Supplemental material for "Quantum transduction is enhanced by single mode squeezing operators"

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A—GAUSSIAN QUANTUM CHANNEL

A Gaussian quantum channel $\mathcal{N}:\rho_{\rm in}\to\rho_{\rm out}$ can be specified by its action on the statistical first and second moments of arbitrary Gaussian state $\hat{\rho}(\bar{\mathbf{x}},\mathbf{V})$. In general, we have

$$\begin{aligned}\bar{\mathbf{x}}&\to\mathbf{T}\bar{\mathbf{x}}+\mathbf{d},\\ \mathbf{V}&\to\mathbf{T}\mathbf{V}\mathbf{T}^T+\mathbf{N},\end{aligned}\tag{1}$$

where $\mathbf{T},\mathbf{N}$ are real matrices satisfying the channel completely positive condition $\mathbf{N}+\mathbf{i\Omega}-\mathbf{iT\Omega T^T}\geq 0$. Specifically, when $\mathbf{N}=\mathbf{0}$, $\mathbf{T}$ is a symplectic matrix, which defines a Gaussian unitary channel. Given a single mode Gaussian channel, the quantum capacity is lower bounded by the following expression

$$Q_{\rm LB}=\begin{cases}\max\{0,\log_2|\frac{\eta}{1-\eta}|-g(n_e)\}, & \eta\neq 1\\ \max\{0,\log_2(\frac{2}{e\sigma^2})\}, & \eta=1,\end{cases}\tag{2}$$

where $g(n_e)=(n_e+1)\log_2(n_e+1)-n_e\log_2 n_e$. η and n_e are the channel transmissivity and added noise, respectively, which are given by

$$\begin{aligned}\eta&=\det\mathbf{T},\\ n_e&=\begin{cases}\frac{\sqrt{\det\mathbf{N}}}{2|1-\eta|}-\frac{1}{2}, & \eta\neq 1\\ \sqrt{\det\mathbf{N}}=\sigma^2, & \eta=1\end{cases}.\end{aligned}\tag{3}$$

Note in some literature, the transmissivity is defined as the quadrature conversion efficiency, which can be varied for different quadratures for none isotropic channel. Clearly, $\eta=1/2$ is the least threshold for a Bosonic loss channel to have non-zero quantum capacity.

B—THE TRANSDUCTION CHANNEL WITH SQUEEZING

In this section, we will derive the Gaussian transduction channel given by the cavity electro-optical system with microwave squeezing. As given in the main text, the Hamiltonian takes the form

$$\hat{H}/\hbar=-\Delta_{\rm o}\hat{a}^\dagger\hat{a}+\Delta_{\rm e}\hat{b}^\dagger\hat{b}+g(\hat{a}^\dagger\hat{b}+\hat{a}\hat{b}^\dagger)+\nu(e^{-i\theta}\hat{b}^{\dagger^2}+e^{i\theta}\hat{b}^2).\tag{4}$$

We write down the Heisenberg-Langevin equations and input–output relations

$$\begin{aligned}\dot{\mathbf{a}}&=\mathbf{A}\,\mathbf{a}+\mathbf{B}\,\mathbf{a}_{\rm in},\\ \mathbf{a}_{\rm out}&=\mathbf{B}^{\rm T}\mathbf{a}-\mathbf{a}_{\rm in},\end{aligned}\tag{5}$$

in which $\mathbf{a}$ is a vector which collects all the mode operators and similarly $\mathbf{a}_{\rm in}$ and $\mathbf{a}_{\rm out}$ collect all the input and output mode operators

