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NON-FUNGIBLE CASH IN THE STOCK MARKET

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ABSTRACT

Investors view cash in their savings accounts differently from cash recycling in their stock brokerage accounts. I propose a novel "temperature" framework for financial resources, in which the former is labeled "cold cash" and the latter "hot cash." I find individual investors tend to buy stocks more cautiously when using cold cash than when using hot cash. Exploiting the quasi-natural experiment of the 2016 Chinese IPO lottery reform, I show the effect of cash temperature on investors' cautiousness in stock selection is causal. To explore the mechanism, I propose a portfolio choice model featuring preferences with temperature-dependent sensitivity to future gains and losses. The model generates the empirical patterns documented in this paper and provides a cash-temperature interpretation for understanding other puzzles in the literature.

CHAPTER 1

Introduction

Fungibility of cash, the feature that any one dollar should always be perceived equal to and exchanged freely with another dollar, is typically taken for granted. As a result, most standard economic models assume the total amount of wealth, rather than its composition, is what matters in the decision-making for financial investment. However, this assumption is untested. Do equity investors actually care about the sources of cash in their brokerage account? If so, how does non-fungibility of cash affect their stock selection?

In this paper, I answer these questions with a novel dataset that contains daily holding records matched to daily brokerage cash transfer records of 46,016 individual investors in the Chinese stock market. My key finding is that investors are more cautious with cash recently transferred into their brokerage account than with cash routinely recycled within their brokerage account. In the following discussion, I label the former "cold cash" and the latter "hot cash."

In the first half of the empirical analysis, I propose two algorithms for constructing the temperature of brokerage cash. The first algorithm assumes cold and hot cash are spent in a mixed and proportional way, whereas the second assumes a pecking order for both cold and hot cash, which means investors keep track of cold and hot cash in separate mental accounts and tap the account that aligns more closely with the desired use of cash.[1] Two algorithms only differ in how they book the usage of brokerage cash, but they share the same key feature that cash temperature increases with more hot cash injected or less cold

[1]For example, when an investor wants to buy a risky asset (e.g., stock), she first turns to hot cash and only uses cold cash if hot cash is depleted. For another example, if the investor plans to transfer some brokerage cash to a savings account, she won't use hot cash until she runs out of cold cash.

cash left. The results under both algorithms show stocks purchased with colder cash are selected more cautiously in the sense that those stocks typically have a lower risk, lower price, better future performance, less attention-grabbing quality, higher probability of being an index component stock, and longer holding horizon.[2]

To rule out alternative factors, in the second half, I exploit the quasi-natural experiment of the Chinese initial public offering (IPO) lottery reform on January 1, 2016. Investors in the Chinese stock market commonly know IPO lotteries are attractive and safe (Chen et al., 2015; Qian et al., 2022; He and Wei, 2022), especially during my identification sample period between June 17, 2014, and December 31, 2016, when IPO stocks were severely underpriced due to the strictly enforced regulatory cap of 23 on the IPO price-to-earnings (PE) ratio. The degree of underpricing is reflected in the fact that returns on the first trading day of all IPO stocks in this period hit the 44% upper limit.[3] As a result, participating in a Chinese IPO lottery in this period was the same as trying to get free money. The variation for identification is given by the change in lottery participation rules. Under the old regime (before January 1, 2016), eligible investors had to submit a cash deposit of the full applied amount to be frozen and safely stored in an IPO cash pool for two trading days, after which the lottery result would be announced and the excess cash would be refunded. Based on the definition of cold cash in the temperature framework introduced in Section 2, the refunded IPO cash was cold cash. By contrast, IPO lotteries under the new regime (after January 1, 2016) do not require cash deposits and refunds never occur, so the brokerage cash of eligible participants does not go through the cooling treatment. Such a policy change allows me to identify the cash-temperature effect using the Difference-in-Differences (DiD) design. The significant and robust results confirm that the effect is causal.

After establishing the cash-temperature effect, I explore a potential mechanism and its implications with a portfolio choice model, in which the sensitivity of an investor's utility

[2]Frydman et al. (2018) find stocks purchased on reinvestment days are more risky. My results in Section 3 to Section 4 are robust to controlling for the reinvestment effect.

[3]Maximum (minimum) daily returns of every stock listed in the Shanghai and Shenzhen stock exchanges are limited to 44% (-44%) on the first trading day after IPO and 10% (-10%) on any day after that.

to future gains and losses is negatively related to her cash temperature. Intuitively, this mechanism means the prospect of losses (gains) is less painful (exciting) to an investor when she uses cash with a higher temperature. In both myopic and dynamic settings, the model successfully generates the patterns whereby a higher cash temperature leads to a higher price and a lower return on the risky asset and a greater amount of risk taken by individual investors. These predictions are consistent with the empirical results and indicate the temperature-sensitivity mechanism can explain the cash-temperature effect. The dynamic setting also predicts two additional patterns, namely temperature smoothing and long-run convergence, for investors who endogenize the effect of risky investment today on cash temperature tomorrow.

This paper makes three contributions. First, it documents the non-fungibility of cash in the Chinese stock market, which is novel to the literature. It also encourages economists to reconsider the three elements in a traditional optimization problem: beliefs (through expectation under uncertainty), preferences (through utility function), and the amount of cash (through budget constraint). The findings in this paper raise the need to add the fourth element: the source of cash.

Second, my findings provide direct evidence that fills a gap in the mental accounting literature, which studies how individuals and households "organize, evaluate, and keep track of financial activities" (Thaler, 1999). Since it was introduced about four decades ago (Thaler, 1985, 1990), mental accounting of money has been applied to a variety of scenarios in debt and expenditures.[4] Mental accounting also plays a role in financial investment scenarios.[5] One strand of literature uses mental accounting as a key element to explain the disposition effect (Shefrin and Statman, 1985; Barberis and Xiong, 2009, 2012), a well-known fact that

[4]For example, mental accounting is used for income-source categorization and differential marginal propensity to consume (Shefrin and Thaler, 1988; Di Maggio et al., 2020), mental money laundering (Imas et al., 2021), simultaneous holding of high-interest debt and low-interest assets (Gross and Souleles, 2002), and debt aversion (Prelec and Loewenstein, 1998).

[5]In the prospect theory framework, mental accounting is sometimes used interchangeably with narrow framing (Barberis and Huang, 2001).

investors sell the winning stocks too early and hold the losing stocks too long.[6] Another strand discusses the risk attitude.[7] Frydman et al. (2018) find evidence that investors may roll over a stock mental account by buying a successor stock immediately after selling one, and the gain (loss) status of the sold stock leads to buying a safer (riskier) successor to preserve the gain (reverse the loss) on the rolled-over stock mental account. Imas (2016) uses the realization effect associated with the stock mental account closing to reconcile the two seemingly contradictory facts that investors take on higher risk after a paper loss and lower risk after a realized loss. However, unlike the debt and expenditures literature, these studies about financial investments only discuss mental accounting of stocks, not mental accounting of money, which is partly due to the lack of high-quality data on the account cash flow records matched to stock holding records.[8] Using a novel dataset, I fill the gap by documenting mental accounting of money in stock investments. The specific patterns identified in this paper are beyond mental accounting predictions.

Third, in addition to generating patterns consistent with my empirical results, the model also predicts the co-existence of no participation and overbuying, overreaction to shocks, and fluctuation in price not driven by change in fundamentals. For limited stock market participation, the existence of friction (Mankiw and Zeldes, 1991; Campbell, 2006), of which the two major types are real costs associated with brokerage account maintenance and the cognitive costs associated with financial planning (Choukhmane and de Silva, 2021), is a crucial assumption.[9] With the widespread FinTech advances, the smartphone trading appli-

[6]The economic intuition of how mental accounting explains the disposition effect is the following. Investors open a new mental account when a stock is purchased and close the account when the stock is sold. The reluctance to close an account at a loss and the desire to close an account at a gain leads to the disposition effect.

[7]Some studies also use mental accounting to explain other anomalies in the market. For example, Barberis and Huang (2001) find that a model with investors' mental accounts maintained for individual stocks explains high mean and excess volatility better than the one with mental accounts opened for the portfolio.

[8]Apart from cash flow records, the individual holdings data have become increasingly available to researchers in the past two decades. For example, Giglio et al. (2021) use holdings data matched to a series of surveys on beliefs of US-based wealthy investors between 2017 and 2020. Grinblatt and Keloharju (2000) and Grinblatt et al. (2012) use daily holdings and trades of all Finnish household investors between 1995 and 2002. Jones et al. (2021) use the daily holdings of 53 million Chinese investors between 2016 and 2019.

[9]In addition to the friction, risk aversion can be another factor for the limited participation in the stock market. Choukhmane and de Silva (2021) propose an identification strategy that decomposes the effects of

cations significantly reduce the real costs, and the complimentary and hassle-free artificial intelligence advising services also reduce the cognitive costs. So, the extent to which friction deters stock investments nowadays is not clear. Moreover, overtrading, the opposite of no participation, is also documented for individual investors (Barber and Odean, 2000), which seems puzzling. In my model, the disparity in cash temperature among investors, rather than the traditional friction or overconfidence channels, can simultaneously generate both no participation of some investors and overbuying of others. Similarly, the temperature-sensitivity mechanism in my model also leads to overreaction (De Bondt and Thaler, 1985) and generates price movement even if the fundamentals of the risky asset hypothetically remain unchanged (Shiller, 1992), which provides an alternative and unifying framework for understanding these stylized facts.

The findings based on individual investors are important. The increasing availability of smartphones and trading applications makes the stock market more accessible to the vast majority of people, so their behaviors become more impactful in both emerging markets and developed markets. In particular, the Chinese stock market is an ideal setting for my research, where retail investors contribute to 85% of the overall daily trading volume (Jones et al., 2021). But my empirical findings and theoretical framework can be generalized to other markets as well.

The findings have policy implications. Recent discussion about direct-cash-transfer programs during the pandemic (Greenwood et al., 2022; Karger and Rajan, 2020; Cox et al., 2020; Chetty et al., 2020) mostly focus on household consumption, but household stock investment may also be important for the economy. Given the cash-temperature effect identified in this paper, a transfer of stock shares to investors (e.g., shares dividends) might work better than an equal amount of transfer of cash (e.g., cash dividends) in encouraging investors to buy stocks more aggressively, since transferred shares end up becoming hot cash when the shares are sold while transferred cash just become cold cash directly. In ad-

friction and risk aversion.

dition, the results also have implications for financial literacy interventions. Although the cash-temperature effect may not directly change the objective aspects of investors' financial sophistication, such as knowledge and skills, it may still have an influence on their realized investment performance by affecting the subjective factors, such as cautiousness. The implication is that the means of receiving transaction revenue from each stock selling, either through a brokerage account or through a deposit directly to a bank account, may affect the well-being of investors.

The rest of the paper is organized as follows. Section 2 formalizes the temperature framework and describes the sample. Section 3 proposes two algorithms for constructing cash temperature, shows the estimates of the cash-temperature effect, and discusses alternative factors that inspire the identification design. Section 4 introduces the institutional details about IPO reforms in China, develops a testable hypothesis based on the temperature framework, and presents the identified causal effect. Section 5 studies a portfolio choice model in both myopic and dynamic settings to demonstrate a potential mechanism and equilibrium asset pricing implications of the cash-temperature effect. Finally, Section 6 concludes.

CHAPTER 2

Conceptual Framework and Data

The temperature of cash, measuring an investor's attitude toward cash along some dimension, is a novel concept that has the potential to unify the documented facts about cash non-fungibility. In this section, I formalize the idea of cash temperature with three desired properties and describe the dataset used to construct and test the measure.

2.1 The Temperature Framework

Many studies agree the nature of a cash source matters (Raghubir and Srivastava, 2008; Imas et al., 2021; Meyer and Pagel, 2022). To be concrete, I consider an agent with some financial resources to allocate. Suppose there are $|\mathcal{K}|$ containers, which can be interpreted as a set of purposeful destinations (e.g., savings account or stock) for financial resources. Each container $k \in \mathcal{K}$ is associated with temperature θ_k^d that measures the agent's attitude along dimension $d \in \mathcal{D}$ toward cash coming out of container k.

To define the measure in a consistent and meaningful way, I discuss three desired properties below.

Property 1. (Invariability of container temperature) *The temperature of container $k \in \mathcal{D}$ along dimension $d \in \mathcal{D}$ is θ_k^d, which is invariable with cash inflows (outflows) from (to) any other container.*

Property 1 ensures agents cannot arbitrarily manipulate the temperature of any container, which is equivalent to assuming the nature of each container is exogenous.

Property 2. (Changeability of cash temperature) *If cash is purposefully allocated to container k, $\forall k \in \mathcal{K}$, then, regardless of its original temperature, cash coming out of container* *k will be assigned temperature* θ_k^d*,* $\forall d \in \mathcal{D}$*.*

Property 2 has two key elements. The first one is purposefulness. To have its temperature changed, cash must be intentionally directed to a container with a purpose, such as investing it in a common stock to increase its value with some risks, or transferring it to a savings account to safely store its value. By contrast, cash temporarily sitting in one's brokerage account, for instance, does not have a salient purpose, so its temperature maintains the level assigned by the previous container.

The second element is Markovian, which means the temperature of cash is only determined by its most recent container, not by those further in the past.

Property 3. (Temperature decay) *Once assigned, the cash temperature decays at a rate of* $\beta \in [0, 1]$*.*

Property 3 requires that temperature 0 is set as the baseline. The decay simply means the differences in cash gradually die out if they are not reinforced, which is closely related to the recency bias and noisy recall (Azeredo da Silveira and Woodford, 2019; Nagel and Xu, 2022). The decay also makes the temperature definition more realistic, because agents are not supposed to clearly distinguish cash from different sources several years after they receive it.

The three properties provide guidelines for constructing the temperature measure. Note that the framework in its most general form does not make any assumptions on what dimension d is. My analysis focuses on a specific and reasonable dimension, which is the stableness of container, but the definition can be extended to other dimensions in future studies.

2.2 Data

The main dataset used in this paper contains the daily holdings of 46,016 individual investors between January 1, 2006, and December 31, 2016, in the Chinese stock market. The novelty of this dataset is that daily cash flows of brokerage accounts are observed and matched to holding records. The representativeness of my sample is shown in Table A.1a, which compares the fraction of five groups of investors based on average portfolio values between my sample and the population.

For any investor i on day t, the dataset has the brokerage cash starting balance ($bal\_start_{i,t}$), net change due to all tradings ($total\_trd_{i,t}$), net change due to bank-brokerage inter-day transfers ($inter\_trsf_{i,t}$) and intra-day transfers ($intra\_trsf_{i,t}$), and ending balance ($bal\_end_{i,t}$), which satisfy

$$\begin{cases} bal\_start_{i,t} = bal\_end_{i,t-1} + inter\_trsf_{i,t}, \\ bal\_end_{i,t} = bal\_start_{i,t} + total\_trd_{i,t} + intra\_trsf_{i,t}. \end{cases} \tag{2.1}$$

Because $inter\_trsf_{i,t}$ happens between the last transaction on day $t-1$ and the first transaction on day t, its influence is only reflected starting from day t. So, I combine the inter- and intra-day transfers to define the total transfer-in on day t by $transfer\_in_{i,t} = \max\{inter\_trsf_{i,t}, 0\} + \max\{intra\_trsf_{i,t}, 0\}$.

In addition, the trading net change $total\_trd_{i,t}$ can be further decomposed into the IPO deposit and refund, the net change in other asset tradings,[1] i.e., $other\_trd_{i,t}$, and net change in stock tradings, i.e., $stock\_trd_{i,t} = stock\_sell_{i,t} - stock\_buy_{i,t}$, where $stock\_sell_{i,t}$ and $stock\_buy_{i,t}$ are holding-implied CNY value of stocks sold and purchased, respectively, by investor i on day t.

Empirically, how important is brokerage cash? The summary statistics in Table A.1b show that investors have 52,820 CNY (approximately 800 USD) in cash sitting in their account every day on average, which is about 18% of their stock portfolio value.

Another observation is that the average brokerage cash reserve is much more than the average immediate flows, i.e., daily stock buys and sells, which makes brokerage cash a quantitatively important factor to explore.

In addition to the proprietary data of holdings and cash transfers, I also use the informa-

[1] In China, investors can use their stock brokerage account to trade exchange-traded funds (ETF), treasury repurchase agreement (repo), listed open-ended funds (LOF), and other assets.

tion of 3,083 public firms listed in the A-share market of Shanghai and Shenzhen exchanges from the WIND database.

The Chinese stock market is a good place to test the predictions of my temperature framework for three reasons. First, the taxation incentives are ruled out, since no capital-gains tax on equity investment exists in China. In addition, the efficient clearing system in China ensures cash balances are accurately reflected in my sample. Finally, the Chinese IPO reform in this sample provides me with a quasi-natural experiment for identification.

CHAPTER 3

Algorithm-Based Empirical Evidence

In this section, I put forth two algorithms for computing the temperature of brokerage cash, present suggestive evidence for the cash-temperature effect, and discuss alternative channels that need to be addressed in Section 4.

3.1 Two Algorithms for Temperature Measurement

To define temperature, I focus on a specific dimension, which is the degree of container stableness. A stable container, assigned low temperature, features value-preserving and predictability, whereas an unstable container, assigned high temperature, features value-fluctuating and unpredictability. This is an obvious distinction one could make about cash sources, with which a testable prediction is that cash coming out of a stable container is invested differently from cash coming out of an unstable container.

Before introducing the temperature measurement to test this prediction, I first clarify the total financial resources available to investor i on day t, denoted by $total\_cash_{i,t}$, which consists of six components, including the IPO refund, transfer-in, stock selling, dividend income, other asset selling and the balance carried over from the previous day, i.e.,

$$total\_cash_{i,t} = refund_{i,t} + transfer\_in_{i,t} + stock\_sell_{i,t} + div_{i,t} + other\_sell_{i,t} + bal\_end_{i,t-1}. \tag{3.1}$$

Note that every term on the right-hand side of (3.1) is non-negative by definition to separate resources from usage. For example, if a transfer is positive, it provides additional cash, so it becomes a part of the total resources on that day; however, if the transfer is negative, it is counted as usage, rather than resource, so it does not enter the calculation for the total cash available in (3.1). The same idea applies to stock tradings and other asset tradings too.

While I do not observe the sequence of actions on intra-day level, it is reasonable to assume that the resources gathering precedes the usage of them, so the $total\_cash_{i,t}$ defined in equation (3.1) is still a good measure of the actual or, at least, the anticipated total resources available to an investor when she makes stock purchasing decisions on that day.

The next task is to assign a temperature value to each of the six items in equation (3.1). I consider a binary temperature system, in which a container is either cold with temperature 0 or hot with temperature 1. There are two reasons for adopting the binary system as opposed to a continuous one. First, the finer the measure is, the more it requires accuracy of cash composition. Since I do not observe the exact ordering of actions on intra-day level, it is impossible to perfectly identify the composition of cash at every moment within a day. Second, people may be heterogeneous when it comes to ranking containers along the stableness dimension. It is possible that one investor thinks a stock is a different container from a mutual fund, while another investor may group those two under the same category of risky investment. A more coarse classification, i.e., the binary system, is less subject to both the intra-day timing and the heterogeneity issues.

In this binary temperature system, IPO cash pools and bank savings accounts are classified as cold containers,[1] and common stocks and other tradable risky assets are labeled hot containers. While the temperature of each container remains constant (Property 1), the temperature of cash leaving a container will be changed (Property 2). In particular, $refund_{i,t}$ from IPO cash pools and $transfer\_in_{i,t}$ from bank savings accounts are cold cash with temperature 0; $stock\_sell_{i,t}$ and $div_{i,t}$ from common stocks and $other\_sell_{i,t}$ from other

[1]The features of IPO cash pools will be introduced in Section 4.

tradable risky assets are hot cash with temperature 1, i.e.,

$$cash_{i,t}^{\text{cold}} = refund_{i,t} + transfer\_in_{i,t},$$
$$cash_{i,t}^{\text{hot}} = stock\_sell_{i,t} + div_{i,t} + other\_sell_{i,t}.$$

The temperature of $bal\_end_{i,t-1}$, however, is unclear as it depends on both the temperature of cash carried over from yesterday and the decay rate (Property 3).[2]

Figure A.1 provides a time series for the decomposition based on equation (3.1). Clearly, $bal\_end_{i,t-1}$ by far has the largest weight at all times, so the main challenge is to pin down its temperature in a coherent way with the help of some reasonable rules.

I propose two algorithms to do this. The first algorithm (A1) assumes that investors mix the cold and hot cash, and spend them proportionally every time. The second algorithm (A2) assumes that investors maintain separate mental accounts for cold and hot cash. When using the balance, they first tap the mental account whose temperature is closer to that of the purpose of spending and only use the next account in the line if the previous one is depleted. Then, the remaining cash is mixed and carried over to the next day as a warm balance. To see how A2 works, a stock purchase spending, for instance, is deducted from hot, warm and cold balances sequentially and only goes to the next source if the previous one is used up on that day. Similarly, a cash transfer from brokerage account to savings account is deducted from cold, warm and hot balances sequentially. Under both A1 and A2, the decay happens overnight, which means the temperature of $bal\_end_{i,t-1}$ on day t is its end-of-day temperature on day $t-1$ multiplied with the decay rate $\beta \in [0, 1]$.

Although A1 and A2 make different assumptions on how spendings are booked, temperature $\theta_{i,t}$ in either case is defined as a weighted average temperature of all cash available to

[2]By Property 2, sitting in the brokerage account without a salient purpose should not change the temperature of cash.

investor i on day t, i.e.,

$$\theta_{i,t} = \frac{1 \times cash^{\text{hot}}_{i,t} + 0 \times cash^{\text{cold}}_{i,t} + \beta \times \theta_{i,t-1} \times bal\_end_{i,t-1}}{total\_cash_{i,t}}.$$

In this way, $\theta_{i,t}$ under A1 is precisely the temperature of cash at the moment of spending; $\theta_{i,t}$ under A2 is positively related to the chances that a spending is booked in the mental account for hot cash.

To understand the differences between the two algorithms, consider a simple numerical example with decay rate $\beta = 3/4$. Suppose investor i opens a brokerage account on day t. On the same day, she transfers \$100 into the account and spends \$20 on stock X, which lead to an ending balance of \$80 on day t that is 100% cold cash, i.e., $\theta_{i,t} = 0$, under both A1 and A2. On day $t + 1$, suppose the price of stock X is doubled, then she sells X and receives \$40 as hot cash. Now, she has \$80 of cold cash and \$40 of hot cash, which means the temperature of her available cash is 1/3 for the subsequent purchases under both A1 and A2. Assume that she invests \$60 on stock Y before the market closes on day $t + 1$. Under A1, the spending of \$60 should be deducted proportionally from both types, so the cold and hot cash balances are adjusted to \$40 and \$20, respectively. Under A2, however, the spending is first deducted from hot cash, which means the two balances are adjusted to \$60 and \$0, respectively. Similarly, suppose the investor also transfers \$30 from brokerage account to savings account after buying stock Y, the two balances become \$20 and \$10 under A1, and \$30 and \$0 under A2. Therefore, the end-of-day temperature is 1/3 under A1 and 0 under A2. On day $t + 2$, after decaying at $\beta = 3/4$ overnight, $\theta_{i,t+2}$ is 1/4 under A1 and 0 under A2.

It is clear from this example that, even though container temperature is binary, cash temperature is a continuous variable on $[0, 1]$ due to averaging and decaying.

To illustrate what the decay does, I plot the half-life for each level of β in Figure A.2.[3]

[3] Half-life is the number of days required to reduce the current temperature to its one-half with the decay parameter β.

The convexity of half-life makes it very sensitive when β is large. Note that the half-life corresponding to $\beta = 0.98$ is about 34 days, which is close to the median stock holding horizon reported in Table A.1b. So, the conjecture is that $\beta = 0.98$ can generate richer cross-sectional variations in $\theta_{i,t}$.

The aggregate trend of $\theta_{i,t}$ can be seen from its monthly mean in Figure A.3a, and the dispersion is given by its median with 25% and 75% percentiles in Figure A.3b. The market index, i.e., SSE Composite Index, is included in all panels for comparison. Both figures show that A1 and A2 are very similar on aggregate, except that A1 assigns a slightly higher temperature than A2 does.[4] In addition, when there is non-zero decay, the mean, or similarly the median, of $\theta_{i,t}$ is pro-cyclical, i.e., positively co-moving with the market index. There are two directions of causality that can give rise to pro-cyclicality. The first starts with improving market conditions, which encourage investors to trade more actively. As a result, they frequently receive hot cash back from selling stocks so that their cash temperature is maintained at a high level. The second, however, implies the opposite. When investors have higher cash temperature, they may be willing to buy stocks at higher prices, which supports a rising market index. In fact, either way can generate pro-cyclicality and both explanations might be true. In my framework, the first direction of causality is mechanically true by construction of the measure. In the rest of this paper, I provide evidence and propose a model for the second direction of causality.

In addition, Figure A.3b shows that no decay ($\beta = 1$) and fast decay ($\beta = 0.9$) both lead to polarized distribution of $\theta_{i,t}$, but moderate decay ($\beta = 0.98$) generates a richer cross-sectional variation, which confirms the conjecture made by comparing half-life for decay and median stock holding horizon. Although there is no reason to reject the polarized distribution, I use $\beta = 0.98$ for the main analysis in the rest of this section to exploit its richer variation. For robustness to different decay levels, see Table B.2 in Appendix A.

