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A Comprehensive Examination of Public and Private Risk Measures for Distinct Commercial Real Estate Asset Classes: Insights from Mixed-Effects Regressions

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1 Abstract

This paper employs mixed-effects regression models (MRMs) to examine the multifaceted components of public and private commercial real estate (CRE) risk. It does so through two exemplar datasets: public Real Estate Investment Trusts (REITs) and private Commercial Mortgage Backed Security (CMBS) loans. For public REITs, traditional MRMs investigate the effects of (1) dividends, (2) market capitalization, (3) acquisitions, and (4) percentage of unsecured debt on the capitalization rates of varying REIT sectors. For private CMBS loans, logistic MRMs assess the effects of (1) occupancy-rates, (2) debt-rates, and (3) principal on loan default and delinquency rates. For both datasets, these risk metrics are implemented as both fixed and random-effects in order to investigate their net impacts and degree of property-type heterogeneity; public market analysis also examines within and between-subject effects.

Public results show that the effects of two risk metrics, dividends and market capitalization, exhibited disparate impacts on the capitalization rates of different REIT sectors. For example, Empirical Bayes random-effect estimates show that dividends significantly influence hotel capitalization rates, while their significance is not observed in the retail sector. Across sectors, most metrics' within-subject fixed-effect component yielded statistically significant results, except for the percentage of unsecured debt. Finally, the significance of acquisition's between-subject effect offers evidence of a potential correlation between higher mean acquisition rates and lower capitalization rates.

Similarly, private market analysis also revealed substantial property-type and region variability in CMBS defaults. However, logistic MRM results indicate that solely the risk metrics' fixed-effect components are significant in predicting CMBS default rates, indicating consistent impacts across sectors. As such, incorporating random-effects for private risk metrics proved unnecessary, yet property-type clustering remained essential. Overall, this paper underscores the nuanced nature of CRE risk assessment, arguing for increased consideration of property-specific dynamics.

2 Acknowledgements

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3 Introduction

In light of the rapidly transforming 2024 CRE landscape, the importance of correctly identifying risk components has become increasingly important. Notably, the CRE market has witnessed a substantial decline in property values, with some investors even anticipating a potential downturn of 40% [21].

To examine the sources of these trends, this paper draws inspiration from Joseph Pagliari – Clinical Professor of Real Estate at the University of Chicago – and his work *Some Thoughts on Real Estate Pricing* [29]. Here, Pagliari demonstrates how CRE's true risk-free rate is the Treasury Inflation-Protected Securities (TIPS) rate, which can then be scaled by the appropriate industry or property-specific risk premium to derive population or property-level capitalization rates (cap rates); cap rates are a signal of a RE investor's required return given expected risk. From this theoretical basis, this paper will attempt to directly examine and quantify the variability and net effects of distinct public and private CRE risk metrics, hypothesizing that different metrics will exert highly variable impacts on distinct property types.

This conjecture is grounded in the fact that CRE is a broad asset class, encompassing multitudes of property types with diverse characteristics. Therefore, it stands to reason that certain risk metrics should prove more effective than others in evaluating risks of specific property types. To investigate this, both public and private markets will be examined, with a shared emphasis on the core research question: *which risk metrics are most indicative of CRE risk, and how do the effects of these metrics vary across different property types?*

Recognizing the correlated nature of CRE data, mixed-effect regression models are implemented to analyze both markets. As such, all model iterations are clustered by property-type. First, traditional mixed-effect regression models (MRMs) are utilized to model the cap rates of twelve distinct REIT sectors. These models, encompassing both fixed and random effects, will aim to quantify which risk metrics have the most significant effects and variances in predicting the implied cap rates. Further, this public-sector analysis also explores each metric's within- and between-subject effects to determine if significance stems from temporal variations or property-inherent disparities in mean metric levels. This paper will then pivot to the private sector, examining risk implications on a 25-year randomized sample of ∼18,000 CMBS loans. Here, iterations of mixed-effect logistic regressions – with 1 indicating a loan default – will again analyze means and variances of CRE risk metrics. Accordingly, private risk is now modeled by default rates instead of cap rates.

4 Literature Review

This paper draws inspiration from both CRE's academic and professional realms. Overall and in addition to presenting its direct results, it aims to advocate for a better recognition of CRE diversity. Moreover, it endeavors to demonstrate the immense value in utilizing MRMs for analyzing CRE. Finally, it hopes to set a precedent, inspiring the further application of mixed models in professional fields.

A fundamental assumption of this paper is that cap and default rates are functions of risk. While the latter is clear, the first is also heavily supported. To begin, a compelling starting point is Jug. and Winkler's 1995 paper *The Capitalization Rate of Commercial Properties and Market Returns* [19] where modern financial theories are extended to cap rate determinations. Notably, the authors explain that cap rates are a function of both debt and equity, the former explained by the weighted average cost of capital (WACC) and the later by the capital-asset pricing model (CAPM)[19].

This paper is primarily concerned with the equity component, evident in the fact that the public market analysis does not incorporate any debt metrics. However, the private analysis does utilize debt rate and principal as risk components. First, debt rate can still be examined as an equity metric, as higher debt rates will simply increase RE investor's required return. Principal value is indeed a debt metric; however, it is insignificant over all model iterations. Therefore, only the equity portion, the CAPM, of Jug and Winkler's work is examined.

$$R_i = R_f + \beta_i(R_m - R_f) \tag{1}$$

Aligning with the assumptions of this paper, the CAPM also incorporates a risk-free rate in its establishment of asset pricing. Further, the equation's second term $\beta_i(R_m - R_f)$ is highly analogous to the risk perspective of this paper. To mathematically visualize this, the required return of real estate is shown [29].

$$r_{re} = r_{\text{risk-free rate}} + \phi \tag{2}$$

ϕ, representing CRE's risk premium, is highly similar to the CAPM's risk premium $\beta_i(R_m - R_f)$. While CAPM's premium designates systematic risk by way of it's β_i term - a security's volatility relative to the market's volatility [26] - Pagliari's risk-metric ϕ is slightly less defined. However, this is logical considering the wide diversity within CRE; ϕ should be expected to vary significantly for different properties, the central focus of this paper.

Before proceeding deeper, it's crucial to emphasize that although cap rates are calculated as net operating income divided by purchase price, interpretations should not solely be based on this definition. Rather, cap rates should be looked at as a measure of real estate's required return. As such, a greater cap rate, alike bond yield, should signal increased risk.

While drawing inspiration from numerous prior studies, this paper notably diverges by utilizing the TIPs rate instead of US treasury rates as CRE's underlying risk-free rate. For example, Jack Corgel's paper *The Effect of an Interest Rate Increase on Hotel Capitalization Rates* [10] argued that a 100 basis point (bp) increase in treasury rates should be coupled by a 28 bp cap rate expansion, thereby advocating for treasury rates as the appropriate risk-free rate. This assertion finds further support in Connor and Liang's research, which identifies a cap rate expansion of 40 bps per a 100 bp treasury rate increase [8]. However, this paper challenges this fixed-rate perspectives by asserting that the treasury inflation-protected securities (TIPs) rate is a more reliable indicator for CRE's risk free rate [28].

To demonstrate this superior relationship, a quick mathematical exposition is performed. As noted, real estate's required return consists of a risk-free rate and risk premium [29].

$$\frac{CF_0}{P_o} = r_{re} = \phi + r_{bonds} \tag{3}$$

Moving forward, this can be converted to year one cash flows by factoring in an inflation rate and inflation pass-through rate.

$$\frac{CF_1}{P_o} = r_{re} \cdot (1 + p) + p \cdot (1 - \lambda) \tag{4}$$

where:

$$p \text{ is the year-1 inflation rate.} \tag{5}$$

$$\lambda \text{ is the inflation pass-through rate} \tag{6}$$

Consequently, when inflation is assumed to flow perfectly into CRE, the second term will fall out.

$$\frac{CF_1}{P_o} = r_{re} \cdot (1 + p) \tag{7}$$

This paper and its respective models indeed assume that inflation uniformly passes through to CRE pricing. Accordingly, if inflation pass-through is highly variable, future model iterations may introduce omitted variable bias. However, given that various prevailing literature's, including the World Bank, assert that pass-through is fairly uniform, it seems that this assumption is justified [27]. This paper adopts that stance and invites future researchers to further validate it. Finally, cash flows are converted to net operating income to arrive at cap rates.

$$\frac{NOI_1}{P_o} = \frac{r_{re} \cdot (1+p)}{b} \tag{8}$$

Evidently, cap rates encompass the risk-premium, risk-free rate, inflation, and maintenance costs. First, this paper fundamentally accounts for maintenance differences, denoted b, by way of the inclusion of between-subject effects. Next, inflation, denoted p, should not have different effects on distinct properties. This uniformity follows by nature of TIPS, which functions as a real-yielding variable rate security, moving in tandem with inflation [29]. Put differently, any inflation variance should be explained by TIPS variance.

Finally, the last term in cap rate determination is r_{re}, a function of the risk-free rate and the risk premium ϕ (2). This paper will begin by statistically establishing TIPS as the optimal risk-free rate and then examine the components of the risk premium ϕ and its variance across property types.

Before proceeding to the empirical analysis, these assumptions are also backed by Paul Mouchakkaa's *Frozen on the Rates: Impact of Interest Rates on Capitalization Rates*. Here, Mouchakkaa argues that the true correlation between interest rates and cap rates is relatively loose [25]. This weakness stems from the fact that treasury rates are a fixed-rate nominal-yielding security while real estate is a variable-rate real-yielding security [29].

