Overview of Λ_c decays

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The decays of the ground-state charmed baryon Λ_c are now close to being completely mapped out. In this paper we discuss some remaining open questions, whose answers can help shed light on weak processes contributing to those decays, calculations of such quantities as transition form factors in lattice QCD, and missing decay modes such as $\Lambda_c \to \Lambda^* \ell^+ \nu_\ell$, where Λ^* is an excited resonance. The discussion is in part a counterpart to a previous analysis of inclusive D_s decays.

I. INTRODUCTION

The lowest-lying charmed baryon Λ_c was discovered more than 40 years ago [1], but its decays have not yet been fully mapped out due to the many available modes. Significant progress toward this goal has been made in the past few years, thanks to advances in particle identification, tracking, and collider luminosity. In the present paper we identify some missing modes of interest, the questions associated with them, and ways of filling the gaps in our knowledge. Some modes involving neutrons cannot be identified directly, so one must resort to models such as isospin statistical models [2–4]. These techniques also apply to modes with many neutral pions. Even when isospin multiplets have been filled, however, there remains a gap. Some of this gap arises from unreported modes with η or η'. In addition, we propose that some of it be filled with semileptonic decays $\Lambda_c \to \Lambda^* \ell^+ \nu_\ell$, where Λ^* is either an excited resonance such as $\Lambda(1405)$ or $\Lambda(1520)$ [5] (the number in parentheses denotes the mass in MeV) or a continuum $I = 0$ state such as $\Sigma\pi$ or $N\overline{K}$.

A global analysis of Λ_c decays is particularly timely now that Belle [6] and BESIII [7] have significantly improved the accuracy of the branching fraction $\mathcal{B}(\Lambda_c \to pK^-\pi^+)$, which has been used to normalize other Λ_c branching fractions. Their results and the resulting Particle Data

Group's (PDG) [8] "fit" value are summarized in Table I. BESIII [7] quotes updated absolute branching fractions for a dozen Λ_c modes, incorporated into the latest PDG averages [5]. Also new is a set of updated branching fractions for $\Lambda_c \to \Sigma\pi\pi$, including the first observation of the mode $\Lambda_c \to \Sigma^+\pi^0\pi^0$ [9]. The situation has greatly improved in the past six years since a plea was issued for improvement of Λ_c absolute branching fractions [10]. The present paper is devoted in part to an update of that analysis. For an early model-dependent discussion of Cabibbo-favored two-body Λ_c decays and for a recent study of singly Cabibbo-suppressed decays, quoting other papers using a similar approach, see Refs. [11,12], respectively.

We review the isospin statistical method in Sec. II, giving examples of its predictions for the $N\overline{K}\pi$ modes in Sec. III, the $\Sigma 2\pi$ modes in Sec. IV, the $N\overline{K}2\pi$ modes in Sec. V, and the $\Sigma 3\pi$ modes in Sec. VI. Some other modes are treated in Sec. VII. We apply the method to identify missing charge modes in Λ_c final states in Sec. VIII. This approach mirrors one applied to D_s decays [4]. Section IX is devoted to suggestions for placing these estimates on a firmer footing, such as identifying missing neutrons and taking account of decays involving η and η'. Section X is devoted to systematic errors associated with possible deviations from the statistical isospin model, such as the dominance of

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TABLE I. Values of $\mathcal{B}(\Lambda_c \to pK^-\pi^+)$.

Source	Reference	Value (%)
Belle	[6]	$6.84 \pm 0.24^{+0.21}_{-0.27}$
BESIII	[7]	$5.84 \pm 0.27 \pm 0.23$
PDG fit	[5]	6.23 ± 0.33

resonant substructure. Part of the remaining shortfall is proposed in Sec. XI to be filled by Λ_c semileptonic decays to excited states. Section XII concludes. An Appendix discusses details of obtaining branching fractions not quoted by Ref. [8].

II. STATISTICAL ISOSPIN MODEL

A multiparticle amplitude may be decomposed into a series of invariant isospin amplitudes depending on particle momenta, as we show by example in the next three sections. The statistical isospin model [2,3] parametrizes one's ignorance of underlying dynamics by assuming that each invariant amplitude contributes equally and incoherently to each decay mode, with relative branching fractions determined only by Clebsch-Gordan coefficients. The squares of each invariant amplitude's coefficients then sum to 1, and for each mode the branching fraction is the sum of squares of each contributing amplitude, divided by the number of invariant amplitudes. The answer does not depend on how the isospin decomposition is performed.

We illustrate this process for a three-body final state ABC produced in a state of definite isospin I and third component I_3. We may first decompose the BC system into isospin amplitudes I^{BC} with $I^B - I^C \leq I^{BC} \leq I^B + I^C$. We then combine the amplitudes I^{BC} with I^A in such a way that the final isospin is the desired value I, while $I_3^{BC} + I_3^A = I_3$. This process then reduces to manipulation of Clebsch-Gordan coefficients.