[4] By construction, A1 tilts toward a larger fraction of hot cash than A2 after a stock purchasing, and the relation is reversed after an outward cash transfer. Since the major activity of the brokerage accounts is buying stocks rather than cashing out, it is reasonable that A1 assigns a higher temperature to the remaining cash than A2 on average.

Of course, there are more than two algorithms that one can propose. How should we think about those algorithms? In fact, $\theta_{i,t}$ under A1 and A2 in my analysis captures similar variations in cash composition and has similar level and trend on aggregate (Figure A.3). Instead of taking a stance on which algorithm matches more closely with investors' real mental process, I present the results of both algorithms and find that the estimates are robust to different choices of algorithms, which suggests the minor differences in measure construction are dominated by the major effect of temperature disparity.

3.2 Suggestive Evidence

In this section, I provide evidence to support the prediction that cold and hot cash are used differently. I focus on the initial purchase record of every investor-stock pair, which on average accounts for over 80% of the maximum holding in the lifecycle of an investor-stock pair (He and Hu, 2022).

To graphically demonstrate the cash-temperature effect, I first compute $\theta_{i,t}$ based on A1 or A2 for every stock j purchased by investor i on day t; then, I group all the initial purchase records into ten bins of $\theta_{i,t}$ evenly sliced on $[0, 1]$; finally, for purchase records in each bin, I compute the mean and 95% confidence interval for nine market-adjusted characteristics of the purchased stocks,[5] which are realized daily return volatility, market-to-book ratio, momentum, short-term performance, long-term performance, abnormal trading volume (Barber and Odean, 2008), last day return, indicator for index component,[6] and number of days the stock stays in an investor's portfolio.

For risk measure, I use realized daily return volatility in the past month instead of market beta, since investors hold under-diversified portfolios with only 2.50 stocks on median (Table A.1b). For momentum and short-term performance, I use realized return in the past month and the next month, respectively. The reason for using a relatively short period of one

[5]The value of each market-adjusted characteristic is obtained by subtracting the market average from the raw value of every stock on every day.

[6]I consider three major indices in the Chinese stock market, including SSE Composite Index, Shenzhen Component Index and CSI 300 Index. The indicator is equal to 1 if the stock belongs to any of the three indices, and 0 otherwise.

month is that it matches with the median holding horizon, which makes the information more relevant for the decision-making of a typical investor in the sample.[7] For long-term performance, I use future return in the next year. To rule out potential effects of outliers, the raw measures are all winsorized at 1% and 99% percentiles.[8]

Figure A.4 shows that the stocks purchased with colder cash display a lower risk (smaller daily return volatility), lower price (lower market-to-book ratio and past return), better future performance (higher future return in both short-term and long-term), less attention-grabbing quality (lower abnormal trading volume and last day return), higher probability of being an index component stock, and longer holding horizon.[9]

The effects on all the nine characteristics point to the same interpretation that cold cash is spent more cautiously than hot cash. Intuitively, it means investors care more about cash from a stable (cold) container than that from an unstable (hot) one. This observation reveals the effect of cash temperature defined along the dimension of container stableness.

Although the graphical results in Figure A.4 control for the time fixed effects by using market-adjusted characteristics, they do not control for account fixed effects and other economically relevant factors. To test the effect more rigorously, consider regression

$$y_{i,j,t} = \alpha \cdot \theta_{i,t} + controls_{i,t} + \lambda_i + \lambda_t + \varepsilon_{i,j,t}, \tag{3.2}$$

where $y_{i,j,t}$ is one of the nine characteristics of stock j purchased by investor i on day t; the key regressor is temperature $\theta_{i,t}$ under A1 or A2; and account (λ_i) and day (λ_t) fixed effects are added to the regression.

One control variable $realized\_loss_{i,t}$, an indicator that equals to 1 if investor i realizes losses on day t, is added to control for the reinvestment effect (Frydman et al., 2018). Another control variable $cml\_paper\_loss_{i,t-1}$, an indicator that equals to 1 if investor i accumulates

[7] The results are robust to alternative horizons. See Table B.3 in Appendix A.

[8] The results are robust to percentile transformation of stock characteristics. See Table B.4 in Appendix A.

[9] The figure is drawn for A1. The patterns are robust to A2. See Figure B.1 in Appendix A.

net gains up to the previous day, is added to control for the house money effect (Thaler and Johnson, 1990) and prospect theory effect (Kahneman and Tversky, 1979). Other control variables include total cash available, i.e., $total\_cash_{i,t}$, and lagged total value of stock holdings, i.e., $total\_holding_{i,t-1}$, of investor i on day t to control for size.

I first run regression (3.2) for the risk measure under both algorithms. The left four columns in Table A.2 show results for raw values of the risk measure, and the right four columns show results for percentile-transformed values of the measure. The estimates of α are positive and significant across all specifications, which confirms the observation in Figure A.4. Quantitatively, when $\theta_{i,t}$ changes from 0 to 1, the effect is shifting the stock selection along the risk dimension by roughly 3 percentiles in the pool of all stocks.

For the other eight characteristics, the estimates of α are presented in Table A.3, which are all consistent with Figure A.4. Specifically, α is positive (negative) for characteristics negatively (positively) related to the degree of investor cautiousness.

Table A.2 and Table A.3 combined show that cash-temperature effect exists even after controlling for account fixed effects and other relevant variables.

3.3 Discussion

While the graphic evidence in Figure A.4 and regression results in Table A.2 and Table A.3 jointly provide evidence supporting the cash-temperature effect, there are some alternative explanations of these results, which are discussed one by one below.

The first alternative explanation is reverse causality. When an investor wants to buy an exciting stock (riskier, better past performance, and more attention-grabbing, etc.) it is possible that the investor becomes less patient and wants to bet immediately, so she cannot wait for the next pay check day and instead sells a stock in her portfolio to raise cash right away. In that case, cash sources in a stock purchase could be driven by stock characteristics.

Another alternative story is that unobserved factors about investors, such as attention and budget constraint, might play a role. For example, when an investor has abundant attention or financial resources, she may transfer more cash into brokerage account and in the meanwhile become more cautious and spend longer time searching for stocks. So, the higher proportion of cold cash and the more cautiousness in stock selections could be simultaneously driven by these hidden factors.

The last issue is the accuracy of temperature algorithms. While A1 and A2 are two reasonable ways to keep track of the composition of brokerage cash balances, the claim that injected cash is always cold cash may not be accurate, since an investor may have multiple brokerage accounts, which means they might be actually transferring hot balances among those brokerage accounts with a bank account being the intermediate stop. Even if the injected cash is indeed all cold cash, one may still cast doubt on the extent to which the algorithms deviate from the real mental processes of investors.

These are valid arguments and need to be addressed. In particular, a convincing identification strategy must ensure that cash flows are not driven by the natures of target assets to rule out reverse causality. It should also control for or cancel out unobserved factors. Finally, it should be algorithm-free.

CHAPTER 4

Algorithm-Free Identification

In this section, I introduce institutional details about the IPO policy reform in the Chinese stock market, which creates an exogenous variation for the identification. Then, I justify the identifying assumptions and propose the testable hypothesis based on the temperature framework. In the end, I present and discuss the results.

4.1 Institutional Details

The Chinese financial markets have experienced several reforms in the past three decades, which provide variations for empirical testings. In particular, I focus on the Chinese IPO system reforms.[1] On the firm side, there are three major phases. In phase one (1992-2000), there was a quota allocation system that assigned the total face value allowed for going public to a limited number of firms. In phase two (2001-2018), the approval system was adopted, in which the underwriter was required to submit an IPO request on behalf of the firm. The Chinese Securities Regulatory Commission (CSRC) reviewed applications and approved a selected subset of them, which could take a couple of months or even years (Piotroski and Zhang, 2014; Cong and Howell, 2021; Lee et al., 2021). Within phase two, an important reform happened on January 12, 2014, when the CSRC imposed an upper bound of 23 on the PE ratio for IPO pricing.[2] See Figure A.5. In phase three (2019-now), the registration system, which resembles the IPO systems in developed markets, has been

[1] See Qian et al. (2022) for a review of Chinese IPO reforms and facts. See He and Wei (2022) for a review of recent literature on the broader Chinese financial system and economy.

[2] The only exception in the shaded period was 603026.SH, which was approved to have an IPO PE ratio of 32.25 to ensure that its IPO price was not lower than the book value per share.

progressively adopted to reduce the authority's intervention on IPO pricing and timing.

On the investor side, the most important reform took place on January 1, 2016. Under the old regime prior to the reform, eligible investors must submit a large amount of deposit to an IPO cash pool on application day to participate in an IPO lottery.[3] All submitted cash deposit would be frozen for two days before the result was announced and the excess deposit was refunded (Li et al., 2021). Under the new regime after the reform, however, investors can participate in an IPO lottery without submitting any cash deposit before the lottery result announcement, so there is no refund of excess deposit. See Figure A.6.

Under both old and new regimes, an investor's eligibility for IPO lottery participation is determined by her holdings in the same way. If investor i wants to enter an IPO lottery on day t, the system will calculate the average value of her end-of-day stock holdings in both Shanghai (SH) and Shenzhen (SZ) exchanges between day $t-22$ and $t-2$, i.e., $\overline{v}_{i,t} = \overline{v}_{i,t}^{\mathrm{SH}} + \overline{v}_{i,t}^{\mathrm{SZ}}$. She can only participate if $\overline{v}_{i,t}$ is not less than 10,000 CNY. If the IPO happens in SH exchange, each ticket for the lottery is $1,000$ shares. Every $10,000$ CNY value of $\overline{v}_{i,t}^{\mathrm{SH}}$ gives the investor eligibility for one ticket, so the total number of shares she can apply for is 1,000 multiplied with the integer part of $\overline{v}_{i,t}^{\mathrm{SH}}/10,000$. Similarly, if the IPO happens in SZ exchange, each ticket is 500 shares. Every $5,000$ CNY value of $\overline{v}_{i,t}^{\mathrm{SZ}}$ gives the investor eligibility for one ticket, so the total number of shares is 500 multiplied with the integer part of $\overline{v}_{i,t}^{\mathrm{SZ}}/5,000$. Lottery winning records are generated on the ticket level. On a day with multiple IPO stocks, the investor's eligibility calculation is additive across all IPO stocks.

To be more concrete, let's look at a numerical example. Suppose investor i holds $\overline{v}_{i,t}^{\mathrm{SH}} = 29,000$ CNY of stocks in SH exchange and $\overline{v}_{i,t}^{\mathrm{SZ}} = 8,500$ CNY of stocks in SZ exchange on average between day $t-22$ and $t-2$. On day t, there are two IPOs in SH exchange (firm A and B) and one IPO in SZ exchange (firm C) open for application with IPO prices $p_{t,A}^{\mathrm{SH}} = 2$, $p_{t,B}^{\mathrm{SH}} = 1$, and $p_{t,C}^{\mathrm{SZ}} = 3$, respectively. The investor is eligible to participate on day t since $\overline{v}_{i,t} = 37,500$

[3] As indicated by Table A.4, IPO events were very popular, and the total submitted cash deposits were much more than what the IPO firm actually needed. As a result, a lottery needed to be run for each IPO event to determine who could receive the shares.

is greater than the threshold of 10,000. She can apply for $1,000 \times \text{floor}\{29,000/10,000\} = 2,000$ shares (two tickets) of each SH IPO firm and $500 \times \text{floor}\{8,500/5,000\} = 500$ shares (one ticket) of the SZ IPO firm. So, the maximum total value that she can apply for is $p^{\text{SH}}_{t,A} \times 2,000 + p^{\text{SH}}_{t,B} \times 2,000 + p^{\text{SZ}}_{t,C} \times 500 = 7,500$ CNY. She can freely choose the firm and number of shares to apply for. Suppose she wants to apply with the maximum value of 7,500 CNY, and wins one ticket, i.e., 1,000 shares, of firm B only. Under the old regime, she needed to submit a deposit of 7,500 CNY on day t to have it frozen until day $t+2$, when she would receive a refund of 6,500 CNY. Under the new regime, she only files the application and pays nothing on day t. On day $t+2$, she simply submits 1,000 CNY for the firm B shares she has won.

My identification design uses the sample period between June 17, 2014, and December 31, 2016, (shadowed period in Figure A.5) for two reasons. First, the reform on IPO participation rules happened in this period, which provides an exogenous variation. Second, the IPO PE ratio upper bound was strictly enforced in this period, leading to a systematic underpricing of IPO firms. Therefore, investors had simple and strong incentives to participate in IPO lotteries, which support the two identifying assumptions discussed in Section 4.2.

To illustrate the degree of underpricing, Figure A.7 plots a set of histograms for returns of IPO stocks on the first nine trading days after they become tradable in the secondary market. The noteworthy fact is that 100% of these IPO stocks hit the daily return upper limits of 44% and 10% on the first and second trading days, and that almost all stocks kept hitting the upper limit of 10% in the next three days.[4] It was almost guaranteed that the shares won through an IPO lottery would double its value within just a few days. Put in another way, participating in IPO lotteries was trying to get free money.

Table A.4 provides more information on the popularity of IPO events. Specifically, the total application value was 248 times the actual needed value on average under the old regime and 3,319 times under the new regime. Under both regimes, IPO stocks hit the upper limit

[4]The fractions of stocks hitting the upper limit on day 3 to day 5 are 99.8%, 98.9%, and 96.7%, respectively.

for over 10 consecutive trading days on average. And there were about 15 to 20 IPO stocks per month in this sample, which means IPO events were not rare or special.

4.2 Identifying Assumptions and Hypothesis Testing

Jointly, the institutional details give rise to the two identifying assumptions below. Assumption 1 generates the testable prediction, and Assumption 2 rules out alternative explanations.

Assumption 1. *The refunded IPO cash under the old regime was cold cash.*

This assumption is consistent with the desired properties in the temperature framework discussed in Section 2. Submitting cash to participate in an IPO lottery was clearly a purposeful and intentional behavior, so the temperature of cash involved in this procedure was assimilated to that of the container, i.e., the IPO cash pool, based on Property 2. Every investor knew that the container would be frozen for two days without uncertainty and that most likely the deposit would be fully refunded, since the winning odds were very low. Even for the lucky investors who did win some shares from the lottery, the majority of their deposit would still be refunded (e.g., winning one ticket out of a hundred tickets submitted) and the part of cash taken away would go to the IPO shares with almost perfectly predictable returns in the first couple of trading days. Furthermore, investors had realistic expectations about the IPO lotteries as they participated in these events routinely. To see the reasoning that supported their repeated and mostly not winning participation, note that they simply locked part of their brokerage cash in an authority-guarded safe for two days in exchange for a small chance of getting a reward of free money, which was a better deal than having the cash sitting idly on the brokerage account. Justifiably, an IPO cash pool was a cold container due to its high level of stableness, and thus IPO refund was cold cash.

Assumption 2. *The extent of participation in an IPO lottery is not driven by an investor's stock choices on the IPO result announcement day or by shocks to the investor's unobserved characteristics that affect stock choices.*

The attractiveness of IPO events also implies that an IPO deposit submission is just driven by the intent to participate in an IPO lottery, rather than by anything else, say the plan to buy certain stocks on the result announcement day, which rules out reverse causality. In addition, degrees of participation are given by both eligibilities and take-up rates.[5] Note that eligibilities are determined by the same rules under both regimes. So, Assumption 2 requires that take-up rates are not affected by shocks to the factors in stock choices. Under the new regime, this is clearly true because participation in an IPO lottery is costless, and it is reasonable to assume every eligible investor participates to the full extent.[6] Under the old regime, investors might not always participate to the full extent due to the requirement of cash deposit, and their participation decisions might be driven by their characteristics. Note that the inclusion of account fixed effects can control for the effects of constant characteristics on their stock choices. Therefore, to rule out the effects of unobserved investor characteristics discussed in Section 3.3, I only need that the shocks to those characteristics around IPOs are not systematically related to both ex-ante IPO participation decisions and ex-post stock choices simultaneously, which is a reasonable assumption.

Now, consider the testable prediction generated by Assumption 1, which is stated as follows.

Hypothesis. *For certain fraction of IPO refund in the total available cash, investors under the old regime, compared to themselves under the new regime, would select stocks more cautiously on result announcement days.*

This hypothesis focuses on the comparison between the two regimes for the same fraction of IPO refund in the total available cash. To see what it means, consider scenario A under the old regime where an investor had 10,000 CNY available, of which 7,500 CNY just arrived at the account as actual IPO refund. In scenario B under the new regime, a similar investor also had 10,000 CNY available and she just learned that her 7,500 CNY of IPO application was

[5]In the numerical example in Section 4.1, the investor's eligibility is the maximum amount she can apply for, which is 7,500 CNY, and her take-up rate is assumed to be 100% in that example.

[6]See the discussion in Section 4.3.

not successful, which could be viewed as a virtual refund. The investors in scenario A and scenario B were in the treatment group and the control group, respectively. In both cases, the IPO refund fraction was 75%, but the difference was that the 7,500 CNY in scenario A actually went through a cash cooling treatment in the IPO cash pool whereas the 7,500 CNY in scenario B did not. So, if investors in two groups at each level of treatment intensity made systematically different decisions, it must be the cash cooling treatment, rather than anything else, that accounted for the difference.

To test the hypothesis, I consider the following DiD regression,

$$y_{i,j,t} = \beta_1 \cdot refund_{i,t} \times regime_t + \beta_2 \cdot refund_{i,t} + controls_{i,t} + \lambda_i + \lambda_t + \varepsilon_{i,j,t}, \tag{4.1}$$

where $y_{i,j,t}$ is the characteristic of stock j purchased by investor i on an IPO result announcement day t; $refund_{i,t}$ is the IPO refund amount standardized by the total cash available; $regime_t$ is an indicator that equals to 1 if day t is under the old regime; account (λ_i) and day (λ_t) fixed effects and control variables are also added to the regression.[7] The $regime_t$ indicator is omitted due to its collinearity with the day fixed effects.

Note that the IPO shares obtained in a lottery won't be available for trading until one to two weeks after its corresponding IPO result announcement day, so the stock j in regression (4.1) is not the IPO stock but just a regular stock traded in the secondary market.

In regression (4.1), the key parameter of interest is the interaction coefficient β_1, which measures the average treatment effect of cash cooling at different intensities. Hypothesis 1 is equivalent to β_1 being negative (positive) for characteristics negatively (positively) related to the degree of investor cautiousness.

4.3 Results and Discussions

In my dataset, I observe IPO lottery deposits and refunds for all investors under the old

[7] The control variables include an indicator for IPO winning and fraction of each of other four cash sources, i.e., $transfer\_in_{i,t}$, $other\_sell_{i,t}$, $div_{i,t}$, $bal\_end_{i,t-1}$, in addition to the same set of control variables used in regression (3.2).

regime, but not under the new regime since there is no such cash flow. To construct $refund_{i,t}$ on the post-reform days, I assume that every eligible investor participates in IPO lotteries to the full extent under the new regime, which enables me to use the holding implied eligible amount as the unobserved application amount for every investor-IPO pair, and to merge it with lottery winning records backed out from holdings to compute the virtual IPO refund amount.

To have a proper sample for identification, I use the stock purchase records on IPO result announcement days to avoid any changes in the temperature of the refund due to delays. To address the multiple accounts issue, I drop the accounts that have any over-application record where the observed actual IPO deposit under the old regime or the win amount under either regime exceeds the holding-implied maximum application amount. In addition, the fraction of IPO virtual refund in the total cash available is constrained on $[0, 1]$ under the new regime to make it comparable to its counterpart under the old regime.[8] The sample has 228,072 observations of 9,666 investors on 238 IPO announcement days.

Table A.5 presents estimates for the same nine stock characteristics as those in Table A.2 and Table A.3 of the algorithm-based analysis. Specifically, Table A.5 shows that receiving cold IPO refund leads to buying stocks that are safer (column 1) and less expensive (column 2-3), have better future performance in the long-term (column 5), have less attention-grabbing quality (column 6-7), are more likely to be an index component stock (column 8), and are held longer in the portfolio (column 9). To see why the effect on short-term future performance is significant in Table A.3 but not here in Table A.5, note that the cold cash under both algorithms in Section 3 is mainly driven by bank-brokerage transfers, which may be subject to reverse causality in the sense that investors inject cold cash because they notice a good investment opportunity for a short horizon. Since the reverse causality is ruled out by the identification design, the effect on short-term future performance is weakened in Table A.5.

[8] About 80% of observations have the fraction of IPO virtual refund on $[0, 1]$ under the new regime without any adjustment, which are the observations used in the main analysis. To check for robustness, I also use all observations, winsorize the measure at 1, and require the amount of available cash is not too small, i.e., no less than 5,000 CNY, and the results are robust.

All estimates, once again, have the same implication that investors use cold cash more cautiously than hot cash for stock purchases, which confirms that the cash-temperature effect is causal. Such consistence is reassuring, because the major source of cold cash in Section 3 (bank-brokerage transfer) and here (IPO refund) are different.

The estimates are robust to alternative specifications, such as different horizons in characteristics construction and non-linear transformation of dependent variables.[9] Quantitatively, the identified cash-temperature effect leads to about 2 to 3 percentiles difference in the characteristics of selected stocks. To connect the magnitude of the effect here to that in Section 3.2, note that β_1 in regression (4.1) corresponds to $-\alpha$ in regression (3.2).[10] With the quasi-natural experiment design, the identified effects in Table A.5 are not subject to the alternative explanations discussed in Section 3.3.

Finally, note that the condition of post-reform full participation may seem a little strong, but it is still a fair assumption. Under the new regime, the trade-off between opportunity cost and winning odds disappears, since no actual deposit is required. Comparing the summary statistics between the two regimes in Table A.4, one can find that the number of participants increased by over ten times after the reform, which suggests the removal of cash deposit requirement significantly encouraged participation. In addition, under the new regime, every investor has three chances in each year to change their mind without any penalty even after winning the lottery. So, there is no reason to miss IPO events that are both flexible and costless after the reform. Even if there are some investors occasionally missing an IPO event for idiosyncratic reasons, it will only underestimate the effect of the virtual IPO refund under the new regime. As shown in Table A.5, such an underestimate will lead to an underestimate of β_1 in regression (4.1), which is the parameter of interest for the cash-temperature effect. The significance and robustness of current estimates under the condition of post-reform full participation suggest the true cash-temperature effect may be even stronger.

[9]See Table B.5 and Table B.6 in Appendix A for results of these robustness tests.

[10]Although not exactly the same, $refund_{i,t}$ under the old regime is very similar to $1 - \theta_{i,t}$ with $\beta = 1$, since the maintained brokerage cash not going through the cooling process of an IPO cash pool is most likely all hot cash when there is no decay.

CHAPTER 5

Model

While the mental accounting framework implies that cash may be non-fungible in financial investment, it does not generate any specific predictions on what the differences are. The causal effect identified in Section 4 goes beyond mental accounting as it shows investors typically buy stocks more cautiously when using colder cash, which is reflected in the characteristics of selected stocks. Now, I need a model to explore the mechanism and to understand the equilibrium asset pricing implications of the cash-temperature effect.

This section proposes a portfolio choice model, in which cash temperature is negatively related to sensitivity of utility with respect to future gains and losses. As discussed below, this temperature-sensitivity mechanism is rooted in existing theories and is the most natural translation of my empirical findings. The model generates three crucial aspects of the cash-temperature effect, which are risk, price, and return.[1] The model also produces predictions that are not directly from my empirical findings but are central in the literature, providing an alternative and unifying framework for understanding several puzzles.

Sections 5.1 to 5.3 present a myopic version of the model to illustrate the key mechanism and predictions. Section 5.4 extends the model to a dynamic setting, compares the two modeling strategies, and discusses additional observations.

[1]Note that the model is built on an economy with only one single-period risky asset. Therefore, by construction it does not target the stylized facts of multi-asset and multi-period settings, such as momentum and attention-grabbing.

5.1 Setup

Consider an economy with infinite horizon in discrete time and a continuum of investors with unit mass. In every period t, there is a single-period risky asset with a unit supply and a next-period payoff d_{t+1}, where $d_{t+1} \sim \mathcal{N}(\mu, \sigma^2)$. Assume that $\{d_{t+1}\}_{t \geq 0}$ are independent and identically distributed (i.i.d.) across time so that the risky assets in all periods have the same fundamental. Investors can also hold uninvested cash with zero net return.

