procyclicality under CECL. Under the representativeness heuristic, good news leads to excessive optimism, which leads to more extensive loan origination and larger output in the economy compared to the rational expectations benchmark. Nevertheless, because the bank underestimates the default risk when using the representativeness heuristic, $h_t^{RE} > h_t^{RH}$, the risk exposure of the bank is higher than the risk exposure in the rational expectations benchmark, i.e., $R_t^{RH} > R_t^{RE}$. Notice that the deviation between rational and biased expectations gives rise to an expected loss overhang of $(h_t^{RE} - h_t^{RH})K_{2t}^{RH} > 0$.

In the case of bad news, overreaction yields too gloomy expectations for the borrower's repayment probability, causing banks to shrink loan supply below the rational level. As a result, output in the economy falls relative to the rational expectation benchmark. At the same time, reported provisions exceed the rational expectation of credit losses.

Figure 4 illustrates how expectations affect lending and the related indicators along the economic cycle. The evolution of the macroeconomic state and bank expectations evolve in line with the simulation presented in Figure 3. As we see, when macroeconomic expectations are based on the representativeness heuristic, bank lending and output are more procyclical than those based on rational expectations.

In summary, these findings indicate that relying on forecasts, particularly when they are susceptible to overreaction to news, can have contrasting effects during expansions and during recessions. In favorable times, this reliance tends to result in excessive lending, potentially leading to a positive output gap but at the expense of accumulating elevated risks within the banking sector. Conversely, in adverse times, the same tendency towards overreacting to news results in excessive precautionary behavior, exacerbating the impact of negative news by causing an excessive contraction in production.

4.3 Interaction between expectations and the minimum capital requirement

When the bank possesses sufficient equity, resulting in a slack in the minimum capital constraint, its lending decisions remain unaffected by the presence of a capital requirement. In this case, the capital requirement does not affect the representativeness heuristic's impact on lending.

The significance of the capital requirement only comes into play when the bank's equity is relatively low, resulting in a binding constraint. Given that the capital requirement hinges on the bank's risk-weighted assets, expressed as $\frac{E_s}{RWA_s} \geq \xi$, for $s = t, t+1$, the constraint exclusively impacts the origination of risky loans. When the constraint is binding, the bank originates the following amount of risky loan: $K_{2t} = \frac{E_t}{\xi + h^\iota(1-\xi)}$.

It is noteworthy that this quantity depends on the bank's estimation of the default probability of the risky entrepreneur, which can deviate from rational expectations. If the estimated default probability falls below the rational one, i.e., $h_t^{RH} < h_t^{RE}$, the constraint becomes more lenient compared to the rational expectations benchmark. On the other hand, if the bank overestimates the borrower's default probability, the constraint tightens. This observation underscores the endogenous nature of the capital constraint. That is, the stringency of the capital requirement under CECL depends on the bank's own evaluation of the risk and, consecutively, on the amount of provisions it accounts for.

Increasing the capital requirement, as shown in Proposition 7, limits the distorting effect of the representativeness heuristic. That is, the higher the minimum capital requirement, the lower the deviation of loan origination under the representativeness heuristic compared to the rational expectations benchmark.

Proposition 7. *Given a minimum capital requirement* ξ*, let* $K_t^{RE}(\xi)$ *and* $K_t^{RH}(\xi)$ *denote the bank lending amount under rational expectations and the representativeness heuristic, respectively. Suppose the capital constraint is binding for both* $K_t^{RE}(\xi)$ *and* $K_t^{RH}(\xi)$*, and* $K_t^{RE}(\xi)$ *and* $K_t^{RH}(\xi)$ *are differentiable.*[1] *Then:*

$$\frac{\partial |K_t^{RH}(\xi) - K_t^{RE}(\xi)|}{\partial \xi} \leq 0,$$

where the equality is obtained only if $K_t^{RE}(\xi) = K_t^{RH}(\xi)$

This proposition demonstrates that bank regulation has the potential to affect the impact of the representativeness heuristic.

1. We need the unconstrained lending amount, which the bank would set in the absence of a capital constraint, to be strictly larger that the lending amount that is constraint by the minimum capital requirement, ξ.

CHAPTER 5

BANK REGULATION UNDER CECL

So far, I have examined how various expectation formation processes impact provisions, lending, and aggregate economic variables while assuming an exogenous capital constraint. In this section, I endogenize the capital constraint and derive the optimal capital requirement considering different expectation formation processes. By doing so, I shed light on novel implications for bank regulation arising from the newly introduced forward-looking provisioning framework.

5.1 Bank failure

An important rationale for bank regulation is the negative externalities stemming from bank failure. I next investigate the role of a regulator who sets the optimal capital requirement by internalizing the social cost arising from bank failure.

Without a capital constraint, the bank originates the following amounts of safe and risky loans, respectively:

$$K_{1t} = \left(\alpha^2 A\right)^{\frac{1}{1-\alpha}} \qquad\qquad K_{2t} = \left(\alpha^2 A(1 - h_t^{\iota})\right)^{\frac{1}{1-\alpha}}$$

Table 5 shows the balance sheet snapshots of the bank over time depending on the realization of the macroeconomic state. Panel (a) shows the balance sheet at time t, while panels (b) and (c) show the balance sheet at time $t+1$ in the two cases depending on whether the risky loan defaults or not. The safe loan generates a return of r_{1t} irrespective of the macroeconomic state at time $t+1$. If the macroeconomic state is favorable, i.e., $x_{t+1} \geq \gamma$, which happens with probability $1 - h_t$, the risky borrower does not default on the loan and generates a return of r_{2t}. In this case, the assets at time $t+1$ reach $(1 + r_{1t})K_{1t} + (1 + r_{2t})K_{2t}$, exceeding the amount of deposits that need to be repaid. Therefore, in the favorable macroeconomic case, the bank does not fail irrespective of the loan origination amounts.

If the macroeconomic state is unfavorable, i.e., $x_{t+1} < \gamma$, which happens with probability h_t, the risky borrower defaults on the loan. In this case, only the safe loan generates a return. The value of the assets at time $t+1$ is $(1+r_{1t})K_{1t}$ and the bank is unable to repay its deposits if $(1+r_{1t})K_{1t} < D_t$, in which case the bank fails. Using the balance sheet equation for time t, $K_{1t} + K_{2t} = E_t + D_t$, we see that in the adverse macroeconomic case, the bank fails if the loss on the risky loan exceeds the initial level of equity and the return on the safe loan, i.e., the bank fails when $K_{2t} > E_t + r_{1t}K_{1t}$.

5.2 The regulator's problem

I now turn to the regulator's problem, which is to maximize the expected surplus in the economy by setting the minimum capital requirement, internalizing the social cost of bank failure. Unlike in the previous section of the paper, where the minimum capital requirement was exogenous and constant over time, I now allow the regulator to set the capital requirement in an optimal way that takes into account the macroeconomic conditions. Consistent with the macroprudential approach to regulation, I posit the existence of negative externalities associated with bank failure, thereby motivating the need for bank regulation. The bank maximizes profits and does not internalize any potential spillover effects that its failure could inflict upon the broader economy. I do not specify the exact nature of the social cost associated with bank failure. For instance, we can conceptualize this externality as the cost incurred when bank failure leads to a loss of confidence in the banking system, triggering negative spillover effects such as fire sales and credit crunches.

It is worth noting that my approach to addressing the regulator's problem is primarily theoretical in nature. Specifically, I explore the optimal capital requirement when the regulator derives the expected surplus using rational expectations, considering the possibility that the bank may base its expectations on the representativeness heuristic. It is crucial to emphasize that my analysis does not delve into potential biases that may arise in the regulator's expectations, even though these are also likely to be present in the real world. It is beyond the scope of this paper whether implementing the optimal regulatory policy is feasible.

The expected surplus in the economy is the following:

$$\mathbb{E}[S|X_t] = \underbrace{AK_{1t}^{\alpha} + (1-h_t)AK_{2t}^{\alpha} - K_{1t} - K_{2t}}_{\text{Surplus of entrepreneurs and bank}} - \underbrace{ch_t \mathbb{1}\{K_{2t} > r_1 K_1 + E_t\}}_{\text{Bank failure cost}}$$

This surplus is generated by the production output of both types of entrepreneurs net of the undergone investment amounts. For the risky project, the production output is realized only in the favorable macroeconomic case, with probability $(1 - h_t)$. If the bank fails, I assume that there is a social cost of c.[1] This happens only in an unfavorable macroeconomic state (with probability h_t) and only if the loss of the bank is high enough to trigger bank failure ($K_{2t} > E_t + r_1 K_{1t}$).

The regulator maximizes the expected surplus in the economy by choosing a minimum capital requirement, ξ_t. The capital requirement only affects the loan amount of the risky loan, hence the problem is equivalent to maximizing $(1 - h_t)AK_{2t}^{\alpha} - K_{2t} - ch_t \mathbb{1}\{K_{2t} > E_t + r_1 K_{1t}\}$.

1. Similar insights are obtained if I assume that the social cost of bank failure is non-decreasing in the size of the bank loss, K_{2t}

Therefore, the regulator's problem is the following:

$$\max_{\xi_t}(1-h_t)AK_{2t}^{\alpha} - K_{2t} - ch_t\mathbb{1}\{K_{2t} > E_t + r_1K_{1t}\}$$

s.t.

$$K_{2t} = \min\left\{\underbrace{(\alpha^2 A(1-h_t^{\iota}))^{\frac{1}{1-\alpha}}}_{\text{Unconstrained: } K^U}, \underbrace{\frac{E}{\xi + h^{\iota}(1-\xi)}}_{\text{Constrained: } K^C}\right\}$$

The problem's constraint reveals the following mechanism: By changing the level of the minimum capital requirement ξ_t, the regulator chooses between two options. The regulator can either opt for the bank to remain unconstrained, thus allowing the bank to set its optimal level of risky loan origination at $K^U \equiv (\alpha^2 A(1-h_t^{\iota}))^{\frac{1}{1-\alpha}}$, or they can raise the minimum capital requirement, thereby constraining the bank to originate risky loans at the amount of $K^C \equiv \frac{E_t}{\xi + h_t^{\iota}(1-\xi)}$. The problem's solution differs based on the bank's expectation approach. Proposition 8 characterizes the solution when the bank uses rational expectations, while Proposition 9 characterizes the solution when the bank relies on the representativeness heuristic.

Proposition 8. *Suppose the bank uses rational expectations in forming expectations.*

Case 1 (high equity)

- *When $E_t \geq \bar{E}$, it is optimal for the regulator to set a non-binding capital constraint $\xi = 0$, and the bank originates $K_{2t} = (\alpha^2 A(1-h_t^{RE}))^{\frac{1}{1-\alpha}}$*

Case 2 (low equity, low expected bank failure cost)

- *When $E_t < \bar{E}$ and $ch_t^{RE} \leq \bar{C}$, it is optimal for the regulator to set a non-binding capital constraint $\xi = 0$, and the bank originates $K_{2t} = (\alpha^2 A(1-h_t^{RE}))^{\frac{1}{1-\alpha}}$*

Case 3 (low equity, high expected bank failure cost)

- *When $E_t < \bar{E}$ and $ch_t^{RE} > \bar{C}$, it is optimal for the regulator to set a binding capital constraint $\xi = \frac{1}{1-h_t^{RE}}\left[\frac{E_t}{r_{1t}K_{1t}+E_t} - h_t^{RE}\right]$, and the bank originates $K_{2t} = r_{1t}K_{1t} + E_t$*

where $\bar{E}$ and $\bar{C}$ denote the following thresholds:

$$
\begin{aligned}
\bar{E} &= \left(\alpha^2 A(1-h_t^{RE})\right)^{\frac{1}{1-\alpha}} - r_{1t}K_{1t} \\
\bar{C} &= (1-h_t^{RE})A\left[(\alpha^2 A(1-h_t^{RE}))^{\frac{\alpha}{1-\alpha}} - (r_{1t}K_{1t}+E_t)^{\alpha}\right] \\
&\quad - \left[(\alpha^2 A(1-h_t^{RE}))^{\frac{1}{1-\alpha}} - r_{1t}K_{1t} - E_t\right]
\end{aligned}
$$

and:

$$r_{1t}K_{1t} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(\alpha A)^{\frac{1}{1-\alpha}}$$

Figure 5 illustrates the optimal level of loan origination as a solution to the regulator's problem under rational expectations. Proposition 8 demonstrates that when the bank uses rational expectations, there is a need for bank regulation when bank equity is low ($E_t < \bar{E}$) and the expected cost of bank failure is high ($ch_t^{RE} > \bar{C}$). In all other cases, it is optimal for the regulator to leave the bank to set its optimal level of lending unconstrained by the capital requirement, which is equivalent to setting the minimum capital adequacy to zero.