$$\begin{aligned}
\mathbf{a} &= \left(\hat{a},\, \hat{a}^{\dagger},\, \hat{b},\, \hat{b}^{\dagger}\right)^{\rm T} \\
\mathbf{a}_{\rm in} &= \left(\hat{a}_{\rm in,c},\, \hat{a}^{\dagger}_{\rm in,c},\, \hat{a}_{\rm in,i},\, \hat{a}^{\dagger}_{\rm in,i},\, \hat{b}_{\rm in,c},\, \hat{b}^{\dagger}_{\rm in,c},\, \hat{b}_{\rm in,i},\, \hat{b}^{\dagger}_{\rm in,i}\right)^{\rm T} \\
\mathbf{a}_{\rm out} &= \left(\hat{a}_{\rm out,c},\, \hat{a}^{\dagger}_{\rm out,c},\, \hat{a}_{\rm out,i},\, \hat{a}^{\dagger}_{\rm out,i},\, \hat{b}_{\rm out,c},\, \hat{b}^{\dagger}_{\rm out,c},\, \hat{b}_{\rm out,i},\, \hat{b}^{\dagger}_{\rm out,i}\right)^{\rm T} \\
\mathbf{A} &= \begin{pmatrix} i\Delta_{\rm o} - \frac{\kappa_{\rm o}}{2} & 0 & -ig & 0 \\ 0 & -i\Delta_{\rm o} - \frac{\kappa_{\rm o}}{2} & 0 & ig \\ -ig & 0 & -i\Delta_{\rm e} - \frac{\kappa_{\rm e}}{2} & -i2\nu e^{-i\theta} \\ 0 & ig & i2\nu e^{i\theta} & i\Delta_{\rm e} - \frac{\kappa_{\rm e}}{2} \end{pmatrix}, \\
\mathbf{B} &= \begin{pmatrix} \sqrt{\kappa_{\rm o,c}} & 0 & \sqrt{\kappa_{\rm o,i}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\kappa_{\rm o,c}} & 0 & \sqrt{\kappa_{\rm o,i}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\kappa_{\rm e,c}} & 0 & \sqrt{\kappa_{\rm e,i}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\kappa_{\rm e,c}} & 0 & \sqrt{\kappa_{\rm e,i}} \end{pmatrix}.
\end{aligned} \tag{6}$$

The lower indices "in,out" denote the "input,output" modes, and the indices "c,i" denote the "coupling,intrinsic" loss ports. Transform all the mode operators into the frequency domain according to the formula,

$$\mathbf{O}[\omega] = \int_{-\infty}^{\infty} \mathbf{O}(t) e^{i\omega t}\, {\rm d}t. \tag{7}$$

Straightforwardly, we obtain that $\mathbf{a}_{\rm in}[\omega]$ and $\mathbf{a}_{\rm out}[\omega]$ is linked through a scattering matrix $\mathbf{S}_a[\omega]$,

$$\mathbf{a}_{\rm out}[\omega] = \mathbf{S}_a[\omega]\, \mathbf{a}_{\rm in}[\omega] = \left[\mathbf{B}^{\rm T} \left(-i\omega \mathbf{D}_4 - \mathbf{A}\right)^{-1} \mathbf{B} - \mathbf{I}_8\right] \mathbf{a}_{\rm in}[\omega], \tag{8}$$

where $\mathbf{I}_8$ denotes the 8-dimensional identity matrix, and $\mathbf{D}_4 = {\rm diag}(1,\,-1,\,1,\,-1)$. The scattering matrix can be rewritten in the quadrature basis using the expression

$$\begin{pmatrix} \hat{x} \\ \hat{p} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{a}^{\dagger} \end{pmatrix}. \tag{9}$$

We thus have a new scattering matrix defined in the quadrature basis

$$\mathbf{x}_{\rm out}[\omega] = \mathbf{S}_x[\omega]\, \mathbf{x}_{\rm in}[\omega] = \mathbf{Q}\, \mathbf{S}_a[\omega]\, \mathbf{Q}^{-1} \mathbf{x}_{\rm in}[\omega], \tag{10}$$

where

$$\mathbf{Q} = \mathbf{I}_4 \otimes \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}. \tag{11}$$