Suppose all existing investors have a probability $\eta \in (0, 1)$ of receiving a liquidity shock and leaving the economy in every period t. They consume all of their accumulated wealth upon exiting. At the same time, a new cohort of investors with mass η will enter the economy to restore the total mass back to 1. Endowed with a homogenous initial wealth w_0, all cohorts allocate their wealth between the risky asset and idle cash in every period. Specifically, cohort τ in period t who has wealth $w_{\tau,t}$ and cash temperature $\theta_{\tau,t}$ observes the risky asset price p_t, and decides the optimal risky asset weights $\{a_{\tau,t'}\}_{t'=t}^{\infty}$ for all future periods based on their expected utility over the period-by-period gains and losses in the periods of staying and over the consumption of terminal wealth in the period of leaving. Mathematically, cohort τ's value function $U_{\tau,t}(w_{\tau,t}; \theta_{\tau,t})$ and optimal risky holding at time $t \geq \tau$ are obtained by solving

$$\max_{\{a_{\tau,t'}\}_{t'=t}^{\infty}} \sum_{t'=t}^{\infty} (1-\eta)^{t'-t} \eta \left(\mathbb{E}_t \left[w_{\tau,t'+1} \right] - \frac{1}{2} \gamma \mathrm{Var}_t \left[w_{\tau,t'+1} \right] + f(\theta_{\tau,t}) \sum_{j=t}^{t'} \mathbb{E}_t \left[v(\Delta w_{\tau,j+1}) \right] \right) \tag{5.1}$$

$$\text{s.t. } w_{\tau,t'+1} = w_{\tau,t} + \sum_{j=t}^{t'} \Delta w_{\tau,j+1} \tag{5.2}$$

where

$$v(\Delta w_{\tau,j+1}) = \begin{cases} \Delta w_{\tau,j+1} & \text{if } \Delta w_{\tau,j+1} \geq 0, \\ \lambda \Delta w_{\tau,j+1} & \text{if } \Delta w_{\tau,j+1} < 0, \end{cases} \text{ with } \lambda > 1, \; \Delta w_{\tau,j+1} = \frac{d_{j+1} - p_j}{p_j} a_{\tau,j}, \tag{5.3}$$

and

$$f\left(\theta_{\tau,t}\right)=\begin{cases}\exp\left\{\frac{\theta_{\tau,t}}{\theta_{\tau,t}-1}\right\} & \text{if } 0\le\theta_{\tau,t}<1,\\ 0 & \text{if } \theta_{\tau,t}=1,\end{cases}\quad\text{with } \theta_{\tau,t+1}=\begin{cases}\frac{\theta_{\tau,t}(w_{\tau,t}-a_{\tau,t})\beta+\frac{a_{\tau,t}}{p_t}d_{t+1}}{(w_{\tau,t}-a_{\tau,t})+\frac{a_{\tau,t}}{p_t}d_{t+1}} & \text{if } t+1>\tau,\\ 0 & \text{if } t+1=\tau.\end{cases}\tag{5.4}$$

The first two terms in the parenthesis of equation (5.1) are the mean-variance preferences over the terminal wealth, where γ measures the aversion to risk; the third term is the sensitivity-adjusted summation of period evaluation of gains and losses based on the prospect theory.[2] I include this prospect theory term because its narrow framing and loss aversion properties have shown success in capturing investor behaviors under uncertainties (Barberis and Huang, 2001; Barberis et al., 2001; Barberis and Xiong, 2009; Barberis et al., 2021). The novelty comes through the function $f\left(\theta_{\tau,t}\right)$ that links the cohort τ's cash temperature $\theta_{\tau,t}$ in period t to the sensitivity with which they evaluate future gains and losses.[3] Note that sensitivity $f\left(\theta_{\tau,t}\right)$ does not have subscript t' or j, which means investors do not anticipate their cash temperature will change in the future. This restriction enables me to simplify the problem to a myopic one by Proposition 1, but it will be relaxed in the dynamic extension presented in Section 5.4, where investors endogenize the effect of risky investment today on their cash temperature in the future.

The negative first-order derivative of $f(\cdot)$ ensures that a higher temperature leads to a lower sensitivity. Intuitively, this means that one dollar loss is less painful with hot cash than with cold cash, and, similarly, one dollar gain is less exciting with hot cash than with cold cash. This assumption finds its foundation in several existing theories. First, it can be motivated by prospective accounting and decoupling introduced by Prelec and

[2]The definition of $v(\cdot)$ in (5.3) adopts a linear version of the prospect theory utility, which has a kink on the curve at zero so that the function is locally concave. The linear simplification still captures the narrow framing and loss aversion properties that are central to the model, which has been used in other papers as well. See Barberis et al. (2001) and Barberis and Huang (2001).

[3]Although there might be other ways to translate the degree of cautiousness into a model parameter, I find the more plausible way is to associate the temperature parameter with the sensitivity with respect to future gains and losses in the utility function. I will show that this setting successfully generates some patterns that I observe in the data.

Loewenstein (1998) in the sense that investors set certain amount of money aside for stock investments, pay the mental costs only once when converting the normal cash (cold cash) into gambling cash (hot cash), and worry less about the value changes going forward. Second, the assumption is also consistent with the expectation-based reference-dependent preferences (Kőszegi and Rabin, 2006). To see the relation, their theory suggests that the expectation of value stableness for cash coming out of an unstable container is low, so investors should not care too much or be too surprised when they see the value of invested money moves up or down in the future, which is exactly the essence of my assumption.

As defined in equation (5.4), the initial cash temperature of any cohort τ is set to $\theta_{\tau,\tau} = 0$, which has a sensitivity of $f(0) = 1$, corresponding to the traditional portfolio choice problem for investors with prospect theory preferences. Starting from period $t = \tau + 1$, there will be cash revenues from the risky asset investment, which increases the cash temperature per algorithm A1 introduced in Section 3 and decreases the sensitivity of utility with respect to value fluctuations going forward. As discussed below, this change in sensitivity plays a crucial role in generating most predictions of the model.

The function $f(\cdot)$ given by equation (5.4) is non-linear. One could alternatively use a linear conversion function, such as $g(\theta) = 1 - \theta$, but $f(\cdot)$ has the desired properties of switching from concave to convex at $\theta = 0.5$ and having negligible level change in the interval close to 1. Therefore, the sensitivity drops faster at lower temperature levels and almost remains constant when the temperature is high enough. See Figure A.8. The function $f(\cdot)$ stabilizes the risky asset price by muting the effect of small fluctuations in cash temperature, which emphasizes the effect of the temperature disparity between entering and existing cohorts.

This model is not the first one introducing history-dependence into the prospect theory preferences. In the model presented by Barberis et al. (2001), the loss aversion parameter λ in equation (5.3) is a function of the size of prior losses to highlight the effect of historical gains and losses on investor's decision-making.[4] However, there is contradicting evidence on

[4] The model proposed by Barberis et al. (2001) assumes that only the sensitivity to losses, not to gains, is dynamic. In my model, however, the effect on sensitivity is symmetric to gains and losses.

what the effects of past gains and losses are (Shiv et al., 2005; Langer and Weber, 2008; Andrade and Iyer, 2009), which implies the actual history-dependence may be broader than gains and losses.[5] In addition, my empirical findings show a strong and novel form of history-dependence through the sources of fundings, which is robust to the inclusion of gains and losses control variables. Therefore, it is reasonable to have the sensitivity being associated with cash composition (i.e., temperature) rather than with gains and losses.

Before proceeding to solve the model, I first show that the maximization problem in (5.1)-(5.2) can be equivalently rewritten as a myopic problem.

Proposition 1. *For cohort τ in period t, $\forall \tau \leq t$, the portfolio choice problem in (5.1)-(5.2) is equivalent to*

$$\max_{a_{\tau,t}} \mathbb{E}_t \left[w_{\tau,t+1} \right] - \frac{1}{2} \gamma \, Var_t \left[w_{\tau,t+1} \right] + f \left(\theta_{\tau,t} \right) \mathbb{E}_t \left[v \left(\Delta w_{\tau,t+1} \right) \right] \tag{5.5}$$

$$s.t. \;\; w_{\tau,t+1} = w_{\tau,t} + \Delta w_{\tau,t+1} \tag{5.6}$$

Proof. See Appendix B1. ■

The intuition behind Proposition 1 is straightforward. In equation (5.1), temperature $\theta_{\tau,t}$ is the only state variable, but by assumption its future changes are not anticipated by investors. So, cohort τ in period t evaluates all future gains and losses under $\theta_{\tau,t}$ without internalizing the effect of their investment today on the cash temperature tomorrow. In other words, $\theta_{\tau,t}$ is not a choice variable in the optimization problem. Consequently, a change in $\theta_{\tau,t}$ later becomes an unexpected shock in every period. In absence of commitment device for future asset allocations, investors need to re-optimize in every period, which makes the portfolio choices myopic and time-inconsistent.

As discussed in Sections 5.2 to 5.3, the myopic setting is useful in illustrating the key

[5] Imas (2016) tries to reconcile the contradiction on the effect of losses by differentiating realized losses from paper losses, but the effect of gains still remains unclear. For example, the "house money effect" proposed by Thaler and Johnson (1990) indicates that prior gains lead to more risk taking, but the prospect theory (Kahneman and Tversky, 1979) suggests that investors are risk averse in the region of gains.

mechanism and generating predictions that either match my empirical findings or address other puzzles in the literature. In Section 5.4, the myopic restriction is relaxed.

5.2 Equilibrium

I start solving the model by further simplifying the objective function (5.5). The linearity of prospect theory term $v(\cdot)$ in equation (5.3) enables me to rewrite $\mathbb{E}_t\left[v\left(\Delta w_{\tau,t+1}\right)\right]$ as a linear function of $a_{\tau,t}$.

Lemma 1. *The prospect theory term in (5.5) is a linear function of* $a_{\tau,t}$*, i.e.,*

$$\mathbb{E}_t\left[v\left(\Delta w_{\tau,t+1}\right)\right] = \frac{g\left(p_t\right)}{p_t} a_{\tau,t}. \tag{5.7}$$

For $a_{\tau,t} > 0$*,* $g\left(p_t\right)$ *is given by*

$$g\left(p_t\right) = \left(\mu - p_t\right)\left[1 + \left(\lambda - 1\right)\Phi\left(-\frac{\mu - p_t}{\sigma}\right)\right] + \phi\left(-\frac{\mu - p_t}{\sigma}\right)\sigma\left(1 - \lambda\right) \tag{5.8}$$

where $\Phi(\cdot)$ *and* $\phi(\cdot)$ *are the cumulative density function (c.d.f.) and probability density function (p.d.f.) of the standard normal distribution, respectively.*

Proof. See Appendix B2. ∎

Lemma 1 indicates that the objective function (5.5) is quadratic in the choice variable $a_{\tau,t}$, which means it can be solved in closed form for all cohorts $\tau \leq t$ in period t.

Proposition 2. *In an economy with short-selling and borrowing constraints, given the risky asset price* p_t *and cash temperature* $\theta_{\tau,t}$*, the demand for the risky asset by cohort* τ *in period* t*,* $\forall \tau \leq t$*, is*

$$a_{\tau,t} = \begin{cases} 0 & \text{if } a^{\star}_{\tau,t} < 0, \\ a^{\star}_{\tau,t} & \text{if } 0 \leq a^{\star}_{\tau,t} \leq w_{\tau,t}, \\ w_{\tau,t} & \text{if } a^{\star}_{\tau,t} > w_{\tau,t}, \end{cases} \tag{5.9}$$

with the unconstrained demand given by

$$a^{\star}_{\tau,t} = \frac{p_t (\mu - p_t)}{\gamma\sigma^2} + \frac{p_t g(p_t)}{\gamma\sigma^2} f(\theta_{\tau,t}), \tag{5.10}$$

where $f(\theta_{\tau,t})$ *and* $g(p_t)$ *are defined in (5.4) and (5.8), respectively.*

Proof. See Appendix B3. ■

The unconstrained demand in equation (5.10) is given by a standard first-order condition. Since the focus is on individual investors, I impose the short-selling and borrowing constraints, as many other models do, to describe investor behaviors more practically.[6] The quadratic and concave functional form of the objective function (5.5) makes the constrained demand tractable.

With demand characterized by Proposition 2, the equilibrium price is obtained by aggregating the demand of all cohorts and setting it equal to the unit supply.

Definition 1. p_t *is an equilibrium of the economy* $\{U_{\tau,t}, w_{\tau,t}, \theta_{\tau,t}\}_{\tau\leq t}$ *in period* t *if and only if there exist* $\{a_{\tau,t}\}_{\tau\leq t}$ *such that the following two conditions are both satisfied:*

(1). Utility maximization: $\{a_{\tau,t}\}_{\tau\leq t}$ *are given by Proposition 2;*

(2). Market clearing: $\{a_{\tau,t}\}_{\tau\leq t}$ *satisfy*

$$\sum_{\tau\leq t} \eta (1-\eta)^{t-\tau} a_{\tau,t} = p_t. \tag{5.11}$$

In the initial period, there is only one cohort with mass 1 and temperature $\theta_{0,0} = 0$. Following Definition 1, the equilibrium price at $t = 0$ is given by Corollary 1.

[6] The short-selling constraint is not critical to the prediction of no participation in the model. To see why it is not, this constraint is only binding when $g(p_t)$ in (5.10) is negative and dominant such that it makes unconstrained demand negative. However, when short-selling constraint is lifted and demand becomes negative, as indicated in (C.25) of the proof for Proposition 4, the sign of $g(p_t)$ will switch to positive under regularity conditions as the gains and losses that define $g(p_t)$ all have their signs flipped, which makes the unconstrained demand positive again. Therefore, the quadratic objective function indicates zero risky holding, i.e., no participation, is optimal.

Corollary 1. *In period $t = 0$, the equilibrium price p_0 is*

$$p_0 = \begin{cases} 0 & \textit{if } p_0^\star < 0, \\ p_0^\star & \textit{if } 0 \leq p_0^\star \leq w_{0,0}, \\ w_{0,0} & \textit{if } p_0^\star > w_{0,0}, \end{cases} \tag{5.12}$$

where

$$p_0^\star = \mu - \gamma\sigma^2 + g\left(p_0^\star\right) \tag{5.13}$$

Proof. See Appendix B4. ■

5.3 Numerical Results

The initial price p_0 in equation (5.12) is the benchmark of traditional models without the cash-temperature effect. Starting from the next period, cohort $\tau = 0$ receives hot cash from their $t = 0$ investment so that their temperature $\theta_{0,1}$ increases and they become less sensitive to gains and losses. To demonstrate how it affects price and holding, I numerically compute the average prices in the next 50 periods and show the difference between that average price and p_0 by a heatmap in Figure A.9a. I also compute the mass of investors who don't buy the risky asset due to short-selling constraint in every period and show the average mass of non-participating investors in Figure A.9b.

The key finding is that the model with a lower γ and higher λ can generate both a larger price increase (overpricing) and a greater fraction of people not investing in the risky asset (no participation). This is true because a lower γ leads to a higher $p_0^\star$ per equation (5.13), which makes the gains and losses evaluation term $g\left(p_0^\star\right)$ more likely to be negative, because a higher price means the probability of losses becomes higher. In the meanwhile, a higher λ increases the penalty on losses, which also reduces the value of $g\left(p_0^\star\right)$. When $g\left(p_0^\star\right) < 0$ holds, the prospect theory term $\mathbb{E}_t\left[v\left(\Delta w_{\tau,t+1}\right)\right]$ in equation (5.5) is a penalty that prevents investors from holding too much risky asset. On one hand, temperature (equivalently sensitivity) changes in later periods will relieve the penalty by a larger extent if the penalty is greater in the first place, so the risky holding and price will go up by more, which explains Figure A.9a. Intuitively, it means that the market price of an risky asset is allowed to be higher and the return is thus lower when people spend their cash less cautiously, which is consistent with the empirical results in Section 3 to Section 4. On the other hand, the later entering cohort still has zero initial temperature, so the temperature disparity makes them unwilling to participate if the price is too high, which explains Figure A.9b.

Although the analysis starting from $t = 0$ is easy to compute numerically and is helpful in providing insights on overpricing and no participation, it does heavily rely on the dynamics of the initial cohort in the first couple of periods due to its uniquely large mass. For the main simulations, I first characterize the steady state of the economy, and then let all simulations start from there instead to remove the impact of the unique initial cohort $\tau = 0$.

Definition 2. *A deterministic steady state* $\left\{\widehat{U}_{\tau,t}, \widehat{w}_{\tau,t}, \widehat{\theta}_{\tau,t}\right\}_{\tau\leq t}$ *is attained in period* t *if and only if*

(1). Cohort τ*'s size is* $\eta\left(1-\eta\right)^{t-\tau}$, $\forall\tau\leq t$;

(2). Price, wealth distribution and holdings distribution remain constant for the risky asset payoffs realized at its mean, i.e., $p_{t+1} = p_t$, $w_{\tau+1,t+1} = w_{\tau,t}$, *and* $a_{\tau+1,t+1} = a_{\tau,t}$ *for* $\forall\tau \leq t$ *if* $d_{t+1} = \mu$.

Note that the economy is subject to aggregate risk in every period, so the steady state in Definition 2 is conditional on the risky payoff being realized at its mean. In my simulations, the risky payoffs are volatile, but the price path is quite flat after reaching the steady state, which can be attributed to the stabilizing property of $f(\cdot)$ as it removes the effect of small fluctuations in temperature when it is close to 1. Therefore, the simulations will emphasize the effect of the temperature shock and weaken the effect of payoff variations.

Now I numerically compute the equilibrium price, holding distribution and cash temperature of each cohort starting from the deterministic steady state, and simulate the economy for different sets of parameters. In particular, I focus on three levels of loss aversion λ ($0.5\lambda_0$, λ_0, $1.5\lambda_0$), risk aversion γ (1, 2, 3), replacement rate η (0.05, 0.1, 0.2), and temperature decay β (1, 0.98, 0.9), with the middle value as the baseline.[7] For each combination of these four parameters,[8] I first find the deterministic steady state per Definition 2; then I impose a shock of 50% reduction in temperature for everyone in the economy, which can be motivated by large bank-brokerage transfers or IPO refunds; finally, I simulate the economy forward by 50 periods and repeat the exercise 100 times for different risky payoff paths that are generated randomly.

Figure A.10 shows the price path of the risky asset. One observation is that in most cases the price goes up as the aggregate temperature recovers from the negative temperature shock, with the only exception when λ takes the smallest value. In addition, prices are negatively related to both λ and γ. These two patterns echo the findings in Figure A.9.

Furthermore, Figure A.10 also shows that the price may overshoot and become volatile for large λ and small γ, which are documented in several empirical studies but are hard to generate within one theoretical framework.

How does the temperature disparity give us both of them? Consider period t when a negative temperature shock hits the economy, the price p_t drops as the existing cohorts ($\forall \tau < t$) reduce their holdings, but the entering cohort ($\tau = t$) is willing to buy more at the lower price since their cash temperature $\theta_{t,t}$ is 0, which is unaffected by the shock. In period $t+1$, cohort $\tau = t$ receives more hot cash back from their risky investment, which increases their cash temperature to a level higher than that in the counterfactual scenario without the shock. This mechanism works through all the entering cohorts until the price returns to its steady state level, where the higher-than-benchmark temperature of those after-shock new cohorts will push them to keep overbuying, leading to further price increase and, consequently, an overreaction to the shock.

[7]The $\lambda_0 = 2.25$ is the value estimated for the median participant in the experiment of Tversky and Kahneman (1992).

[8]In the presented results, I manipulate one parameter at a time and keep the other three parameters at their baseline level.

The dynamics described above also provide an additional channel for price movement not driven by change in fundamentals. Given the overreaction in price, the new cohort who would have bought the risky asset at its steady state price may not buy it if they enter in a period when price goes above the steady state level. For such investors, no participation in their entering period implies no participation in the next period as well, because their cash temperature remains at 0 when there is no hot cash coming back.

Therefore, the price drops as the existing cohorts gradually exit the market until it reaches a point low enough such that all the stand-by new cohorts suddenly decide to buy together, which then pushes the price up again. As shown in Figure A.10, the back and forth price adjustment is more likely to happen when the temperature disparity is more powerful, i.e., larger value of $\mathbb{E}_t \left[v \left(\Delta w_{\tau,t+1} \right) \right]$ due to a higher λ or lower γ.

Not surprisingly, in Figure A.11, the aggregate temperature, i.e., a weighted average across all cohorts, rises after the negative temperature shock. For parameter values that generate additional price movement, they also produce fluctuations in aggregate temperature to make it positively co-move with the price, which is consistent with the pro-cyclical patterns of the aggregate temperature in Figure A.3.

For non-preference parameters, the replacement rate η is negatively related to aggregate temperature, which is mechanically true since a higher replacement rate means a larger fraction of hot cash investors will be replaced by cold cash entrants in every period. Also note that the size of decay parameter's effect is limited, because investors receive relatively large amount of hot cash in every period. Compared to the effect on aggregate temperature (Figure A.11), η and β's effects on the price (Figure A.10) are even smaller, which once again reflects the stabilizing property of function $f(\cdot)$ for temperature at a relatively high value.

To further illustrate the role played by temperature disparity, I draw holding and temperature in time series for the cohort entering in the period of shock and in cross-section for all existing cohorts in the ending period. From Figure A.12a, one can see that it takes about 5 periods for cash temperature to rise from 0 to 1, which maintains at 1 thereafter. In the meanwhile, the holding grows approximately from 0.7 to 1.2 as cash temperature rises. Although not being exactly equivalent, this pattern is consistent with the empirical finding that investors bear greater amount of risk when their cash temperature is higher.[9] From the cross-section perspective, Figure A.12b shows that early cohorts all have similarly high temperature and holdings, whereas younger cohorts have lower both. These observations suggest that it is the large temperature disparity between new and early cohorts, rather than the small disparity among early cohorts, that drives the results.

In summary, the myopic model featuring temperature disparity between new and existing cohorts can explain three patterns related to the empirical findings of this paper. First, higher cash temperature makes investors willing to take more risks. Second, higher cash temperature corresponds to higher price of the risky asset. Third, higher cash temperature leads to lower return of the risky asset, since the distribution of risky asset's payoff remains unchanged. The ability to generate these predictions gives credibility to the mechanism as a plausible theory for the cash-temperature effect.

Furthermore, the model also generates three additional predictions, which are the co-existence of no participation and overbuying, overreaction to shocks, and fluctuation in price not driven by change in fundamentals. Rather than competing against the traditional theories, the temperature-sensitivity mechanism provides an alternative and unifying explanation for these puzzles.

5.4 Extension to Dynamic Setting

Recall that Proposition 1 rewrites the problem in (5.1)-(5.2) as a myopic problem in (5.5)-(5.6), which is possible because $f(\theta_{\tau,t})$ in (5.1) is assumed to be invariant in all future periods. In this section, I lift this restriction and present a dynamic model, in which investors

[9] By assumption, the model has only one risky asset, so it cannot directly replicate any empirical findings about the portfolio choice with multiple risky assets. However, in the model investors bear greater risk in their portfolio when temperature rises, which is similar to selecting a riskier stock documented in Section 3 to Section 4.

endogenize the cash-temperature effect. Then, I compare it with the myopic setting to illustrate temperature smoothing and long-run convergence generated by the dynamic model.

What happens when the cash-temperature effect is taken into account by investors? Note that investors have two sources of utility, of which the first is from exiting wealth in a future period and the second is from period-by-period evaluation of gains and losses. While the utility from exiting wealth encourages investors to hold more risky asset if it is attractive, Figure A.9a and Figure A.10 indicate that the utility from gains and losses evaluation is negative for most parameter values, serving as a penalty that prevents them from holding too much risky asset. Now, suppose investors in the dynamic setting understand that higher weight on risky asset today leads to higher cash temperature and lower sensitivity to the penalty tomorrow. It provides an additional incentive for them to hold the risky asset, especially when their cash temperature is low, because some investment today will make them less worried about value fluctuations tomorrow so that they can maintain sufficient risky holding in the future. This intuition is confirmed by Proposition 4 in this section.

Mathematically, consider a variant of problem (5.1)-(5.2) for cohort τ in period t:

$$\max_{\left\{a_{\tau,t'}\right\}_{t'=t}^{\infty}} \sum_{t'=t}^{\infty} (1-\eta)^{t'-t} \eta \left(\mathbb{E}_t \left[w_{\tau,t'+1} \right] - \frac{1}{2} \gamma \mathrm{Var}_t \left[w_{\tau,t'+1} \right] + \mathbb{E}_t \left[\sum_{j=t}^{t'} f\left(\theta_{\tau,j}\right) v\left(\Delta w_{\tau,j+1}\right) \right] \right) \tag{5.14}$$

$$\text{s.t. } w_{\tau,t'+1} = w_{\tau,t} + \sum_{j=t}^{t'} \Delta w_{\tau,j+1} \tag{5.15}$$

where $v\left(\Delta w_{\tau,j+1}\right)$ and $f\left(\theta_{\tau,t}\right)$ are defined in (5.3) and (5.4), respectively. Compared to (5.1), objective function (5.14) has a dynamically evolving sensitivity term $f\left(\theta_{\tau,j}\right)$, in which cash temperature $\theta_{\tau,j}$ is jointly determined by the path of optimal allocation $\left\{a_{\tau,k}\right\}_{k=t}^{j-1}$, $\forall j \geq t+1$, and cash temperature $\theta_{\tau,t}$ when the problem is being solved. Therefore, the first-order condition for $a_{\tau,t}$ involves partial derivatives $\partial f\left(\theta_{\tau,j}\right) / \partial a_{\tau,t}$ for all future periods $j \geq t+1$, which endogenize the cash-temperature effect and make the problem dynamic.[10]

[10]Note that the objective function (5.14) is no longer quadratic in choice variables and is not guaranteed

The problem is hard to solve, but some reasonable simplifying assumptions can be applied to make it tractable. Importantly, I will illustrate that the solution to the simplified dynamic problem gives quantitatively close and qualitatively equivalent results to those from the original problem in (5.14)-(5.15). Then, I will discuss the takeaways based on the solution to the simplified problem.