This finding aligns with the established understanding of the role of bank regulation in enhancing bank stability. Banking regulations are designed to promote safe and sound banking practices by ensuring banks have enough capital to cover their risks, a rationale that underpins both the minimum capital requirements and stress testing. This result also highlights the mandate for bank regulators to focus on systemically important banks, i.e., banks whose failure can potentially cause big negative spillovers.

Several other observations warrant mentioning. Firstly, the optimal capital requirement depends on the underlying risk of the economy, h_t^{RE}, and in that sense, the solution is time-varying. In the case where there is no default probability of the risky borrower, i.e., $h_t^{RE} = 0$, there is no need for regulation. Noting that $\partial\bar{C}/\partial h_t^{RE} < 0$ and $\partial(ch_t^{RE})/\partial h_t^{RE} > 0$, when the risk increases, the regulator must to shift from a lax regulatory policy to a restrictive one. This stems from the fact that, with a high default probability of the borrower, the probability of bank failure rises without the bank internalizing the welfare implications of its potential collapse. As the risky borrower's default probability further increases, although still constraining, the regulator should lower the minimum capital requirement. This adjustment stems from CECL's mandate for timely loan loss provisioning, which causes an endogenous adjustment of the bank's regulatory capital. Under CECL, as the risk in the economy raises, the bank's expected credit losses also swell, leading to higher provisions that erode the bank's capital and thus tighten the capital constraint. In this context, it is noteworthy that the optimal regulatory capital under the incurred loss framework, where expected losses are not accounted for at the time of loan origination, serves as an upper bound of the optimal capital constraint under CECL.[2] Furthermore, as risk continues to rise, the bank progressively reduces the volume of risky loans it originates, to the point where the bank's equity can absorb any potential loss (note that $\partial \bar{E}/\partial h_t^{RE} < 0$ and as $h_t^{RE} \to 1$, $\bar{E} \to 0$ $E_t > \bar{E}$). In this case, there is no longer need for bank regulation.

Now, I study the regulator's optimal policy when the bank relies on the representativeness heuristic when forming expectations.

Proposition 9. *Suppose the bank uses the representativeness heuristic in expectations.*

Case 1 (high equity)

- *When* $E_t \geq \tilde{E}$ *and* $h_t^{RH} \geq h_0$ *it is optimal for the regulator to set a non-binding capital constraint* $\xi = 0$*, and the bank originates* $K_{2t} = (\alpha^2 A(1 - h_t^{RH}))^{\frac{1}{1-\alpha}}$

- *When* $E_t \geq \tilde{E}$ *and* $h_t^{RH} < h_0$ *it is optimal for the regulator to set a binding capital constraint* $\xi = \frac{1}{1-h_t^{RH}} \left[\frac{E_t}{(\alpha A(1-h_t^{RE}))^{\frac{1}{1-\alpha}}} - h_t^{RH} \right]$*, and the bank originates* $K_{2t} =$

2. See Appendix C for a comparison of loan origination and optimal regulatory capital under ILM and CECL.

$\left(\alpha A(1-h_t^{RE})\right)^{\frac{1}{1-\alpha}}$

Case 2 (low equity, low expected bank failure cost)

- *When* $E_t < \tilde{E}$, $ch_t^{RE} \leq \tilde{C}$ *and* $h_t^{RH} < h_0$, *it is optimal for the regulator to set a binding capital constraint* $\xi = \frac{1}{1-h_t^{RH}}\left[\frac{E_t}{\left(\alpha A(1-h_t^{RE})\right)^{\frac{1}{1-\alpha}}} - h_t^{RH}\right]$, *and the bank originates* $K_{2t} = \left(\alpha A(1-h_t^{RE})\right)^{\frac{1}{1-\alpha}}$

- *When* $E_t < \tilde{E}$, $ch_t^{RE} \leq \tilde{C}$ *and* $h_t^{RH} \in [h_0, h_1] \cup [h_2, 1]$, *it is optimal for the regulator to set a non-binding capital constraint* $\xi = 0$, *and the bank originates* $K_{2t} = (\alpha^2 A(1 - h_t^{RH}))^{\frac{1}{1-\alpha}}$

- *When* $E_t < \tilde{E}$, $ch_t^{RE} \leq \tilde{C}$ *and* $h_t^{RH} \in [h_1, h_2]$ *it is optimal for the regulator to set a binding capital constraint* $\xi = \frac{1}{1-h_t^{RH}}\left[\frac{E_t}{r_1 K_{1t}+E_t} - h_t^{RH}\right]$, *and the bank originates* $K_{2t} = r_{1t}K_{1t} + E_t$

Case 3 (low equity, high expected bank failure cost)

- *When* $E_t < \tilde{E}$ *and* $ch_t^{RH} > \tilde{C}$, *it is optimal for the regulator to set a binding capital constraint* $\xi = \frac{1}{1-h_t^{RH}}\left[\frac{E_t}{r_{1t}K_{1t}+E_t} - h_t^{RH}\right]$, *and the bank originates* $K_{2t} = r_{1t}K_{1t} + E_t$

where $\tilde{E}$, $\tilde{C}$, h_0, h_1 and h_2 are the following thresholds:

$$\tilde{E} = \left(\alpha A(1 - h_t^{RH})\right)^{\frac{1}{1-\alpha}} - r_{1t}K_{1t}$$

$$\tilde{C} = (1 - h_t^{RE})A\left[(\alpha A(1 - h_t^{RE}))^{\frac{\alpha}{1-\alpha}} - (r_{1t}K_{1t} + E_t)^{\alpha}\right]$$

$$- \left[(\alpha A(1 - h_t^{RE}))^{\frac{1}{1-\alpha}} - r_{1t}K_{1t} - E_t\right]$$

$$h_0 = 1 - \frac{1 - h_t^{RE}}{\alpha}$$

h_1 is the smaller root of the equation:

$$(1 - h_t^{RE})A\left[(\alpha^2 A(1 - h_t^{RE}))^{\frac{\alpha}{1-\alpha}} - (r_1 K_{1t} + E_t)^{\alpha}\right]$$

$$- \left[(\alpha^2 A(1 - h_t^{RE}))^{\frac{1}{1-\alpha}} - r_1 K_{1t} - E_t\right] - ch_t^{RE} = 0$$

$$h_2 = 1 - \frac{1}{\alpha^2 A}(r_{1t}K_{1t} + E_t)^{1-\alpha}$$

and

$$r_{1t}K_{1t} = (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}}(\alpha A)^{\frac{1}{1-\alpha}}$$

Figure 6 illustrates this result. The solution in the case when the bank uses the representativeness heuristic in expectations shows that now the optimal level of the minimum capital adequacy ratio relies not only on the borrower's default probability, h_t^{RE}, but also on the probability of default, estimated by the bank, h_t^{RH}. In this sense, bank regulation should take into account and work to undo the biases in the bank's expectations. The following corollaries underscore another important difference compared to the rational expectations benchmark. In particular, there is now a need for a constraining capital requirement even when the bank is not highly leveraged and when the expected cost of bank failure is low.

Corollary 9.1. *When $E_t > \tilde{E}$ and $h^{RH} < h_0$ the regulator sets a binding minimum capital requirement under the representativeness heuristic, but not under rational expectations.*

Corollary 9.2. *When $E_t \leq \bar{E}$, $ch_t^{RE} < \bar{C}$ and $h_t^{RH} \in [0, h_0] \cup [h_1, h_2]$ the regulator sets a binding minimum capital requirement under the representativeness heuristic, but not under* *rational expectations.*

More generally, Propositions 8 and 9 provides evidence that bank regulation should change in response to the introduction of the new accounting regime. These propositions underscore the intricate relationship between the provisioning accounting standard and bank regulation. Furthermore, they demonstrate that the properties of macroeconomic forecasts not only influence provisions and bank lending, but also have a significant impact on the design of optimal bank regulation.

CHAPTER 6

GENERAL IMPLICATIONS

In this paper, I study the role that macroeconomic forecasts play in banks' estimations of expected credit losses within the framework of CECL, and highlight the potential effects biases of macroeconomic forecasts can have on provisions and regulatory capital.

As previously mentioned, CECL mandates that expected credit losses reflect future economic expectations. These expectations potentially encompass not only macroeconomic factors but also various other economic indicators related to bank and borrower future performance. The issues raised in this paper regarding forecasts likely extend beyond macroeconomic predictions to include other types that influence expected credit loss estimation.

There are two main reasons why I choose to emphasize the role of macroeconomic forecasts. Firstly, these forecasts are observable and testable, allowing for explicit examination of their properties and studying potential deviations from rational expectations. Secondly, macroeconomic forecasts play a central role in the provisioning framework under CECL, evident from their prominence in bank disclosures about loan loss allowances. Therefore, studying the implications of biased expectations within this context is particularly important.

However, biases are likely not confined to macroeconomic forecasts alone. This assumption is grounded in the extensive literature on behavioral biases in forecasting, stemming from the inherent challenges of prediction. Existing empirical literature documents issues in forecasting extending to various other variables, including earnings, sales, stock returns, bond yields, and credit spreads.[1] Consequently, systematic errors in expectations may impact provisions not only through macroeconomic forecasts but also through other elements of bank loan loss models, such as forecasts of borrowers' probability of default (PD), loss given default, exposure at default, recovery rates, etc. Although research data on these as-

1. For references, see Ma (2022).

pects may be less readily available, studies like those by Tozzo et al. (2023) and Gaul et al. (2023) provide evidence supporting this hypothesis, highlighting predictable errors in bank PD forecasts.

Furthermore, reliance on forward-looking information not only permeates provisions under CECL but also finds widespread application across accounting practices. Forward-looking estimates constitute an indispensable component in the valuation of reserves, impairment assessments, write-downs, and other. The reliance on forward-looking information assumes particular salience within the domain of fair value accounting, particularly where fair value is based on management's own estimates of the future. The evolution of accounting standards under US GAAP has increasingly included the reliance on forward-looking information in the financial statements, through the expanded use of fair values, as underscored by the augmented utilization of fair values in standards such as SFAS 115, SFAS 119, SFAS 133, and SFAS 159. The investigation into the potential impact of deviations from rational expectations in this forward-looking information, and its broader implications for accounting numbers, remains an open question beyond the scope of this paper.

CHAPTER 7

CONCLUSION

In this paper, I explore the implications of CECL's mandate that provisions be based on future economic conditions. With the increasing reliance of banks on macroeconomic forecasts within the CECL framework, it is crucial to understand how these forecasts affect the provisioning process. I have developed a theoretical model to examine the impact of macroeconomic forecasts on loan loss provisions, bank lending, economic output, and bank stability under CECL. Additionally, I investigate the optimal response of bank regulators in establishing minimum capital requirements. I assume that banks use rational macroeconomic forecasts as a starting point. A key feature of this model is that the return on bank loans is contingent on future macroeconomic conditions, making macroeconomic expectations pivotal for provisioning and bank lending.

I find that the bank plays a disciplinary role in risk-taking by factoring macroeconomic risk into the interest rates it charges and provisioning for expected defaults. As expectations improve, the bank anticipates lower credit losses, resulting in reduced minimum capital requirements, increased loan origination, and expanded exposure to risk. Conversely, in a deteriorating macroeconomic outlook, the opposite occurs. My model also demonstrates that the optimal capital requirement should respond to underlying default risk in the economy, meaning that the optimal capital constraint varies over time. Under rational expectations, effective bank regulation is necessary when bank equity is insufficient or when the bank is systemically important and the social cost of bank failure is significant.

Having established the results under the rational expectations benchmark, I also consider expectations based on Kahneman and Tversky's (1972) representativeness heuristic. I do this in order to account for the empirical observation that macroeconomic forecasts are often non-rational and prone to overreacting to news. Under the representativeness heuristic, favorable news leads to excessive optimism, resulting in heightened loan origination and robust economic activity. However, the bank underestimates default risks and assumes a risk exposure higher than what rational expectations would dictate, increasing the probability of bank failure. On the other hand, overreaction to adverse news triggers excessive precautionary measures during downturns, leading to a welfare loss compared to the rational benchmark. These findings emphasize that reliance on forecasts, especially when they are subject to biases, can yield undesirable outcomes. In particular, the representativeness heuristic contributes to procyclicality in bank lending and risk-taking.

When expectations overreact to news, the optimal capital requirement must adapt to both the inherent default risk in the economy and the biases in bank expectations. The potential for overreaction to news introduces an additional reason for the optimal capital constraint to vary over time. The representativeness heuristic necessitates a binding capital constraint, even when bank equity is high, and the social cost of bank failure is low, fundamentally altering the nature of bank regulation.

My model underscores the significance of examining the properties of macroeconomic forecasts within the CECL framework. More broadly, it highlights the critical connection between accounting standards and bank regulation, shedding light on how bank regulation must evolve in response to CECL.

