

ν scalar in the early Universe and $(g-2)_\mu$ Jia Liu^{1,2,*}, Navin McGinnis^{3,†}, Carlos E. M. Wagner^{4,5,6,‡} and Xiao-Ping Wang^{7,8,§}¹*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*²*Center for High Energy Physics, Peking University, Beijing 100871, China*³*TRIUMF, 4004 Westbrook Mall, Vancouver, British Columbia V6T 2A3, Canada*⁴*High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*⁵*Physics Department and Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA*⁶*Kavli Institute for Cosmological Physics, University of Chicago, Chicago, Illinois 60637, USA*⁷*School of Physics, Beihang University, Beijing 100083, China*⁸*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China*

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We investigate a concrete scenario of a light scalar with a mass around 1 MeV which can be connected to the origin of neutrino masses and simultaneously survive current bounds on relativistic degrees of freedom in the early universe. A feeble coupling to the Standard Model neutrinos can relax stringent bounds on the decays to photons inferred from the measured value of N_{eff} . Interestingly, we find that such a scalar whose diphoton coupling is generated by a tree-level coupling to the muon of similar strength as that of the Standard Model Higgs boson can simultaneously explain the longstanding discrepancy in the measured value of the muon magnetic moment. We present a possible ultraviolet (UV) completion of this scenario providing a link between new physics in the early universe and the generation of neutrino masses.

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I. INTRODUCTION

The theoretical and experimental establishment of the Standard Model (SM) of particle physics [1] and general relativity ushered in the so-called Λ CDM model, which provides an excellent description of nucleosynthesis and cosmic microwave background data [2]. The existence of particles beyond the SM can modify the expansion rate of the universe, impacting both observables [3,4]. Recently, several detailed analyses have explored the constraints on new light particles in the early universe [5–10] where big bang nucleosynthesis (BBN) and Planck data imply strong constraints on (pseudo) scalar particles with an MeV-scale mass and coupling to photons, mainly due to the corresponding contributions to N_{eff} . However, as emphasized, for instance, in Refs. [6,7,11], these bounds are not robust and could be modified either due to the details regarding the generation of

the diphoton coupling or those of the possible modifications to the neutrino or, in general, light particle sector.

New light scalars, as well as new, sterile neutrinos may also be connected to the mechanism generating neutrino masses, which remains an open question in particle physics. Additionally, the current observation of a deviation of the anomalous magnetic moment [12] from its theoretically predicted value in the SM [13–33] (see a recent review [34]) demands an explanation and could be connected to the presence of new scalars coupled to the muon [35–42].

Motivated by these observations, in this paper we present a low-energy model of a real scalar particle with mass $\mathcal{O}(1)$ MeV which, at tree level, couples to the muon and SM neutrinos, and whose UV completion provides the source of neutrino masses. Interestingly, we find that a scalar with mass in this range can simultaneously generate the necessary correction to explain the recently measured value of the muon magnetic moment [12],

$$\Delta a_\mu = (2.51 \pm 0.59) \times 10^{-9}, \quad (1)$$

and, for an appropriate coupling to neutrinos, satisfy all cosmological, astrophysical and laboratory constraints.

II. EFFECTIVE MODEL

We concentrate on the properties of a real singlet scalar which at low energies couples exclusively to the SM neutrinos and the muon

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$$\mathcal{L}_{\text{eff}} \supset -g_\mu \phi \bar{\mu} \mu - \left(\frac{g_{\nu_a}}{2} \phi \nu_a \cdot \nu_a + \text{H.c.} \right), \quad (2)$$

where, ν_a are the active neutrino flavors and, we have used Dirac notation for the first term versus Weyl notation for the second. As anticipated, the ϕ coupling to neutrinos provides the source of the neutrino Majorana masses. The muon mass, instead, comes mostly from Higgs induced, ϕ independent terms within the UV completion of the model.

The coupling to the muon will induce the coupling to photons at one-loop [43]

$$\mathcal{L}_{\text{eff}} \supset -\frac{g_{\gamma\gamma}}{4} \phi F_{\mu\nu} F^{\mu\nu}, \quad (3)$$

with $g_{\gamma\gamma} \approx -(2\alpha g_\mu)/(3\pi m_\mu)$ for $m_\phi \ll m_\mu$ [43].