The input and output quadrature fields are

$$\mathbf{x}_{\rm in} = \left(\hat{x}^a_{\rm in,c},\, \hat{p}^a_{\rm in,c},\, \hat{x}^a_{\rm in,i},\, \hat{p}^a_{\rm in,i},\, \hat{x}^b_{\rm in,c},\, \hat{p}^b_{\rm in,c},\, \hat{x}^b_{\rm in,i},\, \hat{p}^b_{\rm in,i}\right)^{\rm T}, \tag{12}$$

and

$$\mathbf{x}_{\rm out} = \left(\hat{x}^a_{\rm out,c},\, \hat{p}^a_{\rm out,c},\, \hat{x}^a_{\rm out,i},\, \hat{p}^a_{\rm out,i},\, \hat{x}^b_{\rm out,c},\, \hat{p}^b_{\rm out,c},\, \hat{x}^b_{\rm out,i},\, \hat{p}^b_{\rm out,i}\right)^{\rm T}. \tag{13}$$

With the scattering matrix, we easily get the transduction channel. For example, from optical to microwave conversion, we have

$$\begin{pmatrix} \hat{x}^b_{\rm out,c} \\ \hat{p}^b_{\rm out,c} \end{pmatrix} = \mathbf{S}_x[\omega]^{\langle 5::6,1::2\rangle} \begin{pmatrix} \hat{x}^a_{\rm in,c} \\ \hat{p}^a_{\rm in,c} \end{pmatrix} + \mathbf{S}_x[\omega]^{\langle 5::6,3::8\rangle} \cdot \mathbf{x}_{\rm in}^{\langle 3::8\rangle}, \tag{14}$$

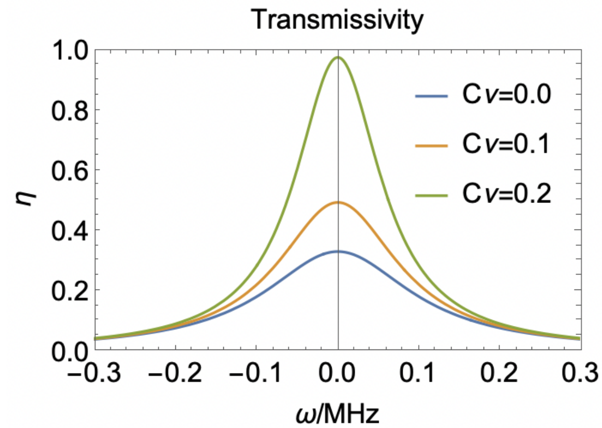

FIG. 1. The transmissivity as a function of the frequency for different squeezing parameters. The transduction bandwidth decreases as we increase the squeezing. The parameter $C_g = 0.1, \zeta_e = \zeta_o = 1$, $\kappa_\mathrm{o} = 100$ MHz, $\kappa_\mathrm{e} = 0.2$ MHz are used.

where the upper index $\langle i :: j, m :: n\rangle$ means the matrix elements from row i to j and column m to n. Obviously, the covariance matrix is transformed according to

$$\mathbf{V}^b_{\mathrm{out}} = \mathbf{T}\mathbf{V}^a_{\mathrm{in}}\mathbf{T}^{\mathrm{T}} + \mathbf{N}, \tag{15}$$

which is exactly the form of a single mode Gaussian channel. Here we define the matrices

$$\begin{aligned} \mathbf{T} =& \mathbf{S}_x[\omega]^{\langle 5::6,1::2\rangle} \\ \mathbf{N} =& \mathbf{S}_x[\omega]^{\langle 5::6,3::8\rangle} \langle \mathbf{x}_{\mathrm{in}}^{\langle 3::8\rangle} \cdot \mathbf{x}_{\mathrm{in}}^{\langle 3::8\rangle^{\mathrm{T}}} \rangle \, \mathbf{S}_x[\omega]^{\langle 5::6,3::8\rangle^{\mathrm{T}}}. \end{aligned} \tag{16}$$

