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GROWTH, RISKS, AND INSTITUTIONAL DESIGN IN THE KNOWLEDGE  
ECONOMY

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To my parents, and Yanran

“The more I learn, the more I realize how much I don’t know.”

– *Albert Einstein*

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## ABSTRACT

This research explores and quantifies the downside of technological innovations, especially the negative externality of an innovation interacting with the stock of existing innovations. Using two novel datasets, I make a novel empirical finding that the varieties of innovation-induced risks (e.g. varieties of side effects caused by FDA-approved new drugs) is quadratic in the number of innovations (e.g. number of FDA-approved new drugs) that caused these risks. Based on this new empirical finding, I further develop a Regulatory Growth Theory: a new endogenous growth model with increasing varieties of innovation-induced risks and with a regulator. I model both the innovation-induced risk generating structure and the regulator's endogenous response. This new theory can help to interpret several empirical puzzles beyond the explanatory power of existing models of innovation and growth: (1) skyrocketing expected R&D cost per innovation; (2) decreasing ratio of Qualified Innovations (i.e. Regulator-approved innovations) to the number of total patents and (3) exponentially increasing regulation over time. Greater expenditures on regulation and corporate R&D are required to assess the net benefit of innovation because of "Risk Externality": negative interaction effects between innovations. Theoretically, this new "Risk Externality" effect counteracts the crucial "Knowledge Spillover" effect in the Endogenous Growth models. The rise of regulation versus litigation, and broader implications for regulatory reform are also discussed.

Secondly, I propose a new theory of rational "Rush", emphasizing the quantity of rational over-investment in contrast to the theory of irrational price "Bubble". I illustrate an important friction when financing breakthrough innovations: non-excludability and spillover of uncertain knowledge due to imperfect IPR (Intellectual Property Rights, e.g. patent) protection. Facing a limited supply of new projects with uncertain return, investors make decisions about when and how many projects to invest. Investors' preemption motive will distort their incentives for patient learning about project return, thus inducing them to "rush in" to finance uncertain projects massively at a premature stage. A small positive

news shock regarding the project return can greatly amplify over-investment and result in large social inefficiency. On the other hand, information externality creates free-rider motive, which can also make under-investment possible. Empirical finding based on sectoral Venture Capital investment shows that weak IPR protection lead to excessively high investment level and more procyclicality. Broader patent rights should be granted when the uncertainty of innovation is high, although the “Rush” prevention can induce more patent race at the early R&D stage, i.e. Rush-Race shifting.

Finally, I propose a Knowledge Theory of Regulation. Knowledge of innovation-induced risks is a public good. Therefore risk knowledge will be undersupplied if only through courts and private litigations. Compared to courts, regulators have their comparative advantage in acquiring and sharing new risk knowledge. Enlarging risk externality induced by growth and innovation accumulation will lead to faster expansion of regulation thanks to regulators’ comparative advantage in risk knowledge acquisition.

# CHAPTER 1

## INTRODUCTION

Innovation and knowledge production are the fountains of modern economic growth, and are oftentimes labeled as the key features of the “Knowledge Economy”. However, innovations can not only bring benefits to consumers, but also bring unexpected adverse effects to the society.

The drug “Thalidomide” provides such a daunting example: it was invented and introduced in the late 1950s to treat nausea for pregnant women, but was later found to be responsible for tens of thousands of babies born with deformities whose mothers had taken Thalidomide. Recently, financial innovations have been widely blamed as one of the important contributors to the 2008 Financial Crisis. Moreover, many adverse effects of innovations can keep unknown for a long time. For example, it takes us more than two hundred years to understand the global warming effect of  $CO_2$  caused by (steam) engines and fossil fuels.

In sum, innovation-induced risks and uncertainties are also essential characteristics of the Knowledge Economy. It thus requires a balanced view and a new research framework for innovation and knowledge production.

### 1.1 Structure and Sources of Innovation-induced Risks

Any innovation or man-made technology has multi-dimensional effects and attributes, positive or negative. Innovators and producers usually focus on the useful dimensions of technology. Some negative dimensions of technology are unknown and can be hidden for a long time. Evaluating the net benefit of a new technology can be complicated and costly.

In addition, technologies can interact with each other or with the environment, and generate compound and complex risks. Measuring these complex innovation-induced risks will be a more challenging task. In this section, I will discuss various sources of complex innovation-induced risks:

## **Complex Interactions between Technologies**

The interactions between products can be complex and generate many unintended consequences. Some adverse dimensions of technology can only be triggered by interactions with future innovations (e.g. Drug-Drug Interactions), but can not be revealed and noticed immediately. As the total number of products in the economy grow, potential combinational interactions between these products are growing much faster. UAVs (Unmanned Aerial Vehicle, sometimes called a drone) have recently threatened and delayed hundreds of flights over several cities of China.

## **Complex Production Technology**

Some modern technologies are essentially complex systems, e.g. nuclear and chemical plants, which are made up of many machines, multiple factories and sub-systems. During the process of production, these interconnected machines and facilities can generate many unintended failures. We can model these complex technology as a combination of many components, while the combination of some components can lead to unanticipated outcomes.

## **Complex Final Products**

Many products are growing to be complicated. For instance, the biggest airplane Airbus A380 has about 4 million parts in total. A product has a large number of attributes, many of which are unknown and hidden ones. However, it can take a fairly long time to notice some of these hidden risks. This type of risk is exemplified by the role of O-ring's failure in the Space Shuttle Challenger disaster.

## **Chain Reactions and Domino Effects**

Some technologies or elements can cause the actions of adjacent technologies. Nuclear disasters, Artificial Intelligence (AI), and Genetic Modification Technology lie in this category.

## **Systemic Risk in the Financial System**

Due to wide connections within the financial system and with the economic system, systemic risk generated in the financial sector can cause extensive damage to the economy.

### **Complex and Huge Projects**

Some huge projects, such as big dams can have large unintended effects (e.g. earthquake) on the environment and economy.

### **Long-run Effects**

Some projects have continuous impacts in the long run, like nuclear plants. Nuclear wastes can have half-life up to tens of thousands of years.

### **Common and Global Hazards**

There can be some common hazards from many technologies, e.g. the  $CO_2$ , together with many other GHGs (Greenhouse Gas). They all have similar effects on warming up the global temperature.

### **Complex Reactions in the Environment**

Our economy is embedded in a large-scale ecological system. The pollutants and other technology-induced technologies can change the complex ecological system and environment in unexpected ways. Until now, researchers have not reached consensus regarding the impact of GHG releases on global warming and climate change. Potential tipping points can amplify the negative side effects of GHGs.

Cai et al (2017) recently find that global GHG emissions have aggravated the haze episodes (pollution of Fine Particulate matter) in Beijing because global warming has changed the large-scale air circulation and weakened the East Asia winter monsoon.

## **1.2 Research Questions**

We now know that an innovation usually has both positive and negative effects. Particularly, the dark side of innovations is still much underexplored. It requires a balanced and comprehensive new research framework regarding innovation and knowledge production. The following research questions and areas should be considered:

Empirical measurement of the adverse effects of innovations, institutional design for mitigating these negative effects; the choice between court (tort and litigation) and regulation



regarding the potential side effects; new growth models that take into consideration the side effects and high uncertainties caused by innovations; and new economic models that can reflect the impact of innovations on macro and financial stability.

### **Measuring the Types of Innovation-induced Risks**

We want to know how many types of risks can be induced by innovations. Accounting of the types of innovation-induced risks should include not only the risks caused an innovation itself (which can be defined as *1<sup>st</sup> – order* risks) but also more complex risks (which can be defined as *high – order* risks) the innovation contributes to. From last section’s analysis, we know many complex risks are very difficult to identify and count. It requires very detailed datasets for this purpose. Then we also want to estimate the relationship between innovations and the types of innovation-induced risks.

### **Representing Innovation-induced Unknowns and Uncertainties**

It is more challenging that many innovation-induced risks are still unknown to the human being. Since these potential risks are still unknown, we can hardly think of ways to measure them. It is even more challenging that many of them lie in the domain of “Unknown unknowns”. Nevertheless, we could try to come up with some creative methods to represent them theoretically.

### **Measuring Useful Innovations**

Only the innovations with positive net benefit should be adopted and produced. Current literature on innovation largely uses patents to measure the quantity of innovations. Accounting the number of patents is insufficient and even misleading because a big portion of the patents can have negative net benefit and be harmful. We want to know which innovations have positive net benefit, and their proportion in all innovations (measured by the number of patents).

### **Implications for Growth Theory**

Canonical growth theories including endogenous growth theories take as given the usefulness and benevolence of innovation and knowledge. Innovation-induced risks matter a lot for

people's wellbeing, and should be included in more sophisticated growth models. Moreover, the number of types of innovation-induced risks should be an important aggregate variable behind such a new growth model.

### **Implications for Macro and Financial Stability**

The 2008 Financial Crisis has inspired a new research literature: "Macro-Finance". Nevertheless, we can study the 2008 crisis together with the 1990s "Internet Bubble" from a different perspective: both turmoils were driven by some important innovations and "over-investment" on them. Admittedly, the irrational bubble theory can offer a decent explanation for both events. But can we also develop a theory that can accounts for the phenomena of over-investment in innovations based on investors' rational choices?

### **Implications for Diversification**

Will the endogenous complexity due to the accumulation of innovations put a limit to diversification? What will be the optimal degree of diversification?

### **Implications for the Theory of Regulation**

Researchers have proposed many new explanations for the fast rise of regulation during the last half century. We want to know the real reasons behind the significant increase in regulation, and further understand the real functions of these regulatory agencies<sup>1</sup>.

The U.S. FDA has regulated the drug and food industry for more than a century since the 1906 Legislation. The establishment of the CFPB (Consumer Financial Protection Bureau) in 2011 illustrate the latest efforts in detecting, disclosing and preventing potential risks in financial products, as a response to the wide blame on financial innovation as a cause of the Subprime Crisis and Great Recession. On June 22, 2016, the Lautenberg Chemical Safety Act was just signed into law, which endowed the EPA with the new power to review and approve any new chemical product.

The practices of these major modern regulators reflect the public responses to the potential risks generated by innovations. This research might help to shed light on the fast

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1. This was suggested to the author by Prof. Gary Becker at the very beginning of this research.

and continuous rise in regulatory expenditure during the last several decades. Hopefully this study can also provide us with a theoretical foundation to build regulatory sector into a macro-growth framework.

### **Implications for Institutional Design**

Knowing the potential risks induced by innovations, what kinds of institutions should be designed to mitigate them? What kinds of institutions should be designed to balance the benefit and potential risks of innovations? What kinds of institutions can efficiently acquire and reveal new knowledge of potential risks?

Patent and IPR (Intellectual Property Rights) laws are intended to foster innovations, but can they also play a role in mitigating potential adverse effects of innovations? Are the current torts and liability system good enough for solving these problems?

## **1.3 Organization of the Dissertation**

This dissertation intends to respond to aforementioned research questions. The rest of the dissertation is organized as the following.

Chapter 2 empirically measures the types of innovation-induced complex risks using the datasets of pharmaceutical innovations, DDI (Drug-drug Interactions), and the FDA's Adverse Events Reporting System. I further develop a Regulatory Growth Theory: a new endogenous growth model with increasing varieties of innovation-induced risks and with a regulator.

Chapter 3 proposes a new theory of rational "Rush", emphasizing the quantity of rational over-investment in contrast to the theory of irrational price "Bubble".

Chapter 4 empirically shows that weak IPR (Intellectual Property Rights) protection can lead to excessively high investment level and more procyclicality, and then uses patent policy as a regulatory tool to control economic and financial risks caused by uncertain innovations.

Chapter 5 illustrates regulator's comparative advantage in acquiring new knowledge of

potential risks, which will be undersupplied by the court system. Then I go on to discuss various institutional choices to correct the inefficiency regarding risk knowledge acquisition, and further classify regulatory regimes according to their information generating process.

Chapter 6 summarizes the major contributions and findings of previous chapters, and concludes.

# CHAPTER 2

## THE DARK SIDE OF INNOVATION: MEASURING, MODELING, AND REGULATING DANGEROUS INTERACTION EFFECTS<sup>1</sup>

### 2.1 Introduction

Undoubtedly innovations are essential for our economic and knowledge growth. On the other hand, is there also a potential downside of innovations? In particular, can there be a negative consequence of an innovation interacting with existing products, that is, stock of past innovations? Is the potential negative externality increasing in the stock of past innovations?

An innovation that by itself is effective at solving a problem could cause a broader negative effect. A recent strict regulatory action on the combined use of opioids with benzodiazepines has just attracted wide public attention<sup>2</sup>. The FDA noticed that the emergency department visits and overdose deaths due to the combined use of opioids and benzodiazepines have tripled between 2004 to 2011. These are two types of frequently prescribed medicines that have significant health benefits when used alone; however, their simultaneous administration generates a new variety of risk that is called Drug-drug interaction (DDI).

In discussing the major contributors to the 2008 Financial Crisis, Kroszner (in Kroszner and Shiller, 2011) points out that Credit Default Swap (CDS) is a valuable hedging device but its interaction with existing financial innovations could lead to “fragile interconnections”

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1. The earliest version of this chapter, Xie(2015a) was presented at Econometric Society 2015 World Congress, Montreal.

2. <http://www.nytimes.com/2016/09/01/health/fda-orders-stronger-warning-on-common-painkiller-sedative-mix.html>

and “systemic risk” that make the system less sound. In other words, the financial innovation is useful individually but could undermine stability of the financial system.

The Great Smog of London in December 1952 killed 12,000 residents. Recent research by Wang et al (2016) suggests the major "Killer" behind the 1952 London Fog was due to the interaction between  $SO_2$  and  $NO_2$ , through combined photochemical and aqueous processes. The authors further show the harmful chemicals in the current severe haze in China are the products from complex interactions between  $SO_2$ ,  $NO_2$ , and  $NH_3$ .

Motivated by the examples above, I take a step forward to measure and quantify these innovation-induced risks, as the first main contribution of this research. I find a new empirical fact: the varieties of innovation-induced interactive risks grow much faster than the varieties of innovations (see Figure 2.1). I bring two novel datasets into the economic literature. One is the DrugBank, which documents detailed chemical and biological information of most existing drugs, including known theoretical DDIs that these drugs can cause. The other is the FDA’s Adverse Event Reporting System (FAERS) database, which records adverse event cases reported from patients. By linking and analyzing these databases, I make a pioneering effort to measure the number of varieties, dynamics and actual occurrences of risks induced by drugs. I emphasize *the varieties of risks* induced by innovations. Empirically, I find that the relationship between the number of different types of DDIs and the number of drugs generating these DDIs are best characterized by a quadratic form function as illustrated by Figure 2.6.

This new empirical fact draws parallel to *Environmental Kuznets Curve (EKC)*. *EKC* points to an *inverted-U* relationship between growth and the quantity of known pollutants (Grossman and Krueger 1995). I explore the relationship between growth and the varieties of risks, which is revealed to be an increasing and quadratic function.

Canonical growth theories take as given the usefulness and benevolence of knowledge and innovations, but it neglects the adverse fact that technological innovations might also create negative externalities to the economy due to its direct side effects as well as complex

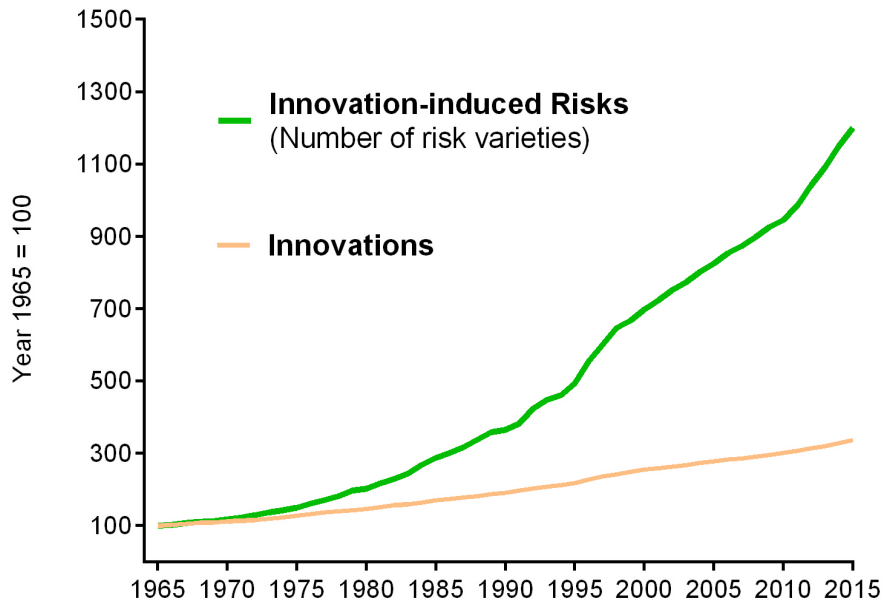


Figure 2.1: Varieties of Innovations vs. Varieties of Innovation-induced Risks

interactions with existing technologies and environment.

As the second major contribution, I develop a new growth theory: Regulatory Growth Theory, which models both the innovation risk generating structure and process and the regulator’s endogenous response. I introduce into growth theory the risk generating process which accompanies innovations, and model the testing and regulatory actions which control the macro risk structure induced by innovations. I build a model regarding the approval process for new products, reflecting an important regulatory function of the FDA on drugs and the EPA on chemicals (per the new 2016 Lautenberg Act). I also model the interactions between regulators and innovators. Innovators will perform testing on their innovations in line with regulatory requirements and then submit the testing outcomes for regulators to review and make final approval decisions.

The implications of my theoretical model are consistent with the following two empirical puzzles that canonical growth models fail to explain:

First, from 1950 to 2010, the expected R&D cost for each FDA-approved drug (NME) has skyrocketed from 20 million to almost 2.5 billion in 2008 constant US dollars, as shown

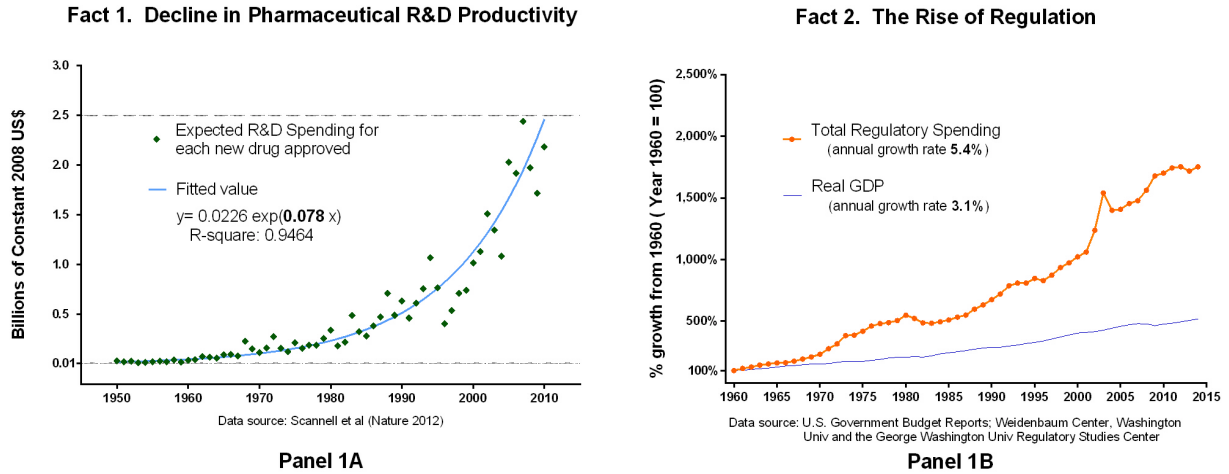


Figure 2.2: Empirical Puzzles

by Figure 2.2A. The R&D spending for each successful drug has increased by more than 100 times during the last sixty years. In contrast, there is only a *five-fold* increase in GDP in the same period. This has been known as the *pharmaceutical R&D productivity crisis*.

Second, the amount of regulation and the size of the regulatory sector have been growing faster than GDP. From 1960 to 2014, the (inflation-adjusted) budget cost for federal regulatory agencies has increased by almost twenty times, while the real GDP only by five times (see Figure 2.2B). Since there is no role for a regulator in canonical growth models, the previous models cannot accommodate such fact.

The key channel in my theoretical model that causes the exponential growth of R&D costs and innovation-related regulation is that innovations are not only complementary in their benefits but also in their risks. The fast growth of risks due to innovations demands more regulatory resources and also reduces the ratio of “qualified” innovations.

As the third major contribution, I put forward a new *knowledge theory of regulation*: new varieties of risks are continuously created by innovations, and the main purpose and function of regulation is to discover and control these new risks. This paper therefore fits in the recent literature which tries to explain the root and the rise of regulation (e.g. Mulligan and Shleifer 2005, Shleifer 2012), but I have different explanations. Discovering new varieties



of risks generated by technological innovations are becoming more costly and important than regulating known risks. Science, technology, and social science knowledge tell us what we can do, i.e. through innovations and new products, as well as the knowledge of risks and what we cannot do, which build the foundation of modern regulation. In this sense, the modern regulatory sector is an institutional setup representing and executing human knowledge about the negative side of human-made technologies and related activities.

FDA has regulated the drug and food industry for more than a century since the 1906 Legislation. The establishment of the CFPB (Consumer Financial Protection Bureau) in 2011 represents the latest efforts in detecting, disclosing and preventing potential risks in financial products, as a response to the wide blame on financial innovations as a cause of the Subprime Crisis and Great Recession. On June 22, 2016, the Lautenberg Chemical Safety Act was just signed into law, which endowed the EPA with the new power to review and approve any new chemical product. Besides the FDA, the newly established CFPB together with the EPA augmented by the 2016 Lautenberg Act allude to an analogous *qualification process* of regulation which inspires the theoretical model developed in this research.

There are several interesting and crucial features that can only be derived from my theory. There will be a divergence between the *General Knowledge*, and the *Qualified Knowledge*. The number of patents represent the *General Knowledge*, while the qualified innovations (regulator-approved innovations) representing the *Qualified Knowledge* which can provide us net benefits. Macro risk structure due to complex interaction effects between innovations in use can make this divergence grow larger, given effective regulation (see Figure 2.13). The total human knowledge (*General Knowledge*) can keep growing fast but the GDP growth can lag much behind it owing to rapid expansion of the varieties of new risks.

The rest of this chapter is organized as follows. Section 2 presents our empirical measurement of interactive risks induced by drugs. Section 3 builds the Regulatory Growth Theory that models the innovation risks and a regulator. Section 4 discusses the implications for corporate R&D, regulation, and potential regulatory reform proposals. Section 5 concludes.

## The Literature

Becker and Murphy (1992) points to the increasing coordination cost due to the deepening of division of labor, and discusses its implications in a growth framework.

Romer (1986,1990) starts the literature on endogenous growth, which emphasizes the benefit of knowledge spillover. Potential hazards of innovation is largely omitted by the literature. However, the spillover of “innovation” can also have widespread adverse effects. Jones (2016) takes a first step forward to also consider potential risk of innovations, and models life-saving innovations.

Weitzman (1998) proposes a "Recombinant growth" model, which shares some features with this research. However, there are several essential differences. Weitzman (1998) does not deal with adverse effects of innovations. The unexpected adverse effects from combinations creates daunting complex interactions in an autonomous and hidden way, and distinguish this research from Weitzman's. After inventing and adopting some new technologies, we are actually forced to play a blind game with Mother Nature, who secretly generates all possible innovation-induced risks behind us.

In environmental research, interactions between different pollutants are called synergistic effects. List and Mason (1999) analyzes a dynamic game for the optimal choice of central and local regulation based on the synergistic effects between different pollutants.

In contrast to the literature of Environmental Macroeconomics (e.g. Acemoglu et al 2012, Golosov et al 2014) which commonly focus on one or several varieties of known pollutants or risks, this research provides a general framework for analyzing risk varieties dynamically generated by continuous innovations in the economy. Discovering new types of risks are an essential part of our framework, while the existing literature is limited to certain known types of risks.

There is a recent literature on Uncertainty shocks and policy uncertainty, pioneered by Bloom (2009), and Baker, Bloom and Davis (2012). This research is complementary to this research line by trying to offer an economic explanation for the origin and rise of uncertainty

over the last several decades.

Barro (2006, 2009) starts the literature of research on rare disasters. The Subprime Crisis advances new interests in tail risk measurement and modeling. However, this literature generally assumes the rare disasters are exogenous. This research proposes a new way to generate tail risk with endogenous complex risks.

Shleifer (2012) documents the rise of regulation relative to litigation. In comparison to the theories advanced in Shleifer (2012) and the classical Capture Theory by Stigler (1971), this research is relatively close to Pigou (1924): but my model integrates the explicit externality generating process with innovation and growth. I emphasize the public good nature of discovering new varieties of risks.

## 2.2 Measuring Innovation-induced Risks

In this section I measure the interactive risks induced by innovations, using the DrugBank and FDA FAERS databases. I characterize and derive the relationship between the varieties of interactive risks caused by drugs (i.e. Drug-drug Interactions) and the varieties of drugs.

### *2.2.1 Prevalence and Regulation of Drug-drug interactions (DDI)*

Bjorkman et al (2002) compile a large dataset of six European countries and find out that up to **46%** of all patients had encountered at least one potential DDIs, **10%** among which were severe ones. Moreover, the DDI occurrence has been rising quickly. Guthrie et al (2015) show that serious DDIs more than doubled in 15 years from 1995 to 2010: **13.1%** of adult patients encountered at least one DDI in 2010, while **5.8%** of adult patients did in 1995.

Huang and Lesko (2004) points out "serious Drug-drug interactions have contributed to half of the recent U.S. market withdrawals and also recent nonapprovals of a few new molecular entities. ... In addition to drug-drug interactions, drug–dietary supplement and drug–citrus fruit interactions, among others, are emerging as important issues to consider in

the evaluation of new drug candidates."

### 2.2.2 *Data Description*

#### The DrugBank Database

DrugBank is a comprehensive database that includes detailed chemical and biological information of most existing drugs in the world. It is deemed as the gold standard in many aspects by Pharmacy researchers. It covers all FDA approved small molecule and biotech drugs (NMEs). In addition, DrugBank documents known DDIs for its covered drugs. These DDI information comes from drug labels, clinical trials, and postmarketing surveillance.

It has standard classification system for drugs based on their molecular structure. This can help to solve the challenging problem of duplicate drug names (many different names correspond to the same drug chemical molecule).

I use the latest DrugBank Version 5.0.1 for this research.

#### The FDA's FAERS Database

The FDA's Adverse Event Reporting System (FAERS) records the reported adverse events and medical errors<sup>3</sup>. FAERS is a crucial part of the post-marketing surveillance infrastructure. The author also gets the full historical time series statistics for the Adverse Events Reports by direct data request to the FDA<sup>4</sup>, shown as in Figure 2.3. The fitted value for the adverse events reporting is close to an exponential curve.

The number and types of risks due to innovations grow much faster than GDP and innovations. This empirical fact is new by itself: I use the drug adverse events reported to the FDA. Figure 2.3 shows the annual number of adverse events per thousand people. Adverse event reports per capita has increased more than 100 times in the last decades.

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3. Detailed data description is provided in Appendix C.

4. The author is grateful for the generous help of the FDA staff.

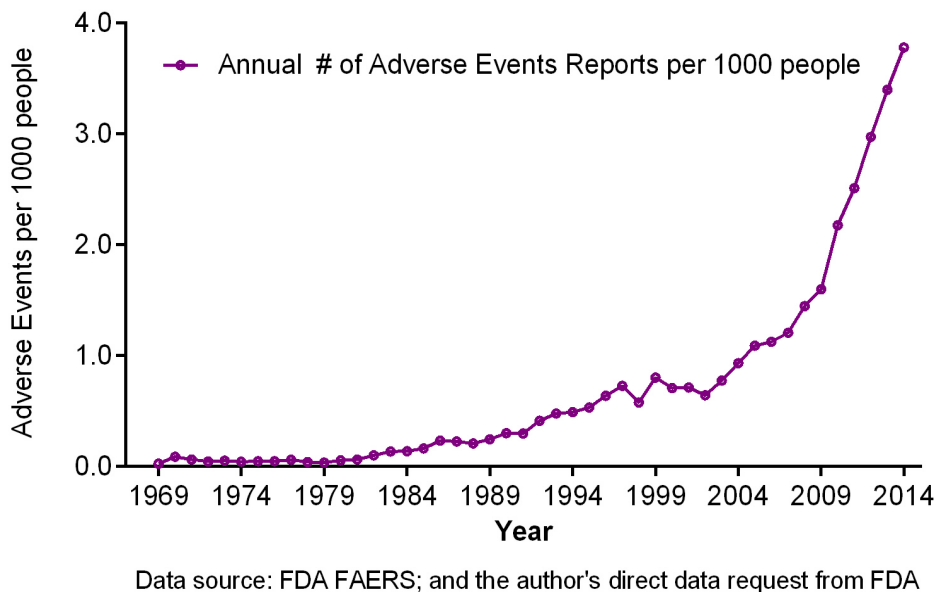


Figure 2.3: The Growth of Drug Adverse Events Reported

FAERS database includes detailed reports for each adverse event entered by the FDA from 2004 to the most recent quarter. For each adverse event case report, all the drugs that the patient was taking are recorded. This is the basis for identifying DDIs from each adverse event case.

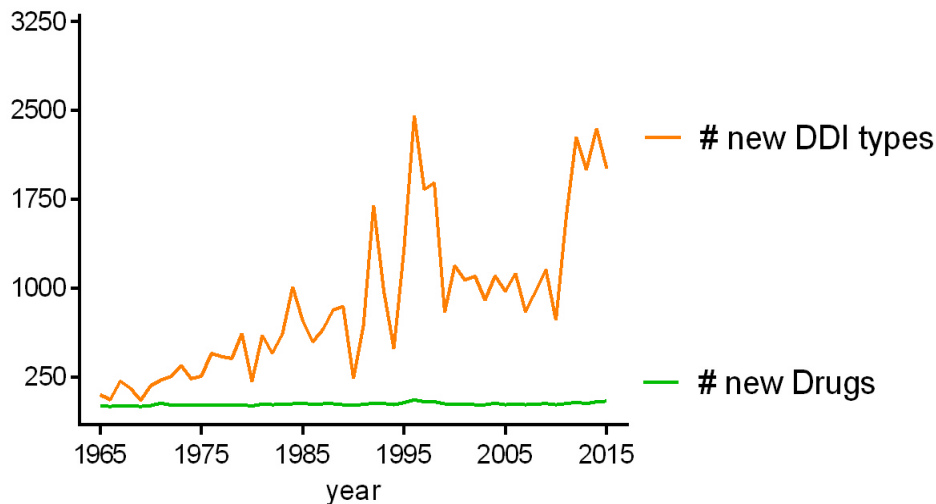
### 2.2.3 *The Dynamics of Incremental DDIs*

DrugBank documents DDIs for each FDA approved NME. I link the Drugbank's DDIs with the FDA approved NMEs for each year. Figure 2.4 displays the annual number of NMEs approved by FDA, in comparison with the new DDIs generated by the new NMEs. That is, the upper curve in Figure 2.4 shows the incremental DDIs that are directly attributable to the new NMEs approved in that year.

### 2.2.4 *The Varieties of DDIs and the Varieties of Drugs*

In the introduction section, Figure 2.1 shows how the number of DDI types grow with the number of drugs. During the last five decades, the number of DDI types have increased by

Figure 2.4: New DDI types v.s. New Drugs (NME) in each year



12 times whereas the total number of NMEs only 3.4 times. DDI types grow at a much higher speed.

We want to estimate a function form of DDI types and drugs:  $DDI = f(drugs)$ . Various specifications have been tried. Figure 2.5 uses a linear function form.

Figure 2.6 adopts a quadratic function form, and can fits the data much better than the linear form.

We can empirically derive a quadratic form function (2.1) :

$$DDI = 0.0172 \cdot (NME)^2 + 0.7618 \cdot NME - 1236.8 \quad (2.1)$$

the high  $R^2$  indicates (2.1) can approximate the data of DDI types and NMEs quite well.

### 2.2.5 Real DDIs Identified from the Adverse Events Database

More importantly, I also want to count the DDIs occurring in reality. For this purpose, I link the FDA's FAERS database with the DrugBank database. I use the drug names and theoretical DDIs documented in the DrugBank to match and identify the cases of DDIs reported to the FAERS. Each adverse event in the FAERS records detailed information for

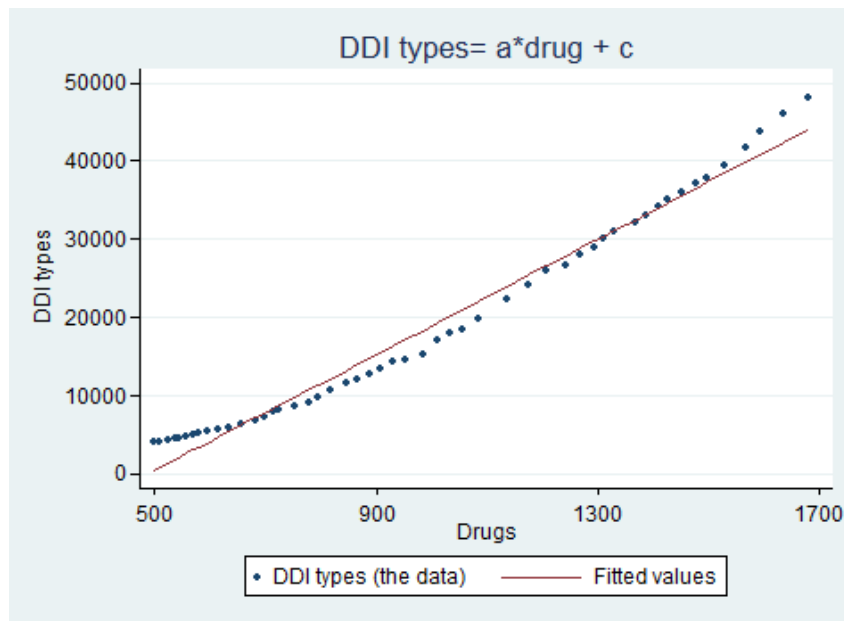


Figure 2.5: Linear Specification

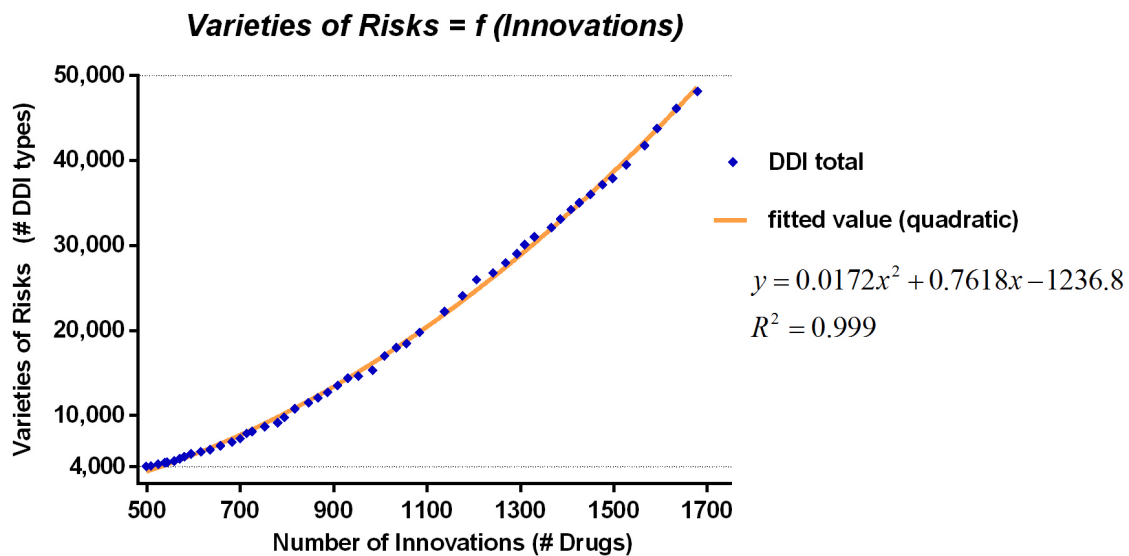


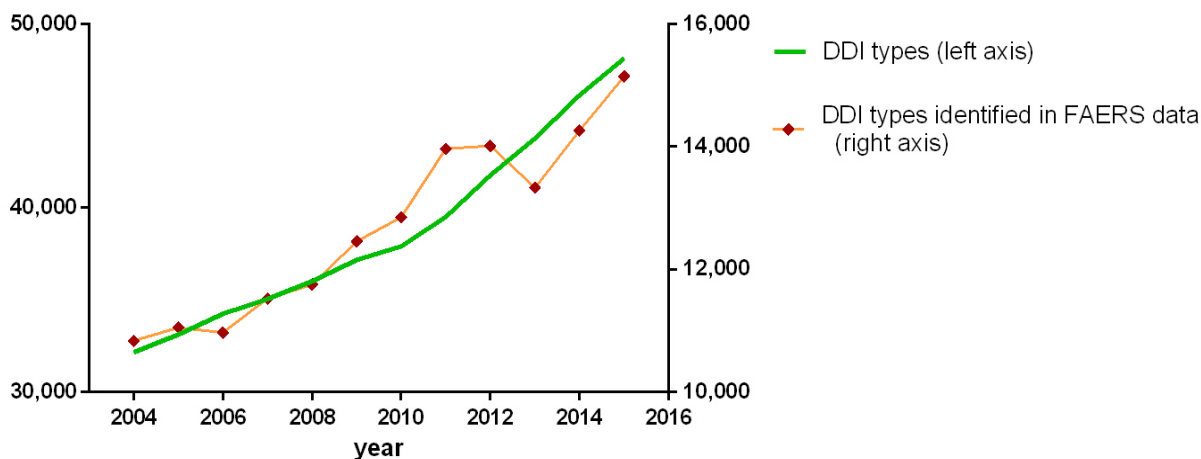
Figure 2.6: Varieties of Innovation-induced Risks as a Function of Innovations

the drugs used by the patient, as well as the patient’s demographic information. If a patient took several drugs simultaneously, the corresponding adverse event report is supposed to document all the drug names (though some drugs might be omitted for reporting).