While a comprehensive literature review is included at the end of this paper, this piece's core aim is to build upon previous studies by utilizing innovative computational approaches [13] to analyze CRE risk. Ultimately, this paper hopes to equip both practitioners and researchers to make more secure investment decisions.

5 Public Analysis

5.1 Public Datasets

The study utilizes public data from three sources: Nareit, Yahoo Finance, and FRED. These sources span from January 2003 to January 2021, during which annual time-series data was collected (Figures 9-10).

Twelve distinct property types are examined: office, industrial, retail shopping centers, retail regional malls, retail free standing, residential apartments, residential manufactured, diversified, health care, lodging resorts, self storage, and data centers.

The Federal Reserve Economic Database was utilized to source TIPs and treasury rate trends [14]. NAREIT's T-Tracker data was used to source data on ∼150 publicly traded REITs (clustered to form population-level data) where the following annual risk metrics were gathered: capitalization rates, dividends, unsecured debt, and acquisitions [5]. Finally, Yahoo Finance's stock database was used to extract various metrics for each of the REITs previously chosen [4]; all metrics were again clustered by property-type.

5.2 Continuous MRMs

In this section, a quick exposition of the utility of mixed-effect regression models (MRMs) is underwent [13]. All explanations stem from Donald Hedeker and Robert Gibbons' textbook *Longitudinal Data Analysis*.

The value of MRMs lies in their incorporation of random effects (REs) to accommodate issues associated with non-independent datasets. In context, MRMs acknowledge the fact that observations within property types are correlated, implementing distinct random effects to account for this. Further, MRMs allow for datasets with missing temporal data; this inclusion is particularly important here as some property types lack much early-2000s data. For example, as data centers are relatively modern, limited data from their early years can be obtained[13].

This paper's MRMs are comprised of level-1 and level-2 models. Here, the level-1 model characterizes the cap rates of 12 property types (indexed by j) over years (i), while the level-2 model determines how separate risk components influence various property types. Random effects, denoted $u_{\#j}$, are included in the level-2 model, while fixed-effects are embedded in the level-1. The complete model is obtained by substituting the level-2 model into the level-1.

The level-1 error term e_{ij} is assumed to be conditionally independently distributed as $N(0, \sigma^2)$, contingent upon each respective property type. Level-2 REs are distributed by $N(0, \sigma_u^2)$. As the expected value of any RE is zero, solely its variance component is important. A risk metric with a high RE variance signals that it exhibits distinct effects on unique properties. Further, large RE variance also signals that clustering by property type is important. Mathematically, MRM's can be expressed in matrix notation[13].

$$Y_i = X_i \cdot B + Z_i \cdot u_i + e_i \tag{9}$$

- Y_i is the $n_i \times 1$ cap rate matrix. (10)
- X_i is the $n_i \times p$ design matrix for the risk metrics' FEs. (11)
- B is a $p \times 1$ vector that contains the FE coefficients. (12)
- Z_i is the $n_i \times r$ design matrix for the risk metrics' REs. (13)
- v_i is a $r \times 1$ vector of each property-type's REs. (14)
- e_i is an $n_i \times 1$ error matrix. (15)

The error terms are distributed normally.

$$e_i \sim N(0, \sigma^2 I_n), \quad u_i \sim N(0, \Sigma_u) \tag{16}$$

From here, the cap rates y_{ij} and the risk metric REs u_{ij} can be shown to follow a joint multivariate distribution with a variance-covariance matrix given by:

$$\begin{bmatrix} Z_i' \Sigma_u Z_i' + \sigma^2 I_n & \Sigma_u Z_i' \\ (\Sigma_u Z_i')^T & \Sigma_u \end{bmatrix} \tag{17}$$

Note: The expectation of y_i can simply be found by taking the expectation of Y_i. Then, as the expectations of u_i and e_i are both 0, the expectation of y_i is $X_i \cdot B$. As stated the expectation of the REs, u_i, is 0.

Empirical Bayes Methods are used to the derive specific random-effects (u_i), while Marginal Likelihood is used to estimate the variances and coefficients, i.e., σ^2, Σ_u, and β. However, these methods are fairly complex, leading to the use of iterative algorithms like Newton-Raphson or Fisher-scoring methods [13]. These algorithms repeatedly test model parameters until they calculate values that minimize the difference between observed and predicted cap rates.

A quick explanation of Empirical Bayes role in random-effect derivation can be shown. First, the random effect u_i is conditioned on the response variable, yielding its posterior distribution (a conditional distribution that is maximized by use of Bayesian prior probabilities). From here, each RE is found[13].

$$\hat{u}_i = [Z_i'(\sigma^2 I_{n_i})^{-1} Z_i + \Sigma_u^{-1}]^{-1} Z_i'(\sigma^2 I_{n_i})^{-1}(y_i - X_i\beta) \tag{18}$$

Such that the posterior covariance matrix and the repeated measures variance-covariance matrix are:

$$\Sigma_{u|y_i} = [Z_i'(\sigma^2 I_{n_i})^{-1} Z_i + \Sigma_u^{-1}]^{-1} \tag{19}$$

$$V(y_i) = Z_i \Sigma_u Z_i' + \sigma^2 I_{n_i} \tag{20}$$

To interpret these REs, the Intraclass Correlation Coefficient (ICC) is a particularly strong tool. In this approach, the variance associated with each RE is normalized by dividing it by the total variance, the sum of the within and between-group variances.

$$ICC = \frac{V_b}{V_b + V_w} \tag{21}$$

This manipulation reveals the proportion of overall variation attributed to variance between property types. Put differently, this value indicates the extent of heterogeneity among groups for a specific RE. Thus, High ICCs signal the RE uniquely impacts the cap rates of distinct property types.

Pivoting to model comparisons, while likelihood ratio tests (LRTs) are employed with respect to ordinary least square regressions, they should not be used in MRMs with different random-effects. To explain, LRTs of MRMs with different REs leads to divergent RE bounding, making them too conservative. As such, this paper chose to undergo model comparisons by using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). A mathematical discussion of the two can be found in section 5.10.

5.3 TIPS Rationale

Building off TIPs theoretical justification, three MRMs models, with each utilizing either the TIPS rate, the five-year treasury rate, or the 10-year treasury rate as a fixed-effect (FE) are now surveyed. As these rates are the most accepted US risk-free rates, other examinations prove unnecessary.

The first iterations solely include random-intercept terms. Looking towards the level-1 and level-2 models, the u_{0j} term illustrates that only between-property variance in 2003 cap rates is permitted; the fixed-effects, temporal rate changes, have equal influence across properties. Notably, the slopes of each property type, changes in cap rates over time, are forced to be constant.

Level-1:

$$\text{CR}_{ij} = b_{0j} + b_1 \times \text{Occ}_{ij} + b_2 \times \text{TIPS}_{ij} + \epsilon_{ij} \tag{22}$$

Level-2:

$$b_{0j} = \beta_0 + u_{0j} \tag{23}$$

$$b_{1j} = \beta_1 \tag{24}$$

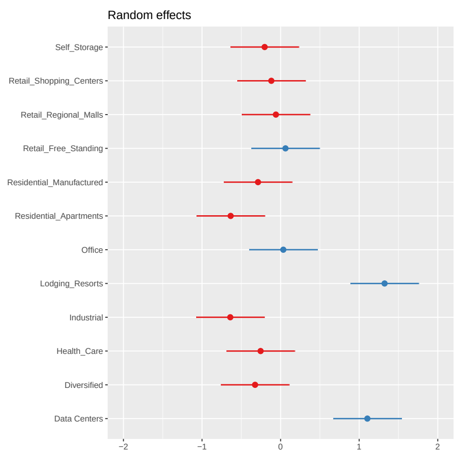
Table 1: Random-Intercept with 3 Rates

Param.	Est.	Std.Err	T-Val.
(Intercept)	7.58	0.28	27.48
Occasion	-0.12	0.02	-7.31
TIPS	0.16	0.08	1.94
(Intercept)	8.27	0.39	21.17
Occasion	-0.16	0.02	-8.92
Treasury-5	-0.11	0.08	-1.31
(Intercept)	8.03	0.66	12.21
Occasion	-0.15	0.03	-5.68
Treasury-10	-0.03	0.13	-0.26

Models: TIPS, Treasury 5, Treasury 10

Table 1 indicates that, in this iteration, TIPS does not achieve significance at the established 5% threshold of the paper. Nevertheless, among the three variables assessed, TIPS emerges as the most significant and influential.

Figure 1: Intercept Heterogeneity



Examining the effects of the random-intercept term (Figure 1), it is clear that distinct properties have strikingly different intercepts. Significantly large u_{0j} terms are observed for Residential-Apartments, Lodging, and Data centers, indicating notable deviations from the grand intercept value β_0. Better, put, these property types exhibited divergent cap rates in 2003 (with TIPs held at 0), suggesting inherent difference in their average cap. High intercept variance was also observed across other starting years!

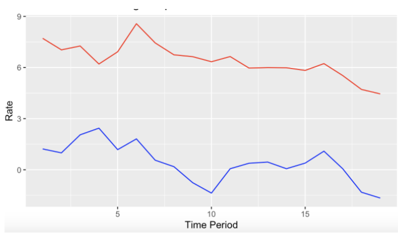
Consequently, the level-2 model is now updated to incorporate a random-effect, denoted u_{1j}, for occasion. This will now allow property-specific temporal variations.