Note the channel transmissivity is obtained as (when on resonance and $\omega = 0$ in rotating frame)

$$\eta_\nu = \det \mathbf{T} = \begin{cases} \frac{4C_g}{(1+C_g)^2 - 4C_\nu}\zeta_\mathrm{o}\zeta_\mathrm{e}, & \nu \neq 0 \\ \frac{4C_g}{(1+C_g)^2}\zeta_\mathrm{o}\zeta_\mathrm{e}, & \nu = 0 \end{cases} \tag{17}$$

which is given in the main text. Note the transduction process is reciprocal, meaning the microwave to optical conversion has the same channel parameters.

Another interesting quantify is the transduction bandwidth as a function of the squeezing. For on resonance detuning, we have

$$\eta_\nu[\omega] = \frac{4C_g\kappa_\mathrm{e}^2\kappa_\mathrm{o}^2\zeta_\mathrm{e}\zeta_\mathrm{o}}{[(1+C_g)^2 - 4C_\nu]\kappa_\mathrm{e}\kappa_\mathrm{o} + 4[(1-4C_\nu)\kappa_\mathrm{e}^2 - 2C_g\kappa_\mathrm{e}\kappa_\mathrm{o} + \kappa_\mathrm{o}^2]\omega^2 + 16\omega^4}. \tag{18}$$

The numerical plot is shown in Fig. 1. We see the squeezing enhances the transmissivity in general, while the transduction bandwidth decreases.

C—THE EULER DECOMPOSITION

In specific, the scattering matrix defined in Eq. 10 can be reduced to a 4×4 matrix when we take a unit extraction ratio

$$\mathbf{S}_x = \begin{pmatrix} \frac{1-C_g+2\sqrt{C_\nu}}{1+C_g+2\sqrt{C_\nu}} & 0 & 0 & \frac{2\sqrt{C_g}}{1+C_g+2\sqrt{C_\nu}} \\ 0 & \frac{1-C_g-2\sqrt{C_\nu}}{1+C_g-2\sqrt{C_\nu}} & \frac{-2\sqrt{C_g}}{1+C_g-2\sqrt{C_\nu}} & 0 \\ 0 & \frac{2\sqrt{C_g}}{1+C_g-2\sqrt{C_\nu}} & \frac{1-C_g+2\sqrt{C_\nu}}{1+C_g-2\sqrt{C_\nu}} & 0 \\ \frac{-2\sqrt{C_\nu}}{1+C_g+2\sqrt{C_\nu}} & 0 & 0 & \frac{1-C_g-2\sqrt{C_\nu}}{1+C_g+2\sqrt{C_\nu}} \end{pmatrix}, \tag{19}$$

which connects the input and output quadratures of the coupling loss ports $\mathbf{x}_{\text{in,c}} = (\hat{x}^a_{\text{in,c}}, \hat{p}^a_{\text{in,c}}, \hat{x}^b_{\text{in,c}}, \hat{p}^b_{\text{in,c}})^{\text{T}}$, $\mathbf{x}_{\text{out,c}} = (\hat{x}^a_{\text{out,c}}, \hat{p}^a_{\text{out,c}}, \hat{x}^b_{\text{out,c}}, \hat{p}^b_{\text{out,c}})^{\text{T}}$. In the main text, we removed all the port index for simplicity. As classified in Ref. [1], this channel belongs to the class $[2,2]$, which is not perfect in general. If we have the squeezing parameter $C_\nu = \frac{1}{4}(1-C_g)^2$, the scattering matrix becomes

$$\mathbf{S}_x = \begin{pmatrix} 1-C_g & 0 & 0 & \sqrt{C_g} \\ 0 & 0 & -\frac{1}{\sqrt{C_g}} & 0 \\ 0 & \frac{1}{\sqrt{C_g}} & -1+\frac{1}{\sqrt{C_g}} & 0 \\ -\sqrt{C_g} & 0 & 0 & 0 \end{pmatrix}, \tag{20}$$