Our *identification strategy* is to use each theoretical DDI from the DrugBank to match real DDI in each FAERS adverse event. Our purpose is to find all DDI occurrences in the real adverse events data. In the DrugBank, the number of all types of theoretical DDIs for the FDA approved drugs is close to 50,000. The FDA’s FAERS has recorded up to 7 million adverse events since 2004. Linking and matching these two database brings us a formidable computational challenge<sup>5</sup>. I have designed an efficient algorithm to reduce the computing time.

Figure 2.7 shows the real DDI types identified from the FAERS data. In general, there are one third of the theoretical DDIs that can be identified from the actual FAERS database. This ratio of one third is relatively constant over the period 2004-2015.

Figure 2.7: Theoretical DDI types and Real DDI types identified in FAERS Database



This result also implies that a significant portion of the theoretical DDI combinations are actually prescribed and used together in the real life. DDIs can impose substantial cost. The recent Black box warning on the DDI of opioids and benzodiazepines by the FDA highlights

5. Here I must thank our supercomputer Acropolis at UChicago.



this challenge.

## 2.3 The Model

Safety and efficacy information of innovative products is costly to acquire and verify. Consumers delegate this task to a group of experts: the regulator. The regulator reviews and evaluates the safety and efficacy information for new products provided by the innovators. The objective of the regulator is to accurately choose the useful and safe products. According to the preference of consumers, regulators require the innovative firms to provide testing information for their innovations. Then the regulator verify the testing information provided by the firms. They compare the benefit and cost of each innovation based on the testing information and decide whether to approve it. The regulator in this model resembles the FDA, and I have a detailed description of this regulatory regime in Appendix A.

### 2.3.1 Consumer Preference

There is one representative consumer in the economy. Her instantaneous utility is represented by  $u_t(C_t - \tilde{D}_t) = \frac{(C_t - \tilde{D}_t)^{1-\gamma}}{1-\gamma}$ . It is composed of two parts: consuming the final goods generates utility  $C_t$  as well as damage (or disutility)  $\tilde{D}_t$  to the consumer. What really matters for consumer is the net util  $C_t - \tilde{D}_t$ . At time  $t$ , consumer only consume one type of final goods.

The infinite-horizon expected utility for the representative consumer is,

$$EU = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \tilde{D}_t)^{1-\gamma}}{1-\gamma} \quad (2.2)$$

### 2.3.2 Final Goods Production

At any time  $t$ , there is a unique final goods  $Y_t$ , which is produced with a range of intermediate products and labor. Production of this final goods is represented by an aggregate production function (2.3) :

$$Y_t = \frac{1}{\alpha} l_t^{1-\alpha} \cdot \sum_{i=1}^{M_t} (x_{i,t})^\alpha \quad (2.3)$$

where  $l_t$  is labor input,  $M_t$  is the number of varieties of intermediate products used to produce the final goods.  $x_i$  is the quantity of intermediate product  $i$  used in aggregate production. Intermediate product is produced directly from the final good and will fully depreciate after use.

The final goods is produced under perfect competition. At any time  $t$ , given intermediate product price  $p_{i,t}$  and wage  $w_t$ , a final goods producer's maximization problem is (2.4),

$$\max_{l_t, [x_{i,t}]_{i \in [0, M_t]}} \frac{1}{\alpha} l_t^{1-\alpha} \cdot \sum_{i=1}^{M_t} (x_{i,t})^\alpha - \sum_{i=1}^{M_t} p_{i,t} x_{i,t} - w_t l_t \quad (2.4)$$

### 2.3.3 Generation of Risk Structure and Damage

When consuming the final goods to get utility  $C_t$ , consumer also bears a damage  $\tilde{D}_t$ . Damage is calculated as the aggregate adverse effects caused by all innovation-induced risks. The risk structure is created by the interactions between different intermediate goods (e.g. the Drug-drug interactions). These interactions happen automatically. Empirical findings from Section 3 point to a quadratic form relationship (2.1) between innovations and their induced risk types.

Higher-order ( $> 2$ ) interactive risks are not ruled out in theory, but in this research I stick to our main empirical evidence<sup>6</sup>. Notice that the measured DDI types and real occurrences are the outcome after FDA's qualification process. The number of risk types and occurrences have been significantly reduced by FDA's requirements and rejections (e.g. the Thalidomide

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6. Appendix E also provides some real cases for Higher-order ( $> 2$ ) DDIs.

case).

## Macro Risk Matrix

I use a 2 – *dimensional* Macro Risk Matrix to represent various types of risks due to interactions between intermediate products<sup>7</sup>. Here I adopt a standard index system for the matrix elements. Intermediate product  $i$  has uncertain interactive side effect with  $j$ , which is represented by potential damage  $\tilde{d}_{ij}$ . The Macro Technology Matrix  $\Xi_M$  with  $M$  intermediate products is represented as below:

$$\Xi_M = \begin{bmatrix} \tilde{d}_{11} & \tilde{d}_{12} & \cdot & \cdot & \tilde{d}_{1,M-1} & \tilde{d}_{1,M} \\ \tilde{d}_{21} & \tilde{d}_{22} & \cdot & \cdot & \tilde{d}_{2,M-1} & \tilde{d}_{2,M} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \tilde{d}_{M-1,1} & \tilde{d}_{M-1,2} & \cdot & \cdot & \tilde{d}_{M-1,M-1} & \tilde{d}_{M-1,M} \\ \tilde{d}_{M,1} & \tilde{d}_{M,2} & \cdot & \cdot & \tilde{d}_{M,M-1} & \tilde{d}_{M,M} \end{bmatrix}$$

which is a 2 – *dimensional* random matrix. Each element  $\tilde{d}_{ij}$  is a random variable, and has the following properties,

1. Symmetry:  $\tilde{d}_{ij} = \tilde{d}_{ji}$
2.  $\tilde{d}_{ii}$  denotes the side effect of intermediate  $i$  itself
3. Matrix elements are independent **but not** necessarily identically distributed
4. The distribution of any matrix element is unknown ex ante

Different matrix element follows disparate distributions. In fact, this is an important cause of complexity. When the number of matrix elements grow, I can not easily infer the probability distribution of new elements from existing ones.

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7. Appendix E presents a generalized Higher-order ( $> 2$ ) *Macro Risk Tensor* representation with random tensor.

$\tilde{d}_{ij}$  will either be 1 (with side effect) or 0 (without side effect) :

$$\tilde{d}_{ij} = \left\{ \begin{array}{l} 1 : i \text{ and } j \text{ has adverse side effect with Probability } q_{ij} \\ 0 : i \text{ and } j \text{ has no side effect with Probability } 1 - q_{ij} \end{array} \right\}$$

$\tilde{d}_{ij}$  follows a Bernoulli distribution with parameter  $q_{ij}$  :  $\tilde{d}_{ij} \sim \text{Bernoulli}(q_{ij})$ .

## Aggregate Risk and Aggregate Damage

I define *aggregate risk* as the total number of elements with value 1 in the Macro Risk Matrix.

Aggregate risk is denoted by  $D_t$ , calculated as the following:

$$D_t = \frac{1}{2} \sum_{i=1}^{M_t} \sum_{j=1}^{M_t} \tilde{d}_{ij} \quad (2.5)$$

The *aggregate damage* is calculated according to the Macro Risk Matrix by equation (2.6),

$$\tilde{D}_t = \frac{1}{2} \sum_{i=1}^{M_t} \sum_{j=1}^{M_t} x_i x_j \cdot \tilde{d}_{ij} \quad (2.6)$$

which quantifies the real damage with the actual intermediate product usage.

### 2.3.4 Intermediates Producer (Stage 1): R&D and Patenting

Each *qualified intermediate product* is produced by one innovative intermediate producer. A potential intermediate producer firstly needs to successfully invent an *intermediate product candidate*, by hiring researchers to conduct R&D. If R&D succeeds, the firm owns the full Intellectual Property of the *intermediate product candidate*. The patent will belong to the firm forever. If the *intermediate product candidate* is approved by the regulator, it becomes a *qualified intermediate product*. An *intermediate product candidate* cannot be produced until it becomes a *qualified intermediate product*. Similarly, final goods can only be produced from *qualified intermediate products*.

$\Delta E_t$  new varieties of *intermediate product candidates* will be invented with human capital input  $h_t$  :

$$\Delta E_t = E_{t+1} - E_t = (\lambda E_t) \cdot h_t \quad (2.7)$$

Here I assume the arrival rate of the number of *intermediate product candidates* follow a poisson process.  $\lambda$  is a parameter for R&D efficiency. Eqn (2.7) implies knowledge spillover effect because the aggregate R&D efficiency  $\lambda E_t$  is augmented by  $E_t$ .

### 2.3.5 The Regulator

The regulator requires intermediates producers to test their new intermediate candidates. Clinical trials requirements for drugs exemplifies this. Intermediates producers need to submit testing information in the form of new product applications, e.g. *NDA*s (New Drug Applications) to FDA, and *PMN*s (Premanufacture Notices) to EPA according to the Lautenberg Act. Then the regulator reviews and verifies these applications. According to the submitted information, the regulator will decide whether the applications can satisfy the standards and be qualified for a final approval.

#### Requiring, Reviewing and Verifying Testing Information

For a new intermediate product candidate  $k$ , the regulator requires the firm to test its benefit  $b_k$ , as well as all the risks  $\tilde{d}_{kj}$ , ( $j = 1, \dots, M_t$ ). Firm can do either *effective testing* or *faked testing*. I assume,

**Assumption 2.1** (Effective testing). *After effective testing, the random variable  $\tilde{d}_{kj}$  will be revealed to be a constant number  $d_{kj} \in \{0, 1\}$ ; After faked testing, there is no more information acquired for  $\tilde{d}_{kj}$ .*

To guarantee firms conduct effective testing, the regulator will review and verify the testing information provided by firms. For a new intermediate candidate  $k$ , the regulator needs

to verify the information set  $I_t = \{b_k, \tilde{d}_{k1}, \tilde{d}_{k2}, \dots, \tilde{d}_{k, M_t}\}$ . Applications with faked testing results will be dismissed immediately.

I make the following assumptions:

**Assumption 2.2.** *With unit labor cost  $\omega$  to review each element in set  $I_t$ , testing results can be accurately verified by the regulator.*

**Assumption 2.3.** *According to the legislative mandates, regulator will review and verify all testing information in the new product applications.*

Labor needed for reviewing and verifying testing information constitutes the main regulatory staff with size  $z_t$ . According to the three assumptions in this subsection, to accurately review  $\Delta E_t$  new product applications at  $t$  will demand a regulatory staff size<sup>8</sup> :

$$z_t = \omega (1 + M_t) \cdot \Delta E_t \quad (2.8)$$

## The Decision Rule for Approval

The regulator follows a simple decision rule (2.9)<sup>9</sup> to approve an application: only approving a new product when the benefit is larger than the sum of risks:

$$\text{approve iff } \left( b_k > \sum_{j=1}^{M_t} d_{kj} \right) \quad (2.9)$$

According to the decision rule (2.9), the number of newly approved drugs at time  $t$  is  $\Delta M_t$  :

$$\Delta M_t = \sum_{i=1}^{\Delta E_t} \left[ \mathbf{1}_{\left\{ b_k > \sum_{j=1}^{M_t} d_{kj} \right\}} \right] \quad (2.10)$$

To characterize the relation between  $\Delta M_t$  and  $\Delta E_t$ , I define a function  $Q_t$  as below:

---

8. Later I will omit the “1” in (2.8) for convenience.

9. This simple decision rule might not be optimal. I can extend this to a generalized optimal decision rule.

**Definition 2.4.** *The Qualified Innovation Ratio  $Q_t$  is defined as (2.11),*

$$Q_t = \frac{\text{new drugs approved at } t}{\text{new drug patents issued at } t} = \frac{\Delta M_t}{\Delta E_t} \quad (2.11)$$

$Q_t$  maps the number of newly issued patents  $\Delta E_t$  to the number of newly approved (or qualified) products  $\Delta M_t$  at time  $t$ . The puzzling fact of a sharply declining  $Q_t$  for drugs over time was presented at the very beginning of this paper.

## Risk Generating and a Binomial Distribution

Each new drug  $k$  has a probability  $q_{kj}$  of interacting with any existing drugs  $j$ . Our finding in the empirical section shows that  $q_{kj}$  is quite constant. Moreover, these probabilities are independent from each other. Then I use  $q$  to approximate  $q_{kj}$ .

Therefore the sum of risks  $\sum_{j=1}^{M_t} \tilde{d}_{kj}$  caused by drug  $k$  (when the number of varieties of existing drugs is  $M_t$ ) follows a Binomial distribution (2.12),

$$\sum_{j=1}^{M_t} \tilde{d}_{kj} \sim B(M_t, q) \quad (2.12)$$

## Probability of Approval and Qualified Innovation Ratio $Q_t$

Because I mainly investigate the dynamics of innovation-induced risks, for simplicity I assume the benefit of a new drug is relatively constant  $b_k \approx b$ . Then the decision rule for approval (2.9) implies that the probability of approving a drug at time  $t$  is equal to a Binomial *CDF* function  $F(b; M_t, q)$  :

$$F(b; M_t, q) = \Pr(\sum_{j=1}^{M_t} \tilde{d}_{kj} < b) \quad (2.13)$$

The Qualified Innovation Ratio  $Q_t$  should be determined by the probability of approving a drug, so I have:

$$Q_t = F(b; M_t, q) \quad (2.14)$$

With this setup,  $Q_t$  is simply equal to the Binomial *CDF* function  $F(b; M_t, q)$ . Moreover, the number of approved drugs at time  $t$  will be:

$$\Delta M_t = F(b; M_t, q) \cdot \Delta E_t \quad (2.15)$$

### 2.3.6 Intermediates Producer (Stage 2): Testing, Application and Production

The innovative intermediates producers will conform to the requirements of the regulator to test the newly invented intermediate candidates. According to the testing results, firm will decide whether to submit an application. If they submit an application and the application gets approved, the firm will produce the intermediate product and sell to the final goods producers.

#### Testing

The firm will test the required set  $I_t = \{b_k, \tilde{d}_{k1}, \tilde{d}_{k2}, \dots, \tilde{d}_{k, M_t}\}$ . Firm can conduct either *effective testing* or *faked testing*. Assuming conducting effective testing for each item in  $I_t$  costs the firm  $c$  units of labor, whereas faked testing costs 0 labor. The firm can choose the total number of items  $\kappa$  for effective testing. The total labor hired by the firm for testing is denoted by  $n_t$ :

$$n_t = c \cdot \kappa \quad (2.16)$$

Then the intermediate producer submits the testing results of all their new intermediate candidates to the Regulator.

#### Production after Approval

If the new intermediate product is approved by the regulator, the innovative firm can produce the new intermediate input. I assume that one unit of any intermediate input is produced



by 1 unit of final goods. Innovative firm monopolizes the production of the new products it just invented. Each intermediate product firm chooses the optimal quantity  $x_{i,t}$  to maximize the profit flow  $\pi_{i,t}$  :

$$\max_{x_{i,t}} \pi_{i,t} = p_{i,t} x_{i,t} - x_{i,t} \quad (2.17)$$

where  $p_{i,t}$  is equal to the marginal productivity of intermediate product  $i$  in final goods production.

Then the Net Present Value (NPV) of a qualified innovation approved at  $t$  is:

$$V_{i,t} = \sum_{\tau=t+1}^{\infty} \frac{\pi_{i,\tau}}{\prod_{s=t+1}^{\tau} (1+r_s)} \quad (2.18)$$

I assume the entry into R&D sector is free. The NPV discounted by the Qualified Innovation Ratio  $Q_t$  should be equal to the sum of R&D, testing cost, and the regulatory staff's compensation<sup>10</sup>. This gives the Research-arbitrage equation (2.19),

$$Q_t \Delta E_t \cdot V_{i,t} \leq w_t (h_t + n_t + z_t) \quad (2.19)$$

If “<” happens in (2.19), there will be no R&D.

### 2.3.7 Resource Constraints

Resource constraint in the final goods sector is,

$$C_t + X_t \leq Y_t \quad (2.20)$$

---

10. According to The Prescription Drug User Fee Act (PDUFA), since 1992, the FDA has been authorized to collect fees from drug companies to fund the new drug approval process. The 2016 Lautenberg Act also granted the EPA similar authority to collect fees to finance the approval process for new chemicals. That is, though the FDA and EPA are independent, fees for reviewing new products are funded by the applicants.

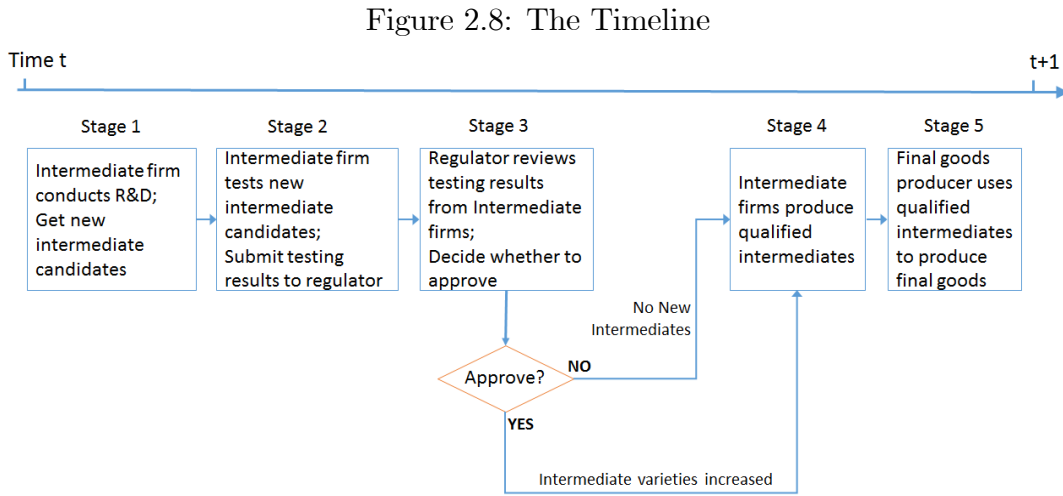
I assume away population growth. At any time, consumers inelastically supply labor at constant quantity  $L$ . The labor market clears at any time  $t$ :

$$l_t + h_t + n_t + z_t \leq L \quad (2.21)$$

where  $l_t$  represents labor for final goods production,  $h_t + n_t$  is the total labor hired by intermediate producers, and  $z_t$  is the staff hired by the regulator.

### 2.3.8 The Timeline and Evolution of Macro Risk Matrix

The Timeline is illustrated by (2.8).



After new intermediate products are invented and finally approved by the regulator, the **Macro Risk Matrix**  $\Xi_{M_t}$  evolves, as the following,



$$\max_{\{h_t, n_t, l_t, C_t, [x_{i,t}]_{i \in [1, M_t]}\}} \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \tilde{D}_t)^{1-\gamma}}{1-\gamma} \quad (2.23)$$

$$\tilde{D}_t = \frac{X_t \cdot D_t}{M_t} \quad (2.24)$$

$$X_t = \sum_{i=1}^{M_t} x_{i,t} \quad (2.25)$$

$$\frac{\Delta E_t}{E_t} = \lambda h_t \quad (2.26)$$

$$\frac{\Delta M_t}{\Delta E_t} = Q_t = F(b, M_t, q) \quad (2.27)$$

$$\Delta D_t = \Delta M_t \times \left[ \int_0^b \nu \cdot f(\nu, M_t, q) d\nu \right] \quad (2.28)$$

$$n_t = c \cdot \Delta E_t \times M_t \quad (2.29)$$

$$C_t + X_t = \frac{1}{\alpha} l_t^{1-\alpha} \cdot \sum_{i=1}^{M_t} (x_{i,t})^\alpha \quad (2.30)$$

$$l_t + h_t + n_t = L \quad (2.31)$$

where  $X_t$  is the total quantify of intermediate products,  $l_t$  denotes labor for final goods production. Intermediate producer hires  $h_t$  researchers to conduct R&D to get new products candidates, and hire additional  $n_t$  researchers to conduct testing regarding the intermediate product candidates. For the optimal allocation problem, I do not need regulator to review testing results.

$F(b, M_t, q)$  denotes a Binomial *CDF*, equal to the Qualified Innovation Ratio  $Q_t$ , or the probability of approval. The Binomial *PDF*  $f(\nu, M_t, q)$  represents the probability of getting exactly  $\nu$  interactions in  $M_t$  trials, with  $q$  as the probability of a single interaction between intermediates. Therefore  $\int_0^b \nu \cdot f(\nu, M_t, q) d\nu$  calculates the average number of adverse effects that a new intermediate candidate will interact with all  $M_t$  existing intermediates at time  $t$ .

## The Static Allocation Problem at Time $t$

It is better to firstly solve the static allocation problem at time  $t$ . This can simplify the grand optimization problem (2.23) by reducing the number of variables. At time  $t$ , the optimal  $x_{i,t}$  can be chosen to maximize the main component of instantaneous utility  $C_t - \tilde{D}_t$ . I know that  $C_t - \tilde{D}_t = \frac{1}{\alpha} l_t^{1-\alpha} \cdot \sum_{i=1}^{M_t} (x_{i,t})^\alpha - \sum_{i=1}^{M_t} x_{i,t} - \frac{X_t \cdot D_t}{M_t}$ .

For simplicity of calculation, I make the following assumption:

**Assumption 2.5.** *At any time  $t$ , all existing intermediate products will be used at the same quantity:  $x_{i,t} = x_t$ .*

Then the optimal  $x_t^*$  is calculated from the following optimization:

$$\max_{x_t} M_t \left( \frac{1}{\alpha} l_t^{1-\alpha} \cdot x_t^\alpha - x_t - x_t \cdot \frac{D_t}{M_t} \right) \quad (2.32)$$

I can solve (2.32) to get  $x_t^*$ :

$$x_t^* = l_t \left( 1 + \frac{D_t}{M_t} \right)^{\frac{1}{\alpha-1}} \quad (2.33)$$

With the solution  $x_t^*$  above, I can represent the time  $t$  instantaneous utility  $u_t$  without  $x_{i,t}$ :

$$u_t = \frac{1-\alpha}{\alpha} \cdot M_t \cdot \underbrace{\frac{1}{\left(1 + \frac{D_t}{M_t}\right)^{\frac{\alpha}{1-\alpha}}}}_{\substack{\text{risk discount factor} \\ \text{risk discounted TFP}}} \cdot l_t \quad (2.34)$$

## Necessary Condition for Growth to Start

From the instantaneous utility (2.34), I can see the part labeled as *Risk discount factor* will determine how useful an extra innovation is to the consumer. The order of  $D_t$  relative to  $M_t$  is determinant.

If  $D_t$  is a linear function of  $M_t$ , e.g.  $D_t = s \cdot M_t$ , then  $u_t$  can be represented by a function

form  $u_t = A \cdot M_t \cdot l_t$ , where  $A$  is a constant. *Risk discounted TFP* in (2.34) is linear in  $M_t$ . Instantaneous utility is linear in the number of varieties of innovations, then the dynamics of (2.23) resembles a classical variety expansion endogenous growth model.

But in another case, if  $\alpha = 0.6$ , and  $D_t = M_t^2$ ,  $u_t \approx \frac{1-\alpha}{\alpha} \frac{1}{\sqrt{M_t}} l_t$ . So the highest instantaneous utility is  $u_0 \approx \frac{1-\alpha}{\alpha} \frac{1}{\sqrt{M_0}} L$ . There will be no growth from  $t = 0$ .

I can derive a more general result as below:

**Proposition 2.1.** *There will be no growth from the beginning iff  $D_t = O(M_t^\sigma)$  and  $\sigma \geq \frac{1}{\alpha}$ .*

When  $D_t = O(M_t^\sigma)$  and  $\sigma \geq \frac{1}{\alpha}$ , the *Risk discounted TFP* is a decreasing function of  $M_t$ . A larger  $M_t$  will reduce utility. No R&D and growth will happen from the beginning.

This results imply when the damage grows too fast relative to the varieties of innovations, there should be no innovation and variety increase in the first place. The condition also depends on the capital share  $\alpha$ . A high capital share will amplify the damage so that the condition for non-growth can be more easily satisfied.

Our empirical work finds a quadratic relationship:  $DDIs = f(drugs)$  in the function form of (2.1). In this case,  $D_t$  is quadratic function of  $M_t$ ,  $D_t = M_t^2$ ,  $\sigma = 2$ . I normally take  $\alpha = 0.3$  in Macroeconomics.  $\sigma = 2 < \frac{1}{0.3} = \frac{1}{\alpha}$ . The condition for non-growth is not satisfied according to Proposition 1. In fact, we have  $u_t = \frac{1-\alpha}{\alpha} \cdot M_t \cdot l_t \cdot \frac{1}{(1+M_t)^{\frac{3}{7}}}$ , which can be simplified to:

$$u_t \approx \frac{7}{3} \cdot M_t^{\frac{4}{7}} \cdot l_t \quad (2.35)$$

Then a larger number of intermediates varieties is beneficial and growth will happen at  $t = 0$ .

## Revised Optimization Problem

I move on to the dynamic optimization problem.

Labor devoted to testing is  $n_t = c \cdot \Delta E_t \times M_t$ . Because  $\Delta E_t = \lambda h_t E_t$ , I can get the

representation of  $n_t$  in  $h_t$  :

$$n_t = c\lambda M_t E_t \cdot h_t \quad (2.36)$$

This also gives us the ratio between R&D staff and testing staff.

From  $n_t + h_t + l_t = L$ , I can derive the representation of  $h_t$  in  $l_t$  :

$$h_t = \frac{L - l_t}{1 + c\lambda M_t E_t} \quad (2.37)$$

With the results derived in the last subsection, I get a simplified optimization problem:

$$\max_{\{l_t\}} \sum_{t=0}^{\infty} \beta^t \cdot \frac{\left[ M_t \cdot l_t \left( 1 + \frac{D_t}{M_t} \right)^{\frac{\alpha}{\alpha-1}} \right]^{1-\gamma}}{1 - \gamma} \quad (2.38)$$

$$\Delta E_t = \lambda \frac{L - l_t}{1 + c\lambda M_t E_t} E_t \quad (2.39)$$

$$\Delta M_t = F(b, M_t, q) \cdot \Delta E_t \quad (2.40)$$

$$\Delta D_t = \Delta M_t \times \left[ \int_0^b \nu \cdot f(\nu, M_t, q) d\nu \right] \quad (2.41)$$

Now there is only one control variable  $\{l_t\}$ , together with three state variables:

$E_t$  : the total patented innovations;

$M_t$  : the total qualified innovations;

$D_t$  : aggregate risk (corresponding to theoretical DDIs).

I can set up the following Lagrangian to solve (2.38)

$$\mathcal{L} = \sum_{t=0}^{\infty} \left\{ \begin{aligned} & \beta^t \cdot \frac{\left[ M_t \cdot l_t \left( 1 + \frac{D_t}{M_t} \right)^{\frac{\alpha}{\alpha-1}} \right]^{1-\gamma}}{1-\gamma} \\ & + \mu_t \cdot \left[ E_{t+1} - E_t - \lambda \frac{L-l_t}{1+c\lambda M_t E_t} E_t \right] \\ & + \xi_t \cdot \left[ M_{t+1} - M_t - F(b, M_t, q) \cdot \lambda \frac{L-l_t}{1+c\lambda M_t E_t} E_t \right] \\ & + \varphi_t \cdot \left[ D_{t+1} - D_t - F(b, M_t, q) \cdot \lambda \frac{L-l_t}{1+c\lambda M_t E_t} E_t \cdot \left( \int_0^b \nu \cdot f(\nu, M_t, q) d\nu \right) \right] \end{aligned} \right\} \quad (2.42)$$

where the costate variables  $\mu_t, \xi_t, \varphi_t$  represent the shadow prices of an additional patent, a qualified innovation, and risk respectively.

From four sets of first order conditions for  $l_t, \mu_t, \xi_t, \varphi_t$ , together with three LOMs for  $E_t, M_t, D_t$ , I can get a system of seven ODEs for (2.42), which are put in the Appendix.

**Proposition 2.2.** *Growth stops at a threshold  $\bar{M}$ . At the steady state, innovation stops.*

In our model, additional new innovations will increase testing cost as well as reduce qualified innovation ratio for the future. In contrast, the canonical endogenous growth models will have an expanding choice set over time. For example, in the Romer model,  $E_{t+1} - E_t = \lambda l_t \cdot E_t < \lambda L \cdot E_t$ . The choice set for  $E_{t+1}$  keeps expanding proportional to  $E_t$ , therefore growth will never stop.

There will be a  $\bar{M}$ , where the expected cost of innovation will be less than the expected benefit of introducing an additional innovation. This results in a steady state where R&D stops. I analyze the existence of such a threshold as follows.

Although it becomes more difficult for an innovation candidate to qualify for approval as the pool of qualified innovations enlarges, there is always a positive probability of qualification since  $F(b, M_t, q) > 0$ . The question of whether R&D stops depend on the total expected cost of R&D and testing.

At time  $t$ , the labor cost of inventing and obtaining a patent is  $\frac{1}{\lambda E_t}$ , and the cost of testing a new innovation candidate is  $c \cdot M_t$ . The probability of the patent being qualified to market



is  $F(b, M_t, q)$ . So the expected total cost of successfully increasing a new qualified innovation is  $\frac{\frac{1}{\lambda E_t} + cM_t}{F(b, M_t, q)}$ .

At the steady state, all labor is allocated to production of the aggregate good:  $l_t = L$ . There will be a switching at the threshold.

### 2.3.10 Equilibrium Growth

In this section, I will characterize the Equilibrium Growth Path of the model.

#### The Equilibrium Definition

An equilibrium of this model is an allocation, such that,

(1) Consumers make their optimal choices between consumption and savings  $[C_t, S_t]_{t=0}^{\infty}$  taking interest rate as given;

(2) Final goods producers choose the quantity of labor  $[l_t]_{t=0}^{\infty}$  and quantities of each intermediate products  $\left[ \{x_{i,t}\}_{i \in [0, M_t]} \right]_{t=0}^{\infty}$  to maximize their profit, taking the wage rate and intermediate price as given;

(3) Each intermediate producer with a qualified intermediate product maximizes its profit by choosing optimal price and quantities  $\left[ \{p_{i,t}, x_{i,t}\}_{i \in [0, M_t]} \right]_{t=0}^{\infty}$  to supply, taking the interest rate as given;

(4) Taking as given wage rate and the price of a qualified intermediate product, a potential intermediate producer chooses the quantity of researchers  $[h_t]_{t=0}^{\infty}$  to hire, the quantities of effective testing which is equivalent to proportionally choosing the size of their testing staff  $[n_t]_{t=0}^{\infty}$ , and submit to regulator the applications of all their new intermediate product candidates; free entry condition will determine the path of varieties of intermediate product candidates  $[E_t]_{t=0}^{\infty}$ ;

(5) The regulator reviews and verified all testing results in the new product applications, and charges the firms a fee to hire regulatory staff with size  $[z_t]_{t=0}^{\infty}$ , which is proportional to the workload of reviewal and verification. The regulator immediately dismisses any applica-

tion with faked testing results, and then decide whether to approve an application according to a decision rule (2.9). Regulator's decisions determine the path of varieties of qualified intermediates  $[M_t]_{t=0}^{\infty}$  ;

(6) Markets clear to determine interest rate and wage rate  $[r_t, w_t]_{t=0}^{\infty}$

The regulator reviews and verifies all testing results in new intermediate product applications. All applications with faked testing results will be rejected. Then the only equilibrium path left in the game is that potential intermediate producers only conduct effective testing and tell true testing outcomes to the regulator, therefore removing all other inefficient equilibrium paths.

## Consumer Optimization

Similar to the canonical growth model, I can solve and get the growth rate of (net) consumption,

$$g_t = \frac{\Delta(C_t - \tilde{D}_t)}{C_t - \tilde{D}_t} = \frac{r_t - \rho}{\gamma} \quad (2.43)$$

where  $\tilde{D}_t$  follows (2.24).

## Equilibrium Factor Prices

The price of intermediate product  $i$  is equal to its marginal productivity in final good production (2.3) :

$$p_{i,t} = x_{i,t}^{\alpha-1} \cdot l_t^{1-\alpha}$$

Wage is equal to marginal productivity of labor in final good production (2.3) :

$$w_t = \frac{1-\alpha}{\alpha} l_t^{-\alpha} \cdot \sum_{i=1}^{M_t} (x_{i,t})^{\alpha}$$

Intermediate producer will choose the optimal output  $x_{i,t}$  to maximize profit, solving

problem (2.9) :

$$x_{i,t} = \alpha^{\frac{1}{1-\alpha}} \cdot l_t \quad (2.44)$$

and the intermediate product *i*'s profit at *t* will be:

$$\pi_{i,t} = \frac{1-\alpha}{\alpha} \alpha^{\frac{1}{1-\alpha}} \cdot l_t \quad (2.45)$$

the profit is proportional to the final goods production labor  $l_t$ .

Applying the solution  $x_{i,t}$  in (2.44) can further simplify  $p_{i,t}$  and  $w_t$  :

$$w_t = \frac{1-\alpha}{\alpha} \alpha^{\frac{\alpha}{1-\alpha}} \cdot M_t \quad (2.46)$$

wage is proportional to the varieties of innovations.

$$p_{i,t} = \frac{1}{\alpha} \quad (2.47)$$

the intermediate product price is constant over time.

## R&D Expenditure in Equilibrium

The Research-arbitrage equation (2.19) implies:

$$Q_t \Delta E_t \cdot \sum_{\tau=t+1}^{\infty} \frac{\pi_{i,\tau}}{\prod_{s=t+1}^{\tau} [1+r_s]} \leq w_t (h_t + n_t + z_t) \quad (2.48)$$

Entry into the R&D sector is free, so the total R&D expenditure at  $t$  should be equal to the present value of  $Q_t \Delta E_t$  qualified innovations generated at  $t$ , which is the sum of the discounted profits from time  $t$  on.

By substituting (2.8), (2.36), and (2.46) into (2.48), we can get

$$\sum_{\tau=t}^{\infty} \frac{1}{[1+r(\tau)]^{\tau-t}} \pi_{i,\tau} \leq \underbrace{\frac{\frac{1-\alpha}{\alpha} \alpha^{\frac{\alpha}{1-\alpha}} M_t [1+(c+\omega)\lambda E_t M_t]}{\lambda E_t \cdot F(b, M_t, q)}}_{ERC} \quad (2.49)$$

**Definition 2.6.** *The Expected R&D Cost (ERC). The RHS of (2.49) denotes the Expected R&D Cost (ERC) : the expected expenditure that an innovative firm needs to spend to get a qualified innovation successfully.*

In the following proposition, I give a lower bound for the Expected R&D Cost in the limit.

**Proposition 2.3.** *The Expected R&D Cost (ERC) has a lower bound  $\underline{ERC}_t$  :  $ERC_t \geq \underline{ERC}_t$ , and  $O(\underline{ERC}_t) = M_t^2 \cdot e^{2q^2 \cdot M_t}$ .*

This proposition gives us an amazing result: in the limit, the Expected R&D Cost grows faster even than exponential. The exponential component comes from the reciprocal of the qualified innovation ratio.

## The Steady State

This subsection discuss the steady state of the decentralized equilibrium.