III. DECAYS $\Lambda_c \to N\overline{K}\pi$

The normalizing branching fraction for many Λ_c decays is $\mathcal{B}(\Lambda_c \to pK^-\pi^+)$. The isospin-partner modes are $n\overline{K}^0\pi^+$ and $p\overline{K}^0\pi^0$. The initial Λ_c has isospin 0, while the $\Delta S = 1$ (Cabibbo-favored) transition is governed by $c \to su\bar{d}$, resulting in a final state with $I = I_3 = 1$. The final $N\overline{K}$ final state can have isospin 0 or 1. If invariant amplitudes A are labeled by this isospin, one finds

$$\mathcal{A}(pK^-\pi^+) = \frac{A_0}{\sqrt{2}} - \frac{A_1}{2}, \qquad \mathcal{A}(n\overline{K}^0\pi^+) = -\frac{A_0}{\sqrt{2}} - \frac{A_1}{2},$$

$$\mathcal{A}(p\overline{K}^0\pi^0) = \frac{A_1}{\sqrt{2}}, \tag{1}$$

satisfying the sum rule

$$\mathcal{A}(pK^-\pi^+) + \mathcal{A}(n\overline{K}^0\pi^+) + \sqrt{2}\mathcal{A}(p\overline{K}^0\pi^0) = 0 \tag{2}$$

as noted by BESIII, the observers of the $n\overline{K}^0\pi^+$ mode [13]. The statistical isospin model postulates equality and incoherence of A_0 and A_1, so that the branching fractions of Λ_c to the above $N\overline{K}\pi$ modes are in the ratio 3/8:3/8:1/4. This prediction is compared with data [8] in Table II. The ratios of branching fractions for the two modes with a

TABLE II. Statistical isospin model predictions for relative branching fractions of Λ_c to $N\overline{K}\pi$ final states and comparison with observation.

Final state	Observed Λ_c branching fraction (%)	Fraction of $N\overline{K}\pi$	Statistical model
$pK^-\pi^+$	6.23 ± 0.33	0.452 ± 0.032	0.375
$n\overline{K}^0\pi^+$	3.64 ± 0.50	0.264 ± 0.038	0.375
$p\overline{K}^0\pi^0$	3.92 ± 0.26	0.284 ± 0.023	0.250
Total $N\overline{K}\pi$	13.79 ± 0.65		

proton are underestimated by 2.4σ and 1.5σ with respect to measurements, while the branching fraction for the $n\overline{K}^0\pi^+$ mode is slightly overestimated by 2.9σ. This gives an idea of the degree to which we can trust the statistical model. Deviations from its predictions are discussed in Sec. X.

One could, if desired, decompose the final states into ones labeled by the isospin of the $\overline{K}\pi$ system, with invariant amplitudes $A_{1/2}$ and $A_{3/2}$. Assuming these two amplitudes are equal in magnitude and incoherent, one arrives at the same result.

IV. DECAYS $\Lambda_c \to \Sigma\pi\pi$

The final state in $\Lambda_c \to \Sigma\pi\pi$ decays must have $I = I_3 = 1$, as noted in the previous subsection. One way to count invariant amplitudes is to designate them by the isospin of the two-pion system, $I_{\pi\pi} = 0$, 1, 2. For each such isospin there is a unique coupling with the Σ (whose isospin is 1) to the $I = 1$ final state. Thus there are three invariant amplitudes A_0, A_1, A_2. The Λ_c decay amplitudes are expressed in terms of them as

$$\mathcal{A}(\Sigma^-\pi^+\pi^+) = \sqrt{\frac{3}{5}}A_2, \tag{3}$$

$$\mathcal{A}(\Sigma^0\pi^+\pi^0) = -\frac{1}{2}\sqrt{\frac{3}{5}}A_2 + \frac{1}{2}A_1, \tag{4}$$

$$\mathcal{A}(\Sigma^0\pi^0\pi^+) = -\frac{1}{2}\sqrt{\frac{3}{5}}A_2 - \frac{1}{2}A_1, \tag{5}$$

$$\mathcal{A}(\Sigma^+\pi^+\pi^-) = \frac{1}{2\sqrt{15}}A_2 - \frac{1}{2}A_1 + \frac{1}{\sqrt{3}}A_0, \tag{6}$$

$$\mathcal{A}(\Sigma^+\pi^-\pi^+) = \frac{1}{2\sqrt{15}}A_2 + \frac{1}{2}A_1 + \frac{1}{\sqrt{3}}A_0, \tag{7}$$

$$\mathcal{A}(\Sigma^+\pi^0\pi^0) = \frac{1}{\sqrt{15}}A_2 - \frac{1}{\sqrt{3}}A_0. \tag{8}$$

Here we quote amplitudes for both orders of differing pion charges, needed for the sum of squares of coefficients of

TABLE III. Statistical isospin model predictions for relative branching fractions of Λ_c to $\Sigma\pi\pi$ final states and comparison with observation.

Final state	Observed Λ_c branching fraction (%)	Fraction of $\Sigma\pi\pi$	Statistical model
$\Sigma^-\pi^+\pi^+$	1.86 ± 0.18	0.177 ± 0.018	0.200
$\Sigma^0(\pi^+\pi^0)$	3.03 ± 0.23	0.288 ± 0.024	0.267
$\Sigma^+(\pi^+\pi^-)$	4.41 ± 0.20	0.419 ± 0.024	0.400
$\Sigma^+\pi^0\pi^0$	1.23 ± 0.12	0.117 ± 0.012	0.133
Total $\Sigma\pi\pi$	10.53 ± 0.37		

each isospin amplitude to add up to 1. The statistical-model predictions are obtained by assuming that invariant amplitudes are equal in magnitude and incoherent. They are compared with experiment [8,9] in Table III. The parentheses around pion pairs denote the sum of both orders. The agreement with the statistical isospin model is quite good.

In Ref. [3] bounds were placed on Λ_c decays to $\Sigma\pi\pi$ final states consisting of all charged particles, with the result

$$\frac{1}{2} \le \frac{\mathcal{B}(\Lambda_c \to \Sigma^-\pi^+\pi^+) + \mathcal{B}(\Lambda_c \to \Sigma^+(\pi^+\pi^-))}{\mathcal{B}(\Lambda_c \to \Sigma\pi\pi)} \le \frac{4}{5}. \quad (9)$$

The quotient in Eq. (9) has the value $3/5$ in the statistical isospin model.