which corresponds to the transducer class $[2,1]$, and can be made to be perfect by squeezing or Homodyne measurement. In the quantum channel language, the quantum capacity diverges. To understand why this is happening, we can Euler decompose the scattering matrix. In principle, any symplectic matrix $\mathbf{S} \in \text{Sp}(2n, R)$ can be decomposed as the product of three matrices

$$\mathbf{S} = \mathbf{O}\mathbf{D}\mathbf{O}' \tag{21}$$

where $\mathbf{O}, \mathbf{O}' \in \text{Sp}(2n, R) \cap O(2n)$ are symplectic orthogonal, and $\mathbf{D}$ is a diagonal matrix with local mode squeezing. This decomposition is called symplectic *Euler decomposition* or *Bloch-Messiah decomposition*. Note the matrix $\mathbf{D}$ only needs local squeezer, and the symplectic orthogonal matrix corresponds to those passive unitary transformations, which can be simply realized by beam splitter and phase shifter. In our case, we have the following clean form of decomposition

$$\mathbf{S}_x = \begin{pmatrix} 0 & \frac{1}{-\sqrt{1+C_g}} & 0 & \frac{\sqrt{C_g}}{\sqrt{1+C_g}} \\ \frac{1}{\sqrt{1+C_g}} & 0 & \frac{-\sqrt{C_g}}{\sqrt{1+C_g}} & 0 \\ \frac{\sqrt{C_g}}{\sqrt{1+C_g}} & 0 & \frac{1}{\sqrt{1+C_g}} & 0 \\ 0 & \frac{\sqrt{C_g}}{\sqrt{1+C_g}} & 0 & \frac{1}{\sqrt{1+C_g}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & 1/r \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{\sqrt{1+C_g}} & \frac{-\sqrt{C_g}}{\sqrt{1+C_g}} & 0 \\ \frac{1}{-\sqrt{1+C_g}} & 0 & 0 & \frac{-\sqrt{C_g}}{\sqrt{1+C_g}} \\ 0 & \frac{\sqrt{C_g}}{\sqrt{1+C_g}} & \frac{1}{\sqrt{1+C_g}} & 0 \\ \frac{-\sqrt{C_g}}{\sqrt{1+C_g}} & 0 & 0 & \frac{1}{\sqrt{1+C_g}} \end{pmatrix}, \tag{22}$$

where we denoted the local effective squeezing $r = \frac{1+C_g+2\sqrt{C_\nu}}{1+C_g-2\sqrt{C_\nu}}$. Obviously, the decomposition gives two beam-splitters sandwiched by a single mode amplifier, and the microwave squeezing parameter C_ν only appears in the $\mathbf{D}$ matrix. If we look at the quadrature transformations from optical $\hat{a}$ to microwave $\hat{b}$, it has the following structure

$$\begin{aligned} \hat{x}^b_{\text{out,c}} &= \frac{\sqrt{C_g}(1+r)}{1+C_g}\hat{p}^a_{\text{in,c}} + \frac{r-C_g}{(1+C_g)}\hat{x}^b_{\text{in,c}}, \\ \hat{p}^b_{\text{out,c}} &= -\frac{\sqrt{C_g}(1+1/r)}{1+C_g}\hat{x}^a_{\text{in,c}} + \frac{1/r-C_g}{1+C_g}\hat{p}^b_{\text{in,c}}. \end{aligned} \tag{23}$$

We see the reflection can be canceled by tuning the squeezing, e.g., to satisfy $r = C_g$ or $1/r = C_g$, which gives the condition $C_\nu = \frac{1}{4}(1+C_g)^2$. Clearly, the introduced squeezing can make one quadrature impedance matched while the other is not, reducing the transducer to the class $[2,1]$ and a perfect transducer can be obtained, with a divergent quantum capacity.