**Proposition 2.4.** *There exists a threshold  $\hat{M}$  as the upper limit of this economic system. R&D and growth stops when the threshold  $\hat{M}$  is reached. A larger labor force  $L$  can lead to a higher threshold  $\hat{M}$ .*

The proof is put in Appendix B. Within this economic setup, there is an upper limit  $\frac{L}{\rho}$  for the value of a qualified patent. Innovators can only earn the monopolistic profit to this limit. The average R&D cost is an increasing function of the varieties of innovations.

This proposition implies that the R&D cost can grow to be so high as to fully stymie R&D in the end.

## Two Externalities

There are two externalities in the decentralized model.

The externality of *knowledge spillover*: new knowledge can reduce the cost of future R&D cost  $\frac{1}{\lambda E_t}$ . This traditional source of externality will discourage private R&D investment due to the public good nature of knowledge.

The externality of *risk spillover*: the cost of testing any new innovation candidate is  $cM_t$ , which increases with the number of existing qualified innovations. The Qualified Innovation Ratio of later innovations will be reduced due to risk spillover of early innovations. This is the type of negative externality I measure and demonstrate in Section 2. Furthermore, I will define “Risk Externality” to capture this negative spillover effect.

**Definition 2.7** (Risk Externality). *Risk Externality is defined as the additional testing, regulatory, and qualification cost imposed by existing products on future innovations*<sup>11</sup>.

### 2.3.11 Comparison with the Romer Model

Eqn (2.50) makes a decomposition of the Expected R&D Cost (*ERC*). We can see there are three factors contributing to the escalation of expected R&D cost. (1) Wage rate grows proportional to  $M_t$ ; (2) Testing and regulatory cost for an innovation at  $t$  is a linear function of  $M_t$ . This includes the testing expenditure that innovative firms directly spend, and regulatory cost that regulators charge firms for reviewing their new product applications; (3) The declining Qualified Innovation Ratio due to the innovation-induced risk structure. Risk Externality is reflected by (2) and (3).

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11. I thank Prof. Roger Myerson for helping to make this definition.

$$\begin{aligned}
ERC_{Xie,t} = & \frac{\underbrace{\frac{1-\alpha}{\alpha} \alpha^{\frac{\alpha}{1-\alpha}} M_t}_{wage}}{\underbrace{\left[ \frac{1}{\lambda E_t} + \underbrace{c \cdot M_t}_{\text{firm's testing cost}} + \underbrace{\omega \cdot M_t}_{\text{regulatory cost}} \right]}_{\substack{F(b, M_t, q) \\ \text{Qualified Innovation Ratio}}}} \\
& \tag{2.50}
\end{aligned}$$

This cost structure has a sharp contrast with the canonical endogenous growth model. For example, the Romer model has the following *ERC* representation (2.51):

$$\begin{aligned}
ERC_{Romer,t} = & \frac{\underbrace{\frac{1-\alpha}{\alpha} \alpha^{\frac{\alpha}{1-\alpha}} E_t}_{wage}}{\underbrace{\left[ \frac{1}{\lambda E_t} + \underbrace{0}_{\text{testing cost}} + \underbrace{0}_{\text{regulatory cost}} \right]}_{\substack{1 \\ \text{Qualified Innovation Ratio}}}} \\
& \tag{2.51}
\end{aligned}$$

Canonical endogenous growth models do not have the qualification process, and so does not distinguish between  $M_t$  and  $E_t$ . We can see from (2.51), the knowledge spillover effect in the denominator can exactly cancel out the wage rate increase in the nominator. Though wage rate increases proportionally to the knowledge growth rate, the R&D productivity grows at the same rate due to knowledge spillover. Moreover, there is no testing cost in the Romer model. Therefore, this results in a neat solution (2.52) for *ERC* :

$$ERC_{Romer,t} = \frac{1}{\lambda} \frac{1-\alpha}{\alpha} \alpha^{\frac{\alpha}{1-\alpha}} \tag{2.52}$$

The Expected R&D Cost *keeps constant over time in the canonical endogenous growth model*. This is a crucial feature behind the canonical endogenous growth model. However, empirical evidence in the following subsection contradicts this prediction very firmly. The comparison between (2.50) and (2.51) illustrates a key difference between this regulatory growth model and the canonical endogenous growth model.

### 2.3.12 Simulation and Quantitative Results

In this section, I provide some numerical results for the problem (2.38). This is challenging due to the 3 – *dimensional* structure of the state space, which is exploding. I have designed an efficient algorithm to conquer this.

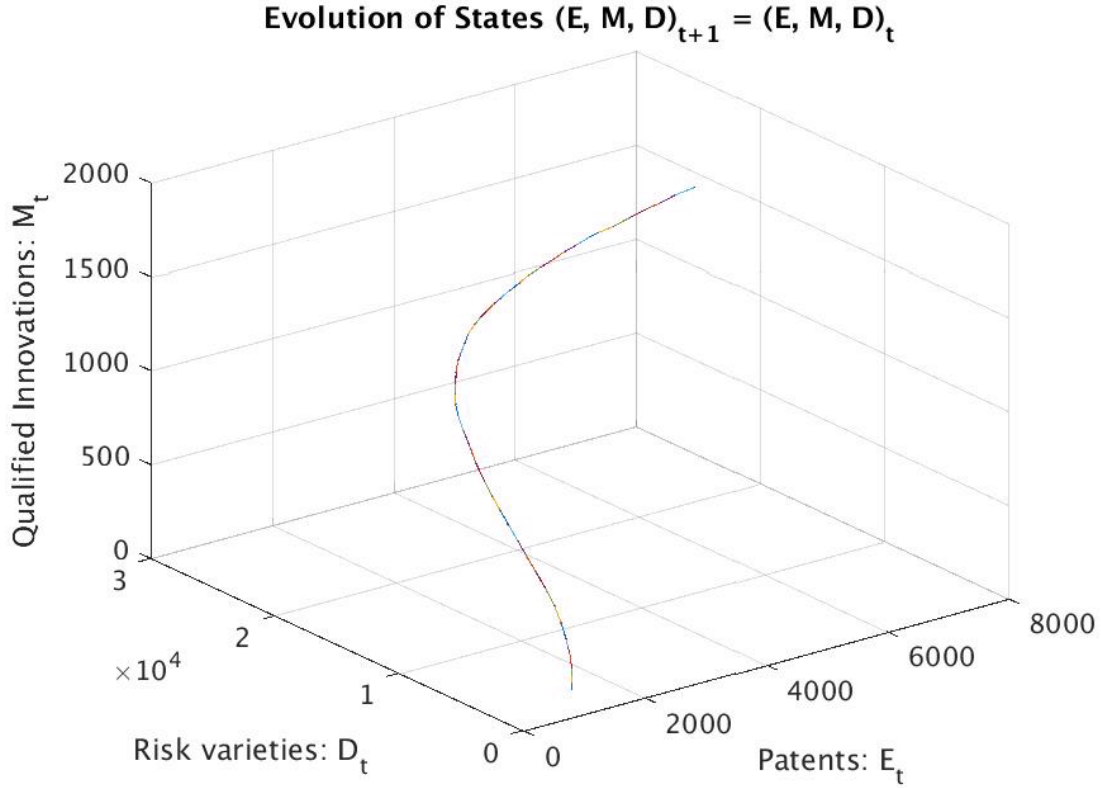


Figure 2.9: State Evolution

The following Figure (2.9) displays the evolution of states  $(E, M, D)_t$  using a well-designed Value Function Iteration algorithm. Figure (2.9) shows the optimal growth path, evolving from initial state  $(E, M, D)_0$  to  $(E, M, D)_t$ .

Some calibration parameters are shown in Table D.1.

The qualified innovations  $M_t$  is a concave function of the number of patents  $E_t$ . Innovation finally reaches a steady state and growth stops. Risk varieties  $D_t$  is initially a convex function of the number of patents and switches to a concave function at a higher level.

**Definition 2.8.** The reachable set of states  $\Theta$  from initial state  $(E, M, D)_0$  :

$$\Theta = \{(E, M, D)_i \mid \text{there is a feasible transition path from } (E, M, D)_0 \text{ to } (E, M, D)_i\}.$$

Although the whole state space is huge, the reachable set of states from  $(E, M, D)_0$  is relatively small. As the first step, our algorithm recursively calculate the reachable set of states  $\Theta$  for the initial state  $(E, M, D)_0$ . The first step of the algorithm stops until no new state is added into  $\Theta$ .

Moreover, for each state  $(E, M, D)_i$  in  $\Theta$ , there exists a State Transition Set  $\Psi_i$  :

**Definition 2.9.** *State Transition Set  $\Psi_i$  from state  $(E, M, D)_i$  :*

$$\Psi_i = \{(E, M, D)_j \mid \text{there is a feasible direct transition from } (E, M, D)_i \text{ to } (E, M, D)_j\}.$$

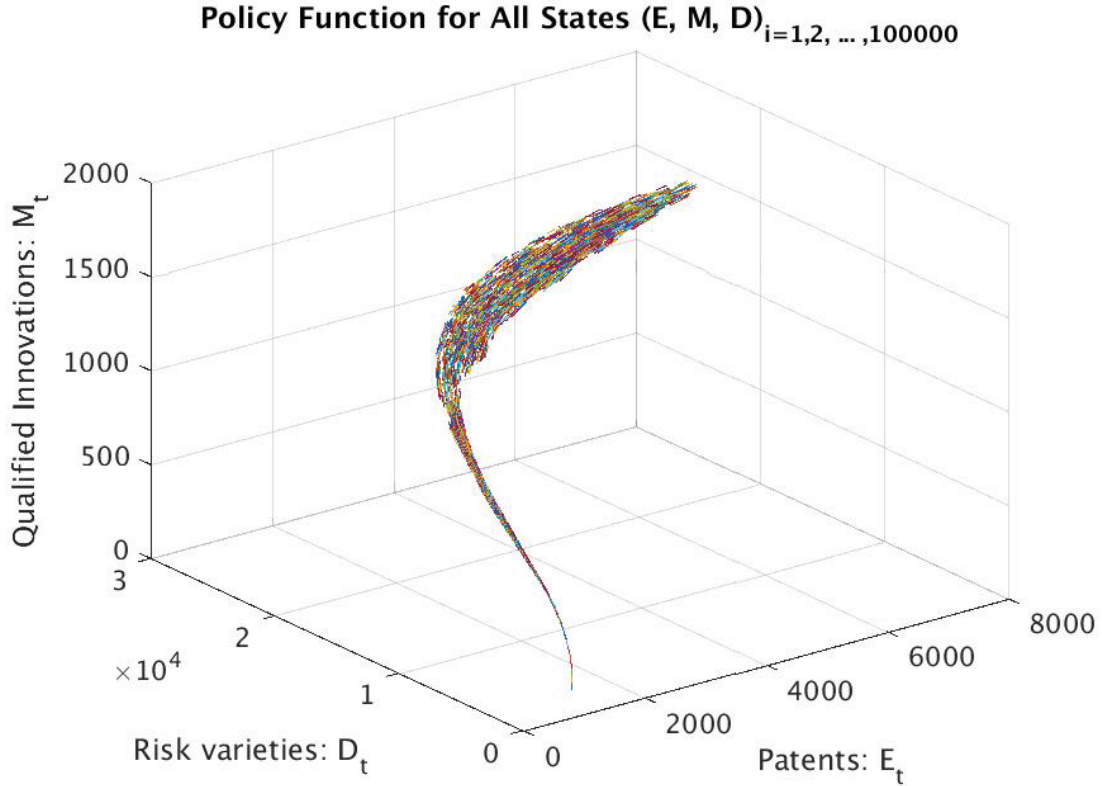


Figure 2.10: The Policy Function

Therefore, as the second step of our algorithm, value function iteration for each state  $i$  in  $\Theta$  is based on its specific State Transition Set  $\Psi_i$  :

$$V_i = \max_{j \in \Psi_i} \{u(c_j) + \beta \cdot V_j\} \quad (2.53)$$



$\Psi_i$  for each state is small. With this algorithm, I derive and depict a specific form of the policy function shown in Figure (2.10). In Figure (2.10), from each state  $i \in \Theta$ , I draw a line from state  $i$  to state  $j$ ,  $j = P(i)$ , where  $P(i)$  is the policy function.

As we can see, although the whole 3 – *dimensional* state space is huge, the reachable set from the initial state is only a small slice of the full space.

## 2.4 Implications for R&D and Regulation

In this section, I discuss the implications of our theoretical model for R&D and regulation, and provide additional empirical evidence. There will be greater expenditures on regulation and compliance.

### 2.4.1 *The Composition of Regulation*

A direct implication of our new empirical fact of Section 2 and new growth model of Section 3 is that regulation will not only grow fast but also become more and more risk-oriented.

Resources spent on regulations have grown significantly during the last several decades. The long-run trend of regulatory spending is displayed by Figure 2.2B. When regulatory spending is decomposed into categories (see Figure 2.11), we can see an obvious trend. The rise of regulation is mainly caused by the disproportionate increase in the social and economic risk regulations. This offers support for our previous argument and model that the fast growing new varieties of risks are the main driving force of regulation growth.

Moreover, traditional economic regulation, illustrated as “Industry and Business regulation” in Figure 2.11, stays relatively stagnant. In the traditional view of economists, regulators mainly deal with inefficiencies attributable to monopoly. In contrast, social regulations cope with Safety, Health, and Environmental issues, with an aim to identifying, controlling and removing the risks induced by technologies. In the same sense, the purpose of Finance and Banking regulation is also to identify and control the risks in the financial

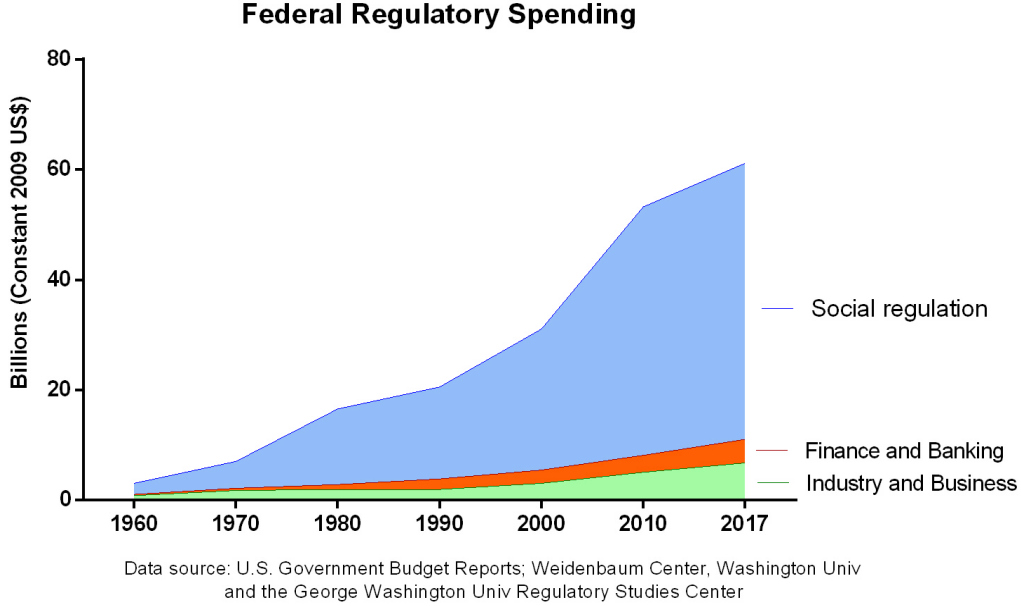


Figure 2.11: Decomposed Budget of Federal Regulatory Agencies (Inflation adjusted)

and economic system. Therefore, I propose to group the Social Regulation, and Finance and Banking regulation into one grand category: the Social and Economic Risk regulations (or *abbr. Risk regulations*).

### 2.4.2 Statistics for the Interactive Risk Generating Process

From the FAERS and Drugbank datasets, I estimated that each new drug  $k$  has a probability  $q_{kj}$  of interacting with any existing drugs  $j$ . Our statistics shows that  $q_{kj}$  is quite constant around 3.5%<sup>12</sup>. Moreover, these probabilities are independent from each other. Then I use  $q$  to approximate  $q_{k,j}$  :

$$q_{kj} \approx 3.5\% = q \quad (2.54)$$

This reflects the interactive risk generating process behind innovations. Innovation-induced risks demand the increase in regulatory expenditures to assess the NET benefits

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12. Admittedly, this statistics comes from the data of approved drugs. I do not have data for the unapproved drugs. It can be reasonably conjectured the interaction probability between unapproved drugs and approved drugs will be higher than 3.5%.

of an innovation, that is, interactions of drugs for health, and systemic risk in the financial system.

### 2.4.3 *Pharmaceutical R&D*

#### Skyrocketing Expected R&D Spending for New Drugs

As an important empirical puzzle, Figure (2.2A) shows, from 1950 to 2010, the expected R&D spending for each FDA-approved drug has increased by more than 100 times, in contrast to a 5 – *fold* increase in real GDP. This is also an important factor contributing to the fast increase in health care expenditure.

The canonical endogenous growth model obviously fails to explain this fact. Nevertheless, this Pharmaceutical R&D Productivity puzzle is consistent with the prediction of our Proposition 4: the Expected R&D Cost (*ERC*) for each successful innovation grow faster even than an exponential function of the number of existing innovations.

According to the Regulatory Growth Theory proposed in Section 3, the rise of ERC mainly comes from two sources: the decline in Qualified Innovation Ratio, and the rise of testing cost. Following subsection discusses the cost of clinical trials. Qualified Innovation Ratio will be discussed in the next subsection.

#### Clinical Trials as Increasing Share of R&D Cost

The decomposition of *ERC* of our model points to three major contributors to this huge increase. Besides the declining Qualified Innovation Ratio, I will provide further evidence on the rise in testing (e.g. clinical trials) cost.

As documented by various Pharmaceutical Industry report<sup>13</sup>, clinical trials has been growing to make up a major share of the total R&D cost for drugs.

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13. For example, PhRMA 2016 Biopharmaceutical Research Industry Profile.

<b>Function</b>	<b>Million US\$</b>	<b>Share</b>
Pre-Human/Pre-Clinical	11,272.7	21.2%
Phase I	4,722.0	8.9%
Phase II	5,697.8	10.7%
Phase III	15,264.4	28.7%
Approval	2,717.7	5.1%
Phase IV	8,827.0	16.6%
Uncategorized	4,751.5	8.9%
<b>TOTAL R&amp;D</b>	<b>53,253.2</b>	<b>100.0%</b>
<b>Phase I,II,III,IV, Approval</b>	<b>37,228.9</b>	<b>70.0%</b>

Source: Pharmaceutical Research and Manufacturers of America  
PhRMA Annual Membership Survey 2016.

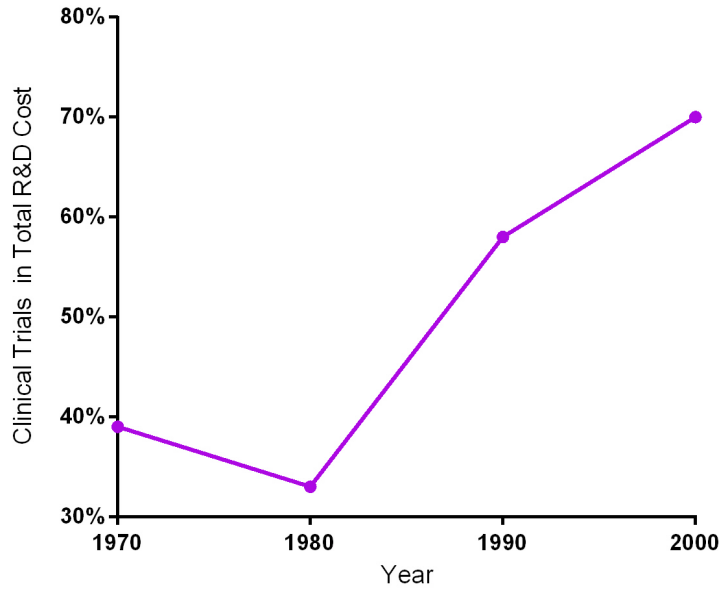
Table 2.1: 2014 R&D by Function, PhRMA Member Companies

Table 2.1 shows that clinical trials plus approval now take up 70% of all R&D expenditures for the PhRMA Member Companies (all major Pharmaceutical companies have been included). The share of clinical trials plus approval correspond to the component  $(c + \omega) \lambda E_t \cdot M_t$  in our decomposition of the expected R&D cost (2.50).

The share of clinical trials cost in total pharmaceutical R&D expenditure has increased significantly for the last several decades. Figure (2.12) displays such a trend. From 1970s to 2000s, the share of clinical trials has increased from below 40% to more than 70%.

#### 2.4.4 *The Declining Qualified Innovation Ratio*

Since 1938, although annual patent issuance for new drugs has grown quickly, the FDA kept approving only a stagnant number of new drugs for each year: varying around 30 New Molecular Entities (NMEs). Our first puzzling fact is the sharply declining Qualified Innovation Ratio ( $= \frac{\text{new products approved}}{\text{new patents issued}}$ ), from around 10% in the 1950s down to 0.2% in the 2010s (see Figure 2.13). Existing growth theories focus on patents, and did not make a distinction between innovations and qualified innovations. In this section I will try to match our model with some empirical evidence, and explain the declining Qualified Innovation



Source: Hansen, DiMasi et al (1979, 1991, 2003)  
PhRMA Annual Membership Surveys.

Figure 2.12: The Share of Clinical Trials Cost in Total R&D

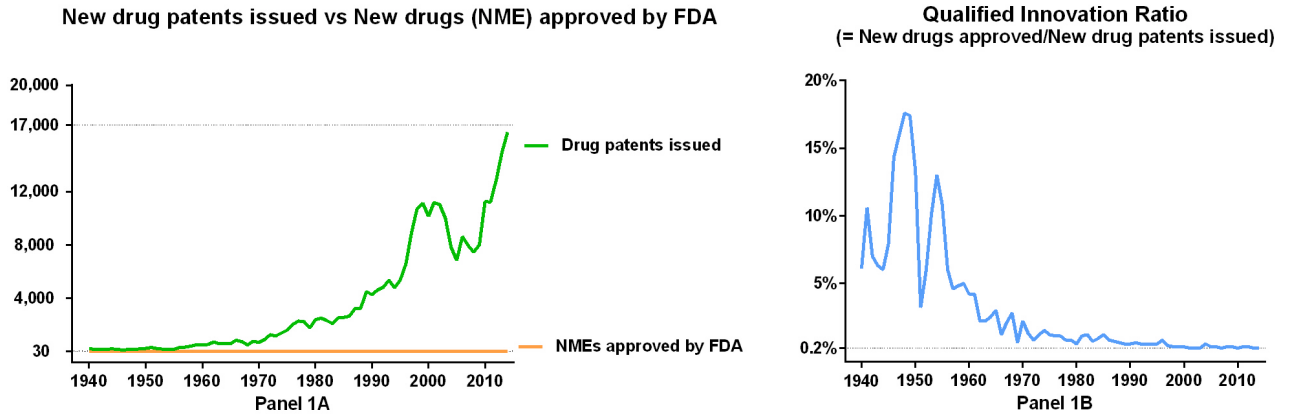
Ratio.

### Dynamics of Qualified Innovation Ratio $Q_t$

The *CDF* function  $F(b; M_t, q)$  is a decreasing function of  $M_t$  : with the number of existing drugs  $M_t$  increasing, the probability of meeting the standards of approval is decreasing. A smaller and smaller share of new products can be qualified to meet the standards.

If the benefit is fixed, which means the threshold of a adopted drugs' adverse effects is fixed, the *Qualified Innovation Ratio*  $Q_t$  will decrease with an increasing  $M_t$ . Admittedly, there will always be a positive share of new product candidates that can satisfy the decision rule (2.9).  $Q_t$  is always a positive number, though  $Q_t$  is decreasing in  $M_t$ .

## Declining Ratio of Qualified Innovations



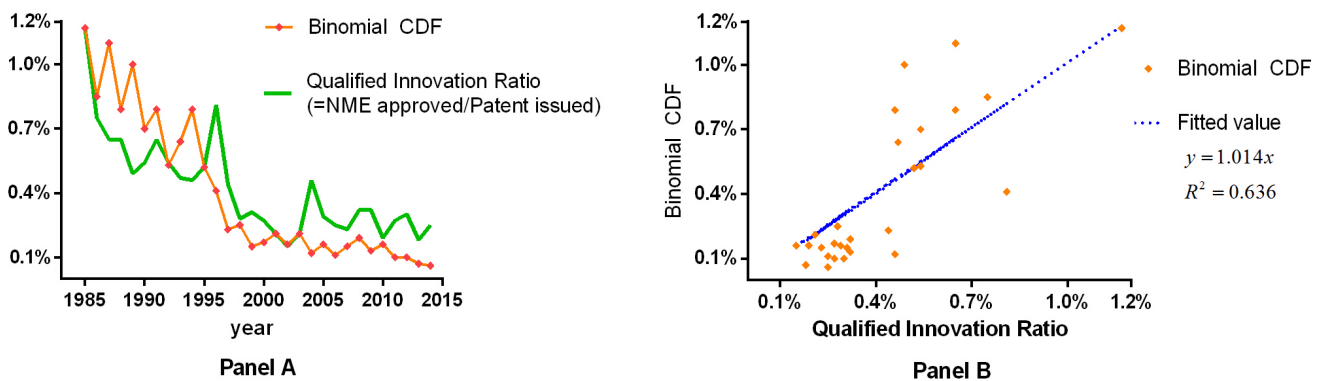
Data Source: USPTO Historical Patent Data; NBER patent data; FDA historical NME approval data

Figure 2.13: Patents Issued vs. New Products Approved by Regulator

### Explaining Declining Qualified Innovation Ratio

The declining Qualified Innovation Ratio is our first puzzling fact as displayed by Figure 2.13. In the previous subsection, I have argued that based on our model, the Qualified Innovation Ratio  $Q_t$  is equal to a Binomial CDF  $F(b; M_t, q)$ .

Figure 2.14: Empirical match of the Binomial CDF and Qualified Innovation Ratio



The Qualified Innovation Ratio  $Q_t$  is directly calculated by (2.11), with the NBER patent data (category 31: pharmacy patents issued) and FDA's NME approval data.

I calibrate a Binomial CDF curve  $F(b; M_t, q)$ , using the estimated  $q$  in (2.54) and the

total number of approved NME drugs as  $M_t$ . To fit the Qualified Innovation Ratio better, I also allow a slight upward slope in the threshold of aggregate risks.

Figure 2.14 shows the result. The left panel demonstrates the trend of the two curves. The right panel plots the Binomial CDF as a function of  $Q_t$ , and the fitted value has a reasonable  $R^2$  value. We can claim that the model proposed before has some explanatory power for the declining Qualified Innovation Ratio.

## 2.5 Conclusions

This research makes three main contributions to the literature. This is the first research to empirically measure the “dark side” of innovations, in particular, risk externality caused by existing innovations. I document a new empirical fact about the relationship between innovations and innovation-induced risks.

Then I develop a new endogenous growth model with innovation risks and with a regulator. I explicitly model the qualification process of regulator who aims to control the risks induced by innovations. This new theoretical model can help us to understand better how we can regulate innovations, thus providing a link between the innovations literature and the regulation literature.

This new theory can explain the rise of R&D cost for new drugs as well as greater expenditures on regulation and compliance. Fast-growing new innovation risks are the key driving force behind all these. The risk structure induced by previous innovations make the qualification of later innovations more and more difficult.

Above model can be extended and applied in various ways. As direct applications, it can be used to study environmental risks under the new Lautenberg Act that requires testing for all existing and new chemicals. Financial innovations oftentimes cause complex risks due to their linkages among themselves as well as other sectors of the economy. Therefore, this framework is ready for study the regulation of Consumer Financial Protection Bureau

(CFPB) and financial innovations.

In Appendix E, I also characterize a generalized *High – Order* Risk Space beyond the 2 – *dimensional* case discussed in this paper. In Xie (2015a), I generalize the risk space as well as the functions of the regulatory sector. This framework can also be extended to study endogenous disaster as in Xie (2016a).

Our model and fact might be helpful to understand the link between innovations and innovation-induced macro risk structure, potentially useful to bridge two separate literatures of Macroeconomics: growth and business cycle. The new stylized fact enriches our understanding of the structure of shocks which are endogenously generated, in contrast to the assumption of the Real Business Cycle (RBC) models that TFP shocks are exogenous like a black box. This research might shed some light on a new analytical framework for growth and business cycles. Xie (2016b) models “Regulatory Cycles” as such an attempt.



# CHAPTER 3

## A RATIONAL RUSH THEORY OF FINANCING INNOVATIONS<sup>1</sup>

### 3.1 Introduction

New financial products have been widely blamed as an important cause of the 2008 financial crisis. Report issued by the FCIC (Financial Crisis Inquiry Commission) claims “there was an explosion in risky subprime lending and securitization ... the GSEs (Government-sponsored enterprise) participated in the expansion of subprime and other risky mortgages, but they followed rather than led Wall Street and other lenders in the rush for fool’s gold”. As early as 2007, the subprime sector started to experience serious delinquencies, triggering fire sales and a credit crunch eventually. Therefore, a central question is whether, why, and how can innovation lead to over-investment and social welfare loss?

This chapter makes a linkage between innovation-induced economic booms with the friction of imperfect protection of Intellectual Property Rights (IPR). I develop a new theory of “Rush” in financing new technologies. *By “Rush”, I mean the premature massive investments in uncertain new technologies, which may be eventually proved to be futile, wrong or even harmful.* “Rush” emphasizes a sharp increase in the *quantity* of investments, in contrast with the widely used term “Bubble” which focuses on high asset price. I also illustrate an amplification mechanism through which a small shock can induce a big “Rush”. Market structure matters for the scale of amplification and inefficiency.

I argue for a view of “Rational Rush”, rather than “Irrational Bubble”. In fact, *the market equilibrium price for innovations might be too low, not too high.* Property rights

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1. The earliest version of this chapter, Xie (2014) was invited for presentation at 2014 Annual Meeting of the Society for Economic Dynamics (SED), Toronto.

for innovations especially the breakthrough ones are oftentimes not perfectly assigned, and consequently the innovators are usually underpaid by investors. In the market equilibrium, over-investment oftentimes happens in the sense that uncertainty of innovations is undervalued due to investors' tradeoff of seizing a larger share of these underpriced innovations.

Then I point to an important friction: imperfect IPR (Intellectual Property Rights) protection and missing knowledge market especially for breakthrough innovations and General Purpose Technologies (*GPT*). Innovation is generally a favorable public good, which can broaden the human knowledge base, provide new products to consumers, and introduce new profitable opportunities for entrepreneurs and investors. On the other hand, innovations still bear uncertainties. Therefore the new technology needs to be examined and refined patiently. Some of them will eventually prove to be fool's gold, and should be discarded efficiently. Premature massive adoption might create big social hazards. From the social planner's view, it is optimal to firstly invest in the new products on a small scale, wait and examine the outcomes carefully, and decide whether to continue to finance them on a large scale at a later stage.

*Excludability* due to perfect patent protection is assumed by Romer-type innovation literature, but here I adopt an opposite assumption: *non-excludability*. Despite well-established IPR laws, there are many innovations that IPR laws cannot be applied to. Abstract ideas and business method are generally not patentable. For example, patent is rarely granted to financial innovations, though first movers of financial innovations usually can catch a larger market share, according to Tufano (1989). Due to *non-excludability*, investors face a tradeoff between the precision of learning and market share grabbing. This is an important reason why they will be inclined to deviate from the socially efficient investment in learning of a new technology.

In contrast to the assumption of *nonrival* knowledge usage in the endogenous growth literature, there is actually *rivalry* in financing these new projects. New investment opportunities are often supplied in a limited quantity. Therefore, investors have strong incentive

to capture a lion's share and preempt others. This will distort the incentives for patient learning and generate inefficiency and unnecessary hazards. There are strong complementarity between investors because they compete for limited resources in a common pool. A small news shock to the expected return of new technology can be greatly amplified. A larger number of investors will exacerbate this amplification mechanism. This mechanism for "Rush-in" bears a comparative analogy to "Bank run" in that depositors withdraw funds from a common pool of deposits. Similar to coordination problem in bank runs, investors rush in to invest in new projects, giving up the benefits of waiting and learning. This reminds us of the "Tragedy of the Commons" in an uncertain and competitive environment. Fish in the "Uncertain open-access pond" might be poisonous, and poisoning can spread too quickly due to coordination failure. I name this type of inefficiency "*The Tragedy of the Uncertain Commons*".

This research makes a linkage between innovation and social risk of innovation, which the endogenous growth literature, as well as the current experimentation and preemption game literature have missed. Besides the well-known public good property of innovation, I emphasize innovations' "public bad" potential due to its innate uncertainty. Therefore, this research reveals another benefit that IPR protection have contributed implicitly: by granting Patent, it can give innovators' enough incentives to patiently learn and reduce the potential hazards of innovations to the society.

Information externality from the learning of the innovation also creates free-rider motive for investors. This makes under-investment possible especially when the expected return of an innovation is low ex ante. I derive a sharp threshold of expected return below which under-investment occurs and above which there will be over-investment. Larger number of investors will amplify over-investment and under-investment on both sides.

The friction identified and emphasized in this research is an important one: missing knowledge market and the spillover of uncertain knowledge. This has broad policy implications: anti-trust, patent design, monetary policy and social hazards regulation (e.g. FDA,

EPA, CFPB). Particular attention should be paid to policies that can alleviate the coordination problem and mitigate the “Tragedy of the Uncertain Commons”.

## Related Literature

*Endogenous Growth and Innovation-induced Social Hazards* Romer (1986,1990) starts the literature on endogenous growth. This line of research emphasizes the *Nonrivalry and Excludability* feature of innovation and knowledge. This paper makes opposite assumptions: “*Rivalry*” and “*Non-excludability*” when financing new technologies.

Moreover, the potential hazards of innovation is largely omitted by the literature. Knowledge spillover is generally treated as a spread of good thing. However, the spillover of “innovation” can also have widespread adverse effects. Jones (2014) takes a first step forward to also consider potential risk of innovations. In my another paper, Xie (2015a) endogenizes hazards generation and regulation in a growth framework. This paper emphasizes the effect of market structure on amplifying uncertainty and potential hazards of innovations.

*Information Acquisition, Bubbles and Crisis* Chari and Kehoe (2003) shows that information is important for financial crisis, and illustrates the herding effect on crisis. Angeletos, Hellwig and Pavan (2006) highlights that endogenous information generated by policy intervention can move global game from unique equilibrium to multiple equilibria. In this paper, the information structure is endogenously determined by the market structure and actions of agents.

*Entry, Competition and Inefficiency* Mankiw and Winston (1986) discuss the social inefficiency due to over-entry of firms. Hsieh and Moretti (2003) provides empirical evidence on this channel.

*Preemption game and Patent race* Reinganum (1981), and Fudenberg and Tirole (1985) apply preemption game to market entry and technology adoption. Firm will make a tradeoff between entering earlier with the possibility of acquiring a patent or a significant share of the

new market and waiting for a reduced uncertainty and entry cost. Hopenhayn and Squintani (2011) extends this line of research by adding heterogeneous information to each firm.

*Experimentation, learning, and Learning-by-doing (LBD)* Jovanovic and Lach (1989) shows that the Learning-by-doing mechanism can result in the S-shaped diffusion across firms. Bolton and Harris (1999) extend the two-armed bandit problem to a multi-agents dynamic game, and show the coexistence of a discouraging free-rider effect and a counteracting encourage effect for experimentation.

Pastor and Veronesi (2009) argue technology bubble may be efficient, and their analysis is under the assumption of social efficient learning. An essential difference from our research is that they assume the *nonrivalry* of technological innovation and unlimited number of new projects available for investing. With this assumption, there will not be distorted incentives to induce early entry, and competitive learning in equilibrium delivers the same efficient outcome as the social optimum. However in reality, *there is significant rivalry for the limited new investment opportunities*. As revealed by the 2008 Subprime Crisis, investors had competed intensively to enter a new market too early and massively than the socially optimal level before the uncertainty of new technology is sufficiently reduced.

The paper proceeds as follows. Section 2 describes the environment and model setup. Section 3 derives the optimal allocation. Section 4 discusses ownership and market structure for new projects, and how they are related to inefficiency. Section 5 analyzes the equilibria in a decentralized economy. Section 6 analyzes the amplification mechanism of rush, and tries to derive a shadow price corresponding to the observed investment quantity. Section 7 discusses inefficiencies in both cases of over- and under-investment. Section 8 concludes the paper and points to some future extensions.

## 3.2 The Model

The model has three periods  $t = 0, 1, 2$ , as shown by Figure (3.1).

### 3.2.1 Types of Projects

There are two types of projects: (i) the new and illiquid projects and (ii) liquidity.