The scalar decay widths to photons and neutrinos are then given by

$$\Gamma_{\gamma\gamma} = \frac{g_{\gamma\gamma}^2 m_\phi^3}{64\pi}, \quad \Gamma_{\nu\nu} = \sum_a \frac{g_{\nu_a}^2 m_\phi}{32\pi}, \quad (4)$$

respectively. It is clear that the same parameters, g_μ and m_ϕ , which fix the coupling to photons, simultaneously determine the contribution of ϕ to the muon anomalous magnetic moment which at one-loop is given by

$$\Delta a_\mu = \frac{g_\mu^2}{8\pi^2} \int_0^1 dz \frac{(1-z)^2(1+z)}{(1-z)^2 + z(m_\phi/m_\mu)^2}. \quad (5)$$

Thus, the experimentally determined range for Δa_μ , Eq. (1), leads directly to a prediction for $g_{\gamma\gamma}$. For $m_\phi \lesssim 1$ MeV, this is achieved at the $2\text{-}\sigma$ level if $g_\mu \simeq m_\mu/v$, similar to the Higgs coupling to muons in the SM [41].

In Fig. 1, we show the allowed $g_{\gamma\gamma}$ for a scalar with mass $\sim \mathcal{O}(0.1 - 100)$ MeV where constraints are presented from both laboratory and astrophysical searches, as presented

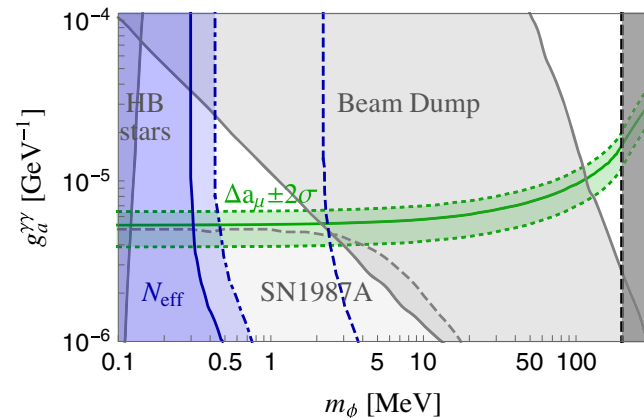


FIG. 1. Allowed region of $g_{\gamma\gamma}$ [GeV $^{-1}$] vs m_ϕ [MeV] with constraints taken from [7]. The green shaded region shows the values of $g_{\gamma\gamma}$ consistent with Δa_μ within 2σ . The dashed line shows the bound based on the standard cosmology. Dot-dashed and solid lines show the effects of extrarelativistic d.o.f. and a neutrino chemical potential, respectively.

in [7]. Let us emphasize that, although Ref. [7] concentrates in the case of pseudoscalars, as these constraints are derived with respect to the lifetime and mass of these particles, they will be equally stringent for scalar particles. While the constraints from beam dumps and HB stars (shown in the gray shaded regions with solid-line boundaries) should be considered robust, we note that the bound from SN1987A (gray shaded region with a dashed-line boundary) has recently received critical skepticism in the literature and could very well be much weaker [44–46]. On the other hand, it was emphasized recently that this region of parameters could be excluded from supernova (SN) bounds resulting from muon-specific SN models [42]. We have not included their explosion energy bounds as the addition of the neutrino coupling will modify the trajectory of ϕ within the neutrino sphere itself and the muonic density profile in current SN models do not include the possible new physics effects.

The green shaded region in Fig. 1 shows the predicted values of $g_{\gamma\gamma}$ when g_μ is fixed to satisfy Δa_μ within 2σ . The bounds from N_{eff} constrain masses up to $m_\phi \simeq 2$ MeV assuming the standard cosmological evolution. The blue shaded regions show the weaker constraints associated with a modification of the cosmological evolution by the addition of extra relativistic degrees of freedom (d.o.f.) or a neutrino chemical potential. Taking into account the possible modification of N_{eff} leaves an open region for m_ϕ in 1–2 MeV, which has been referred to as *the cosmological triangle*. In contrast to [7], the scenario we consider leads to a similar variation of N_{eff} without any additional new physical degree of freedom by considering additional decays of ϕ to SM neutrinos.