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FIGURES AND TABLES

Figure 1: Reference to macroeconomic forecasts in bank financial statements

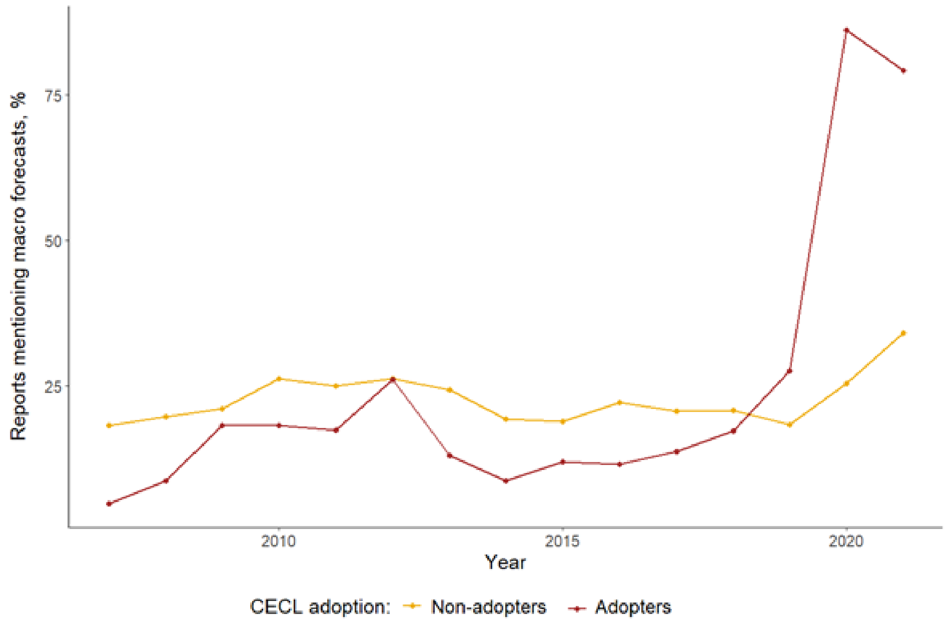
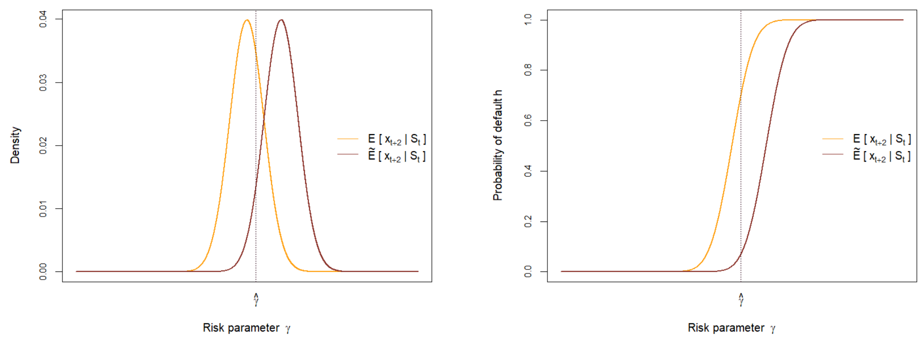
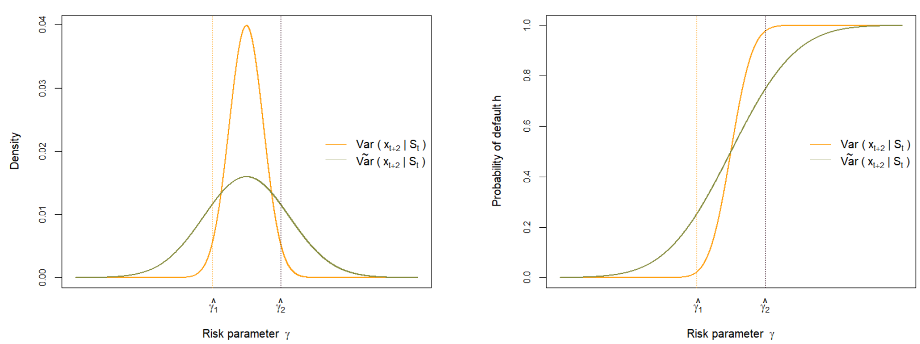


Figure 2: Default probability under changes in the moments of the conditional distribution $f(x_{t+1}|X_t)$



(a) Density under a shift in the first moment

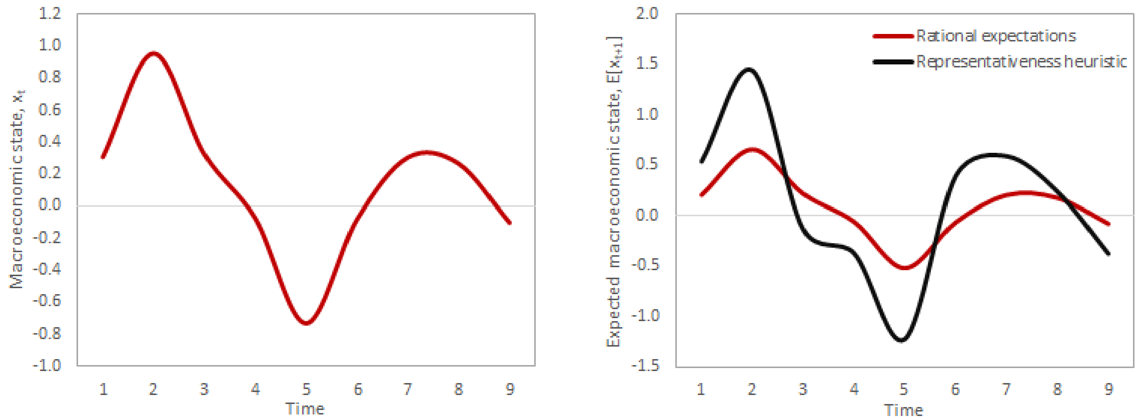
(b) Probability of default under a shift in the first moment



(c) Density under a shift in the second moment

(d) Probability of default under a shift in the second moment

Figure 3: Current macroeconomic state and macroeconomic expectations

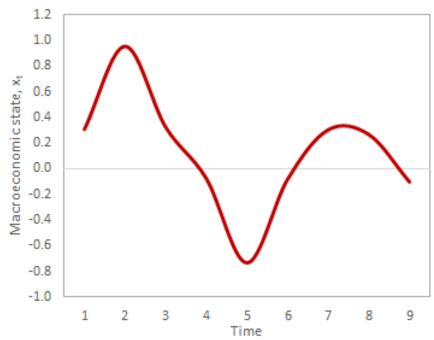


(a) Current macroeconomic state

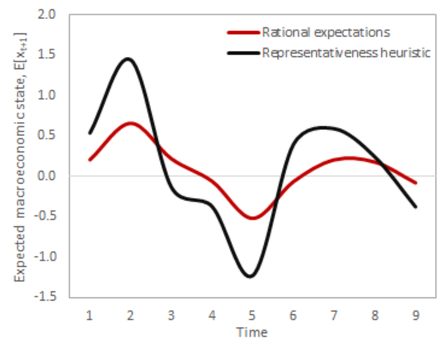
(b) Expected future macro state, $\mathbb{E}^{\iota}(x_{t+1}|X_t)$

Simulation under parameters: $\rho = 0.7$, $\alpha = 0.8$, $\xi = 0.2$, $\sigma_u = 0.5$, $E_t = 4$, $\gamma = -0.7$, $\theta = 0.5$

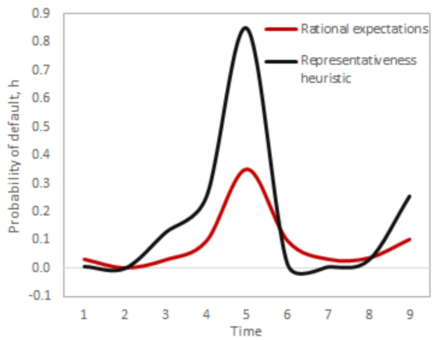
Figure 4: Banking sector indicators under changes in expectation $E^{\iota}[x_{t+1}|X_t]$



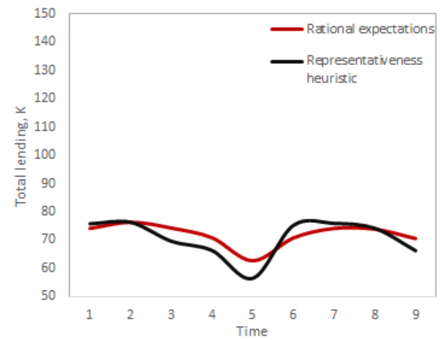
(a) Current macroeconomic state



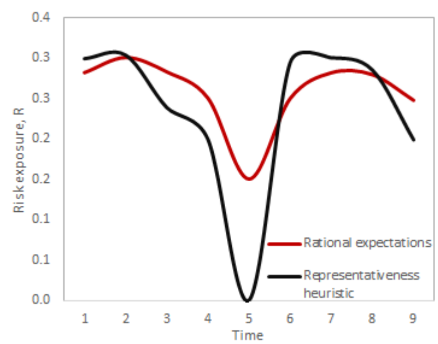
(b) Expected macroeconomic state



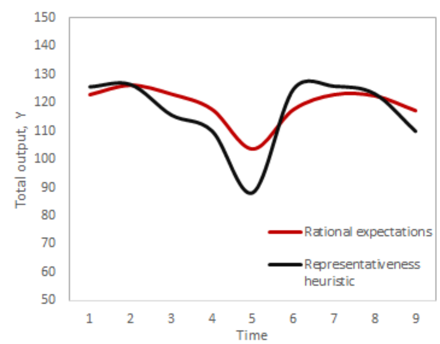
(c) Probability of default, h



(d) Total lending, K



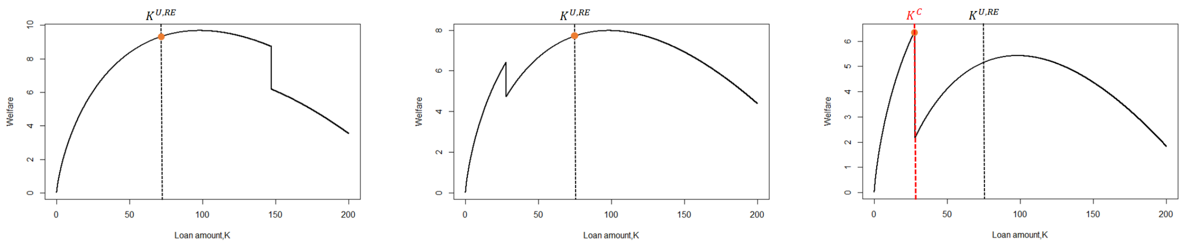
(e) Risk exposure, "R"



(f) Total output, "Y"

Simulation under parameters: $\rho = 0.7$, $\alpha = 0.8$, $\xi = 0.2$, $\sigma_u = 0.5$, $E_t = 4$, $\gamma = -0.7$, $\theta = 0.5$

Figure 5: Optimal loan origination under rational expectations

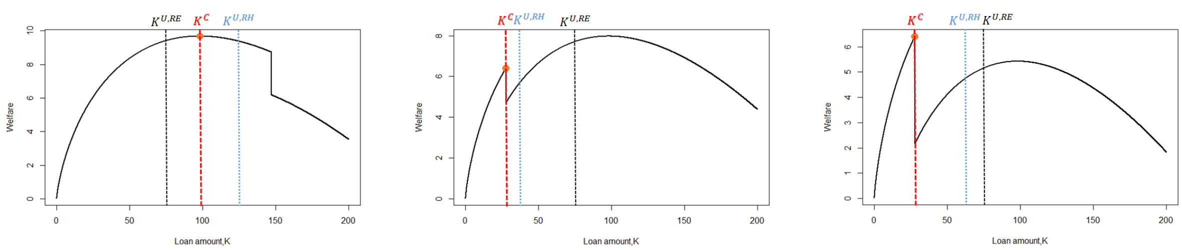


(a) High equity

(b) Low equity, low expected bank failure cost

(c) Low equity, high expected bank failure cost

Figure 6: Optimal loan origination under the representativemness heuristic



(a) High equity

(b) Low equity, low expected bank failure cost

(c) Low equity, high expected bank failure cost

Table 1: Test of Rational Expectations

	Dependent variable: Forecast error			
	Full sample		Banks	
	(1)	(2)	(3)	(4)
Forecast revision	-0.132^{***}	-0.163^{***}	-0.068^{**}	-0.094^{***}
	(0.023)	(0.023)	(0.032)	(0.032)
Constant	0.118^{***}	0.471^{**}	0.092^{*}	-0.063
	(0.036)	(0.205)	(0.051)	(0.162)
Fixed effects:				
Macro variable	Yes	Yes	Yes	Yes
Forecast horizon	Yes	Yes	Yes	Yes
Forecaster	No	Yes	No	Yes
Observations	47,302	47,302	20,814	20,814
R^2	0.002	0.016	0.001	0.014
Adjusted R^2	0.002	0.011	0.001	0.010
F Statistic	19.857^{***}	3.566^{***}	4.215^{***}	3.232^{***}