To see how the problem can be simplified, in Figure A.13a, I plot the sensitivity $f(\theta_{\tau,\tau+j})$ of cohort τ in period $\tau+j$ ($j=1,2,3,4$) against risky holding $a_{\tau,\tau}$ in the initial period.[11] An important observation is that

$$\frac{\partial f(\theta_{\tau,\tau+j})}{\partial a_{\tau,\tau}} = 0, \ \forall j \geq 2, \tag{5.16}$$

approximately holds for the range of $a_{\tau,\tau}$ marked in gray that investors choose from.

Condition (5.16) implies the portfolio choice in the initial period only has significant impact on sensitivity in the next period. The reason for having this pattern is revealed in Figure A.13b, which shows that the temperature smoothing term $\xi_{\tau,\tau+1}$ defined in (5.19) is negative with a large absolute value when the initial risky holding $a_{\tau,\tau}$ is too small to increase the cash temperature in period $\tau+1$. In this way, entrants in period τ, who realize the cash-temperature effect, will have a negative term $\xi_{\tau,\tau+1}$ in their demand function tomorrow ($t=\tau+1$) to counteract the large sensitivity $f(\theta_{\tau,\tau+1})$ if their cash temperature $\theta_{\tau,\tau+1}$ is low due to small risky holding $a_{\tau,\tau}$ today ($t=\tau$). As a result, the cash temperature

to be a single-peak function. The following discussion focuses on the interior solution, which is typically attained under some regularity conditions discussed in Appendix B7. But in cases where corner solutions (when short-selling or borrowing constraint is binding) are not ruled out, solution to (5.14)-(5.15) can be obtained by comparing the utility levels associated with all-in risky investment, no participation, and interior solution, which is not central to the discussion.

[11] The results in Figure A.13 are approximate due to two adjustments for computational convenience. First, steady state price used for computation is obtained in myopic setting. This adjustment does not have strong impact on the results since the true steady state price in dynamic setting is very close to that in myopic setting when η is small, as implied by the long-run convergence property in Proposition 4. Second, the next period optimal risky holding $\hat{a}^{\star}_{\tau,\tau+1}$is set equal to the initial optimal risky holding $\hat{a}^{\star}_{\tau,\tau}$ when computing $\xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t}, \hat{a}^{\star}_{\tau,t+1}; \theta_{\tau,t}, w_{\tau,t}\right)$ given by (5.19). With this adjustment, the results in Figure A.13 underestimate the magnitude of $\xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t}, \hat{a}^{\star}_{\tau,t+1}; \theta_{\tau,t}, w_{\tau,t}\right)$ and thus underestimate the speed of convergence of $f(\theta_{\tau,\tau+j})$ to 0, $\forall j \geq 2$. So, observation in (5.16) holds.

starting from $t = \tau + 2$, regardless of the initial period $a_{\tau,\tau}$, will quickly rise to almost 1 to make cohort τ's sensitivity close to 0 due to the stabilizing property of $f(\cdot)$.[12] As shown in Figure A.13a, condition (5.16) is very close to be true.[13]

When an investor holds the risky asset for a few more periods, the magnitude of current risky holding's effect on future sensitivities is even smaller with higher cash temperature. So, (5.16) can be generalized to

$$\frac{\partial f\left(\theta_{\tau,t+j}\right)}{\partial a_{\tau,t}} = 0, \ \forall j \geq 2 \text{ and } \forall t \geq \tau, \tag{5.17}$$

which is closer to be true. So, simplifying the dynamic problem with (5.17) leads to results that are quantitatively close to those from the original problem.

In the following discussion, Proposition 3 solves the problem (5.14)-(5.15) with condition (5.17), and Proposition 4 compares the optimal holding paths with and without endogenizing the cash-temperature effect. Qualitatively, the conclusion of that comparison, as the main takeaway from this exercise, holds even without the simplifying condition (5.17).[14]

Proposition 3. *For cohort τ in period t, $\forall t \geq \tau$, given the deterministic steady state price $\hat{p}$ of the risky asset and cash temperature $\theta_{\tau,t}$, the interior solution to problem (5.14)-(5.15) with $\beta = 1$ and condition (5.17) is characterized by*

$$\hat{a}^{\star}_{\tau,t} = \frac{\hat{p}\left(\mu - \hat{p}\right)}{\gamma\sigma^2} + \left(1 + \xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t}, \hat{a}^{\star}_{\tau,t+1}; \theta_{\tau,t}, w_{\tau,t}\right)\right)\frac{\hat{p}g\left(\hat{p}\right)}{\gamma\sigma^2} f\left(\theta_{\tau,t}\right), \tag{5.18}$$

[12] Speed of cash temperature convergence to 1 is largely determined by the amount of initial wealth as cold cash. Note that the "wealth" in the model corresponds to the brokerage account total value in the data. So, it is reasonable to set the initial wealth in a way such that the optimal weight of risky asset in the model matches the empirical weight of risky asset in the sample, so that the speed of cash temperature convergence in the model matches that in the data.

[13] Figure A.13a and A.13b use baseline parameter values to demonstrate the patterns, which also hold for other parameter values. See Appendix B5.

[14] To see why this is true, take cohort τ's optimal risky holding in the initial period as an example. Note that $\partial f\left(\theta_{\tau,\tau+j}\right)/\partial a_{\tau,\tau}$ is negative for any $j \geq 1$, since cash temperature increases in a cumulative way. So, relaxing condition (5.17) will, under similar regularity conditions to those in Proposition 4, only add more negative terms to the coefficient of penalty $g\left(\hat{p}\right)$ in (5.18), which means the optimal initial risky holding in the dynamic setting, with or without condition (5.17), is greater than that in the myopic setting.

where the temperature smoothing term $\xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t},\hat{a}^{\star}_{\tau,t+1};\theta_{\tau,t},w_{\tau,t}\right)$ *is given by*

$$
\begin{aligned}
&\xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t},\hat{a}^{\star}_{\tau,t+1};\theta_{\tau,t},w_{\tau,t}\right)\\
&=(1-\eta)\,\hat{a}^{\star}_{\tau,t+1}\exp\left\{\frac{\mu\hat{a}^{\star}_{\tau,t}}{\hat{p}\left(\theta_{\tau,t}-1\right)\left(w_{\tau,t}-\hat{a}^{\star}_{\tau,t}\right)}+\frac{\sigma^{2}\left(\hat{a}^{\star}_{\tau,t}\right)^{2}}{2\hat{p}^{2}\left(\theta_{\tau,t}-1\right)^{2}\left(w_{\tau,t}-\hat{a}^{\star}_{\tau,t}\right)^{2}}\right\}\\
&\quad\times\left[\frac{\mu}{\hat{p}\left(\theta_{\tau,t}-1\right)}+\frac{\sigma^{2}\hat{a}^{\star}_{\tau,t}}{\hat{p}^{2}\left(\theta_{\tau,t}-1\right)^{2}\left(w_{\tau,t}-\hat{a}^{\star}_{\tau,t}\right)}\right]\frac{w_{\tau,t}}{\left(w_{\tau,t}-\hat{a}^{\star}_{\tau,t}\right)^{2}}.
\end{aligned}
\tag{5.19}
$$

Proof. See Appendix B6. ■

The only difference between the dynamic solution $\hat{a}^{\star}_{\tau,t}$ given by equation (5.18) and the myopic solution $a^{\star}_{\tau,t}$ given by equation (5.10) is an additional term $\xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t},\hat{a}^{\star}_{\tau,t+1};\theta_{\tau,t},w_{\tau,t}\right)$ in the coefficient of the penalty term $g(\hat{p})$. So, the comparison between $\hat{a}^{\star}_{\tau,t}$ and $a^{\star}_{\tau,t}$ is determined by the sign of $\xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t},\hat{a}^{\star}_{\tau,t+1};\theta_{\tau,t},w_{\tau,t}\right)$, which is characterized below.

Proposition 4. *Under regularity conditions,* $\left\{\hat{a}^{\star}_{\tau,t}\right\}_{t=\tau}^{\infty}$ *given by (5.18) and* $\left\{a^{\star}_{\tau,t}\right\}_{t=\tau}^{\infty}$ *given by (5.10) at the deterministic steady state satisfy:*

(1) Temperature smoothing: $\hat{a}^{\star}_{\tau,\tau}>a^{\star}_{\tau,\tau}$*; and* $\exists t_0>\tau$ *such that* $\hat{a}^{\star}_{\tau,t_0}<a^{\star}_{\tau,t_0}$*;*

(2) Long-run convergence: $\lim_{\theta_{\tau,t}\to 1}\hat{a}^{\star}_{\tau,t}=\lim_{\theta_{\tau,t}\to 1}a^{\star}_{\tau,t}$.

Proof. See Appendix B7. ■

The condition $\hat{a}^{\star}_{\tau,\tau}>a^{\star}_{\tau,\tau}$ corresponds to $\xi_{\tau,\tau}\left(\hat{a}^{\star}_{\tau,\tau},\hat{a}^{\star}_{\tau,\tau+1};\theta_{\tau,\tau},w_{\tau,\tau}\right)<0$, which means that investors are willing to reduce the penalty $g(\hat{p})$ when sensitivity is the highest and to buy more risky asset in the initial period compared to their portfolio choice in the myopic benchmark. In some later period t_0, the opposite condition $\hat{a}^{\star}_{\tau,t_0}<a^{\star}_{\tau,t_0}$ corresponds to $\xi_{\tau,t_0}\left(\hat{a}^{\star}_{\tau,t_0},\hat{a}^{\star}_{\tau,t_0+1};\theta_{\tau,t_0},w_{\tau,t_0}\right)>0$, which means investors increase the penalty $g(\hat{p})$ when sensitivity is low and buy less risky asset than they would do in the myopic setting. Therefore, by anticipating the effect of their portfolio choice today on cash temperature tomorrow, investors counteract the cash-temperature effect and make their risky investment trajectory smoother.

Long-run convergence shows that the dynamic solution $\hat{a}^{\star}_{\tau,t}$ degenerates to the myopic solution $a^{\star}_{\tau,t}$ after investors stay in the market for a while such that their cash temperature reaches 1, which implies that the myopic setting discussed in Section 5.1 to Section 5.3 is powerful enough to generate predictions of the cash-temperature effect for the majority of continuing investors.

There are three insights from the comparison of dynamic and myopic models. First, the myopic model predicts different behaviors of entering and existing cohorts due to disparity in their cash temperature, but the dynamic model implies such differences should be smaller since investors endogenize and counteract the cash-temperature effect. So, whether an investor anticipates the cash-temperature effect is revealed by the smoothness of her risky holding trajectory. If most investors make significantly different investment decisions after a cold cash injection, as what I find empirically, it indicates that the effect is not well-understood by those investors.

Second, even though understanding the effect helps improve time consistency in behaviors, it does not eliminate the effect.[15] The dynamic model predicts that people with low cash temperature will behave similarly to those with high cash temperature. So, compared to a counterfactual where the cash temperature is always 0, it is still true in the dynamic model that investors will hold riskier portfolio and that risky asset price (return) will be higher (lower) due to the cash-temperature effect, which are the three key patterns generated by the myopic model that are consistent with the empirical findings of this paper.

Finally, that understanding the effect does not eliminate it also inspires policy interventions. Although an one-time cold cash injection cannot change investor behaviors if the effect is well-understood, regular cold cash arrivals can still make a difference. For example, an investor may trade more cautiously if revenues of stock selling are always first deposited into her savings account instead of her brokerage account.[16]

[15] This is reasonable since the cash-temperature effect works through preferences rather than beliefs or information channels, so revealing the effect does not eliminate it as long as preferences remain the same.

[16] There are two types of brokerage account systems in Singapore, of which one has cash distributions directly paid to the specified bank account while the other has them paid to the brokerage account.

CHAPTER 6

Conclusion

In this paper, I study how the sources of available cash affect an investor's selections in subsequent investments. In particular, I propose a novel temperature framework in which the stableness of a cash container determines the temperature of cash coming out of it. In this framework, cash received as an IPO refund or transferred from a savings account to a brokerage account is cold cash, and cash raised from selling stocks or other risky assets is hot cash.

To show the existence of the cash-temperature effect, I construct a new measure of brokerage cash temperature based on two algorithms. The results for nine characteristics not only show the existence of the effect, but also reveal its nature whereby investors use cold cash more cautiously than hot cash when selecting stocks. To address the alternative explanations, I exploit the quasi-natural experiment of the Chinese IPO reform on January 1, 2016, and use the DiD design to show the effect is causal. Quantitatively, the effect corresponds to 2 to 3 percentiles difference in characteristics of selected stocks.

In addition to documenting the non-fungibility of brokerage cash, the paper presents a model to explore the underlying mechanism of the cash-temperature effect, which also offers a novel explanation for three other puzzles documented in the literature: the co-existence of no participation and overbuying, overreaction to shocks, and fluctuation in price not driven by change in fundamentals. Moreover, the dynamic extension of the model inspires policy interventions related to transaction-revenue arrangements.

More broadly, this paper inspires a future research agenda on both empirical and theoretical sides. From the empirical perspective, the next step is to compare temperature measures with different decays and algorithms. A better temperature measure should be able to predict investor behavior more accurately. Specifically, a measure of fit, such as sum of squared residuals, based on a simple functional form (e.g., linear or quadratic) that relates characteristics of selected stocks to cash temperature should be evaluated to determine which decay and algorithm fit best with data. Furthermore, the effect identified in this paper can be tested with other dimensions that define cash temperature and other scenarios that involve various sources of funding. For more dimensions, one can test whether the degree of effort associated with earning the money defines a useful temperature measure for cash that has effects on how investors use the cash. For other scenarios, one can investigate whether the composition of capital available to mutual fund managers has any effect on their capital allocation and performance.

From the theoretical perspective, the model of temperature-dependent sensitivity in this paper is the first attempt to incorporate cash composition into optimization problems as the fourth element in addition to beliefs, preferences, and cash amount. More attempts can be made along this line. Moreover, the model itself can be extended to a more general setting with multiple risky assets to generate richer predictions that can match more of the empirical patterns identified in this paper. Finally, the temperature framework, in addition to the traditional mechanisms such as trade-off theories and information asymmetry, may be useful in modeling firm capital structure.

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CHAPTER A

Figures & Tables

Figure A.1: Source Decomposition for the Brokerage Cash

The figure shows the monthly decomposition of total funding sources $total\_cash_{i,t}$ given by equation (3.1) for the sample of 46,016 investors from January 2006 to December 2016. For the six sources, "IPO Refund" refers to $refund_{i,t}$; "Transfer" refers to $transfer\_in_{i,t}$; "Stock Selling" refers to $stock\_sell_{i,t}$; Dividend" refers to $div_{i,t}$; "Other Selling" refers to $other\_sell_{i,t}$; and "Previous Balance" refers to $cash_{i,t-1}$. The presented weight of each source is the monthly average of all investors in the sample.

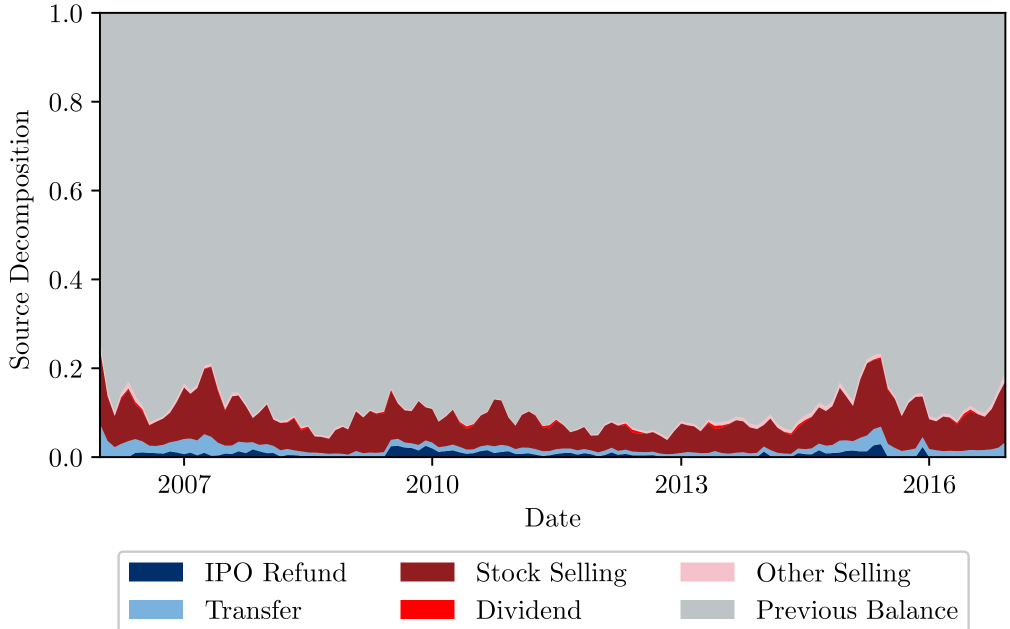


Figure A.2: Temperature Decay

The figure shows the half-life against the daily decay rate $\beta \in [0, 1]$. Half-life is the number of days required for the temperature to decay to one-half of its current level, i.e., T that satisfies $\beta^T = \frac{1}{2}$.

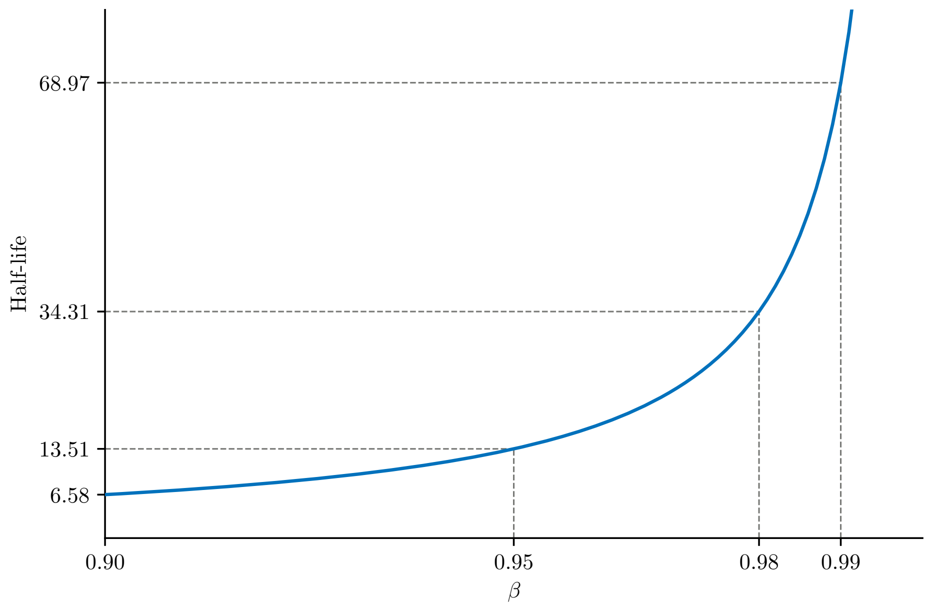
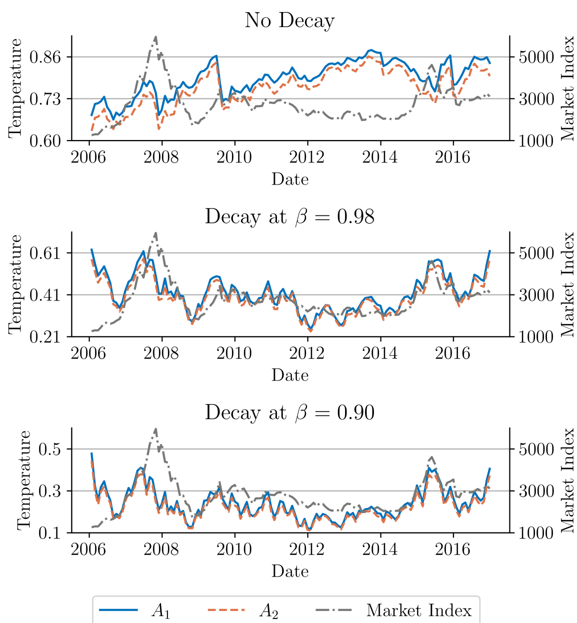
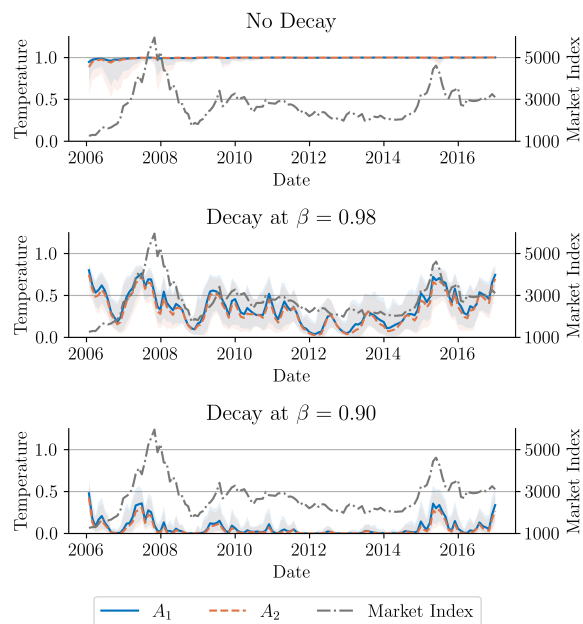


Figure A.3: Aggregate Temperature

The figure shows the monthly time series of the aggregate temperature measures for the sample of 46,016 investors from January 2006 to December 2016. Panel (a) plots the monthly mean (solid and dashed lines) of temperature under A1 and A2 with no decay, decay at $\beta = 0.98$ and decay at $\beta = 0.9$, respectively. Panel (b) plots the monthly median (solid and dashed lines) and 25%-75% percentile interval (color band) of temperature under A1 and A2 with the same three decay rates. The monthly SSE Composite Index (dash-dot line) is added to each panel for comparison.



(a) Mean



(b) Dispersion

Figure A.4: Cash Temperature and Stock Features

The figure shows the relation between stock features and cash temperature under which the stock is purchased. For every stock purchased by investor i on day t, the temperature $\theta_{i,t}$ is computed under A1 with decay at $\beta = 0.98$. Then, stocks are grouped into 10 evenly sliced bins on $[0, 1]$ based on the value of $\theta_{i,t}$. Each panel presents the mean (solid line) and 95% confidence interval (band between dashed lines) of the market-adjusted value of a characteristic, which is defined as the raw value, winsorized at the 1% and 99% percentiles, subtracting the daily average of all listed stocks. The nine characteristics are the realized daily return volatility in the past month, market-to-book ratio, momentum (realized return in the past month), short-term performance (return in the next month), long-term performance (return in the next year), abnormal trading volume used in Barber and Odean (2008), last day return, indicator for component of SSE Composite Index, Shenzhen Component Index or CSI 300 Index, and number of days the stock stays in the portfolio.

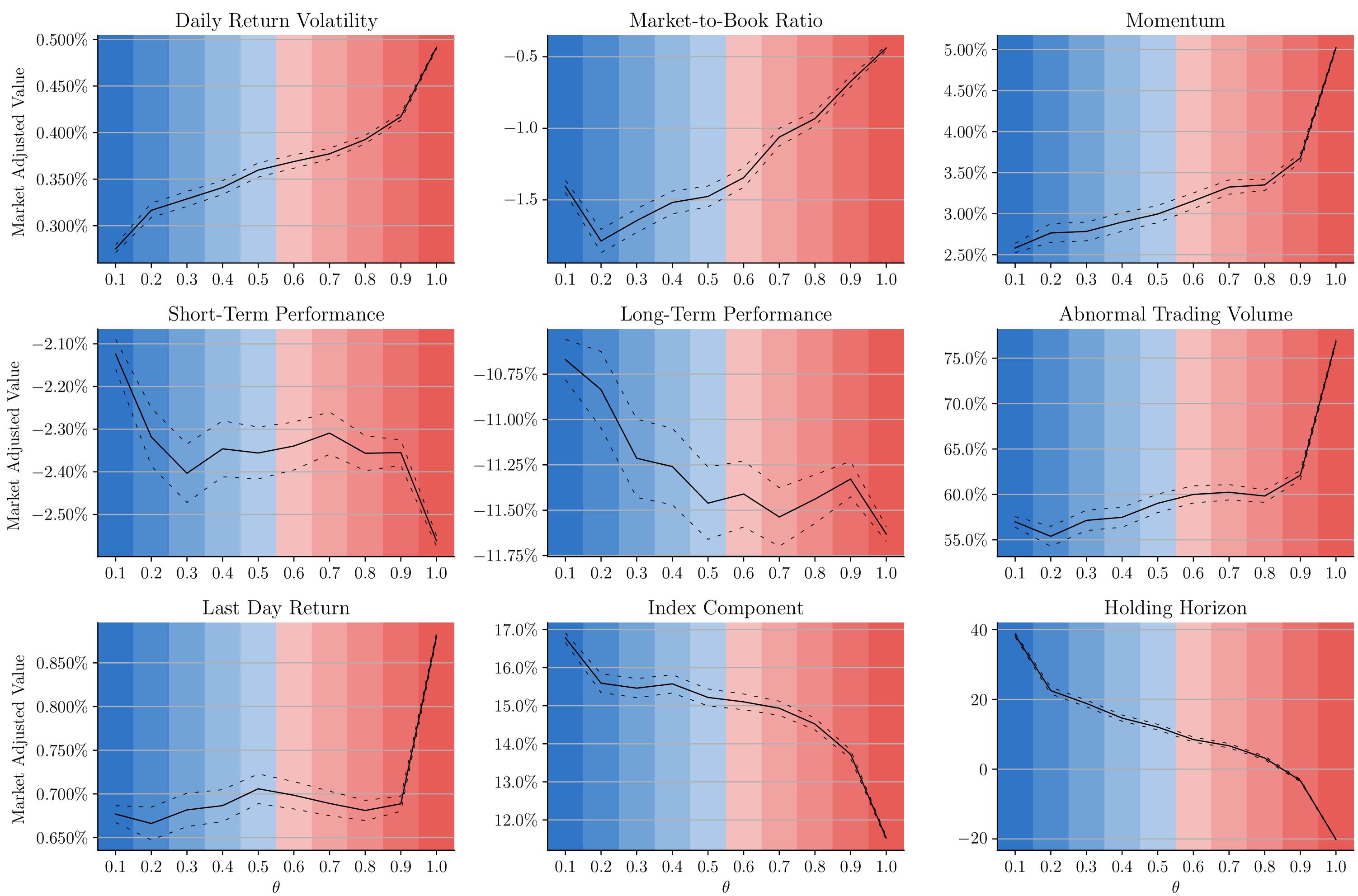


Figure A.5: IPO Underpricing

The figure shows the IPO PE ratio of 1,732 firms between January 1, 2006, and December 31, 2016, in the Chinese stock market. The horizontal black dashed line indicates the PE ratio of 23, which is a regulatory cap announced on January 12, 2014. The grey band marks the identification sample period between June 17, 2014, and December 31, 2016, in which the PE cap of 23 was strictly enforced.