Finally, it should be emphasized that there exists a window of masses $m_\phi \simeq \mathcal{O}(100)$ MeV which is currently not subject to either SN or N_{eff} constraints. Rather, this region is bounded from below by beam dump experiments, and above by $2m_\mu$ where the threshold for pair production of muons sets an upper bound where recent constraints from BABAR and Belle-II become relevant [47,48].

III. N_{eff} CALCULATION

In the standard neutrino cosmology picture, the effective number of neutrino species can be parameterized by the ratio between neutrino and photon densities after neutrino decoupling and electron-positron annihilation in the thermal background [1]

$$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8} N_{\text{eff}} \left(\frac{T_\nu}{T_\gamma} \right)_{\text{SM}}^4, \quad (6)$$

where the SM value, $(T_\nu/T_\gamma)_{\text{SM}} = (4/11)^{1/3}$, is determined by requiring that the neutrino and electromagnetic plasma separately conserve comoving entropy.

In our model, additional production mechanisms, e.g. $\mu^+ \mu^- \leftrightarrow \gamma \phi$, $\gamma \ell^\pm \leftrightarrow \phi \ell^\pm$ and $\gamma\gamma, \nu\nu \leftrightarrow \phi$, where ℓ^\pm stand for the charged leptons, make ϕ fully thermalized before BBN. After neutrino decoupling, ϕ will inject entropy

separately into the neutrino and electromagnetic plasmas, whose entropies themselves will evolve independently. N_{eff} may be determined by the requirement of conservation of comoving entropy alone [49]. Thus, rather than resort to a calculation of the scalar abundance from neutrino interactions, as in [50–54], we may evaluate the modified neutrino decoupling temperature T_D , which will in general be different than $T_D^{\text{SM}} = 2.3$ MeV in the SM.

The presence of new particles in thermal equilibrium with photons, electrons or neutrinos can alter N_{eff} through modifications of T_ν/T_γ [49]. For particles with mass $\mathcal{O}(1)$ MeV it follows that [49,55–57]

$$N_{\text{eff}} = N_\nu \left(\frac{4}{11}\right)^{-4/3} \left(\frac{T_\nu}{T_\gamma}\right)^4. \quad (7)$$

The requirement of conservation of comoving entropies after neutrino decoupling further determines the ratio T_ν/T_γ [57]

$$\frac{T_\nu}{T_\gamma} = \left(\frac{(g_s^*)_\nu}{(g_s^*)_\gamma}\right)_{T_D} \left(\frac{g_s^*}{g_s^*}\right)^{1/3}, \quad (8)$$

where $(g_s^*)_\nu$ and $(g_s^*)_\gamma$ denote the effective number of relativistic degrees of freedom contributing to the neutrino and photon entropy densities, respectively.

The neutrino decoupling temperature, T_D , is modified for scalar masses $\mathcal{O}(1)$ MeV via the presence of the s-channel resonant exchange $\nu\nu \leftrightarrow \gamma\gamma$ which delays neutrino decoupling, leading to lower T_D . The t-channel exchange $\gamma\nu \leftrightarrow \gamma\nu$ has a negligible contribution because of small couplings. The Boltzmann equation for the process $\nu\nu \leftrightarrow \phi^{(*)} \leftrightarrow \gamma\gamma$ is given by

$$\frac{dn_\nu}{dt} + 3Hn_\nu = \langle\sigma v\rangle_{\text{res}}(n_{\nu,\text{eq}}^2 - n_\nu^2), \quad (9)$$

where the thermal averaged, resonant cross section is given by

$$\langle\sigma v\rangle_{\text{res}} = \frac{16\pi^2}{9\xi(3)^2} \Gamma_\phi \text{BR}_{\gamma\gamma} \text{BR}_{\nu\nu} \frac{x^2}{T^3} K_1(x). \quad (10)$$

