ν solution to the strong *CP* problem

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We present a solution to the strong *CP* problem in which the imaginary component of the up quark mass, $\mathcal{I}[m_u]$, acquires a tiny, but nonvanishing value. This is achieved via a Dirac seesaw mechanism, which is also responsible for the generation of the small neutrino masses. Consistency with the observed value of the up quark mass is achieved via instanton contributions arising from QCD-like interactions, as is the case in the closely related massless up-quark solution to the strong *CP* problem. In our framework, however, the value of the neutron electric dipole moment is directly related to $\mathcal{I}[m_u]$, which, due to its common origin with the neutrino masses, implies that the neutron electric dipole moment is likely to be measured in the next round of experiments. We also present a supersymmetric extension of this Dirac seesaw model to stabilize the hierarchy among the scalar mass scales involved in this mechanism.

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I. INTRODUCTION

The Standard Model (SM) has been highly successful in describing all experimental observations [1]. The observed flavor and *CP*-violating effects originate from the weak interactions via the dependence of the charged currents on the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{CKM}. There is, however, another potential source of *CP* violation in the SM, associated with the strong interaction. After the diagonalization of the quark masses, the QCD Lagrangian density contains the terms

$$\mathcal{L} \supset -\frac{\theta g_s^2}{32\pi^2} G_{\mu\nu,a}\tilde{G}^{\mu\nu,a} - \sum_q (m_q \bar{q}_L q_R + \mathrm{H.c.}), \quad (1)$$

where g_s is the strong gauge coupling, $G_{\mu\nu,a}$ is the QCD field strength tensor, $\tilde{G}_{\mu\nu,a} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta,a}$ is its dual, and m_q are the quark masses. Due to the QCD chiral anomaly, the value of θ can be modified by a phase redefinition of the chiral quark fields, but the physical value

$$\theta_{\mathrm{QCD}} = \theta + \arg\,[\det[M_q]], \quad (2)$$

where $\det[M_q] = \prod m_q$, remains invariant. As will be discussed in detail later on, a non-vanishing value of θ_{QCD} leads to QCD induced *CP*-violating effects, like the neutron electric dipole moment (nEDM), which is as yet unobserved. The current bound on the nEDM, $d_n < 3.0 \times 10^{-26}$ e cm [2,3], leads to the constraint $\theta_{\mathrm{QCD}}(1\ \mathrm{GeV}) \lesssim 1.3 \times 10^{-10}$. The dynamical origin of such small values of θ_{QCD} is the so-called strong *CP* problem.

The θ term in Eq. (1) may be eliminated by a proper phase redefinition of the quark fields. For a nonzero θ_{QCD}, at least one of the quark masses, for instance the up quark mass, would become a complex quantity, with argument $\theta_{\mathrm{QCD}} \sim \mathcal{I}[m_u]/|m_u|$. Hence in such a case, all the QCD-induced *CP*-violating effects would be associated with $\mathcal{I}[m_u]$, and would vanish in the limit of zero up quark mass. This is the well known massless up quark solution to the strong *CP* problem [4–10].

We shall denote as the *canonical basis*, the basis in which $\theta = 0$ and θ_{QCD} is the argument of the up quark mass. Using the value of the up quark mass determined in the framework of chiral perturbation theory, $|m_u(1\ \mathrm{GeV})| \simeq 5$ MeV [11], the bound on θ_{QCD} becomes equivalent to

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$$\mathcal{I}[m_u(1\ \text{GeV})] \lesssim 6.5\times 10^{-4}\ \text{eV}. \tag{3}$$

The relevant question then becomes, can one dynamically generate a value of $\mathcal{I}[m_u(1\ \text{GeV})]$ consistent with such a stringent bound, while the real part, $\mathcal{R}[m_u(1\ \text{GeV})]$, is of the order of a few MeV?