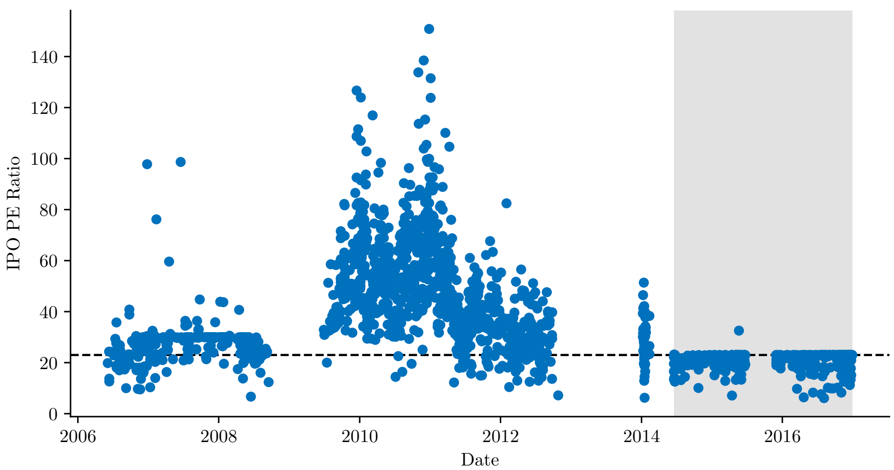


Figure A.6: Two IPO Regimes

The figure exhibits the procedures for participating in an IPO lottery in China. Under the old regime presented in the upper panel, a participant is required to submit a deposit $f_{ipo,t}$ to an IPO cash pool on the application day t; the pool is frozen during the pending period; and the deposit net of the win amount $f_{win,t}$, if any, will be refunded on the result announcement day. Under the new regime presented in the lower panel, the deposit and refund are both 0. The participant only files an application on day t, and pays the win amount $f_{win,t}$ if any shares are won in the lottery.

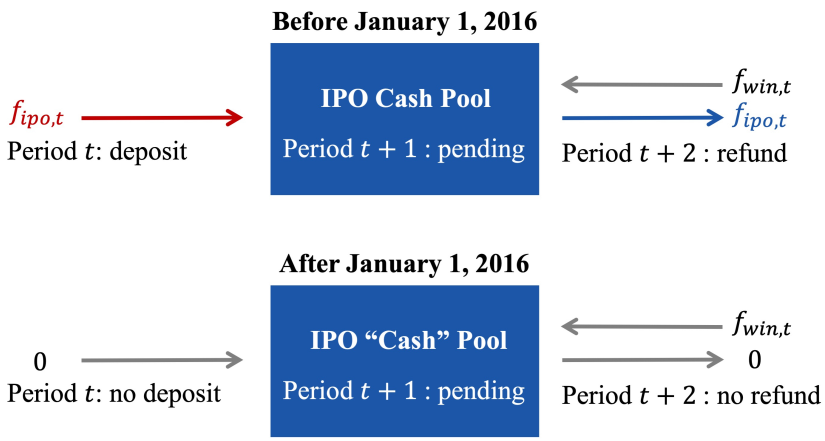


Figure A.7: Performance after IPO

The figure shows the distribution of the daily returns of IPO stocks in their first 9 trading days. The sample includes 539 IPO stocks between June 17, 2014, and December 31, 2016, in the Chinese stock market, where all stocks' daily returns are subject to the 10% (-10%) upper (lower) limit starting from the second trading day after the IPO, and the limit is 44% (-44%) on the first trading day. Limiting values at -10%, 10% and 44% are labeled on the x-axis.

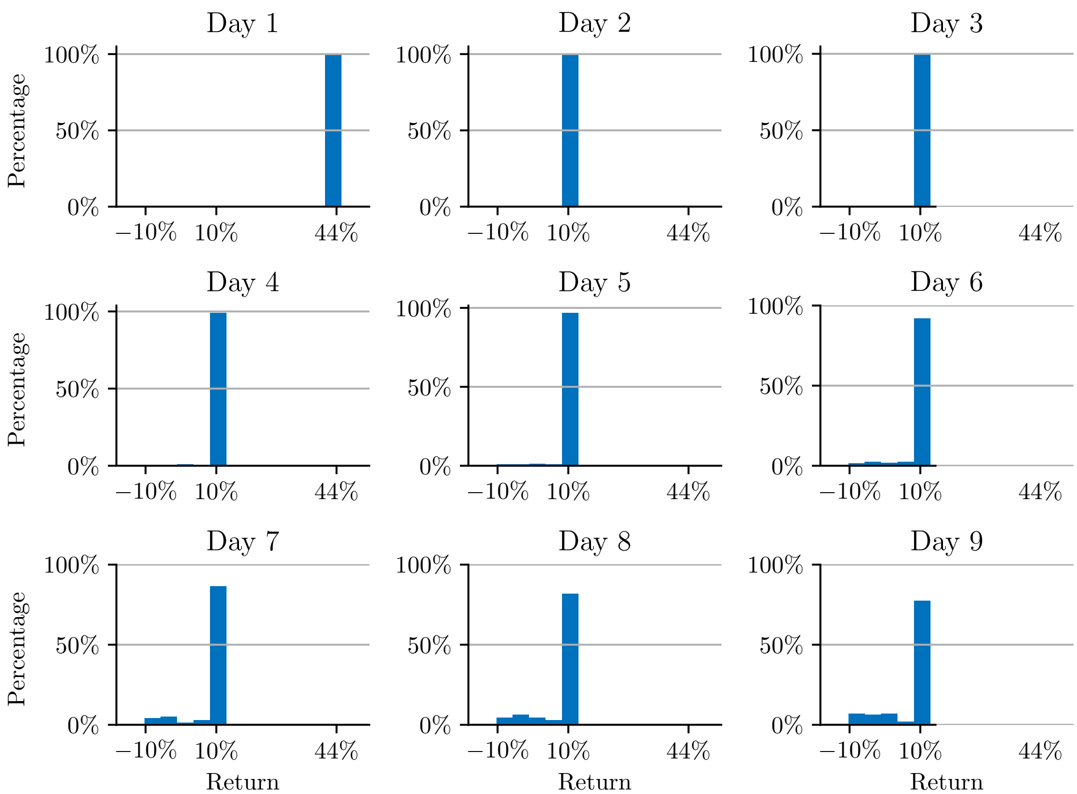


Figure A.8: Temperature and Sensitivity

The figure plots the function $y = f(\theta)$ in solid line, where $f(\cdot)$ is defined in equation (5.4), and $y = 1 - \theta$ in dashed line as the linear baseline. Function $f(\theta)$ is concave for $\theta \in (0, 0.5)$ and convex for $\theta \in (0.5, 1)$.

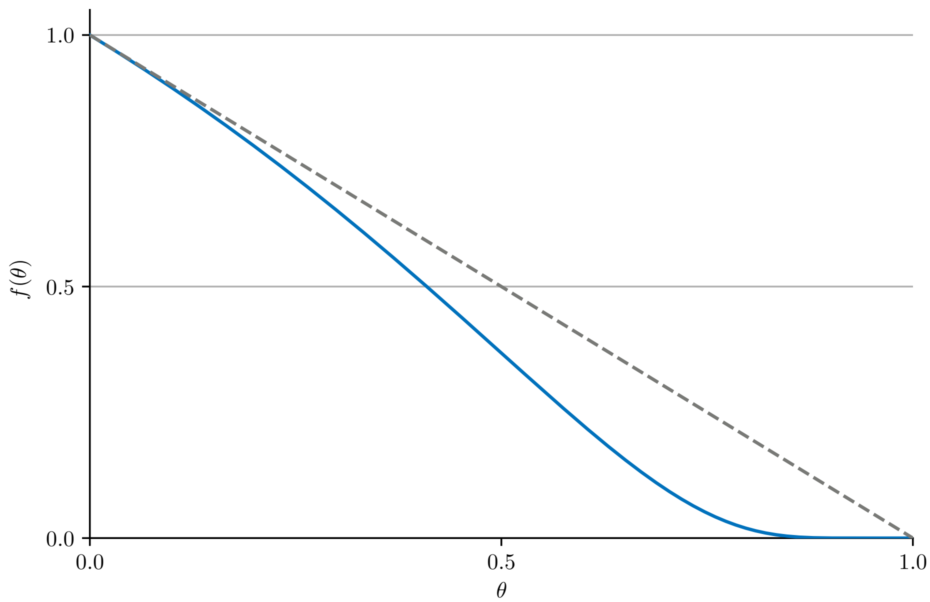
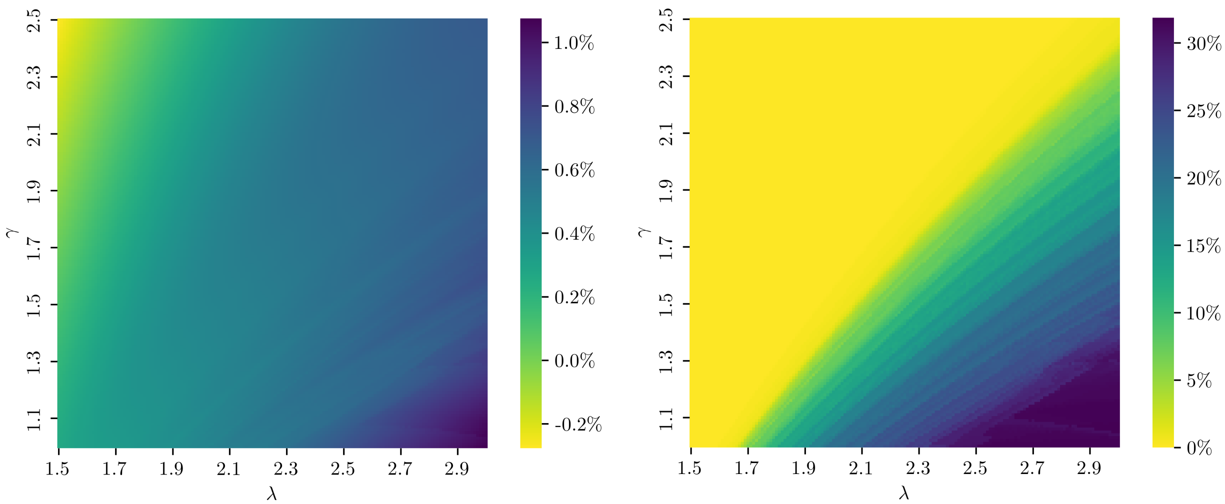


Figure A.9: Preference Parameters

The figure shows the sensitivity tests for loss aversion $\lambda \in [1.5, 3.0]$ and risk aversion $\gamma \in [1.0, 2.5]$. I simulate the economy for 50 periods starting from the initial period, when the price p_0 is given by Corollary 1. For other parameters, the initial wealth is set to $w_0 = 2$; and the mean and standard deviation of the risky asset are set to $\mu = 1.012$ and $\sigma = 0.09$, respectively, which are the mean and standard deviation of the monthly returns of SSE Composite Index between January 1, 2016, and December 31, 2016. Panel (a) exhibits the overpricing for each pair of (λ, γ), which is defined as the difference between the average price in 50 periods and p_0 standardized by p_0. Panel (b) exhibits the mass of investors who place zero weight on the risky asset on average across 50 periods.



(a) Overpricing (b) No Participation

Figure A.10: Equilibrium Price

The figure shows the equilibrium price from simulations that start from the deterministic steady state with a shock of 50% reduction in cash temperature for everyone in the economy. With the baseline values $\lambda = \lambda_0$ (i.e., 2.25), $\gamma = 2$, $\eta = 0.1$ and $\beta = 0.98$, the upper left panel explores two other values of loss aversion, i.e., $\lambda = 0.5\lambda_0$ and $\lambda = 1.5\lambda_0$; the upper right panel explores two other values of risk aversion, i.e., $\gamma = 1$ and $\gamma = 3$; the lower left panel explores two other values of replacement rate, i.e., $\eta = 0.05$ and $\eta = 0.2$; and the lower right panel explores two other values of temperature decay, i.e., $\beta = 1$ and $\beta = 0.9$. For other parameter values, the initial wealth is set to $w_0 = 2$; and the mean and standard deviation of the risky asset are set to $\mu = 1.012$ and $\sigma = 0.09$, respectively, which are the mean and standard deviation of the monthly returns of SSE Composite Index between January 1, 2016, and December 31, 2016. In every setting, the simulation is repeated 100 times for randomly generated risky payoff paths, and the mean and 95% confidence interval are shown in the figure.

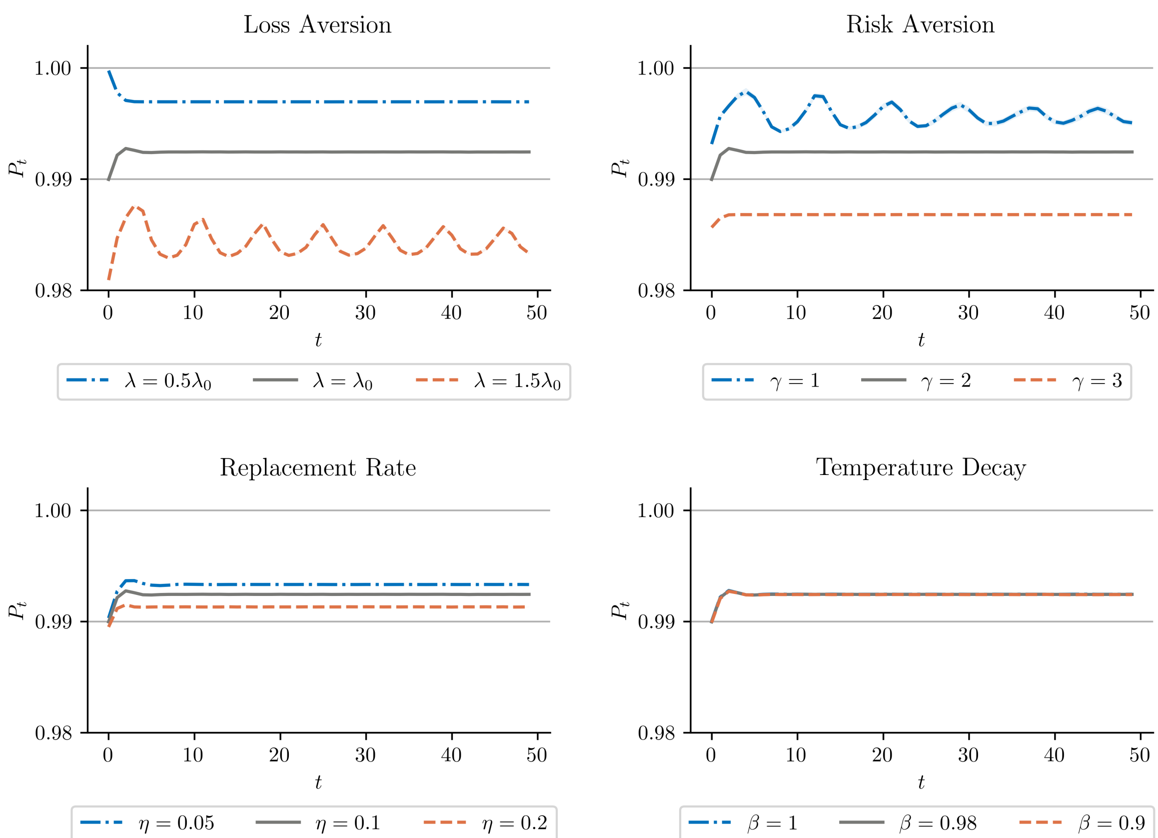


Figure A.11: Aggregate Temperature

The figure shows the aggregate weighted average temperature from simulations, of which the setting and methodology are the same as those described in Figure A.10. The mean and 95% confidence interval are shown in the figure.

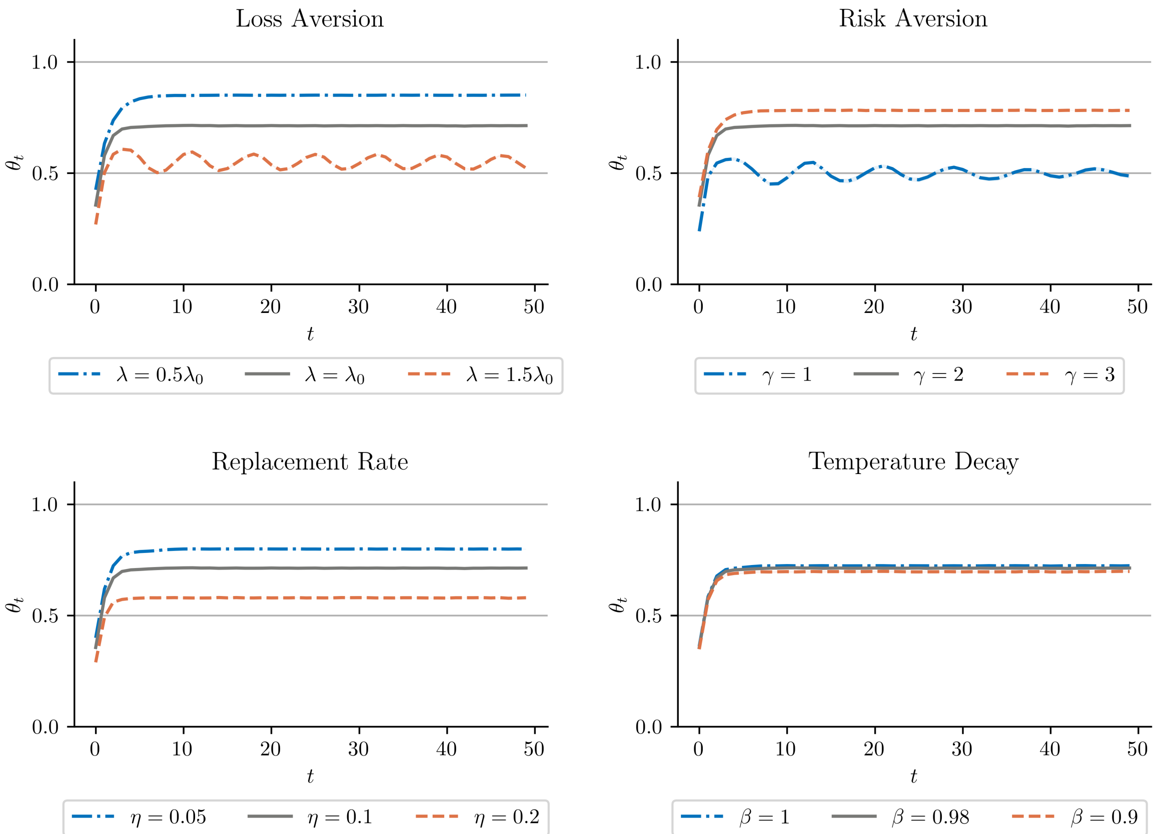
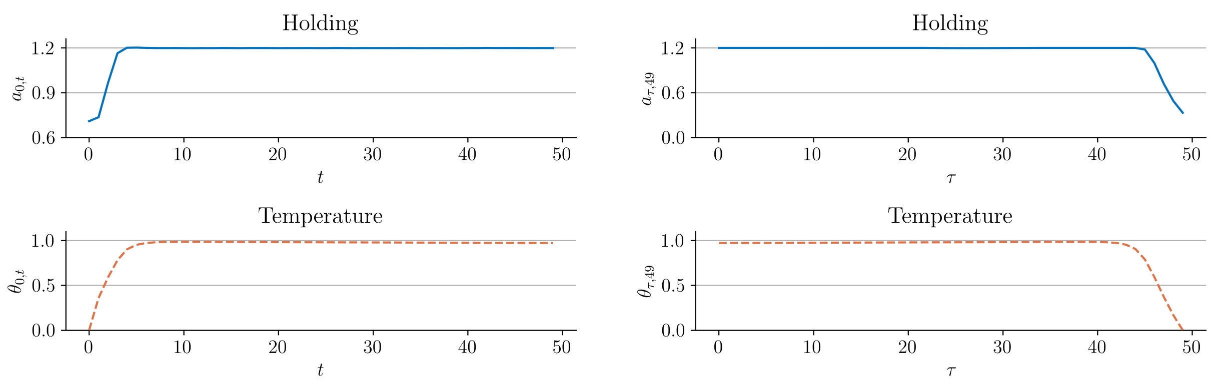


Figure A.12: Initial Cohort and Cross-Section

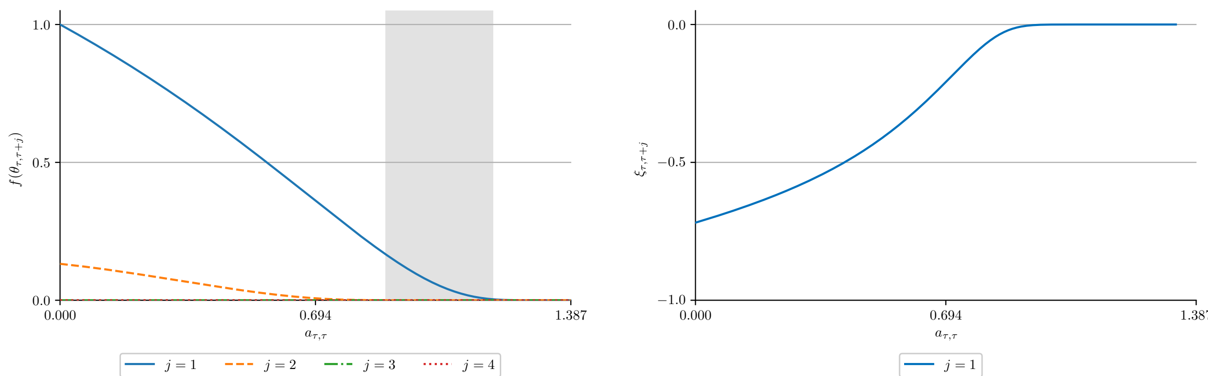
The figure shows the time series of the initial cohort in 50 periods and the cross-section of all cohorts in the 50th period from simulations that start from the deterministic steady state with a shock of 50% reduction in cash temperature for everyone in the economy. The model parameters take the baseline values, i.e., $\lambda = \lambda_0$ (i.e., 2.25), $\gamma = 2$, $\eta = 0.1$ and $\beta = 0.98$. Panel (a) plots the time series of the initial cohort in 50 periods, of which the upper graph shows the size of the risky asset holding and the lower graph shows the cash temperature $\theta_{0,t}$, $\forall t \in \{0, 1, \ldots, 49\}$. Panel (b) plots the cross-section in the 50th period, of which the upper graph shows the size of the risky asset holding of each cohort τ and the lower graph shows the cash temperature $\theta_{\tau,49}$, $\forall \tau \in \{0, 1, \ldots, 49\}$. For other parameters, the initial wealth is set to $w_0 = 2$; and the mean and standard deviation of the risky asset are set to $\mu = 1.012$ and $\sigma = 0.09$, respectively, which are the mean and standard deviation of the monthly returns of SSE Composite Index between January 1, 2016, and December 31, 2016. The simulation is repeated 100 times for randomly generated risky payoff paths. The mean and 95% confidence interval are shown in the figure.