V. DECAYS $\Lambda_c \to N\overline{K}\pi\pi$

One convenient way to define invariant amplitudes for the $N\overline{K}\pi\pi$ final state is to couple the $N\overline{K}$ pair to isospin $I_{N\overline{K}} = 0, 1$ and the pion pair to $I_{\pi\pi} = 0, 1, 2$. When $I_{N\overline{K}} = 0$, only $I_{\pi\pi} = 1$ can lead to a final state with $I = 1$; we call the corresponding reduced amplitude A_{1a}. When $I_{N\overline{K}} = 1$, the $I = 1$ final state receives contributions from $I_{\pi\pi} = 0, 1, 2$; we call the corresponding reduced amplitudes A_0, A_{1b}, A_2. The decomposition of Λ_c decay amplitudes in terms of these reduced amplitudes is

$$\mathcal{A}(pK^-\pi^+\pi^0) = -\frac{1}{2}\sqrt{\frac{3}{10}}A_2 + \frac{1}{2}A_{1a} + \frac{1}{2\sqrt{2}}A_{1b}, \quad (10)$$

$$\mathcal{A}(pK^-\pi^0\pi^+) = -\frac{1}{2}\sqrt{\frac{3}{10}}A_2 - \frac{1}{2}A_{1a} - \frac{1}{2\sqrt{2}}A_{1b}, \quad (11)$$

$$\mathcal{A}(n\overline{K}^0\pi^+\pi^0) = -\frac{1}{2}\sqrt{\frac{3}{10}}A_2 - \frac{1}{2}A_{1a} + \frac{1}{2\sqrt{2}}A_{1b}, \quad (12)$$

$$\mathcal{A}(n\overline{K}^0\pi^0\pi^+) = -\frac{1}{2}\sqrt{\frac{3}{10}}A_2 + \frac{1}{2}A_{1a} - \frac{1}{2\sqrt{2}}A_{1b}, \quad (13)$$

$$\mathcal{A}(p\overline{K}^0\pi^0\pi^0) = \frac{1}{\sqrt{15}}A_2 - \frac{1}{\sqrt{3}}A_0, \quad (14)$$

TABLE IV. Statistical isospin model predictions for relative branching fractions of Λ_c to $N\overline{K}\pi\pi$ final states and comparison with observation. Quantities in brackets are inferred from implied total average.

Final state	Statistical model	Λ_c branching fraction (%)	Implied total $\mathcal{B}(\Lambda_c \to N\overline{K}\pi\pi)$ (%)
$pK^-(\pi^+\pi^0)$	$9/40 = 0.225$	4.42 ± 0.31	19.64 ± 1.38
$n\overline{K}^0(\pi^+\pi^0)$	$9/40 = 0.225$	$[3.07 \pm 0.16]$	$\cdots$
$p\overline{K}^0\pi^0\pi^0$	$1/10 = 0.100$	$[1.36 \pm 0.07]$	$\cdots$
$p\overline{K}^0(\pi^+\pi^-)$	$3/10 = 0.300$	3.18 ± 0.24	10.60 ± 0.80
$nK^-\pi^+\pi^+$	$3/20 = 0.150$	$[2.05 \pm 0.11]$	$\cdots$
Average			12.88 ± 0.69[a]

[a]Error to be multiplied by a scale factor of 5.67 in the final total.

$$\mathcal{A}(p\overline{K}^0\pi^+\pi^-) = \frac{1}{2\sqrt{15}}A_2 - \frac{1}{2}A_{1b} + \frac{1}{\sqrt{3}}A_0, \quad (15)$$

$$\mathcal{A}(p\overline{K}^0\pi^-\pi^+) = \frac{1}{2\sqrt{15}}A_2 + \frac{1}{2}A_{1b} + \frac{1}{\sqrt{3}}A_0, \quad (16)$$

$$\mathcal{A}(nK^-\pi^+\pi^+) = \sqrt{\frac{3}{5}}A_2. \quad (17)$$

The implications of this decomposition for the statistical isospin model are summarized in Table IV. Only two modes are measured, and they imply quite different values of the total $\mathcal{B}(\Lambda_c \to N\overline{K}\pi\pi)$. We bear this uncertainty in mind when evaluating the accuracy of our predictions. Using the average value of the total branching fraction, we predict the branching fractions for as yet unseen decay modes in brackets in the table.

The average of implied total branching fractions involves two very disparate values, so we apply a scale factor of 5.67 to the error, giving 3.92 to be quoted in the summary table. Possible deviations from the statistical isospin model are noted in Sec. X.

TABLE V. Statistical model predictions and observed branching fractions for $\Lambda_c \to \Sigma 3\pi$ decays. For each mode, all permutations of pions are implied.

Final state	Statistical model	Λ_c branching fraction (%)	Implied total $\mathcal{B}(\Lambda_c \to \Sigma 3\pi)$ (%)
$\Sigma^0 2\pi^+\pi^-$	$1/5 = 0.200$	1.10 ± 0.30	5.5 ± 1.5
$\Sigma^-\pi^0 2\pi^+$	$1/5 = 0.200$	2.1 ± 0.4	10.5 ± 2.0
$\Sigma^+\pi^+\pi^-\pi^0$	$2/5 = 0.400$	(a)	$\cdots$
$\Sigma^0\pi^+ 2\pi^0$	$3/20 = 0.150$	$\cdots$	$\cdots$
$\Sigma^+ 3\pi^0$	$1/20 = 0.050$	$\cdots$	$\cdots$
Average			7.3 ± 1.2(b)

(a)$B(\Lambda_c \to \Sigma^+\omega) = (1.69 \pm 0.21)\%$ counted separately.
(b)Error to be multiplied by a scale factor of 2.0 in the final total.