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2: Distribution of macroeconomic states, an example

	State		
	Recession	No change	Boom
GDP growth	-5%	0%	5%
	Pr(State|News)		
No news	0.10	0.50	0.40
Good news	0.05	0.45	0.50
Bad news	0.20	0.60	0.20
	Representativeness factor		
No news	1.0	1.0	1.0
Good news	0.5	0.9	1.3
Bad news	2.0	1.2	0.5

Table 3: Bank balance sheet

Assets	K_{1t}, K_{2t}
Liabilities	D_t
Equity	E_t

Table 4: Expectation formation

Expectation formation setting	$\mathbb{E}^{\iota}[x_{t+1}\|X_t]$	$Var^{\iota}(x_{t+1}\|X_t)$
Rational expectations	ρx_t	σ_u^2
Representativeness heuristic	$\rho(x_t + \theta u_t)$	σ_u^2

Table 5: Bank balance sheet over time

Assets	K_{1t}, K_{2t}
Deposits	D_t
Equity	E_t

(a) Time t

Assets	$(1+r_{1t})K_{1t}, (1+r_{2t})K_{2t}$
Deposits	D_t
Equity	$E_t + r_{1t}K_{1t} + r_{2t}K_{2t}$

(b) Time $t+1$, favorable macroeconomic state

Assets	$(1+r_{1t})K_{1t}$
Deposits	D_t
Equity	$E_t + r_{1t}K_{1t} - K_{2t}$

(c) Time $t+1$, unfavorable macroeconomic state

Appendices

APPENDIX A

DISCLOSURE ON LOAN LOSS ALLOWANCES

Figure 7: Excerpt from Form 10K of Bank of America Corporation for the fiscal year ended Dec 31, 2021

Allowance for Credit Losses

The allowance for credit losses includes the allowance for loan and lease losses and the reserve for unfunded lending commitments. Our process for determining the allowance for credit losses is discussed in *Note 1 – Summary of Significant Accounting Principles* and *Note 5 – Outstanding Loans and Leases and Allowance for Credit Losses* to the Consolidated Financial Statements.

The determination of the allowance for credit losses is based on numerous estimates and assumptions, which require

a high degree of judgment and are often interrelated. A critical judgment in the process is the weighting of our forward-looking macroeconomic scenarios that are incorporated into our quantitative models. As any one economic outlook is inherently uncertain, the Corporation uses multiple macroeconomic scenarios in its ECL calculation, which have included a baseline scenario derived from consensus estimates, an adverse scenario reflecting an extended moderate recession, a downside scenario reflecting persistent inflation and interest rates above the baseline scenario, a tail risk scenario similar to the severely adverse scenario used in stress testing and an upside scenario that considers the potential for improvement above the baseline scenario. The overall economic outlook is weighted 95 percent towards a recessionary environment in 2023, with continued inflationary pressures leading to lower gross domestic product (GDP) and higher unemployment rate expectations as compared to the prior year. Generally, as the consensus estimates improve or deteriorate, the allowance for credit losses will change in a similar direction. There are multiple variables that drive the macroeconomic scenarios with the key variables including, but not limited to, U.S. GDP and unemployment rates. As of December 31, 2021, the weighted macroeconomic outlook for the U.S. average unemployment rate was forecasted at 5.2 percent, 4.7 percent and 4.3 percent in the fourth quarters of 2022, 2023 and 2024, respectively, and the weighted macroeconomic outlook for U.S. GDP was forecasted to grow at 2.1 percent, 1.9 percent and 1.9 percent year-over-year in the fourth quarters of 2022, 2023 and 2024, respectively. As of December 31, 2022, the latest consensus estimates for the U.S. average unemployment rate for the fourth quarter of 2022 was 3.7 percent and U.S. GDP was forecasted to grow 0.4 percent year-over-year in the fourth quarter of 2022, reflecting a tighter labor market and depressed growth expectations compared to our macroeconomic outlook as of December 31, 2021, and were factored into our allowance for credit losses estimate as of December 31, 2022. In addition, as of December 31, 2022, the weighted macroeconomic outlook for the U.S. average unemployment rate was forecasted at 5.6 percent and 5.0 percent in the fourth quarters of 2023 and 2024, and the weighted macroeconomic outlook for U.S. GDP was forecasted to contract 0.4 percent and grow 1.2 percent year-over-year in the fourth quarters of 2023 and 2024.

Figure 8: Excerpt from Form 10K of Comerica Incorporated for the fiscal year ended Dec 31, 2021

Allowance for Credit Losses

The allowance for credit losses includes both the allowance for loan losses and the allowance for credit losses on lending-related commitments. As a percentage of total loans, the allowance for credit losses was 1.26 percent at December 31, 2021, compared to 1.90 percent at December 31, 2020. Excluding PPP loans, which are guaranteed by the Small Business Administration, the allowance for credit losses was 1.27 percent of total loans at December 31, 2021, compared to 2.03 percent at December 31, 2020. The allowance for credit losses covered 2.3 times and 2.8 times total nonperforming loans at December 31, 2021 and December 31, 2020, respectively.

The allowance for credit losses decreased by $374 million from $992 million at December 31, 2020 to $618 million at December 31, 2021, primarily reflecting strong credit quality, a sustained recovery by the U.S. economy from the COVID-19 pandemic and benefits of mitigating actions by the U.S. government. In addition to $2.8 trillion of stimulus spending approved by Congress between December 2020 and March 2021, the rollout of the COVID vaccine, strong business spending and improved labor markets contributed to an overall improved economic outlook. However, there continues to be uncertainty related to the impact of emerging COVID-19 variants and vaccine efficacy, supply chain constraints, future monetary and fiscal support and inflationary pressures.

These factors shaped the 2-year reasonable and supportable forecast used by the Corporation in its CECL modeled estimate at December 31, 2021. The U.S. economy is expected to grow at a moderate pace through the first half of 2022 before normalizing into historical growth rates. Forecasts for other key economic variables, including the unemployment rate, are generally in line with Gross Domestic Product (GDP) projections. Oil prices are expected to decrease from current elevated levels. Interest rates are expected to increase, reflecting the Federal Reserve's revised expectations, while corporate bond spreads reflect normalized default risk. The following table summarizes select economic variables representative of the economic forecasts used to develop the allowance for credit losses estimate at December 31, 2021.

Economic Variable	Base Forecast
Real GDP growth	Gradually normalizes to a long-term growth rate of 2 to 3 percent by third quarter 2022.
Unemployment rate	Current levels decrease to 4 percent by second quarter 2022, remaining between 3.5 percent and 4 percent through the remainder of the forecast period.
Corporate BBB bond to 10-year Treasury bond spreads	Spreads remain below 2 percent throughout the forecast period.
Oil Prices	Prices gradually decline from current levels to $62 by second quarter 2023.

Figure 9: Excerpt from Form 10K of JPMorgan Chase & Co for the fiscal year ended Dec 31, 2022

ALLOWANCE FOR CREDIT LOSSES

The Firm's allowance for credit losses represents management's estimate of expected credit losses over the remaining expected life of the Firm's financial assets measured at amortized cost and certain off-balance sheet lending-related commitments. The Firm's allowance for credit losses comprises:

- the allowance for loan losses, which covers the Firm's retained loan portfolios (scored and risk-rated) and is presented separately on the Consolidated balance sheets,
- the allowance for lending-related commitments, which is reflected in accounts payable and other liabilities on the Consolidated balance sheets, and
- the allowance for credit losses on investment securities, which is reflected in investment securities on the Consolidated balance sheets.

Discussion of changes in the allowance

The allowance for credit losses as of December 31, 2022 was $22.2 billion, reflecting a net addition of $3.5 billion from December 31, 2021, consisting of:

- $2.3 billion in wholesale, driven by deterioration in the Firm's macroeconomic outlook and loan growth, predominantly in CB and CIB, and
- $1.2 billion in consumer, predominantly driven by Card Services, reflecting higher outstanding balances and deterioration in the Firm's macroeconomic outlook, partially offset by a reduction in the allowance related to a decrease in uncertainty associated with borrower behavior as the effects of the pandemic gradually recede.

Deterioration in the Firm's macroeconomic outlook included both updates to the central scenario in the fourth quarter of 2022, which now reflects a mild recession, as well as the impact of the increased weight placed on the adverse scenarios beginning in the first quarter of 2022 due to the effects associated with higher inflation, changes in monetary policy, and geopolitical risks, including the war in Ukraine.

The Firm's allowance for credit losses is estimated using a weighted average of five internally developed macroeconomic scenarios. The adverse scenarios incorporate more punitive macroeconomic factors than the central case assumptions provided in the table below, resulting in a weighted average U.S. unemployment rate peaking at 5.6% in the second quarter of 2024, and a 1.2% lower U.S. real GDP exiting the second quarter of 2024.

The Firm's central case assumptions reflected U.S. unemployment rates and U.S. real GDP as follows:

	Assumptions at December 31, 2022		
	2Q23	**4Q23**	**2Q24**
U.S. unemployment rate[(a)]	**3.8 %**	**4.3 %**	**5.0 %**
YoY growth in U.S. real GDP[(b)]	**1.5 %**	**0.4 %**	**— %**

	Assumptions at December 31, 2021		
	2Q22	4Q22	2Q23
U.S. unemployment rate[(a)]	4.2 %	4.0 %	3.9 %
YoY growth in U.S. real GDP[(b)]	3.1 %	2.8 %	2.1 %

(a) Reflects quarterly average of forecasted U.S. unemployment rate.

(b) The year over year growth in U.S. real GDP in the forecast horizon of the central scenario is calculated as the percentage change in U.S. real GDP levels from the prior year.

Subsequent changes to this forecast and related estimates will be reflected in the provision for credit losses in future periods.

Figure 10: Excerpt from the 10K of Goldman Sachs Group Inc for the fiscal year ended Dec 31, 2022

Forecast Model Inputs as of December 2022

When modeling expected credit losses, the firm employs a weighted, multi-scenario forecast, which includes baseline, adverse and favorable economic scenarios. As of December 2022, this multi-scenario forecast was weighted towards the baseline and adverse economic scenarios.

The table below presents the forecasted U.S. unemployment and U.S. GDP growth rates used in the baseline economic scenario of the forecast model.

	As of December 2022
U.S. unemployment rate	
Forecast for the quarter ended:	
June 2023	**4.2** %
December 2023	**4.6** %
June 2024	**4.6** %
Growth in U.S. GDP	
Forecast for the year:	
2023	**0.4** %
2024	**1.3** %
2025	**1.7** %

The adverse economic scenario of the forecast model reflects a global recession in 2023 and a more aggressive tightening of monetary policy by central banks, resulting in an economic contraction and rising unemployment rates. In this scenario, the U.S. unemployment rate peaks at approximately 7.4% during the first quarter of 2024 and the maximum decline in the quarterly U.S. GDP relative to the fourth quarter of 2022 is approximately 2.7%, which occurs during the fourth quarter of 2023.

In the table above:

- U.S. unemployment rate represents the rate forecasted as of the respective quarter-end.
- Growth in U.S. GDP represents the year-over-year growth rate forecasted for the respective years.
- While the U.S. unemployment and U.S. GDP growth rates are significant inputs to the forecast model, the model contemplates a variety of other inputs across a range of scenarios to provide a forecast of future economic conditions. Given the complex nature of the forecasting process, no single economic variable can be viewed in isolation and independently of other inputs.

APPENDIX B

LATENT CLASS ANALYSIS

In this section,following Bordalo at al.(2020), I run the test of rational expectations separately for each forecasting institution. Specifically, I estimate Equation (1) separately for each forecasting institution $i = 1, 2, \dots I$:

$$x_{t+h} - F_t x_{t+h,i} = \alpha^i + \beta^i (F_t x_{t+h,i} - F_{t-1} x_{t+h,i}) + \delta_h + \delta_i + \delta_x + e_{t,h,i} \tag{1}$$

I do not pool the data for all forecasters into a single regression, as this would impose the same coefficient of reaction to news, which might not be a reasonable assumption in case of heterogeneity in the forecasting properties across forecasters. The histograms in Figure 11 summarize the estimated coefficients β^i across forecasters. The first panel is based on the forecasts of real GDP growth, while the second panel is based on the forecasts of inflation.[1] Both panels show that there is a large mass of the distribution for which β^i differs from zero. The median coefficients, -0.29 for real GDP growth and -0.31 for inflation, reveal that the majority of forecasting institutions overreact to news.

Even though the regression results summarized in the histograms demonstrate great heterogeneity in forecasting behavior, it is still unclear what part of this heterogeneity is important and what the predominant patterns of forecasting behavior are. To uncover the primary emerging forecasting patterns, I utilize a data-driven approach, specifically latent class analysis (LCA). This method groups firms into clusters with homogeneous characteristics based on the sign and significance of the association between forecast revisions and forecast errors. The LCA method is especially suitable here since it accounts for the fact that the estimated coefficient connecting forecast revisions and forecast errors may vary in magnitude, statistical significance, and even sign across different subsets of firms.