To analyze this question, one should remember that the up quark mass at scales of the order of 1 GeV receives contributions not only from its tree-level Higgs Yukawa interaction, which we will denote as m_u^H, but also from instanton contributions, m_u^{inst}. Hence in general,

$$m_u(1\ \text{GeV}) = m_u^{\text{inst}} + m_u^H. \tag{4}$$

In the case of QCD, the instanton contributions to the up quark mass depend on the masses of the other quarks in the theory. In a general basis, the light quark contributions are given by [4,6],

$$m_u^{\text{inst}} = \frac{\exp(-i\theta)(m_d^H m_s^H)^*}{\Lambda}, \tag{5}$$

where Λ is a scale which characterizes the size of these contributions, and m_d^H and m_s^H are the tree-level Higgs induced down and strange quark masses.

In the canonical basis, m_u^{inst} is a real contribution, implying that $\mathcal{I}[m_u^H] = \mathcal{I}[m_u]$. The physical CP-violating phase, Eq. (2), then reads

$$\theta_{\text{QCD}}(1\ \text{GeV}) \simeq \sin\theta_{\text{QCD}}^{\text{H}} \frac{|m_u^H|}{|m_u|}(1\ \text{GeV}), \tag{6}$$

where $\theta_{\text{QCD}}^{\text{H}} = \arg[m_u^H]$ and we have assumed that $|m_u^H| \ll |m_u^{\text{inst}}|$. This expression is consistent with $\theta_{\text{QCD}} = \arg[m_u]$. The small imaginary components of the instanton induced strange and down quarks masses, proportional to $m_d^H(m_u^H)^*/\Lambda$ and $m_s^H(m_u^H)^*/\Lambda$ respectively, induce a subdominant effect that becomes negligible in the one instanton approximation. From Eqs. (3) and (6), we conclude that a strong CP problem solution would be provided if values of

$$|m_u^H(1\ \text{GeV})| \sin\theta_{\text{QCD}}^{\text{H}} \lesssim 6.5\times 10^{-4}\ \text{eV}, \tag{7}$$

could be dynamically generated while maintaining consistency with the observed up quark mass.

Interestingly, it has been argued that the m_u^{inst} contribution induced by the standard QCD interactions may be as large as a few MeV [4–10], and hence be able to explain the observed up quark mass value. This is allegedly in tension with the lattice determination of the up quark mass at scales where the instanton contribution should be negligible, namely, $m_u^{\overline{\text{MS}}}(2\ \text{GeV}) = 2.1$ MeV [12,13]. Alternatively, it has been postulated that similar contributions may come from instantons in some ultraviolet gauge extensions [14]. A possible ultraviolet configuration is that each generation is sensitive to a different $SU(3)$ gauge interaction, with a gauge group $SU(3)^3 = SU(3)_1\times SU(3)_2\times SU(3)_3$ that is spontaneously broken to the diagonal group $SU(3)$ at a scale of the order of hundreds of TeV. Assuming that the tree-level Higgs induced strange and bottom quark masses are equal to zero, the instanton contributions in each sector would be responsible for bringing these masses to their observed values (via contributions proportional to the charm and top quark masses, respectively). In such a case, the low energy CP-violating interactions will be governed by expressions similar to Eq. (6), with the only difference that m_u^{inst} will include the ultraviolet instanton contributions. Hence, any tension of the up quark mass with lattice determinations would be eliminated.

Irrespective of its origin, provided m_u^{inst} can lead to the observed up quark mass at scales of the order of 1 GeV, m_u^H can be arbitrarily small. One would naturally expect the Higgs induced CP-violating phase $\theta_{\text{QCD}}^{\text{H}}$ to be larger than $\sim 10^{-2}$. In such a case, from Eq. (7), $|m_u^H|$ would be of the order of or smaller than 4×10^{-2} eV. This implies values of $|m_u^H|$ similar in magnitude to the small neutrino masses [1]. Our proposed solution of the strong CP problem is associated with the dynamical generation of precisely such small values of $|m_u^H|$.