VI. DECAYS $\Lambda_c \to \Sigma 3\pi$

The number of invariant amplitudes may be counted by noting the number of 3π amplitudes with each isospin and then coupling them up with the $I = 1$ Σ to a final state with $I = 1$. The multiplicities of three-pion amplitudes are 1 for $I = 0$, 3 for $I = 1$, 2 for $I = 2$, and 1 for $I = 3$. Each of these except the $I = 3$ amplitude can couple up with the Σ to form final isospin 1. Thus there are a total of six reduced amplitudes.

$$\frac{3}{5} \le \frac{\mathcal{B}(\Lambda_c \to \Sigma^- 2\pi^+ \pi^0) + \mathcal{B}(\Lambda_c \to \Sigma^+ \pi^+ \pi^- \pi^0) + \mathcal{B}(\Lambda_c \to \Sigma^0 2\pi^+ \pi^-)}{\mathcal{B}(\Lambda_c \to \Sigma 3\pi)} \le 1. \tag{18}$$

The quotient in Eq. (18) has the value 4/5 in the statistical isospin model.

VII. SOME OTHER CABIBBO-FAVORED MODES

We use a standard format, extracting estimates of the branching fraction to the sum of all charge states for a given mode and averaging where there is more than one measured charge state. The statistical-model fractions are taken from Table IV of Ref. [3].

A. $N\bar{K}3\pi$

Only one mode ($K^- p 2\pi^+ \pi^-$) is used in estimating the total, as the branching fraction for $K^- p\pi^+ 2\pi^0$ is suspiciously large in comparison with the all-charged-particle mode. It bears watching, however (see Table VI).

B. $\Lambda 3\pi$

So far only the mode with no neutral pions has been detected. Reference [3] obtains the bounds

$$\frac{1}{2} \le \frac{\mathcal{B}(\Lambda_c \to \Lambda 2\pi^+ \pi^-)}{\mathcal{B}(\Lambda_c \to 3\pi)} \le \frac{4}{5}, \tag{19}$$

TABLE VI. Statistical model predictions and observed branching fractions for $\Lambda_c \to N\bar{K}3\pi$ decays. For each mode, all permutations of pions are implied.

Final state	Statistical model	Λ_c branching fraction (%)	Implied total $\mathcal{B}(\Lambda_c \to N\bar{K}3\pi)$ (%)
$K^- p 2\pi^+ \pi^-$	$1/6 = 0.167$	0.14 ± 0.09	0.84 ± 0.54
$\bar{K}^0 n 2\pi^+ \pi^-$	$1/6 = 0.167$	$\cdots$	$\cdots$
$\bar{K}^0 p\pi^+ \pi^- \pi^0$	$4/15 = 0.267$	$\cdots$	$\cdots$
$K^- n 2\pi^+ \pi^0$	$2/15 = 0.133$	$\cdots$	$\cdots$
$K^- p\pi^+ 2\pi^0$	$7/60 = 0.117$	(a)	$\cdots$
$\bar{K}^0 n\pi^+ 2\pi^0$	$7/60 = 0.117$	$\cdots$	$\cdots$
$\bar{K}^0 p 3\pi^0$	$1/30 = 0.033$	$\cdots$	$\cdots$
Average			0.84 ± 0.54

(a)The PDG value of 1.0 ± 0.5 is ignored but bears watching.

The statistical model's predictions of relative branching fractions for Cabibbo-favored decays have been given in Ref. [3]. For $\Sigma 3\pi$ final states of Λ_c we show the results in Table V. In averaging the two values leading to different implied total fractions we multiply the uncertainty of 1.2% by a scale factor of 2 to give a final uncertainty of 2.4%. Deviations from the statistical isospin model are discussed in Sec. X. Bounds on Λ_c decays to $\Sigma 3\pi$ final states with three charged particles [3] are

where the value of the quotient in the statistical isospin model is 3/5 (see Table VII).

C. $\Lambda 4\pi$

The mode with a single neutral pion is the only one detected. With three neutral pions the missing mode is unlikely to be confirmed soon. Reference [3] finds the bounds

$$\frac{3}{5} \le \frac{\mathcal{B}(\Lambda_c) \to \Lambda \pi^- \pi^0 2\pi^+)}{\mathcal{B}(\Lambda_c \to \Lambda 4\pi)} \le 1, \tag{20}$$

with the statistical isospin model giving 4/5 for the quotient (see Table VIII).

TABLE VII. Statistical model predictions and observed branching fractions for $\Lambda_c \to \Lambda 3\pi$ decays. For each mode, all permutations of pions are implied.

Final state	Statistical model	Λ_c branching fraction (%)	Implied total $\mathcal{B}(\Lambda_c \to \Lambda 3\pi)$ (%)
$\Lambda \pi^- 2\pi^+$	$3/5 = 0.600$	3.61 ± 0.29	6.02 ± 0.48
$\Lambda \pi^+ 2\pi^0$	$2/5 = 0.400$	$\cdots$	$\cdots$
Average			6.02 ± 0.48

TABLE VIII. Statistical model predictions and observed branching fractions for $\Lambda_c \to \Lambda 4\pi$ decays. For each mode, all permutations of pions are implied.

Final state	Statistical model	Λ_c branching fraction (%)	Implied total $\mathcal{B}(\Lambda_c \to \Lambda 4\pi)$ (%)
$\Lambda \pi^- \pi^0 2\pi^+$	$4/5 = 0.800$	2.2 ± 0.8	2.75 ± 1.00
$\Lambda \pi^+ 3\pi^0$	$1/5 = 0.200$	$\cdots$	$\cdots$
Average			2.75 ± 1.00

VIII. IDENTIFYING MISSING MODES

In the previous sections we have used the isospin statistical model to estimate missing charge states for Λ_c decay modes due to the Cabibbo-favored process $c \to su\bar{d}$, populating final states with strangeness $S = -1$ and isospin $I = I_3 = 1$. The results are shown in Table IX. Also shown are much rougher estimates of branching fractions to $S = 0$ final states, populated by the singly Cabibbo-suppressed transitions $c \to du\bar{d}$ and $c \to su\bar{s}$.