D—THE BOGOLIUBOV TRANSFORMATION

In this section, we give the derivation of the Bogoliubov transformation used in the main text. Let's consider the dynamics of the microwave squeezing

$$\hat{H}_\nu = \hbar\Delta_{\text{e}}\hat{b}^\dagger\hat{b} + \hbar\nu(e^{-i\theta}\hat{b}^{\dagger 2} + e^{i\theta}\hat{b}^2). \tag{24}$$

Including the dissipation, the mode dynamics follows the Langevin equation $d\mathbf{V}/dt = \mathbf{A}\mathbf{V}$, expanded as

$$\frac{d}{dt}\begin{pmatrix} \hat{b} \\ \hat{b}^\dagger \end{pmatrix} = \begin{pmatrix} -i\Delta_{\text{e}} - \frac{\kappa_{\text{e}}}{2} & -i2\nu e^{-i\theta} \\ i2\nu e^{i\theta} & i\Delta_{\text{e}} - \frac{\kappa_{\text{e}}}{2} \end{pmatrix}\begin{pmatrix} \hat{b} \\ \hat{b}^\dagger \end{pmatrix}. \tag{25}$$

The right hand side defines a non-Hermitian dynamical matrix $\mathbf{A}$, which can be diagonalized according to $\mathbf{A} = \mathbf{W^{-1}DW}$ with the transformation matrix

$$\mathbf{W} = \begin{pmatrix} \cosh(r) & e^{-i\theta}\sinh(r) \\ e^{i\theta}\sinh(r) & \cosh(r) \end{pmatrix} \tag{26}$$

and the corresponding eigenvalue matrix $\mathbf{D} = \mathrm{Diag}[-\frac{\kappa_e}{2} - i\sqrt{\Delta_e^2 - (2\nu)^2}, -\frac{\kappa_e}{2} + i\sqrt{\Delta_e^2 - (2\nu)^2}]$. The parameter r is the *effective squeezing* used in the transformation which satisfies $\tanh(2r) = 2\nu/\Delta_e$ (note this effective squeezing is different from the microwave mode squeezing, although related). It is worth mentioning that we need $\Delta_e > 2\nu$ to guarantee a unitary Bogoliubov transformation. Practically, the detuning can be chosen to to smaller than the squeezing, although a Bogoliubov transformation can still be defined, the transformation is generally not unitary [2], and identifying the corresponding quantum channel is an interesting topic for the future. Define $\beta \equiv 2\nu/\Delta_e$. When $\beta \sim 1$, the system gets to the amplification threshold. The effective squeezing factor r quickly increases near the threshold ($\beta \sim 1$) which as shown in the main text is the condition of boosting the transduction performance.

The diagonalization enables us to define a new eigen-mode $\hat{b}_s$ through the transformation

$$\hat{b}_s = \cosh(r)\hat{b} + e^{-i\theta}\sinh(r)\hat{b}^\dagger \tag{27}$$

with the corresponding mode frequency $\omega_s = \sqrt{\Delta_e^2 - (2\nu)^2}$, and the mode dissipation rate stays the same $\kappa_s = \kappa_e$. This new mode is called the Bogoliubov squeezing mode as we used in the main text. The system total Hamiltonian can be written in the following form in terms of the Bogoliubov mode

$$\hat{H}_s/\hbar = -\Delta_o\hat{a}^\dagger\hat{a} + \omega_s\hat{b}_s^\dagger\hat{b}_s + g_s(\hat{a}^\dagger\hat{b}_s + \hat{a}\hat{b}_s^\dagger) \tag{28}$$

which is the expression shown in the main text with the rotating wave approximation applied.

