Scaling of *P*-wave excitation energies in heavy-quark systems

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A simple regularity in anticipating P-wave excitation energies of states with heavy quarks is noted. It can apply to systems such as the negative-parity Σ_c, Σ_b, and Ω_c, $\bar{Q}Q$ quarkonia, and the bottom-charmed meson B_c. When one subtracts a term accounting for phenomenological energies of heavy quarks binding with one another in S-waves, the residual excitation energies display an approximately linear behavior in the reduced mass of constituents, all the way from the Λ to the Υ.

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I. INTRODUCTION

The LHCb experiment, working at the CERN Large Hadron Collider, has observed a number of new baryons containing heavy quarks, including a series of five excited $\Omega_c = css$ resonances [1] and a new $\Xi_b^- = bsd$ resonance [2]. These have been interpreted, though not uniquely, as, respectively, P-wave excitations of the ground state Ω_c [3–8] and one or more P-wave excitations of the ground state Ξ_b^- [9]. We seek simple methods for confirming these assignments. Furthermore, it has been of interest to estimate the P-wave excitation energies for Σ_c and Σ_b states [10] as well as for the $B_c = b\bar{c}$ system (see, e.g., [11–15]).

Spurred by these developments, we asked whether there is a simple way of estimating P-wave excitation energies without the use of the two-body or three-body Schrödinger equation, its relativistic analogue, or other methods such as lattice quantum chromodynamics. To our surprise, there appears to be an approximate method which, while not perfect, probably suffices as a guideline to whether a given state is a P-wave candidate.

The method builds upon a constituent-quark treatment which was used to predict successfully [16] the mass of the $\Xi_{cc}^{++} = ccu$ baryon subsequently discovered by LHCb [17]. Account was taken of quark masses, hyperfine interactions, and S-wave binding terms $B(q_1 q_2)$ involving any quark pairs where one quark is heavier than u, d and the other heavier than s. These binding terms are obtained phenomenologically by comparing masses of hadrons containing a single heavy quark (e.g., q_1 or q_2) with ones containing two heavy quarks (e.g., $q_1\bar{q}_2$). We find that when these binding terms are taken into account in calculating S-P mass differences, the residual energy differences ΔE_R depend approximately linearly on the reduced mass $\mu_{12} = m_1 m_2/(m_1 + m_2)$ of the pair. This behavior extends from the $\Lambda = uds$ baryon all the way up to the $\Upsilon(1S)$ and their respective P-wave excitations.

We lay out the tools for our estimates in Sec. II, describing assumed quark masses and binding terms. The ground rules for quoting S-P splittings are also given. We quote the observed S-P splittings for a number of pairs in Sec. III. The effects of binding terms, if any, are considered in Sec. IV, giving rise to residual energy differences ΔE_R which are plotted as functions of reduced mass. An approximately linear dependence is seen. In baryonic cases the problem is reduced to a two-body one by assuming one quark is excited with respect to two others which remain in a relative S-wave.

The linear dependence of ΔE on reduced mass is used in Sec. V to predict several quantities which were only crudely estimated before. These include P-wave excitation energies for Σ_c and Σ_b states [10] and for Ω_c states [3]. Predictions for Ξ_b and B_c are also given and compared with others in the literature. Section VI is devoted to a discussion of the possible source of the observed regularity, and a brief conclusion.

II. TOOLS

We use separate constituent-quark masses for mesons and baryons [16]. They are summarized in Table I. Analysis of S-wave mesons and baryons makes use of binding terms $B(q_1 q_2)$, also from Ref. [16], summarized in Table II.

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TABLE I. Quark masses in MeV used in this analysis.

Quark	In a meson	In a baryon
u, d	$m^m_{u,d} = 310$	$m^b_{u,d} = 363$
s	$m^m_s = 483$	$m^b_s = 538$
c	$m^m_c = 1663.3$	$m^b_c = 1710.5$
b	$m^m_b = 5003.8$	$m^b_b = 5043.5$

TABLE II. Pair binding terms $B(q_1q_2)$ in MeV used in this analysis.