The sum of the two sets of branching fractions is $(89.6 \pm 5.0)\%$. Thus there is a hint, though not statistically compelling at present, that about 10% of Λ_c decays remain to be accounted for. We suggest that this could be due in

TABLE X. Final states in Λ_c decay leading directly to a Λ (left column) or through a Σ^0 (right column).

State	$\mathcal{B}(\%)$	State	$\mathcal{B}(\%)$
$\Lambda\pi^+$	1.29 ± 0.07	$\Sigma^0\pi^+$	1.28 ± 0.07
$\Lambda\pi^+\pi^0$	7.0 ± 0.4	$\Sigma^0\pi^+\pi^0$	3.03 ± 0.23
$\Lambda 3\pi$	6.02 ± 0.48	$\Sigma^0\pi^-2\pi^+$	1.10 ± 0.30
$\Lambda 4\pi$	2.75 ± 1.00	$\Sigma^0\pi^+2\pi^0$	1.10 ± 0.18[a]
$\Lambda K^+\bar{K}^0$	0.56 ± 0.11	$\Sigma^0 K^+$	0.051 ± 0.008
$\Lambda e^+\nu_e$	3.63 ± 0.43	$\Sigma^0 K^+\pi^0$	0.21 ± 0.06[b]
$\Lambda\mu^+\nu_\mu$	3.49 ± 0.53	$\Sigma^0 K^0\pi^+$	0.21 ± 0.06[b]
Total	24.74 ± 1.37	Total	6.98 ± 0.43

[a]See Table V: $(3/20) \cdot (7.3 \pm 1.2)$.
[b]Assuming equal to measured $\mathcal{B}(\Lambda_c \to \Sigma^+ K^+\pi^-)$ [8].

part to semileptonic Λ_c decays to excited states such as $\Lambda(1405)$, $\Lambda(1520)$, or continuum $\Sigma\pi$ and/or $N\bar{K}$ states.

The estimates for the $\Delta S = 0$ transitions are very rough, as many of them rely on the assumption that each charge mode is equally populated. What one sees in the statistical model, instead, is that the modes with the most neutral pions tend to be populated the least. Thus the total branching fraction for $\Delta S = 0$ decays may in fact be an upper bound.

A recent BESIII determination of inclusive Λ production in Λ_c decays [18] finds $\mathcal{B}(\Lambda_c \to \Lambda + X) = (38.2^{+2.8}_{-2.2} \pm 0.8)\%$. We can compare this result with the sum of contributing entries in Table IX. Table X shows the final states directly leading to a Λ, and separately gives those leading to a Σ^0, which decays 100% of the time to $\Lambda\gamma$. The sum of these totals is $(31.72 \pm 1.44)\%$, a shortfall of 2.4σ. What could fill the gap? Possible candidates are underestimates of modes $\Lambda n\pi(n = 3, 4)$ using the statistical model, modes $\Lambda n\pi(n > 4)$ or $\Sigma^0 n\pi(n > 3)$, and semileptonic decays to hadronic final states consisting of Λ accompanied by other particles. Examples are $\Lambda(1405, 1520) \to \Sigma^0\pi^0 \to \Lambda\gamma\pi^0$, $\Lambda(1690) \to \Lambda 2\pi$, and Λ in nonresonant continuum.

TABLE IX. Observed and extrapolated branching fractions $\mathcal{B}$ for Λ_c decays, in %. Unless shown otherwise, the statistical isospin model has been used to extrapolate to unseen charge states.

$\Delta S = -1$ transitions		$\Delta S = 0$ transitions	
Mode	$\mathcal{B}$	Mode	$\mathcal{B}$
$p\bar{K}^0$	3.16 ± 0.16	$p\eta$	0.124 ± 0.030
$N\bar{K}\pi$	13.79 ± 0.65	$N\pi\pi$	1.26 ± 0.12[a]
$p\bar{K}^0\eta$	1.6 ± 0.4	$N3\pi$	1.22 ± 0.30[b]
$N\bar{K}2\pi$	12.88 ± 3.92	$N4\pi$	1.10 ± 0.70[a]
$N\bar{K}3\pi$	0.84 ± 0.54	$NK\bar{K}$	0.30 ± 0.12[a]
$\Lambda\pi^+$	1.29 ± 0.07	ΛK^+	0.06 ± 0.012
$\Lambda\pi^+\pi^0$	7.0 ± 0.4	ΣK	0.102 ± 0.016[a]
$\Lambda 3\pi$	6.02 ± 0.48	$\Sigma K\pi$	1.05 ± 0.30[a]
$\Lambda 4\pi$	2.75 ± 1.00	$ne^+\nu_e$	0.41 ± 0.03[c]
$\Sigma\pi$	2.52 ± 0.12	$n\mu^+\nu_\mu$	0.40 ± 0.03[c]
$\Sigma\eta$	0.69 ± 0.23	$p\pi^0$	0.008[d]
$\Sigma 2\pi$	10.53 ± 0.37	$n\pi^+$	0.027[d]
$\Sigma 3\pi$	7.3 ± 2.4		
$\Sigma\omega$	1.69 ± 0.21		
$\Lambda K^+\bar{K}^0$	0.56 ± 0.11		
$\Sigma K\bar{K}$	1.36 ± 0.16[a]		
$\Xi^0 K^+$	0.55 ± 0.07[e]		
$\Xi K\pi$	1.86 ± 0.18[f]		
$\Lambda e^+\nu_e$	3.63 ± 0.43[g]		
$\Lambda\mu^+\nu_\mu$	3.49 ± 0.53[h]		
Total $\Delta S = -1$	83.51 ± 4.92	Total $\Delta S = 0$	6.06 ± 0.84