1. The specification of both variables is the following: annualized q-o-q rate based on the seasonally adjusted annual time series.

The LCA model assumes that the data can be characterized by:

$$x_{t+h} - F_t x_{t+h,i} = \alpha + \beta^c (F_t x_{t+h,i} - F_{t-1} x_{t+h,i}) + \delta_i + \delta_h + \delta_v + e_{t,h,i},\ c = 1, 2, \ldots C \quad (2)$$

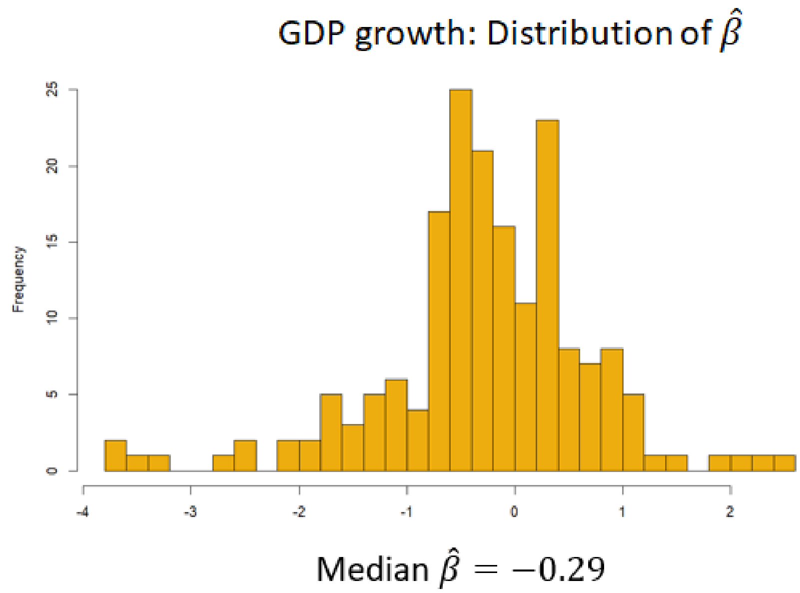
where δ_i, δ_h and δ_v capture forecasting institution, forecasting horizon and macroeconomic variable of interest (real GDP growth or inflation) fixed effects, $c = 1, 2, \ldots, C$ indexes the number of clusters and the the coefficient on forecast revisions β^c varies across clusters. I choose the number of clusters based on the Bayesian Information Criterion (BIC) (Nylund et al., 2007). I increase the number of clusters until there is no further sizable benefit in term of lowering BIC. The estimation procedure maximizes the likelihood function:

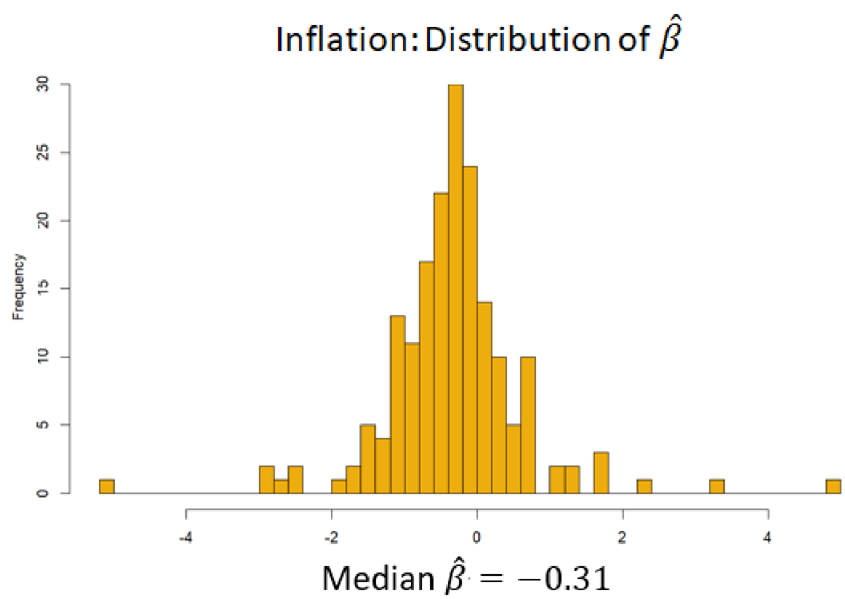
$$L = \prod_{i=1}^{I} \prod_{t=1}^{T} \prod_{h=1}^{5} \left[\sum_{c=1}^{C} \lambda_c \frac{1}{\sqrt{2\pi\sigma_c^2}} \exp\left(-\frac{(f_{i,t,c,h})^2}{2\sigma_c^2} \right) \right]$$

where $f_{i,t,c,h} = x_{t+h} - F_t x_{t+h,i} - (\alpha + \beta^c (F_t x_{t+h,i} - F_{t-1} x_{t+h,i}))$, λ_c is the unknown proportion of the sample that is contained in cluster c, σ_c is the standard deviation of the error term withing the cluster, and β^c is the coefficient representing reaction to news within the cluster c.

Following Larcker et al. (2019), I estimate Equation (2) in two steps, partialling out the fixed effects. The benefit of this two-stage procedure is making sure that only differences in the reaction to news rather than potential differences in the fixed effects determine the clusters, which is my primary interest. In the first step, I partial out the fixed effects for the macroeconomic variables, forecast horizons and forecasting institution (denoted by δ_v, δ_h and δ_i). Second, I run the LCA analysis on the estimated residuals.

Figure 11: Estimated coefficient: forecast error on forecast revision





In the first stage, I estimate the following two regressions for the forecast error (FE) and forecast revision (FR) as dependent variables:

$$x_{t+h} - F_t x_{t+h,i} = \alpha^{FE} + \delta_v^{FE} + \delta_h^{FE} + \delta_i^{FE} + e_{t,h,i}^{FE}$$

$$F_t x_{t+h,i} - F_{t-1} x_{t+h,i} = \alpha^{FR} + \delta_v^{FR} + \delta_h^{FR} + \delta_i^{FR} + e_{t,h,i}^{FR}$$

In the second stage, I estimate the following regression, using the LCA methodology described above:

$$\hat{e}^{FE}_{t,h,i} = \alpha + \beta^c \hat{e}^{FR}_{t,h,i} + \varepsilon_{t,h,i} \tag{3}$$

The estimated coefficient β^c in the second-stage regression is equivalent to the coefficient of interest β^c in Equation (2). Moreover, I restrain the LCA optimization problem so that all observations pertaining to the same banks are a part of the same cluster. The reason for using this constraint is that the forecasting properties are found to be persistent over time, and incorporating this information leads to a more efficient estimation procedure.[2]

Table 6: Latent class analysis of the predictability of forecast errors

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. LCA estimation results						
	Dependent variable: Forecast error					
	Full sample		Banks		Prof. forecasters	
	Cluster 1	Cluster 2	Cluster 1	Cluster 2	Cluster 1	Cluster 2
Forecast revision	-0.1123**	-0.1507***	-0.0278	-0.1035**	0.1861	-0.5395***
	(0.0372)	(0.0251)	(0.0514)	(0.0383)	(0.1211)	(0.1212)
Panel B. Cluster size						
Number of forecasters	60	146	23	62	5	7
% of total forecasters	29%	71%	27%	73%	42%	58%

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Panel A of Table 6 reports the results from the LCA analysis, while Panel B shows the size of each cluster. I find evidence that the whole sample of forecasters (columns 1 and 2) consists of a mixture of two clusters, both of which exhibit significant overreaction to news. This is evident by the negative and statistically significant coefficients on the forecast revision term in both clusters: -0.11 and -0.15 respectively. Turning the attention

2. As a robustness check, I compare the LCA classification based on the whole sample (up to February 2023) with that based on the samples up to 2019, and up to 2009. I find that that 97.5% and 80% respectively of the forecasters are classified in the same clusters based on the data up to 2019, or using the data up to 2009, supporting the argument that forecasting behavior is persistent.

to banks only (columns 3 and 4), we see that the sample of banks is partitioned into two clusters. Approximately 27% of banks comprise the first cluster where forecast errors are not predictable by forecast revisions. This is evident by the statistically insignificant estimated coefficient $\hat{\beta}^c = -0.03$, which aligns with rational expectations for this group of banks. The remaining 73% of banks, however, show overreaction to news, as indicated by the negative and statistically significant coefficient on forecast revision ($\hat{\beta}^c = -0.10$).[3] As evident by bank financial statements, some banks opt to purchase macroeconomic forecasts from professional forecasters rather than creating them in-house. My dataset includes data from 12 prominent professional forecasters[4]. With the caveat that the sample size is limited, Columns (5) and (6) suggest that professional forecasts can be classified into two clusters. For the first cluster I cannot reject the hypothesis of rational expectations, while the second, slightly larger cluster shows signs of overreaction to news. Overall, for the whole sample, as well as for the sample of banks and professional forecasters, we see that the predominant expectation formation process is not rational and subject to overreaction to news.

3. Table 7 in Appendix A shows key descriptive statistics for the two clusters of banks based on their 10K filings on Compustat for the fiscal years ending after March 31, 2022. Although the sample is small to draw definitive conclusions (19 banks), the summary statistics indicate that the banks with expectations close to rational tend to be bigger, more profitable, and more likely to be incorporated in the US compared to the group that exhibits overreaction to news.

4. Including Moody's, S&P, Oxford Economics, E&Y Parthenon, KPMG

Table 7: Comparison between banks by cluster

	Cluster 1 Rational expectations	Cluster 2 Overreaction to news
Total Assets (in log)	13.49	12.86
EBIT/Sales ratio	0.46	0.42
Share of banks listed on major exchange	0.80	0.83
Share of banks incorporated in the US	0.60	0.50

APPENDIX C

ACCOUNTING FOR LOAN LOSS PROVISIONS

I now provide an illustration of the accounting for loan loss provisions under CECL. Under CECL, the banks should recognize expected future losses at the time of loan origination, and the allowance for credit losses should reflect the difference between the amortized cost basis and the present value of the expected cash flows, where cash flows are discounted by the loan's effective interest rate.

To illustrate this, consider the following simple example, which is in line with the model: a one-year term loan with a principal value of \$100 that pays a coupon of 20% with 90% probability. With a 10% probability, however, the borrower defaults, in which case the bank suffers a loss at the amount of the provided loan. The expected cash flow of this loan is $0.9 \times 120 = 108$. The loan loss allowance is the difference between the amortized cost basis, \$100, and the expected cash flow, \$108, discounted by the effective interest rate, 20%. Following these steps, the allowance is $100 - \frac{108}{1.2} = 10$. Therefore, the net recognized loan amount is \$90.

I now present this procedure in the context of the model. The bank starts with equity E and originates a loan at the amount L at the interest rate r, facing a probability of default h. Therefore, expected cash flows from the loan are: $(1-h)(1+r)L$. Loan loss allowances are difference between the amortised cost basis, L, and expected cash flows $(1-h)(1+r)L$, discounted by the effective interest rate r. I obtain that the allowance is: $L - \frac{(1-h)(1+r)L}{(1+r)} = hL$, which represents the expected loss on the loan. The corresponding net recognized loan amount is $(1-h)L$. Therefore, the capital ratio of the bank is $\frac{E-hL}{(1-h)L}$.

APPENDIX D

PROOFS OF PROPOSITIONS

D.1 Proof of Proposition 1

The macroeconomic process is the following: $x_t = \rho x_{t-1} + u_t$ and using forward substitution we obtain $x_{t+T} = \rho^T x_t + \sum_{i=1}^{T-1} \rho^i u_{t+T-1}$ for $T \geq 1$. Therefore, conditional on x_t, x_{t+T} is a sum of normally distributed random variables. Under precise information, conditioning on x_t and conditioning on S_t is equivalent in this case. Therefore, $x_{t+T}|S_t$ is normal. To characterize the mean and variance:

$$\begin{aligned}
\mathbb{E}[x_{t+T}|S_t] &= \mathbb{E}\left[\rho^T x_t + \sum_{i=1}^{T-1} \rho^i u_{t+T-1}|x_t\right] = \rho^T x_t \\
Var(x_{t+T}|S_t) &= Var\left(\rho^T x_t + \sum_{i=1}^{T-1} \rho^i u_{t+T-1}|x_t\right) \\
&= \sum_{i=1}^{T-1} \rho^i \sigma_u^2 = \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2
\end{aligned}$$

Proof of Proposition 2

The distribution perturbed by the representativeness heuristic is:

$$f^\theta(x_{t+T}|x_t = \hat{x}_t) = f(x_{t+T}|x_t = \hat{x}_t)\left[\frac{f(x_{t+T}|x_t = \hat{x}_t)}{f(x_{t+T}|x_t = \rho\hat{x}_{t-1})}\right]^\theta \frac{1}{Z}$$

From Proposition 1 we know that $f(x_{t+T}|x_t = \hat{x}_t)$ is a normal distribution with:

$$\begin{aligned}
\mathbb{E}[x_{t+T}|x_t = \hat{x}_t] &= \rho^T \hat{x}_t \\
Var(x_{t+T}|x_t = \hat{x}_t) &= \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2
\end{aligned}$$

Similarly, $f(x_{t+T}|x_t = \rho\hat{x}_{t-1})$ is a normal distribution with:

$$\mathbb{E}[x_{t+T}|x_t = \rho\hat{x}_{t-1}] = \rho^{T+1}\hat{x}_{t-1}$$
$$Var(x_{t+T}|x_t = \rho\hat{x}_{t-1}) = \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2$$

Denote $\mathbb{E}[x_{t+T}|x_t = \hat{x}_t] \equiv m_1$, $\mathbb{E}[x_{t+T}|x_t = \rho\hat{x}_{t-1}] \equiv m_2$ and $Var(x_{t+T}|x_t = \hat{x}_t) = Var(x_{t+T}|x_t = \rho\hat{x}_{t-1}) \equiv \sigma_T^2$.