E—SQUEEZING INDUCED NOISE ELIMINATION

The enhancement of coupling strength comes with a cost—the Bogoliubov mode will see a noise bath amplified by the squeezing. This noise could destroy all the quantum feature of the transduction channel. We write the squeezing-amplified noise operator as $\hat{b}_\nu = \cosh(r)\hat{b}_{th} + e^{-i\theta}\sinh(r)\hat{b}_{th}^\dagger$, where $\hat{b}_{th}$ and $\hat{b}_{th}^\dagger$ are the noise operators of microwave thermal bath with thermal photon n_{th}. For vacuum bath $n_{th} = 0$, which is assumed in the main text (Note for thermal temperature $T \sim 1$ mK and the microwave frequency $\omega_e \sim 10$ GHz, the thermal photon $n_{th} \sim 10^{-200}$, which is negligible). Obviously, the noise photon of the squeezed bath is $n_\nu = \cosh(2r)n_{th} + \sinh^2(r)$, as shown in the main text.

To suppress this noise, we may input a broadband squeezed vacuum to the microwave port with squeezing factor λ. Effectively, the squeezed-vacuum field can be considered as the thermal bath couple to the microwave mode with thermal photon $n_{th} = \sinh^2\lambda$. The squeezed noise operator $\hat{b}_\nu$ should be written in the following form

$$\begin{aligned} \hat{b}_\nu =& \cosh(r)[\cosh(\lambda)\hat{b}_{th} + e^{-i\phi}\sinh(\lambda)\hat{b}_{th}^\dagger] + e^{-i\theta}\sinh(r)[\cosh(\lambda)\hat{b}_{th}^\dagger + e^{i\phi}\sinh(\lambda)\hat{b}_{th}] \\ =& [\cosh(r)\cosh(\lambda) + e^{-i(\theta-\phi)}\sinh(r)\sinh(\lambda)]\hat{b}_{th} + [e^{-i\phi}\cosh(r)\sinh(\lambda) + e^{-i\theta}\sinh(r)\cosh(\lambda)]\hat{b}_{th}^\dagger. \end{aligned} \tag{29}$$

Note $\hat{b}_{th}$ is still the vacuum or thermal noise operator that the microwave mode originally couples to. This means the Bogoliubov mode will see a thermal reservoir with thermal photon (assuming $n_{th} = 0$)

$$\begin{aligned} n_\nu =& [e^{i\phi}\cosh(r)\sinh(\lambda) + e^{i\theta}\sinh(r)\cosh(\lambda)][e^{-i\phi}\cosh(r)\sinh(\lambda) + e^{-i\theta}\sinh(r)\cosh(\lambda)] \\ =& \cosh^2(r)\sinh^2(\lambda) + \sinh^2(r)\cosh^2(\lambda) + \frac{1}{2}\cos(\theta-\phi)\sinh(2r)\sinh(2\lambda). \end{aligned} \tag{30}$$

Obviously, if we pick $r = \lambda$ and $\theta - \phi = \pm k\pi$ with $k = 1, 3, 5, ...$, we can get $n_\nu = 0$. It means the squeezed noise can be totally eliminated. The Bogoliubov mode is effectively couples to a vacuum reservoir. This effect can be understood as the phase matching: the original vacuum bath is first squeezed along the direction $\phi/2$ with squeezing factor λ, then this squeezing is totally cancelled by Bogoliubov mode squeezing along the direction $\theta/2 = \phi/2 \pm k\pi/2$ with squeezing factor $r = \lambda$. In the phase space, the elongated noise distribution becomes a circle again as seen by the Bogoliubov mode.