Table 2: TIPS Random-Int and Slope

TIPs	Est.	Std.Err	T-Val.
(Intercept)	7.58	0.39	19.56
Occasion	-0.12	0.02	-4.97
FiveTIPS	0.16	0.08	2.07

Table 2 shows that TIPS has now achieved statistical significance. It's coefficient of 0.16, indicates that a unit increase in TIPS should lead to a 0.16 cap rate expansion. However, the observed positive correlation, though consistent with theoretical models, is notably low, suggesting a negative relationship between TIPS and risk premiums. To fully agree with the theoretical models, a unit-increase in TIPS must be correlated with a risk premium contraction that, in turn, leads to a 0.84 cap rate expansion.

Figure 2: Rate Trends



TIPS in Blue, Cap in Red

This diminished correlation is backed by historical trends (Figure 2). To explain, TIPS rose from 2003 to 2007, primarily declined from 2007 to 2013, experienced an upward trend until approximately 2019, and then sharply dropped with the onset of Covid-19. However, Cap Rates clearly did not follow this same trend, evident in their 2004-2006 divergence. These results combine to underscore the theoretical imperfections. Despite these challenges, this section presents compelling evidence supporting the use of the TIPs rate over Treasuries as the true risk-free rate in commercial real estate.

5.4 Public Risk Metrics

This paper chose to utilize four measures of REIT risk: Dividends, Acquisitions, Market Capitalization, and Percent of Unsecured Debt. First, dividends play a crucial role in risk, as REITs must distribute at least 90% of earnings to shareholders. This paper examines variations of these dividend distributions. Next, acquisitions indicate REIT expansion or contraction, with an intuitive assumption that expansionary practices signal growth. Market capitalization has a diverse relationship with risk as a greater market cap makes shares more expensive, but, also serve as a signal of increased market presence. Put differently, this variable is of particular importance as cap rates are expected to be inversely related to REIT valuations while also positively correlated with stock prices. Unsecured Debt, which refers to the proportion of REIT debt not secured by specific properties, is indicative of REIT liquidity. Greater liquidity should, in turn, lead to increased operational flexibility. Consequently, it seems that higher levels of unsecured debt should lead to lower cap rates.

To ensure proper scaling, grand-mean centered was utilized for all variables. This process functions by subtracting the mean of each risk metric (across properties) from each property's metric at each occasion. As such, both within-subject and between-subject effects are still permissible. Further, market cap and dividends were log-transformed to account for potential skews and stabilize large variances.

5.5 Risk Metrics as FEs

First, the risk-metrics were only included as fixed-effects. Consequently, this initial model assumes that each metric has a uniform effect across properties; random-intercepts and trends were allowed.

Level-1:

$$\begin{aligned} \mathrm{CR}_{ij} = b_{0j} + b_{1j} \times \mathrm{Occ}_{ij} + b_2 \times \mathrm{TIPS}_{ij} \\ + b_3 \times \mathrm{Dvd}_{ij} + b_4 \times \mathrm{Acq}_{ij} \\ + b_5 \times \mathrm{MCap}_{ij} + b_6 \times \mathrm{Unsd}_{ij} + e_{ij} \end{aligned} \tag{25}$$

Level-2:

$$b_{0j} = \beta_0 + u_{0j} \tag{26}$$

$$b_{1j} = \beta_1 + u_{1j} \tag{27}$$

$$b_{2-6} = \beta_{2-6} \tag{28}$$

For context, a variance-covariance matrix of the intercept and occasion REs is shown.

$$\Sigma_u = \begin{bmatrix} \sigma^2_{u_0} & \sigma_{u01} \\ \sigma_{u01} & \sigma^2_{u_1} \end{bmatrix} \tag{29}$$

Table 3: Risk-Metric as FEs

Fixed Effects	Est.	Std. Err	T-Val.
(Intercept)	6.81	0.56	10.79
Occasion	-0.04	0.03	-1.44
Five	0.04	0.07	0.56
Dividends	0.75	0.13	5.68
Acquisitions	-0.09	0.03	-2.73
MarketCap	-1.59	0.18	-9.09
Unsecured	0.01	0.01	-1.33

The FEs of dividends, acquisitions, and market cap emerge as statistically significant. Although the intercept term is too, this value is non-interpretable as it is the average cap rate when all metrics are zero.

Before proceeding, it is crucial to note that any dividend or market cap interpretation should account for their logarithmic nature. For MRMs that regress a continuous dependent on a logarithmically transformed independent, a 1% change in the independent is associated with a 0.01b dependent shift. Thus, for the dividend's β of 0.75, the expected cap rate shift associated with a 1% increase in dividends can be calculated as $0.75 \div 100$; a 1% increase in dividends is correlated with a 0.0075% cap rate expansion. Interpretations of market cap follow a congruent methodology.

Since acquisitions are on a billion-dollar scale, their FE coefficient signals that a billion-dollar acquisition is expected to compress cap rates by 0.09%. Finally, percent of unsecured debt was found to be statistically insignificant, indicating market participants don't see it as a strong indicator of risk.

A key result lies in the relationship between dividends and market caps. Beginning with market caps, definitional interpretations imply that they should be inversely correlated with cap rates. However, by financial theory, increased market caps indicate increased risk (rising share prices). As the latter does not agree with the negative coefficient, the former is accepted. Interpreting dividends definitionally, increased dividends should expand cap rates. However, by efficient market hypothesis, increased dividends should lower risk, prompting investors to purchase more stock to restore risk equilibrium and, consequently, balance cap rates. The former is also accepted as β_3 is positive.

Next, the two have a negative correlation of -0.428, an increase in one should decrease the other. This is of particular importance as investors usually view REIT's high dividend return as their most attractive quality. It follows that increased dividends should lead to investors buying more of the REIT's shares; however, by these variables negative coefficient, the opposite is true. Moreover, market cap has a fixed-effect coefficient twice as influential as dividends. As such, it seems that investors prioritize market caps or, similarly, property valuations over dividend (cash flow) returns.

The cause of this preference is left to future researchers. However, to posit some thoughts, the average REIT investor may believe:

1. Long-term property appreciation is preferable to short-term income.

2. Increased dividend yields lead to increased return volatility.

3. High dividends indicate low growth potential; concurring with the -0.152 acquisitions/dividends correlation!

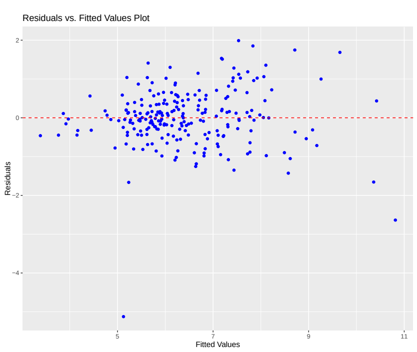
Table 4: ICCs for Int & trend

Groups	RE	Var	Std.Dev.
Property	(Inter)	3.101843	1.76121
	Occasion	0.003101	0.05569
Residual		0.628481	0.79277

Turning to REs, the ICC for the intercept term is first calculated, $\frac{3.10}{3.10+0.628}$, as 0.83152. 83.152% cap rate variance is attributed to between properties differences. Put differently, a lot of variability in starting cap rates can be explained by market's attributing different underling (mean) cap rates to distinct properties. Most notably, these findings persisted across different starting years, suggesting that certain property types have inherently different cap rates and risk perceptions! This further underscores the importance of clustering by property type.

On the other hand, the random-occasion (year) effect only led to an ICC of 0.5%, indicating that property temporal cap rate changes are fairly consistent; occasion's RE may not be essential.

Figure 3: Residual Values



Assessing the model's fit involves comparing predicted versus observed values, with Figure 3 indicating relatively poor data alignment. Moreover, it is clear that residual variance increases over time; the model is less predictive of cap rates in later time periods than early ones. As such, the assumption of a homoskedastic error variance is violated.

5.6 Within & Between-Effects

The previous model assumed equivalence of the within and between-subject effects, suggesting average metric levels and changes in metrics are equally predictive of cap rates. These two will now be delineated. This is demonstrated by incorporating the dividends metric into previous models; all other metrics follow an identical process.

$$\text{Dividends}_{ij} = (\text{Dvds}_{ij} - \bar{\text{Dvds}}_j) + \bar{\text{Dvds}}_i \tag{30}$$

The first term represents the within-subject effect while the second represents the between-subject effect. As such, the within-subject effect refers to the deviation of each property type's metric from the group mean of that metric while the between-subject effect relates to the average level of that metric across time, the group-mean. These effects were indistinguishable in the previous model. *Note again that the actual model included the within and between-effect decomposition for each metric.*

Level-1 Example:

$$\begin{aligned} \Delta\text{CapRate}_{ij} = b_{0j} &+ b_1 \times \text{Occ}_{ij} \\ &+ b_2 \times \text{TIPS}_{ij} \\ &+ b_3 \times (\text{Dvds}_{ij} - \bar{\text{Dvds}}_j) \\ &+ \epsilon_{ij} \end{aligned} \tag{31}$$

Level-2 Example:

$$b_{0j} = \gamma_{00} + \gamma_{01} \times \overline{\text{Dvds}}_j + u_{0j} \tag{32}$$
$$b_{1j} = \gamma_{10} + u_{1j} \tag{33}$$

This paper hypothesized that both within and between-subject effects would be significant. Between-subject intuition stems from the fact that as unique properties have different underlying cap rates, their associated risk metric levels should have naturally different means. Additionally, the validity of risk metrics as measures of REIT risk affirms the significance of within-subject effects. For instance, hotels typically exhibit higher cap rates as they have large capital expenditure / maintenance demands. Moreover, they are extremely sensitive to macroeconomic trends and, in turn, should react more strongly than others to dividend cuts.