II. A DIRAC SEESAW MODEL

We present a model which realizes a seesaw mechanism for the dynamical generation of m_u^H and of small Dirac neutrino masses [15–18] (see Refs. [19–21] for an alternative formulation relating θ_{QCD} to the neutrino masses.). To realize this idea, we assume the presence of a $\mathcal{Z}_4$ discrete symmetry that forbids the direct coupling of the up quark and neutrinos to the Higgs field. While the right-handed up quark and the right-handed neutrinos have charge 1, all other SM fields carry zero charge under this symmetry. In addition, we introduce a heavy scalar doublet Φ with $\mathcal{Z}_4$ charge 1 and hypercharge 1/2, and a singlet S of charge -1 under the $\mathcal{Z}_4$ symmetry, such that ν_R Majorana masses are forbidden.

The Lagrangian for the Yukawa interactions of the up quark and the neutrinos is given by:

$$\mathcal{L} = Y_\nu \bar{\ell}_L \tilde{\Phi}\nu_R + Y_u \bar{q}_L \tilde{\Phi} u_R + \text{H.c.}, \tag{8}$$

where $\tilde{\Phi} = i\sigma_2\Phi^*$ carries charge -1 under $\mathcal{Z}_4$. All the other SM fermions have standard Yukawa interactions with the Higgs doublet H, which are not shown here. The potential involving the heavy scalar fields relevant for our discussion reads

$$V = m_\Phi^2 \Phi^\dagger\Phi + (\rho S H^\dagger \Phi + \text{H.c.}) + \lambda_{\Phi,1}\Phi^\dagger\Phi H^\dagger H + \lambda_{\Phi,2}\Phi^\dagger\Phi|S|^2 + \lambda_{S,1}|S|^4 + (\lambda_{S,2}S^4 + \text{H.c}) + \cdots. \tag{9}$$

Here the term $\lambda_{S,2}S^4$ is allowed by the discrete symmetry but would not be allowed by a global Peccei-Quinn $U(1)$ symmetry [22]. Hence, there is no axionlike Goldstone boson [23,24]. It is easy to prove that to ensure a vacuum expectation value (vev) in the real direction and stability of the potential, we need $\lambda_{S,2} < 0$ and $(\lambda_{S,1} + \lambda_{S,2}) > 0$. We will assume that $m_\Phi \gg m_S, m_H$, so that one can integrate it out by the equation of motion $\Phi \simeq -\frac{1}{m_\Phi^2}\rho S^* H$, where we have assumed that ρ is real. The effective Yukawa interactions for the up quark and neutrinos, represented in Fig. 1, are given by:

$$\mathcal{L}_{\mathrm{eff}} \simeq -Y_\nu \frac{\rho}{m_\Phi^2} S\bar{\ell}_L \tilde{H}\nu_R - Y_u \frac{\rho}{m_\Phi^2} S\bar{q}_L \tilde{H} u_R + \mathrm{H.c.} \tag{10}$$

After the singlet and the neutral component of the SM Higgs field acquire vevs, $\langle S\rangle = v_S/\sqrt{2}$, $\langle H^0\rangle = v/\sqrt{2}$, the Dirac masses of the up quark and neutrinos read:

$$m_\nu \sim Y_\nu \frac{\rho v_S v}{2m_\Phi^2}, \qquad m_u^H \sim Y_u \frac{\rho v_S v}{2m_\Phi^2}. \tag{11}$$