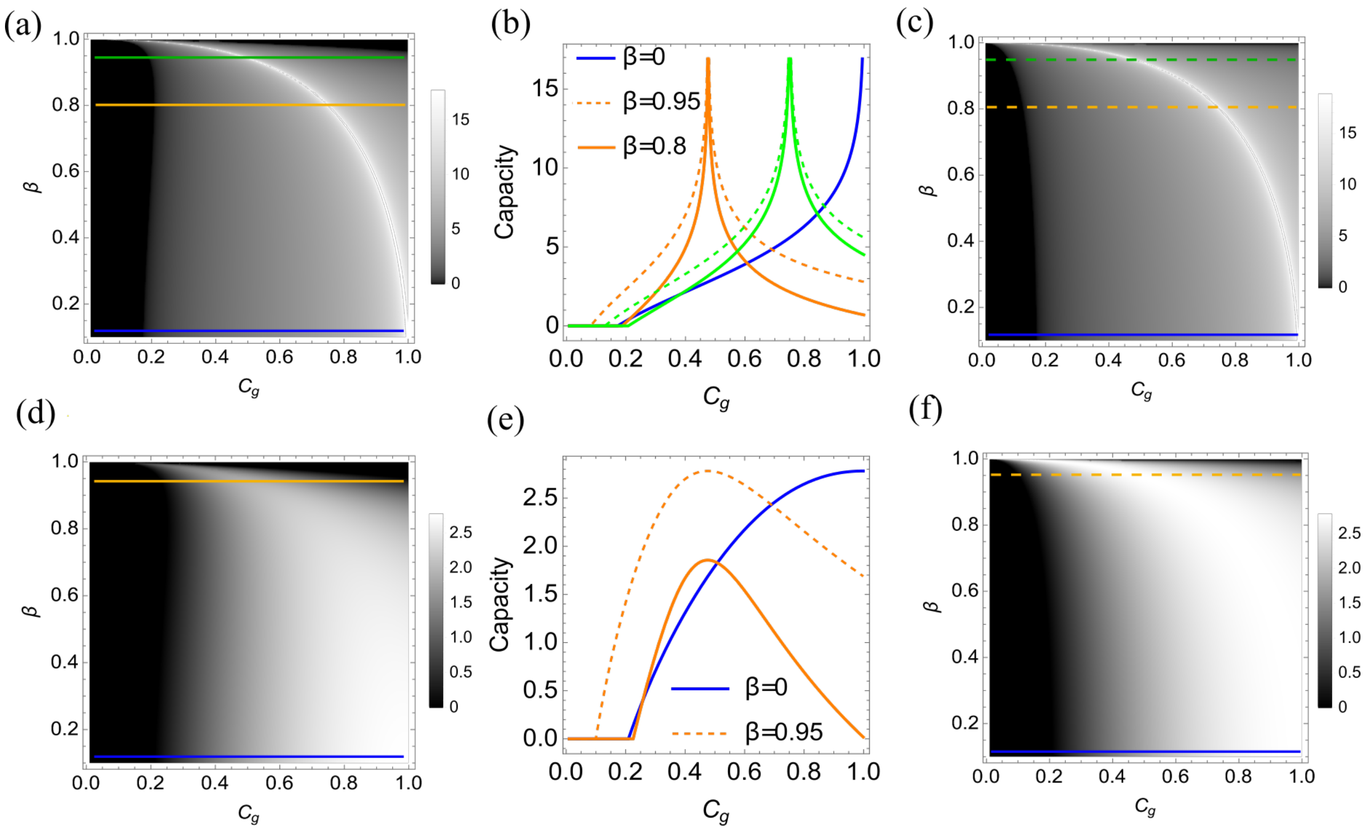

FIG. 2. The capacity lower bound by scanning the parameter C_g and β. The unit extraction ratios are use in (a),(b) and (c), while $\zeta_e = 0.97, \zeta_o = 0.9$ for (d), (e) and (f). Noise eliminations are done in (c) and (f). The curves in (b) and (e) trace the solid and dashed line in (a)(c) and (d)(f), correspondingly.

Figure 2 shows more data about the capacity enhancement with and without squeezing amplified noise elimination. The Fig. 2(b) and (c) are shown in the main text. Note the data becomes inaccurate when β really approaches one because the Bogoliubov framework will fail in the regime, as discussed in the main text.

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