Running this model, the previous hypothesis of between-subject significance is rejected. The model identified significant within-subject effects for dividends, acquisitions, and market cap, indicating that changes in these metrics significantly impact cap rates across properties. However, average risk metric levels showed no statistically significant correlation to cap rates.

Nevertheless, acquisition's between-subject effect yielded a T-value of -1.795. While this is not statistically significant, it suggests a potential correlation between average acquisition rates and cap rates that warrants further investigation. Finally, it is noted that both RE variances are lower. This reduction is expected as the decomposition enhances the model's ability to discern between temporal and cross-sectional dynamics of varying CRE sectors, reducing RE variances. Future models, will incorporate REs to examine how changes in these metrics affect different property types.

5.7 Effect Homogeneity?

Comparing the composed vs. decomposed models shows the latter has lower AIC & BIC (section 4.9) values. Thus, the decomposed model achieves a better balance between complexity and goodness-of-fit compared to the latter. This is even more insightful as the decomposed model had twice as many variables as the first! *Thus, the assumption of homogeneous between and within-subject effect is rejected.*

5.8 Metrics as REs

This section addresses a key question: to what extent do distinct risk metrics uniquely impact different property types? Therefore, each risk metric now includes a random-effect term, enabling it to have unique effects on each property's cap rates; fixed-effect interpretations remain unchanged.

Level-1:

$$\begin{aligned}\text{CapRate}_{ij} = b_{0j} &+ b_{1j} \times \text{Occ}_{ij} + b_2 \times \text{TIPS}_{ij}\\ &+ b_{3j} \times \text{Dividends}_{ij}\\ &+ b_{4j} \times \text{Unsecured}_{ij}\\ &+ b_{5j} \times \text{Acquisitions}_{ij}\\ &+ b_{6j} \times \text{MarCap}_{ij} + e_{ij}\end{aligned} \tag{34}$$

Level-2:

$$b_{0j} = \beta_0 + u_{0j} \tag{35}$$

$$b_{1j} = \beta_1 + u_{1j} \tag{36}$$

$$b_{2j} = \beta_2 + u_{2j} \tag{37}$$

$$b_{3j} = \beta_3 + u_{3j} \tag{38}$$

$$b_{4j} = \beta_4 + u_{4j} \tag{39}$$

$$b_{5j} = \beta_5 + u_{5j} \tag{40}$$

$$b_{6j} = \beta_6 + u_{6j} \tag{41}$$

Table 5: Updated Fixed-Effects

Fixed Effects	Est.	Std. Err	T-Val.
(Intercept)	6.303	0.640	9.847
Occasion	-0.015	0.036	-0.431
TIPS	0.054	0.057	0.960
Dividends	0.300	0.233	1.286
Acquisitions	-0.075	0.035	-2.163
MarketCap	-1.522	0.227	-6.713
Unsecured	0.706	0.573	1.232

Unlike the first models, the dividends FE is no longer significant. This reduction occurs since once a RE is incorporated for dividends, their effects are now better captured by this RE component. In other words, the significance of the dividends variable shifts to specific property types where it has a notable impact. As such, dividend changes likely influence distinct property type's differently!

As expected, dividends have a notably high ICC value, signaling that 51.6% percent of the total variance in dividend effects on Cap Rates can be found in how they affect property types

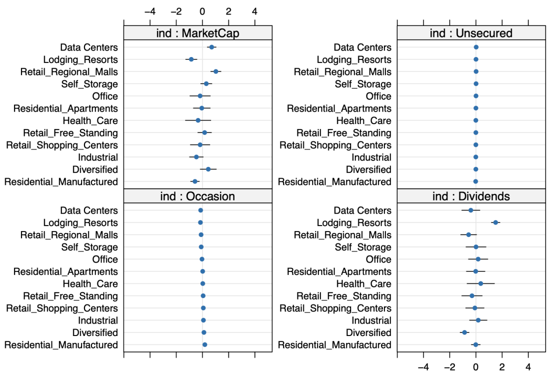
Table 6: Risk-Metrics' REs

Random-Effect	Var	Std.De	ICC
Intercept	2.279	1.509	0.839
Occasion	0.012	0.109	0.027
Acquisitions	0.005	0.072	0.012
MarketCap	0.392	0.626	0.475
Dividends	0.464	0.682	0.516
Unsecured	0.0001	0.012	0.0003

differently. These results highlight the varied effects of risk metrics, such as dividends and market capitalization, on impacting distinct property types.

To validate previous discussions, dividends' non significant FE shows that this metric is likely to be highly significant in affecting some properties while insignificant in others. To validate this, Empirical Bayes RE estimates are obtained.

Figure 4: Empirical Bayes Estimates



Looking to the Empirical Bayes (EB) estimates, it is now possible to make statements about how risk metrics affect specific property types. Beginning with dividends, the EB estimates show that dividends positively affects Lodging / Resorts. Note that this sector is synonymous with the hotel sector in private RE. On the other hand, dividends EB estimate for Diversified signals that Dividends are negatively correlated with the cap rates of diverse REITs. This sector refers to REITs that invest in a variety of property types.

5.9 Another Decomposition

After applying a within and between-subject decomposition to this model's fixed and random effects, the results for the FEs were highly similar to those of the previous model. However, acquisition's between-subject fixed-effect is now significant, $T = -2.20$; higher average acquisition rates are correlated to lower average cap rates. Further, its -0.074577 coefficient signals that, for a billion-dollar increase in acquisitions, average cap-rates should be expected to decrease by 0.0745% across properties.

Additionally, unsecured debt is nearing significance, $T = 1.898$, indicating a potential correlation where a percentage increase in average unsecured debt corresponds to a 0.70% expansion in cap rates. This finding contradicts the previous hypothesis that higher unsecured debt implies lower risk. As such, further investigation is required. Finally, the within-subject effects of dividends and acquisitions are now insignificant. Similar to the previous interpretation, this suggests that their REs likely exhibit large variances.

Overall, by incorporating a RE for each metric, the model more effectively distinguishes the true between and within-subject effects. Note that this section above was in reference to the FEs; the REs also have their own between and within-subject effects.

Table 7: Random-Effect Decomposition

REs	Var	Std.Dev	Resi	ICC
Intercept	0.039	0.199	0.416	0.087
Occasion	0.010	0.10	0.416	0.023
W_Div	0.435	0.659	0.416	0.511
B_Div	0.267	0.517	0.416	0.391
W_MCap	0.063	0.251	0.416	0.131
B_MCap	0.458	0.677	0.416	0.524
W_Unsrd	.0006	0.0241	0.416	0.001
B_Unsrd	.0001	0.012	0.416	.0003
W_Acqu	0.005	0.072	0.416	.0124
B_Acq	0.0239	0.154	0.416	0.054

Building on the earlier model's high dividends ICC, this significance can now be directly linked to both its between and within-subject effects, each of which demonstrates a sufficiently large ICC. While the previous interpretations of market cap's RE holds, this second decomposition allows the model to better attribute this significance to market cap's between-subject effect. To conclude, these models offer compelling evidence that different risk metrics exert diverse effects on the cap rates of different properties. This illustrates the importance of integrating random effects for these high variance metrics.

5.10 Fixed vs. Random

To analyze whether the risk metrics should be included as REs, the model that solely incorporated them as FEs can be compared to the one that allowed both FEs & REs. Consequently, AIC & BIC criterion is utilized.

Table 8: Model Comparisons

Model	AIC	BIC
FEs	618.66	656.39
FEs & REs	580.22	679.67

As expected, the model incorporating the risk metrics as REs shows a lower AIC compared to the first model. However, the latter model has a higher BIC value than the former. To explain this, the mechanics of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are quickly touched upon [20].

BIC functions by maximizing posterior model probability while AIC intends to minimize the Kullback-Leibler divergence between that of the chosen model and the true distribution. As such, they have different intentions and, in turn, different interpretations.

Given a model with parameters θ and a dataset D, AIC and BIC are defined as follows:
Akaike Information Criterion:

$$AIC = -2\ln(L(\hat{\theta}|D)) + 2k \tag{42}$$

- $L(\hat{\theta}|D)$ is the maximized value of the likelihood function given the dataset.
- k is the number of parameters in the model.

Bayesian Information Criterion:

$$BIC = -2\ln(L(\hat{\theta}|D)) + k\ln(n) \tag{43}$$

- n is the sample size.

By comparing the two equations, it's evident that these values converge through distinct penalty terms. Essentially, BIC imposes a heavier penalty on the number of parameters compared to AIC, owing to the $k\ln(n)$ term, which increases the penalty as the sample size (n) grows. Thus, BIC favors more parsimonious models compared to AIC. The trade-off between the two lies in BIC's effectiveness in identifying the true model, while AIC excels in prediction.

In light of these findings, as the later model with REs possesses significantly more parameters than the former, the BIC value strongly critiques the inclusion of these risk metrics. To address the counterargument, one might contend that since some of these REs exhibited extremely high variances, their inclusion should have improved the BIC score despite the penalty. While this idea is logical, the higher BIC value in the later model is primarily due to the unnecessary inclusion of random effects for acquisitions and unsecured debt, which exhibit very small variances and ICC values.

To recap these findings, the high ICC values for market cap and dividends indicate significant variation in property-type relationships, justifying the implementation of REs here. Conversely, the low ICC values for percent unsecured and acquisitions suggest that adding REs for them would only increase model complexity without adding strong predictive benefits. These criterion comparisons show the importance of acknowledging how *certain* risk metrics exhibit varying property-type effects, while *others* have consistent property-type effects.