Pair q_1q_2	$B(q_1q_2)$	$B(q_1\bar{q}_2)$
cs	35.0	70.0
bs	41.8	83.6
cc	129	258
bc	170.8	341.5
bb	281.4	562.8

These terms were calculated by comparing the masses of spin-averaged S-wave bound states (e.g., for charmonium) with the sum of their constituent-quark masses as determined from hadrons containing a single heavy quark (e.g., Λ_c).

III. S-P SPLITTINGS

A. Baryons

Unless otherwise specified, we take all masses from the 2018 Particle Data Group listings [18]. We consider baryons with excitation of a spinless (scalar) diquark except in the case of $\Omega_c = css$, where we consider the spin-1 ss diquark to be excited by one unit of orbital angular momentum with respect to the charmed quark [3].

We take the masses listed in Table III to calculate the spin-averaged S-P splittings shown. The masses of excited states are calculated using averages $\bar{M}_P$ weighted by $2J+1$ factors, where J is the spin of the resonance. Small uncertainties in masses are not quoted.

TABLE III. Masses of ground state baryons and their orbital excitations ΔE_{P-S}, in MeV. Here ΔE_{P-S} denotes the difference between spin-weighted average P-wave and S-wave masses.

State	$1/2^+$	$1/2^-$	$3/2^-$	$\bar{M}_P$	ΔE_{P-S}
Λ	1115.683	1405.1	1519.5	1481.37	365.68
Λ_c	2286.46	2592.25	2628.11	2616.16	329.70
Λ_b	5619.60	5912.20	5919.92	5917.35	297.75
Ξ_c	2469.37[a]	2792.2[a]	2818.4[a]	2809.6	340.3
Ω_c[b]	2742.33	See note[c]		3079.94	337.61

[a]Error-weighted isospin average.
[b]Spin-averages of ground state and assumed P-wave states from Ref. [3].
[c](2,2,1) states with $J = (1/2, 3/2, 5/2)$, cf. Ref. [3].

B. Mesons

We consider only those systems for which the spin-averaged ground state and P-wave masses can be calculated. They are $c\bar{s}$ ("D_s"), $c\bar{c}$, and $b\bar{b}$. For $c(\bar{u},\bar{d})$ not all candidates for the 1P level are firmly established, while for $b(\bar{u},\bar{d})$ a spin-zero meson and one of two predicted spin-1 mesons are still missing (see Sec. V B). For $b\bar{c}$ ("B_c") no P-wave states have been seen, but their masses have been predicted (see Sec. V F). The relevant masses are shown in Table IV. Spin averaged masses are

$$\bar{M}_S \equiv [M(^1S_0) + 3M(^3S_1)]/4,$$
$$\bar{M}_P \equiv [M(^3P_0) + 3M(^3P_1) + 3M(^1P_1) + 5M(^3P_2)]/12. \quad (1)$$

IV. RESIDUAL ENERGY DIFFERENCES ΔE_R

We now calculate residual energy differences $\Delta E_R \equiv \Delta E_{P-S} - \sum B$ for the above systems, where $\sum B$ denotes the sum of $B(q_1q_2)$ over all relevant heavy quarks q_1 and q_2 (cf. Table II). The results are shown in Table V. Here $[q_1q_2]$ denotes a spinless color-antitriplet diquark, while (ss) denotes a spin-1 color-antitriplet diquark. We quote isospin-averaged masses where appropriate, letting q stand for u or d.

Whereas the quantities ΔE_{P-S} are not monotonic functions of the reduced mass μ_{12}, when the binding energies B are subtracted from them, the residual energies ΔE_R are crudely arranged along a straight line, as shown in Fig. 1. A linear fit to the eight experimentally known values in Table V gives the result

$$\Delta E_R = (417.37\text{–}0.2141\mu_{12})\ \text{MeV}. \quad (2)$$

TABLE IV. Masses of ground state mesons and their orbital excitations, in MeV.