[a]Branching fraction for one observed charge mode multiplied by a number of charge states.
[b]Branching fraction to $p\pi^+\pi^0\pi^-$ taken as $(0.304 \pm 0.076)\%$ (geometric mean of $p\pi^+\pi^-$ and $p2\pi^+2\pi^-$ modes), multiplied by 4 for a total number of charge states.
[c]Lattice QCD calculation [14].
[d]Theoretical estimate from Ref. [12].
[e]New value of $(0.59 \pm 0.09)\%$ [15] averaged with PDG value $(0.49 \pm 0.12)\%$ [8].
[f]We multiply $\mathcal{B}(\Lambda_c \to \Xi^- K^+\pi^+) = (0.62 \pm 0.06)\%$ [8] by 3 to include charge states $\Xi^0 K^+\pi^0$ and $\Xi^0 K^0\pi^+$. Reference [15] measures $\mathcal{B}(\Lambda_c \to \Xi^0(1530)K^+) = (0.50 \pm 0.10)\%$, accounting for part but not all of the $\Xi^- K^+\pi^+$ final state.
[g]Reference [16].
[h]Reference [17].

IX. SUGGESTIONS FOR IMPROVEMENT

A. Modes needing further attention

The uncertainty on the $\Delta S = -1$ transitions is dominated by the disagreement with the isospin statistical model in the $N\bar{K}\pi\pi$ modes. The ratio $\mathcal{B}(\Lambda_c \to pK^-\pi^+\pi^0)/\mathcal{B}(\Lambda_c \to p\bar{K}^0\pi^+\pi^-)$ is measured to be 1.39 ± 0.11, whereas in the isospin statistical model it is predicted to be $(9/40)/(3/10) = 3/4$. The corresponding total $N\bar{K}2\pi$ branching ratio is very different depending on which mode one uses to estimate it. Measurement of further $N\bar{K}2\pi$ modes might help to resolve the ambiguity.

The singly Cabibbo-suppressed ($\Delta S = 0$) modes are not readily amenable to a statistical treatment, as the final states are a mixture of $I = 1/2$ and $I = 3/2$. Thus measuring their branching fractions in the widest possible cases is called for instead.

B. Neutron identification

BESIII has recently reported observation of the first Λ_c mode containing a neutron [13]. The method used was to ensure production of a Λ_c using a combination of single and double tags at a center-of-mass energy in e^+e^- collisions just above the $\Lambda_c^+\Lambda_c^-$ threshold. The neutron was then inferred from kinematic reconstruction. In principle, this method could be applied to many states in the $N\overline{K}2\pi$ and $N\overline{K}3\pi$ modes. The presence of a neutron in a kinematically constrained fit could be confirmed if there were a calorimetric signal (resembling the interaction of a K_L^0) in the outer layer of a detector such as BESIII or Belle.

C. Inclusive η, η' branching fractions

Although some portion of decay modes involving η or η' appears in multipion final states, the inclusive η and η' branching fractions have not been reported. It would be very helpful to have them, in the same manner that inclusive measurements were very helpful in sorting out D_s decays [4].

X. DEVIATIONS FROM THE STATISTICAL MODEL

In all decays involving three or more final-state particles, pairwise associations in resonant substructures can lead to deviations from the statistical isospin model. However, in their high-statistics studies of Λ_c decays, neither BESIII [7] nor Belle [9] shows Dalitz plots or one-dimensional plots of pairwise effective masses. Consequently, we have to anticipate possible deviations from the statistical isospin model without the help of experiment. We hope this situation changes in the near future.

In Fig. 1 we give three examples of processes contributing to Λ_c decays. These have the potential of populating final states in a manner differing from the statistical isospin model, giving rise to characteristic resonant substructures. We estimate the corresponding uncertainties for a series of final states. In cases where all charge states are allowed, we comment on how well the statistical isospin model is

obeyed, but do not assign any uncertainty to the branching fractions.

A. $N\overline{K}\pi$

All branching fractions have been observed. Defining $R_1 = \mathcal{B}(pK^-\pi^+)$, $R_2 = \mathcal{B}(n\overline{K}^0\pi^+)$, $R_3 = \mathcal{B}(p\overline{K}^0\pi^0)$, amplitudes in Eq. (1) (up to a common factor) are related by

$$|A_0|^2 = R_1 + R_2 - 2R_3 = 5.95 \pm 0.65, \qquad (21)$$

$$|A_1|^2 = 2R_3 = 7.84 \pm 0.52, \qquad (22)$$

$$\text{Re}(A_0^*A_1) = (R_2 - R_1)/\sqrt{2} = -1.83 \pm 0.42. \qquad (23)$$

The two amplitudes A_0 and A_1 are unequal in magnitude and have some degree of coherence, in contrast to the statistical isospin model which would have them equal in magnitude and out of phase with one another.

One can qualitatively anticipate the violation of the statistical isospin model by reference to the three diagrams of Fig. 1. The relatively short lifetime of the Λ_c, about 0.2 ps [8], can be ascribed in large part to the contribution of Fig. 1(a), which leads to a final state suu resembling a $J = 1/2$ excited Σ^{*+}. This can hadronize by production of either a $u\overline{u}$ or $d\overline{d}$ pair. In the former case one produces a configuration $(s\overline{u})(uuu)$ that can materialize as $K^-\Delta^{++} = K^-p\pi^+$. The latter case leads to a configuration $(s\overline{d})(duu)$ that can materialize to $\overline{K}^0n\pi^+$ or $\overline{K}^0p\pi^0$. In this example, $K^-\pi^+$ accounts for 50% of $\Lambda_c \to N\overline{K}\pi$ decays, exhibiting the likely direction of deviation from the statistical isospin model.