5.11 Public Summary

Public Market results have fully echoed the diverse nature of CRE. The first model, solely incorporating FEs, found that dividends, market capitalization, and acquisitions were significant in predicting cap rates across properties. These metrics were then broken down into their within and between-subject effects, where it was determined that solely the within-subject effects were significant. However, there did exist some evidence, albeit not statistically significant, that between-subject effects of acquisitions were related to cap rate levels.

Next, random effects (REs) were introduced for each metric to explore their distinct impacts across properties. Initially, the REs for dividends and market caps exhibited large ICCs, indicating highly variable effects on different property types. Furthermore, the fixed-effect term for dividends lost significance, illustrating that dividends affect cap rates significantly in some properties but not in others. Empirical Bayes RE estimation revealed, for instance, that dividends have a substantial impact on hotels but only a minor effect on regional malls.

Another within vs between-subject analysis was then conducted for the REs. This revealed significant variability in both components of dividends and the between-subject component of market capitalization. Consequently, both the average levels of dividends and changes in dividends have significantly disparate effects on distinct property types, while it is primarily the mean levels of market cap that has disparate effects. Additionally, the model provided statistically significant evidence to argue for a correlation between higher mean acquisition levels and lower cap rates across all properties (FE).

Despite unsecured debt having a moderately high between-subject effect in the third model, it failed to yield significant results in all model iterations. As such, it seems that investors do not view this metric as very significant in REIT valuation and risk; this is an interesting and counter intuitive result that deserves additional research.

6 Market Comparisons

The importance of implementing MRMs and REs when examining public CRE has been established. Consequently, a clear expansion of this analysis is to now examine the heterogeneity of risk in private CRE. This pivot, undergone in the next section, will be performed by way of a randomized sample of ~18,000 Commercial Mortgage Backed Securities. However, before proceeding to these logistic MRMs, a broad comparison of the two markets is conducted.

To examine which property types are perceived as the riskiest in each market market, temporal cap rate and default rate averages are obtained. However, note that public and private datasets cover slightly different time spans. Therefore, interpret these results thoughtfully, but do not attempt to make any direct conclusions.

Table 9: Public Cap Rates (2003-2021)

Property	Cap Rate
Data Centers	7.68
Diversified	6.07
Health_Care	6.14
Industrial	5.71
Lodging_Resorts	7.93
Office	6.47
Residential_Apartments	5.71
Residential_Manufactured	6.11
Retail_Free_Standing	6.50
Retail_Regional_Malls	6.36
Retail_Shopping_Centers	6.30
Self_Storage	6.20

Public markets deem data centers, lodging (Hotels), and retail as the riskiest sectors. Conversely, they categorize residential apartments (multifamily) and industrial sectors as the least risky.

Table 10: Average Status by Property Type

Property Type	Default Rate
Healthcare	0.0392
Hospitality	0.241
Industrial	0.209
Mixed Use	0
Mobile Home Park	0.181
Multifamily	0.138
Office	0.311
Other	0.154
Retail	0.294
Self-Storage	0.171
Various	0.164

Default histories indicates that the office, retail, and hospitality (hotel/lodging) sectors are the riskiest, while multifamily (residential apartments) and healthcare sectors are the least risky. Mixed-use and diverse sectors were not examined due to unclear public connections.

In both public and private sectors, the multifamily (or residential apartment) sector is considered low-risk, while the hotel sector tends to be viewed as higher risk. Nevertheless, there are notable differences across markets; for instance, industrial properties in the public sector are generally perceived as less risky than the default rates of private sector loans would indicate.

To quantify this relationship, a Pearson correlation coefficient of 0.3603 is obtained using the following sectors: Healthcare, Industrial, Lodging (Hotel), Office, Residential Apartments (Multifamily), Retail (Averaged), & Self-Storage. This coefficient is moderately positive and corresponds to an R^2 value of 0.129. Thus, while some similarity exists in public and private risk assessments, the correlation falls short, with only 12.98% of default rate variability attributable to cap rate variability. In summary, initial findings show commonalities between the markets, but numerous factors hinder more direct associations.

Average region default rates are finally calculated. It seems that their exists considerable default rate variability by region.

Table 11: Region Default Rates

Region	Default Rate
Midwest	0.326
Northeast	0.167
Southeast	0.240
Southwest	0.199
West	0.157

7 Private Analysis

7.1 Private Datasets

A randomized sample of ~1,800 US CMBS loans originated in the last 25 years was gathered. To efficiently collect a vast amount of data, a web scraping algorithm, coded in Rust (Figure 8), was built to randomly collect loans from the CRE data provider CMBS.com's website [2]. The following metrics were gathered: deal name, note status, remaining term, original loan amount, debt interest rate, first payment date, maturity date, and occupancy rate (Figures 11-12).

This study exclusively encompasses non-current loans, encompassing those that have defaulted, undergone special servicing, become delinquent, been repaid, or been defeased. Defeased refers to a process of replacing collateral (the property) and interest income with another security, like US treasuries, to fulfill all future debt payments. Specially serviced refers to the assignment of a distressed loan to a special servicer to come up with possible ways for borrowers to repay the loan or to pursue court action. As specially serviced loans are basically delinquent loans (loans with overdue debt payments overdue), they are viewed as such in this paper.

Turning to logistic configurations, the dependent variable, loan status, is assigned a 1 if the loan went delinquent or was specially serviced and a 0 if not. Unlike their public counterparts, these logistic MRMs are not longitudinal. Rather, the level-1 units are now the individual loans while the level-2 units are property type. The following risk metrics were examined: occupancy rate, debt rate, and loan principal. While the first two are clear, this section posits that smaller loans may be riskier than larger loans.

7.2 Logistic MRMs

Alike the previous models, when datasets contain non-independent data, simple logistic regression models fail to account for within-group dependencies[13]. Therefore, logistic MRMs also integrate REs to address this, enabling loan-specific risk metrics to exert varying impacts on default rates across different property types. Methodology is also drawn from Donald Hedeker and Robert Gibbons' textbook *Longitudinal Data Analysis*.

The first logistic iteration will only implement a random intercept term, while the later will then incorporate REs for each metric. Looking to the random-intercept model, the logistic MRM will have only one RE: u_i.

$$\log(\frac{p_{ij}}{1-p_{ij}}) = x'_{ij}b + u_i \tag{44}$$

x_{ij} is the FE covariate vector, B contains the FE coefficients, and u_i is the random intercept term. Further, the dependent variable (the logit transformed response) has a conditional variance of $\sigma_v^2 + \sigma_e^2$ given x_{ij}. The conditional σ_e^2 variance is assumed to be constant at $\pi^2/3$. As future models will also incorporate multiple REs, the model is now transformed.

$$\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = x'_{ij}b + z'_{ij}T\Theta_i \tag{45}$$

where the random effects, u_i, have a variance-covariance matrix of Σ_u. The random effects are standardized by setting $v_i = T\Theta_i$, where $TT' = \Sigma_u$ represents the Cholesky decomposition of Σ_u. Essentially, the Cholesky factor T acts as the square root of the variance-covariance matrix [13].

The key characteristic of logistic MRMs is that all estimates are conditional, conditional on the random (property) effects [13]. To explain this conditional interpretation, the random-intercept model will be utilized. By showing the random-intercept model can only have a conditional interpretation, it follows that the model with more random-effects will also be conditional. The random intercept model can be expressed in conditional form.

$$g[P(Y_{ij} = 1 \mid \Theta_i)] = x'_{ij}b + u_i \tag{46}$$

$$P(Y_{ij} = 1 \mid \Theta_i) = g^{-1}(x'_{ij}b + u_i) \tag{47}$$

To standardize the REs, let $u_i = \sigma_v \Theta_i$ and then take the expectation of this equation to derive the values.

$$\mu_{ij} = E(Y_{ij}) = \tag{48}$$

$$E[E(Y_{ij} \mid \Theta_i)] = \int_\Theta g^{-1}(x'_{ij}b + u_i) \cdot f(\Theta)\, d\Theta$$

where $\Theta_i \sim N(0,1)$ and $g(\cdot)$ is the logit link function. As this equation shows, the these expectations are conditional.

Put differently, this expected value is conditional on the random-effect (indexed by i) and, as such, will differ for each respective property type [13]; this is evident by the logit link function. To explain this conditionality better, in continuous MRMs the residual variance can be estimated so that the total variance, like that of simple OLS models, is not inflated by the addition of REs. On the other hand, logistic MRMs assume that the residual variance is constant at $\frac{\pi^2}{3}$. As such, the addition of REs inflates the total variances of these models.

Moving forward, as the models only produce FE coefficients on the logit scale, they must be examined under logit interpretations. However, these interpretations are inherently difficult to understand and interpret. Although one may argue that these values can simply be converted to probabilities, these probabilities can still only be interpreted conditionally if they are not population average estimates.

As a case in point of this conditionality, a probability coefficient of 0.25 doesn't uniformly translate to a 0.25 probability increase with a one-unit increase in the independent variable; rather, the effect varies based on the unit's position and the grouping variable's level.

Thus, a better way to interpret these coefficients is to examine each risk metric's average marginal probability [15]. This approach functions by computing average probability changes of the dependent variable (throughout clusters) across a range of values for the designated independent predictor variable.

In this paper, R was used to obtain these graphs. First, a sequence of risk metric levels was chosen to be predictors. Then, the correlated linear predictor is utilized to predict the probability of the dependent variable such that the other covariates are randomly sampled. Once all predicted probabilities are obtained, a graph is constructed to examine how the average of these marginal predicted probabilities vary over predictor levels.