The diagnostic distribution is then:

$$\begin{aligned}
f^\theta(x_{t+T}|x_t = \hat{x}_t) &= f(x_{t+T}|x_t = \hat{x}_t)\left[\frac{f(x_{t+T}|x_t = \hat{x}_t)}{f(x_{t+T}|x_t = \rho\hat{x}_{t-1})}\right]^\theta \frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2}\right)\left[\frac{\frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2}\right)}{\frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)}\right]^\theta \frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2}\right)\left[\frac{\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2}\right)}{\exp\left(-\frac{(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)}\right]^\theta \frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2} - \theta\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2} + \theta\frac{(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)\frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(1+\theta)(x_{t+T}-m_1)^2 - \theta(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)\frac{1}{Z}
\end{aligned}$$

We can express the numerator in the exponential as:

$$\begin{aligned}
&(1+\theta)(x_{t+T}-m_1)^2-\theta(x_{t+T}-m_2)^2 = \\
&= (1+\theta)(x_{t+T}^2-2x_{t+T}m_1+m_1^2)-\theta(x_{t+T}^2-2x_{t+T}m_2+m_2^2) \\
&= x_{t+T}^2+x_{t+T}(-2(1+\theta)m_1+2\theta m_2)+(1+\theta)m_1^2-\theta m_2^2 \\
&= x_{t+T}^2-2x_{t+T}((1+\theta)m_1-\theta m_2)+(1+\theta)m_1^2-\theta m_2^2 \\
&= x_{t+T}^2-2x_{t+T}((1+\theta)m_1-\theta m_2)+((1+\theta)m_1-\theta m_2)^2 \\
&\quad -((1+\theta)m_1-\theta m_2)^2+(1+\theta)m_1^2-\theta m_2^2 \\
&= (x_{t+T}-((1+\theta)m_1-\theta m_2))^2 \\
&\quad -\underbrace{((1+\theta)m_1-\theta m_2)^2+(1+\theta)m_1^2-\theta m_2^2}_{c} \\
&= (x_{t+T}-((1+\theta)m_1-\theta m_2))^2-c
\end{aligned}$$

Therefore, $f^\theta(x_{t+T}|x_t=\hat{x}_t)$ becomes:

$$\begin{aligned}
f^\theta(x_{t+T}|x_t=\hat{x}_t) &= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(1+\theta)(x_{t+T}-m_1)^2-\theta(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)\frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-((1+\theta)m_1-\theta m_2))^2}{2\sigma_T^2}\right)\frac{\exp(c/2\sigma_T^2)}{Z}
\end{aligned}$$

We can set $\exp(c/2\sigma_T^2)=Z$. As we see, the diagnostic distribution is also normal, with:

$$\begin{aligned}
\mathbb{E}^\theta[x_{t+T}|x_t=\hat{x}_t] &= (1+\theta)m_1-\theta m_2 \\
&= (1+\theta)\mathbb{E}(x_{t+T}|x_t=\hat{x}_t)-\theta\mathbb{E}(x_{t+T}|x_t=\rho\hat{x}_{t-1}) \\
&= (1+\theta)\mathbb{E}(x_{t+T}|x_t=\hat{x}_t)-\theta\mathbb{E}(x_{t+T}|x_{t-1}=\hat{x}_{t-1}) \\
Var^\theta(x_{t+T}|x_t=\hat{x}_t) &= \sigma_T^2=\frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2
\end{aligned}$$

In particular, when $T = 1$:

$$E^{\theta}[x_{t+1}|x_t] = (1+\theta)\mathbb{E}(x_{t+1}|x_t) - \theta\mathbb{E}(x_{t+1}|x_{t-1})$$

D.2 Proof of Proposition 3

The Lagrangian for the problem is as follows:

$$\mathcal{L} = r_1 K_1 + (1-h) r_2 K_2 - h K_2 + \lambda\left[E_t - (h + \xi(1-h))K_2\right]$$

where:

$$K_i = \left(\frac{1+r_i}{\alpha^2 A}\right)^{\frac{1}{\alpha-1}}, \; i = 1, 2$$

FOC wrt r_1:

$$K_1 + r_1 \frac{dK_1}{dr_1} = 0$$

Rearranging:

$$K_1\left(1 + \frac{r_1}{1+r_1}\frac{1}{\alpha-1}\right) = 0$$
$$r_1 = \frac{1-\alpha}{\alpha}, \quad K_1 = 0$$

There are two solutions to the FOC: $r_1 = \frac{1-\alpha}{\alpha}$ and $K_1 = 0$, but only the former satisfies the second order condition, hence the maximum is obtained at $r_1 = \frac{1-\alpha}{\alpha}$.

Turning attention to the solution for r_2, in the unconstrained case $\lambda = 0$:

$$\text{FOC: } (1-h)K_2 + (1-h)r_2\frac{dK_2}{dr_2} - h\frac{dK_2}{dr_2} = 0$$

Rearranging:

$$K_2\left((1-h)K_2 + \frac{(1-h)r_2 - h}{(\alpha-1)(1+r_2)}K_2\right) = 0$$
$$r_2 = \frac{1}{(1-h)\alpha} - 1, \quad K_2 = 0$$

There are two solutions to the FOC: $r_2 = \frac{1}{(1-h)\alpha} - 1$ and $K_2 = 0$, but only the former satisfies the second order condition, hence the maximum in the unconstrained case is obtained at $r_2 = \frac{1}{(1-h)\alpha} - 1$. The corresponding loan amount is $K_2 = \left(\alpha^2 A(1-h)\right)^{\frac{1}{1-\alpha}}$

Turning attention to the case when the bank is constrained by the minimum capital requirement, the solution is determined by the following system:

$$\lambda > 0$$
$$E_t - (h + \xi(1-h))K_2 = 0$$
$$(1-h)K_2 + (1-h)r_2\frac{dK_2}{dr_2} - h\frac{dK_2}{dr_2} - \lambda(h+\xi(1-h))\frac{dK_2}{dr_2} = 0$$

Therefore, $K_t = \frac{E_t}{(h+\xi(1-h))}$, and the corresponding interest rate stemming from the demand curve is $r_2 = \left(\alpha A\left(\frac{h+\xi(1-h)}{E}\right)^{1-\alpha} - 1\right)$ and $\lambda = \frac{\alpha(1-h)(1+r_2)-1}{h+\xi(1-h)} > 0$.

The condition $\lambda > 0$ shows when the problem is constrained by the minimum capital requirement. It is equivalent to $r_2 > \frac{1}{\alpha(1-h)} - 1$. Plugging in the solution for $r_2 = \left(\alpha A\left(\frac{h+\xi(1-h)}{E}\right)^{1-\alpha} - 1\right)$, I obtain that the problem is constrained when:

$$\left(\alpha^2 A(1-h)\right)^{\frac{1}{1-\alpha}} > \frac{E}{h+\xi(1-h)}$$

D.3 Proof of Proposition 4

As shown in Propositions 1 and 2, the conditional distribution of $x_{t+2}|St$ is normal. For a normal distribution $N(\mu, \sigma^2)$ with density $f_X(x)$, let ϕ and Φ denote the probability and cumulative density functions of the standard normal distribution. In this case the repayment probability is:

$$\int_{\gamma}^{\infty} f_X(x)dx = 1 - \int_{-\infty}^{\gamma} f_X(x)dx$$

First, let us derive the derivative of the repayment probability with respect to the first moment of the distribution, μ:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$
$$\frac{\partial f_X(x)}{\partial \mu} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \frac{x-\mu}{\sigma^2}$$

Using Leibniz rule:

$$\frac{\partial \int_{-\infty}^{\gamma} f_X(x)dx}{\partial \mu} = \int_{-\infty}^{\gamma} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \frac{x-\mu}{\sigma^2}$$

Denote $z = \frac{x-\mu}{\sigma}$:

$$\int_{-\infty}^{\gamma} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \frac{x-\mu}{\sigma^2} dx = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\gamma} z \exp\left(-\frac{z^2}{2}\right) dz$$
$$= \frac{1}{\sqrt{2\pi}\sigma} \left[- \exp\left(\frac{z^2}{2}\right) \Big|_{-\infty}^{\gamma}\right] = -\frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{(\gamma-\mu)^2}{2\sigma^2}\right)$$
$$= -\frac{1}{\sigma}\phi\left(\frac{\gamma-\mu}{\sigma}\right) \leq 0$$

Therefore, for the repayment probability we obtain:

$$\frac{\partial \int_{\gamma}^{\infty} f_X(x)dx}{\partial \mu} = \frac{1}{\sigma}\phi\left(\frac{\gamma-\mu}{\sigma}\right) \geq 0$$

Second, let us derive the derivative of the repayment probability with respect to σ:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$
$$\frac{\partial f_X(x)}{\partial \sigma} = -\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) + \frac{(x-\mu)^2}{\sqrt{2\pi}\sigma^4} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Using Leibniz rule:

$$\frac{\partial \int_{-\infty}^{\gamma} f_X(x)dx}{\partial \sigma} =$$
$$-\int_{-\infty}^{\gamma} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx + \int_{-\infty}^{\gamma} \frac{(x-\mu)^2}{\sqrt{2\pi}\sigma^4} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

Simplifying the first term:

$$-\int_{-\infty}^{\gamma} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = -\frac{1}{\sigma}\Phi\left(\frac{\gamma-\mu}{\sigma}\right)$$

Simplifying the second term, let $z \equiv \frac{x-\mu}{\sigma}$ and using integration by parts with $u = z, v = -\exp\left(-\frac{z^2}{2}\right), v' = z\exp\left(-\frac{z^2}{2}\right)$:

$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\gamma} \frac{(x-\mu)^2}{\sigma^2} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\gamma} z^2 \exp\left(-\frac{z^2}{2}\right) dz =$$
$$= \frac{1}{\sqrt{2\pi}\sigma}\left[-z\exp\left(-\frac{z^2}{2}\right)\Big|_{-\infty}^{\gamma} + \int_{-\infty}^{\gamma} \exp\left(-\frac{z^2}{2}\right)\right]$$
$$= -\frac{\gamma-\mu}{\sigma^2}\phi\left(-\frac{\gamma-\mu}{\sigma}\right) + \frac{1}{\sigma}\Phi\left(\frac{\gamma-\mu}{\sigma}\right)$$

Combining all terms:

$$\frac{\partial \int_{-\infty}^{\gamma} f_X(x)dx}{\partial \sigma} = -\frac{1}{\sigma}\Phi\left(\frac{\gamma-\mu}{\sigma}\right) - \frac{\gamma-\mu}{\sigma^2}\phi\left(\frac{\gamma-\mu}{\sigma}\right) + \frac{1}{\sigma}\Phi\left(\frac{\gamma-\mu}{\sigma}\right)$$
$$= -\frac{\gamma-\mu}{\sigma^2}\phi\left(\frac{\gamma-\mu}{\sigma}\right)$$

Therefore, for the repayment probability I obtain:

$$\frac{\partial \int_{\gamma}^{\infty} f_X(x)dx}{\partial \sigma} = \frac{\gamma-\mu}{\sigma^2}\phi\left(\frac{\gamma-\mu}{\sigma}\right) < 0 \Leftrightarrow \gamma < \mu$$

Similarly, I can show that:

$$\frac{\partial \int_{\gamma}^{\infty} f_X(x)dx}{\partial \sigma^2} = \frac{1}{2}\frac{\gamma-\mu}{\sigma^3}\phi\left(\frac{\gamma-\mu}{\sigma}\right) < 0 \Leftrightarrow \gamma < \mu$$