**(i) New and illiquid projects** Each project needs one unit of investment. These new projects can be started at either  $t = 0$  or  $t = 1$ . No matter whether a project is financed at  $t = 0$  or  $1$ , it will mature at  $t = 2$ . After maturity, each project will produce  $R$  units of output. Return  $R$  will be the same for all projects.  $R$  is unknown, follows a normal distribution, with a prior  $R \sim N(R_0, \alpha_0^{-1})$ , shared by all players.

**Assumption 3.1.** *Project invested at  $t = 0$  cannot be liquidated at date 1.*

There will be a total number  $N$  of new projects.  $N$  is deterministic and known to all players.

**(ii) Liquidity** liquidity asset, which can return 1 for 1 unit of investment at any time when needed. It is equal to holding cash.

#### Ownership of New Projects

Because of knowledge spillover, there is no perfect ownership defined for the new projects. They look like common-pool resource to the economic agents. In Section 4, I will discuss in details the nonexcludability problem of innovation and the “Tragedy of the Uncertain Commons” as a result. Moreover, under imperfect property rights for innovations, we need to propose an allocation mechanism for these new projects.

### 3.2.2 Investors

There are  $M$  symmetric investors in total, indexed by  $i = 1, 2, \dots, M$ . Each investor  $i$  is indifferent to consuming at  $t = 1$  or  $2$ , and has a linear preference as follows,

$$u(c_{i,1} + c_{i,2}) = c_{i,1} + c_{i,2} \tag{3.1}$$

Each investor has an endowment of  $k$  units of capital at date 0. Investors can choose to

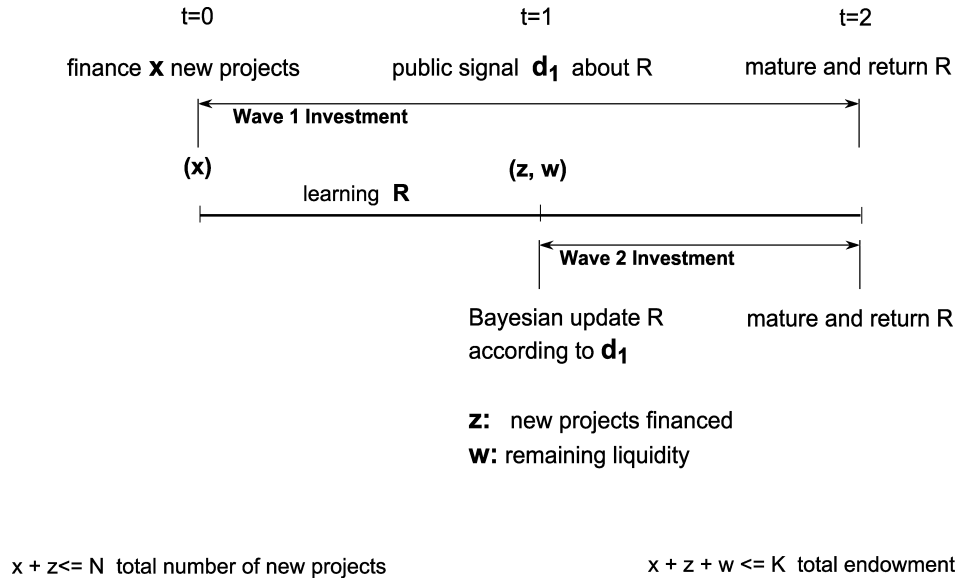


Figure 3.1: Timing

invest in new projects at two dates: either  $t = 0$  or  $t = 1$ , or both dates. Investors essentially select a portfolio composed of illiquid projects and liquidity.

### 3.2.3 Allocation mechanism

At  $t = 0$ , all investors make a simultaneous move. There is no pre-assigned property rights for all new projects at  $t = 0$  (but later I will discuss Patent as a special allocation mechanism which grants certain degree of property rights to the innovator). Each investor  $i$  can pose a request for  $x_i$  new projects. Denote the aggregate requests of all Investors as  $X$ , so  $X = \sum_{i=1}^M x_i$ . The total requests from all investors can be larger than the total number of new projects  $N$ . This is also true for  $t = 1$ . So we need a rationing and allocation mechanism, defined as the following,

**Definition 3.2.** *An allocation mechanism allocates new projects to each investor  $i$  at  $t = 0$  and 1, according to each investor's individual requests as well as all investors' aggregate requests of projects. The allocation follows two related allocation functions  $h^0(\cdot)$  for  $t = 0$  and  $h^1(\cdot)$  for  $t = 1$  respectively.*

Allocation at  $t = 0$

I use  $x_i^a$  to denote the number of projects that investor  $i$  will actually receive from an allocation mechanism at  $t = 0$ . Then we have the following definition for the vector-valued function  $h^0(\cdot)$  at  $t = 0$ ,

**Definition 3.3.** An allocation function  $h^0(\cdot)$ , takes the investors' request vector  $\langle x_1, x_2, \dots, x_M \rangle$  as input and return a unique allocation vector  $(x_1^a, x_2^a, \dots, x_M^a)$  as the function value:

$$\langle x_1^a, x_2^a, \dots, x_M^a \rangle = h^0(\langle x_1, x_2, \dots, x_M \rangle) \quad (3.2)$$

in addition, I use  $h_i^0(\cdot)$  to denote the  $i^{\text{th}}$  element of  $\langle x_1^a, x_2^a, \dots, x_M^a \rangle$ , i.e. number of projects allocated to the  $i^{\text{th}}$  investor. And  $h^0(\cdot)$  must satisfy the following constraint (3.3),

$$\sum_{i=1}^M h_i^0(\langle x_1, x_2, \dots, x_M \rangle) \leq N \quad (3.3)$$

I also impose the following assumption for all allocation mechanisms,

**Assumption 3.4.** As a commitment to participating in the allocation mechanism, investor is required to finance all its allocated projects at  $t = 0$  and 1.

Then there will be  $N_1 \geq 0$  new projects left for date  $t = 1$ ,

$$N_1 = N - \sum_{i=1}^M h_i^0(\langle x_1, x_2, \dots, x_M \rangle) \quad (3.4)$$

Allocation at  $t = 1$

At  $t = 1$ , each investor  $i$  can pose a request for  $z_i$  new projects. Denote the aggregate requests of all investors as  $Z$ , so  $Z = \sum_{i=1}^M z_i$ . The remaining  $N_1$  projects will be allocated according to a vector-valued function  $h^1(\cdot)$  at  $t = 0$ ,

**Definition 3.5.** An allocation function  $h^1(\cdot)$ , takes the investors' request vector  $\langle z_1, z_2, \dots, z_M \rangle$



as input and return a unique allocation vector  $\langle z_1^a, z_2^a, \dots, z_M^a \rangle$  as the function value:

$$\langle z_1^a, z_2^a, \dots, z_M^a \rangle = h^1(\langle z_1, z_2, \dots, z_M \rangle) \quad (3.5)$$

subject to the following constraint (3.4) and (3.6),

$$\sum_{i=1}^M h_i^1(\langle z_1, z_2, \dots, z_M \rangle) \leq N_1 \quad (3.6)$$

With  $h^0(\cdot)$  and  $h^1(\cdot)$ , I can formally describe various allocation mechanisms.

### 3.2.4 Investors' Strategy Space and Strategy Profile

Each investor  $i$  choose a 3-tuple  $(x_i, z_i, w_i)$ .  $x_i$  denotes her requested new projects at  $t = 0$ ;  $z_i$  is additional requests of new projects at  $t = 1$ ;  $w_i$  is the investments in liquidity. The strategy profile of  $M$  investors is thus  $\langle (x_1, z_1, w_1), (x_2, z_2, w_2), \dots, (x_M, z_M, w_M) \rangle$ .

### 3.2.5 Information Structure

At  $t = 0$ , all players have the same prior for the return of new projects:  $R \sim N(R_0, \alpha_0^{-1})$ .

Assume at  $t = 0$ , there is a total number of investments  $x$ . At  $t = 1$ , consumers and Investors receive an aggregate public signal  $d_1$  about  $R$ ,

$$d_1 = R + \epsilon_1 \quad (3.7)$$

$d_1$  is a realization of the return of new projects financed at  $t = 0$ . The signal  $d_1$  is not perfect because there is a component noise  $\epsilon_1$ .

The endogenous noise  $\epsilon_1$  follows,

$$\epsilon_1 \sim N\left(0, \frac{1}{x^\theta}\right) \quad (3.8)$$

where  $x^\theta$  is the precision of the noise  $\epsilon_1$ . Investing in more projects can reduce the variance of  $\epsilon_1$ .

Due to (3.7), we have  $d_1|R \sim N\left(R, (x^\theta)^{-1}\right)$ .  $R$  and  $\epsilon_1$  are two independent random variables with normal distributions. The sum of them, i.e.  $d_1$  also follows a normal distribution as follows,

$$d_1 \sim N\left(R_0, (x^\theta)^{-1} + (\alpha_0)^{-1}\right) \quad (3.9)$$

### 3.3 Optimal Allocation

In this section, I start from the description of the social planner's problem and then derive the optimal allocation.

Social planner will allocate the total endowment of capital  $K$  to a 3-tuple  $(x, z, w)$ .  $x$  is the capital invested in risky new projects at  $t = 0$ . Because each illiquid project requires one unit of investment, this means that number  $x$  risky projects are invested at  $t = 0$ . At the same time, the choice  $x$  also determines the signal  $d_1$  for  $t = 1$ , whose distribution follows  $d_1 \sim N\left(R_0, (x^\theta)^{-1} + (\alpha_0)^{-1}\right)$ .

At  $t = 1$ , an additional decision is to choose  $z$ : the number of additional risky projects to invest. The total number of risky projects available for investment is  $N$ , so I have,

$$x + z \leq N \quad (3.10)$$

This choice of  $z$  depends on the new information received at  $t = 1$ , i.e. signal  $d_1$  of projects' return invested last period. There will be Bayesian Updating according to signal  $d_1$ , and the posterior belief follows (3.11) and (3.12),

$$E_1 [R|d_1] = \frac{\alpha_0 R_0 + d_1 x^\theta}{\alpha_0 + x^\theta} \quad (3.11)$$

$$V_1 [R|d_1] = (\alpha_0 + x^\theta)^{-1} \quad (3.12)$$

If the signal of project return is high enough, more risky projects will be invested.  $z = 0$  if the signal is below some threshold. The remaining endowment will be the leftover liquidity  $w$ . Therefore, we have a binding resource constraint (3.13),

$$x + z + w = K \tag{3.13}$$

$x$  plays the role of learning about risky project's return. However, if the return of risky project turns out to be very low, it will impose a cost due to the investment on bad projects at the beginning.  $x$  is chosen to make an optimal tradeoff between information acquisition and potential welfare loss due to investments in uncertain new projects.

### 3.3.1 Social Planner's Problem

The social planner's problem is recursively described by (3.14) and (3.15),

$$V_0 = \max_x \mathbb{E}_0 [Rx + V_1] \tag{3.14}$$

where  $V_1$  is the continuation value at  $t = 1$ , described by (3.15)

$$V_1 = \max_{z,w} \mathbb{E}_1 [Rz + w | d_1] \tag{3.15}$$

$$s.t. \quad (3.9), (3.10), (3.11), (3.12), (3.13)$$

At  $t = 0$ , the expected return of risky projects is  $E_0 [R] = R_0$ , which is just the prior of the return at the beginning. Whereas at  $t = 1$ , the information set will include a new signal  $d_1$ . Thus the choice of  $z$  at  $t = 1$  depends on this newly generated signal  $d_1$ , which is affected by the choice  $x$  last period, according to (3.9).

Additional new projects will be financed if and only if  $E_1 [R | d_1] > 1$ . Because the total endowment is significantly larger than the number of all risky projects, there can be some

leftover liquidity  $w$ , even if all the remaining risky projects have been financed at  $t = 1$ .

The grand optimization problem (3.14) need to be solved by backward induction.

### 3.3.2 Allocation Problem at $t = 1$

At  $t = 1$ , the optimization problem is (3.15).The public signal  $d_1$  has been generated. The belief about return  $R$  is updated according to signal  $d_1$ .

The optimal choice for  $(z^*, w^*)$  is given by (3.16),

$$\begin{cases} z^* = N - x & w^* = K - N & \text{if } E_1 [R|d_1] \geq 1 \\ z^* = 0 & w^* = K - x & \text{if } E_1 [R|d_1] < 1 \end{cases} \quad (3.16)$$

The threshold value  $\bar{d}_1$  of choice is when  $E_1 [R|d_1] = \frac{\alpha R_0 + d_1 x^\theta}{\alpha_0 + x^\theta} = 1$ ,and is given by (3.17),

$$\bar{d}_1 = \frac{\alpha_0 + x^\theta - \alpha_0 R_0}{x^\theta} \quad (3.17)$$

Given the optimal choice (3.16) at  $t = 1$ , maximized total utility derived from  $t = 1$  investment now becomes,

$$\begin{cases} \frac{\alpha R_0 + d_1 x^\theta}{\alpha_0 + x^\theta} N + (K - N) & \text{if } d_1 \geq \bar{d}_1 \\ \frac{\alpha R_0 + d_1 x^\theta}{\alpha_0 + x^\theta} x + (K - x) & \text{if } d_1 < \bar{d}_1 \end{cases} \quad (3.18)$$

Then the planner can take the result (3.18) as given, and make decisions at  $t = 0$ .

### 3.3.3 Allocation Problem at $t = 0$

Back to the very beginning, the decision problem is to choose the optimal signal  $d_1$  for the next period.

At  $t = 0$ , we only know  $d_1 = R + \epsilon_1$ . Because  $R$  and  $\epsilon_1$  are two independent, we have (3.9). Therefore, we know signal  $d_1$ 's probability distribution function  $f(d_1)$  follows (3.19),

$$f(s) = \frac{1}{\sqrt{(x^\theta)^{-1} + (\alpha_0)^{-1}}} \phi \left( \frac{s - R_0}{\sqrt{(x^\theta)^{-1} + (\alpha_0)^{-1}}} \right) \quad (3.19)$$

Notice here the signal's probability distribution function  $f(d_1)$  is a function of  $x$ .

Then I can rewrite (3.14) as the optimization problem (3.20),

$$\max_{\{x\}} \left\{ \begin{aligned} & \int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} \left( \frac{\alpha_0 R_0 + s \cdot x^\theta}{\alpha_0 + x^\theta} x + K - x \right) f(s) ds \\ & + \int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} \left( \frac{\alpha_0 R_0 + s \cdot x^\theta}{\alpha_0 + x^\theta} N + K - N \right) f(s) ds \end{aligned} \right\} \quad (3.20)$$

s.t. (3.19)

**Proposition 3.1.** *There is a unique solution to the optimal allocation problem (3.20). The optimal solution  $x$  to (3.20) is given by (3.21),*

$$\frac{\Phi \left( (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + x^\theta)}{x^\theta}} \right)}{\phi \left( (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + x^\theta)}{x^\theta}} \right)} = \frac{1}{(1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + x^\theta)}{x^\theta}}} \left( \frac{\theta \alpha_0 (N - x)}{2x (\alpha_0 + x^\theta)} - 1 \right) \quad (3.21)$$

*Proof.* The detailed derivation and proof is provided in the appendix. □

**Corollary 3.1.**  *$x^*$  is larger than 0.*

Experimentation at the beginning is efficient because the option value for learning is greater than 0. The new projects are essentially a kind of real option and provide an opportunity of financing new projects with potentially higher return. Therefore, investments at this stage can generate more accurate information by trying a small number of new projects. Information acquisition at the first stage can help to make better decisions later.

**Corollary 3.2.**  *$x^*$  is less than  $N$  in the optimal allocation.*

In general, financing all risky projects at  $t = 0$  are not socially optimal. This is due to precaution regarding an uncertain new technology.

**Corollary 3.3.** *When  $R_0 = 1$ ,  $\theta = 1$ , I have a unique closed-form solution for  $x^*$ ,*

$$x^* = \frac{1}{4} \sqrt{\alpha_0 (8N + 9\alpha_0)} - \frac{3}{4} \alpha_0$$

*and in the limit  $\frac{dx^*}{d\alpha_0}$  converges to 0 when  $\alpha_0$  grows to be large,*

$$\lim_{\alpha_0 \rightarrow \infty} \frac{dx^*}{d\alpha_0} = 0$$

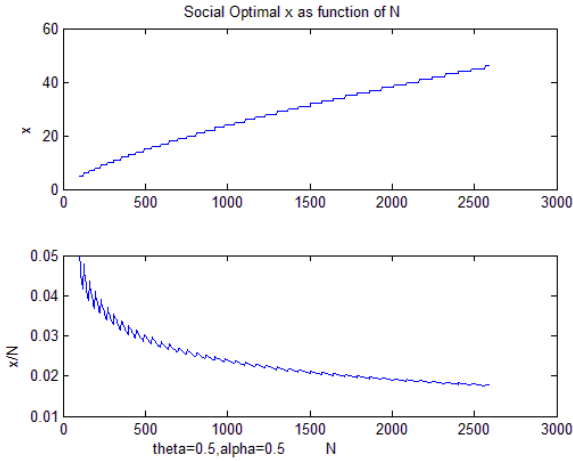
### 3.3.4 Properties and discussions

#### Number of New Projects $N$

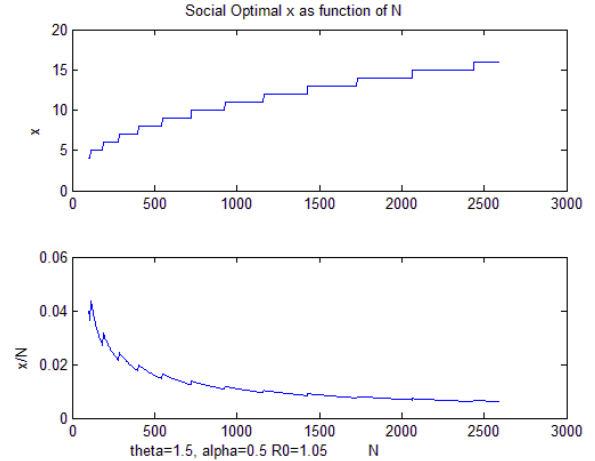
As shown in the Figure 3.2, the optimal  $x^*$  is an increasing function of  $N$ , but the fraction  $x/N$  is a decreasing function of  $N$ . This means that the relative learning cost is declining with respect to the total number of new projects.

**Proposition 3.2.** *The social optimal investments  $x^*$  at  $t = 0$  is an increasing function of  $N$ , but the ratio  $x/N$  is a decreasing function of  $N$ .*

With a larger number of new projects, the social planner tends to be more cautious because the total social value and therefore the stake of learning will increase with  $N$ . However, the relative number of projects  $x/N$  used for learning purpose decreases with  $N$ . On the other hand, this can imply that market equilibrium can impose relatively higher social cost when  $N$  grows larger.



Panel A

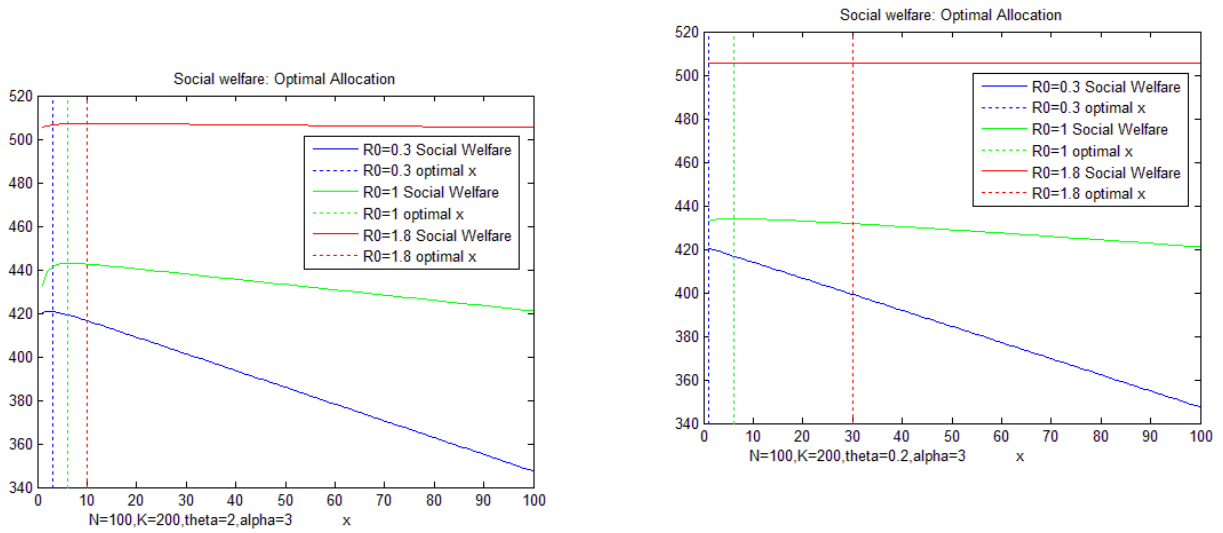


Panel B

Figure 3.2: The number of new projects  $N$

### Learning efficiency $\theta$

We want to see the effect of learning efficiency on the optimal  $x^*$ . We can see from Figure 3.3, that less efficient learning, as in Panel B will demands more learning when the prior of return is high, but less learning when the prior of return is low. Although the cost of learning rises with a lower  $\theta$ , it is worth more investment because of a better outlook of return.



Panel A

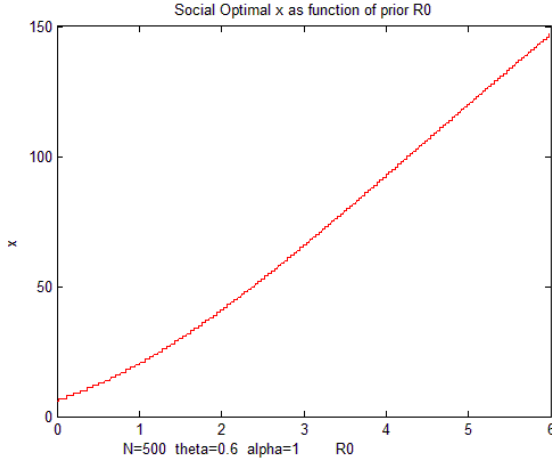
Panel B

Figure 3.3: Learning efficiency

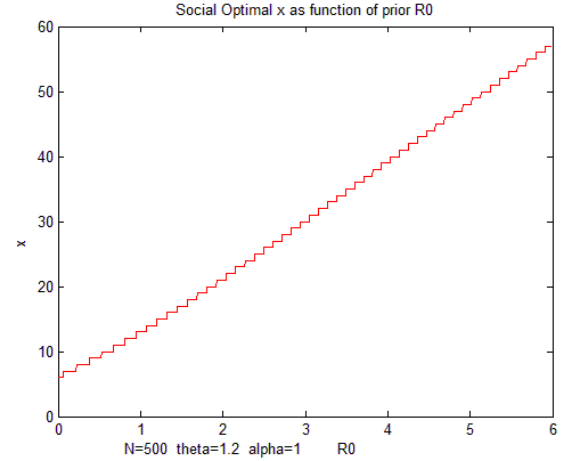
### Prior return $R_0$

Figure 3.4 shows how the socially optimal  $x$  responds to different priors of  $R_0$ . With disparate learning efficiency ( $\theta = 0.6$  for the left panel,  $\theta = 1.2$  for the right panel), the optimal  $x$  is smooth increasing function of  $R_0$ . Even the prior  $R_0$  rises to as high as 6 (600% return for investment), the first period investment only increases smoothly, without occurrence of any “Rush”. You may now imagine market equilibrium can generate very different result than the social optimum.





Panel A



Panel B

Figure 3.4: Optimal  $x$  as a function of prior  $R_0$

### 3.4 Nonexcludability, Rivalry, and Uncertainty of Innovation: The Tragedy of the Uncertain Commons

In this section, I discuss the ownership and market structure for new technologies. Exclusive ownership of new technology encourages patient learning of new technology which can reduce potential hazards to the society. Conversely, imperfect IPR (Intellectual Property Rights) protection can aggravate the coordination failure of massive premature investments in those new but uncertain technologies.

#### 3.4.1 Nonexcludability

The classical “Tragedy of the Commons” denotes a situation that individuals tend to “*overgraze*” some common pool of resource because the social cost will be shared by the group whereas an individual can keep the benefit for herself. *Nonexcludability* and the lack of property rights is one important cause of the Tragedy of the Commons.

IPR, e.g. patent, assigns exclusive property rights to the inventors, for the purpose of providing incentive to invest on *R&D*. The endogenous growth literature assumes the full patent rights to innovations. However, in reality, due to the public good nature of innovation, excludability of IPR can be easily violated in various ways. Therefore property rights are more often ill-defined for these “*New Commons*”, e.g. new technologies and new business opportunities. I will discuss several situations of nonexcludability.

## Nonpatentability by the Patent Law

There are many types of new ideas that cannot be protected by the current Patent law. For example, “*abstract idea*” and “obvious idea” are not patentable. Most recently, the business method “surge-pricing” of *Uber* is thought to be nonpatentable: “This application is really seeking to claim the basic idea of pricing and service, which is a concept Adam Smith discussed 200 years ago.” It is very difficult for *Uber* to discourage other competitors to copy its business method.

In general, financial innovations are also not eligible for patent application. In a recent case, *Alice Corp. v. CLS Bank* (2014), the U.S. Supreme Court made a final decision that “a computer-implemented, electronic escrow service for facilitating financial transactions covered abstract ideas ineligible for patent protection. The patents were held to be invalid because the claims were drawn to an *abstract idea*, and implementing those claims on a computer was not enough to transform that idea into patentable subject matter.” Moreover, after this Supreme Court decision, the U.S. Patent and Trademark office has indeed *stopped* granting business method patents.

## Design-arounds

It is also possible to bypass an existing patent. By borrowing the idea of a major patent, competitors can patent slightly modified ideas and bypass the existing one. This enroaches the monopoly of existing patent and “steals” the IPR in an implicit way. Kremer (2001)

points out the relative easiness to design around vaccines patents.

## GPT and Technological Revolutions

The “Commons” problem is especially prominent for the GPT (General Purpose Technology) and technological revolutions. A breakthrough in GPT often injects huge knowledge spillover to the whole economy. A GPT, like the Internet, can spur the invention of a lot of new products. The original inventor of the GPT can only capture a very small share of all the profitable opportunities built on the general technology. “*New Commons*” will naturally emerge from such technological revolutions. This implies the supply of a large quantity of “free lunches”.

### *3.4.2 Nonrivalry of ideas v.s. Rivalry of Investment Opportunities*

On the other hand, limited profit opportunities for a new technology results in *Rivalry* for financing new technologies. This is contrary to the “*Nonrivalry of knowledge use*” assumption in the endogenous growth literature. Knowledge or ideas indeed has the nonrival nature, especially for long run growth. Future generations can freely reuse the same idea for infinite times without depreciation. Investment opportunities for new idea or new technology go in the other way: it will disappear very quickly, and its profitability can be completely lost when the idea becomes a pure public knowledge. The opportunities of investing in profitable innovations is in a very limited supply.

### *3.4.3 Deep Uncertainty of Innovations*

Some innovations have potential hazards, even very lethal ones. Many problems of innovations can be revealed only in the market. Lab experiment and *FDA-like* pre-testing can only detect a limited amount of potential hazards. Therefore, investment in a moderate scale is necessary to generate better and more subtle information for innovations.

In general, idea and knowledge spillover is believed to be beneficial to the society. However, the hasty proliferation of bad ideas or wrong models can be extremely detrimental, because the full spectrum of an innovation's benefits and risks can not be completely understood during a very short period.

**Example 3.6.** *The drug “Thalidomide” is a thrilling case. “Shortly after Thalidomide was sold in 1957, in Germany, between 5,000 and 7,000 infants were born with phocomelia (malformation of the limbs). Only 40% of these children survived.” Globally, there were more than 10,000 reported malformations due to usage of Thalidomide.*

#### 3.4.4 *The Tragedy of the Uncertain Commons*

A property with the nature of nonexcludability and rivalry is called open-access common property. Without perfect IPR protection, the tragedy of the Commons will come up. There does not exist a complete price system to discourage the massive entry into financing certain new technologies. The limited supply of profitable opportunities creates strategic complementarity between investors which incentivizes them to rush into the new market. The fear of losing market share to competitors will dilute the investor's concern for the uncertainty of innovations. Investors will finance new projects too quickly even when there are still a lot of uncertainties remaining. “Overgrazing” these *new* and “*Uncertain Commons*” can result in high social costs ex post. This leads to the *Tragedy of the Uncertain Commons*.

In general, “rush” phenomenon is often a consequence of ill-defined property rights and missing market. With uncertainty, rush will generate larger inefficiencies than under the traditional Tragedy of the Commons.

### 3.5 The $M$ – investor Equilibria

In this section, I discuss the market equilibrium of  $M$  symmetric investors ( $M$  – investor Equilibria). I denote  $X_{-i}$  as the total number of new projects requested by other investors at  $t = 0$  :  $X_{-i} = \sum_{j \neq i}^M x_j$ . The aggregate requests is  $X = \sum_{j=1}^M x_j$ . Denote  $Z_{-i}$  as the total number of new projects financed by other investors at  $t = 1$ , i.e.  $Z_{-i} = \sum_{j \neq i}^M z_j$ .

#### 3.5.1 Investor's Problem with General-form Allocation Mechanism

The optimization problem of investor  $i$  is described by (3.22),

$$V_{i,0} = \max_{x_i} \mathbb{E}_{i,0} [R \times x_i + V_{i,1}] \quad (3.22)$$

where  $V_{i,1}$  is investor  $i$ 's continuation value at  $t = 1$ , defined by (3.23)

$$V_{i,1} = \max_{z_i, w_i} \mathbb{E}_{i,1} [R \times z_i + w_i | d_1] \quad (3.23)$$

$$s.t. \quad h_i^0 (\langle x_1, x_2, \dots, x_M \rangle) + h_i^1 (\langle z_1, z_2, \dots, z_M \rangle) + w_i \leq k \quad (3.24)$$

$$x_i, z_i, w_i \geq 0 \quad (3.25)$$

$$E_1 [R | d_1] = \frac{\alpha_0 R_0 + d_1 \left( \sum_{i=1}^M h_i^0 (\langle x_1, x_2, \dots, x_M \rangle) \right)^\theta}{\alpha_0 + \left( \sum_{i=1}^M h_i^0 (\langle x_1, x_2, \dots, x_M \rangle) \right)^\theta} \quad (3.26)$$

$$V_1 [R | d_1] = \left( \alpha_0 + \left( \sum_{i=1}^M h_i^0 (\langle x_1, x_2, \dots, x_M \rangle) \right)^\theta \right)^{-1} \quad (3.27)$$

$$d_1 \sim N(R_0, \left( \sum_{i=1}^M h_i^0 (\langle x_1, x_2, \dots, x_M \rangle) \right)^{-\theta} + (\alpha_0)^{-1}) \quad (3.28)$$

Investor's problem has been described above with the most general-form allocation mechanism. In the next section, I will introduce the baseline allocation mechanism.

### 3.5.2 Baseline Allocation Mechanism: ECPR

I firstly define an allocation mechanism which embodies rivalry and nonexcludability as discussed in the previous section. Under this mechanism, each investor receives her requested projects up to an equal share limit at both dates. This is formally defined by the following **ECPR** Allocation Mechanism (Equal Opportunity for the Common-pool Resource),

**Allocation Mechanism 3.7** (Baseline: ECPR). *Allocation functions  $h^0(\cdot)$  and  $h^1(\cdot)$  are defined by (3.4), (3.29), and (3.30),*

$$h_i^0(\langle x_1, x_2, \dots, x_M \rangle) = \begin{cases} x_i & \text{if } x_i \leq \frac{N}{M} \\ \frac{N}{M} & \text{if } x_i > \frac{N}{M} \end{cases} \quad (3.29)$$

$$h_i^1(\langle z_1, z_2, \dots, z_M \rangle) = \begin{cases} z_i & \text{if } z_i \leq \frac{N_1}{M} \\ \frac{N_1}{M} & \text{if } z_i > \frac{N_1}{M} \end{cases} \quad (3.30)$$

Notice that (3.4), (3.29), and (3.30) have automatically satisfied constraints (3.3) and (3.6).

#### Common-pool Resource problem

In the ECPR mechanism, the Common-pool Resource problem is embodied by the inter-period constraint (3.4). The remaining projects will be reallocated equally at  $t = 1$ . Other investors' increase in investments at  $t = 0$  will reduce the leftover quantity  $N_1$  at  $t = 1$ . This raises the cost of delaying investments. Therefore, this creates complementarity between investors'  $x$  choices at  $t = 0$ .

### 3.5.3 A Model of Venture Capital (VC)

I argue the current model with the ECPR mechanism is consistent with the Venture Capital's (VC) investment pattern. Think each investor as a Venture Capitalist. At  $t = 0$ , each VC builds their specific product based on a technological breakthrough, and stays in their own

niche for the first period. This is consistent with (3.29). Take the Shared-economy as a recent example of technological breakthrough, Uber and AirBnB have applied the same idea to different areas. But at  $t = 1$ , investors are allowed to enter each other's niches, as illustrated by (3.30).

With the ECPR mechanism, constraints in the general-form (3.24),(3.25),(3.26),(3.27),and (3.28) can be reduced to the following,

$$x_i + z_i + w_i \leq k \quad (3.31)$$

$$x_i, z_i, w_i \geq 0 \quad (3.32)$$

$$x_i \leq \frac{N}{M} \quad (3.33)$$

$$z_i \leq \frac{N - x_i - X_{-i}}{M} \quad (3.34)$$

$$E_1 [R|d_1] = \frac{\alpha_0 R_0 + d_1 (x_i + X_{-i})^\theta}{\alpha_0 + (x_i + X_{-i})^\theta} \quad (3.35)$$

$$V_1 [R|d_1] = \left( \alpha_0 + (x_i + X_{-i})^\theta \right)^{-1} \quad (3.36)$$

$$d_1 \sim N(R_0, (x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}) \quad (3.37)$$

In comparison with social planner's problem (3.14), the decentralized problem (3.22) for each investor  $i$  have additional constraints (3.33), (3.34), (3.35), (3.36) and (3.37). These constraints will move the optimal allocation to opposite directions.

As a mapping to the allocation functions (3.4), (3.29), and (3.30) in the last subsection, (3.33) and (3.34) create complementarity between investors'  $x$  choices.

The other three constraints (3.35), (3.36) and (3.37) reflect *Information externality*, i.e. the public good nature of the signal generated by all investments at  $t = 0$ , including other Investors'. Here I have assumed all Investors' investments contribute equally to the generation of the public signal  $d_1$ . Only the aggregate number of new projects financed at  $t = 0$ , the  $X_a$ ,

matters for the precision of  $d_1$ . Thus other Investors' investment at  $t = 0$  can directly benefit investor  $i$  at  $t = 1$ . This information externality encourages Investors to delay investment and free ride on others' effort.

Other Investors' investments can reveal more information, but this will also reduce the stock of new projects available for financing. These two effects work in contradictory directions.

### 3.5.4 Subgame Perfect Equilibrium of $M$ -investor game

The Equilibrium concept for the  $M$ -investor game is Subgame Perfect Equilibrium (*SPE*).

**Definition 3.8.** (*Subgame Perfect Equilibrium*) *In an equilibrium of  $M$ -investor game, each investor  $i$  chooses its optimal vector  $(x_i^*, z_i^*, w_i^*)$  and will not deviate from it, given all other Investors' optimal strategies  $\left\{ \left( x_j^*, z_j^*, w_j^* \right)_{j=1 \& j \neq i}^M \right\}$ ; and this also applies to every proper subgame.*

Similar to the solution method for the social planner's problem, I use backward induction to derive the decentralized solution.

### 3.5.5 Investor $i$ 's problem at $t = 1$

At  $t = 1$ , the decision problem is whether to finance additional new risky projects, after perceiving the public signal  $d_1$  about project return. All Investors see the same signal  $d_1$ .

There will exist a signal threshold  $\bar{d}_1$  above which more investments will be made. No more investments will be made if  $d_1 < \bar{d}_1$ . After Bayesian updating, the expected return of new projects is given by  $\frac{\alpha_0 R_0 + d_1 (x_i + X_{-i})^\theta}{\alpha_0 + (x_i + X_{-i})^\theta}$ .