(a) Initial Cohort (b) Cross-Section

Figure A.13: Sensitivity Dynamics

Panel (a) plots sensitivity $f(\theta_{\tau,\tau+j})$ of cohort τ in period $\tau+j$ ($j = 1,2,3,4$) against risky holding $a_{\tau,\tau}$ in the initial period, where $f(\cdot)$ is defined in equation (5.4). The results are based on the deterministic steady state with decay rate $\beta = 1$ and baseline parameter values given by $\lambda = \lambda_0$ (i.e., 2.25), $\gamma = 2$, and $\eta = 0.1$. The initial wealth is set to $w_{\tau,\tau} = 1.387$ such that the optimal idle cash holding of existing cohorts in the model matches the ratio of cash in brokerage accounts in the sample. The gray band marks the interval $\left[\underline{a}_{\tau,\tau}, \overline{a}_{\tau,\tau}\right]$, where $\underline{a}_{\tau,\tau} = 0.884$ and $\overline{a}_{\tau,\tau} = 1.176$ are the minimum and maximum optimal risky asset holding corresponding to cash temperature 0 and 1, respectively. Under the same settings, Panel (b) plots temperature smoothing term $\xi_{\tau,\tau+1}$ of cohort τ in period $\tau+1$ defined in (5.19) against risky holding $a_{\tau,\tau}$ in the initial period.



(a) Future Sensitivities

(b) Temperature Smoothing

Table A.1: Sample

The two panels present information about the sample of 46,016 individual brokerage accounts in the Chinese stock market from January 1, 2006, to December 31, 2016. Panel (a) compares the sample against the population of 53 million accounts registered at the Shanghai Stock Exchange between 2016 and 2019 (Jones et al., 2021). The number and share of accounts are reported for each of the five groups sorted on account average holding value in CNY, namely Smallest (less than 100K CNY), Small (between 100K and 500K CNY), Medium (between 500K and 3M CNY), Large (between 3M and 10M CNY), and Largest (above 10M CNY). Panel (b) provides summary statistics for six variables about investors' cash balance and stock holdings. Columns "Cash Bal." and "Stock Bal." are the account average brokerage cash balance and stock holding balance, respectively, at the end of each trading day. Columns "Buy Value" and "Sell Value" are the account average daily stock buying and selling values, respectively. Column "Stock Num." is the account average number of stocks in the portfolio at the end of each trading day. Column "Holding Horizon" is the account average number of holding days for all stocks ever held. Columns 1-4 are in thousands of CNY.

	Sample		Population	
	Number	Share	Number	Share
Smallest [0, 100K)	25,083	54.51%	31,410,000	58.72%
Small [100K, 500K)	15,017	32.63%	15,282,000	28.57%
Medium [500K, 3M)	5,226	11.36%	5,827,000	10.89%
Large [3M, 10M)	551	1.20%	735,000	1.37%
Largest [10M, ∞)	139	0.30%	235,000	0.44%
Total	46,016	100%	53,489,000	100%

(a) Representativeness

	Cash Bal.	Stock Bal.	Buy Value	Sell Value	Stocks Num.	Holding Horizon
Mean	52.82	294.38	15.85	18.81	3.45	57.06
S.D.	857.82	1,646.67	125.09	630.51	4.33	89.97
Min	0.03	0.00	0.00	0.00	0.00	1.00
25%	3.89	23.55	0.84	0.82	1.68	12.49
Median	11.10	66.72	2.86	2.82	2.50	28.08
75%	32.40	198.29	9.34	9.23	3.95	64.57
Max	132,154.22	192,921.97	13,103.28	132,328.63	392.89	2023.00
Obs.	46,016	46,016	46,016	46,016	46,016	45,905

(b) Summary Statistics

Table A.2: Algorithm-Based Regression: Risk

This table reports the estimates from the ordinary least squares (OLS) regressions of stock risk measures on cash temperature $\theta_{i,t}$ and controls when the stock is initially purchased. For investor i on day t, $\theta_{i,t}$ is the temperature under A1 or A2 with decay at $\beta = 0.98$; $realized\_loss_{i,t}$ is an indicator that equals to 1 if investor i realizes losses on day t; $cml\_paper\_loss_{i,t-1}$ is an indicator that equals to 1 if investor i accumulates net losses up to the previous day; $total\_cash_{i,t}$ is the total cash available to investor i on day t; and $total\_holding_{i,t-1}$ is the total value of the stock holdings of investor i on the previous day. The dependent variable in column (1) - (4) is the realized daily return volatility in the past month in raw values winsorized at the 1% and 99% percentiles; the dependent variable in column (5) - (8) is the same risk measure transformed into percentiles sorted on every day. Column (1) - (2) and (5) - (6) report the estimate for $\theta_{i,t}$ under A1; column (3) - (4) and (7) - (8) report the estimate for $\theta_{i,t}$ under A2. Account and day fixed effects are added to all specifications. The sample of 46,016 investors from January 1, 2006, to December 31, 2016, is used. Standard errors are clustered on account and day. Robust t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Risk: Raw				Risk: Percentile			
	Algorithm 1		Algorithm 2		Algorithm 1		Algorithm 2	
$\theta_{i,t}$	0.115***	0.101***	0.107***	0.093***	3.424***	3.039***	3.185***	2.801***
	(42.09)	(36.93)	(40.82)	(35.64)	(43.93)	(38.98)	(42.76)	(37.73)
$realized\_loss_{i,t}$		0.038***		0.039***		1.006***		1.025***
		(27.36)		(27.78)		(26.44)		(26.93)
$cml\_paper\_loss_{i,t-1}$		−0.005***		−0.005***		−0.120**		−0.121**
		(−2.84)		(−2.85)		(−2.41)		(−2.43)
$total\_cash_{i,t}$		0.000		0.000		0.006		0.006
		(0.53)		(0.52)		(0.66)		(0.65)
$total\_holding_{i,t-1}$		0.000		0.000		0.014		0.018
		(0.07)		(0.19)		(0.46)		(0.57)
Account FE	Y	Y	Y	Y	Y	Y	Y	Y
Day FE	Y	Y	Y	Y	Y	Y	Y	Y
Adj. R^2	0.5509	0.5515	0.5509	0.5515	0.1390	0.1396	0.1389	0.1395
Obs.	6,340,338	6,298,595	6,340,338	6,298,595	6,340,338	6,298,595	6,340,338	6,298,595

Table A.3: Algorithm-Based Regression: More Dimensions

This table reports the estimates from the ordinary least squares (OLS) regressions of eight characteristics on cash temperature $\theta_{i,t}$ and controls when the stock is initially purchased. For investor i on day t, $\theta_{i,t}$ is the temperature under A1 with decay at $\beta = 0.98$; the controls are the same to those in Table A.2, except that the industry dummies are added to the market-to-book ratio regression in column (1). The dependent variables in column (1) - (8) are market-to-book ratio, momentum (realized return in the past month), short-term performance (return in the next month), long-term performance (return in the next year), abnormal trading volume used in Barber and Odean (2008), last day return, indicator for component of SSE Composite Index, Shenzhen Component Index or CSI 300 Index, and number of days the stock stays in the portfolio, respectively. All dependent variables are in raw values winsorized at the 1% and 99% percentiles. Account and day fixed effects are added to all specifications. The sample of 46,016 investors from January 1, 2006, to December 31, 2016, is used. Standard errors are clustered on account and day. Robust t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Price		Performance		Attention		Others	
	M-B	MOM	Short	Long	Abn. Trd.	Last Ret.	Index	Horizon
$\theta_{i,t}$	15.188***	1.538***	−0.390***	−0.818***	12.903***	0.135***	−1.956***	−19.778***
	(16.67)	(36.50)	(−14.40)	(−10.08)	(29.04)	(15.12)	(−18.43)	(−38.61)
Controls	Y	Y	Y	Y	Y	Y	Y	Y
Account FE	Y	Y	Y	Y	Y	Y	Y	Y
Day FE	Y	Y	Y	Y	Y	Y	Y	Y
Adj. R^2	0.2457	0.3836	0.4961	0.5674	0.2881	0.3081	0.1127	0.2070
Obs.	6,297,959	6,298,595	6,397,288	6,378,921	5,871,995	6,371,335	6,398,999	6,399,860

Table A.4: Summary Statistics for IPOs in China

This table presents the summary statistics of 539 IPO stocks between June 17, 2014, and December 31, 2016, in the Chinese stock market. In both Panel (a) and Panel (b), the first column reports the statistics for the total number of accounts (in thousands) participating in the IPO lottery; the second column reports statistics for the ratio of the total application amount submitted by all market participants divided by the actual amount issued by the IPO firm; the third column reports the statistics for the winning odds (in percentage) defined as one over the overbuy multiple in column 2; the fourth column reports the statistics for the number of consecutive days that IPO stock hits the upper limit since the first trading day; the last column reports the statistics for the number of IPO stocks in each month. Panel (a) reports the summary statistics under the old regime between June 17, 2006, and December 31, 2015. Panel (b) reports the summary statistics under the new regime between January 1, 2016, and December 31, 2016.

	App. Account	Overbuy	Winning Odds	Up Limit Days	IPOs Per Month
Mean	1204.39	248.12	0.53	10.65	15.63
S.D.	729.87	134.78	0.30	5.43	14.82
Min	381.62	55.64	0.12	1.00	0.00
25%	717.62	151.45	0.32	6.00	1.50
Median	984.18	216.00	0.46	10.00	11.00
75%	1488.11	317.00	0.66	14.00	24.00
Max	7301.31	808.78	1.80	29.00	48.00
Obs.	297	297	297	297	19.00

(a) June 17, 2014 to December 31, 2015

	App. Account	Overbuy	Winning Odds	Up Limit Days	IPOs Per Month
Mean	11234.08	3318.66	0.05	12.69	20.17
S.D.	2122.57	1772.70	0.05	4.84	13.10
Min	6698.16	213.99	0.01	2.00	4.00
25%	9612.50	2168.65	0.02	9.00	13.75
Median	11365.64	2963.95	0.03	12.50	15.00
75%	12876.49	4173.97	0.05	16.00	26.50
Max	15262.79	8568.76	0.47	29.00	49.00
Obs.	242	242	242	242	12

(b) January 1, 2016 to December 31, 2016

Table A.5: Difference-in-Differences

This table reports the estimates from a DiD design that relies on the quasi-natural experiment of the IPO lottery reform on January 1, 2016. Variable $refund_{i,t}$ is the IPO refund amount standardized by the total cash available for investor i on day t; $regime_t$ is an indicator that equals to 1 if the IPO lottery result announced on day t is under the old regime, and equals to 0 if it is under the new regime; The control variables include an indicator for IPO winning and fraction of other four cash sources, i.e., $transfer\_in_{i,t}$, $other\_sell_{i,t}$, $div_{i,t}$, and $bal\_end_{i,t-1}$, in addition to the same set of control variables used in regression (3.2). For the market-to-book ratio regression in column (2), the industry dummies are added to controls. The dependent variable in column (1) is the realized daily return volatility in the past month in raw values winsorized at the 1% and 99% percentiles; the dependent variables in column (2) - (9) are the same as those in columns (1) - (8) of Table A.3, respectively. Account and day fixed effects are added to all specifications. The sample of 9,666 investors on 238 IPO announcement days between June 17, 2014, and December 31, 2016, is used. Standard errors are clustered on account and day. Robust t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Risk	Price		Performance		Attention		Others	
	Realized Vlt.	M-B	MOM	Short	Long	Abn. Trd.	Last Ret.	Index	Horizon
$refund_{i,t} \times regime_t$	-0.077^{***}	-26.827^{**}	-1.535^{***}	0.210	2.404^{**}	-11.214^{***}	-0.264^{***}	3.259^{***}	9.870^{***}
	(-2.63)	(-2.25)	(-3.18)	(0.86)	(2.47)	(-2.73)	(-3.26)	(2.98)	(4.66)
$refund_{i,t}$	0.080^{***}	16.468^{*}	0.825^{***}	-0.150	-2.177^{***}	9.253^{***}	0.141^{***}	-1.759^{**}	-3.840^{***}
	(3.98)	(1.85)	(2.95)	(-0.85)	(-2.98)	(3.54)	(2.67)	(-2.42)	(-4.93)
Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y
Account FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Day FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Adj. R^2	0.4490	0.2632	0.2533	0.5349	0.4730	0.3438	0.2899	0.1170	0.2775
Obs.	224,628	224,559	224,628	226,982	226,374	211,969	226,965	227,018	227,018

CHAPTER B

Supplemental Material to Chapter 3 & 4

In the main analysis, I use algorithm A1 with decay at $\beta = 0.98$ to construct the temperature measure $\theta_{i,t}$ for account i on day t, and regress the raw values of nine stock characteristics on the measure. To show robustness of the cash-temperature effect, I make several modifications to the construction of the temperature measure and stock characteristics for the algorithm-based results. In particular, Figure B.1 and Table B.4 use the same methodology as in Figure A.4 and Table A.3, respectively, but report results based on algorithm A2 instead of A1. Table B.2 reports regression results for alternative decay levels. Table B.4 reports regression results for different horizons of a subset of stock characteristics. Finally, Table B.3 reports regression results for a percentile transformation of stock characteristics.

Similarly, for the algorithm-free identification, I check different horizons of a subset of stock characteristics and conduct a percentile transformation of stock characteristics. The results are presented in Table B.5 and Table B.6, respectively.

Figure B.1: Cash Temperature and Stock Features: A2

The figure shows the relation between stock features and cash temperature under which the stock is purchased. For every stock purchased by investor i on day t, the temperature $\theta_{i,t}$ is computed under A2 with decay at $\beta = 0.98$. Then, stocks are grouped into 10 evenly sliced bins on $[0, 1]$ based on the value of $\theta_{i,t}$. Each panel presents the mean (solid line) and 95% confidence interval (band between dashed lines) of the market-adjusted value of a characteristic, which is defined as the raw value, winsorized at the 1% and 99% percentiles, subtracting the daily average of all listed stocks. The nine characteristics are the realized daily return volatility in the past month, market-to-book ratio, momentum (realized return in the past month), short-term performance (return in the next month), long-term performance (return in the next year), abnormal trading volume used in Barber and Odean (2008), last day return, indicator for component of SSE Composite Index, Shenzhen Component Index or CSI 300 Index, and number of days the stock stays in the portfolio.

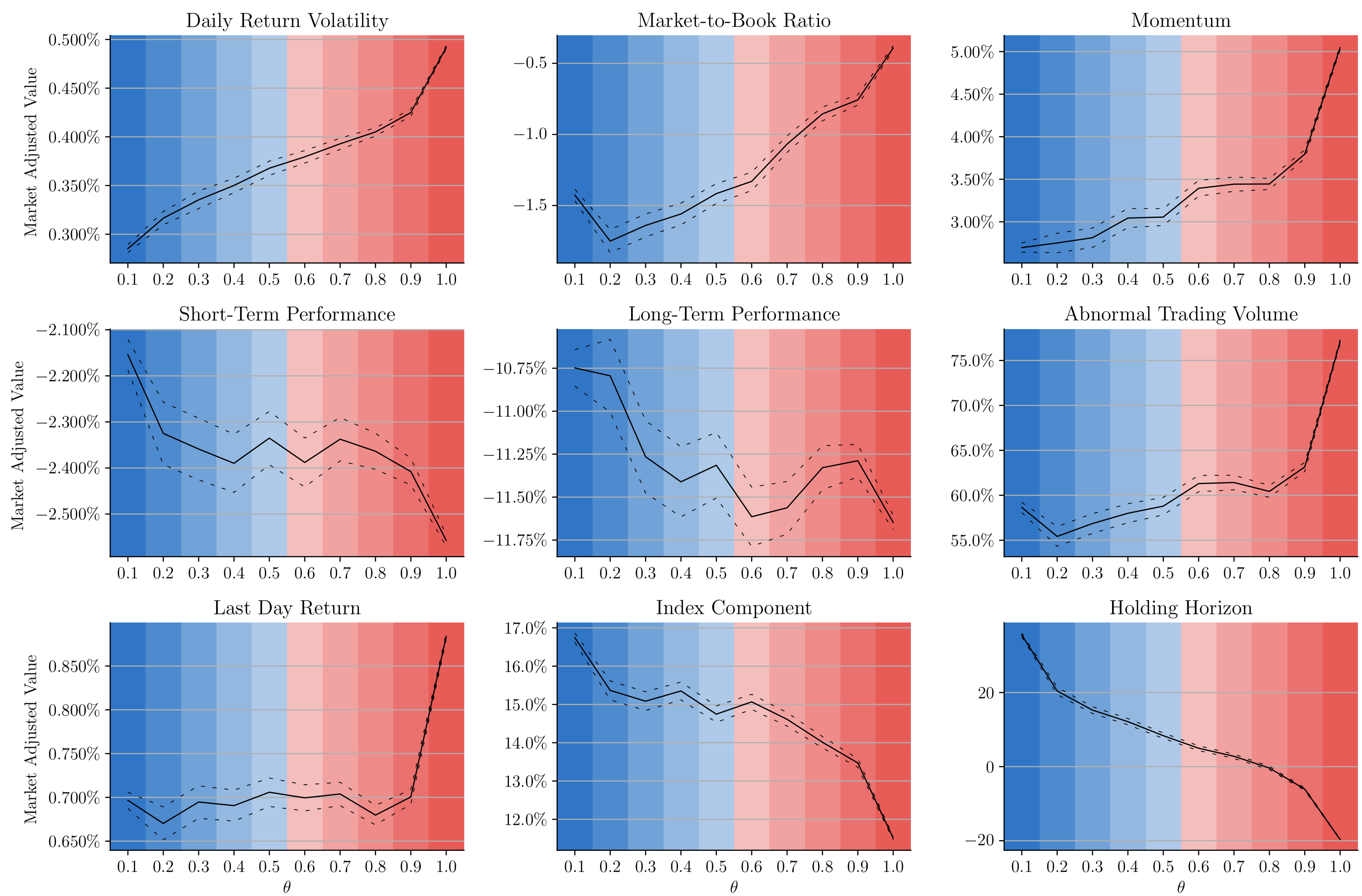


Table B.1: Algorithm Robustness: A2

This table reports the estimates from the ordinary least squares (OLS) regressions of eight characteristics on cash temperature $\theta_{i,t}$ and controls when the stock is initially purchased. For investor i on day t, $\theta_{i,t}$ is the temperature under A2 with decay at $\beta = 0.98$; the controls are the same to those in Table A.3. The dependent variables in column (1) - (8) are market-to-book ratio, momentum (realized return in the past month), short-term performance (realized return in the next month), long-term performance (realized return in the next year), abnormal trading volume used in Barber and Odean (2008), last day return, indicator for component of SSE Composite Index, Shenzhen Component Index or CSI 300 Index, and number of days the stock stays in the portfolio, respectively. All dependent variables are in raw values winsorized at the 1% and 99% percentiles. Account and day fixed effects are added to all specifications. The sample of 46,016 investors from January 1, 2006, to December 31, 2016, is used. Standard errors are clustered on account and day. Robust t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	M-B	MOM	Short	Long	Abn. Trd.	Last Ret.	Index	Horizon
$\theta_{i,t}$	15.056***	1.390***	−0.372***	−0.782***	11.099***	0.110***	−1.835***	−17.587***
	(16.82)	(34.52)	(−14.08)	(−9.89)	(25.64)	(12.80)	(−17.93)	(−37.08)
Controls	Y	Y	Y	Y	Y	Y	Y	Y
Account FE	Y	Y	Y	Y	Y	Y	Y	Y
Day FE	Y	Y	Y	Y	Y	Y	Y	Y
Adj. R^2	0.2457	0.3836	0.4961	0.5674	0.2881	0.3080	0.1127	0.2068
Obs.	6,297,959	6,298,595	6,397,288	6,378,921	5,871,995	6,371,335	6,398,999	6,399,860

Table B.2: Algorithm Robustness: Different Decays

This table reports the estimates from the ordinary least squares (OLS) regressions of nine characteristics on cash temperature $\theta_{i,t}$ and controls when the stock is initially purchased. For investor i on day t, $\theta_{i,t}$ is the temperature under A1 with decay at $\beta = 1$ in Panel (a) and at $\beta = 0.9$ in Panel (b); the controls are the same to those in Table A.3. The dependent variables in column (1) - (9) are realized daily return volatility in the past month, market-to-book ratio, momentum (realized return in the past month), short-term performance (return in the next month), long-term performance (return in the next year), abnormal trading volume used in Barber and Odean (2008), last day return, indicator for component of SSE Composite Index, Shenzhen Component Index or CSI 300 Index, and number of days the stock stays in the portfolio, respectively. All dependent variables are in raw values winsorized at the 1% and 99% percentiles. Account and day fixed effects are added to all specifications. The sample of 46,016 investors from January 1, 2006, to December 31, 2016, is used. Standard errors are clustered on account and day. Robust t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Realized Vlt.	M-B	MOM	Short	Long	Abn. Trd.	Last Ret.	Index	Horizon
$\theta_{i,t}$	0.060***	8.436***	0.850***	−0.272***	−0.506***	5.928***	0.046***	−1.153***	−12.143***
	(24.81)	(9.43)	(21.50)	(−10.61)	(−6.63)	(14.50)	(5.43)	(−11.46)	(−26.39)
Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y
Account FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Day FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Adj. R^2	0.5513	0.2456	0.3834	0.4961	0.5674	0.2879	0.3080	0.1126	0.2062
Obs.	6,298,595	6,297,959	6,298,595	6,397,288	6,378,921	5,871,995	6,371,335	6,398,999	6,399,860

(a) $\beta = 1$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Realized Vlt.	M-B	MOM	Short	Long	Abn. Trd.	Last Ret.	Index	Horizon
$\theta_{i,t}$	0.108***	18.789***	1.851***	−0.423***	−0.829***	16.633***	0.213***	−2.279***	−20.319***
	(40.27)	(20.43)	(42.31)	(−15.21)	(−10.03)	(36.17)	(23.11)	(−21.29)	(−41.61)
Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y
Account FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Day FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Adj. R^2	0.5516	0.2457	0.3838	0.4961	0.5675	0.2883	0.3082	0.1127	0.2072
Obs.	6,298,595	6,297,959	6,298,595	6,397,288	6,378,921	5,871,995	6,371,335	6,398,999	6,399,860

(b) $\beta = 0.9$

Table B.3: Algorithm Robustness: Different Horizons

This table reports the estimates from the ordinary least squares (OLS) regressions of four characteristics on cash temperature $\theta_{i,t}$ and controls when the stock is initially purchased. For investor i on day t, $\theta_{i,t}$ is the temperature under A1 with decay at $\beta = 0.98$; the controls are the same to those in Table A.3. The dependent variables in column (1) - (4) are risk (realized daily return volatility in the past three months), momentum (realized return in the past three months), short-term performance (return in the next three months), and long-term performance (return in the next six months), respectively. All dependent variables are in raw values winsorized at the 1% and 99% percentiles. Account and day fixed effects are added to all specifications. The sample of 46,016 investors from January 1, 2006, to December 31, 2016, is used. Standard errors are clustered on account and day. Robust t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)	(4)
	Realized Vlt.	MOM	Short Pfm.	Long Pfm.
$\theta_{i,t}$	0.079^{***}	2.779^{***}	-0.524^{***}	-0.768^{***}
	(35.88)	(38.51)	(-11.96)	(-12.63)
Controls	Y	Y	Y	Y
Account FE	Y	Y	Y	Y
Day FE	Y	Y	Y	Y
Adj. R^2	0.5967	0.4625	0.5379	0.5436
Obs.	6,206,986	6,206,986	6,392,659	6,387,511

Table B.4: Algorithm Robustness: Percentile Transformation

This table reports the estimates from the ordinary least squares (OLS) regressions of six characteristics on cash temperature $\theta_{i,t}$ and controls when the stock is initially purchased. For investor i on day t, $\theta_{i,t}$ is the temperature under A1 with decay at $\beta = 0.98$; the controls are the same to those in Table A.3. The dependent variables in column (1) - (6) are market-to-book ratio, momentum (realized return in the past month), short-term performance (return in the next month), long-term performance (return in the next year), abnormal trading volume used in Barber and Odean (2008), and last day return, respectively. All dependent variables are in percentiles sorted daily. Account and day fixed effects are added to all specifications. The sample of 46,016 investors from January 1, 2006, to December 31, 2016, is used. Standard errors are clustered on account and day. Robust t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
	M-B	MOM	Short	Long	Abn. Trd.	Last Ret.
$\theta_{i,t}$	1.551^{***}	2.861^{***}	-1.070^{***}	-1.008^{***}	2.848^{***}	0.837^{***}
	(27.43)	(37.87)	(−17.61)	(−15.61)	(35.28)	(12.98)
Controls	Y	Y	Y	Y	Y	Y
Account FE	Y	Y	Y	Y	Y	Y
Day FE	Y	Y	Y	Y	Y	Y
Adj. R^2	0.3217	0.1168	0.0472	0.0499	0.1098	0.0671
Obs.	6,297,959	6,298,595	6,397,288	6,378,921	5,871,995	6,371,335