If v_S is the order of the EW scale $v = 246$ GeV, and ρ is of order m_Φ, one gets an effective seesaw suppression of the up quark and neutrino masses $|m_u|, |m_\nu| \sim v^2/m_\Phi$. Hence, as assumed, one sees the need for large values of m_Φ,

$$m_\Phi \simeq 6\times 10^{12}\ \mathrm{GeV}\left(\frac{Y_\nu}{0.1}\right)\left(\frac{\rho}{0.1 m_\Phi}\right)\left(\frac{v_S}{v}\right)\left(\frac{0.05\ \mathrm{eV}}{m_\nu}\right), \tag{12}$$

to get an observational consistent mass for the heavier neutrino, where we have assumed Y_ν to be real. Given the bound on $\mathcal{I}[m_u^H]$ in Eq. (7), one obtains the bound on the up quark Yukawa at the scale of m_Z:

$$|Y_u(m_Z)| < 0.05 Y_\nu \left(\frac{0.1}{\sin\theta_{\mathrm{QCD}}^H}\right), \tag{13}$$

where we have taken into account the running of the up quark mass due to QCD interactions $|m_u(m_Z)|/|m_u(1\ \mathrm{GeV})| \sim 0.4$. For the $SU(3)^3$ instanton configuration [14], assuming a similar Higgs and flavor structure to generate the proper CKM mixing angles, the required vanishing tree-level Yukawa coupling of strange and bottom quarks to the H and Φ Higgs fields may be simply ensured by assigning s_R and b_R the same $\mathcal{Z}_4$ charge as the one for u_R.

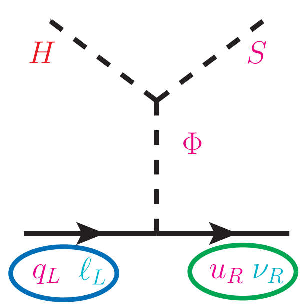

FIG. 1. A diagrammatic representation of the Dirac seesaw mechanism for the up quark and neutrino masses.

As pointed out in Ref. [14], after the generation of the proper CKM mixing angles, one obtains flavor violating effects that demand the $SU(3)^3$ breaking scale to be larger than a few 100's of TeV. Moreover, the corresponding off-diagonal Yukawa couplings lead to instanton corrections to the imaginary component of the quark masses. These corrections modify the value of θ_{QCD} at the $SU(3)^3$ instanton scale, and, if they are evaluated at the scale $\Lambda_i \sim \mathcal{O}$ (few 100 TeV), they are of the order of 10^{-11}, and hence an order of magnitude smaller than the current bound on θ_{QCD}. One potential problem of the formulation presented is that the hierarchy between m_Φ and the electroweak scale is not stable in the presence of $\lambda_{\Phi,1}, \lambda_{\Phi,2}, \rho$. To address this problem, in the next section we present a supersymmetric extension of this scenario.

III. SUPERSYMMETRIC EXTENSION

In the case of supersymmetry (SUSY), we assume the presence of a $\mathcal{Z}_3$ symmetry, and charges $\Phi_u: -1$, $\Phi_d: 1$, $u_R^c: 1$, $\nu_R^c: 1$, $S: -1$. All other fields are neutral under the discrete $\mathcal{Z}_3$ symmetry. The corresponding superpotential is given by

$$\begin{aligned} W = {} & -Y_\nu^* L\Phi_u \nu_R^c - Y_u^* Q\Phi_u u_R^c - y_e^* L H_d e_R^c - y_d^* Q H_d d_R^c \\ & + \mu H_u H_d + m_\Phi \Phi_u \Phi_d + \lambda H_u \Phi_d S + \frac{\kappa}{3} S^3. \end{aligned} \tag{14}$$