So the threshold  $\bar{d}_1$  will make  $\frac{\alpha_0 R_0 + \bar{d}_1 (x_i + X_{-i})^\theta}{\alpha_0 + (x_i + X_{-i})^\theta} = 1$ , and is thus determined by (3.38),

$$\bar{d}_1 = \frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha_0 R_0}{(x_i + X_{-i})^\theta} \quad (3.38)$$



The optimal choice  $(z_i^*, w_i^*)$  of investor  $i$  is thus given by the following equations,

$$\begin{cases} z_i^* = \frac{N-x_i-X_{-i}}{M} & w_i^* = k - \left(x_i + \frac{N-x_i-X_{-i}}{M}\right) & \text{if } d_1 \geq \bar{d}_1 \\ z_i^* = 0 & w_i^* = k - x_i & \text{if } d_1 < \bar{d}_1 \end{cases} \quad (3.39)$$

### 3.5.6 Investor $i$ 's problem at $t = 0$

Given the  $t = 1$  solution (3.39), Investor  $i$ 's optimization problem at  $t = 0$  becomes,

$$\max_{\{x_i\}} \left\{ \begin{aligned} & \int_{-\infty}^{\bar{d}_1} \left( \frac{\alpha_0 R_0 + s \cdot (x_i + X_{-i})^\theta}{\alpha_0 + (x_i + X_{-i})^\theta} x_i + k_i - x_i \right) f(s) ds \\ & + \int_{\bar{d}_1}^{\infty} \left( \frac{\alpha_0 R_0 + s \cdot (x_i + X_{-i})^\theta}{\alpha_0 + (x_i + X_{-i})^\theta} \left( x_i + \frac{N-x_i-X_{-i}}{M} \right) + k_i - \left( x_i + \frac{N-x_i-X_{-i}}{M} \right) \right) f(s) ds \end{aligned} \right\} \quad (3.40)$$

$$\text{s.t. (3.38), (3.41)}$$

where  $d_1$ 's probability distribution function  $f(\cdot)$  follows (3.41),

$$f(s) = \frac{1}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}} \phi \left( \frac{s - R_0}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}} \right) \quad (3.41)$$

After careful integration, the optimization problem finally becomes,

$$\max_{x_i} \left\{ \begin{aligned} & k_i + (R_0 - 1) \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) \\ & + \frac{N-x_i-X_{-i}}{M} (1 - R_0) \Phi \left( \frac{(1-R_0) [\alpha_0 + (x_i + X_{-i})^\theta]}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}} \right) \\ & + \frac{N-x_i-X_{-i}}{M} \frac{(x_i + X_{-i})^{\frac{\theta}{2}}}{\alpha_0^{\frac{1}{2}} (\alpha_0 + (x_i + X_{-i})^\theta)^{\frac{1}{2}}} \phi \left( \frac{(1-R_0) [\alpha_0 + (x_i + X_{-i})^\theta]}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}} \right) \end{aligned} \right\} \quad (3.42)$$

### 3.5.7 Best response correspondence

**Proposition 3.3.** Equation (3.43) gives the best response correspondence regarding the optimization problem (3.22).

$$M - 1 + \Phi \left( \frac{\frac{(1-R_0)[\alpha_0+(x_i+X_{-i})^\theta]}{(x_i+X_{-i})^\theta}}{\sqrt{(x_i+X_{-i})^{-\theta}+(\alpha_0)^{-1}}} \right) =$$

$$\frac{(x_i+X_{-i})^{\frac{1}{2}\theta-1}}{2(R_0-1)\left(\alpha_0+(x_i+X_{-i})^\theta\right)^{\frac{3}{2}}\alpha_0^{\frac{1}{2}}} \begin{pmatrix} 2(x_i+X_{-i})^{\theta+1}+2\alpha_0(x_i+X_{-i}) \\ -N\theta\alpha_0+\theta\alpha_0(x_i+X_{-i}) \end{pmatrix} \quad (3.43)$$

$$\times \phi \left( \frac{\frac{(1-R_0)[\alpha_0+(x_i+X_{-i})^\theta]}{(x_i+X_{-i})^\theta}}{\sqrt{(x_i+X_{-i})^{-\theta}+(\alpha_0)^{-1}}} \right)$$

Equation (3.43) characterizes the best response of Investor  $i$  to the aggregate choice of all other Investors.

We can see (3.43) has an additional item  $M - 1$  in comparison with social planner's solution (3.21). This can move the equilibrium solution to either direction, over-invest or under-invest, conditional on the sign of the middle item (3.43). This implies market equilibrium level of early investment  $x_i$  can be above or below the social optimal level.

Solution (3.43) is compatible with either pure-strategy symmetric equilibrium or mixed-strategy (*leader, follower*) equilibrium.

Figure 3.5 shows a best response function with 2 Investors. This will give a unique equilibrium.

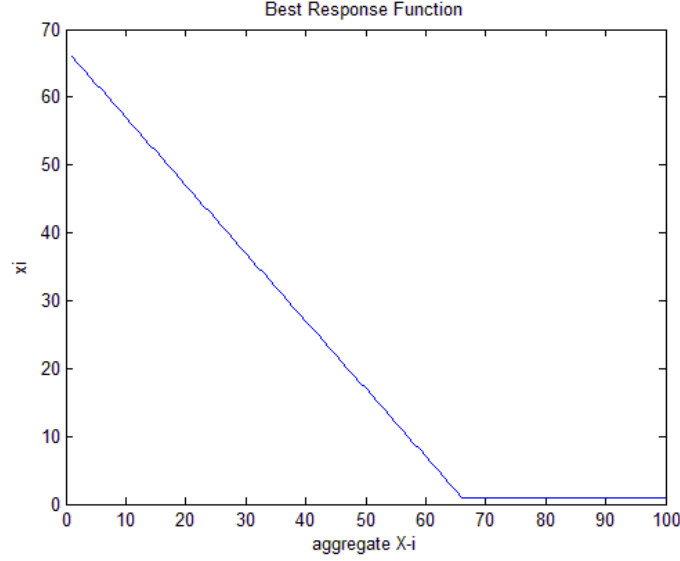


Figure 3.5: Best Response Correspondence

### 3.5.8 Symmetric Equilibrium

I have assumed all Investors are symmetric at the beginning, and then can get the following proposition.

**Proposition 3.4.** *There exists a unique symmetric equilibrium, and the closed-form solution of  $x$  is given by (3.44),*

$$\begin{aligned}
 & (M - 1) + \Phi \left( \frac{(1 - R_0) \sqrt{\alpha} \left( (Mx)^\theta + \alpha \right)^{\frac{1}{2}}}{(Mx)^{\frac{\theta}{2}}} \right) = \\
 & \frac{1}{2(R_0 - 1)} \frac{(Mx)^{\frac{1}{2}\theta - 1}}{\alpha^{\frac{1}{2}} \left( \alpha + (Mx)^\theta \right)^{\frac{3}{2}}} \begin{pmatrix} 2Mx\alpha + 2(Mx)^{\theta+1} \\ -N\theta\alpha + \theta\alpha Mx \end{pmatrix} \phi \left( \frac{(1 - R_0) \sqrt{\alpha} \left( (Mx)^\theta + \alpha \right)^{\frac{1}{2}}}{(Mx)^{\frac{\theta}{2}}} \right)
 \end{aligned} \tag{3.44}$$

In the symmetric equilibrium, it is possible that all  $x_i = 0$ , i.e. all Investors wait and intend to free-ride on others' information acquisition.

At first glance, it seems there can exist multiple symmetric equilibria within some parameter ranges. Assume there exist two symmetric equilibria,  $(x_i^H)$ ,  $(x_i^L)$ , where  $x_i^H > x_i^L$ . A *high*  $-x$  equilibrium  $(x_i^H)$  can be justified due to preemption motive. A *low*  $-x$  equilibrium  $(x_i^L)$  can be also justified due to the value of learning and free-rider motive. Multiple equilibria can exist when these two forces are close to a balance. However, uncertainty removes the multiplicity.

**Corollary 3.4.** *When  $R_0 = 1$ ,  $\theta = 1$ , we have a unique closed-form solution for  $x^*$ ,*

$$x^* = \frac{\sqrt{\alpha_0(8N + 9\alpha_0)} - 3\alpha_0}{4M}$$

and in the limit  $\frac{dx^*}{d\alpha_0}$  converges to 0 when  $\alpha_0$  grows to be large,

$$\lim_{\alpha_0 \rightarrow \infty} \frac{dx^*}{d\alpha_0} = 0$$

We can see that at  $R_0 = 1$ , the decentralized solution coincides with the social optimum.

### 3.6 Amplification and "Rush"

In this section I will discuss how a "Rush" can happen and the amplification mechanism in market equilibrium.

I firstly use the following numerical example to demonstrate the degree of sensitivity around some critical value of  $R_0$ .

**Example 3.9.**  $\theta = 1.25$ ; *prior precision*  $\alpha_0 = 1$ ;  $R_0 = 1.275$ ; *total number of new projects*  $N=500$ ; *two Investors*  $M=2$

when  $\Delta R_0 = 0.001$ ; the change of  $x_i$  is from 75 to 325,  $\frac{\Delta x_i}{x_i} = 3.333$ ,  $\frac{\Delta R_0}{R_0} = 0.0008$ . Then we have a sensitivity  $\frac{\left(\frac{\Delta X}{X}\right)}{\left(\frac{\Delta R_0}{R_0}\right)} = 4250$ .

This means some tiny belief change  $\frac{\Delta R_0}{R_0} = 0.0008$  can trigger a huge move in the market. Investors will respond with more than four thousand times amplification in early investment. A "Rush" occurs.

I will formally define "**Rush**" as the following,

**Definition 3.10.** *A Rush: The equilibrium  $\hat{X}$ , the first period investment, is dramatically increased by some small shock.*

A typical small shock is News shocks. For example,  $\Delta R_0 > 0$ , for some good news;  $\Delta R_0 < 0$  for some bad news. A mathematical measure of "Rush" is defined by the following Amplification function,

**Definition 3.11.** *Amplification Function  $S(R_0)$  :*

$$S(R_0) = \frac{\frac{d\hat{X}}{dR_0}}{\frac{dX^*}{dR_0}}(R_0) \quad (3.45)$$

### 3.6.1 In Comparison with the Social Optimum

With Figure 3.4 in section 3, I have shown the social optimal  $X^*$  is a smooth function of prior  $R_0$ . There will never be a "rush". Social planner always prefer to conduct some small-scale experiment to learn the new projects patiently, before any massive investments. We call the social optimum choice of  $X^*$  "smooth learning" for investment. But market equilibrium will behavior in very different fashion.

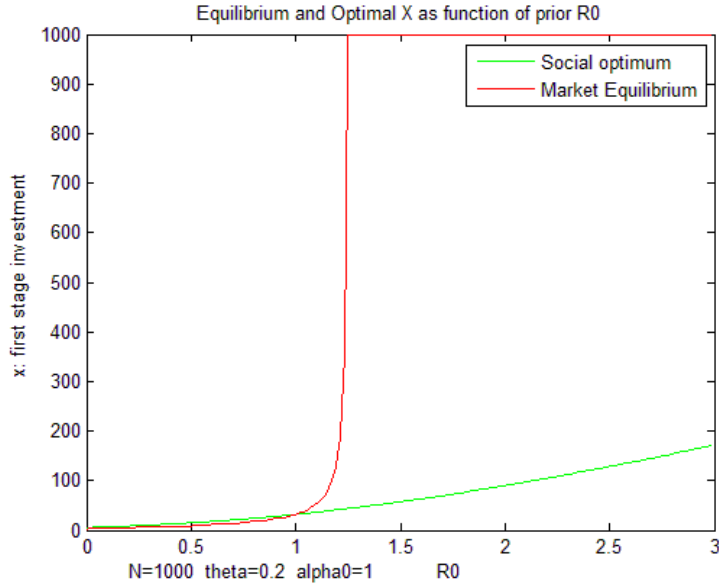


Figure 3.6.: The generation of a "Rush"

Figure 3.6 compares the social optimum with a 2-*Investor* market equilibrium outcome. We can see a rush indeed occurs for the 2-*Investor* equilibrium around some threshold value 1.2. The amplification function  $S(R_0)$  is an increasing function of  $R_0$  until all the new projects are exhausted.

It is also worth mentioning that in this market equilibrium it is not a direct switching between small-scale experiment and full-scale investment. It is still a continuous transition though the amplification effect grows very fast. The remaining uncertainty prevents Investors from taking a vertical jump to finance all new projects.

The amplification function  $S(R_0)$  and its derivative  $S'(R_0)$  increase in  $R_0$ . And the fact amplification function  $S(R_0)$  is increasing in  $R_0$  also brings about the concern for social welfare loss.

### 3.6.2 Amplification as a Function of $N$

At any given  $R_0$ , if the prior is good enough, a higher  $N$  will add more impetus for an investor to preempt the market.

Figure 3.7 displays the case of a huge  $N = 10000$ . A huge number of new projects can be exhausted instantaneously because the amplification function  $S(R_0)$  keeps growing very quickly.

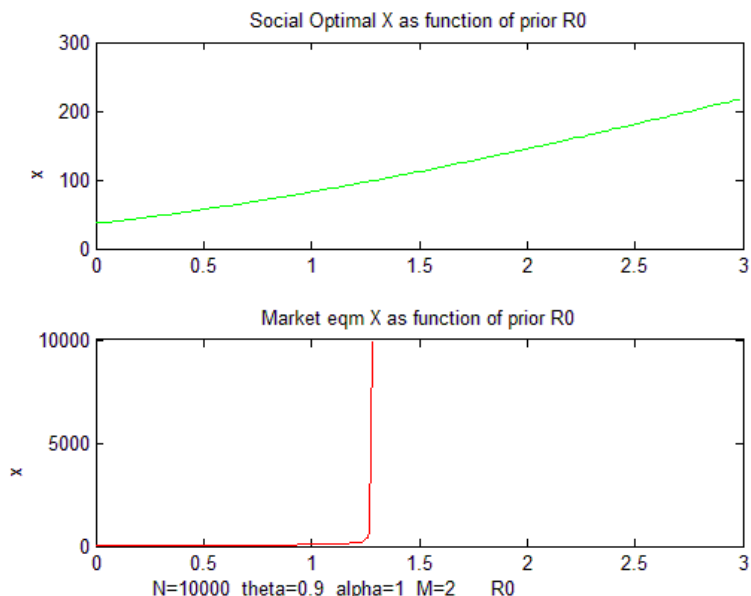


Figure 3.7: "Rush" for huge  $N$

When there is over-investment, I can prove  $\lim_{N \rightarrow \infty} S(R_0)|_{\hat{X}=N} = \infty$ ; but  $S(R_0) < \infty$ , for any  $R_0$ , when  $N < \infty$ .

This implies that larger quantity of new projects, can be drained in an accelerated manner. Without constraint in available deposit, higher  $N$  will lead to expeditious investments. The total social welfare loss is an increasing function of  $N$  because the upper limit  $N$  can be reached very fast no matter how large is  $N$ . The uncertainty of innovation will be magnified by the number of new projects.

### 3.6.3 Amplification as a function of $M$

Previous discussions focus on the 2-Investor equilibrium. We can easily generalize the amplification function to  $M$  – Investor setup:  $S(R_0, M)$ .  $S(R_0, M)$  is non-decreasing in  $M$ , and welfare loss is also non-decreasing in  $M$ .

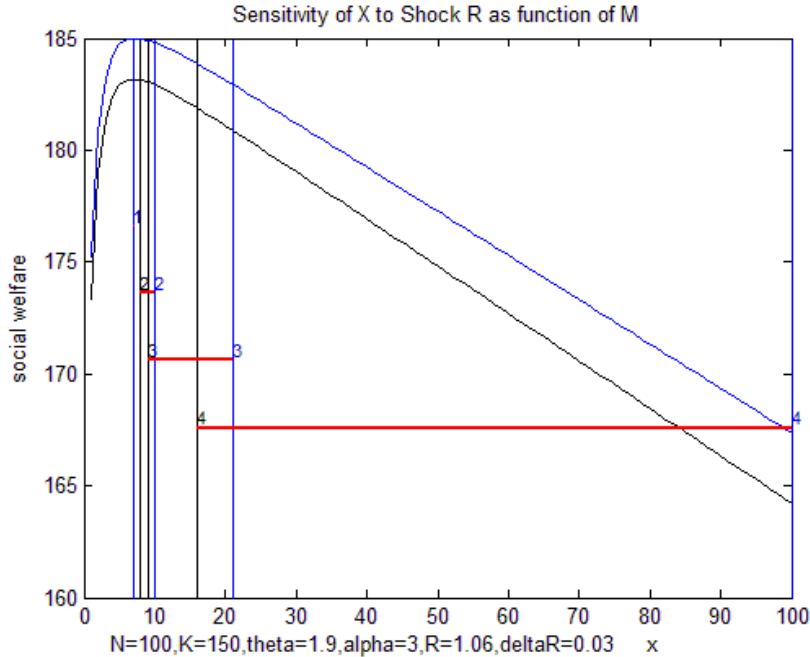


Figure 3.8: Amplification and M

Figure 3.8 shows the different  $x$  for  $M$  – *Investor* equilibria ( $M = 1, 2, 3, 4$ ). After a small shock to  $R_0$ , the new equilibrium  $x$  is illustrated with blue lines. The horizontal lines indicate the increase in  $x$  for different  $M$  after the shock. The increase in equilibrium  $x$  is an increasing function of Investor number  $M$ .

### 3.6.4 "Pricing" a Rush: Shadow Asset Price

Since the property rights for innovations are not perfectly defined, the investment market for innovations is somewhat missing. Therefore the market price for financing innovations might be just misleading. Investors usually underpay for innovations.

In this section, I try to derive the "shadow price" (i.e. the shadow return) for new technologies. We cannot observe the real price but can see the equilibrium quantity  $\hat{x}$ . A thought experiment is to back out the shadow return of new projects by referring to the optimal allocation. For example, in Figure 3.7, the top panel shows the relationship between



optimal  $x^*$  and prior mean return  $R_0$ , while the bottom panel shows the relationship between equilibrium  $\hat{x}$  and prior mean return  $R_0$ .

Denote the socially optimal solution  $x^* = X(R_0)$  as a function of  $R_0$ , and the its inverse function  $R = X^{-1}(x)$  gives the corresponding  $R$  value. Then I substitute the market equilibrium  $\hat{x}$  into the inverse function  $X^{-1}(\cdot)$  and get the shadow return in (3.46),

$$\hat{R} = X^{-1}(\hat{x}) \tag{3.46}$$

The solution to (3.46) can be finally derived by combining (3.44) and (3.21).

Corresponding to the super high level of equilibrium  $\hat{x}$ , the prior mean return  $R_0$  must reach an excessively high level  $X^{-1}(\hat{x})$ .

"Bubble" usually refers to the excessively high asset price. But I argue for the view of "**Rational Rush**", rather than "**Irrational Bubble**". In fact, *the market equilibrium price for innovations is too low, not too high*. The property rights for innovations are often not perfectly defined, and consequently the innovators are usually underpaid. In the market equilibrium, over-investment oftentimes happens in the sense that uncertainty of innovations is underpriced due to the tradeoff of seizing a larger share of the underpriced innovations.

### 3.7 Inefficiencies: Over-investment or Under-investment

In this section, I will discuss the causes and conditions for inefficiencies in the market equilibria.

There are two kind of externalities embedded in the model: **(i)** imperfect property rights and common pool resource problem, and **(ii)** information externalities.

*(i) Common-pool resource and coordination cost*

The limited supply of new investment opportunities creates complementarities between Investors. The ownership of these new investment opportunities is not clearly defined and

results in the "Commons" problem. Without well-defined property rights, price cannot work properly to impede a rush into the new and uncertain market. This will lead to over-investment and "the Tragedy of the Uncertain Commons". The Option value of learning and discarding bad innovations is reduced due to over-investment at an early stage.

(ii) *Information externality*

Information externality is generated because the public signal  $d_1$  of project return at  $t = 1$  is equally contributed by all Investor's investments at  $t = 0$ . Given that  $d_1$  can be perfectly seen by all Investors, each of them wants to delay investment and wait for the signal created by other Investors' efforts. This free-rider motive always exist but will play a more evident role when the prior of return  $R_0$  is low.

Interestingly, there is a watershed between over-investment and under-investment, and we have the following proposition,

**Proposition 3.5.** *There exists a threshold  $\bar{R}_0 = 1$ ,*

- (i) *above which decentralized investors over-invest relative to the social optimum;*
- (ii) *below which decentralized Investors under-invest in equilibrium.*

See the appendix for the details of proof.

### 3.7.1 *Welfare loss due to over-investment in early stage*

Competition between Investors will incentivize them to concern more about the market share of new projects than their uncertain return. Figure 3.9 compares the social optimal investment with a 2 – Investor market equilibrium. The social optimal  $x^*$  is 8, while each investor will invest in 49 new projects in equilibrium. The total market equilibrium investments are 98, close to exhausting all the new investment opportunities at the very beginning. The top panel of Figure 3.9 shows that social welfare (the  $y$  axis) is achieved at the highest level when  $x = x^*$  ( $= 8$ ). It contrasts with a *sub – optimal* social welfare level (the dotted line) corresponding to the equilibrium  $x$  ( $= 98$ ). The coordination cost

dominates the information externality. The welfare loss is mainly due to the abandoned option value of learning.

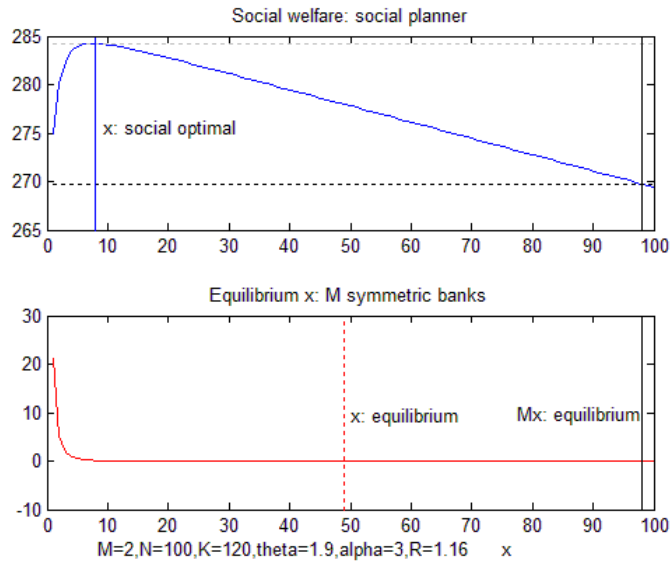


Figure 3.9: Over-investment in 2-Investor Equilibrium

### 3.7.2 Welfare loss due to under-investment in early stage

There are also circumstances that information externality plays a major role so that under-investment happens. In particular, when the prior of mean return  $R_0$  is less than 1, Investors have much less incentive to invest early in the new market, because the benefit of preemption is relatively small. Investors tend to delay investments and wait to watch the public signal. Figure 3.10 shows that the 2-Investor equilibrium has lower  $x$  than the social optimal  $x^*$ . Admittedly, we can see from the top panel that social welfare loss is not very significant in this under-investment situation.

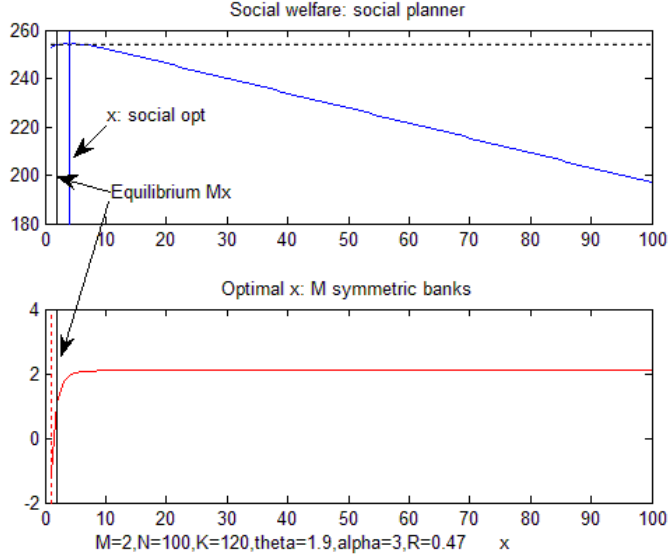


Figure 3.10: Under-investment in 2-Investor Equilibrium

### 3.7.3 Amplification at the threshold value $\bar{R}_0$

The switch between under-investment and over-investment will increase the curvature of amplification function  $S(R_0)$  at the threshold value  $\bar{R}_0$ . The free-rider motive will amplify a news shock around the  $\bar{R}_0$ .

### 3.7.4 Welfare loss and $M$

Increase in the number of Investors will exacerbate both the coordination problem and information externality. For the over-investment circumstance, coordination cost is aggravated more severely than information free-riding problem. Therefore, the net over-investment incentive is magnified by a larger  $M$ . We have the following proposition.

**Proposition 3.6** (Inefficiency and  $M$ ). *For the over-investment circumstance:*

(i) *The aggregate quantity of illiquid projects  $X$  financed at  $t = 0$  is a non-decreasing function of the number of banks  $M$ ;*

(ii) *Social welfare is a non-increasing function of the number of banks  $M$ ;*

(iii) *The limiting case: there exists a threshold  $\bar{M}$ , above which all  $N$  illiquid projects will be financed at  $t = 0$ .*

This can be easily proved as an extension to the proof of Proposition 5.

A mirror proposition can also be proved for the case of under-investment.

Figure 3.11 shows how the equilibrium  $x$  increases with  $M$ , and correspondingly how the social welfare decreases with  $M$ .

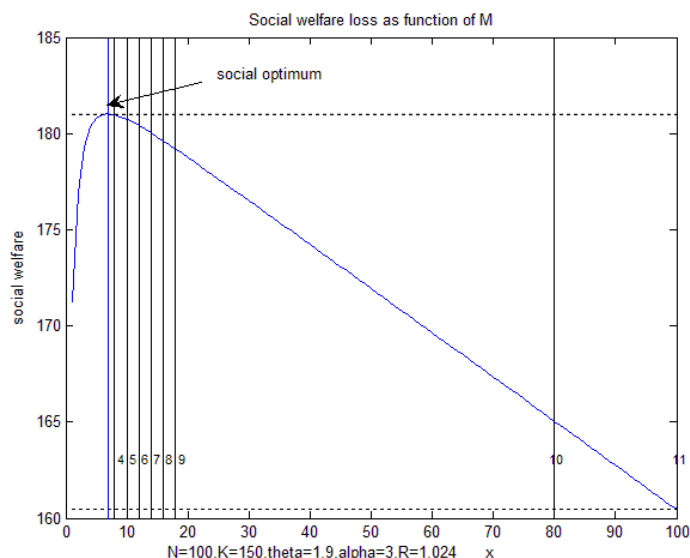


Figure 3.11: Welfare loss as a function of  $M$

### 3.7.5 The Number of New Projects $N$

The number of new projects also matter for the total welfare loss. From the previous discussion of social planner's problem and Figure 3.2 we know the relative cost of learning  $\frac{x}{N}$  is a decreasing function of  $N$ . This implies the benefit of learning increases with  $M$ . On the other hand, from the Proposition of Amplification Function and Figure 3.7, we know around the threshold level  $R_0$ , any large number of projects can be exhausted very quickly due to the property of the amplification function. The welfare loss due to "**Rush**" will be multiplied by the number of projects  $N$ .

### 3.8 Conclusions

Innovations not only create better investment opportunities but also bear uncertainties and potential hazards. Patient learning are necessary to reduce the uncertainties to a socially optimal level. However, this can be distorted by Investor’s incentive to capture a lion’s share of the new market. Facing a limited supply of new projects, investors prefer to enter the market prematurely. An inefficient “**Rush**” often happens after the launch of some breakthrough technology.

I assume the knowledge of learning can be publicly seen. Interestingly, the incentive for free-riding the knowledge generated by other investors’ early investments can mitigate the coordination problem to some extent. On the other hand, this information externality can lead to under-investment at early stage especially when the prior of mean return is low.

Our findings imply that granting some monopolistic power can help to remove inefficiencies in decentralized learning and investments in innovations. Financial innovations are generally not protected by patent laws. The traditional literature of innovation and patent emphasizes the benefit and public good nature of innovations. This research calls attention to the other side of innovation: uncertainty and potential social hazards. Premature diffusion and rush in financing an innovation can bring about significant and unexpected “public bad”. Appropriate mechanism needs to be designed to make Investors internalize the negative externalities.

This “Rush” model of investment can also be combined with a “Run” model of disinvestment as in Xie (2015b), which shows that “Rush” to financing innovations can lead to higher probability of “Run” on these new projects.

## CHAPTER 4

### PATENT DESIGN FOR ECONOMIC AND FINANCIAL RISK

#### 4.1 Introduction

This chapter explores the relationship between the uncertainty of innovation and patent design. I use the Optimal Patent Design method to correct the inefficiency of rush. Traditional Patent Design has not take into consideration the uncertainty and premature adoption of innovations. To discourage over-investment and too much entry when uncertainty is still high, broader patent rights should be granted to the inventor. This can mitigate social welfare loss due to over-entry into an uncertain technology.

I present and analyze the Robust Patent Design problem when the prior of the expected return is unknown. Moreover, I embed the rush mechanism in a simple endogenous growth framework, which allows endogenous choices of R&D investment, as well as endogenous choice of technology adoption. Rush prevention can induce more patent race at the early R&D stage but this *rush – race shifting* can still improve social welfare in total, because in general rush inefficiency dominates race inefficiency.

I also provide multiple pieces of new empirical evidence on investment “Rush” for technological innovations. I document several historical “Rush” episodes. A new wave of “*Unicorn Rush*” has just emerged during the last few years. Using cross-industry Venture capital investment data, I illustrate a linkage between the degree of sectoral IPR protection and investment volatility.

#### Related Literature

*Information Acquisition, Bubbles and Crisis* Chari and Kehoe (2003) shows that information is important for financial crisis, and illustrates the herding effect on crisis. Angeletos, Hellwig and Pavan (2006) highlights that endogenous information generated by policy intervention can move global game from unique equilibrium to multiple equilibria. In this paper,

the information structure is endogenously determined by the market structure and actions of agents.

*Entry, Competition and Inefficiency* Mankiw and Winston (1986) discuss the social inefficiency due to over-entry of firms. Hsieh and Moretti (2003) provides empirical evidence on this channel.

*Preemption game and Patent race* Reinganum (1981), and Fudenberg and Tirole (1985) apply preemption game to market entry and technology adoption. Firm will make a tradeoff between entering earlier with the possibility of acquiring a patent or a significant share of the new market and waiting for a reduced uncertainty and entry cost. Hopenhayn and Squintani (2011) extends this line of research by adding heterogeneous information to each firm.

*Experimentation, learning, and Learning-by-doing (LBD)* Jovanovic and Lach (1989) shows that the Learning-by-doing mechanism can result in the S-shaped diffusion across firms. Bolton and Harris (1999) extend the two-armed bandit problem to a multi-agents dynamic game, and show the coexistence of a discouraging free-rider effect and a counteracting encourage effect for experimentation.

The paper proceeds as follows. Section 2 provides some empirical evidence on Rush and patent protection. Section 3 analyzes the optimal patent design problem to correct the inefficiency. Section 4 focuses on the robust patent design problem. Section 5 extends the baseline model of section 3 by allowing endogenous R&D, and analyzes the new optimal patent design problem. Section 6 concludes the paper and points to some future extensions.

## 4.2 Empirical Evidence on “Rush”

Kindleberger (1978) cites Minsky’s argument that any speculative bubble and crisis starts with a "displacement" or innovation or some exogenous macroeconomic shock. This will grow to be a speculative bubble, over-investment, and eventual crash. The mechanism modeled in this research is consistent with the empirical descriptions of Kindleberger and Minsky. I



emphasize the rush in financing innovations and inefficiencies due to coordination failure.

In this section, I study several historical cases and provide some suggestive evidence on rush in terms of quantity of investment as well as the linkage between the degree of patent protection and over-investment.

### 4.2.1 The 1990's "Internet Rush"

Figure 4.1 compares several scaled time series during the 1990's Internet "Rush", with all variables indexed by 100 for the year 1995.

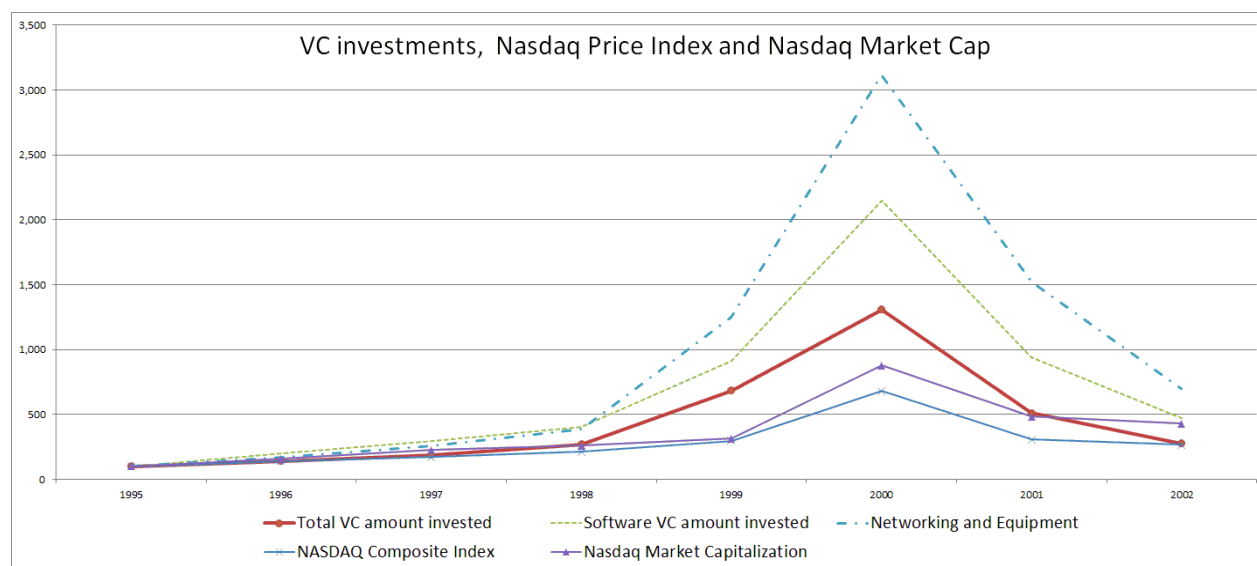


Figure 4.1: VC investments v.s. Nasdaq Price and Market Cap

Price, as illustrated by the Nasdaq index, cannot capture all the abrupt increase in investment. If we look at the quantity of Venture Capital investments, we see VC investments have grown much faster than Nasdaq price index and the Nasdaq Market Cap. From 1995 to 2000, the total amount of venture capital investment increased by 12 times, while the NASDAQ price index only quadrupled.

The new technologies had actually been undervalued due to knowledge spillovers and non-excludability in applying the General Purpose Technology, the Internet.

### 4.2.2 *The 2000's "Subprime Rush"*

The cause of the 2008 Financial Crisis was largely attributed to excessive subprime mortgage lending and securitization. Subprime mortgage and securitization are financial innovations, which are not patentable in general. We have seen a large number of subprime mortgage lenders enter the market and compete heavily to lend to new subprime borrowers. Nevertheless, both securitization and subprime mortgage contract have revealed to have innate defects in the design. This "Subprime Rush" engendered excessive lending and eventually ended up with the Great Recession.

According to the principle illustrated in the Optimal Patent design section, more patents should be granted to the financial innovations. The uncertainty of financial innovations can easily raise systemic risk in the economy if a rush happens.

### 4.2.3 *The 2010's "Unicorn Rush"*

Most recently, we are witnessing a new (potential) rush, the phenomenon of the rise of "Unicorns". "Unicorns" refer to technology startups with at least \$1 billion valuation, based on fundraising. Figure 4.2 displays the trend of Unicorns from 2009 to Nov 2015<sup>1</sup>

Among the list of Unicorns<sup>2</sup>, dominating ones are based on social media, Mobile technology, and shared economy. The top Unicorn is Uber, with 51 \$Billion till November 2015, and the second one is Xiaomi, a new Chinese mobile phone company. Airbnb takes the third position, Airbnb shares the same fundamental technology with Uber, but has the application in a different niche. This is consistent with our theoretical model above.

Unicorns have blurred the borderline between early VC investment and IPO, and they might have grown too big relative to the social optimal size. From Figure 4.2, we see an accelerated growth of unicorns from 2013 on: both the number and aggregate value of the

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1. Data Source: CB Insights and Thomson Reuters.