State	$M(^1S_0)$	$M(^3S_1)$	$\bar{M}_S$[a]	$M(^3P_0)$	$M(^3P_1)$	$M(^1P_1)$	$M(^3P_2)$	$\bar{M}_P$[b]	ΔE_{P-S}
D_s	1968.34	2112.2	2076.2	2317.7	2459.5[c]	2535.1[c]	2569.1	2512.3	436.0
$c\bar{c}$	2983.4	3096.9	3068.5	3414.71	3510.67	3525.38	3556.17	3525.3	456.8
$b\bar{b}$	9399.0	9460.3	9445.0	9859.44	9892.78	9899.73	9912.21	9899.7	454.8

[a]Spin-averaged ground state mass.
[b]Spin-averaged P-wave mass.
[c]Orthogonal mixtures of $^3P_1$ and $^1P_1$ states.

TABLE V. Residual energy differences ΔE_R and corresponding reduced masses, in MeV.

System	q_1	q_2	m_1	m_2	μ_{12}	ΔE_{P-S}	$\sum B$	ΔE_R
Λ	$[ud]$	s	576.0	538	278.2	365.7	0	365.7
Λ_c	$[ud]$	c	576.0	1710.5	430.9	329.7	0	329.7
Λ_b	$[ud]$	b	576.0	5043.5	517.0	297.8	0	297.8
Ξ_c	$[qs]$	c	799.8	1710.5	545.0	340.3	35.0	305.3
Ω_c	(ss)	c	1098.8	1710.5	669.0	337.6	70.0	267.6
D_s	c	s	1663.3	483	374.3	436.0	70.0	366.0
$c\bar{c}$	c	c	1663.3	1663.3	831.6	456.8	258.0	198.8
$b\bar{b}$	b	b	5003.8	5003.8	2501.9	454.8	563	–108.2

The root-mean-square deviation of the data from this fit is 18.7 MeV. We discuss some consequences of this regularity, if it is to be taken seriously, in the next section.

V. CONSEQUENCES AND PREDICTIONS

A. Σ_c and Σ_b baryons

In Ref. [10] a linear extrapolation of excitation energy was used to estimate the S-P wave splittings for Σ_c and Σ_b baryons. The present discussion gives support to that assumption. The parameters of the present linear fit give slightly different values of ΔE_R, as shown in Table VI. For the states in this table, there are no B terms, so $\Delta E_{P-S} = \Delta E_R$.

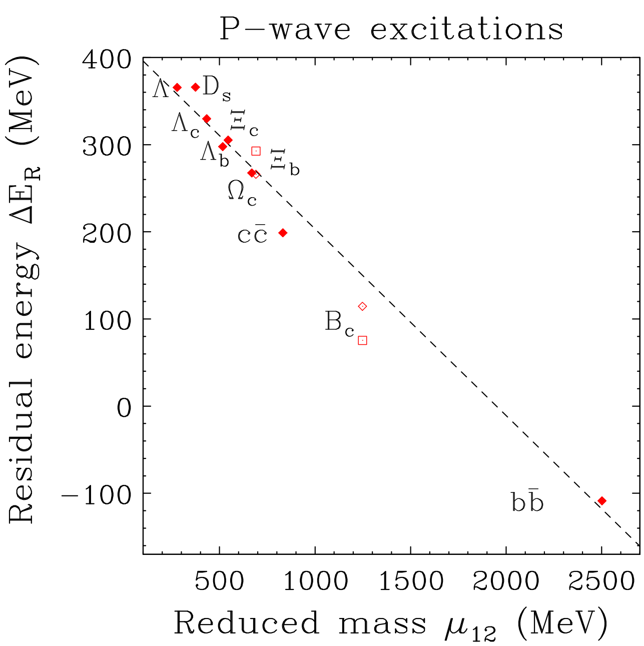

FIG. 1. Residual energies ΔE_R as functions of reduced mass μ_{12}. Dashed line: fit of Eq. (2). Filled diamonds denote data used in the fit. Theoretical predictions for Ξ_b (Sec. V E) and B_c (Sec. V F) systems are plotted as hollow diamonds and hollow squares.

TABLE VI. Values of $\Delta E_R = \Delta E_{P-S}$ predicted by linear fit of Eq. (2) compared with those of Ref. [10].