Right-handed neutrino Majorana masses generated by the singlet S are forbidden by the holomorphicity of the superpotential. In addition, we have imposed R-parity which forbids terms like $(\nu_R^c)^3$. The SUSY invariant potential for the Higgs fields reads:

$$\begin{aligned} V_{\mathrm{SUSY}} = {} & |\mu|^2|H_u|^2 + |\mu H_d + \lambda\Phi_d S|^2 + |m_\Phi \Phi_u + \lambda H_u S|^2 \\ & + |m_\Phi|^2|\Phi_d|^2 + |\kappa S^2 + \lambda H_u \Phi_d|^2, \end{aligned} \tag{15}$$

where μ is the conventional μ term. In the following, we will take $m_\Phi \gg \mu \sim$ TeV. After SUSY-breaking, we have the following soft-breaking interaction terms:

$$\begin{aligned} V_{\mathrm{soft}} = {} & m_{\Phi_u}^2 |\Phi_u|^2 + m_{\Phi_d}^2 |\Phi_d|^2 + m_S^2 S^* S + \cdots \\ & + (\lambda a_\lambda H_u \Phi_d S + b_\lambda \Phi_u^\dagger H_u S + a_\kappa S^3 + \cdots + \mathrm{H.c.}), \end{aligned}$$

where we have omitted terms not relevant for our discussion. Note that in the limit of $\kappa = a_\kappa = 0$ there would be a $U(1)$ global symmetry which would make the singlet CP-odd scalar massless. More specifically, a global $U(1)$ Peccei-Quinn symmetry [22] is broken by the $S^2(H_u\Phi_d)^*$

and S^3 terms, which are proportional to $\kappa\lambda$ or a_κ. Since Φ_d acquires a very small vev, the mass of the CP-odd scalar predominantly originates from a negative a_κ.

By assuming that the SUSY-invariant mass m_Φ is much larger than all the soft masses, one can integrate out the heavy scalar fields $\Phi_{u,d}$: $\Phi_u \sim -\frac{\lambda}{m_\Phi} H_u S$, $\Phi_d \sim -\frac{1}{|m_\Phi|^2}(\mu\lambda^* H_d S^* + \lambda a_\lambda \tilde{H}_u S^*)$, and obtain the low-energy effective Lagrangian for the Yukawa interactions in the Dirac fermion notation:

$$\mathcal{L}^y_{\rm eff} = -Y_\nu \frac{\lambda^* S^*}{m^*_\Phi} \bar{\ell}_L \tilde{H}_u \nu_R - Y_u \frac{\lambda^* S^*}{m^*_\Phi} \bar{q}_L \tilde{H}_u u_R + \cdots \tag{16}$$

from which we can read off the neutrino and up quark masses:

$$m_\nu \sim \left(Y_\nu \frac{\lambda^* v^*_S}{\sqrt{2} m^*_\Phi}\right)\frac{v_u}{\sqrt{2}}, \quad m^H_u \sim \left(Y_u \frac{\lambda^* v^*_S}{\sqrt{2} m^*_\Phi}\right)\frac{v_u}{\sqrt{2}}, \tag{17}$$

where we assumed v_u to be real, and the expression between parenthesis on the left- and right-hand side of Eq. (17) defines the low energy Yukawa couplings y_ν and y_u, respectively. The necessary values of $|m_\Phi|$ and $|Y_{u,\nu}|$ can be extracted from Eqs. (12) and (13) after replacing $|\rho/m_\Phi|$ by $|\lambda|$, and v by v_u. For the $SU(3)^3$ case, as in the non-SUSY scenario, the required vanishing tree-level Yukawa coupling of strange and bottom quarks to the H_d and Φ_d Higgs fields may be simply ensured by assigning s^c_R and b^c_R the same $\mathcal{Z}_3$ charge as the one for u^c_R.

Generically supersymmetric extensions lead to additional contributions to the electric dipole moments. In the absence of flavor violation in the scalar mass parameters, they are proportional to the phases $\Phi^{if}_A = \arg[M_i A^*_f]$, $\Phi_B = \arg[M^*_{\tilde{g}} \mu^*(B\mu)]$, where $y_f A_f$ are the scalar trilinear couplings, $M_{\tilde{g}}$ is the mass of the gluino, M_i the gaugino masses, and $B\mu$ the $H_u H_d$ bilinear mass parameter. The one-loop SUSY corrections to the nEDM, controlled by Φ^{if}_A and Φ_B, may be parametrized as [25]