Γ_ϕ is the total width of ϕ , the Riemann zeta function $\xi(3) = 1.202$, $x \equiv m_\phi/T$, $K_1(x)$ is the modified Bessel function of the second kind, $\text{BR}_{\nu\nu} \equiv \text{BR}(\phi \rightarrow \nu\nu)$ and $\text{BR}_{\gamma\gamma} \equiv 1 - \text{BR}_{\nu\nu}$. For $T \lesssim m_\phi$, we compare the exchange rate $R \equiv n_\nu^{\text{eq}} \langle\sigma v\rangle_{\text{res}}$ with the Hubble expansion rate and find the neutrino decoupling temperature

$$x_D \approx \log\left(\frac{1.67 m_{\text{PL}} \Gamma_\phi}{g_*^{1/2} m_\phi^2} \text{BR}_{\gamma\gamma} \text{BR}_{\nu\nu}\right) + \frac{7}{2} \log(x_D). \quad (11)$$

In the left panel of Fig. 2, we show the comparison of R to the Hubble rate for values of the neutrino branching ratio $\text{BR}_{\nu\nu} \simeq \mathcal{O}(10^{-2}) - \mathcal{O}(10^{-3})$. A larger branching ratio to neutrinos leads to lower values of T_D . In general, a lower T_D than that in the SM will lead to entropy sharing between the photon and neutrino baths, in particular after e^+e^-

annihilation if $T_D \lesssim m_e$. The resulting slower reheating of the photon bath from e^+e^- annihilation will cause the baryon number density to increase leading to strong constraints from BBN, $T_D \gtrsim 0.3$ MeV [6].

At temperatures below T_D , the decay modes of ϕ will inject entropy into both the electromagnetic and neutrino sectors, while the annihilation of e^+/e^- will further provide injection to the photon entropy. Requiring that the entropy in each sector is conserved, we can then evaluate the ratio T_ν/T_γ for a given branching ratio $\text{BR}_{\nu\nu}$ and temperature, $T \leq T_D$,

$$\frac{T_\nu}{T_\gamma} \approx \left(\frac{2 + \frac{7}{2} F\left(\frac{m_e}{T_\gamma}\right) + \frac{7}{8} \text{BR}_{\gamma\gamma} F\left(\frac{m_\phi}{T_\gamma}\right)}{2 + \frac{7}{2} F\left(\frac{m_e}{T_D}\right) + \frac{7}{8} \text{BR}_{\gamma\gamma} F\left(\frac{m_\phi}{T_D}\right)} \cdot \frac{N_\nu + \frac{1}{2} \text{BR}_{\nu\nu} F\left(\frac{m_\phi}{T_D}\right)}{N_\nu + \frac{1}{2} \text{BR}_{\nu\nu} F\left(\frac{m_\phi}{T_\gamma}\right)}\right)^{1/3},$$

$$F(y) = \frac{30}{7\pi^4} \int_y^\infty dx \frac{(4x^2 - y^2) \sqrt{x^2 - y^2}}{e^x \pm 1}, \quad (12)$$

where the plus (minus) signs correspond to fermion (boson) particles.

In the right panel of Fig. 2, we show the evolution of T_ν/T_γ with respect to T_γ . Considering values of g_μ which are consistent with Δa_μ , the decay width to photons leads to a decay rate higher than the Hubble rate at $T = \mathcal{O}(1)$ MeV, heating the photon plasma. This is compensated for by lowering T_D when $\text{BR}_{\nu\nu} \neq 0$ driving T_ν/T_γ back to values consistent with ΛCDM [58], $N_{\text{eff}} = 2.9913 \pm 0.1690$. We see in Fig. 2 that this can be achieved with a small branching ratio to neutrinos $\text{BR}_{\nu\nu} \simeq \mathcal{O}(10^{-2}) - \mathcal{O}(10^{-3})$, corresponding to a coupling to neutrinos $g_{\nu a} \simeq \text{few} \times 10^{-10}$ if we assume approximately equal couplings to all neutrino flavors, for $m_\phi = 1-2$ MeV. When $\text{BR}_{\nu\nu} \ll 1$, the entropy injection of the scalar into the neutrino sector is negligible and its impact is implicit through the value of T_D .