						ΔE_R	
State	q_1	q_2	m_1 (MeV)	m_2 (MeV)	μ_{12} (MeV)	Ref. [10]	Eq. (2)
Σ	(uu)	s	776	538	317.7	357.5	349.3
Σ_c	(uu)	c	776	1710.5	533.8	290.9	303.1
Σ_b	(uu)	b	776	5043.5	672.5	238.8	273.4

B. Charm and bottom mesons

The reduced masses for D and B mesons are displayed in Table VII. They lead to predictions via Eq. (2) of ΔE_R, which is equal to ΔE_{P-S} because the binding correction B is zero.

In order to compare these predictions with experiment, one must know the masses of all four P-wave states. Our partial information is summarized in Table VIII. The D and B mass eigenstates have j (the vector sum of light-quark spin and orbital angular momentum) equal to $1/2$ or $3/2$. Those with $j = 3/2$ (total $J = 1$, 2) decay predominantly via D waves, are narrow, and are firmly established [18]. Those with $j = 1/2$ ($J = 0$, 1) are expected to decay via S waves and are very broad, with consequent mass uncertainty. The $j = 1/2$ D mesons would satisfy the linear fit if their widths, exceeding 200 MeV, were included as error bars. No candidates for the $j = 1/2$ B states have been identified. They would have to be considerably lighter than the $j = 3/2$ states if they were to obey the prediction in Table VII. The outlier nature of D and B states is further discussed in Sec. VI.

The predicted spin-averaged P-wave mass for B_s is low enough that the $j = 1/2$ B_s P-wave states are probably below the respective BK and B^*K thresholds for the $J = 0$ and $J = 1$ states. Thus, like the $D_{s0}(2317)$ and $D_{s1}(2460)$ (see below), they are expected to be very narrow, decaying only via $B_{s0} \to \gamma B_s^*$ and $B_{s1} \to \gamma B_s$ or γB_s^*, or with isospin-violating processes involving π^0 emission. The properties of these states have been discussed in Refs. [19,20].

C. D_s mesons

The observed masses of $D_{s0}(2317)$ and $D_{s1}(2460)$ were considerably below predictions of potential models, leading to some initial surprise. The present regularity (Fig. 1)

TABLE VII. Calculation of ΔE_R and $\Delta E_{P-S} = \Delta E_R + B(q_1 q_2)$ for D, B, and B_s mesons, based on linear fit of Eq. (2). Masses in MeV.

State	q_1	q_2	m_1	m_2	μ_{12}	ΔE_R	$B(q_1 q_2)$	ΔE_{P-S}
D	c	q	1663.3	310	261.3	361.4	0	361.4
B	b	q	5003.8	310	291.9	354.9	0	354.9
B_s	b	s	5003.8	483	440.5	323.1	83.6	406.7

TABLE VIII. Masses for calculating S-P splitting in charmed and bottom mesons. Error-weighted averages over charge states unless otherwise indicated.

State	$M(^1S_0)$	$M(^3S_1)$	$\bar{M}_S$ [a]	$\bar{M}_P$ [b] (predictions)	$M(^3P_0)$	$M(J=1)$ $j=1/2$	$M(J=1)$ $j=3/2$	$M(^3P_2)$
D	1867.24	2008.56	1973.23[a]	2334.6	2349.2[c]	Note [d]	2420.9	2461.1
B	5279.48	5324.65	5313.36[a]	5668.2	...	...	5726.0	5738.4
B_s	5366.89	5415.4	5403.3[a]	5810.0	...	...	5828.63	5839.85

[a]Spin-averaged ground state mass.
[b]Spin-averaged P-wave mass predicted from Eq. (2).
[c]Error-weighted isospin average width 235.7 MeV.
[d]Neutral candidate: $M = 2427 \pm 40$ MeV, $\Gamma = 384^{+130}_{-110}$ MeV.

indeed supports the picture of these states as lying below those predictions.

D. Ω_c baryons

The residual energy ΔE_R for the five narrow Ω_c states observed by LHCb [1] lies right on the linear fit, supporting their assignment as five P-wave states [3–8] and disfavoring an alternate assignment (see, e.g., [3]) in which the two highest states are 2S excitations and two lower-mass P-wave states remain to be discovered.

E. Ξ_b baryons

In Table IX we compare a recent prediction [9] for the masses of P-wave excitations of the scalar $[sq]$ quark in Ξ_b baryons (Fig. 1, hollow diamond), with an earlier one ([21], Fig. 1, hollow square), and with the result of the linear fit for reduced mass $\mu_{bc} = 690.3$ MeV. The fit is more consistent with the later prediction.