$$d^{\rm SUSY}_n \simeq 2\left(\frac{100\ {\rm GeV}}{m_{\rm SUSY}}\right)^2 \Phi^{if}_{A,B} 10^{-23}\ {\rm e\,cm}, \tag{18}$$

where $m_{\rm SUSY}$ denotes a common soft supersymmetry breaking mass scale. There are also relevant contributions at the two-loop level, that lead to a somewhat more complicated dependence on the SUSY and Higgs spectrum, as well as to possible cancellations between one and two loop contributions [26,27]. These contributions will be suppressed well below the current bounds without fine-tuning the CP-violating phases if the masses of the gluino, squark and heavy Higgs boson masses are larger than 10 TeV.

An important consideration is that after integrating out the SUSY particles, the low energy Yukawa couplings are affected by nondecoupling and CP violating contributions, proportional to $\Phi^{if}_{A,B}$ [28]. Hence, if the instanton scale is above the supersymmetric particle mass scale, the proposed solution to the strong CP problem will be invalidated by the appearance of new phases in the Yukawa couplings. In addition, in the presence of colored Majorana gluinos, the instanton contribution to the up-quark Yukawa coupling will be suppressed by an additional factor $M^3_{\tilde{g}_1}/\Lambda^3_1$ compared with the non-SUSY case. Therefore, we must demand the supersymmetry particle masses to be above the instanton scale. Moreover, for the up-quark Yukawa coupling to remain small after supersymmetry particle corrections, we should demand that the supersymmetry breaking mechanism preserves the $\mathcal{Z}_3$ symmetry.

If the instanton effects come from regular $SU(3)$ interactions, the supersymmetry particle and heavy Higgs boson masses are naturally much larger than the QCD instanton scale. However, for the $SU(3)^3$ scenario our proposed solution of the strong CP problem is only viable if heavy Higgs and colored SUSY particle masses are of the order of or larger than the characteristic $SU(3)_i$ instanton scales Λ_i. As discussed above, assuming a similar flavor and Higgs structure for the generation of the CKM mixing angles, Λ_i must be $\sim\mathcal{O}$ (few 100 TeV) [14]. This suppresses all the CP-violating and flavour-changing effects induced by the heavy Higgs and SUSY particles in Eq. (18). On the other hand, it introduces a little hierarchy problem, which will not be addressed further in this work.

IV. NEUTRON ELECTRIC DIPOLE MOMENT

A notable outcome of our framework is that a nonzero nEDM is induced by the nonvanishing value of $\theta_{\rm QCD}$. We can calculate the contribution to the nEDM from current algebra [29,30]; the result reads:

$$\frac{d_n}{e} \sim \frac{g_{\pi NN}\bar{g}_{\pi NN}}{4\pi^2 M_N} \ln\frac{M_N}{m_\pi}, \tag{19}$$

where $M_N \sim 940$ MeV is the nucleon mass, $m_\pi \sim 140$ MeV is the pion mass and $|g_{\pi NN}| \sim 13.4$ is the usual CP conserving pion-nucleon coupling. The CP violating coupling $\bar{g}_{\pi NN}$ is given by:

$$\bar{g}_{\pi NN} \sim \theta_{\rm QCD} \frac{m_{\rm eff}}{F_\pi}, \tag{20}$$

with $m_{\rm eff} \equiv |m_u m_d m_s|/(|m_u m_d| + |m_u m_s| + |m_d m_s|)$, $F_\pi \sim 93$ MeV is the pion decay constant, and the masses of the quarks and the strong CP phase are evaluated at the scale $Q \sim 1$ GeV. Using the currently determined values for $|m_{u,d,s}|$ [1], this result becomes consistent with the calculation of Refs. [31,32] by using the QCD sum rules,

$$d_n \sim \theta_{\rm QCD} \times (2.4 \pm 0.7) \times 10^{-16}\ {\rm e\,cm}, \tag{21}$$