Finally, we define N_{eff} at $T < T_D$ as the effective number of SM neutrinos that would lead to the same energy density contribution, including the electron and scalar ϕ contribution. However, for $T_\gamma \ll \text{MeV}$, the contributions from e and ϕ are exponentially suppressed and one can simply use Eq. (12) in Eq. (7). For $\text{BR}_{\nu\nu} \ll 1$ and $T_\gamma \ll m_e, m_\phi$, we have

$$N_{\text{eff}} \approx N_\nu \left(\frac{11/4}{1 + \frac{7}{4} F\left(\frac{m_e}{T_D}\right) + \frac{7}{16} F\left(\frac{m_\phi}{T_D}\right)}\right)^{4/3}. \quad (13)$$

We emphasize that, for a given scalar mass, T_D is determined by Eq. (11). In Fig. 3, we show the range of acceptable values of T_D and m_ϕ varying $\text{BR}_{\nu\nu}$ to achieve N_{eff} within $2\text{-}\sigma$ of the measured value. We see that for a scalar in the cosmological triangle, values of T_D between 0.4 MeV and 0.8 MeV are needed to be consistent with the Planck measurements. Note that this range is safely above the lower bound from BBN, $T_D \gtrsim 0.3$ MeV [6].

IV. UV MODEL

In this section, we present a UV completion to the effective model. We start by introducing a SM scalar