2. see the latest updates from <http://fortune.com/unicorns/> and <http://graphics.wsj.com/billion-dollar-club/?co=Square>

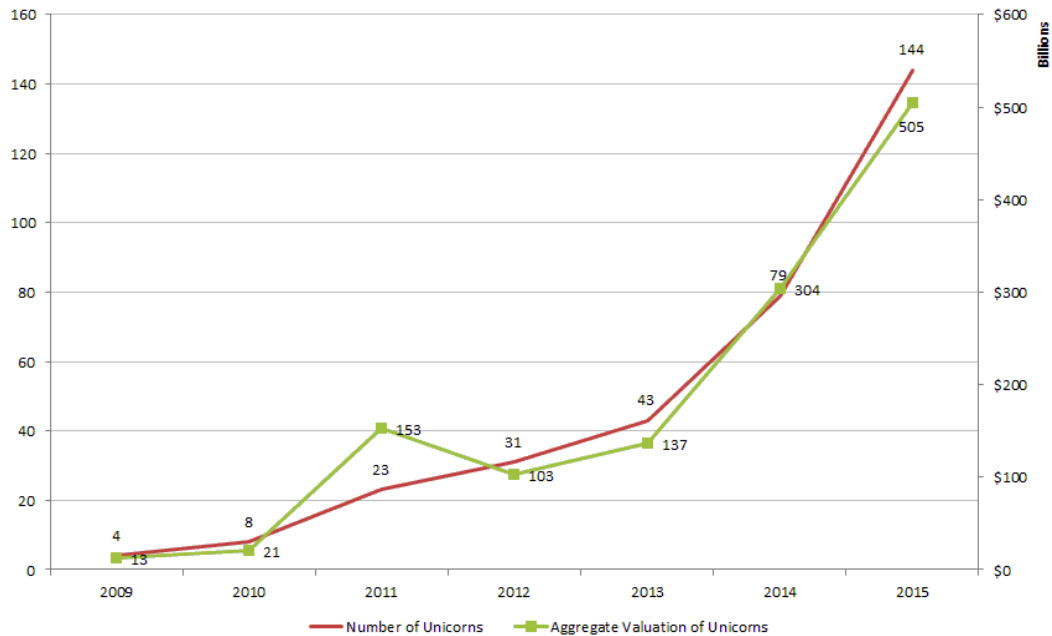


Figure 4.2: The Trend of Unicorns

unicorns almost quadrupled in only 2 years. This reminds us the Internet Rush and a jump in VC investments from 1998, as shown in Figure 4.1.

#### 4.2.4 Cross-industry VC investments and Patent protection

According to the theoretical part of this research, we will expect sectors with less de facto IPR protection will be more procyclical and display more severe “Rush”.

Figure 4.3 compares the Venture capital investments across several industries. We can see from Figure 4.3 that biotech and medical devices were rushed on a much smaller scale than software. The volatility of VC investments in software industry is huge relative to other sectors. This is because the idea of software is easy to copy and duplicate, even though it can be patented. Therefore, a concept of "De facto IPR protection" is important. The new idea of a software can be easily designed around and so it is not protected as well as pharmacy although both of them have de jure patents.

Within the biotech industry, different types of drugs also have variations in de facto

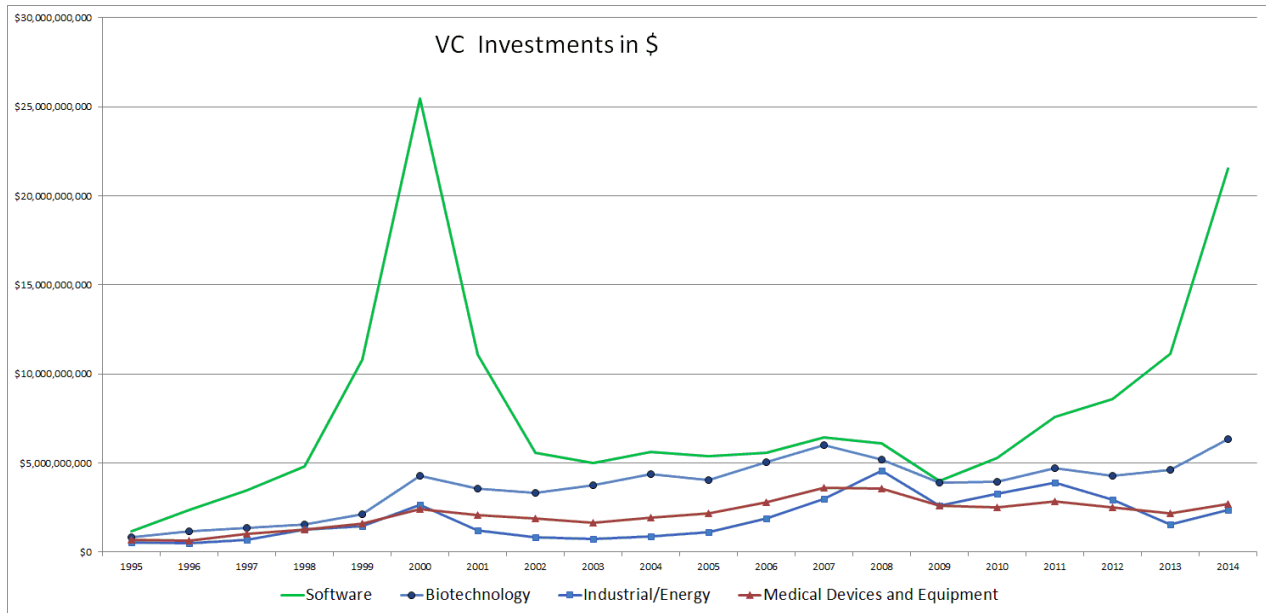


Figure 4.3: VC investments by sectors

patent protection. Chemical drugs (based on Chemical molecule) are relatively easy to design around or copy with small changes, while bimolecular drugs are difficult to copy because the production process has subtleties, which can be thought of as business secret in some sense.

### 4.3 Optimal Macro-Patent Design

In this section, I will discuss the *Patent Design problem*, as a correction to the inefficiency of over-investment. At the end of the section, I will also prescribe the *Robust Patent Design problem* for a generalized setup with ambiguous priors.

Model in this section is based on the basic model setup in the previous chapter (3.2).

#### 4.3.1 Flexible Patent Mechanism

I assume the investor indexed by  $s$  is the innovator who creates the breakthrough technology, and is guaranteed the ownership of a fixed shares  $(\mu_0, \mu_1)$  of new projects based on that

breakthrough technology. The vector  $(\mu_0, \mu_1)$  embodies the Patent policy.  $\mu_0$  and  $\mu_1$  represent the monopolistic share for the innovator at  $t = 0$  and  $t = 1$  respectively, as illustrated by the following allocation mechanism. Under full patent protection,  $\mu_0 = \mu_1 = 1$ . However, full patent protection is not socially optimal with the possibility of sequential innovation (or with deadweight loss of monopoly).

**Allocation Mechanism 4.1** (Flexible Patent). *Allocation functions  $h^0(\cdot)$  and  $h^1(\cdot)$  are defined by (3.4), (4.1), (4.2), with the patent policy  $(\mu_0, \mu_1)$ ,*

$$h_i^0((x_1, x_2, \dots, x_M)) = \begin{cases} x_i & \text{if } x_{i=s} \leq \mu_0 N \\ \mu_0 N & \text{if } x_{i=s} > \mu_0 N \\ x_i & \text{if } x_{i \neq s} \leq \frac{(1-\mu_0)N}{M-1} \\ \frac{(1-\mu_0)N}{M-1} & \text{if } x_{i \neq s} > \frac{(1-\mu_0)N}{M-1} \end{cases} \quad (4.1)$$

$$h_i^1((z_1, z_2, \dots, z_M)) = \begin{cases} z_i & \text{if } z_{i=s} \leq \mu_1 N_1 \\ \mu_1 N_1 & \text{if } z_{i=s} > \mu_1 N_1 \\ z_i & \text{if } z_{i \neq s} \leq \frac{(1-\mu_1)N_1}{M-1} \\ \frac{(1-\mu_1)N_1}{M-1} & \text{if } z_{i \neq s} > \frac{(1-\mu_1)N_1}{M-1} \end{cases} \quad (4.2)$$

This Patent Mechanism is flexible in that disparate degrees of monopolistic power are allowed for different stages.

### 4.3.2 Correcting Rush Inefficiency with Patent

I want to use patent to discourage over-investment and reduce the inefficiency due to rush. Patent as a monopolistic power also imposes cost, which I will model as the barrier to sequential innovation.

For the following discussion, I focus on the 2 – investor case for simplicity.

At  $t = 0$ : First-wave experimental investment in the market

I assume at  $t = 0$ , one investor succeeds in making a breakthrough innovation and gets the patent. I call her the leader. The other investor is then named a copycat. This breakthrough will directly generate  $N$  new projects available for financing. Each of the project has return  $R$ .

### **Patent Policy**

According to the Flexible Patent Allocation Mechanism, there is a total share  $\mu_0$  of  $N$  new projects reserved for the leader for  $t = 0$ . The other investor can only stay in her own niche of  $(1 - \mu_0) N$  new projects at  $t = 0$ .

### **Sequential Innovation**

The copycat (but in this sense, not a pure copycat) can combine the breakthrough technology with her own innovation. The sequential innovation will generate a multiplier effect by expanding the number of projects he financed by a factor  $\chi$ , and  $\chi > 1$ . This can be thought as a "buy one get  $\chi - 1$  free" bonus due to sequential innovation. This captures the social welfare gain of sequential innovation which expands the applications of the technological breakthrough. Too strong patent protection can discourage sequential innovation and reduce social welfare.

At  $t = 1$ : Second-wave investment and systemic adoption decision

In this stage, the leader can finance up to  $\mu_1 \cdot N_1$  new projects. The copycat can only invest in his own niche of  $(1 - \mu_1) N_1$  projects.

### *4.3.3 The Optimal Patent Design Problem*

The patent policy is defined by the vector  $(\mu_0, \mu_1)$ .

Copycat's Problem at  $t = 0$

$$V_{c,0} = \max_{x_c} \mathbb{E}_{c,0} [R \times \chi x_c + V_{c,1}] \quad (4.3)$$

$$V_{c,1} = \max_{z_c, w_c} \mathbb{E}_{c,1} [R \times \chi z_c + w_c | d_1] \quad (4.4)$$

subject to

$$x_c + z_c + w_c \leq k - y_c$$

$$x_c, z_c, w_c \geq 0$$

$$x_c \leq (1 - \mu_0)N \quad (4.5)$$

$$z_c \leq (1 - \mu_1)(N - x_s - x_c) \quad (4.6)$$

$$E_1 [R | d_1] = \frac{\alpha_0 R_0 + d_1 (x_c + x_s)^\theta}{\alpha_0 + (x_c + x_s)^\theta} \quad (4.7)$$

$$V_1 [R | d_1] = \left( \alpha_0 + (x_c + x_s)^\theta \right)^{-1} \quad (4.8)$$

$$d_1 \sim N(R_0, (x_c + x_s)^{-\theta} + (\alpha_0)^{-1}) \quad (4.9)$$

Leader (patent holder)'s Problem at  $t = 0$

$$V_{s,0} = \max_{x_s} \mathbb{E}_{s,0} [R \times x_s + V_{s,1}] \quad (4.10)$$

$$V_{s,1} = \max_{z_s, w_s} \mathbb{E}_{s,1} [R \times z_s + w_s | d_1] \quad (4.11)$$

subject to

$$x_s + z_s + w_s \leq k - y_s$$

$$x_s, z_s, w_s \geq 0$$

$$x_s \leq \mu_0 N \tag{4.12}$$

$$z_s \leq \mu_1 (N - x_s - x_c) \tag{4.13}$$

and (4.7),(4.8),(4.9)

### The Choice of Optimal Patent Policy

Finally, designing optimal patent is just to choose the vector  $(\mu_0, \mu_1)$  to maximize the value function  $V_0$  at  $t = 0$ ,

$$\max_{\{(\mu_0, \mu_1)\}} \{V_0\} \tag{4.14}$$

$$V_0 = \omega V_{s,0} + (1 - \omega) V_{c,0} \tag{4.15}$$

where  $V_0$  is the weighted sum of utilities of the two types of players.  $\omega$  is the patent holder's population share or the successful probability of becoming a patent holder. Here I simply assume equal probability of becoming a leader and a copycat,  $\omega = 0.5$ . In a later subsection where I will explicitly model the R&D investment, these probabilities will be endogenously determined by the choice of early R&D investments.



### 4.3.4 Solution Method

At  $t=1$

The copycat's threshold  $\bar{d}_{c,1}$  will make  $\chi \frac{\alpha_0 R_0 + \bar{d}_1 (x_c + x_s)^\theta}{\alpha_0 + (x_c + x_s)^\theta} = 1$ , and is thus determined by (4.16),

$$\bar{d}_{c,1} = \frac{\alpha_0 + (x_c + x_s)^\theta - \chi \alpha_0 R_0}{\chi (x_c + x_s)^\theta} \quad (4.16)$$

The optimal choice  $(z_c^*, w_c^*)$  of the *copycat* is thus given by the following equations,

$$\begin{cases} z_c^* = (1 - \mu_1)(N - x_s - x_c) & w_i^* = k - x_c - z_c^* & \text{if } d_1 \geq \bar{d}_{c,1} \\ z_c^* = 0 & w_i^* = k - y_c - x_c & \text{if } d_1 < \bar{d}_{c,1} \end{cases} \quad (4.17)$$

The leader's threshold  $\bar{d}_{s,1}$  will make  $\frac{\alpha_0 R_0 + \bar{d}_1 (x_c + x_s)^\theta}{\alpha_0 + (x_c + x_s)^\theta} = 1$ , and is thus determined by (4.18),

$$\bar{d}_{s,1} = \frac{\alpha_0 + (x_c + x_s)^\theta - \alpha_0 R_0}{(x_c + x_s)^\theta} \quad (4.18)$$

The optimal choice  $(z_s^*, w_s^*)$  of the *leader* is thus given by the following equations,

$$\begin{cases} z_s^* = \mu_1(N - x_s - x_c) & w_s^* = k - x_s - z_s^* & \text{if } d_1 \geq \bar{d}_{s,1} \\ z_s^* = 0 & w_s^* = k - y_s - x_s & \text{if } d_1 < \bar{d}_{s,1} \end{cases} \quad (4.19)$$

At  $t=0$

Given the  $t = 1$  solution (4.17), *copycat's* optimization problem at  $t = 0$  becomes,

$$V_{c,0} = \max_{\{x_c\}} \left\{ \int_{-\infty}^{\bar{d}_{c,1}} \left( \chi \frac{\alpha_0 R_0 + s \cdot (x_c + x_s)^\theta}{\alpha_0 + (x_c + x_s)^\theta} x_c + k - y_c - x_c \right) f(s) ds \right. \\ \left. + \int_{\bar{d}_{c,1}}^{\infty} \left( \chi \frac{\alpha_0 R_0 + s \cdot (x_c + x_s)^\theta}{\alpha_0 + (x_c + x_s)^\theta} (x_c + (1 - \mu_1)(N - x_s - x_c)) \right. \right. \\ \left. \left. + k - (y_c + x_c + (1 - \mu_1)(N - x_s - x_c)) \right) f(s) ds \right\} \quad (4.20)$$

s.t. (4.16), (4.21), (4.5)

where  $d_1$ 's probability distribution function  $f(\cdot)$  follows (4.21),

$$f(s) = \frac{1}{\sqrt{(x_c + x_s)^{-\theta} + (\alpha_0)^{-1}}} \phi \left( \frac{s - R_0}{\sqrt{(x_c + x_s)^{-\theta} + (\alpha_0)^{-1}}} \right) \quad (4.21)$$

Similarly, *leader's* problem at  $t = 0$  becomes,

$$V_{s,0} = \max_{\{x_s\}} \left\{ \begin{array}{l} \int_{-\infty}^{\bar{d}_{s,1}} \left( \frac{\alpha_0 R_0 + s \cdot (x_c + x_s)^\theta}{\alpha_0 + (x_c + x_s)^\theta} x_s + k - y_s - x_s \right) f(s) ds \\ + \int_{\bar{d}_{s,1}}^{\infty} \left( \frac{\alpha_0 R_0 + s \cdot (x_c + x_s)^\theta}{\alpha_0 + (x_c + x_s)^\theta} (x_s + \mu_1 (N - x_s - x_c)) \right. \\ \left. + k - (y_s + x_s + \mu_1 (N - x_s - x_c)) \right) f(s) ds \end{array} \right\} \quad (4.22)$$

*s.t.* (4.18), (4.21), (4.12)

### 4.3.5 Numerical Result

In this subsection, I will numerically solve the optimal patent problem (4.14).

Figure 4.4 illustrates a numerical solution to (4.14). The x-axis corresponds to  $\mu_1$ , and the y-axis corresponds to  $\mu_0$ . Function values denote the social welfare. With a setup  $R_0 = 1.1, \alpha_0 = 3, \chi = 1.01, \theta = 1$ , we get the optimal patent  $(\mu_0^* = 0.76, \mu_1^* = 0.65)$ . That is,  $\mu_0^* \geq \mu_1^*$ .

In fact, this result is robust to various parameter combinations. In general, I present the following result (4.23),

**Result 4.2** (Uncertainty of Innovation and Patent Design).

$$\mu_0^* \geq \mu_1^* \quad (4.23)$$

This implies that higher uncertainty demands stricter patent protection. At an early

stage of technology adoption, larger monopolistic power should be granted to the innovator or first-mover, and discourage the rush of other investors.

At first glance, you may think  $\mu_1^*$  should be equal to  $1/2$ , which means no monopolistic power for the leader. But if so, the leader will over-invest at  $t = 0$  because she expects decreased investment opportunities at  $t = 1$ .

Granting monopolistic power is a way to prevent the premature spread of uncertain knowledge which can potentially cause damages. After uncertainty dwindles with learning, the monopolistic power of the patent holder should be reduced to facilitate sequential innovation.

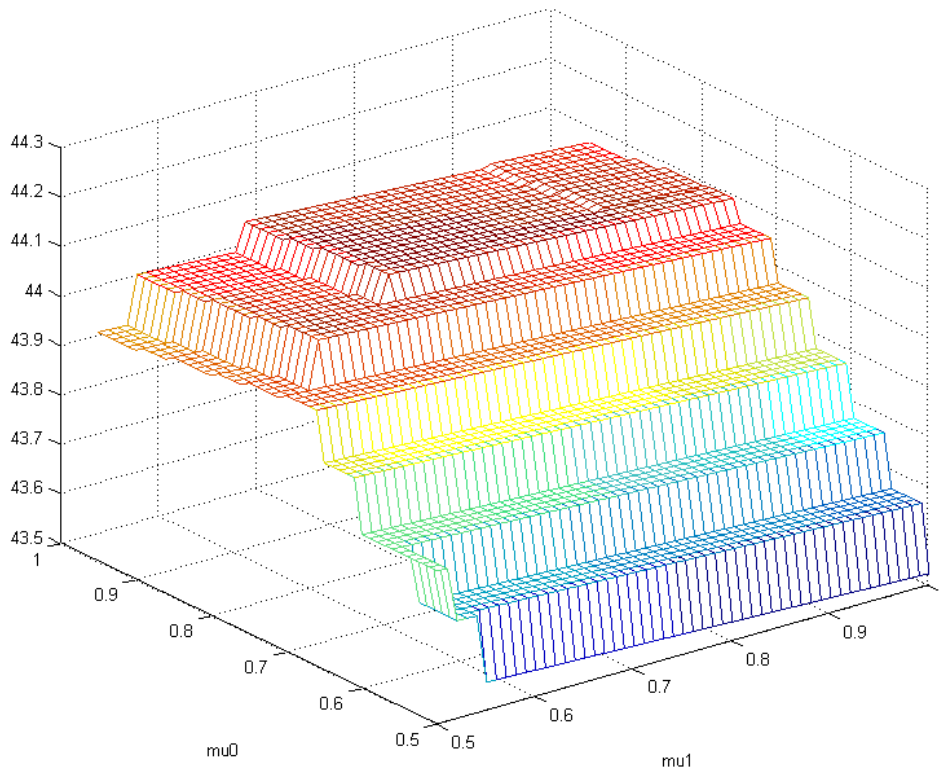


Figure 4.4: 2-Stage Optimal Patent

### 4.3.6 Tradeoffs in a sum

Better Patent protection leads to,

1. (+) lower cost of over-investment under uncertainty;
2. (−) lower sequential innovation;

Number (1) is a new force that traditional patent design has not taking into account. This will imply a stricter patent protection than the traditional patent literature.

## 4.4 Robust Macro-Patent Design

Another challenge facing patent designer is the existence of uncertainty even for the prior return of a new technology. The prior return  $R_0$  can also be a random variable, following certain distribution  $R_0 \sim F$ . Here we can assume  $R_0 \sim Uniform(\underline{R}, \bar{R})$  for simplicity.

Assume the social planner (patent designer) needs to consider the worst-case scenario. This results in the following Robust Patent Design problem,

$$\max_{\{(\mu_0, \mu_1)\}} \min_{\{R_0\}} \{V_0\} \quad (4.24)$$

where  $V_0$  is defined by (4.15) as before.

The problem (4.24) takes into consideration extreme cases of both low and high  $R_0$ . The possibility of a very high  $R_0$  requires more stringent patent protection to prevent excessive investment. This implies determining a high upper bound of monopolistic power to deter big "rush".

On the other hand, a low  $R_0$  implies weak private incentive to explore the new technology relative to the social optimum. This also demands better patent protection, and even with necessary government subsidies like the Orphan drug case.



investor is named *copycat*, denoted with subscript  $c$ .

The probabilities for all four possible contingencies is,

$$\left\{ \begin{array}{ll} (1 - p(y_i)) (1 - p(y_j)) & \text{no investor makes a breakthrough} \\ p(y_i) (1 - p(y_j)) & \text{only investor } i \text{ makes a breakthrough} \\ p(y_j) (1 - p(y_i)) & \text{only investor } j \text{ makes a breakthrough} \\ p(y_i)p(y_j) & \text{both } i \text{ and } j \text{ make a breakthrough} \end{array} \right. \quad (4.25)$$

When both investors make the breakthrough, the planner randomly assigns patent to one of them.

I assume the return of all innovations are,

$$\left\{ \begin{array}{ll} r_i = 1 & i \neq s \\ r_i = R & i = s \end{array} \right. \quad (4.26)$$

Investor's problem at  $t = -1$

The two investors are identical at  $t = -1$ .

The probability for investor  $i$  to successfully get the patent of a breakthrough innovation is  $p(y_i) (1 - p(y_j)) + \frac{p(y_i)p(y_j)}{2}$ ;

the probability for the other investor  $j$  get the patent is  $p(y_j) (1 - p(y_i)) + \frac{p(y_i)p(y_j)}{2}$

the probability of no breakthrough innovation is  $(1 - p(y_i)) (1 - p(y_j))$ .

The aggregate successful probability of a breakthrough for the society is,

$$p(y_i) + p(y_j) - p(y_i)p(y_j) \quad (4.27)$$

So we can write investor  $i$ 's optimization problem as (4.28),

$$V_{-1} = \max_{y_i} \mathbb{E}_{-1} \left[ \begin{array}{l} \left( p(y_j) (1 - p(y_i)) + \frac{p(y_i)p(y_j)}{2} \right) V_{c,0} \\ + \left( p(y_i) (1 - p(y_j)) + \frac{p(y_i)p(y_j)}{2} \right) V_{s,0} \\ + (1 - p(y_i)) (1 - p(y_j)) V_{n,0} \\ - y_i \end{array} \right] \quad (4.28)$$

Value of the case of no-breakthrough  $V_{n,0}$  is,

$$V_{n,0} = N \times 1 \quad (4.29)$$

where the project return is just equal to 1.

## Rush and Patent Race

The Generalized Optimal Patent design problem is a tradeoff between the following forces. Better Patent protection leads to,

1. (+) higher aggregate R&D expenditures, and higher probability of a breakthrough;
2. (−) lower sequential innovation;
3. (+) lower cost of over-investment under uncertainty;
4. (−) patent race at the very beginning

More stringent patent protection can discourage rush and over-investment at a later stage ( $t = 0$ ), but it will aggravate the traditional "patent race" at the very beginning. Despite this *Rush – Race* shifting, the welfare loss due to patent race in *R&D* will be at a smaller scale than the loss due to rush in investments at a later stage. There is very few empirical evidence on *Patent Race* although there have been many theoretical research on it. In contrast, the magnitude of over-investment at later stage is usually much larger, as shown by the empirical evidence of the next section.

## 4.6 Conclusions

Traditional tradeoff of patent design is between reducing monopolistic cost and providing enough R&D incentive. Accordingly, financial innovations and most business methods such as Uber are generally not protected by patent laws. However, empirical findings in this chapter imply that granting some monopolistic power can help to remove inefficiencies in decentralized learning and investments in innovations.

This chapter calls attention to the other side of innovation: uncertainty and potential social hazards. Premature diffusion and rush in financing an innovation can bring about significant and unexpected “public bad”. Appropriate mechanism needs to be designed to make Investors internalize the negative externalities. Broader patent rights should be granted when the uncertainty of the innovation is high.

Monetary policy plays a big role in controlling credit to firms and investors. It has been pointed out that loose monetary policy and thus cheap credit was culprit for the major financial crises. However, technological breakthroughs often inject too much “liquidity” of underpriced knowledge, due to its nature of public good. Relative to the monetary policy which broadly affects the whole economy, can a counter-cyclical IPR policy be more accurate in targeting the sectors which market failure of IPR is most significant?

In sum, to target the exact market failure in the economy implies that IPR policy might also be an effective policy tool for business cycle management.



# CHAPTER 5

## A KNOWLEDGE THEORY OF REGULATION

### 5.1 Introduction

In this chapter I put forward a *Knowledge Theory of Regulation*: new varieties of risks are continuously created by innovations, and the main purpose and function of regulation is to discover, acquire knowledge about and control these new potential risks. This chapter therefore fits in the recent literature which tries to explain the root and the rise of regulation (e.g. Mulligan and Shleifer 2005, Shleifer 2012), but I have different explanations. With economic growth and accumulation of innovations, discovering and acquiring **Risk Knowledge**, i.e. the knowledge about new varieties of risks generated by technological innovations are becoming more costly and important than regulating known risks.

Science, technology, and social science knowledge tell us what we can do, i.e. through innovations and new products, as well as the knowledge of risks and what we cannot do, which build the foundation of modern regulation. In this sense, the modern regulatory sector is an institutional setup representing and executing human knowledge about the negative side of human-made technologies and related activities.

The public good nature of risk knowledge will result in its underproduction if only through private litigations and courts: victims of a potential risk tend to wait and see the litigation results of a firstmover, given the uncertain compensation to a risk ex ante and the testing cost to prove the causation of the potential risk to real damage. Relative to court, regulator as a public R&D institution for discovering new risks, have comparative advantage in acquiring and sharing new risk knowledge. Risk externality analyzed in Chapter 2 will aggravate information inefficiency over time. Therefore, modern economic growth and fast-growing innovation-induced risks will lead to the relative rise of regulator thanks to its comparative advantage in acquiring and sharing risk knowledge.

FDA has regulated the drug and food industry for more than a century since the 1906

Legislation. The establishment of the CFPB (Consumer Financial Protection Bureau) in 2011 represents the latest efforts in detecting, disclosing and preventing potential risks in financial products, as a response to the wide blame on financial innovations as a cause of the Subprime Crisis and Great Recession. On June 22, 2016, the Lautenberg Chemical Safety Act was just signed into law, which endowed the EPA with the new power to review and approve any new chemical product. Besides the FDA, the newly established CFPB together with the EPA augmented by the 2016 Lautenberg Act allude to an analogous *qualification process* of regulation which inspires the theoretical model developed in this research.

There are several interesting and crucial features that can only be derived from my theory. There will be a divergence between the *General Knowledge*, and the *Qualified Knowledge*. The number of patents represent the *General Knowledge*, while the qualified innovations (regulator-approved innovations) representing the *Qualified Knowledge* which can provide us net benefits. Macro risk structure due to complex interaction effects between innovations in use can make this divergence grow larger, given effective regulation (see Figure 2.13). The total human knowledge (*General Knowledge*) can keep growing fast but the GDP growth can lag much behind it owing to rapid expansion of the varieties of new risks.

This chapter is organized as follows. Section 2 analyzes a toy model illustrating the information underprovision problem under a pure liability and litigation system. Section 3 compares several solutions for the inefficiency of risk information underprovision. Section 4 classifies regulatory regimes according to the data generating process. Section 5 concludes the paper.

## 5.2 A Model of Litigation Timing

Why do we need regulator given the existence of the court system? This section analyzes and discusses where the court system (Tort and liability) can fail with a simple theoretical model. Then I will analyze the necessity of using regulatory agency to deal with the inefficiency implied by the model.

Many innovations and patents can have negative net value if their benefits and adverse effects are thoroughly accounted. For example, Thalidomide has caused huge damage and the total cost could have reached hundreds of billion US\$ without the FDA’s wise decision to deny its approval.

In this section, I will first point out an important inefficiency caused by information externality of new risk knowledge, using a toy model. Then I discuss four possible ways to correct the inefficiency: the current tort law practice, tort with a “Negative Patent System”, private information producers, and the regulator.

### 5.2.1 Basic Model Setup

This model is built for the court system, particularly following the liability and litigation rule under the Tort law.

Assume there are  $n$  victims, who were all harmed by a specific risk potentially caused by the product of a firm. Victims can all sue the firm for damages that the firm’s products have potentially caused.

For each victim, the fixed cost of hiring a lawyer to bring a lawsuit is  $c_0$ .

There is uncertainty in terms of the compensation for such a liability lawsuit. Before any lawsuit is brought, the compensation is only known to be a random variable  $b \sim Unif(0, B)$ , following a uniform distribution. This compensation (which will be decided by the court) depends on scientific testing which imposes a nontrivial cost  $c_1$  on the plaintiff who brings the first case regarding this type of product liability.

Assume after the first court decision on this risk, the compensation  $b$  will be revealed to a same constant  $b_1 \in [0, B]$  for all victims, and publicly known to all. No more testing is needed for future cases of this type. We can reasonably assume  $c_0 < B$ .

So for a victim who will bring a lawsuit before all others, her expected net benefit  $EV_{first}$  will be:

$$EV_{first} = E_0 [b] - c_0 - c_1 \tag{5.1}$$

For a victim who waits for other victims to sue firstly, and prosecutes only if the ex post value  $b_1$  is larger than a threshold, she will get expected value  $EV_{follower}$ :

$$EV_{follower} = E_0 [\max (0, b_1 - c_0)] \quad (5.2)$$

### 5.2.2 Results

We can easily prove the following proposition:

**Proposition 5.1.**  $EV_{follower} > EV_{first}$ . The Expected Gain of Waiting  $EG_{wait}$  is

$$EG_{wait} = \frac{c_0^2}{2B} + c_1 \quad (5.3)$$

*Proof.* From (5.1) we know

$$EV_{first} = \frac{B}{2} - c_0 - c_1 \quad (5.4)$$

As a follower,  $EV_{follower} = E [\max (0, b_1 - c_0)] = \int_{c_0}^B \frac{x-c_0}{B} dx = \frac{1}{B} \left( \frac{x^2}{2} - c_0 x \right) \Big|_{c_0}^B = \frac{1}{B} \left( \frac{B^2}{2} - c_0 B + \frac{c_0^2}{2} \right) = \frac{B}{2} - c_0 + \frac{c_0^2}{2B}$

$$EV_{follower} = \frac{B}{2} - c_0 + \frac{c_0^2}{2B} \quad (5.5)$$

we can see from (5.4) and (5.5) that  $EV_{follower} > EV_{first}$ . And the net expected gain of waiting is calculated by  $EG_{wait} = EV_{follower} - EV_{first} = \frac{c_0^2}{2B} + c_1$ .  $\square$

Waiting and freeriding on other's lawsuit information and outcome will dominate bringing a lawsuit first by herself. From the expected gain of waiting  $EG_{wait} = \frac{c_0^2}{2B} + c_1$ , we can see the value of delaying litigation comes from two sources: (1) the option value  $\frac{c_0^2}{2B}$ , which is created by avoiding bad realizations of  $b$ ; and (2) cost saving from freeriding on the firstmover's testing expenditure  $c_1$ . We notice there still exists a value of waiting even the testing cost  $c_1 = 0$ .

The structure of this problem is similar to the classic problem of R&D and innovation. Incentives must be provided to encourage investment in knowledge provision, for example, patent rights. Without intellectual property rights protection for “risk knowledge”, there will be underinvestment on litigation.

### 5.3 Solutions to the Inefficiency Problem

There are several ways to solve the inefficiency problem in last section.

#### 5.3.1 *Solution 1: Class Action lawsuits or Mass Torts*

A collective lawsuits can share the cost  $c_0$  and  $c_1$  among a group of victims. This can help mitigate the information freeriding problem to some extent. However, there will be additional coordination cost to organize the group. Some victims may still choose not to join a Mass Torts case, and instead wait to see the outcome of others’. Forming a group for Mass Torts is on a voluntary basis.

Moreover, a significant number of victims might even do not know it is this specific risk having done harm to them, and then have never thought about a litigation.

#### 5.3.2 *Solution 2: Tort Law plus a “Negative Patent System”*

Patent system intends to protect the intellectual property of innovators. The real value of a patent depends on how useful an innovation is. As discussed in last section, an innovation can bear multiple types of risks and can even interact with other innovations to generate additional risks. These risks are rarely written in the patent because it is the utility of an innovation can generate monetary return.

Although a victim can get certain compensation from the producer through the court system (torts), she cannot be rewarded enough due to externality of the knowledge of risks. Discovering and confirming harmful risks through the court system can be too costly for a

victim who brings the first lawsuit. The knowledge generated by a successful law suit has public good nature so that a victim has less incentive to sue than the social optimal.

The court system plus a “*Negative Patent System*” can help to mitigate this inefficiency. A negative patent system should compensate the plaintiff who sues firstly. To cover her evidence discovery cost  $c_1$ , as well as the information externality of revealing  $b$  to  $b_1$ . Later plaintiffs should pay sufficient information premium to the first plaintiff.

Admittedly, this system is difficult to implement in reality.

### *5.3.3 Solution 3: Private Information Producers like Credit Rating Agency or Bank*

A Credit Rating Agency (CRA) evaluates and rates the risk and return of various securities. Banks typically evaluates and monitors loan risks. Both CRAs and banks can produce information about risks. However, product risks that CRAs and Banks evaluates are more idiosyncratic compared to innovation risks discussed in this research. For example, the risk of Thalidomide is common, regardless of specific manufacturers. It is the common chemical properties rather than the brand name that matter for consumer welfare. With such a strong public good nature, a CRA or Bank alike private evaluator is difficult to makes ends meet.

### *5.3.4 Solution 4: Regulator for Risk Knowledge Acquisition and Sharing*

A regulator can be delegated the task of providing more detailed information about new products, particularly their adverse effects. The cost will be levied on producers. This can help to eliminate the problems of both information externality and coordination failure. The FDA is such a regulator which evaluates detailed information of new drugs and only allows qualified drugs to enter the market. It is equally important that the FDA makes public all the testing information of new drugs and sends warning messages to the people if necessary.

Beyond the toy model above, the situation of each victim might differ, and evidence

discovery to prove a causation between the risk and harm in each case is very costly and difficult (e.g. an Agent Orange case<sup>1</sup>). Distinguishing the risk from other potential factors as a causation for each case can impose additional social costs. In contrast, a regulator who requires firms to produce necessary information with *controlled experiments* (e.g. clinical trials) ex ante can save a lot of information cost ex post for proving the causation linkage with the “*dirty data*” from the real life of every victim. From the view of social efficiency, conducting controlled experiments ex ante are more cost-efficient than discovering evidence and proving causation ex post from uncontrolled data.

Regulators can also play an important role of educating the general public about known risks. They can publicize the risk knowledge to the people for precautionary and preventive purposes.

### 5.3.5 *The Implication of Growth and Innovation-induced Risks for Litigation versus Regulation*

Regulators have comparative advantage in acquiring new risk knowledge relative to courts, as discussed in the last two sections. Moreover, chapter 2 shows that innovation-induced risks and uncertainties grow faster than innovations, due to **Risk Externality**. Because of the synthesis of these two mechanisms, modern economic growth will lead to the relative rise of regulator to the court.

At the early stage of development, innovation-induced risks play a relatively minor role when the stock of innovations is low. Courts were sufficient to deal with this externality on a small scale. With more innovations being invented and entering the market, the inefficiency of risk knowledge underprovision will impose larger and larger costs on the economy. This enlarging inefficiency therefore demands more regulation than court service over long-run growth, due to regulator’s comparative advantage in providing risk knowledge.

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1. <http://www.law.harvard.edu/publications/evidenceiii/cases/agent.htm>

## 5.4 Classifying Regulatory Regimes by Data Generating Process

Information regarding potential risks of a new product can come either from premarketing tests or postmarketing surveillance. As described in Appendix A, the FDA's regulatory activities indeed cover the full lifespan of a new product: (1) in the research lab; (2) clinical trials; (3) adverse events reports from the market. Other regulators' activities usually cover or resemble some stages of this full-range regulatory process.