What if the spectator diagram (b) were dominant? The spectator ud pair would remain in an isospin-zero final state, implying equal branching fractions for $n\overline{K}^0\pi^+$ and $pK^-\pi^+$, and no contribution to $p\overline{K}^0\pi^0$, far from the observed situation.

B. $\Sigma\pi\pi$

Here the statistical isospin model works surprisingly well. Both the internal conversion and spectator processes can contribute. They both can excite resonances such as $\Lambda(1405)(J^P = 1/2^-)$ and $\Lambda(1520)(J^P = 3/2^-)$, which in turn can couple to all charge states of $\Sigma\pi$ and $N\overline{K}$. [The
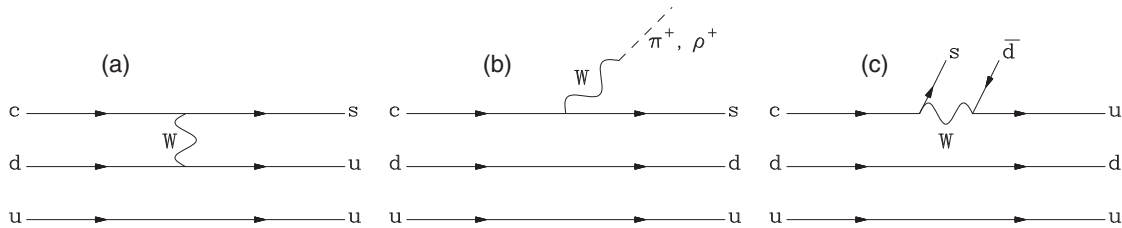

FIG. 1. Cabibbo-favored processes contributing to Λ_c hadronic decays. (a) Internal conversion involving the subprocess $cd \to su$ with W exchange. (b) Spectator process with $c \to (\pi^+, \rho^+)s$ with isospin-zero ud pair as a spectator. (c) Color-suppressed process involving $c \to (s\overline{d})u$, where $s\overline{d} \to \overline{K}^0, \overline{K}^{*0}, \ldots$ For Cabibbo-suppressed modes, replace s with d.

contribution to the $N\overline{K}$ amplitude of the $\Lambda(1405)$ involves an intermediate off-shell state.] Either hyperon resonance decays to an $I = 0$ final state, so $\Lambda(1405) \to (\pi^+\Sigma^-)$, $(\pi^0\Sigma^0)$, $(\pi^-\Sigma^+)$ in equal proportions. The spectator process does not contribute to $\Lambda_c \to \Sigma^+\pi^0\pi^0$, which is the smallest $\Sigma\pi\pi$ branching fraction, both predicted by the isospin statistical model and observed.

C. $N\overline{K}\pi\pi$

The ratio of branching ratios to $pK^-\pi^+\pi^0$ and $p\overline{K}^0\pi^+\pi^-$ is $(4.42 \pm 0.31)/(3.18 \pm 0.24) = 1.39 \pm 0.14$, very far from the statistical isospin model's prediction of $(9/40)/(3/10) = 3/4$. The $p\overline{K}^0\pi^+\pi^-$ amplitude would be suppressed if the $N\overline{K}$ amplitude were predominantly $I = 0$, as if dominated by $\Lambda(1405)$ and $\Lambda(1520)$. In that case the only nonzero amplitude in Sec. V would be A_{1a}, and the mode $pK^-\pi^+\pi^0$ would be $1/4$ of the $N\overline{K}2\pi$ total. Thus for the $N\overline{K}2\pi$ total one would have $4 \times (4.42 \pm 0.31)\% = (17.68 \pm 1.24)\%$. Taking this value rather than $(12.88 \pm 0.69)\%$ on the bottom line of Table IV one sees a difference of 4.80%, which could be substituted for the scaled error of $(0.69)(5.67) = 3.92\%$ ascribed to this mode.

D. $\Sigma3\pi$

The statistical model predicts equal branching fractions for $\Lambda_c \to \Sigma^02\pi^+\pi^-$ and $\Lambda_c \to \Sigma^-\pi^02\pi^+$, whereas the first $[(1.10 \pm 0.30)\%]$ is only about half the second $[(2.1 \pm 0.4)\%]$. (See Table V.) The second process can receive a contribution from the spectator subprocess $c \to \rho^+s$ with $\rho^+ \to \pi^+\pi^0$; the first process has no π^0. Suppose (to visualize the effect of hypothetical resonant substructure) we assume the process were dominated by the final-state $\rho^+\Lambda(1405) \to (\pi^+\pi^0) + (\pi^+\Sigma^-, \pi^0\Sigma^0, \pi^-\Sigma^+)$. Then the second, third, and fourth branching fractions in Table V would all be $1/3$, and the implied total $\mathcal{B}(\Lambda_c \to \Sigma3\pi)$ would be $3 \times (2.1 \pm 0.4)\% = (6.3 \pm 1.2)\%$, not that far from the value of $(7.3 \pm 2.4)\%$ quoted in Table IX. Although this is not a complete model for the $\Sigma3\pi$ final state, it illustrates the importance of experimentally determining resonant substructure in multibody Λ_c final states.

XI. SEMILEPTONIC Λ_c DECAYS

The only semileptonic Λ_c decays that have been reported are those to a $\Lambda\ell^+\nu_\ell$ final state. A hint that there may be other semileptonic final states is provided by a calculation assuming pointlike Λ_c and Λ, whose rate is predicted to be

$$\Gamma(\Lambda_c \to \Lambda e^+\nu_e) = \frac{G_F^2M(\Lambda_c)^5}{192\pi^3}f([M(\Lambda)/M(\Lambda_c)]^2)$$
$$= 2.52 \times 10^{-13} \text{ GeV}, \qquad (24)$$

where $f(x) \equiv 1 - 8x + 8x^3 - x^4 + 12x^2\ln(1/x) = 0.1763$ for $M(\Lambda) = 1115.68$ GeV and $M(\Lambda_c) = 2286.46$ GeV.