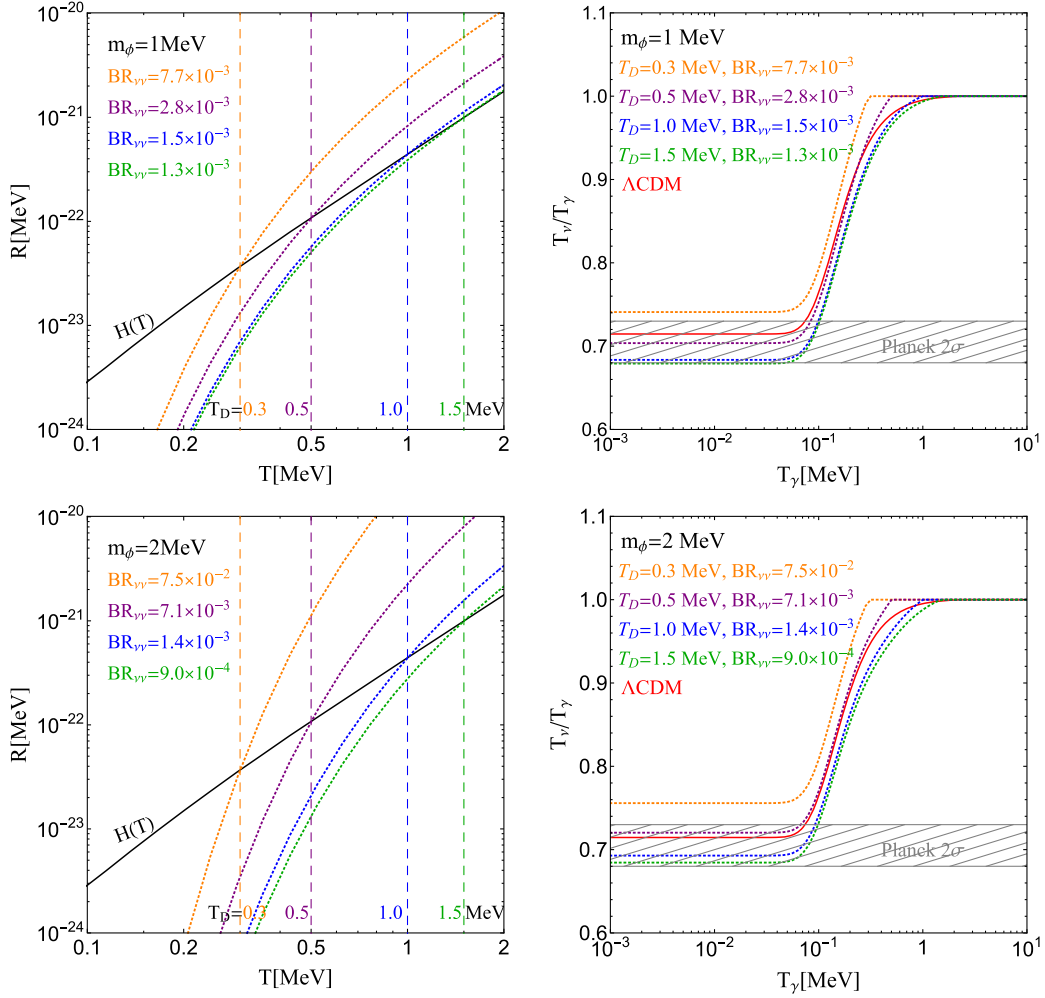


FIG. 2. Left panel: comparison of the rate, R , of the process $\nu\nu \leftrightarrow \phi^{(*)} \leftrightarrow \gamma\gamma$ with that of Hubble, $H(T)$. Examples of T_D , determined via vertical dashed lines where $R \simeq H(T)$, are shown for different $BR_{\nu\nu}$. Right panel: evolution of T_ν/T_γ as a function of the photon temperature for different $BR_{\nu\nu}$. The red solid line and hatched gray region shows the evolution in Λ CDM and 2σ Planck limits, respectively. The top (bottom) panels are for $m_\phi = 1$ (2) MeV respectively.

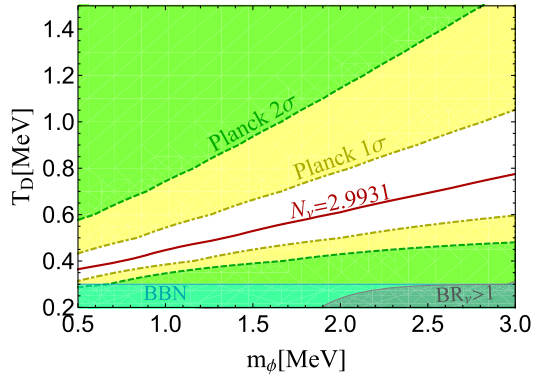


FIG. 3. Limits on T_D and m_ϕ from the Planck measurement of N_{eff} [58]. The color shaded regions are excluded at 1σ and 2σ respectively. The lower bound $T_D > 0.3$ MeV [6] is plotted as shaded cyan.

singlet ϕ and doublet H' with equal charges under a parity Z_2 symmetry and two singlet Weyl fermions (N, N'), only one of which is charged under Z_2 . In addition, we assume that the right-handed muon field is also charged under Z_2 . The particle content and charges relevant for our discussion are summarized in Table I and the Lagrangian reads,

$$\begin{aligned} \mathcal{L}_{\text{UV}} = & y_\mu \bar{L}_\mu H' \mu_R + y_{N,i} (L_i \cdot H) N + y_{N',j} (L_j \cdot H') N' \\ & + \lambda_N N N' \phi + \mu_\phi H'^{\dagger} H \phi + \sum_{f=N,N'} \frac{m_f}{2} \bar{f} f + \text{H.c.} \end{aligned} \quad (14)$$

The Z_2 symmetry may be softly broken by mass terms in the scalar potential, to allow proper electroweak symmetry breaking with the required vacuum expectation value (vev) of the new doublet H' .

The effective coupling of ϕ to muons is given by

$$g_\mu = y_\mu \frac{v\mu_\phi}{\sqrt{2}m_{H'}^2}. \quad (15)$$

TABLE I. Particle content for the UV completion of the effective model. Added to the SM are a singlet (doublet) scalar ϕ (H') and two singlet fermions N and N' . A Z_2 parity symmetry is added. μ_R is the only particle in the SM which carries a Z_2 charge.

Field	μ_R	ϕ	H	H'	N	N'
$SU(2)_L$	1	1	2	2	1	1
$U(1)_Y$	-1	0	1/2	1/2	0	0
Z_2	-1	-1	+1	-1	+1	-1

Hence, taking for instance $y_\mu \simeq 10^{-1}$ and $\mu_\phi \simeq 10$ GeV, g_μ is in the same order of the SM Higgs muon coupling for $m_{H'} \simeq 700$ GeV. In order to obtain the proper muon mass, the vev of H' should be $v'/\sqrt{2} \simeq 1$ GeV. Such a scalar, which couples mostly to muons and neutrinos will not be detectable at current colliders, but could easily be seen at a future high energy muon collider with center of mass energies larger than $m_{H'}$.