F. B_c states

One can obtain a value of ΔE_{P-S} for the B_c system by interpolating between the nearly equal values for the $c\bar{c}$ and $b\bar{b}$ systems, as one might expect if the interquark potential is close to the logarithmic one proposed in Ref. [22]. One thus obtains $\Delta E_{P-S} = 456$ MeV, corresponding to the open diamond in Fig. 1 when a binding term of 341.5 MeV is taken into account. An early potential-model prediction [11] was $\Delta E_{P-S} = 417$ MeV, corresponding to the open square in Fig. 1. Subsequent calculations of ΔE_{P-S} gave 430, 427, and 427 MeV in Refs. [12–14], respectively. The prediction of Eq. (2), using a reduced mass of $m_c m_b/(m_c + m_b) = 1248.3$ MeV, is $\Delta E_R = 150.1$ MeV, or $\Delta E_{P-S} = 491.6$ MeV, considerably larger than any of the above values.

TABLE IX. Values of ΔE_{P-S} and ΔE_R for Ξ_b states from models compared with predictions of linear fit (2). Masses in MeV.

$1/2^+$	$1/2^-$	$3/2^-$	$\bar{M}_P$	ΔE_{P-S}	ΔE_R
5792.19[a]	6096[b]	6102[b]	6100	307.8	266.0
	6120[c]	6230[c]	6126.7	334.5	292.7
	Calculated from Eq. (2)			311.4	269.6

[a]Spin-averaged ground state mass.
[b]Ref. [9].
[c]Ref. [21].

VI. DISCUSSION AND CONCLUSIONS

The binding terms B used to calculate ΔE_R represent corrections to the picture of spectra due to constituent-quark masses and hyperfine terms [23], when quarks are heavy enough to experience the short-distance Coulomb-like force of single gluon exchange. In a purely Coulombic potential $V(r) = -(4/3)\alpha_s/r$ the energy levels are given by $E_n = -[(4/3)\alpha_s]^2\mu/(2n^2)$. In the simplest approximation the P-wave excitation energy is given by $\Delta E_{P-S} = E_2 - E_1$. We have subtracted the S-wave binding energy B from this P-wave excitation energy to obtain the residual energy difference

$$\Delta E_R = \Delta E_{P-S} - B. \tag{3}$$

In our convention this S-wave binding energy is positive. In this simple example it is just the minus the ground state energy, $-E_1$. The upshot is that here the residual excitation energy is just the energy eigenvalue of the P-wave,

$$\Delta E_R = E_2 - E_1 - B = E_2 - E_1 + E_1 = E_2 = -[(4/3)\alpha_s]^2\mu/8. \tag{4}$$

So in this case the slope in Fig. 1 is just $-[(4/3)\alpha_s]^2\mu/8$.

In a more realistic potential with a confining piece the slope will be different and there is likely to be also a constant term. For light quarks (u, d, s) the use of constituent-quark masses means that it is not necessary to subtract a B term; the constituent-quark masses already embody such a term. Nonetheless, the negative slope in the relation between residual energy and reduced mass is generic. It just reflects the fact that the P-wave energy (as opposed to energy splitting) is negative.

This is surprising, as relativistic corrections (important even for systems as heavy as bottomonium) do not depend purely on the reduced mass. This is true for quantum electrodynamics, as shown by the comparison between positronium and the hydrogen atom [24]. The linear dependence of residual energy must be the result of compensating effects, not some fundamental relation. What we have done is to construct a phenomenological "bridge" between confinement and short-distance Coulomb-like behavior. This picture then explains why the B and D mesons are outliers. Their radii are of order $1/\Lambda_{\mathrm{QCD}}$, rather than $1/(\alpha_s\mu)$. The fact that α_s runs between $\mu = 500$ MeV and 2500 MeV will make the slope slightly scale dependent.

The potential for learning about P-wave excitations of heavy-quark baryons and mesons makes the present discussion timely. Consequences have been noted for charmed and bottom-flavored baryons and mesons. It will be interesting to see if some of these regularities are further supported by experiment.

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