Table B.5: DiD Robustness: Different Horizons

This table reports the estimates from a DiD design that relies on the quasi-natural experiment of the IPO lottery reform on January 1, 2016. Variable $refund_{i,t}$ is the IPO refund amount standardized by the total cash available for investor i on day t; $regime_t$ is an indicator that equals to 1 if the IPO lottery result announced on day t is under the old regime, and equals to 0 if it is under the new regime; The control variables are the same to those in Table A.5. The dependent variables in column (1) - (4) are risk (realized daily return volatility in the past three months), momentum (realized return in the past three months), short-term performance (return in the next three months), and long-term performance (return in the next six months), respectively. All dependent variables are in raw values winsorized at the 1% and 99% percentiles. Account and day fixed effects are added to all specifications. The sample of 9,666 investors on 238 IPO announcement days between June 17, 2014, and December 31, 2016, is used. Standard errors are clustered on account and day. Robust t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)	(4)
	Realized Vlt.	MOM	Short Pfm.	Long Pfm.
$refund_{i,t} \times regime_t$	-0.046^{**}	-1.360^{*}	0.337	1.633^{**}
	(-2.13)	(-1.85)	(0.73)	(2.58)
$refund_{i,t}$	0.051^{***}	1.164^{**}	-0.126	-1.106^{***}
	(3.14)	(2.36)	(-0.42)	(-2.73)
Controls	Y	Y	Y	Y
Account FE	Y	Y	Y	Y
Day FE	Y	Y	Y	Y
Adj. R^2	0.4730	0.4235	0.6121	0.5246
Obs.	220,950	220,950	226,787	226,524

Table B.6: DiD Robustness: Percentile Transformation

This table reports the estimates from a DiD design that relies on the quasi-natural experiment of the IPO lottery reform on January 1, 2016. Variable $refund_{i,t}$ is the IPO refund amount standardized by the total cash available for investor i on day t; $regime_t$ is an indicator that equals to 1 if the IPO lottery result announced on day t is under the old regime, and equals to 0 if it is under the new regime; The control variables are the same to those in Table A.5. The dependent variables in column (1) - (7) are risk (realized daily return volatility in the past month), market-to-book ratio, momentum (realized return in the past month), short-term performance (return in the next month), long-term performance (return in the next year), abnormal trading volume used in Barber and Odean (2008), and last day return, respectively. All dependent variables are in percentiles sorted daily. Account and day fixed effects are added to all specifications. The sample of 9,666 investors on 238 IPO announcement days between June 17, 2014, and December 31, 2016, is used. Standard errors are clustered on account and day. Robust t-statistics are presented in parentheses. ***, **, and * indicate statistical significance at 1%, 5% and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Realized Vlt.	M-B	MOM	Short	Long	Abn. Trd.	Last Ret.
$refund_{i,t} \times regime_t$	−2.251***	−1.563***	−2.834***	1.026*	2.434***	−3.735***	−2.113***
	(−2.64)	(−2.62)	(−3.83)	(1.82)	(2.70)	(−4.43)	(−3.43)
$refund_{i,t}$	2.317***	0.972**	1.800***	−0.600	−2.308***	3.203***	1.000**
	(4.02)	(2.37)	(3.59)	(−1.24)	(−3.18)	(5.30)	(2.19)
Controls	Y	Y	Y	Y	Y	Y	Y
Account FE	Y	Y	Y	Y	Y	Y	Y
Day FE	Y	Y	Y	Y	Y	Y	Y
Adj. R^2	0.1707	0.4367	0.1384	0.0570	0.0804	0.1319	0.0712
Obs.	224,628	224,559	224,628	226,982	226,374	211,969	226,965

CHAPTER C

Supplemental Material to Chapter 5

C.1 Proof of Proposition 1

Proof. First, combine (5.2) and (5.3) to have

$$w_{\tau,t'+1} = w_{\tau,t} + \sum_{j=t}^{t'} \frac{d_{j+1} - p_j}{p_j} a_{\tau,j}. \tag{C.1}$$

Then, plug (C.1) into (5.1) to have

$$\begin{aligned} \max_{\{a_{\tau,t'}\}_{t'=t}^{\infty}} & \sum_{t'=t}^{\infty} (1-\eta)^{t'-t} \eta \mathbb{E}_t \left[w_{\tau,t} + \sum_{j=t}^{t'} \frac{d_{j+1} - p_j}{p_j} a_{\tau,j} \right] \\ & - \sum_{t'=t}^{\infty} (1-\eta)^{t'-t} \eta \frac{1}{2} \gamma \mathrm{Var}_t \left[w_{\tau,t} + \sum_{j=t}^{t'} \frac{d_{j+1} - p_j}{p_j} a_{\tau,j} \right] \\ & + f(\theta_{\tau,t}) \sum_{t'=t}^{\infty} (1-\eta)^{t'-t} \eta \sum_{j=t}^{t'} \mathbb{E}_t \left[v \left(\frac{d_{j+1} - p_j}{p_j} a_{\tau,j} \right) \right]. \end{aligned} \tag{C.2}$$

By the linearity property of expectation operator and the independence of d_j's for $t+1 \leq j \leq t'$, the first term in (C.2) can be rewritten as

$$\begin{aligned}
&\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\eta\mathbb{E}_t\left[w_{\tau,t}\right]+\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\eta\left(\sum_{j=t}^{t'}\mathbb{E}_t\left[\frac{d_{j+1}-p_j}{p_j}\right]a_{\tau,j}\right)\\
=&\frac{\eta\mathbb{E}_t\left[w_{\tau,t}\right]}{1-(1-\eta)}+\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\frac{\eta\mathbb{E}_t\left[\frac{d_{t'+1}-p_{t'}}{p_{t'}}\right]a_{\tau,t'}}{1-(1-\eta)}\\
=&\mathbb{E}_t\left[w_{\tau,t}\right]+\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\mathbb{E}_t\left[\frac{d_{t'+1}-p_{t'}}{p_{t'}}\right]a_{\tau,t'}. \qquad \text{(C.3)}
\end{aligned}$$

Similarly, the second and third terms in (C.2) can be rewritten as

$$-\frac{1}{2}\gamma\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\left(\mathrm{Var}_t\left[\frac{d_{t'+1}-p_{t'}}{p_{t'}}\right]a_{\tau,t'}^2\right), \qquad \text{(C.4)}$$

and

$$f\left(\theta_{\tau,t}\right)\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\mathbb{E}_t\left[v\left(\frac{d_{t'+1}-p_{t'}}{p_{t'}}a_{\tau,t'}\right)\right]. \qquad \text{(C.5)}$$

Finally, plug (C.3)-(C.5) back into (C.2) to have

$$\begin{aligned}
\max_{\{a_{\tau,t'}\}_{t'=t}^{\infty}}\ &\mathbb{E}_t\left[w_{\tau,t}\right]+\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\mathbb{E}_t\left[\frac{d_{t'+1}-p_{t'}}{p_{t'}}\right]a_{\tau,t'}\\
&-\frac{1}{2}\gamma\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\left(\mathrm{Var}_t\left[\frac{d_{t'+1}-p_{t'}}{p_{t'}}\right]a_{\tau,t'}^2\right)\\
&+f\left(\theta_{\tau,t}\right)\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\mathbb{E}_t\left[v\left(\frac{d_{t'+1}-p_{t'}}{p_{t'}}a_{\tau,t'}\right)\right]. \qquad \text{(C.6)}
\end{aligned}$$

Since the choice variable $a_{\tau,t'}$, $\forall t' \geq t$, only appears once in each of the three summations in (C.6), the problem can be equivalently expressed as independent problems

$$\max_{a_{\tau,t'}}\mathbb{E}_t\left[\frac{d_{t'+1}-p_{t'}}{p_{t'}}\right]a_{\tau,t'}-\frac{1}{2}\gamma\mathrm{Var}_t\left[\frac{d_{t'+1}-p_{t'}}{p_{t'}}\right]a_{\tau,t'}^2+f\left(\theta_{\tau,t}\right)\mathbb{E}_t\left[v\left(\frac{d_{t'+1}-p_{t'}}{p_{t'}}a_{\tau,t'}\right)\right] \qquad \text{(C.7)}$$

in every period t', $\forall t' \geq t$. For the decision on $a_{\tau,t}$, (C.7) is equivalent to (5.5)-(5.6). ■

C.2 Proof of Lemma 1

Proof. By definition in (5.3), $\mathbb{E}\left[v\left(\Delta w_{\tau,t+1}\right)\right]$ can be rewritten as

$$\begin{aligned}\mathbb{E}_t\left[v\left(\Delta w_{\tau,t+1}\right)\right] =& \mathbb{E}_t\left[\Delta w_{\tau,t+1} \mid \Delta w_{\tau,t+1} \geq 0\right] \cdot \mathbb{P}\left\{\Delta w_{\tau,t+1} \geq 0\right\} \\ &+ \mathbb{E}_t\left[\lambda \Delta w_{\tau,t+1} \mid \Delta w_{\tau,t+1} < 0\right] \cdot \mathbb{P}\left\{\Delta w_{\tau,t+1} < 0\right\}. \end{aligned} \tag{C.8}$$

Note that I have $\Delta w_{\tau,t+1} = \frac{d_{t+1}-p_t}{p_t} a_{\tau,t}$ and $d_{t+1} \sim \mathcal{N}\left(\mu, \sigma^2\right)$, so it is true that

$$\Delta w_{\tau,t+1} \sim \mathcal{N}\left(\frac{\mu - p_t}{p_t} a_{\tau,t}, \left(\frac{a_{\tau,t}}{p_t}\right)^2 \sigma^2\right).$$

For a normally distributed random variable $X \sim \mathcal{N}\left(m, v^2\right)$, the truncated expectation follows

$$\mathbb{E}\left[X \mid a < X < b\right] = m + \frac{\phi\left(\frac{a-m}{v}\right) - \phi\left(\frac{b-m}{v}\right)}{\Phi\left(\frac{b-m}{v}\right) - \Phi\left(\frac{a-m}{v}\right)} v, \tag{C.9}$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are the c.d.f. and p.d.f. of the standard normal distribution, respectively.

Equation (C.9) implies that

$$\mathbb{E}\left[X \mid X \geq 0\right] = m + \frac{\phi\left(\frac{-m}{v}\right)}{1 - \Phi\left(\frac{-m}{v}\right)} v, \tag{C.10}$$

$$\mathbb{E}\left[X \mid X < 0\right] = m + \frac{-\phi\left(\frac{-m}{v}\right)}{\Phi\left(\frac{-m}{v}\right)} v. \tag{C.11}$$

Now, plug $m = \frac{\mu - p_t}{p_t} a_{\tau,t}$ and $v^2 = \left(\frac{a_{\tau,t}}{p_t}\right)^2 \sigma^2$ into (C.10) and (C.11) for $a_{\tau,t} > 0$, and rewrite (C.8) as

$$
\begin{aligned}
&\mathbb{E}_t\left[v\left(\Delta w_{\tau,t+1}\right)\right] \\
=&\left(m+\frac{\phi\left(\frac{-m}{v}\right)}{1-\Phi\left(\frac{-m}{v}\right)}v\right)\cdot\left(1-\Phi\left(\frac{-m}{v}\right)\right)+\lambda\left(m+\frac{-\phi\left(\frac{-m}{v}\right)}{\Phi\left(\frac{-m}{v}\right)}v\right)\cdot\Phi\left(\frac{-m}{v}\right) \\
=&\left(\frac{\mu-p_t}{p_t}a_{\tau,t}+\frac{\phi\left(-\frac{\mu-p_t}{\sigma}\right)}{1-\Phi\left(-\frac{\mu-p_t}{\sigma}\right)}\frac{a_{\tau,t}}{p_t}\sigma\right)\cdot\left(1-\Phi\left(-\frac{\mu-p_t}{\sigma}\right)\right) \\
&+\lambda\left(\frac{\mu-p_t}{p_t}a_{\tau,t}+\frac{-\phi\left(-\frac{\mu-p_t}{\sigma}\right)}{\Phi\left(-\frac{\mu-p_t}{\sigma}\right)}\frac{a_{\tau,t}}{p_t}\sigma\right)\cdot\Phi\left(-\frac{\mu-p_t}{\sigma}\right) \\
=&a_{\tau,t}\frac{1}{p_t}\left[\left(\mu-p_t\right)\left[1+\left(\lambda-1\right)\Phi\left(-\frac{\mu-p_t}{\sigma}\right)\right]+\phi\left(-\frac{\mu-p_t}{\sigma}\right)\sigma\left(1-\lambda\right)\right]
\end{aligned}
$$

which is (5.7) with (5.8) plugged in. ■

C.3 Proof of Proposition 2

Proof. Let's first solve for $a^{\star}_{\tau,t}$, i.e., the unconstrained demand by cohort τ in period t.

Rewrite the objective function (5.5) to have

$$\max_{a_{\tau,t}} \frac{\mu-p_t}{p_t}a_{\tau,t}-\frac{1}{2}\gamma\frac{\sigma^2}{p_t^2}a_{\tau,t}^2+f\left(\theta_{\tau,t}\right)\frac{g\left(p_t\right)}{p_t}a_{\tau,t}, \tag{C.12}$$

and the first-order condition for $a_{\tau,t}$ is

$$
\begin{aligned}
0&=\frac{\mu-p_t}{p_t}-\gamma\frac{\sigma^2}{p_t^2}a^{\star}_{\tau,t}+f\left(\theta_{\tau,t}\right)\frac{g\left(p_t\right)}{p_t} \\
\Rightarrow a^{\star}_{\tau,t}&=\frac{p_t\left(\mu-p_t\right)}{\gamma\sigma^2}+\frac{p_t g\left(p_t\right)}{\gamma\sigma^2}f\left(\theta_{\tau,t}\right)
\end{aligned}
$$

which gives me exactly (5.10).

Now consider the case with short-selling and borrowing constraint. Since the objective function (C.12) is quadratic and concave in $a_{\tau,t}$, the closer $a_{\tau,t}$ is to the $a^{\star}_{\tau,t}$, the greater objective function (C.12) will be. Therefore, when $a^{\star}_{\tau,t}<0$, the closest feasible $a_{\tau,t}$ will be

$a_{\tau,t} = 0$ under the short-selling constraint. Similarly, when $a^{\star}_{\tau,t} > w_{\tau,t}$, the closest feasible $a_{\tau,t}$ will be $a_{\tau,t} = w_{\tau,t}$ under the borrowing constraint. ■

C.4 Proof of Corollary 1

Proof. For cohort $\tau = 0$ in period $t = 0$ with $\theta_{0,0} = 0$, (5.10) implies

$$a^{\star}_{0,0} = \frac{p^{\star}_0 \left(\mu - p^{\star}_0\right)}{\gamma\sigma^2} + \frac{p^{\star}_0 g\left(p^{\star}_0\right)}{\gamma\sigma^2}, \tag{C.13}$$

where $p^{\star}_0$ is the market price when the interior solution is attained.

Since $\tau = 0$ is the only existing cohort with unit mass, the market clearing condition in (5.11) becomes $a^{\star}_{0,0} = p^{\star}_0$. Finally, I replace $a^{\star}_{0,0}$ with $p^{\star}_0$ on the left side of (C.13) to have

$$p^{\star}_0 = \frac{p^{\star}_0 \left(\mu - p^{\star}_0\right)}{\gamma\sigma^2} + \frac{p^{\star}_0 g\left(p^{\star}_0\right)}{\gamma\sigma^2} \quad \Rightarrow \quad p^{\star}_0 = \mu - \gamma\sigma^2 + g\left(p^{\star}_0\right)$$

which completes the proof for the interior solution. Corner solutions are given by (5.11). ■

C.5 Other Parameter Values for Figure A.13

Figure C.1: Future Sensitivities: Other Parameter Values

The figure plots the sensitivity $f\left(\theta_{\tau,\tau+j}\right)$ of cohort τ in period $\tau+j$ $(j=1,2,3,4)$ against risky holding $a_{\tau,\tau}$ in the initial period, where $f\left(\cdot\right)$ is defined in equation (5.4). Each panel is based on the deterministic steady state with $\beta=1$ and a set of values for λ, γ and η as specified. The initial wealth is set such that the optimal idle cash holding of existing cohorts in the model matches the ratio of cash in brokerage accounts in the sample. The gray band marks the interval $\left[\underline{a}_{\tau,\tau},\overline{a}_{\tau,\tau}\right]$, where $\underline{a}_{\tau,\tau}$ and $\overline{a}_{\tau,\tau}$ are the minimum and maximum optimal risky asset holding corresponding to cash temperature 0 and 1, respectively.

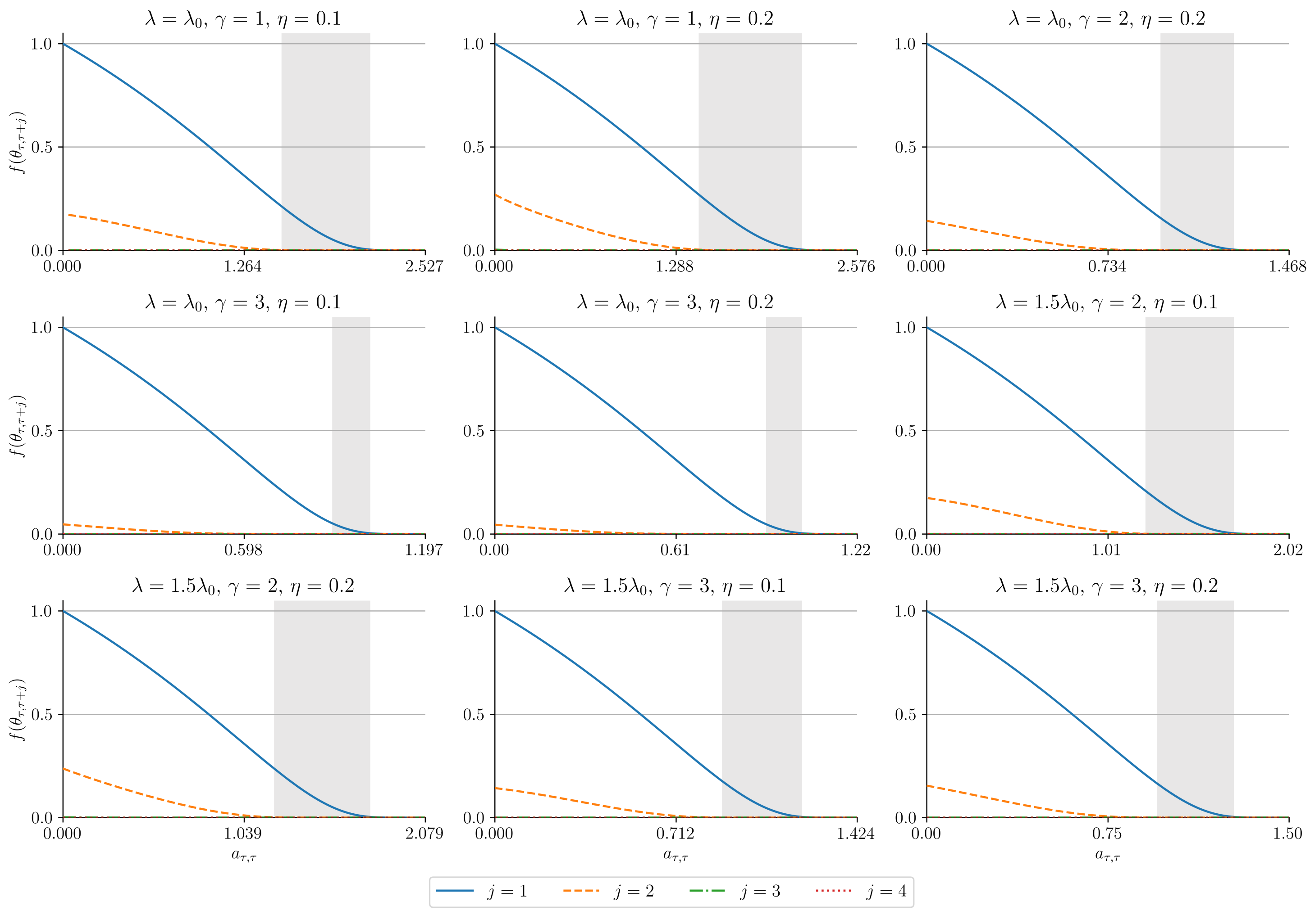
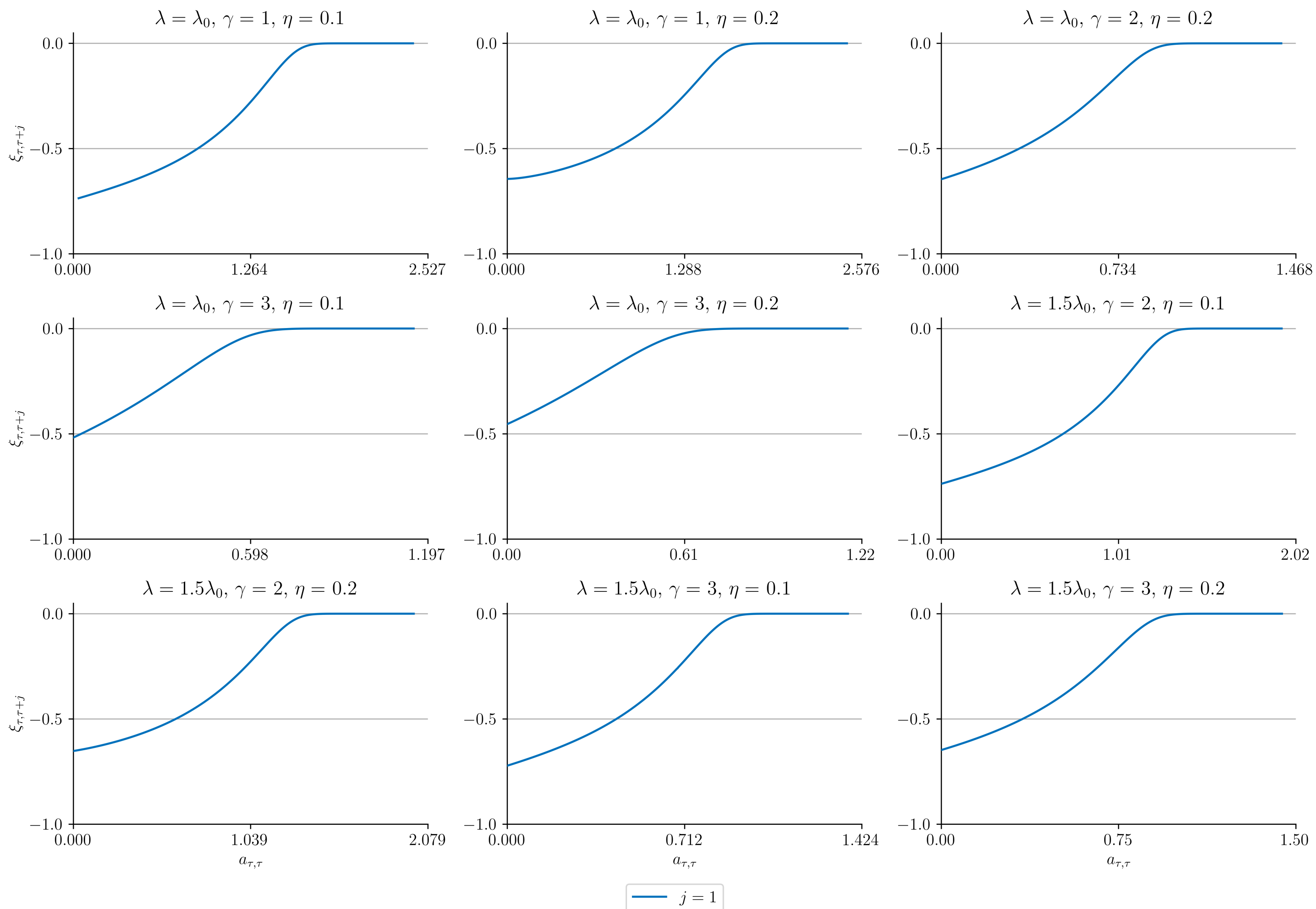


Figure C.2: Temperature Smoothing: Other Parameter Values

The figure plots temperature smoothing term $\xi_{\tau,\tau+1}$ of cohort τ in period $\tau+1$ defined in (5.19) against risky holding $a_{\tau,\tau}$ in the initial period. Each panel is based on the deterministic steady state with $\beta = 1$ and a set of values for λ, γ and η as specified. The initial wealth is set such that the optimal idle cash holding of existing cohorts in the model matches the ratio of cash in brokerage accounts in the sample.



C.6 Proof of Proposition 3

I first show two lemmas, which are useful in the proof of Proposition 3. Lemma 2 rewrites the objective function (5.14).

Lemma 2. *With condition (5.17), the objective function (5.14) for cohort τ in period t is equivalent to*

$$\max_{\{a_{\tau,t'}\}_{t'=t}^{\infty}} \mathbb{E}_t\left[w_{\tau,t}\right] + \sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\,\mathbb{E}_t\left[\frac{d_{t'+1}-\hat{p}}{\hat{p}}\right]a_{\tau,t'} - \frac{1}{2}\gamma\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\left(Var_t\left[\frac{d_{t'+1}-\hat{p}}{\hat{p}}\right]a_{\tau,t'}^2\right)$$
$$+ f\left(\theta_{\tau,t}\right)\mathbb{E}_t\left[v\left(\frac{d_{t+1}-\hat{p}}{\hat{p}}a_{\tau,t}\right)\right] + (1-\eta)\,\mathbb{E}_t\left[f\left(\theta_{\tau,t+1}\right)\right]\mathbb{E}_t\left[v\left(\frac{d_{t+2}-\hat{p}}{\hat{p}}a_{\tau,t+1}\right)\right], \tag{C.14}$$

where $\hat{p}$ is the deterministic steady state price of the risky asset.