### 5.4.1 *Premarketing Risk Assessment*

Premarketing tests help to acquire risk knowledge before a product enters the market. It is precautionary and preventive. FDA's clinical trials (now includes stages I, II, III) requirement had successfully prevented the Thalidomide tragedy in the United States. These premarketing tests are mainly conducted in labs. To qualify an approval, regulator generally requires testing results to be sufficient to prove the safety and efficacy of a new product.

#### **Case 5.1.** *EPA and the 2016 Lautenberg Act*

Traditionally, the regulatory function of the EPA mainly lies in the Postmarketing Surveillance stage. The EPA only responded to a product after it was found to be the cause of a risk. For example,  $CO_2$  emission was identified to be the major cause of global warming two hundred years later than the first introduction of steam engines. DDT (dichlorodiphenyltrichloroethane) was banned for environmental protection purpose after being globally used for half of a century.

However, there has just been a major overhaul of the Chemical regulation in 2016: the Lautenberg Chemical Safety for the 21st Century Act starts to require safety testing for new chemicals before they are permitted to enter the market. The EPA is granted the new power to approve any new chemical. Chemical manufactures are required to submit a PMNs (Premanufacture Notices) for any of their new chemical products to the EPA for reviewal and approval according to the Lautenberg Act.



**Case 5.2.** *“Regulatory Sandbox” of the British FCA*

The “Regulatory Sandbox” was proposed by the British FCA (Financial Conduct Authority) in 2015 to provide a testing environment for new “Fintech” before it can go to the market. New “Fintech” will be tested in a simulated market environment. The “Regulatory Sandbox” resembles the FDA premarketing clinical trials, but with richer experimental environments.

*5.4.2 Postmarketing Surveillance*

The FDA still monitors a new drug after it enters the market. Pharmaceutical companies are required to conduct Phase IV clinical trials in this stage. Adverse events regarding new drugs will be reported to and collected by the FDA. The FDA will analyze these postmarketing data and try to identify signals of potential new risks.

**Case 5.3.** *The CFPB*

The CFPB (Consumer Financial Protection Bureau) was established in 2011 by the Dodd–Frank Act. It largely adopts a regulatory regime of Postmarketing Surveillance. The CFPB records and traces consumers’ numerous complaint reports about financial products on the market, and try to analyze and discover potential risks behind the heavily complained financial products.

**Case 5.4.** *The Regulation of the UAV*

UAV (Unmanned Aerial Vehicle, or drone) was not regulated initially. However, UAV-related accidents called for regulations. In the U.S., a UAV hit several buildings in Manhattan in 2013. The U.S. FAA announced UAV regulation in 2015, which requires the registration of all UAVs with the agency. In 2017, UAVs have threatened the safety and delayed hundreds of flights over several Chinese airports. China is thus planning for strict UAV regulation to avoid UAV-induced plane crash.

### *5.4.3 Proposal for a New Regulatory Regime*

As discussed before, current regulatory regimes mainly use ex ante lab experiments and ex post market data generating to monitor and control risks. The FDA covers both methods, the CFPB only uses market data, and the EPA is making a transition from using sole market data to a mixed regime. Nevertheless, current regulatory regimes are not effective in detecting and regulating interactive risks and systemic risks discussed in chapter 2.

Many financial products can pose unexpected risks due to their interlinkages with other financial products and economic sectors. Lab experiment is insufficient to detect this type of complex and systemic risks, and sole postmarketing surveillance might be too late to prevent disasters.

A potential new regulatory regime to conduct field experiment<sup>2</sup> regarding new products can be adopted by regulators for discovering interactive, complex, and environment-contingent risks. Conducting field experiment for financial innovations might be appropriate: designing experiments and collecting data for new financial products in some typical scenarios and economic environments.

For Drug-drug Interactions and other types of interactive risks, it becomes more and more costly to test increasing number of interactions due to the accumulation of innovations. Therefore choosing the most likely scenarios to test will be more cost-effective.

## **5.5 Conclusions**

Knowledge of potential new risks is essentially a public good. Private litigations through courts tend to undersupply such risk knowledge. Therefore, more and more modern regulatory agencies are functioning as public R&D centers to acquire and share the risk knowledge.

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2. Pioneered by John List

The rise of regulation over courts could be the response to the fast-growing risk externality induced by continuous innovative activities and economic growth. Regulators are more efficient than courts and litigations to provide public knowledge of these innovation-induced risks. I further classify current regulatory regimes in terms of the data generating processes, and propose to use field experiment method for some complex interactive risks and specific scenarios.

## CHAPTER 6

### CONCLUSION

This dissertation has made contributions to the topic in the following areas.

#### **Provides New Empirical Evidence**

This research empirically measures the dark side of innovations, in particular, negative externality caused by existing innovations. I document a new empirical fact about the relationship between innovations and the types of innovation-induced risks, using the datasets of Drug-drug Interactions and FDA's adverse events reporting system. Types of innovation-induced risks are illustrated to grow at a higher speed than innovations.

Qualified innovation ratio, measured by the ratio of qualified innovations (new products approved by the regulator) to all patents, is found to be declining over time, at least in the pharmaceutical industry. Future empirical work can be conducted among broader industries.

#### **Represents Innovation-induced Unknowns and Uncertainty**

I use the **Macro Risk Tensor (MRT)** to represent the innovation-induced unknown and known risks. Complex interactions between  $M$  technologies can be represented by an  $M - dimensional$  random tensor (e.g. Chapter 2 uses a simplified 2-dimensional random matrix to illustrate high-order risks and unknowns). Elements of the MRT are random variables ex ante, whose distributions are even unknown to us. Over time, some elements of the MRT will be revealed to be known risks with active or passive information acquisition, whereas the information of a significant portion of MRT elements still remains unknown to us. This method of representation for innovation-induced unknowns and risks is very useful for theoretical modeling.

Appendix E provides general discussions on this.

#### **Develops a New Growth Theory**

Then I develop a new endogenous growth model with innovation-induced risks and with a regulator. I explicitly model the qualification process of regulator who aims to control the risks induced by innovations. The risk externality induced by previous innovations make the

qualification of later innovations more and more difficult. Risk externality proposed in this research counteracts the famous positive knowledge spillover effect proposed by Paul Romer.

### **Proposes A Knowledge Theory of Regulation**

The new theoretical model can also help us to understand better how we can regulate innovations, thus providing a link between the innovations literature and the regulation literature. The new theory can explain the rise of R&D cost per new drug as well as greater expenditures on regulation and compliance.

Knowledge of the adverse effects of innovations is public good. More and more modern regulatory agencies are functioning as public R&D centers to acquire and share the knowledge of innovations' and technologies' adverse effects. Regulators are more efficient than courts and litigations to provide public knowledge of those innovation-induced risks.

### **Towards an “Innovation, Risk, and Regulation” Framework**

This model can be extended to a more general “Innovation, Risk, and Regulation” framework, which could work as an alternative to the current Macro-Finance literature for the purpose of explaining the recent financial crisis and man-made disasters. This "Innovation, Risk, and Regulation" framework can be used to model and explain (endogenous) crisis due to the imbalance or deficiency of resource deployed in the regulatory sector. I explore cyclical patterns thanks to this mechanism in another working paper (Xie 2016b).

### **Studies Patent Design Problem for Macro and Financial Stability**

In addition, I propose a new theory of rational “Rush”, emphasizing the quantity of rational over-investment in contrast to the theory of irrational price “Bubble”. I illustrate an important friction when financing breakthrough innovations: non-excludability and spillover of uncertain knowledge due to imperfect IPR (Intellectual Property Rights, e.g. patent) protection. Facing a limited supply of new projects with uncertain return, investors make decisions about when and how many projects to invest. Investors' preemption motive will distort their incentives for patient learning about project return, thus inducing them to "rush in" to finance uncertain projects massively at a premature stage. A small positive news shock

regarding the project return can greatly amplify over-investment and result in large social inefficiency. On the other hand, information externality creates free-rider motive, which can also make under-investment possible.

The empirical finding based on sectoral Venture Capital investment shows that weak IPR protection lead to excessively high investment level and more procyclicality. Broader patent rights should be granted when the uncertainty of innovation is high, although the “Rush” prevention can induce more patent race at the early R&D stage, i.e. Rush-Race shifting.

## APPENDIX A

### DESCRIPTION OF FDA'S REGULATORY REGIME

The modern FDA is responsible for ensuring the safety and efficacy of all drugs. The FDA will request and review the Clinical trials information for new drug candidates before they can enter the market. Moreover, the EPA is starting to adopt a very similar qualification process for new chemicals conforming with the Lautenberg Act passed in June 2016.

After successful drug discovery, pharmaceutical firms will get some new drug candidates and patent them. Then the firms will conduct pre-clinical investigations for these new drug candidates. If the preclinical results are good enough, the firm will submit an *Investigational New Drug* (IND) application to the FDA.

After review, the FDA will give the firm a permission (IND approval) for the high-quality drug candidates to conduct human clinical trials. After three stages of clinical trials (*I, II, III*) with satisfying testing results, firms may choose to submit a *New Drug Application* (NDA) to the FDA. The FDA will review the NDA information carefully and reach final decision of approval or rejection. Even after approval by the FDA and entering the market, some drugs can still be withdrawn from the market if the FDA finds they can cause intolerable adverse effects during the postmarketing surveillance stage.

#### A.1 Guidances

Guidances for IND and NDA applications inform the industry of detailed requirements about what items to test, what protocols, procedures and standards to follow. The FDA has also issued specific guidance for testing and controlling the risks of DDIs.

#### A.2 Submission, Reviewal and Approval

Innovative firms firstly conduct their drug discovery and then do preclinical-trial investigations. Firms clearly know the requirement and standards of the FDA. So rational firms will

only submit final applications with fair testing results which can almost surely get approval. The FDA reviews the preclinical-trial information (for IND) and clinical trials information (for NDA). Then the FDA evaluates the total benefit and cost of a drug candidate and decide its qualification for marketing.

### **A.3 Labeling**

Drug labels provide important information to consumers about all the possible side effects and DDIs. The FDA has very detailed requirements for the format of the labels. All drugs are required to have a specific section on the label to list all important DDIs.

*The Blackbox Warnings* The most severe warnings about side effects and DDIs are emphasized with black box in the product label.

### **A.4 Postmarketing Surveillance**

The FDA still has the responsibility of monitoring drugs after they enter the market. This includes the FAERS system, the Medwatch, and the REMS conforming to the FDA Amendments Act (FDAAA) of 2007. Pharmaceutical companies' Phase IV clinical trial is also an essential part of the Postmarketing surveillance.

### **A.5 Withdrawal from the Market**

If there are severe adverse effects being found for some drugs, they will be recalled and required to withdraw from the market.

The U.S. FDA is actually the first regulator who conducts Risk regulation.



## APPENDIX B

### MODEL PROOFS

#### B.1 Proof for Proposition 2.1

*Proof.* From  $u_t = \frac{1-\alpha}{\alpha} M_t \cdot l_t \frac{1}{\left(1 + \frac{D_t}{M_t}\right)^{\frac{\alpha}{1-\alpha}}} = \frac{1-\alpha}{\alpha} \cdot \underbrace{\frac{M_t}{\left(1 + \frac{D_t}{M_t}\right)^{\frac{\alpha}{1-\alpha}}}}_{\text{Risk discounted TFP}} \cdot l_t$  I assume  $\frac{D_t}{M_t} \gg 1$ , and

omit the 1 in  $1 + \frac{D_t}{M_t}$ . Then the Risk discounted TFP  $\frac{M_t}{\left(1 + \frac{D_t}{M_t}\right)^{\frac{\alpha}{1-\alpha}}} \approx \frac{M_t}{\left(\frac{D_t}{M_t}\right)^{\frac{\alpha}{1-\alpha}}} = \frac{M_t^{\frac{\alpha}{1-\alpha}+1}}{D_t^{\frac{\alpha}{1-\alpha}}} =$

$\frac{M_t^{\frac{1}{1-\alpha}}}{D_t^{\frac{\alpha}{1-\alpha}}} = \left(\frac{M_t^{\frac{1}{\alpha}}}{D_t}\right)^{\frac{\alpha}{1-\alpha}}$ . Iff  $D_t = O(M_t^\sigma)$ ,  $\sigma \geq \frac{1}{\alpha}$ , the Risk discounted TFP is decreasing in  $\frac{M_t^{\frac{1}{\alpha}}}{D_t}$ . □

#### B.2 ODEs for the Optimal Allocation Problem

For the optimization problem (2.38), I derive the following first order conditions:

$$\begin{aligned}
 [l_t] : \\
 & \beta^t \cdot M_t \left(1 + \frac{D_t}{M_t}\right)^{\frac{\alpha}{\alpha-1}} \cdot \left[ M_t \cdot l_t \left(1 + \frac{D_t}{M_t}\right)^{\frac{\alpha}{\alpha-1}} \right]^{-\gamma} \\
 & + \mu_t \cdot \frac{\lambda E_t}{1+c\lambda M_t E_t} \\
 & + \xi_t \cdot F(b, M_t, q) \cdot \frac{\lambda E_t}{1+c\lambda M_t E_t} \\
 & + \varphi_t \cdot \left[ \int_0^b \nu \cdot f(\nu, M_t, q) d\nu \right] \cdot F(b, M_t, q) \cdot \frac{\lambda E_t}{1+c\lambda M_t E_t} = 0
 \end{aligned}$$

$$\begin{aligned}
 [E_{t+1}] : \\
 & \mu_t - \mu_{t+1} \cdot \left(1 + \lambda(L - l_{t+1}) \cdot \frac{d}{dE_{t+1}} \left( \frac{E_{t+1}}{1+c\lambda M_{t+1} E_{t+1}} \right) \right) \\
 & - \xi_{t+1} \cdot F(b, M_{t+1}, q) \cdot \lambda(L - l_{t+1}) \cdot \frac{d}{dE_{t+1}} \left( \frac{E_{t+1}}{1+c\lambda M_{t+1} E_{t+1}} \right) \\
 & - \varphi_{t+1} \cdot \left[ \int_0^b \nu \cdot f(\nu, M_{t+1}, q) d\nu \right] \cdot F(b, M_{t+1}, q) \cdot \lambda(L - l_{t+1}) \cdot \frac{d}{dE_{t+1}} \left( \frac{E_{t+1}}{1+c\lambda M_{t+1} E_{t+1}} \right) \\
 & = 0
 \end{aligned}$$

$$[M_{t+1}] :$$

$$\begin{aligned}
& \xi_t + \beta^{t+1} l_{t+1} \cdot \left[ M_{t+1} \cdot l_{t+1} \left( 1 + \frac{D_{t+1}}{M_{t+1}} \right)^{\frac{\alpha}{\alpha-1}} \right]^{-\gamma} \cdot \frac{d}{dM_{t+1}} \left( M_{t+1} \cdot \left( 1 + \frac{D_{t+1}}{M_{t+1}} \right)^{\frac{\alpha}{\alpha-1}} \right) \\
& - \mu_{t+1} \cdot \lambda E_{t+1} (L - l_{t+1}) \cdot \frac{d}{dM_{t+1}} \left( \frac{1}{1+c\lambda M_{t+1} E_{t+1}} \right) \\
& - \xi_{t+1} \cdot \left( 1 + \lambda E_{t+1} (L - l_{t+1}) \cdot \frac{d}{dM_{t+1}} \left( \frac{F(b, M_{t+1}, q)}{1+c\lambda M_{t+1} E_{t+1}} \right) \right) \\
& - \varphi_{t+1} \cdot \lambda E_{t+1} (L - l_{t+1}) \cdot \frac{d}{dM_{t+1}} \left( \frac{F(b, M_{t+1}, q)}{1+c\lambda M_{t+1} E_{t+1}} \left[ \int_0^b \nu \cdot f(\nu, M_{t+1}, q) d\nu \right] \right) \\
& = 0
\end{aligned}$$

$[D_{t+1}] :$

$$\varphi_t - \varphi_{t+1} + \beta^{t+1} \cdot M_{t+1} \cdot l_{t+1} \cdot \left[ M_{t+1} \cdot l_{t+1} \left( 1 + \frac{D_{t+1}}{M_{t+1}} \right)^{\frac{\alpha}{\alpha-1}} \right]^{-\gamma} \cdot \frac{d}{dD_{t+1}} \left( \left( 1 + \frac{D_{t+1}}{M_{t+1}} \right)^{\frac{\alpha}{\alpha-1}} \right) =$$

0

Together with three LOMs:

$$E_{t+1} - E_t = \lambda \frac{L - l_t}{1 + c\lambda M_t E_t} E_t \tag{B.1}$$

$$M_{t+1} - M_t = F(b, M_t, q) \cdot \lambda \frac{L - l_t}{1 + c\lambda M_t E_t} E_t \tag{B.2}$$

$$D_{t+1} - D_t = F(b, M_t, q) \cdot \lambda \frac{L - l_t}{1 + c\lambda M_t E_t} E_t \cdot \left( \int_0^b \nu \cdot f(\nu, M_t, q) d\nu \right) \tag{B.3}$$

At the steady state,

$$l_t = L; L - l_{t+1} = 0$$

$$[E_{t+1}] : \mu_t - \mu_{t+1} = 0$$

$$[M_{t+1}] : \xi_t - \xi_{t+1} + \beta^{t+1} L \cdot \frac{d}{dM_{t+1}} \left( M_{t+1} \cdot \left( 1 + \frac{D_{t+1}}{M_{t+1}} \right)^{\frac{\alpha}{\alpha-1}} \right) = 0$$

$$[D_{t+1}] : \varphi_t - \varphi_{t+1} + \beta^{t+1} \cdot M_{t+1} \cdot L \cdot \frac{d}{dD_{t+1}} \left( \left( 1 + \frac{D_{t+1}}{M_{t+1}} \right)^{\frac{\alpha}{\alpha-1}} \right) = 0$$

### B.3 Proof for Proposition 2.2

**Proof.**

With and without one more intermediate, the sum of discounted utilities after  $T$ :

$$\sum_{t=T+1}^{\infty} \beta^t \cdot \frac{1-\alpha}{\alpha} \bar{M} \left( 1 + \frac{\bar{D}}{\bar{M}} \right)^{\frac{\alpha}{\alpha-1}} \cdot L$$

$$\sum_{t=T+1}^{\infty} \beta^t \cdot \frac{1-\alpha}{\alpha} (\bar{M} + 1) \left( 1 + \frac{(\bar{D} + [\int_0^b \nu \cdot f(\nu, \bar{M}, q) d\nu])}{(\bar{M} + 1)} \right)^{\frac{\alpha}{\alpha-1}} \cdot L$$

With  $ETC$  spent at  $t$ , there will be one additional qualified innovation introduced.

Before this, a large portion of labor is devoted to R&D. After reaching the threshold or time  $T$ , no labor will be devoted to R&D.

From patents' law of motion equation (2.39), because  $l_t > 0$  we can derive that  $E_{t+1} - E_t < \frac{\lambda E_t L}{(1+c\lambda M_t E_t)} = \frac{L}{\frac{1}{\lambda E_t} + c M_t} < \frac{L}{c M_t}$ . That is, the choice set for  $E_{t+1}$  at  $t$  is bounded by:

$$E_{t+1} - E_t < \frac{L}{c M_t} \quad (\text{B.4})$$

When  $M_t$  grows large enough,  $\frac{L}{c M_t}$  can shrink to such a small level that the choice set for  $E_{t+1}$  is even smaller than 1. This implies that R&D will stop.

■

## B.4 Proof for Proposition 2.3

**Proof.**

For the Binomial CDF  $F(b, M_t, q)$ , if  $b \leq M_t \cdot q$ , which can be easily satisfied when  $M_t$  is large enough, there is the **Hoeffding's inequality** that gives an upper bound:

$$F(b, M_t, q) \leq \frac{1}{e^{\frac{2(q \cdot M_t - b)^2}{M_t}}} \quad (\text{B.5})$$

Combine (B.5) and (2.49), we can derive a lower bound  $\underline{ERC}$  for  $ERC$ :

$$ERC \geq \underline{ERC} = \frac{\frac{1-\alpha}{\alpha} \alpha^{\frac{\alpha}{1-\alpha}} M_t [1 + (c + \omega) \lambda E_t M_t]}{\lambda E_t} \cdot e^{\frac{2(q \cdot M_t - b)^2}{M_t}} \quad (\text{B.6})$$

We can further derive the limiting property for ERC:

$$\lim_{M_t \rightarrow \infty} \underline{ERC} = \frac{1-\alpha}{\alpha} \alpha^{\frac{\alpha}{1-\alpha}} (c+\omega) \cdot e^{-4qb} \cdot M_t^2 \cdot e^{2q^2 \cdot M_t} \quad (\text{B.7})$$

therefore we have the following result<sup>1</sup> for ERC :

$$O(\underline{ERC}) = M_t^2 \cdot e^{M_t} \quad (\text{B.8})$$

■

## B.5 Proof for Proposition 2.4

**Proof.**

By substituting (2.45) into (2.48), we can get

$$\sum_{\tau=t+1}^{\infty} \frac{l_{\tau}}{\prod_{s=t+1}^{\tau} [1+r_s]} \leq \frac{M_t (1 + (c + \omega) \lambda E_t M_t)}{\alpha \lambda E_t \times F(b, M_t, q)} \quad (\text{B.9})$$

We know  $l_{\tau} \leq L$ , and  $r_s \geq \rho > 0$ . So  $\sum_{\tau=t+1}^{\infty} \frac{l_{\tau}}{\prod_{s=t+1}^{\tau} [1+r_s]} \leq \sum_{\tau=t+1}^{\infty} \frac{L}{\prod_{s=t+1}^{\tau} [1+\rho]} = \frac{L}{\rho}$ .

Then we have the following constraint:

$$\frac{M_t (1 + (c + \omega) \lambda E_t M_t)}{\alpha \lambda E_t \times F(b, M_t, q)} \leq \frac{L}{\rho} \quad (\text{B.10})$$

$\frac{M_t(1+(c+\omega)\lambda E_t M_t)}{\alpha \lambda E_t \times F(b, M_t, q)}$  is an increasing function of  $M_t$ . It is bounded above by  $\frac{L}{\rho}$ .

Therefore there must exist  $\hat{M}$ , so that  $M_t < \hat{M}$ , satisfying (B.10).

Eqn (B.9) will have strict “<” when  $M_t > \hat{M}$ , and R&D stops.

Moreover, a larger labor force  $L$  can relax the *RHS* of the constraint (B.10) and admits

---

1. in terms of big O notation

a higher  $\hat{M}$ .



# APPENDIX C

## DATA DESCRIPTION

### C.1 FDA's FAERS Database

The FAERS data mainly include the following main files

1. Demographic information of patients;
2. Drug information ;
3. Patient outcome information;
4. Drug Therapy Start/End Dates

#### *C.1.1 Demographic Information File*

primaryid: Unique number for identifying a FAERS report

event\_dt: Date the adverse event occurred or began

age: Numeric value of patient's age at event

sex: Code for patient's sex

wt: Numeric Value of Patient's Weight

occpr\_cod: Abbreviation for the reporter's type of occupation

occr\_country: The country where the event occurred

#### *C.1.2 Drug Information File*

is what we are particularly interested in

##### **Major Info**

primaryid: is the Unique code for identifying a FAERS case report.

drug\_seq: is the Unique number for identifying a drug in a FAERS case

each FAERS case report includes one or multiple drugs

drugname: Name of medicinal product

prod\_ai: Product Active Ingredient

role\_cod: Code for drug's reported role in a case.

Codes including following roles:

PS Primary Suspect Drug

SS Secondary Suspect Drug

C Concomitant

I Interacting

however, the role information is very rough

### **Other Info**

val\_vbm: Code for source of DRUGNAME

route: The route of drug administration

dose\_vbm: Verbatim text for dose, frequency, and route, exactly as entered on report

cum\_dose\_chr: Cumulative dose to first reaction

cum\_dose\_unit: Cumulative dose to first reaction unit

dechal: Dechallenge code, indicating if reaction abated when drug therapy was stopped

rechal: Rechallenge code, indicating if reaction recurred when drug therapy restarted

lot\_num: Lot number of the drug

exp\_dt: Expiration date of the drug

nda\_num: NDA number

dose\_amt: Amount of drug reported

dose\_unit: Unit of drug dose

dose\_form: Form of dose reported

dose\_freq: Code for Frequency

### *C.1.3 Patient Outcomes File*

primaryid: Unique number for identifying a FAERS case report

outc\_cod: Code for a patient outcome CODE MEANING

DE Death

LT Life-Threatening

HO Hospitalization - Initial or Prolonged

DS Disability

CA Congenital Anomaly

RI Required Intervention to Prevent permanent Impairment/Damage

OT Other Serious (Important Medical Event)

### *C.1.4 Drug Therapy Start/End Dates*

primaryid: Unique number for identifying a FAERS report

caseid: Number for identifying a FAERS case (example. 3123456)

dsg\_drug\_seq: Drug sequence number for identifying a drug for a Case

start\_dt: A date therapy was started (or re-started) for this drug

start\_dt\_num: A date therapy was started (or re-started) for this drug

end\_dt long: A date therapy was stopped for this drug. (YYYYMMDD)

end\_dt\_num: A date therapy was stopped for this drug. (YYYYMMDD)

dur: Numeric value of the duration (length) of therapy

dur\_cod: Unit abbreviation for duration of therapy



## C.2 Drugbank Database Description

The DrugCard for each drug (molecule) has >200 data fields. Half of the information is drug/chemical data and the other half is drug target or protein data. I list the useful data categories as below:

DrugBank ID (Primary Accession Number)      Unique DrugBank accession number consisting of a 2 letter prefix (DB) and a 5 number suffix.

Brand Names      Brand names from different manufacturers

Synonyms      Alternate names of the drug

Patents      The first and last drug patent, including approval and expiry dates

Chemical Formula      Chemical formula describing atomic or elemental composition

FDA Label      Food and Drug Administration approval label (if it exists)

Indication      Description or common names of diseases that the drug is used to treat

Toxicity      Lethal dose (LD50) values from test animals, description of side effects and toxic effects seen in humans

Contraindications      Cautions or conditions indicating why or when the drug should not be taken

Drug Interactions      Drugs that are known to interact, interfere or cause adverse reactions when taken with this drug

Food Interactions      Foods that are known to interact, interfere or cause adverse reactions when taken with this drug

## C.3 Drug Name Mapping and Standardization

Drug names are messy in the FAERS database. Different proprietary names, brand names, and abbreviations can correspond to one NME (a common molecular entity).

Drugbank categorizes drugs based on the molecular entity, and has a set of disparate drug names correspond to each molecule. I use this set of drug names to identify and map the drug names in the FAERS to a standard molecule.

# APPENDIX D

## CALIBRATION PARAMETERS

		Value	Notes
$\gamma$	risk aversion	2	Standard
$\beta$	Discount factor	0.96	Standard
$\lambda$	R&D efficiency	0.001	
$q$	probability of interaction	3.5%	my estimation

Table D.1: Some Calibration parameters

# APPENDIX E

## GENERALIZATION OF THE HIGH-ORDER RISK SPACE AND NP-COMPLETE PROBLEM

In the most general case, any combinational interaction between any number of intermediate inputs can possibly generate adverse effects. These complex interactions can be represented by an  $M - dimensional$  random tensor. Each element is represented by  $\tilde{d}_{i_1, i_2, \dots, i_M}$ , a random variable indexed by  $i_1, i_2, \dots, i_M$ . Each subscript  $i_k \in \{0, 1\}$ , where  $k$  is the  $k^{th}$  intermediate product. For example,  $\tilde{d}_{1,0,1,1,0,0, \dots, 0}$ , where  $i_1 = 1, i_2 = 0, i_3 = 1, i_4 = 1, i_5 = 0, \dots, i_M = 0$ , stands for the adverse effect due to the interaction between intermediate products #1, #3 and #4.

I define this random tensor as **Macro Risk Tensor**.

**Proposition E.1.** *The number of elements in the Macro Technology Tensor is  $O(2^M)$ .*

The total number of elements in the Macro Technology Matrix is equal to all possible interactions between  $M$  intermediate products. I can easily derive this by summing up all the possible combinations from 1 to  $M$  intermediate products:  $\sum_{k=1}^M \binom{M}{k} = 2^M - 1$ . I deduct one here from  $2^M$  for the case of no hazards for any combination.

**Proposition E.2.** *If allowing multiple copies of each technology up to number  $N$ , the number of elements in the Macro Risk Tensor is  $O(N^M)$ .*

When I allow replicas of each technology, there will be much more possible combinations.

This  $M - dimensional$  random matrix can be very sparse. Nevertheless, as  $M$  increases, the total number of elements in this Macro Technology Tensor will explode exponentially. This captures the essential idea of **Complex Uncertainty**, which describes increasing externality due to technological progress. The curse of dimension might work against innovation and adoption of new intermediate products in the long run.

There has been some empirical evidence showing the existence of Higher-order risks, for example, (*Cyclophosphamide, Prednisone, Vincristine*)

and (*Cyclophosphamide, Doxorubicin, Prednisone, Rituximab*), exemplified by Harpaz et al (2010).

# APPENDIX F

## PROOFS IN CHAPTER 3

### F.1 Proof of Proposition 3.1

Given the optimization problem,

$$\max_{\{x\}} \left\{ \begin{aligned} & \int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} \left( \frac{\alpha R_0+s \cdot x^\theta}{\alpha+x^\theta} x + K - x \right) f(s) ds \\ & + \int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} \left( \frac{\alpha R_0+s \cdot x^\theta}{\alpha+x^\theta} N + K - N \right) f(s) ds \end{aligned} \right\} \quad (\text{F.1})$$

where the distribution  $f(s)$  follows,

$$f(s) = \frac{1}{\sqrt{(x^\theta)^{-1} + (\alpha_0)^{-1}}} \phi \left( \frac{s - R_0}{\sqrt{(x^\theta)^{-1} + (\alpha_0)^{-1}}} \right) \quad (\text{F.2})$$

There are three major steps. Firstly integrate (F.1), and secondly take first-order conditions to get a closed-form solution of  $x$ . Thirdly, we prove the uniqueness of social planner's solution.

#### F.1.1 Step 1: Integration

(i) Integrate the lower part of (F.1)

$$\begin{aligned} & \int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} \left( \frac{\alpha R_0+s \cdot x^\theta}{\alpha+x^\theta} x + K - x \right) f(s) ds \\ &= \int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} \left( \frac{\alpha R_0+s \cdot x^\theta}{\alpha+x^\theta} x - x + K \right) f(s) ds \\ &= \int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} \left( \frac{s \cdot x^\theta}{\alpha+x^\theta} x \right) f(s) ds + \left( K - x + \frac{x \alpha R_0}{\alpha+x^\theta} \right) \int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} f(s) ds \end{aligned}$$

(ii) Integrate the lower part of (F.1)

$$\begin{aligned} & \int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} \left( \frac{\alpha R_0+s \cdot x^\theta}{\alpha+x^\theta} N + K - N \right) f(s) ds \\ &= \int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} \left( \frac{s \cdot x^\theta}{\alpha+x^\theta} N \right) f(s) ds + \left( K - N + \frac{\alpha R_0 N}{\alpha+x^\theta} \right) \int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} f(s) ds \end{aligned}$$

rearrange the sum of the two outcomes above, we have

$$\begin{aligned} & \left( \left( K - x + \frac{x \alpha R_0}{\alpha+x^\theta} \right) \int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} f(s) ds + \left( K - N + \frac{\alpha R_0 N}{\alpha+x^\theta} \right) \int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} f(s) ds \right) \\ & + \frac{x^\theta}{\alpha+x^\theta} \frac{1}{1-\lambda} \left( x \int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} s f(s) ds + N \int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} s f(s) ds \right) \end{aligned}$$

and we integrate the two parts respectively,

(a) The first part

$$\left( K - x + \frac{x \alpha R_0}{\alpha+x^\theta} \right) \int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} f(s) ds + \left( K - N + \frac{\alpha R_0 N}{\alpha+x^\theta} \right) \int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} f(s) ds$$

because  $f(s) = \frac{1}{\sqrt{(x^\theta)^{-1} + (\alpha_0)^{-1}}} \phi \left( \frac{s - R_0}{\sqrt{(x^\theta)^{-1} + (\alpha_0)^{-1}}} \right)$ ,

we have  $\int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} f(s)ds = \Phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)\Big|_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}$

$$\begin{aligned} & \text{therefore } \left(K-x+\frac{x\alpha R_0}{\alpha+x^\theta}\right)\int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} f(s)ds + \left(K-N+\frac{\alpha R_0 N}{\alpha+x^\theta}\right)\int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} f(s)ds \\ &= \frac{E-x+\frac{x\alpha R_0}{\alpha+x^\theta}}{1-\lambda}\Phi\left(\frac{(\alpha+x^\theta)(1-R_0)}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) + \frac{E-N+\frac{\alpha R_0 N}{\alpha+x^\theta}}{1-\lambda}\left[1-\Phi\left(\frac{(\alpha+x^\theta)(1-R_0)}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)\right] \\ &= \frac{E-N+\frac{\alpha R_0 N}{\alpha+x^\theta}}{1-\lambda} + \frac{N-x+\frac{x\alpha R_0-N\alpha R_0}{\alpha+x^\theta}}{1-\lambda}\Phi\left(\frac{(\alpha+x^\theta)(1-R_0)}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \\ &= \frac{E-N+\frac{\alpha R_0 N}{\alpha+x^\theta}}{1-\lambda} + \frac{(N-x)}{1-\lambda}\left(\frac{\alpha+x^\theta-\alpha R_0}{\alpha+x^\theta}\right)\Phi\left(\frac{(\alpha+x^\theta)(1-R_0)}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \end{aligned}$$

(b) The second part

$$\frac{x^\theta}{\alpha+x^\theta}\frac{1}{1-\lambda}\left(x\int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} sf(s)ds + N\int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} sf(s)ds\right)$$

we need to firstly derive  $\int sf(s)ds$

$$\begin{aligned} \int sf(s)ds &= \int s\phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)d\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \\ &= \sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}\int\frac{s}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)d\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \\ &= \sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}\int\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)d\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \\ &+ \sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}\int\frac{R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)d\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \\ &= \sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}\int\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)\phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)d\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \\ &+ R_0\int\phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)d\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \\ &= \left(-\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}\right)\phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)\Big| + R_0\Phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)\Big| \end{aligned}$$

then

$$\begin{aligned} & x\int_{-\infty}^{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}} sf(s)ds \\ &= -x\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}\phi\left(\frac{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) + xR_0\Phi\left(\frac{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \end{aligned}$$

and

$$\begin{aligned} & N\int_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} sf(s)ds \\ &= N\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}\phi\left(\frac{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) + NR_0\Phi\left(\frac{s-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right)\Big|_{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}}^{\infty} \\ &= NR_0 + N\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}\phi\left(\frac{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) - NR_0\Phi\left(\frac{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \end{aligned}$$

Add up the two parts' integration results, we have the final integration result:

$$\begin{aligned} & \frac{E-N+\frac{\alpha R_0 N}{\alpha+x^\theta}}{1-\lambda} + \frac{(N-x)}{1-\lambda}\left(\frac{\alpha+x^\theta-\alpha R_0}{\alpha+x^\theta}\right)\Phi\left(\frac{(\alpha+x^\theta)(1-R_0)}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \\ &+ \frac{x^\theta}{\alpha+x^\theta}\frac{NR_0}{1-\lambda} + \frac{x^\theta}{\alpha+x^\theta}\frac{N-x}{1-\lambda}\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}\phi\left(\frac{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) - \frac{x^\theta R_0}{\alpha+x^\theta}\frac{N-x}{1-\lambda}\Phi\left(\frac{\frac{\alpha+x^\theta-\alpha R_0}{x^\theta}-R_0}{\sqrt{(x^\theta)^{-1}+(\alpha_0)^{-1}}}\right) \end{aligned}$$

which can be simplified to

$$\frac{1}{1-\lambda} (E + N (R_0 - 1)) + \frac{1}{1-\lambda} (N - x) (1 - R_0) \Phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) + \frac{1}{1-\lambda} \frac{x^{\frac{\theta}{2}} (N-x)}{\alpha_0^{\frac{1}{2}} (\alpha_0+x^\theta)^{\frac{1}{2}}} \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right)$$

### F.1.2 Step 2: First-order condition

now the optimization problem becomes,

$$\max_x \left\{ \begin{array}{l} K + N (R_0 - 1) \\ + (N - x) (1 - R_0) \Phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \\ + \frac{x^{\frac{\theta}{2}} (N-x)}{\alpha_0^{\frac{1}{2}} (\alpha_0+x^\theta)^{\frac{1}{2}}} \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \end{array} \right\} \quad (\text{F.3})$$

the derivative of  $\frac{(1-R)(\alpha+x^\theta)^{\frac{1}{2}} \alpha^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}$

$$\frac{d}{dx} \left\{ \frac{(1-R)(\alpha+x^\theta)^{\frac{1}{2}} \alpha^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right\} = \frac{1}{2x^{\frac{1}{2}\theta+1}} \alpha^{\frac{3}{2}} \frac{\theta}{\sqrt{\alpha+x^\theta}} (R-1)$$