D.4 Proof of Proposition 5

In the unconstrained case, $K_{2t} = \left(\alpha^2 A(1-h_t)\right)^{\frac{1}{1-\alpha}}$. Using the result of Proposition 4 that $\frac{dh_t}{d\mathbb{E}[x_{t+1}|X_t]} \leq 0$:

$$\begin{aligned}
\frac{dK_{2t}}{dE[x_{t+1}|X_t]} &= \frac{dK_{2t}}{dh_t}\frac{dh_t}{d\mathbb{E}[x_{t+1}|X_t]} = -\frac{K_{2t}}{(1-\alpha)(1-h_t)}\frac{dh_t}{dE[x_{t+1}|X_t]} \geq 0 \\
\frac{dY_{2t}}{dE[x_{t+1}|X_t]} &= \frac{dY_{2t}}{dh_t}\frac{dh_t}{d\mathbb{E}[x_{t+1}|X_t]} = -\frac{\alpha Y_{2t}}{(1-\alpha)(1-h_t)}\frac{dh_t}{dE[x_{t+1}|X_t]} \geq 0 \\
\frac{dR_t}{dE[x_{t+1}|X_t]} &= \frac{dR_t}{dK_{2t}}\frac{dK_{2t}}{d\mathbb{E}[x_{t+1}|X_t]} = \frac{K_{1t}}{(K_{1t}+K_{2t})^2}\frac{dK_{2t}}{d\mathbb{E}[x_{t+1}|X_t]} \geq 0
\end{aligned}$$

In the constrained case, $K_{2t} = \frac{E_t}{h_t+(1-\xi)h_t}$. Again, using the result of Proposition 4 that $\frac{dh_t}{d\mathbb{E}[x_{t+1}|X_t]} \leq 0$, I obtain the following:

$$\begin{aligned}
\frac{dK_{2t}}{dE[x_{t+1}|X_t]} &= \frac{dK_{2t}}{dh_t}\frac{dh_t}{d\mathbb{E}[x_{t+1}|X_t]} = -\frac{(1-\xi)K_{2t}}{h_t+\xi(1-h_t)}\frac{dh_t}{dE[x_{t+1}|X_t]} \geq 0 \\
\frac{dY_{2t}}{dE[x_{t+1}|X_t]} &= \frac{dY_{2t}}{dh_t}\frac{dh_t}{d\mathbb{E}[x_{t+1}|X_t]} = -\frac{\alpha(1-\xi)Y_{2t}}{h+\xi(1-h_t)}\frac{dh_t}{dE[x_{t+1}|X_t]} \geq 0 \\
\frac{dR_t}{dE[x_{t+1}|X_t]} &= \frac{dR_t}{dK_{2t}}\frac{dK_{2t}}{d\mathbb{E}[x_{t+1}|X_t]} = \frac{K_{1t}}{(K_{1t}+K_{2t})^2}\frac{dK_{2t}}{d\mathbb{E}[x_{t+1}|X_t]} \geq 0
\end{aligned}$$

In both cases, K_{1t} is constant. Therefore, the aggregate variables K_t and Y_t move in the same direction as the variables corresponding to the risky loan: $\frac{dK_t}{dE[x_{t+1}|X_t]} \geq 0$ and

$\frac{dY_t}{dE[x_{t+1}|X_t]} \geq 0$.

D.5 Proof of Proposition 6

From Proposition 4 it follows that when $\mathbb{E}^{RH}[x_{t+1}|X_t] > \mathbb{E}^{RE}[x_{t+1}|X_t]$, $h_t^{RH} < h_t^{RE}$. Therefore, for the loss overhang the following holds: $H_t^{RH} = (h_t^{RE} - h_t^{RH})K_t^{RH} < 0$. From Proposition 5 it follows that if $\mathbb{E}^{RH}[x_{t+1}|X_t] > \mathbb{E}^{RE}[x_{t+1}|X_t]$, then: $K_t^{RH} > K_t^{RE}$, $Y_t^{RH} > Y_t^{RE}$ and $R_t^{RH} > R_t^{RE}$.

D.6 Proof of Proposition 7

Case 1. Suppose $h^{RH} < h^{RE}$ and let us denote the difference by Δ, so that $h^{RH} + \Delta = h^{RE}$ and $\Delta > 0$. From Proposition 6, we already know that in this case $K^{RH} > K^{RE}$, hence $|K^{RH}(\xi) - K^{RE}(\xi)| = K^{RH}(\xi) - K^{RE}(\xi)$. By assumption, K^{RE} and K^{RH} are constrained, so:

$$K^{RH}(\xi) - K^{RE}(\xi) = \frac{E}{\xi + h^{RH}(1-\xi)} - \frac{E}{\xi + h^{RE}(1-\xi)}$$

Therefore:

$$\begin{aligned}
\frac{\partial(K^{RH}(\xi) - K^{RE}(\xi))}{\partial \xi} &= -\frac{E(1-h^{RH})}{(\xi + h^{RH}(1-\xi))^2} + \frac{E(1-h^{RE})}{(\xi + h^{RE}(1-\xi))^2} \\
&= -\frac{E(1-h^{RH})}{(\xi + h^{RH}(1-\xi))^2} + \frac{E(1-h^{RH}-\Delta)}{(\xi + h^{RE}(1-\xi))^2} \\
&< -\frac{E(1-h^{RH})}{(\xi + h^{RH}(1-\xi))^2} + \frac{E(1-h^{RH})}{(\xi + h^{RE}(1-\xi))^2} \\
&= -(E(1-h^{RH}))\frac{(h^{RE}-h^{RH})(1-\xi)}{(\xi + h^{RH}(1-\xi))^2(\xi + h^{RE}(1-\xi))^2} \\
&= -\frac{E(1-h^{RH})(1-\xi)\Delta}{(\xi + h^{RH}(1-\xi))^2(\xi + h^{RE}(1-\xi))^2} < 0
\end{aligned}$$

Case 2. Suppose $h^{RH} > h^{RE}$ and let us denote the difference by Δ , so that $h^{RH} = h^{RE} + \Delta$ and $\Delta > 0$. From Proposition 6, we know that in this case $K^{RH} < K^{RE}$, hence $|K^{RH}(\xi) - K^{RE}(\xi)| = K^{RE}(\xi) - K^{RH}(\xi)$ By assumption, K^{RE} and K^{RH} are constrained, so:

$$K^{RE}(\xi) - K^{RH}(\xi) = \frac{E}{\xi + h^{RE}(1-\xi)} - \frac{E}{\xi + h^{RH}(1-\xi)}$$

$$\begin{aligned}
\frac{\partial(K^{RE}(\xi) - K^{RH}(\xi))}{\partial\xi} &= -\frac{E(1-h^{RE})}{(\xi + h^{RE}(1-\xi))^2} + \frac{E(1-h^{RH})}{(\xi + h^{RH}(1-\xi))^2} \\
&= -\frac{E(1-h^{RH}+\Delta)}{(\xi + h^{RE}(1-\xi))^2} + \frac{E(1-h^{RH})}{(\xi + h^{RH}(1-\xi))^2} \\
&< -\frac{E(1-h^{RH})}{(\xi + h^{RE}(1-\xi))^2} + \frac{E(1-h^{RH})}{(\xi + h^{RH}(1-\xi))^2} \\
&= -E(1-h^{RH})\frac{(1-\xi)(h^{RH}-h^{RE})}{(\xi + h^{RE}(1-\xi))^2(\xi + h^{RH}(1-\xi))^2} \\
&= -\frac{E(1-h^{RH})(1-\xi)\Delta}{(\xi + h^{RE}(1-\xi))^2(\xi + h^{RH}(1-\xi))^2} < 0
\end{aligned}$$

Case 3. Suppose $h^{RE} = h^{RH}$, then $K^{RE}(\xi) = K^{RH}(\xi)$ and $\frac{\partial(K^{RE}-K^{RH})}{\partial\xi} = 0$.

D.7 Proof of Proposition 8

Notation:

$$\begin{aligned}
K^U &= \left(\alpha^2 A(1-h_t)\right)^{\frac{1}{1-\alpha}} \\
K^C &= \frac{E_t}{\xi + (1-\xi)h_t} \\
K^{opt} &= (\alpha A(1-h_t))^{\frac{1}{1-\alpha}} \\
\bar{K} &= E_t + r_{1t}K_{1t}
\end{aligned}$$

Preliminaries:

Denote the optimization function as follows: $S(K_{2t}) \equiv (1-h_t)AK_{2t}^{\alpha} - K_{2t} - ch_t\mathbb{1}\{K_{2t} > E_t + r_1K_{1t}\}$. The regulator chooses to constrain the bank's lending to $K^C = \frac{E_t}{\xi+(1-\xi)h_t}$ if $S(K^C) > S(K^U)$. Note that if the regulator sets a binding capital constrained, so that $K^C < K^U$, the bank will choose to originate lending at the maximum amount that does not violate the capital requirement, i.e., K^C. This stems from the fact that $\frac{d\pi(K^C)}{dK^C} > 0$ where $\pi(.)$ denotes the bank profit function.

First, let me study the optimum of the surplus function when $ch_t\mathbb{1}\{K_{2t} > E_t + r_1K_{1t}\} = 0$: $\tilde{S}(K_{2t}) = (1-h_t)AK_{2t}^{\alpha} - K_{2t}$. FOC wrt K_{2t} yields: $K^{opt} = (\alpha A(1-h_t))^{\frac{1}{1-\alpha}}$. If the bank is unconstrained, it sets the following level of lending: $K^U = \left(\alpha^2 A(1-h_t)\right)^{\frac{1}{1-\alpha}}$. Note that $K^U < K^{opt}$.

Case 1: Suppose $K^U \leq E_t + r_1K_{1t}$

This case is equivalent to:

$$\left(\alpha^2 A(1-h_t)\right)^{\frac{1}{1-\alpha}} < E_t + r_1K_{1t}$$

i.e., $E_t > \bar{E}$, where $\bar{E} = \left(\alpha^2 A(1-h_t)\right)^{\frac{1}{1-\alpha}} - r_1K_{1t}$.

The regulator's problem is:

$$\max_{K^C \in [0,K^U]} (1-h_t)AK_{2t}^{\alpha} - K_{2t}$$

In this case, for $K^C \in [0, K^U]$ the surplus function is continuous and increasing:

$$\frac{d\tilde{S}(K^C)}{dK} = (1-h)\alpha A(K^C)^{\alpha-1} - 1 > 0$$

Therefore, the optimal level of lending that maximizes the expected surplus is $K^C = K^U$.

This is equivalent to leaving the bank unconstrained, and setting $\xi = 0$.

Case 2: Suppose $K^U > \bar{K}$

In this case, for $K \in [0, \bar{K}]$ the surplus function is continuous and increasing. Similarly, in the interval $K \in (\bar{K}, K^U)$ the surplus function is continuous and increasing. Furthermore, $\lim_{K \to \bar{K}^-} S(\bar{K}) > \lim_{K \to \bar{K}^+} S(\bar{K})$. Therefore, the two candidate values of K that maximize the regulator's problem are $\bar{K}$ and K^U, so the regulator only needs to compare $S(\bar{K})$ and $S(K^U)$.

If $S(\bar{K}) > S(K^U)$, it is optimal for the regulator to set a binding capital requirement, such that $K^C = \bar{K}$:

$$\frac{E_t}{\xi + (1-\xi)h_t} = \bar{K}$$

The optimal capital constraint is: $\xi = \frac{1}{h_t}\left[\frac{E_t}{\bar{K}} - h_t\right]$.
Note that $S(\bar{K}) > S(K^U)$ when:

$$(1-h)\bar{K}^\alpha - \bar{K} > (1-h)(K^U)^\alpha - K^U - ch_t$$

This is equivalent to:

$$ch_t > \underbrace{\Big((1-h_t)(K^U)^\alpha - K^U - ((1-h_t)\bar{K}^\alpha - \bar{K})\Big)}_{\bar{C}}$$

Alternatively, if $S(\bar{K}) < S(K^U)$, i.e., if $ch_t \leq \bar{C}$, it is optimal for the regulator to leave the bank unconstrained: $\xi = 0$. In this case, the bank would set $K_{2t} = K^U$.

D.8 Proof of Proposition 9

Notation:

$$
\begin{aligned}
K^U &= \left(\alpha^2 A(1-h_t^{RH})\right)^{\frac{1}{1-\alpha}} \\
K^C &= \frac{E_t}{\xi + (1-\xi)h_t^{RH}} \\
K^{opt} &= \left(\alpha A(1-h_t^{RH})\right)^{\frac{1}{1-\alpha}} \\
\bar{K} &= E_t + r_{1t}K_{1t}
\end{aligned}
$$

Note that the most significant difference compared to Proposition 8 is that the unconstrained level of lending, K^U, can exceed the optimum one, K^{opt}, due to biased expectations of the bank. Whenever this is the case, it is optimal for the regulator to limit lending up to K^{opt}.