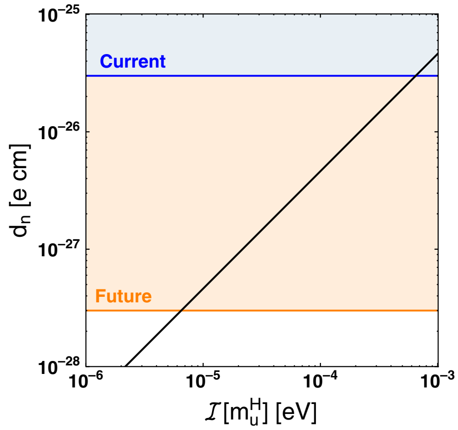

FIG. 2. The neutron EDM as a function of the imaginary part of the up quark mass. We have also shown the current 90% C.L. bound [2] and prospective sensitivity from the future neutron EDM measurements [34–40].

and also with a recent lattice calculation [33]. In the canonical basis, where $\theta_{\rm QCD} \sim \mathcal{I}[m_u^H]/|m_u|$, and normalizing the value of the nEDM to the present bound [2], we obtain

$$d_n = \frac{\mathcal{I}[m_u^H]}{(6.5 \pm 2.0) \times 10^{-4}\ \mathrm{eV}} \times 3.0 \times 10^{-26}\ \mathrm{e\,cm}. \qquad (22)$$

Figure 2 shows the nEDM as a function of the imaginary part of the up quark mass. While the current measurement leads to a bound on $\mathcal{I}[m_u^H] < (6.5 \pm 2.0) \times 10^{-4}$ eV, future nEDM experiments [34–40] will be able to improve the present sensitivity by two orders of magnitude $\sim 3 \times 10^{-28}$ e cm [37], and hence will be able to probe $\mathcal{I}[m_u^H]$ up to about 6×10^{-6} eV. Note that even for a phase $\theta^H_{\rm QCD} \simeq 10^{-2}$, the values of $|m_u^H|$ that will be probed are much smaller than the ones that naturally arise from the relation of m_u^H and the neutrino masses. Hence, it is natural to expect a measurement of the nEDM by the next generation of experiments within this framework.

Finally, we should comment on additional contributions to the nEDM. As discussed above, they can either come from sources of *CP* violation associated with the new physics introduced to stabilize the scale hierarchies, Eq. (18), or, in the $SU(3)^3$ scenario [14], from instanton contributions to the imaginary part of the quark masses, arising after the generation of off-diagonal Yukawa couplings. While the former are suppressed by the square of the new particle masses, the latter are about an order of magnitude smaller than the current bound on the nEDM. Although these corrections may potentially break the correlation between the nEDM and the neutrino masses, barring an unlikely strong cancellation, they reinforce the expectation of a measurement of the nEDM in the near future. Moreover, as pointed out in Ref. [41], if $\theta_{\rm QCD}$ is the dominant source of *CP* violation in the strong sector, there will be strong correlations between the nEDM and the measurable EDMs of light nuclei and atoms, which can be used to distinguish different contributions to the nEDM.

V. CONCLUSION

In this article, we have explored the Dirac seesaw mechanism to provide a common origin of two small scales $\mathcal{I}[m_u]$ and m_ν, which are related to two different physical phenomena: strong *CP*-violation and neutrino oscillations. We propose a dynamical generation of the small nonzero imaginary part of the up quark mass, naturally solving the strong *CP* problem. Similar to the case of the related massless up-quark solution to the strong *CP* problem, the real part of the up quark mass obtains additive renormalization from instanton effects above the chiral symmetry breaking scale ∼1 GeV. However, irrespective of the detailed origin of this additive instanton contribution, the novel part of our construction is that the neutrino mass scale is strongly correlated with the static nonzero value of the neutron EDM, with predicted values that are expected to be probed by the next generation of experiments.

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