7.3 Logistic w. Random Int

Private market examinations will commence with only a random-intercept. Each property type can have its own baseline default rate when other variables are zero. This intercept term, while not directly interpretable, enables differences in default rates across property types. In all future models, the level-2 units refer to property types (subscript i) while level-1 units refer to the individual loans (j). As such, there are 10 level-2 units and 18,180 level-1 units. The metrics are included as fixed effects.

Level-1:

$$\begin{aligned}\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = & b_{0i} + b_1\text{Occupancy}_{ij} + \\ & b_2\text{InterestRate}_{ij} + \\ & b_3\text{Principal}_{ij}\end{aligned} \tag{49}$$

Level-2:

$$b_{0i} = \beta_{00} + u_{0i} \tag{50}$$

$$b_{1-3} = \beta_{1-3} \tag{51}$$

As stated, the random subject effect $u_{0i} \sim N(0, \frac{\pi^2}{3})$. This term represents each property

Table 12: Fixed Effects

Variable	Est.	Std.Err	P-Val	Odds
Intercept	5.6381	1.1774	<0.001	
Occupancy	-0.0227	0.0034	<0.001	0.993
Int-Rate	-0.9056	0.0601	<0.001	0.878
Principal	0.1055	0.0610	0.084	1.021

type's deviation from the grand property default rate.

The intercept coefficient of 5.638 signals the logit of a default when all other variables are zero. However, interpretation of this coefficient is illogical as the risk metrics will never be zero, emphasized in it yielding a probability of 0.996. Turning to the FEs, their conditional nature is acknowledged, and only their signs are considered in this section. As expected, increased occupancy rates are correlated to decreased default rates. However, increased interest rates unexpectedly led to lower default rates, an interesting result to be examined later. Occupancy and interest rates are statistically significant, whereas principal value is not, indicating that loan size does not predict defaults.

Finally, the metrics marginal (population-averaged) odds ratios are obtained. To achieve this, the population-averaged coefficients and their corresponding standard errors are first calculated. These values are then exponentiated to compute the confidence intervals for each marginal odds ratio.

Starting with occupancy, the 95% confidence interval for the marginal odds ratio is [0.9801, 1.008]; note that, within this interval, the estimated or expected odds ratio for occupancy is 0.994. As expected and given that occupancy values range from 0 to 100, a unit increase in occupancy rate is not expected to significantly decrease the probability of a default. Next, the expected odds ratio for the debt rate is 0.878, with a confidence interval of [0.8637, 0.8937]. A unit increase in the debt rate is anticipated to significantly reduce default rates, more so than a similar change in occupancy. This is rational, as the debt rate only ranges from around 2% to 12%, making a 1% increase a proportionally larger shift compared to occupancy. Finally, the principal has an expected marginal odds ratio of 1.02 with a confidence interval of [1.0204, 1.0224].

Consistent with previous interpretations, this suggests that while increases in occupancy or debt rates generally lead to a decrease in default rates, higher principal values are associated with an increase in defaults.

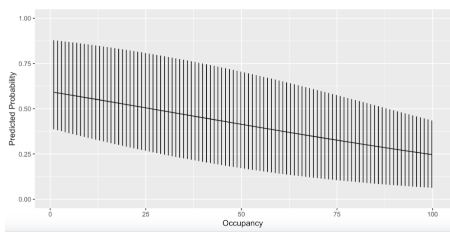
Turning to random effects, the ICC of the intercept term is computed using it's variance of 0.8268 and the fact that residual variance is always $\pi^2/3$: $0.8268/((\pi^2/3)+0.8268)$. This yields a value of 0.2010, indicating that 20.10% of the variation in default rates is explained by differences in property types. This substantial proportion highlights the importance of clustering by property type.

Table 13: Comparison with Simple OLS

Model	Par	AIC	BIC
Not Clustered	4	2381.9	2404.9
Random-Int	5	2006.0	2034.8

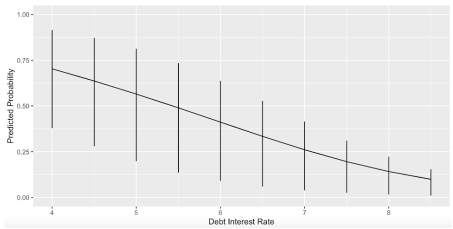
Finally, property-type clustering yields significantly lower AIC & BIC values than a simple non-clustered model. This further supports the fact that property-type clustering is essential. To proceed, each risk metric's average marginal probabilities is now calculated. The occupancy range is divided into 100 points, increasing incrementally from 1% to 100% in steps of 1%. Then average marginal probability theory is implemented to calculated the 100 corresponding default probabilities; quartiles are also found to indicate a range in which 50% of the predicted default rates lie.

Figure 5: Occupancy Avg. Marg Prob



This graph shows a pronounced linear relationship with respect to occupancy rates. Consequently, the logistic model's coefficient for occupancy rate is exponentiated, resulting in a value of 0.9776. This corresponds to an initial reduction of 2.25% in the default rate for each unit increase in occupancy rate, a trend that seems to remain fairly consistent across rates. Subsequently, this same analysis can be calculated for the debt rate, spanning from 4% to 8.5% in intervals of 0.5%.

Figure 6: Debt Rate's Avg. Marg Prob



Strikingly, as interest rates increase, default probabilities decrease. This phenomenon is likely due to the cyclical nature of CRE underwriting practices. To explain, during low rate periods, underwriters tend to exhibit aggressive underwriting standards, financing loans for properties with a high probability of default at lower rates. Then, once defaults start to manifest, underwriting practices become more conservative, utilizing higher interest rates and more stable properties with lower default probabilities. As such, this study reaffirms previous theoretical and empirical research, notably [11], which argues that CRE underwriters and investors tend to over correct; they are either too aggressive or too conservative.

7.4 Region Effects

As a corollary to previous results, region impacts are now examined, with state-to-region designations shown in figure 13. This FE term is added to the level-1 model.
Level-1:

$$\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = b_0 + \ldots + b_4 \text{Region}_{ij} + e_{ij} \tag{52}$$

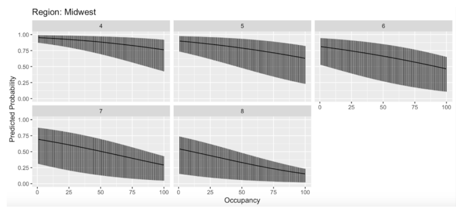
As region is a dummy-coded categorical variable, unit-change interpretations are precluded. Instead, interpretations are specific to each region and are compared relative to the Midwest, which serves as the baseline with the highest default rate. Therefore, the coefficients for each regional category are analyzed to rank the regions by default rate: Midwest, Northeast, Southeast, West, and Southwest. Further, the regions Southeast, Southwest, and West are statistically significant, signaling that there is evidence to conclude that their *true* default rates are truly different from the *true* default rate in the Midwest region.

Table 14: Region as a Fixed Effect

Variable	Estimate	Std. Err	Z
(Intercept)	6.189	1.195	5.18
Occupancy	-0.022	0.003	-6.41
Debt Rate	-0.908	0.061	-14.99
Principal	0.095	0.061	1.54
Northeast	-0.242	0.205	-1.18
Southeast	-0.327	0.184	-1.77
Southwest	-0.866	0.216	-4.01
West	-0.610	0.201	-3.026

To examine how regional differences affect default rates under varying interest and occupancy rates, region effects are modeled conditionally. As such, the average marginal probability of a Midwest default, conditional on debt rate (4% to 8%) and occupancy rate is calculated (Figure 7).

Figure 7: Conditional Midwest Defaults



This Midwest graph shows that higher occupancy rates consistently yield lower default probabilities, especially at higher interest rates. This means that the effect of occupancy on reducing defaults is more significant when debt rates are elevated. These results are echoed by all other regions.

7.5 RE Implementations

Private risk metrics are now incorporated as both FEs and REs, allowing them to uniquely influence the default rates of different property types. The level-2 model is updated to allow this.

$$b_{0i} = \beta_0 + v_{0i} \tag{53}$$

$$b_{1i} = \beta_1 + v_{1i} \tag{54}$$

$$b_{2i} = \beta_2 + v_{2i} \tag{55}$$

$$b_{3i} = \beta_3 + v_{3i} \tag{56}$$

As a quick note, the population-level or marginal odds ratios were highly similar to that of the previous logistic MRM. As such, it seems that the model's predictive capacity did not change much. This also provides evidence that the REs may not have large variances.

Table 15: Logistic REs

Random Effects	Variance	Std. Dev.
(Intercept)	1.839e+00	1.356
Occupancy	7.335e-05	0.0086
Interest Rate	2.979e-02	0.1726
Principal	2.335e-02	0.1528

This prediction is true. To explain, when looking at the effects ICCs, only the intercept term has a large effect. The intercept variance, leading to an ICC of 35.87%, indicates that different property types maintain heterogeneity in default rates. However, all of the other covariates had extremely small ICC values, less than 1%, suggesting that incorporating REs for these metrics might be redundant. Put differently, the small ICCs suggest that these metrics have highly similar effects across properties. To confirm, this model was compared against the initial logistic model.

Table 16: Logistic AIC and BIC

Model	AIC	BIC
Random-Intercept	2006.0	2034.8
Random-Trends	1997.2	2077.7

Consistent with previous interpretations, these AIC and BIC values indicate that implementing random-effects for the metrics does not significantly enhance the model. Occupancy and debt rates are only influential at the population level, shown in the significance of their FEs. This means that, while these metrics are crucial in predicting CMBS default rates, they do not appear to affect distinct property types differently!