Case 1: Suppose $K^{opt} \leq \bar{K}$

This case is equivalent to $\left(\alpha A(1-h_t^{RH})\right)^{\frac{1}{1-\alpha}} < E_t + r_1K_{1t}$ or $E_t > \tilde{E}$, where $\tilde{E} = \left(\alpha A(1-h_t^{RH})\right)^{\frac{1}{1-\alpha}} - r_1K_{1t}$.

If $K^U > K^{opt}$, it is optimal for the regulator to constrain lending up to K^{opt}. Therefore, in this case, the minimum capital requirement is such that:

$$
\underbrace{\frac{E_t}{\xi + h_t^{RH}(1-\xi)}}_{K^C} = \underbrace{(\alpha A(1-h_t^{RE}))^{\frac{1}{1-\alpha}}}_{K^{opt}}
$$

This yields:

$$
\xi = \frac{1}{1-h_t^{RH}}\left[\frac{E_t}{(\alpha A(1-h_t^{RE}))^{\frac{1}{1-\alpha}}} - h_t^{RH}\right]
$$

Note that $K^U > K^{opt}$ is equivalent to $\alpha^2 A(1-h^{RH}) > \alpha A(1-h^{RE})$, i.e. $h_t^{RH} < h_0$, where $h_0 \equiv 1 - \frac{1-h_t^{RE}}{\alpha}$.

If $K^U \leq K^{opt}$, then it is optimal for the regulator to leave the bank unconstrained. The reason is that in the interval $K \in [0, K^{opt}]$, including the interval $K \in [0, K^U] \subset [0, K^{opt}]$, the surplus function is continuous and increasing.

Case 2A: Suppose $K^{opt} > \bar{K}$ and $S(K^{opt}) < S(\bar{K})$

This case occurs when $E < \tilde{E}$ and:

$$(1-h_t^{RE})A\left(\alpha A(1-h_t^{RE})\right)^{\frac{\alpha}{1-\alpha}} - \left(\alpha A(1-h_t^{RE})\right)^{\frac{1}{1-\alpha}} - ch_t^{RE} < (1-h_t^{RE})A\bar{K}^{\alpha} - \bar{K}$$

The latter is equivalent to:

$$ch_t^{RE} > \tilde{C}$$

where:

$$\tilde{C} \equiv (1-h_t^{RE})A\left((\alpha A(1-h_t^{RE}))^{\frac{\alpha}{1-\alpha}} - (r_{1t}K_{1t}+E_t)^{\alpha}\right)$$
$$-\left(\left(\alpha A(1-h_t^{RE})\right)^{\frac{1}{1-\alpha}} - (r_{1t}K_{1t}+E_t)\right)$$

Note that: $S(\bar{K}) > S(K)$ for any $K \in (\bar{K}, \infty)$. Also, the surplus is increasing in the interval $K \in [0, \bar{K}]$; hence, $S(\bar{K}) > S(K)$ for any $K \in (\bar{K}, \infty)$. Therefore, $S(\bar{K})$ maximizes the surplus function. The regulator sets $K^C = \bar{K}$, by requiring minimum capital adequacy ratio $\xi = \frac{1}{1-h^{RH}}\left(\frac{E_t}{r_{1t}K_{1t}-h_t^{RH}}\right)$.

Case 2B: Suppose $K^{opt} > \bar{K}$ and $S(K^{opt}) > S(\bar{K})$

Now, there are four noteworthy intervals. For $K \in [0, \bar{K}]$, the surplus function is continuous and increasing. Let h_1 is the smallest root of the equation $(1-h_t^{RE})A\bar{K}^{\alpha} - \bar{K} = (1 -$

$h_t^{RE})A(\alpha A(1-h_t^{RE}))^{\frac{\alpha}{1-\alpha}} - (\alpha A(1-h_t^{RE}))^{\frac{1}{1-\alpha}} - ch_t^{RE}$, considered as a function of h_t^{RE}. Let $K(h_1) = (\alpha A(1-h_t^{RE}))^{\frac{1}{1-\alpha}}$. In the interval $K \in \big(\bar{K}, K(h_1)\big)$, $S(\bar{K}) > S(K)$. Therefore, in this interval, it is optimal for the regulator to constrain loan origination to $\bar{K}$. In the interval $K \in [K(h_1), K^{opt}]$, the surplus function is increasing. Therefore, if $K^U \in [K(h_1), K^{opt}]$, it is optimal for the regulator to leave the bank unconstrained by the capital requirement, i.e., $\xi = 0$. Last, in the interval $K \in (K^{opt}, \infty)$, the surplus function is falling. Because of this, if $K^U \in (K^{opt}, \infty)$, it is optimal for the regulator to constrain lending up to K^{opt}, by setting ξ in the following way:

$$\xi = \frac{1}{1-h_t^{RH}} \left(\frac{E_t}{(\alpha A(1-h_t^{RE}))^{\frac{1}{1-\alpha}}} - h_t^{RH} \right)$$

Proof of Corollaries 9.1 and 9.2

Note that $\tilde{C} \geq \bar{C}$ and $\tilde{E} \geq \bar{E}$. Then, apply Propositions 8 and 9.

APPENDIX E

COMPARISON OF THE MODEL'S SOLUTION UNDER ILM AND CECL

I consider two aspects in which the incurred loss and forward-looking provisioning regimes differ; namely, the timing of provisioning and the process used in expectation formation about credit losses. First, under the incurred loss framework, no loan loss provisions are accounted for at the time of loan origination, t. This resembles the property that, in an incurred loss provisioning system, loan loss provisions are driven by non-performing loans when there is evidence that losses are likely to occur. In contrast, under CECL, expected credit losses are accounted for more timely, based on reasonable and supportable forecasts of future economic conditions.

E.1 Timely provisioning

Under the incurred credit loss method, the bank does not recognize losses at the time of loan origination;[1] hence, it is subject to the following constraint: $\frac{E_t}{K_{2t}} \geq \xi$. To compare, the constraint under CECL is: $\frac{E_t - h_t K_{2t}}{(1-h_t)K_{2t}} \geq \xi$. Apart from the capital adequacy constraint, the bank problem is equivalent under both accounting methods.

Table 8 summarizes the amount of loan origination under the two provisioning methods. We can make the following observations. When the bank is not constrained by the capital requirement, loan origination is the same under the two regimes. This is expected, as the difference between the accounting regimes is that they affect the timing of loan loss provisions and, therefore, the timing of potential violations of the capital requirement. However, if the bank has enough equity not to worry about the capital requirement, then the timing of LLP

1. I assume that the bank would not originate the loan if a loss is likely at this time, as it will not be profitable to originate the loan on expectation.

does not matter. Thus, when the bank has enough equity, the provisioning method does not impact bank lending, nor any of the bank or economy-level indicators presented in Section 3.4.1. In particular, if the bank is subject to behavior biases in its forecasts, the capital requirement does nothing to limit its overreaction to news.

Table 8: Lending to the risky borrower under the two accounting regimes

Accounting framework	Constrained	Unconstrained	Constrained when
Incurred loss	$K_{2t} = \frac{E_t}{\xi}$	$K_{2t} = \left(\frac{1}{\alpha^2 A}\frac{1}{1-h_{2t}}\right)^{\frac{1}{\alpha-1}}$	$\alpha^2 A(1-h_{2t})\left(\frac{\xi}{E_t}\right)^{1-\alpha} > 1$
Forward-looking	$K_{2t} = \frac{E_t}{\xi+h_{2t}(1-\xi)}$	$K_{2t} = \left(\frac{1}{\alpha^2 A}\frac{1}{1-h_{2t}}\right)^{\frac{1}{\alpha-1}}$	$\alpha^2 A(1-h_{2t})\left(\frac{\xi+(1-\xi)h_{2t}}{E}\right)^{1-\alpha} > 1$

On the other hand, when banks are constrained by the capital adequacy ratio, the accounting regime affects the amount of loan origination. Loan origination under the forward-looking regime coincides with that under the incurred loss regime only when the loan is risk-free, i.e. when $h_t \equiv \mathbb{E}[\mathbb{1}\{x_{t+1} < \gamma\}|X_t] = 0$. However, in the more realistic case where the loan default probability is non-zero, loan origination under the forward-looking regime is smaller. This result echoes the argument expressed by some banks and politicians that CECL can increase the cost and lower the availability of credit. Overall, the bank is more conservative under the forward-looking provisioning method. This can limit the amount of excessive lending under good news, but it can exacerbate the problem of excessive pessimism when bad macroeconomic news arrives.[2] In other words, by stimulating precautionary behavior, the forward-looking regime can better limit pro-cyclical lending in good times, but also limit lending and exacerbate the loss in welfare in bad times.

2. Notice that under bad news, and more generally under bad macroeconomic states, the optimal level of lending to the risky entrepreneur is small. In this case, the capital adequacy ratio increases and the capital constraint may become slack. Therefore, the capital constraint only matters under positive macroeconomic states and under less severe macroeconomic contractions, but it does not have a bite when the macro state is so low that banks optimally shrink their risky lending.

Furthermore, when banks are constrained by the capital adequacy requirement, loan origination under the forward-looking regime depends on expectations about the macroeconomy through the default probability of the risky loan h_t. In contrast, loan origination under the incurred loss is only determined by the minimum capital requirement, which is assumed to be constant. Hence, under the incurred loss provisioning, loan origination is not affected by the expected macroeconomic conditions. This distinction highlights an intended consequence of using the forward-looking regime, as the name suggests: making bank behavior more sensitive to expectations of the economy rather than focusing on the current state alone. This, however, also shows that the forward-looking regime opens room for more sensitivity of bank behavior to the properties of the forecasts, when the bank is constrained by the capital requirement.

Lastly, note that that the conditions of when the bank is constrained by the capital requirement under CECL and ILM are the following:

$$C^{FL} \equiv \alpha^2 A(1-h_t)\left(\frac{\xi+(1-\xi)h_t}{E_t}\right)^{1-\alpha} - 1 > 0$$
$$C^{IL} \equiv \alpha^2 A(1-h_t)\left(\frac{\xi}{E_t}\right)^{1-\alpha} - 1 > 0$$

Also note that $C^{FL} \geq C^{IL}$. This expression shows that whenever the bank problem is constrained under the incurred loss regime, it is also constrained under the forward-looking one. In other words, IFRS 9/CECL makes the capital requirement more stringent. By this, forward-looking provisioning is more effective in limiting excessive lending in good times, when macroeconomic expectations can be too optimistic.

E.2 Change in the expectation formation process

One method of estimating expected credit losses which is commonly used in practice and also suggested in the literature (Harris et al., 2018), is to use a constant ratio between charge-offs and total loan amount. In my framework, this method is equivalent to assuming the default rate at $t + 1$ will remain unchanged from the one observed at the current date t, which is consistent with the following macroeconomic expectation formation process, which I call extrapolation:

$$E^{extr}[x_{t+1}|x_t] = x_t$$

This method is subject to criticism, which was commonly expressed in relation to the incurred credit loss regime, according to which banks relied too much on current and past information, and did not pay enough attention to expectations about the future. To overcome this criticism, CECL and IFRS9 explicitly state that estimates about expected credit losses should reflect reasonable and supportable forecasts of future economic conditions. This stipulation might change the way expectations are formed: from relying on simple extrapolation to relying on macroeconomic forecast. Therefore, it is interesting the study the implications for lending from such a shift, which I discuss in this section.

Suppose the state is positive and good news have arrived. Rational expectations imply that $E[x_{t+1}] = \rho x_t$. In this case, if banks are using extrapolation to predict the probability of default, they are overly optimistic: $E^{extr}[x_{t+1}|X_t] = x_t > E^{RE}[x_{t+1}|X_t] = \rho x_t$. In this case, banks would be underestimating the probability of default, and lending excessively to the risky entrepreneur. It would be optimal to bring the expectations closer to the rational expectations. However, if banks become more reliant on macroeconomic forecast, which are subject to the representativeness bias, there are parameters under which $E^{RH}[x_{t+1}|X_t] > E^{extr}[x_{t+1}|X_t] > E^{RE}[x_{t+1}|X_t]$. In particular this is the case when $\rho\theta u_t > (1 - \rho)x_t$, i.e. when the economy is very persistent (ρ is high) and subject to high overreaction (θ is high). Bordalo et al. (2020) find that the diagnostic parameter θ in around 0.5 on average over 20 macroeconomic variables. Therefore, if x_t fluctuates around the expected value of 0 and $\rho > 0$, whenever there is a positive macroeconomic shock, the expectation under the representativeness heuristic will exceed the extrapolation and the rational one. Under negative shocks, overreaction can make expectations worse that a simple extrapolation and worse than a rational forecast: $E^{RH}[x_{t+1}|X_t] < E^{extr}[x_{t+1}|X_t] < E^{RE}[x_{t+1}|X_t]$. This shows that moving to a more forward-looking framework can exacerbate the procyclicality of the banking sector, even if the benchmark is not the perfect Bayesian forecasting, but the simple extrapolation benchmark.