The total decay rate of Λ_c for a lifetime of 200 fs is 3.29×10^{-12} GeV, so the pointlike prediction corresponds to a branching fraction of 7.6%, about twice the observed rate. This suggests that a form factor is present, which can be interpreted as indicating the presence of excited final hadronic states. A similar conclusion can be drawn from a quark model cartoon of charm semileptonic decay [19]. With $(m_c, m_s, m_{u,d}) = (1710, 536, 364)$ MeV [20], a slightly higher branching fraction is obtained (see the Appendix), again indicating that semileptonic Λ_c decays are not saturated by the $\Lambda\ell^+\nu_\ell$ final state.

A calculation in lattice QCD [21] finds $\mathcal{B}(\Lambda_c \to \Lambda e^+\nu_e) = (3.80 \pm 0.19 \pm 0.11)\%$ and $\mathcal{B}(\Lambda_c \to \Lambda\mu^+\nu_\mu) = (3.69 \pm 0.19 \pm 0.11)\%$, where the first error comes from lattice QCD and the second from the uncertainty in the Λ_c lifetime. The agreement with experiment further confirms the need for a form factor and, indirectly, hints at a role for excited final states in Λ_c semileptonic decays.

The detection of $\Lambda(1405)$ or $\Lambda(1520)$ in the final state of Λ_c semileptonic decays may not be straightforward. The former decays only to $\Sigma\pi$, while the latter decays both to $\Sigma\pi$ and to $N\overline{K}$. Thus branching fractions are spread over many final states. There is also a measurement [22] $\mathcal{B}(\Lambda_c \to e^+ + \text{anything}) = (4.5 \pm 1.7)\%$ from 1982 that needs to be reexamined if our proposal is to account for a significant portion of the missing Λ_c decays.

XII. CONCLUSIONS

We have investigated Λ_c decays from a global standpoint, finding impressive progress in mapping out branching fractions. Nevertheless, there remains the possibility of a shortfall of about 10%. We have suggested that this could be filled [at least in part] by semileptonic decays to excited final states, not just the Λ. To reduce the uncertainty in the total observed branching fraction, we urge more studies of modes containing neutrons, greater investigation of the singly Cabibbo-suppressed modes, and inclusive studies of η and η'. Determination of resonant substructure is a crucial ingredient in filling gaps only partially addressed by an imperfect isospin statistical model. The fact that such progress has already been made for charmed meson decays [8] should serve as an encouragement for similar advances in our understanding of charmed baryon decays.

Note added.—The BESIII Collaboration has recently measured $\mathcal{B}(\Lambda_c \to Xe^+\nu_e) = (3.95 \pm 0.34 \pm 0.09)\%$ [23], significantly limiting the contribution from $X \neq \Lambda$.

APPENDIX: BRANCHING FRACTION DETAILS

We give some details of how branching fractions are quoted in Table IX if they are not taken directly from Ref. [8].

Modes with $\overline{K}^0$ are inferred from those quoted in [8] for K_S^0 by multiplying by 2. We renormalize $\mathcal{B}(\Lambda_c \to \Sigma^+ \pi^0 \pi^0)$, first measured by Belle [9] using a PDG (2016) value $\mathcal{B}(\Lambda_c \to pK^-\pi^+) = 6.35 \pm 0.33\%$, by the slightly smaller value of this normalizing branching ratio given in Table I. Using the same normalization for decays to $\Sigma^+\pi^+\pi^-$ and $\Sigma^0\pi^+\pi^0$ we also include early measurements of these branching ratios.

For the $N\overline{K}\pi$ and $\Sigma\pi\pi$ modes, all charge states have been measured, so the experimental totals in Tables II and III are transcribed in Table IX. For the $N\overline{K}2\pi$ and $\Sigma 3\pi$ modes, not all charge states are measured, so totals implied by the statistical model are averaged and quoted (with a scale factor for the $N\overline{K}2\pi$ average) in Table IX. Finally, the modes described in Sec. VII have only one charge state, which is used to estimate the missing modes, with the inferred total quoted in Table IX.

An alternative way of estimating the total contribution of singly Cabibbo-suppressed decays to Λ_c branching fractions is to use free-quark estimates. For a crude calculation we may consider only the subprocess $c \to du\overline{d}$, neglecting contributions from $c \to su\overline{s}$ by virtue of phase space suppression. We take effective quark masses from Ref. [20]: $m_c = 1710$ Mev, $m_s = 536$ MeV, $m_{u,d} = 364$ MeV, implying a phase space enhancement of 1.46 for $c \to du\overline{d}$ relative to $c \to su\overline{d}$. The corresponding ratio of squared CKM matrix elements is $|V_{cd}/V_{cs}|^2 = (0.2265/0.974)^2 = 0.0541$, implying a total branching fraction for subprocesses dominated by $c \to du\overline{d}$ of $88.6\% \times 1.46 \times 0.0541 = 7.0\%$. Thus we could be missing a few $\Delta S = 0$ modes, not to mention those governed by $c \to su\overline{s}$.

In parallel with the lattice QCD calculations mentioned earlier for $\Lambda_c \to \Lambda\ell^+\nu_\ell$ [21], there appeared recently one for the Cabibbo-suppressed $\Delta S = 0$ processes $\Lambda_c \to n\ell^+\nu_\ell$ [14]. The corresponding branching fractions are displayed in the right-hand column of Table IX.

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