7.6 Private Market Results

Private market MRMs have determined that occupancy rate and debt rate's FE component is significant in predicting default rates. First, the coefficient on occupancy rate indicates that higher occupancy rates correlate with lower default rates across property types. On the other hand, lower debt rates are associated with higher default rates; this inverted conclusion stems from the cyclical nature of CRE underwriting standards. Next, this paper finds no correlation between loan principal and default rate. Finally, this section conclusively demonstrates that the model clustered by property type significantly outperforms the unclustered model.

Although these metrics were significant across properties, this paper found that these metrics tend to have similar impacts across different property types. This is quantitatively backed by their low ICCs. While RE significance is not obtained, this paper does confirm that different property types exhibit significantly varied default probabilities, as evidenced by the superior performance of the clustered model.

Finally, there is considerable variation in default rates by U.S. region. Notably, this section provides evidence that true default rates differ between the Midwest and both the West and Southwest regions. Consequently, the paper suggests that Southern CMBS properties are less likely to default than those in the North.

8 Conclusion

This paper and its methodology are inspired by the viewpoint that commercial real estate comprises an incredibly diverse asset class. As a result, it aimed to emphasize the necessity of employing varied metrics to analyze different CRE property types.

Broadly, this paper establishes two simple yet important results. First, it establishes the superiority of the Treasury Inflation-Protected Securities rate over Treasury rates in predicting capitalization rates. Secondly, it emphasizes the significance of clustering by property type and, in turn, highlights the necessity of utilizing mixed-effect regression models in this domain.

Turning to comparisons between public and private market results, this paper reveals that certain public metrics have notably diverse effects on different properties. However, the private metrics appear to impact different property types similarly. As such, there seems to exist increased correlations within REIT sectors than CMBS sectors. These differences likely arise from inherent market confounders, such as distinct regulatory frameworks and reporting requirements. Specific risk-metric results can be found in the market result sections.

In conclusion, this paper aims to broaden perspectives within the CRE industry while also providing robust quantitative models that warrant future implementations.

9 Literature Review

9.1 Capitalization Rates

This paper fundamentally assumes that a property (REITs) capitalization rate (cap rate) is a measure of risk. This assumption is not unique and is backed by multitudes of previous research. Firstly, it's important to recognize that cap rates not only reflect an investor's return on a real estate investment but also act as indicators of market-specific or geographical-specific risk. Therefore, consistent with broad economic theory, higher returns should be associated with higher levels of risk. To examine previous literature on this topic, a strong starting point is Jug and Winkler's 1995 paper *The Capitalization Rate of Commercial Properties and Market Returns* [19].

In this piece, the authors build upon Copeland and Weston's 1988 work *Financial Theory and Corporate Policy* by asserting that cap rates are both a function of the weighted average cost of capital (WACC) and the capital asset pricing model (CAPM) [9]. As such, the authors argue that cap rates have both equity and debt characteristics. Cumulatively, cap rates should then signal the overall risk-adjusted return of a real-estate investment. Further, this notion is backed by Froland's 1987 study which analyzed quarterly capitalization rates across four core asset classes. Here, Froland provides evidence for the correlation between cap rates, ten-year bond rates, and earnings/price [12]. As such, this correlation argues that as Treasury rates increase, capitalization rates should also increase [12].

Diving deeper, Joseph Pagliari highlights that cap rates are convex instruments, such that shifts in capitalization rates have greater effects when they are at low values than when they are high. Put differently, the influence of cap rate shifts decreases at higher levels [28]. This convexity highlights the fact the real estate is a real-yielding security and, as such, cannot be based upon a fixed-yield.

Pagliari also highlights that true property cash-flow yields serve as a superior indicator of CRE return compared to capitalization rates, as the latter fail to account for property maintenance costs [29]. However, capitalization rates remain a key indicator of risk, being influenced by inflation; if net operating income (NOI) grows at the same rate as valuations, capitalization rates stay constant [29]. In essence, as other risk metrics increase, capitalization rates also rise, scaling the risk-free rate by the risk premiums associated with property ownership. This explains why cap rates, rather than cash flows, were utilized in this paper.

Finally, Brent Ambrose, Professor of Real Estate at Penn State, and his paper *Factors Influencing Capitalization Rates* serve as a key inspiration for this thesis as he also clusters CRE data by property type[6]. Ambrose contends that employing property average cap rates overlooks significant information, obscuring genuine trends in the CRE industry. However, Ambrose's study yields markedly divergent results and employs slightly different regression models: Seemingly Unrelated Regression (SUR) and cross-sectional/time-series regression (panel data). While the latter closely resembles the MRMs used in this paper, the former does not[6].

9.2 Fixed and Variable-Rate Securities

Subsequently, a natural evolution of these pieces is Connor and Liang's 2004 paper on *The Complex Interaction between Real Estate Cap Rates and Interest Rates* in which they apply bond mathematics to CRE capitalization rates [8]. Notably, they focus on the duration, from a fixed-income perspective, of capitalization rates to build a model that estimates the effects of higher interest rates on real estate values. They examined a national average apartment building index that led them to conclude that a 100 basis point (bp) increase in rates is expected to be coupled by a 40 bp increase in cap rates. Building upon these approaches, a contemporary addition is Jack Corgel's research paper *The Effect of an Interest Rate Increase on Hotel Capitalization Rates* [10]. Corgel's statistical approach estimates a 100 bp increase in interest rates should be coupled by a 28 bp in capitalization rates (12 bps off Connor and Liang)[8]. Further, Corgel applied a Gordon Growth based regression analysis to find a 0.68 correlation coefficient and an interest rate elasticity of 0.25 (100 bp increased coupled by 25 bp increase). Corgel held the following variables constant: risk premiums, NOI growth rates, investor sentiment, and credit availability [10].

Finally, Martha Peyton - managing director in Global Real Estate at CREF - explains that corporate bond shifts are highly correlated to cap rate trends. Notably, *the mid-2007 shift in corporate bond spreads did foreshadow the subsequent widening of cap rates* [30]. As such, Peyton argues the importance of monitoring bond market dynamics for predicting commercial real estate trends.

Although these subsequent studies argue for a correlation between interest rates and capitalization rates, the section on Treasury Inflation Protected Securities and Variable-Rate Securities will explain how this relationship lacks both qualitative and quantitative backing in today's academic and professional climate. A statistical rationale for the increased influence of TIPS is presented in section 5.3.

9.3 Volatility

This paper takes the perspective that risk premium measures are a better approach in predicting cap rates than interest rates volatility level. However, as this notion is popular in many academic fields, its basis will quickly be examined. Thus, the next section reviews literature on interest rate volatility fluctuations, followed by an explanation of how Risk Premium metrics better enhance predictive power.

Many papers have argued for a direct association between volatility and the spread on CRE CMBS loans. However, these analyses fail to account for the non-uniform or, similarly, non-linear nature of this spread. Rather, capitalization rates are contingent on the perceived risk of a property's cash flows and its capacity to meet debt service obligations. Similarly, it is a loan's LTV that will act hand-in-hand with the rate. These loan-specific effects highly outweigh any of the effects of volatility.

As one justification against this approach, Paul Mouchakkaa's *Frozen on the Rates: Impact of Interest Rates on Capitalization Rates* argues that the true correlation between interest rates and cap rates is relatively loose [25]. He argues that increased rates would not always lead to increased cap rates by arguing that cap rates are more directly influenced by a combination of factors that include credit availability, supply-demand dynamics, inflation, and spreads. While this paper does not prioritize volatility as a primary determinant of cap rates, it represents an early academic contribution that questions the interest rate hike approach. Notably, challenging the interest rate approach inherently challenges the interest rate volatility approach. This assertion aligns with mathematical reasoning, as a function that is not influenced by a variable implies that its derivative will also have no effect from that variable.

9.4 TIPS and CRE Return

Pagliari's 2017 piece, *Some Thoughts on Real Estate Pricing*, explains how, contrary to popular sentiment, there fails to exist a true correlation between interest rates and capitalization rates [29]. He substantiates this assertion by presenting empirical and theoretical evidence indicating that the anticipation of high future inflation more accurately determines the spread between interest rates and capitalization rates. However, implicit in his argument is the assumption that the economy is experiencing a period of high inflation; this correlation is not applicable to periods of low inflation [29]. Although, at the time of publishing, this idea between high and low inflation was not backed by particularly strong historical evidence, recent market trends (2021-2023) have served as direct evidence of Pagliari's point.

Second, Pagliari demonstrates how there exists an inherent limitation of comparing interest rates to cap rates. To look at this from a financial perspective, capitalization rates are a function of net operating income which does not take into account the costs of maintaining the properties. To logistically refute this correlation, Pagliari notes how an interest-rate to capitalization-rate comparison is fundamentally flawed as the first is a fixed-rate nominal-yield security while the latter is a variable-rate real-yielding security [29]; rather one must base real estate's real yielding risky return by using the basis of a real-yielding riskless security, Treasury Inflation Protected Securities.

To elucidate this, the return on real estate should be a combination of three factors: the current TIPS yield, the fee differential of TIPS v. core real estate, and the fair risk premium for purchasing real estate. The TIPS yield represents the risk-free rate while the risk premium represents the additional required return for holding a more risky investment scaled up for additional fees [29]. This return, in essence, represents the cash flow yield of real estate; This cash flow yield then must be scaled up by property costs to then arrive at the market's required capitalization rate.

$$\frac{CF_1}{P_o} = r_{re} \cdot (1 + p) \tag{57}$$

Notably: The term $1 + p$ converts from year 0 to 1.

Real-Estate Required Return $r_{re} = \phi + r_{bonds}$ and Capitalization Rate

$$\frac{NOI_1}{P_o} = \frac{r_{re} \cdot (1 + p)}{b} \tag{58}$$

This equation provides two solutions: setting the capitalization rate to determine the required return, or the reverse. As such, treasury rates do not affect the capitalization rates or returns of real estate investments [29].