(i) derivative of  $(N-x)(1-R_0) \Phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right)$

$$\begin{aligned} & \frac{d}{dx} \left\{ (N-x)(1-R_0) \Phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \right\} \\ &= (R_0 - 1) \Phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) - (N-x)(1-R_0)^2 \frac{1}{2x^{\frac{1}{2}\theta+1}} \alpha^{\frac{3}{2}} \frac{\theta}{\sqrt{\alpha+x^\theta}} \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \end{aligned}$$

(ii) derivative of  $\frac{x^{\frac{\theta}{2}} (N-x)}{\alpha_0^{\frac{1}{2}} (\alpha_0+x^\theta)^{\frac{1}{2}}} \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right)$

$$\begin{aligned} & \frac{d}{dx} \left\{ \frac{x^{\frac{\theta}{2}} (N-x)}{\alpha_0^{\frac{1}{2}} (\alpha_0+x^\theta)^{\frac{1}{2}}} \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \right\} \\ &= -\frac{1}{2} \frac{x^{\frac{1}{2}\theta-1}}{\sqrt{\alpha}(\alpha+x^\theta)^{\frac{3}{2}}} (2x\alpha + 2xx^\theta - N\alpha\theta + x\alpha\theta) \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \\ & \quad - \frac{x^{\frac{\theta}{2}} (N-x)}{\alpha_0^{\frac{1}{2}} (\alpha_0+x^\theta)^{\frac{1}{2}}} \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \frac{1}{2x^{\frac{1}{2}\theta+1}} \alpha^{\frac{3}{2}} \frac{\theta}{\sqrt{\alpha+x^\theta}} (R-1) \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \\ &= -\frac{1}{2} \frac{x^{\frac{1}{2}\theta-1}}{\sqrt{\alpha}(\alpha+x^\theta)^{\frac{3}{2}}} (2x\alpha + 2xx^\theta - N\alpha\theta + x\alpha\theta) \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \\ & \quad + \frac{1}{2x^{\frac{1}{2}\theta+1}} \alpha^{\frac{3}{2}} \theta \frac{N-x}{\sqrt{\alpha+x^\theta}} (R-1)^2 \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \end{aligned}$$

then the sum of these two derivatives, and equalizes it with 0,

$$(R_0 - 1) \Phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) - (N-x)(1-R_0)^2 \frac{1}{2x^{\frac{1}{2}\theta+1}} \alpha^{\frac{3}{2}} \frac{\theta}{\sqrt{\alpha+x^\theta}} \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right)$$



$$\begin{aligned}
& -\frac{1}{2} \frac{x^{\frac{1}{2}\theta-1}}{\sqrt{\alpha}(\alpha+x^\theta)^{\frac{3}{2}}} (2x\alpha + 2xx^\theta - N\alpha\theta + x\alpha\theta) \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \\
& + \frac{1}{2x^{\frac{1}{2}\theta+1}} \alpha^{\frac{3}{2}} \theta \frac{N-x}{\sqrt{\alpha+x^\theta}} (R-1)^2 \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) = 0
\end{aligned}$$

rearrange items to get,

$$\begin{aligned}
& \Phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) (R_0 - 1) \\
& = \phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \left\{ \begin{aligned} & \frac{1}{2} \frac{x^{\frac{1}{2}\theta-1}}{\sqrt{\alpha}(\alpha+x^\theta)^{\frac{3}{2}}} (2x\alpha + 2xx^\theta - N\alpha\theta + x\alpha\theta) \\ & + (N-x)(1-R_0)^2 \frac{1}{2x^{\frac{1}{2}\theta+1}} \alpha^{\frac{3}{2}} \frac{\theta}{\sqrt{\alpha+x^\theta}} \\ & - \frac{1}{2x^{\frac{1}{2}\theta+1}} \alpha^{\frac{3}{2}} \theta \frac{N-x}{\sqrt{\alpha+x^\theta}} (R-1)^2 \end{aligned} \right\}
\end{aligned}$$

which can be further simplified to,

$$\begin{aligned}
& \frac{\Phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right)}{\phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right)} \\
& = \frac{1}{2} \frac{1}{R_0-1} \left\{ \frac{x^{\frac{1}{2}\theta-1}}{\sqrt{\alpha}(\alpha+x^\theta)^{\frac{3}{2}}} (2x\alpha + 2xx^\theta - N\alpha\theta + x\alpha\theta) \right\} \\
& = \frac{1}{2} \frac{1}{R_0-1} \left\{ \frac{2x^{\frac{\theta}{2}}}{\sqrt{\alpha}(\alpha+x^\theta)} - \frac{x^{\frac{1}{2}\theta-1} \theta \sqrt{\alpha}}{(\alpha+x^\theta)^{\frac{3}{2}}} (N-x) \right\}
\end{aligned}$$

finally we have,

$$\frac{\Phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right)}{\phi \left( \frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right)} = \frac{1}{2} \frac{1}{R_0-1} \left\{ \frac{2x^{\frac{\theta}{2}}}{\sqrt{\alpha}(\alpha+x^\theta)} - \frac{x^{\frac{1}{2}\theta-1} \theta \sqrt{\alpha}}{(\alpha+x^\theta)^{\frac{3}{2}}} (N-x) \right\} \quad (\text{F.4})$$

### F.1.3 Step 3. Uniqueness

denote  $G = (1-R_0) \sqrt{\frac{\alpha_0(\alpha_0+x^\theta)}{x^\theta}}$

$\sqrt{\frac{\alpha_0(\alpha_0+x^\theta)}{x^\theta}} = \sqrt{\frac{\alpha_0^2}{x^\theta} + \alpha_0}$  is always a decreasing function of  $x$ , when  $x \geq 1$

Equation (F.4) can be rearranged to be

$$G \frac{\Phi(G)}{\phi(G)} = \frac{\theta\alpha_0(N-x)}{2x(\alpha_0+x^\theta)} - 1 \quad (\text{F.5})$$

$\frac{\theta\alpha_0(N-x)}{2x(\alpha_0+x^\theta)}$  is always decreasing in  $x$  for  $x > 0$  and  $x < N$ .

$$\frac{d}{dG} \left( \frac{\Phi(G)}{\phi(G)} \right) = 1 + \Phi(G) \left( \frac{1}{\phi(G)} \right)' = 1 + \Phi(G) \cdot \left( -\frac{1}{\phi^2(G)} \right) \cdot (-G\phi(G)) = 1 + G \frac{\Phi(G)}{\phi(G)}$$

$$\frac{d \left( \frac{G \cdot \Phi(G)}{\phi(G)} \right)}{dG} = \frac{d \left( G \cdot \frac{\Phi(G)}{\phi(G)} \right)}{dG} = \frac{\Phi(G)}{\phi(G)} + G \cdot \left( 1 + G \frac{\Phi(G)}{\phi(G)} \right)$$

$$= G + \frac{\Phi(G)}{\phi(G)} (G^2 + 1)$$

For any  $G > 0$ ,  $\frac{d \left( \frac{G \cdot \Phi(G)}{\phi(G)} \right)}{dG} > 0$ ;

and in fact, for any  $G > \underline{G} = -7.48$ ,  $\frac{d \left( \frac{G \cdot \Phi(G)}{\phi(G)} \right)}{dG} > 0$ .

$\underline{G}$  is tiny enough, and  $G = (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + x^\theta)}{x^\theta}}$  will never be smaller than  $\underline{G}$  in reasonable way.

So generally  $\frac{G \cdot \Phi(G)}{\phi(G)}$  is an increasing function of  $G$ .

**When**  $R_0 > 1, (1 - R_0) < 0,$

$G(x) = (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + x^\theta)}{x^\theta}}$  is increasing function of  $x$ ;

so  $LHS = \frac{G(x) \cdot \Phi(G(x))}{\phi(G(x))}$  is increasing function of  $x$ .

$RHS$  is decreasing function of  $x$ .

there is a unique solution at their intersection.

**When**  $R_0 < 1, (1 - R_0) > 0,$

$G(x) = (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + x^\theta)}{x^\theta}}$  is decreasing function of  $x$ ;

so  $LHS = \frac{G(x) \cdot \Phi(G(x))}{\phi(G(x))}$  is decreasing function of  $x$ .

and  $RHS$  is decreasing function of  $x$ .

However, the slope of the  $RHS$  is higher than the  $LHS$  when  $x$  is large enough. The  $LHS$  and  $RHS$  curve will cross once and only once, and (F.5) has a unique solution  $x$ .

■

## F.2 Proof of Corollary 3.1

$$\max_x \left\{ \begin{array}{l} K + N(R_0 - 1) \\ + (N - x)(1 - R_0) \Phi \left( \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \\ + (N - x) \frac{x^{\frac{\theta}{2}}}{\alpha_0^{\frac{1}{2}} (\alpha_0 + x^\theta)^{\frac{1}{2}}} \phi \left( \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \end{array} \right\} \quad (F.6)$$

$$\lim_{x \rightarrow 0} \left\{ \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right\} = \begin{cases} \infty & \text{if } R_0 < 1 \\ -\infty & \text{if } R_0 \geq 1 \end{cases}$$

therefore

(i) if  $R_0 < 1, x \rightarrow 0$

$$\lim_{x \rightarrow 0} \left\{ \Phi \left( \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \right\} = 1;$$

$$(N - x)(1 - R_0) \Phi \left( \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \rightarrow N(1 - R_0) = (N - 1)(1 - R_0) + (1 - R_0)$$

$$\frac{x^{\frac{\theta}{2}}(N - x)}{\alpha_0^{\frac{1}{2}} (\alpha_0 + x^\theta)^{\frac{1}{2}}} \phi \left( \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \rightarrow 0$$

if  $x = 1,$

$$\text{then } (1 - R_0) \Phi \left( \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) = (1 - R_0) \Phi \left( (1 - R_0)(\alpha_0 + 1)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}} \right)$$

$$\frac{x^{\frac{\theta}{2}}}{\alpha_0^{\frac{1}{2}} (\alpha_0 + x^\theta)^{\frac{1}{2}}} \phi \left( \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) = \frac{1}{\alpha_0^{\frac{1}{2}} (\alpha_0 + 1)^{\frac{1}{2}}} \phi \left( (1 - R_0)(\alpha_0 + 1)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}} \right)$$

(ii) if  $R_0 > 1, x \rightarrow 0$

$$\lim_{x \rightarrow 0} \left\{ \Phi \left( \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \right\} = 0; (N - x)(1 - R_0) \Phi \left( \frac{(1 - R_0)(\alpha + x^\theta)^{\frac{1}{2}} \alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}} \right) \rightarrow 0$$

$$\frac{x^{\frac{\theta}{2}}(N-x)}{\alpha_0^{\frac{1}{2}}(\alpha_0+x^\theta)^{\frac{1}{2}}}\phi\left(\frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}}\alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}\right)\rightarrow 0$$

but any  $0 < x < N$  will make  $(N-x)(1-R_0)\Phi\left(\frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}}\alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}\right) > 0$ ;

$$\frac{x^{\frac{\theta}{2}}(N-x)}{\alpha_0^{\frac{1}{2}}(\alpha_0+x^\theta)^{\frac{1}{2}}}\phi\left(\frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}}\alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}\right) > 0$$

so  $x = 0$  is not optimal.

■

### F.3 Proof of Corollary 3.2

$$\max_x \left\{ \begin{array}{l} K + N(R_0 - 1) \\ + (N-x)(1-R_0)\Phi\left(\frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}}\alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}\right) \\ + \frac{x^{\frac{\theta}{2}}(N-x)}{\alpha_0^{\frac{1}{2}}(\alpha_0+x^\theta)^{\frac{1}{2}}}\phi\left(\frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}}\alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}\right) \end{array} \right\} \quad (\text{F.7})$$

at  $x = N$ , both  $(N-x)(1-R_0)\Phi\left(\frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}}\alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}\right)$  and  $\frac{x^{\frac{\theta}{2}}(N-x)}{\alpha_0^{\frac{1}{2}}(\alpha_0+x^\theta)^{\frac{1}{2}}}\phi\left(\frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}}\alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}\right)$  are equal to 0.

because  $\Phi\left(\frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}}\alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}\right) > 0$  and  $\phi\left(\frac{(1-R_0)(\alpha+x^\theta)^{\frac{1}{2}}\alpha_0^{\frac{1}{2}}}{x^{\frac{\theta}{2}}}\right) > 0$

when  $(1-R_0) < 0$ , any  $x < N$  will have a higher utility than when  $x = N$ .

■

### F.4 Proof of Corollary 3.3

$$(1-R_0)\frac{\Phi\left((1-R_0)\sqrt{\frac{\alpha_0(\alpha_0+x^\theta)}{x^\theta}}\right)}{\phi\left((1-R_0)\sqrt{\frac{\alpha_0(\alpha_0+x^\theta)}{x^\theta}}\right)} = \frac{1}{\sqrt{\frac{\alpha_0(\alpha_0+x^\theta)}{x^\theta}}}\left(\frac{\theta\alpha_0(N-x)}{2x(\alpha_0+x^\theta)} - 1\right)$$

When  $R_0 \rightarrow 1$ ,

$$\lim_{R_0 \rightarrow 1} (1-R_0)\frac{\Phi\left((1-R_0)\sqrt{\frac{\alpha_0(\alpha_0+x^\theta)}{x^\theta}}\right)}{\phi\left((1-R_0)\sqrt{\frac{\alpha_0(\alpha_0+x^\theta)}{x^\theta}}\right)} = 0 = \lim_{R_0 \rightarrow 1} \frac{1}{\sqrt{\frac{\alpha_0(\alpha_0+x^\theta)}{x^\theta}}}\left(\frac{\theta\alpha_0(N-x)}{2x(\alpha_0+x^\theta)} - 1\right)$$

so we have  $\frac{\theta\alpha_0(N-x)}{2x(\alpha_0+x^\theta)} - 1 = 0$

$$\theta\alpha_0(N-x) = 2x(\alpha_0+x^\theta)$$

That is,  $x$  is the solution to  $2x^{\theta+1} + \alpha_0(2+\theta)x - \theta\alpha_0N = 0$

When  $\theta = 0$ ,  $2x + 2\alpha_0x = 0$ ; so  $x = 0$ .

When  $\theta = 1$ ,

we have  $2x^2 + 3\alpha_0x - \alpha_0N = 0$ , and the solution is given by,

$\left(\frac{1}{4}\sqrt{\alpha_0(8N+9\alpha_0)} - \frac{3}{4}\alpha_0, -\frac{1}{4}\sqrt{\alpha_0(8N+9\alpha_0)} - \frac{3}{4}\alpha_0\right)$   
 finally we have the following unique positive solution for  $x$ ,  
 $x = \frac{1}{4}\sqrt{\alpha_0(8N+9\alpha_0)} - \frac{3}{4}\alpha_0$

In the limit

$$\lim_{\alpha_0 \rightarrow \infty} \frac{dx}{d\alpha_0} = \lim_{\alpha_0 \rightarrow \infty} \frac{8N+18\alpha_0}{8\sqrt{9\alpha_0^2+8N\alpha_0}} - \frac{3}{4} = \lim_{\alpha_0 \rightarrow \infty} \frac{8N+18\alpha_0}{24\alpha_0} - \frac{3}{4} = 0$$

■

## F.5 Proof of Proposition 3.2

### F.5.1 $x$ with respect to $N$

When  $(1 - R_0) < 0$ , the *LHS* of (F.5) is a decreasing function of  $x$ . The *RHS* of (F.5) is an increasing function of  $x$ .

Because for  $\sqrt{\frac{x^\theta}{\alpha_0(\alpha_0+x^\theta)}} \frac{\theta\alpha_0}{2x(\alpha_0+x^\theta)} (N-x)$ ,  $\sqrt{\frac{x^\theta}{\alpha_0(\alpha_0+x^\theta)}} > 0$  and  $\frac{\theta\alpha_0}{2x(\alpha_0+x^\theta)} > 0$ ,  $1 - R_0 < 0$ , so an increase in  $N$  will shift down the *RHS*.

The intersection  $x$  will rise.

This outcome also applies when  $(1 - R_0) > 0$ .

### F.5.2 $x/N$ with respect to $N$

the *RHS* of (F.5) can be rearranged to be  $\sqrt{\frac{x^\theta}{\alpha_0(\alpha_0+x^\theta)}} \left( \frac{\theta\alpha_0 \left( \left[ \frac{N}{x} \right] - 1 \right)}{2(\alpha_0+x^\theta)} - 1 \right)$ .

■

## F.6 Proof of Proposition 3.3

Derivation of the best response function

The threshold of signal:  $\bar{d}_1 = \frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha R_0}{(x_i + X_{-i})^\theta}$

$$\max_{\{x_i\}} \left\{ \int_{-\infty}^{\frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha R_0}{(x_i + X_{-i})^\theta}} \left( \frac{\alpha R_0 + s \cdot (x_i + X_{-i})^\theta}{\alpha + (x_i + X_{-i})^\theta} x_i + k - x_i \right) f(s) ds \right. \\ \left. + \int_{\frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha R_0}{(x_i + X_{-i})^\theta}}^{\infty} \left( \frac{\alpha R_0 + s \cdot (x_i + X_{-i})^\theta}{\alpha + (x_i + X_{-i})^\theta} \left( x_i + \frac{N - x_i - X_{-i}}{M} \right) + \left( k - \left( x_i + \frac{N - x_i - X_{-i}}{M} \right) \right) \right) f(s) ds \right\} \quad (\text{F.8})$$

with  $f(s) = \frac{1}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}} \phi \left( \frac{s - R_0}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}} \right)$

Integration Part

$$\int_{-\infty}^{\frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha R_0}{(x_i + X_{-i})^\theta}} \left( \frac{\alpha R_0 + s \cdot (x_i + X_{-i})^\theta}{\alpha + (x_i + X_{-i})^\theta} x_i + k - x_i \right) f(s) ds$$

$$\begin{aligned}
&= \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} \left( \frac{\alpha R_0+s \cdot (x_i+X_{-i})^\theta}{\alpha+(x_i+X_{-i})^\theta} x_i - x_i + k \right) f(s) ds \\
&= \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} \left( \frac{x_i(x_i+X_{-i})^\theta}{\alpha+(x_i+X_{-i})^\theta} \right) sf(s) ds + \left( k - x_i + \frac{x_i \alpha R_0}{\alpha+(x_i+X_{-i})^\theta} \right) \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} f(s) ds
\end{aligned}$$

and

$$\begin{aligned}
&\int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} \left( \frac{\alpha R_0+s \cdot (x_i+X_{-i})^\theta}{\alpha+(x_i+X_{-i})^\theta} \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) + \left( k - \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) \right) \right) f(s) ds \\
&= \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} \left( \frac{(x_i+X_{-i})^\theta}{\alpha+(x_i+X_{-i})^\theta} \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) \right) sf(s) ds \\
&+ \left( k - \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) + \frac{\alpha R_0 \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right)}{\alpha+(x_i+X_{-i})^\theta} \right) \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} f(s) ds
\end{aligned}$$

**(1) The coefficient on  $\int f(s) ds$**

$$\begin{aligned}
&\int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} f(s) ds = \Phi \left( \frac{\frac{(1-R_0)[\alpha_0+(x_i+X_{-i})^\theta]}{(x_i+X_{-i})^\theta}}{\sqrt{(x_i+X_{-i})^{-\theta}+(\alpha_0)^{-1}}} \right) \\
&\left( k - x_i + \frac{x_i \alpha R_0}{\alpha+(x_i+X_{-i})^\theta} \right) \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} f(s) ds \\
&+ \left( k - \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) + \frac{\alpha R_0 \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right)}{\alpha+(x_i+X_{-i})^\theta} \right) \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} f(s) ds \\
&= \left( k - x_i + \frac{x_i \alpha R_0}{\alpha+(x_i+X_{-i})^\theta} \right) \Phi \left( \frac{\frac{(1-R_0)[\alpha_0+(x_i+X_{-i})^\theta]}{(x_i+X_{-i})^\theta}}{\sqrt{(x_i+X_{-i})^{-\theta}+(\alpha_0)^{-1}}} \right) \\
&+ \left( k - \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) + \frac{\alpha R_0 \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right)}{\alpha+(x_i+X_{-i})^\theta} \right) \left( 1 - \Phi \left( \frac{\frac{(1-R_0)[\alpha_0+(x_i+X_{-i})^\theta]}{(x_i+X_{-i})^\theta}}{\sqrt{(x_i+X_{-i})^{-\theta}+(\alpha_0)^{-1}}} \right) \right) \\
&= k - \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) + \frac{\alpha R_0 \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right)}{\alpha+(x_i+X_{-i})^\theta} + \Phi \left( \frac{\frac{(1-R_0)[\alpha_0+(x_i+X_{-i})^\theta]}{(x_i+X_{-i})^\theta}}{\sqrt{(x_i+X_{-i})^{-\theta}+(\alpha_0)^{-1}}} \right) \left( \frac{N-X}{M} - \frac{N-X}{M} \frac{\alpha R_0}{\alpha+(x_i+X_{-i})^\theta} \right)
\end{aligned}$$

**(2) The coefficient on  $\int sf(s) ds$**

$$\begin{aligned}
&\int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} \left( \frac{x_i(x_i+X_{-i})^\theta}{\alpha+(x_i+X_{-i})^\theta} \right) sf(s) ds + \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} \left( \frac{(x_i+X_{-i})^\theta}{\alpha+(x_i+X_{-i})^\theta} \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) \right) sf(s) ds \\
&= \frac{X^\theta}{\alpha+X^\theta} \left( x_i \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} sf(s) ds + \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) \int_{-\infty}^{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0}{(x_i+X_{-i})^\theta}} sf(s) ds \right) \\
&= \frac{X^\theta}{\alpha+X^\theta} \frac{\left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) R_0}{1-\lambda} + \frac{X^\theta}{\alpha+X^\theta} \frac{\left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) - x_i}{1-\lambda} \sqrt{(X^\theta)^{-1} + (\alpha_0)^{-1}} \phi \left( \frac{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0 - R_0}{(x_i+X_{-i})^\theta}}{\sqrt{(X^\theta)^{-1}+(\alpha_0)^{-1}}} \right) \\
&- \frac{X^\theta}{\alpha+X^\theta} \frac{\left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) - x_i}{1-\lambda} R_0 \Phi \left( \frac{\frac{\alpha_0+(x_i+X_{-i})^\theta-\alpha R_0 - R_0}{(x_i+X_{-i})^\theta}}{\sqrt{(X^\theta)^{-1}+(\alpha_0)^{-1}}} \right)
\end{aligned}$$

**(3) The final integration result**

$$= k - \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) + \frac{\alpha R_0 \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right)}{\alpha+(x_i+X_{-i})^\theta} + \frac{X^\theta}{\alpha+X^\theta} \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) R_0$$

$$\begin{aligned}
& + \Phi \left( \frac{(1-R_0)[\alpha_0 + (x_i + X_{-i})^\theta]}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}}} \right) \left( -\frac{X^\theta}{\alpha + X^\theta} \left( \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) - x_i \right) R_0 \right) \\
& + \frac{X^\theta}{\alpha + X^\theta} \left( \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) - x_i \right) \sqrt{(X^\theta)^{-1} + (\alpha_0)^{-1}} \phi \left( \frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha R_0 - R_0}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(X^\theta)^{-1} + (\alpha_0)^{-1}}}} \right)
\end{aligned}$$

and it can be simplified to the following optimization problem,

$$\max_x \left( \begin{aligned} & k + (R_0 - 1) \left( \frac{M-1}{M} x_i + \frac{N-X_{-i}}{M} \right) \\ & + \frac{N-x_i-X_{-i}}{M} (1-R_0) \Phi \left( \frac{(1-R_0)[\alpha_0 + (x_i + X_{-i})^\theta]}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}}} \right) \\ & + \frac{N-x_i-X_{-i}}{M} \frac{(x_i + X_{-i})^{\frac{\theta}{2}}}{\alpha^{\frac{1}{2}} (\alpha + (x_i + X_{-i})^\theta)^{\frac{1}{2}}} \phi \left( \frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha R_0 - R_0}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(X^\theta)^{-1} + (\alpha_0)^{-1}}}} \right) \end{aligned} \right)$$

Taking First-order Conditions with respect to  $x$

$$\frac{d}{dx} \left\{ (1-R_0) \frac{((x_i + X_{-i})^\theta + \alpha_0)^{\frac{1}{2}}}{(x_i + X_{-i})^{\frac{\theta}{2}}} \sqrt{\alpha_0} \right\} = \frac{R-1}{2} \frac{\alpha^{\frac{3}{2}} \theta}{(x + X_{-i})^{\frac{1}{2}\theta + 1} \sqrt{\alpha + (x + X_{-i})^\theta}}$$

(i) Part 1 derivative

$$\begin{aligned}
& \text{derivative for } \frac{N-x_i-X_{-i}}{M} (1-R_0) \Phi \left( \frac{(1-R_0)[\alpha_0 + (x_i + X_{-i})^\theta]}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}}} \right) \\
& = \frac{R_0-1}{M} \Phi \left( \frac{(1-R_0)[\alpha_0 + (x_i + X_{-i})^\theta]}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}}} \right) - \frac{N-x_i-X_{-i}}{M} \frac{(R-1)^2}{2} \frac{\alpha^{\frac{3}{2}} \theta}{(x + X_{-i})^{\frac{1}{2}\theta + 1} \sqrt{\alpha + (x + X_{-i})^\theta}} \phi \left( \frac{(1-R_0)[\alpha_0 + (x_i + X_{-i})^\theta]}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}}} \right)
\end{aligned}$$

(ii) Part 2 derivative

$$\begin{aligned}
& \text{derivative for } \frac{N-x_i-X_{-i}}{M} \frac{(x_i + X_{-i})^{\frac{\theta}{2}}}{\alpha^{\frac{1}{2}} (\alpha + (x_i + X_{-i})^\theta)^{\frac{1}{2}}} \phi \left( \frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha R_0 - R_0}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(X^\theta)^{-1} + (\alpha_0)^{-1}}}} \right) \\
& = -\frac{(X_{-i} + x)^{\frac{1}{2}\theta - 1}}{2(\alpha + (X_{-i} + x)^\theta)^{\frac{3}{2}} M \alpha^{\frac{1}{2}}} \left( \begin{aligned} & 2X_{-i}(X_{-i} + x)^\theta + 2x(X_{-i} + x)^\theta \\ & + 2X_{-i}\alpha + 2x\alpha - N\theta\alpha \\ & + X_{-i}\theta\alpha + x\theta\alpha \end{aligned} \right) \phi \left( \frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha R_0 - R_0}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(X^\theta)^{-1} + (\alpha_0)^{-1}}}} \right) \\
& - \frac{1}{2M} \theta \frac{\alpha^{\frac{3}{2}}}{\sqrt{\alpha + (X_{-i} + x)^\theta}} \frac{(R-1)^2}{(X_{-i} + x)^{\frac{1}{2}\theta + 1}} (X_{-i} - N + x) \phi \left( (1-R_0) \sqrt{\alpha_0} \frac{((x_i + X_{-i})^\theta + \alpha_0)^{\frac{1}{2}}}{(x_i + X_{-i})^{\frac{\theta}{2}}} \right)
\end{aligned}$$

(iii) The final FOC result with respect to  $x$

$$\begin{aligned}
M-1 + \Phi \left( \frac{(1-R_0)[\alpha_0 + (x_i + X_{-i})^\theta]}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}}} \right) & = \frac{(x_i + X_{-i})^{\frac{1}{2}\theta - 1}}{2(R_0 - 1) (\alpha_0 + (x_i + X_{-i})^\theta)^{\frac{3}{2}} \alpha_0^{\frac{1}{2}}} \\
& \times \left( \begin{aligned} & 2(x_i + X_{-i})^{\theta+1} + 2\alpha_0(x_i + X_{-i}) \\ & - N\theta\alpha_0 + \theta\alpha_0(x_i + X_{-i}) \end{aligned} \right) \phi \left( \frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha R_0 - R_0}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}}} \right) \tag{F.9}
\end{aligned}$$

■

## F.7 Proof of Proposition 3.4

from the Best-response function of  $x$  derived in Proposition 3.2,

$$\begin{aligned} M - 1 + \Phi \left( \frac{(1-R_0)[\alpha_0 + (x_i + X_{-i})^\theta]}{\frac{(x_i + X_{-i})^\theta}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}}} \right) \\ = \frac{(x_i + X_{-i})^{\frac{1}{2}\theta - 1}}{2(R_0 - 1)(\alpha_0 + (x_i + X_{-i})^\theta)^{\frac{3}{2}} \alpha_0^{\frac{1}{2}}} \left( \begin{array}{c} 2(x_i + X_{-i})^{\theta + 1} + 2\alpha_0(x_i + X_{-i}) \\ -N\theta\alpha_0 + \theta\alpha_0(x_i + X_{-i}) \end{array} \right) \phi \left( \frac{\frac{\alpha_0 + (x_i + X_{-i})^\theta - \alpha_0 R_0}{(x_i + X_{-i})^\theta} - R_0}{\sqrt{(x_i + X_{-i})^{-\theta} + (\alpha_0)^{-1}}} \right) \end{aligned}$$

for the symmetric equilibrium  $x = x_i = \frac{X_{-i}}{M-1}$

then the final M-Investor symmetric equilibrium is,

$$\begin{aligned} (M - 1) + \Phi \left( \frac{(1-R_0)\sqrt{\alpha_0}((Mx)^\theta + \alpha_0)^{\frac{1}{2}}}{(Mx)^{\frac{\theta}{2}}} \right) \\ = \frac{1}{R_0 - 1} \frac{1}{2} \frac{1}{\alpha_0^{\frac{1}{2}}} \frac{(Mx)^{\frac{1}{2}\theta - 1}}{(\alpha_0 + (Mx)^\theta)^{\frac{3}{2}}} \left( 2Mx\alpha_0 + 2(Mx)^{\theta + 1} - N\theta\alpha_0 + \theta\alpha_0 Mx \right) \phi \left( \frac{(1-R_0)\sqrt{\alpha_0}((Mx)^\theta + \alpha_0)^{\frac{1}{2}}}{(Mx)^{\frac{\theta}{2}}} \right) \end{aligned}$$

This can be simplified to (F.10),

$$(M - 1) + \Phi \left( (1 - R_0) \sqrt{\frac{\alpha_0 (\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}} \right) = \phi \left( (1 - R_0) \sqrt{\frac{\alpha_0 (\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}} \right) \times \frac{1}{(1 - R_0) \sqrt{\frac{\alpha_0 (\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}}} \left[ \frac{\theta\alpha_0 (N - Mx)}{2Mx (\alpha_0 + (Mx)^\theta)} - 1 \right] \quad (\text{F.10})$$

### F.7.1 Uniqueness

denote  $G(x) = (1 - R_0) \sqrt{\frac{\alpha_0 (\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}}$

$\sqrt{\frac{\alpha_0 (\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}} = \sqrt{\frac{\alpha_0^2}{(Mx)^\theta} + \alpha_0}$  is always a decreasing function of  $x$ , when  $x \geq 1$

Equation (F.10) can be rearranged to be,

$$\underbrace{(M - 1) \left[ (1 - R_0) \sqrt{\frac{\alpha_0 (\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}} \right]}_{\text{LHS1}} + \underbrace{\left[ (1 - R_0) \sqrt{\frac{\alpha_0 (\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}} \right] \Phi \left( (1 - R_0) \sqrt{\frac{\alpha_0 (\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}} \right)}_{\text{LHS2}} = \underbrace{\left[ \frac{\theta\alpha_0 (N - Mx)}{2Mx (\alpha_0 + (Mx)^\theta)} - 1 \right]}_{\text{RHS}}$$

and further simplified to (F.11),

$$\underbrace{\frac{(M - 1)G}{\phi(G)}}_{\text{LHS1}} + \underbrace{\frac{G\Phi(G)}{\phi(G)}}_{\text{LHS2}} = \underbrace{\left[ \frac{\theta\alpha_0 (N - Mx)}{2Mx (\alpha_0 + (Mx)^\theta)} - 1 \right]}_{\text{RHS}} \quad (\text{F.11})$$

**Monotonicity of  $\frac{G}{\phi(G)}$**

$$\frac{d\left(\frac{G}{\phi(G)}\right)}{dx} = \frac{1}{\phi(G)} - \frac{G}{(\phi(G))^2} [-G\phi'(G)] = \frac{1}{\phi(G)} + \frac{G^2}{\phi(G)} = \frac{1 + G^2}{\phi(G)} > 0$$

$\frac{G}{\phi(G)}$  is always an increasing function of  $G$ , and does not depend on the sign of  $G$ .

**Monotonicity of  $\frac{G\Phi(G)}{\phi(G)}$**

As proved before, generally  $\frac{G \cdot \Phi(G)}{\phi(G)}$  is an increasing function of  $G$ .

### Monotonicity of RHS

**RHS** =  $\frac{\theta \alpha_0 (N - Mx)}{2Mx(\alpha_0 + (Mx)^\theta)}$  is always decreasing in  $x$  for  $x > 0$  and  $x < N$ .

**When**  $R_0 > 1, (1 - R_0) < 0$ ,

$G(x) = (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}}$  is an increasing function of  $x$ ;

so  $LHS = LHS1 + LHS2$  is increasing functions of  $x$ .

$RHS$  is decreasing function of  $x$ . There is a unique solution at their intersection.

**When**  $R_0 < 1, (1 - R_0) > 0$ ,

$G(x) = (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}}$  is an decreasing function of  $x$ ;

so  $LHS$  is a decreasing function of  $x$ , and  $RHS$  is also a decreasing function of  $x$ .

However, the slope of the  $RHS$  is larger than the  $LHS$  when  $x$  grows large enough. The  $LHS$  and  $RHS$  curve will cross once and only once, and (F.5) has a unique solution  $x$ .

■

## F.8 Proof of Proposition 3.5

The Threshold  $R_0$

$$\underbrace{(M-1) \left[ (1-R_0) \sqrt{\frac{\alpha_0(\alpha_0+(Mx)^\theta)}{(Mx)^\theta}} \right]}_{LHS1} + \underbrace{\left[ (1-R_0) \sqrt{\frac{\alpha_0(\alpha_0+(Mx)^\theta)}{(Mx)^\theta}} \right] \Phi \left( (1-R_0) \sqrt{\frac{\alpha_0(\alpha_0+(Mx)^\theta)}{(Mx)^\theta}} \right)}_{LHS2} = \underbrace{\left[ \frac{\theta \alpha_0 (N - Mx)}{2Mx (\alpha_0 + (Mx)^\theta)} - 1 \right]}_{RHS}$$

denote  $G(x) = (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}}$ , then we have,

$$\underbrace{\frac{(M-1)G}{\phi(G)}}_{LHS1} + \underbrace{\frac{G\Phi(G)}{\phi(G)}}_{LHS2} = \underbrace{\left[ \frac{\theta \alpha_0 (N - Mx)}{2Mx (\alpha_0 + (Mx)^\theta)} - 1 \right]}_{RHS}$$

**When**  $R_0 > 1, (1 - R_0) < 0$ ,

As have been proved before,  $LHS1, LHS2$  are increasing functions of  $x$ , and  $LHS = LHS1 + LHS2$  is also an increasing function of  $x$ .

But  $G(x) = (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}} < 0$ , and  $LHS1 < 0$ .

$LHS2$  is an increasing functions of  $x$ ; adding  $LHS1$  to it will shift the original  $LHS2$  curve downwards.

Compare to the social planner's solution  $x$ , the equilibrium  $\hat{x}$  will be larger, and there will be overinvestment.

**When**  $R_0 < 1, (1 - R_0) > 0$ ,

$G(x) = (1 - R_0) \sqrt{\frac{\alpha_0(\alpha_0 + (Mx)^\theta)}{(Mx)^\theta}} > 0$ , and it is an decreasing function of  $x$ ;



so *LHS* is a decreasing function of  $x$ , and *RHS* is also a decreasing function of  $x$ .

However, the slope of the *RHS* is larger than the *LHS* when  $x$  grows large enough. The *LHS* and *RHS* curve will cross once and only once, and (F.5) has a unique solution  $x$ . *RHS* curve will cross the *LHS* curve from above once and only once.

$LHS1 > 0$ . So adding *LHS1* to *LHS2* will shift the original *LHS2* curve up.

This will make the interaction point  $\hat{x}$  between *LHS* and *RHS* smaller. That is, there will be underinvestment relative to the social optimal level.

■

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