We assume that the Dirac mass term $\lambda_N v_\phi$, coupling the sterile neutrinos N and N' , is much larger than the Majorana mass terms m_N and $m_{N'}$. In this limit, the mass eigenvalues for the three light active neutrinos and two sterile neutrinos N and N' are approximately given by

$$\tilde{m}_{\nu_1} = 0, \quad (16)$$

$$\tilde{m}_{\nu_{2,3}} = \frac{vv'}{\sqrt{2}\lambda_N v_\phi} (|\vec{y}_N||\vec{y}_{N'}| \mp \vec{y}_N \cdot \vec{y}_{N'}) \sim 0.1 \text{ eV}, \quad (17)$$

$$\tilde{m}_{N,N'} \approx \frac{\lambda_N v_\phi}{\sqrt{2}} \pm \frac{m_N + m_{N'}}{2} + \mathcal{O}\left(\frac{1}{\lambda_N v_\phi}\right), \quad (18)$$

where \vec{y}_N and $\vec{y}_{N'}$ are the vectors of Yukawa couplings in flavor space for the three generations of active neutrinos. One massless active neutrino exists because the mass matrix is rank four. In a block-diagonal intermediate basis (see Supplemental Materials [59]), the active neutrino mass matrix is then given by

$$\mathcal{M}_{ij} = (y_{N,i}y_{N',j} + y_{N,j}y_{N',i}) \frac{\sqrt{2}vv'}{2\lambda_N v_\phi}. \quad (19)$$

After diagonalizing this matrix, one obtains the eigenvalue masses of neutrinos and the mixing angles, which follow the usual seesaw expressions. Since the active neutrino mass is proportional to $1/v_\phi$, the $\phi - \nu$ coupling is, apart from an irrelevant sign,

$$g_{\nu_a} = \frac{\tilde{m}_{\nu_a}}{v_\phi}. \quad (20)$$

In our setup the neutrino spectrum must be hierarchical, with $m_{\nu_1} \simeq 0$, $m_{\nu_2} \simeq 8.7 \times 10^{-3}$ eV and $m_{\nu_3} \simeq 0.059$ eV, to be consistent with the current neutrino oscillation data and the present neutrino mass bounds from laboratory data and cosmology [1,58]. The scalar will dominantly decay into the heaviest neutrino, ν_3 , with a coupling

$$g_{\nu_3} \simeq 2.5 \times 10^{-10}, \quad (21)$$

to be consistent with values of $\text{BR}_{\nu\nu}$ needed for N_{eff} , leading to $v_\phi \simeq 240$ MeV. Thus, in Eq. (19)

$$\frac{y_{N,i}y_{N',j}v'}{\lambda_N} \simeq 10^{-13} \text{ GeV}, \quad (22)$$

implying that $y_{N,i}, y_{N',i}$ must be small. Taking $\lambda_N = \mathcal{O}(1)$ allows us to identify the neutrino masses with v_ϕ . In such a case, the sterile neutrinos are significantly heavier than 1 MeV. Further, if in addition $y_{N,i}, y_{N',i}$ are of order of a few 10^{-7} the mixing between the sterile and active neutrinos will be considerable so that the sterile neutrinos will decay fast enough to avoid constraints from BBN. Finally, the scalar ϕ can couple to electrons via mixing with the SM Higgs, but the coupling can be easily suppressed down to values of order $\sim 10^{-10}$, thus not affecting this scenario.

V. CONCLUSION

We have presented a model of a real scalar, with a mass in the MeV range, that leads to the active neutrino mass generation and to an explanation of the observed muon anomalous magnetic moment. Neutrino masses are generated through the addition of sterile neutrinos, with masses in the hundreds of MeV range. This model is interesting phenomenologically, while also providing an example of the avoidance of the cosmological constraints on MeV scalar particles coupled to photons. The relevant modification of the cosmological history is provided by the coupling to neutrinos, which should be in a range leading to a branching ratio to neutrinos of order (0.1–1)% in order to be consistent with observed number of relativistic degrees of freedom. It is worth mentioning that as the scalar we consider has a long lifetime its presence at colliders will appear in the form of missing energy. If the scalar potential contains a coupling to the SM Higgs boson through a quartic coupling this will lead to a contribution to the invisible decay of the Higgs, and therefore a complementary collider signal of the scenario we have presented.

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