9.5 The 2023 CMBS Market

The 2023 commercial real estate market has grappled with persistent challenges over the past year, and these difficulties are anticipated to persist in the foreseeable future. This price decline, as previously discussed, is being highly attributed (by the professional world) to rising interest rates, tighter bank lending standards, bank failures, and more. Notably, in Erica Jiang's piece *U.S. Bank Fragility to Credit Risk in 2023: Monetary Tightening and Commercial Real Estate Distress*, the authors note a massive decline in banks market value of assets; The authors then proceed to illustrate how this decline has led to increased CRE lending hurdles for both regional and global banks [18]. Further, the authors show that there exists the possibility of a 10% to 20% default rate on CRE assets, a default level note seen since the Great Financial Crisis (GFC). From here, the authors state the effects of these defaults could severely cripple many US banks [18].

Turning to a broader themes in CRE, the US office sector has recently faced significant challenges, notable in Fitch Ratings' deteriorating outlook [22]. In particular, these problems are highly visible in CMBS SASB deals. To quantify this, Fitch reports that 56% of conduit transactions involve over 20% office exposure, and another 8% of SASB transactions exceed 30% office exposure [22].

Pivoting from these more extreme views, Rich Hill - Head of RE Strategy & Research at Cohen & Steers - argues that, although there is increased CRE market risk, this risk is diversified across the banking industry and potential default rates are over exaggerated. Notably, Hill explains how CRE underwriting standards have become highly conservative since the 2008 crisis which, in turn, mitigates much of the risk in falling property values [17]. To illustrate this diversification, Hill shows the respective CRE loan exposure by lender and property type as percentage values in Figure 4 of the appendix [17]. To further push back on fear of a CRE crash, Hill shows that most loans are currently backed by properties that can cover their debt service [17].

Overall, existing literature recognizes that the CRE industry has faced challenges over the past year, with the future of the market subject to intense debate. Given this uncertainty, the identification of risk metrics and their property-specific effects has become increasingly crucial in portfolio management and investment decisions.

10 Appendix

This paper utilized ChatGPT and other LLMs to fix grammatical and language issues. However, these tools were not used in any other way.

Figure 8: Rust Code

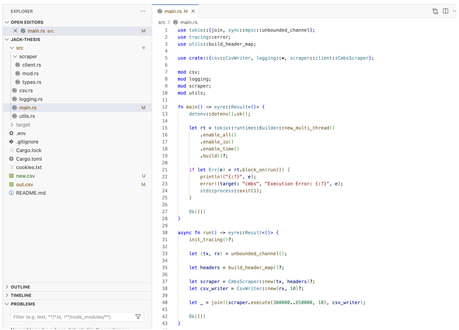


Figure 9: Public Market 1

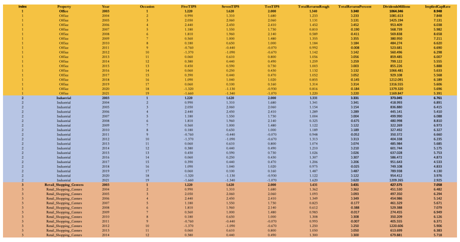


Figure 10: Public Market 2

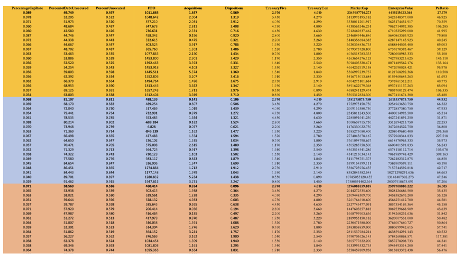
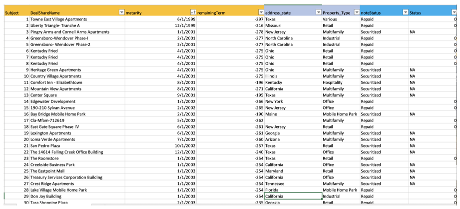
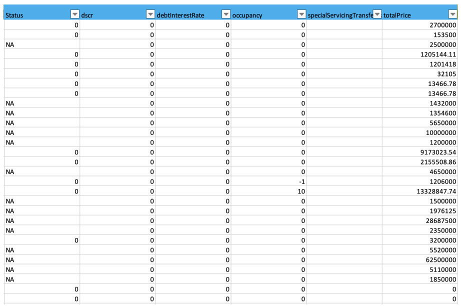


Figure 11: Private Market 1



Subject	DealShareName	maturity	remainingTerm	address_state	Property_Type	noteStatus	Status
1	Towne East Village Apartments	6/1/1999	-297	Texas	Various	Repaid	0
2	Liberty Triangle- Tranche A	12/1/1999	-216	Missouri	Retail	Repaid	0
3	Pingry Arms and Cornell Arms Apartments	1/1/2001	-278	New Jersey	Multifamily	Securitized	NA
4	Greensboro-Wendover Phase-I	2/1/2001	-277	North Carolina	Industrial	Repaid	0
5	Greensboro- Wendover Phase-2	2/1/2001	-277	North Carolina	Industrial	Repaid	0
6	Kentucky Fried	4/1/2001	-275	Ohio	Retail	Repaid	0
7	Kentucky Fried	4/1/2001	-275	Ohio	Retail	Repaid	0
8	Kentucky Fried	4/1/2001	-275	Ohio	Retail	Repaid	0
9	Heritage Green Apartments	4/1/2001	-275	Ohio	Multifamily	Securitized	NA
10	Country Village Apartments	4/1/2001	-275	Illinois	Multifamily	Securitized	NA
11	Comfort Inn - Elizabethtown	8/1/2001	-196	Kentucky	Hospitality	Securitized	NA
12	Mountain View Apartments	8/1/2001	-271	California	Multifamily	Securitized	NA
13	Center Square	9/1/2001	-195	Texas	Multifamily	Securitized	NA
14	Edgewater Development	1/1/2002	-266	New York	Office	Repaid	0
15	190-210 Sylvan Avenue	2/1/2002	-265	New Jersey	Office	Repaid	0
16	Bay Bridge Mobile Home Park	2/1/2002	-190	Maine	Mobile Home Park	Securitized	NA
17	Cla-Mfam-712619	5/1/2002	-262		Multifamily	Repaid	0
18	East Gate Square Phase IV	6/1/2002	-261	New Jersey	Retail	Repaid	0
19	Lexington Apartments	6/1/2002	-261	Georgia	Multifamily	Securitized	NA
20	Loma Verde Apartments	7/1/2002	-260	Arizona	Multifamily	Securitized	NA
21	San Pedro Plaza	10/1/2002	-257	Texas	Retail	Securitized	NA
22	The 14614 Falling Creek Office Building	12/1/2002	-240	Texas	Office	Securitized	NA
23	The Roomstore	1/1/2003	-254	Texas	Retail	Repaid	0
24	Creekside Business Park	1/1/2003	-254	California	Office	Securitized	NA
25	The Eastpoint Mall	1/1/2003	-254	Maryland	Retail	Securitized	NA
26	Treasury Services Corporation Building	1/1/2003	-254	California	Office	Securitized	NA
27	Crest Ridge Apartments	1/1/2003	-254	Tennessee	Multifamily	Securitized	NA
28	Lake Village Mobile Home Park	1/1/2003	-254	Florida	Mobile Home Park	Repaid	0
29	Don Joy Building	1/1/2003	-254	California	Industrial	Repaid	0
30	Tara Shopping Plaza	2/1/2003	-235	Georgia	Retail	Repaid	0

Figure 12: Private Market 2



Status	dscr	debtInterestRate	occupancy	specialServicingTransfe	totalPrice
0	0	0	0		2700000
0	0	0	0		153500
NA	0	0	0		2500000
0	0	0	0		1205144.11
0	0	0	0		1201418
0	0	0	0		32105
0	0	0	0		13466.78
0	0	0	0		13466.78
NA	0	0	0		1432000
NA	0	0	0		1354600
NA	0	0	0		5650000
NA	0	0	0		10000000
NA	0	0	0		1200000
0	0	0	0		9173023.54
0	0	0	0		2155508.86
NA	0	0	0		4650000
0	0	0	-1		1206000
0	0	0	10		13328847.74
NA	0	0	0		1500000
NA	0	0	0		1976125
NA	0	0	0		28687500
NA	0	0	0		2350000
0	0	0	0		3200000
NA	0	0	0		5520000
NA	0	0	0		62500000
NA	0	0	0		5110000
NA	0	0	0		1850000
0	0	0	0		0
0	0	0	0		0

Figure 13: Region Designation

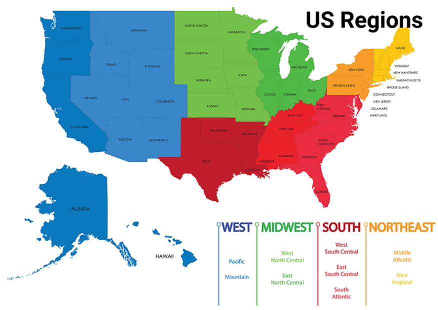


Figure 14: Unsecured Debt by Class

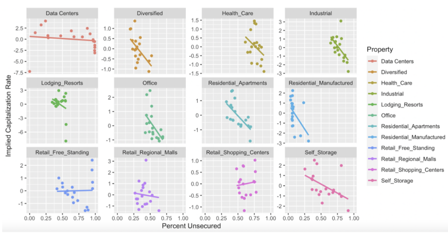


Figure 15: Cap Rate Trends

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