Proof. The first three terms in (C.14) correspond to the mean-variance utility over exiting consumption and are given by (C.6) in the proof of Proposition 1. To obtain the last two terms in (C.14), rewrite the prospect theory utility term in (5.14), i.e.,

$$\begin{aligned}
&\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\eta\mathbb{E}_t\left[\sum_{j=t}^{t'}f\left(\theta_{\tau,j}\right)v\left(\Delta w_{\tau,j+1}\right)\right]\\
=&\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\eta f\left(\theta_{\tau,t}\right)\mathbb{E}_t\left[v\left(\Delta w_{\tau,t+1}\right)\right] + \sum_{j=1}^{\infty}\sum_{t'=t+j}^{\infty}(1-\eta)^{t'-t}\eta\mathbb{E}_t\left[f\left(\theta_{\tau,t+j}\right)v\left(\Delta w_{\tau,t+j+1}\right)\right]\\
=&f\left(\theta_{\tau,t}\right)\mathbb{E}_t\left[v\left(\Delta w_{\tau,t+1}\right)\right] + \sum_{j=1}^{\infty}(1-\eta)^{j}\,\mathbb{E}_t\left[f\left(\theta_{\tau,t+j}\right)v\left(\frac{d_{t+j+1}-\hat{p}}{\hat{p}}a_{\tau,t+j}\right)\right]\\
=&f\left(\theta_{\tau,t}\right)\mathbb{E}_t\left[v\left(\Delta w_{\tau,t+1}\right)\right] + \sum_{j=1}^{\infty}(1-\eta)^{j}\,\mathbb{E}_t\left[f\left(\theta_{\tau,t+j}\right)\right]\mathbb{E}_t\left[v\left(\frac{d_{t+j+1}-\hat{p}}{\hat{p}}a_{\tau,t+j}\right)\right]
\end{aligned} \tag{C.15}$$

where the last step is true because the returns of risky assets in different periods are i.i.d. by assumption. Note that

$$\frac{\partial\mathbb{E}_t\left[v\left(\frac{d_{t+j+1}-\hat{p}}{\hat{p}}a_{\tau,t+j}\right)\right]}{\partial a_{\tau,t}} = 0$$

holds for $\forall j \geq 1$. So, condition (5.17) implies

$$\frac{\partial \mathbb{E}_t\left[f\left(\theta_{\tau,t+j}\right)\right] \mathbb{E}_t\left[v\left(\frac{d_{t+j+1}-\hat{p}}{\hat{p}} a_{\tau,t+j}\right)\right]}{\partial a_{\tau,t}} = \begin{cases} \frac{\partial \mathbb{E}_t\left[f\left(\theta_{\tau,t+j}\right)\right]}{\partial a_{\tau,t}} \mathbb{E}_t\left[v\left(\frac{d_{t+j+1}-\hat{p}}{\hat{p}} a_{\tau,t+j}\right)\right] & \text{if } j = 1, \\ 0 & \text{if } j > 1. \end{cases}$$

Therefore, the terms in (C.15) with $j \geq 2$ can be dropped when solving for $\hat{a}^{\star}_{\tau,t}$ as they will not appear in the first-order condition, which leads to objective function (C.14). ■

Lemma 3 provides an expression for $\mathbb{E}_t\left[f\left(\theta_{\tau,t+1}\right)\right]$ in objective function (C.14).

Lemma 3. *The period t expectation of sensitivity in the next period is given by*

$$\mathbb{E}_t\left[f\left(\theta_{\tau,t+1}\right)\right] = f\left(\theta_{\tau,t}\right) \exp\left\{\frac{1}{\theta_{\tau,t}-1} \frac{\frac{\mu}{p_t} a_{\tau,t}}{w_{\tau,t}-a_{\tau,t}} + \frac{1}{2}\left(\frac{1}{\theta_{\tau,t}-1}\right)^2 \frac{a^2_{\tau,t}}{\hat{p}^2\left(w_{\tau,t}-a_{\tau,t}\right)^2}\sigma^2\right\},$$

where $\hat{p}$ is the deterministic steady state price of the risky asset.

Proof. The law of motion of cash temperature in equation (5.4) implies

$$f\left(\theta_{\tau,t+1}\right) = \exp\left\{\frac{\theta_{\tau,t}\beta\left(w_{\tau,t}-a_{\tau,t}\right) + \frac{a_{\tau,t}}{\hat{p}} d_{t+1}}{\left(\theta_{\tau,t}\beta - 1\right)\left(w_{\tau,t}-a_{\tau,t}\right)}\right\}. \tag{C.16}$$

With simplifying assumption $\beta = 1$, (C.16) can be rewritten as

$$f\left(\theta_{\tau,t+1}\right) = \underbrace{\exp\left\{1 + \frac{1}{\theta_{\tau,t}-1}\right\}}_{=f\left(\theta_{\tau,t}\right)} \exp\left\{\frac{\frac{d_{t+1}}{\hat{p}} a_{\tau,t}}{\left(\theta_{\tau,t}-1\right)\left(w_{\tau,t}-a_{\tau,t}\right)}\right\}.$$

So, its expectation in period t satisfies

$$\begin{aligned} \mathbb{E}_t\left[f\left(\theta_{\tau,t+1}\right)\right] &= f\left(\theta_{\tau,t}\right) \mathbb{E}_t\left[\exp\left\{\frac{\frac{d_{t+1}}{\hat{p}} a_{\tau,t}}{\left(\theta_{\tau,t}-1\right)\left(w_{\tau,t}-a_{\tau,t}\right)}\right\}\right] \\ &= f\left(\theta_{\tau,t}\right) \exp\left\{\frac{\mu a_{\tau,t}}{\hat{p}\left(\theta_{\tau,t}-1\right)\left(w_{\tau,t}-a_{\tau,t}\right)} + \frac{\sigma^2 a^2_{\tau,t}}{2\hat{p}^2\left(\theta_{\tau,t}-1\right)^2\left(w_{\tau,t}-a_{\tau,t}\right)^2}\right\}, \end{aligned} \tag{C.17}$$

where the last step is true because $\exp\left\{\frac{\frac{d_{t+1}}{\hat{p}}a_{\tau,t}}{(\theta_{\tau,t}-1)(w_{\tau,t}-a_{\tau,t})}\right\}$ follows a log-normal distribution.

■

Now, I combine Lemma 2 and Lemma 3 to show Proposition 3.

Proof. The distribution of risky asset payoff implies

$$\mathbb{E}_t\left[\frac{d_{t'+1}-\hat{p}}{\hat{p}}\right] = \frac{\mu-\hat{p}}{\hat{p}},\ \forall t' \geq t, \tag{C.18}$$

$$\mathrm{Var}_t\left[\frac{d_{t'+1}-\hat{p}}{\hat{p}}\right] = \frac{\sigma^2}{\hat{p}^2},\ \forall t' \geq t. \tag{C.19}$$

With Lemma 1, I can rewrite the prospect theory terms as

$$\mathbb{E}_t\left[v\left(\frac{d_{t+1}-\hat{p}}{\hat{p}}a_{\tau,t}\right)\right] = \frac{g(\hat{p})}{\hat{p}}a_{\tau,t} \tag{C.20}$$

$$\mathbb{E}_t\left[v\left(\frac{d_{t+2}-\hat{p}}{\hat{p}}a_{\tau,t+1}\right)\right] = \frac{g(\hat{p})}{\hat{p}}a_{\tau,t+1} \tag{C.21}$$

where (C.21) is true because $a_{\tau,t+1}$ is a choice variable determined in period t.

Then, I plug (C.18)-(C.21) and expression (C.17) for $\mathbb{E}_t\left[f\left(\theta_{\tau,t+1}\right)\right]$ into (C.14) to equivalently rewrite objective function (5.14) as

$$\begin{aligned}
&\max_{\{a_{\tau,t'}\}_{t'=t}^{\infty}} \sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\frac{\mu-\hat{p}}{\hat{p}}a_{\tau,t'} - \frac{1}{2}\gamma\sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\left(\frac{\sigma^2}{\hat{p}^2}a_{\tau,t'}^2\right) \\
&\qquad + f\left(\theta_{\tau,t}\right)\frac{g(\hat{p})}{\hat{p}}a_{\tau,t} + (1-\eta)\,\mathbb{E}_t\left[f\left(\theta_{\tau,t+1}\right)\right]\frac{g(\hat{p})}{\hat{p}}a_{\tau,t+1} \\
\Leftrightarrow &\max_{\{a_{\tau,t'}\}_{t'=t}^{\infty}} \sum_{t'=t}^{\infty}(1-\eta)^{t'-t}\left(\frac{\mu-\hat{p}}{\hat{p}}a_{\tau,t'} - \frac{1}{2}\gamma\frac{\sigma^2}{\hat{p}^2}a_{\tau,t'}^2\right) + f\left(\theta_{\tau,t}\right)\frac{g(\hat{p})}{\hat{p}}a_{\tau,t} + (1-\eta)\, f\left(\theta_{\tau,t}\right) \\
&\qquad \times \exp\left\{\frac{\mu a_{\tau,t}}{\hat{p}\left(\theta_{\tau,t}-1\right)\left(w_{\tau,t}-a_{\tau,t}\right)} + \frac{\sigma^2 a_{\tau,t}^2}{2\hat{p}^2\left(\theta_{\tau,t}-1\right)^2\left(w_{\tau,t}-a_{\tau,t}\right)^2}\right\}\frac{g(\hat{p})}{\hat{p}}a_{\tau,t+1}
\end{aligned}$$

and take partial derivative with respect to $a_{\tau,t}$ to obtain the following first-order condition:

$$
\begin{aligned}
0 =& \frac{\mu-\hat{p}}{\hat{p}} - \gamma\frac{\sigma^2}{\hat{p}^2}\hat{a}^\star_{\tau,t} + \frac{g\left(\hat{p}\right)}{\hat{p}} f\left(\theta_{\tau,t}\right) + \left(1-\eta\right)\frac{g\left(\hat{p}\right)}{\hat{p}}\hat{a}^\star_{\tau,t+1} f\left(\theta_{\tau,t}\right) \\
&\times \underbrace{\exp\left\{\frac{\mu\hat{a}^\star_{\tau,t}}{\hat{p}\left(\theta_{\tau,t}-1\right)\left(w_{\tau,t}-\hat{a}^\star_{\tau,t}\right)} + \frac{\sigma^2\left(\hat{a}^\star_{\tau,t}\right)^2}{2\hat{p}^2\left(\theta_{\tau,t}-1\right)^2\left(w_{\tau,t}-\hat{a}^\star_{\tau,t}\right)^2}\right\}}_{\equiv A_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right)} \\
&\times \underbrace{\left[\frac{\mu}{\hat{p}\left(\theta_{\tau,t}-1\right)} + \frac{\sigma^2\hat{a}^\star_{\tau,t}}{\hat{p}^2\left(\theta_{\tau,t}-1\right)^2\left(w_{\tau,t}-\hat{a}^\star_{\tau,t}\right)}\right]}_{\equiv B_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right)} \underbrace{\frac{w_{\tau,t}}{\left(w_{\tau,t}-\hat{a}^\star_{\tau,t}\right)^2}}_{\equiv C_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right)}
\end{aligned}
\tag{C.22}
$$

where $\hat{a}^\star_{\tau,t}$ and $\hat{a}^\star_{\tau,t+1}$ are optimal portfolio weight on the risky asset in period t and $t+1$, respectively. Finally, I obtain (5.18) by rearranging (C.22), i.e.,

$$
\begin{aligned}
\hat{a}^\star_{\tau,t} =& \frac{\hat{p}\left(\mu-\hat{p}\right)}{\gamma\sigma^2} + \frac{\hat{p}g\left(\hat{p}\right)}{\gamma\sigma^2} f\left(\theta_{\tau,t}\right) \\
&\times \left[1 + \underbrace{\left(1-\eta\right)\hat{a}^\star_{\tau,t+1} A_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right) B_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right) C_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right)}_{=\xi_{\tau,t}\left(\hat{a}^\star_{\tau,t},\hat{a}^\star_{\tau,t+1};\theta_{\tau,t},w_{\tau,t}\right)}\right],
\end{aligned}
\tag{C.23}
$$

where $A_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right)$, $B_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right)$, and $C_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right)$ are defined in (C.22). ■

C.7 Proof of Proposition 4

Proof. There are four regularity conditions for Proposition 4. The first requires $g\left(\hat{p}\right) < 0$ for positive risky holding, which holds if λ is not too small for a given pair of (μ,σ). The second requires $a^\star_{\tau,\tau} > 0$, which implies the optimal risky holdings under the myopic setting are always positive. The third and fourth both require that wealth is not too small, which are given by (C.27) and (C.37), respectively.

Note that $A_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right) > 0$ and $C_{\tau,t}\left(\hat{a}^\star_{\tau,t},\theta_{\tau,t},w_{\tau,t}\right) > 0$ are trivially true. Given the regularity condition $g(\hat{p}) < 0$ for positive risky holding, it is true by definition of $g(\cdot)$ in (5.8) that

$$
\begin{aligned}
&(\mu - \hat{p}) \left[1 + (\lambda - 1) \Phi \left(-\frac{\mu - \hat{p}}{\sigma}\right)\right] + \phi \left(-\frac{\mu - \hat{p}}{\sigma}\right) \sigma (1 - \lambda) < 0 \\
\Rightarrow &(\mu - \hat{p}) \left[-1 + (1 - \lambda) \Phi \left(-\frac{\mu - \hat{p}}{\sigma}\right)\right] + \phi \left(-\frac{\mu - \hat{p}}{\sigma}\right) \sigma (\lambda - 1) > 0.
\end{aligned}
\tag{C.24}
$$

When the risky holding becomes negative, $g(\hat{p})$ can be obtained by the same algebra in Lemma 1, i.e.,

$$
(\mu - \hat{p}) \left[\lambda + (1 - \lambda) \Phi \left(-\frac{\mu - \hat{p}}{\sigma}\right)\right] + \phi \left(-\frac{\mu - \hat{p}}{\sigma}\right) \sigma (\lambda - 1) > 0, \tag{C.25}
$$

which is true because the left-hand side of inequality (C.25) is greater than the left-hand side of inequality (C.24) with $\mu - \hat{p} > 0$ implied by the first and second regularity conditions given at the beginning of this proof. So, $\hat{a}^{\star}_{\tau,t+1} g(\hat{p}) < 0$ always holds. Without loss of generality, let $\hat{a}^{\star}_{\tau,t+1} > 0$, which is consistent with the temperature smoothing pattern to be shown in the rest of this proof. Now, it is clear that the sign of $\xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t}, \hat{a}^{\star}_{\tau,t+1}; \theta_{\tau,t}, w_{\tau,t}\right)$ is determined by the sign of $B_{\tau,t}\left(\hat{a}^{\star}_{\tau,t}, \theta_{\tau,t}, w_{\tau,t}\right)$ in (C.23). To proceed, I set

$$
B_{\tau,t}\left(\hat{a}^{\star}_{\tau,t}, \theta_{\tau,t}, w_{\tau,t}\right) < 0,
$$

which is equivalent to

$$
\hat{a}^{\star}_{\tau,t} < \hat{a}^{\text{cutoff}}_{\tau,t}, \text{ where } \hat{a}^{\text{cutoff}}_{\tau,t} = \frac{\mu \hat{p} (1 - \theta_{\tau,t}) w_{\tau,t}}{\sigma^2 + \mu \hat{p} (1 - \theta_{\tau,t})}. \tag{C.26}
$$

With $\theta_{\tau,\tau} = 0$ and $g(\hat{p}) < 0$, (C.26) holds for $t = \tau$ if and only if

$$\hat{a}^{\star}_{\tau,\tau} < \frac{\mu \hat{p} w_{\tau,\tau}}{\sigma^2 + \mu \hat{p}}$$

$$\Leftrightarrow w_{\tau,\tau} > a^{\star}_{\tau,\tau} \left(1 + \frac{\sigma^2}{\mu \hat{p}} \right) \tag{C.27}$$

which gives me the third regularity condition. So, $g(\hat{p}) < 0$ and (C.27) combined lead to $\xi_{\tau,\tau}\left(\hat{a}^{\star}_{\tau,\tau}, \hat{a}^{\star}_{\tau,\tau+1}; \theta_{\tau,\tau}, w_{\tau,\tau}\right) < 0$.

Then, by way of contradiction, I show that $\exists t_0 > \tau$ such that

$$\hat{a}^{\star}_{\tau,t_0} > \hat{a}^{\text{cutoff}}_{\tau,t_0} \;\Leftrightarrow\; \xi_{\tau,t_0}\left(\hat{a}^{\star}_{\tau,t_0}, \hat{a}^{\star}_{\tau,t_0+1}; \theta_{\tau,t_0}, w_{\tau,t_0}\right) > 0. \tag{C.28}$$

Suppose the claim does not hold, then it means that

$$\hat{a}^{\star}_{\tau,t} < \frac{\mu \hat{p} \left(1 - \theta_{\tau,t}\right) w_{\tau,t}}{\sigma^2 + \mu \hat{p} \left(1 - \theta_{\tau,t}\right)}, \;\forall t \geq \tau, \tag{C.29}$$

which implies $\hat{a}^{\star}_{\tau,t} > a^{\star}_{\tau,t}$, $\forall t \geq \tau$. Note that the simplifying assumption $\beta = 1$ ensures monotonicity of $\{\theta_{\tau,t}\}_{t=\tau}^{\infty}$. Thus, I have

$$\hat{a}^{\star}_{\tau,t} > a^{\star}_{\tau,t} > a^{\star}_{\tau,\tau} > 0, \;\forall t \geq \tau, \tag{C.30}$$

which implies

$$\lim_{t \to \infty} \theta_{\tau,t} = 1. \tag{C.31}$$

With (C.31), I also have

$$\lim_{t \to \infty} \hat{a}^{\text{cutoff}}_{\tau,t} = 0, \tag{C.32}$$

where $\hat{a}^{\text{cutoff}}_{\tau,t}$ is defined in (C.26). Now, combine (C.30) and (C.32) to conclude that $\exists t_0 > \tau$ such that (C.28) holds, which contradicts with (C.29).

Finally, I show long-run convergence, i.e., $\lim_{\theta_{\tau,t} \to 1} \hat{a}^{\star}_{\tau,t} = \lim_{\theta_{\tau,t} \to 1} a^{\star}_{\tau,t}$. To see why

$\lim_{\theta_{\tau,t}\to 1} \hat{a}^{\star}_{\tau,t}$ exists, first assume

$$\lim_{\theta_{\tau,t}\to 1} \hat{a}^{\star}_{\tau,t} < \lim_{\theta_{\tau,t}\to 1} \hat{a}^{\text{cutoff}}_{\tau,t}, \tag{C.33}$$

which means

$$\lim_{\theta_{\tau,t}\to 1} \hat{a}^{\star}_{\tau,t} > \lim_{\theta_{\tau,t}\to 1} a^{\star}_{\tau,t} = \frac{\hat{p}\left(\mu - \hat{p}\right)}{\gamma\sigma^2} > 0. \tag{C.34}$$

The last inequality in (C.34) holds because of the first and second regularity condition, i.e., $g\left(\hat{p}\right) < 0$ and $a^{\star}_{\tau,\tau} > 0$. However, by (C.26),

$$\lim_{\theta_{\tau,t}\to 1} \hat{a}^{\text{cutoff}}_{\tau,t} = 0. \tag{C.35}$$

(C.33) and (C.35) combined imply

$$\lim_{\theta_{\tau,t}\to 1} \hat{a}^{\star}_{\tau,t} < \lim_{\theta_{\tau,t}\to 1} \hat{a}^{\text{cutoff}}_{\tau,t} = 0,$$

which contradicts with (C.34). Therefore, it must be the case that

$$\lim_{\theta_{\tau,t}\to 1} \hat{a}^{\star}_{\tau,t} \geq \lim_{\theta_{\tau,t}\to 1} \hat{a}^{\text{cutoff}}_{\tau,t} = 0,$$

and therefore

$$0 = \lim_{\theta_{\tau,t}\to 1} \hat{a}^{\text{cutoff}}_{\tau,t} \leq \lim_{\theta_{\tau,t}\to 1} \hat{a}^{\star}_{\tau,t} \leq \lim_{\theta_{\tau,t}\to 1} a^{\star}_{\tau,t} = \frac{\hat{p}\left(\mu - \hat{p}\right)}{\gamma\sigma^2}, \tag{C.36}$$

i.e., $\lim_{\theta_{\tau,t}\to 1} \hat{a}^{\star}_{\tau,t}$ is bounded. I introduce the last regularity condition, i.e.,

$$w_{\tau,t} > \frac{\hat{p}\left(\mu - \hat{p}\right)}{\gamma\sigma^2},\ \forall t \geq \tau \tag{C.37}$$

to ensure that $w_{\tau,t} > \hat{a}^{\star}_{\tau,t}$ always holds and that

$$\lim_{\theta_{\tau,t}\to 1} \frac{\hat{a}^{\star}_{\tau,t}}{w_{\tau,t} - \hat{a}^{\star}_{\tau,t}} \text{ and } \lim_{\theta_{\tau,t}\to 1} \left(\frac{\hat{a}^{\star}_{\tau,t}}{w_{\tau,t} - \hat{a}^{\star}_{\tau,t}}\right)^2$$

are both bounded, which is crucial in proving the existence of $\lim_{\theta_{\tau,t}\to 1} \hat{a}^{\star}_{\tau,t}$ as it guarantees that q_1, q_2, and q_4 in (C.38) are positive and bounded real numbers. Condition (C.36) and (C.37) also mean that the interior solution is attained when cash temperature is close to 1.

Then, (5.18) implies

$$\lim_{\theta_{\tau,t}\to 1} \hat{a}^{\star}_{\tau,t} = \lim_{\theta_{\tau,t}\to 1} a^{\star}_{\tau,t} + \lim_{\theta_{\tau,t}\to 1} \frac{\hat{p}g\left(\hat{p}\right)}{\gamma\sigma^2} \xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t}, \hat{a}^{\star}_{\tau,t+1}; \theta_{\tau,t}, w_{\tau,t}\right) f\left(\theta_{\tau,t}\right).$$

Since $f\left(\theta_{\tau,t}\right)$ is bounded on $[0,1]$, it suffices to show $\lim_{\theta_{\tau,t}\to 1} \xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t}, \hat{a}^{\star}_{\tau,t+1}; \theta_{\tau,t}, w_{\tau,t}\right) = 0$. To see why this is true, I decompose it as follows,

$$\begin{aligned}
&\lim_{\theta_{\tau,t}\to 1} \xi_{\tau,t}\left(\hat{a}^{\star}_{\tau,t}, \hat{a}^{\star}_{\tau,t+1}; \theta_{\tau,t}, w_{\tau,t}\right) \\
&= \lim_{\theta_{\tau,t}\to 1} \left(1-\eta\right) \hat{a}^{\star}_{\tau,t+1} \frac{w_{\tau,t}}{\left(w_{\tau,t} - \hat{a}^{\star}_{\tau,t}\right)^2} \\
&\times \lim_{\theta_{\tau,t}\to 1} \exp\left\{ \underbrace{\frac{1}{\theta_{\tau,t}-1}}_{\equiv x} \underbrace{\frac{\mu \hat{a}^{\star}_{\tau,t}}{\hat{p}\left(w_{\tau,t} - \hat{a}^{\star}_{\tau,t}\right)}}_{\equiv q_1} + \frac{1}{\left(\theta_{\tau,t}-1\right)^2} \underbrace{\frac{\sigma^2\left(\hat{a}^{\star}_{\tau,t}\right)^2}{2\hat{p}^2\left(w_{\tau,t} - \hat{a}^{\star}_{\tau,t}\right)^2}}_{\equiv q_2} \right\} \\
&\qquad \times \left[\frac{1}{\theta_{\tau,t}-1} \underbrace{\frac{\mu}{\hat{p}}}_{\equiv q_3} + \frac{1}{\left(\theta_{\tau,t}-1\right)^2} \underbrace{\frac{\sigma^2 \hat{a}^{\star}_{\tau,t}}{\hat{p}^2\left(w_{\tau,t} - \hat{a}^{\star}_{\tau,t}\right)}}_{\equiv q_4} \right]
\end{aligned} \tag{C.38}$$

where q_1, q_2, q_3, and q_4 are finite positive real numbers.

Then, the second limit on the right-hand side of (C.38) can be rewritten as

$$\begin{aligned}
&\lim_{x\to-\infty} \exp\left\{q_1 x + q_2 x^2\right\}\left(q_3 x + q_4 x^2\right) \\
=&\lim_{x\to-\infty} \frac{q_3 x + q_4 x^2}{\exp\{-q_1 x - q_2 x^2\}} \\
[\because \text{L'Hôpital's rule}] =&\lim_{x\to-\infty} \frac{q_3 + 2q_4 x}{\exp\{-q_1 x - q_2 x^2\}(-q_1 - 2q_2 x)} \\
[\because \text{L'Hôpital's rule}] =&\lim_{x\to-\infty} \frac{2q_4}{\exp\{-q_1 x - q_2 x^2\}\left[(-q_1 - 2q_2 x)^2 - 2q_2\right]} \\
=&0.
\end{aligned}$$

Therefore, I have shown

$$\lim_{\theta_{\tau,t}\to 1} \xi_{\tau,t}\left(\hat{a}^\star_{\tau,t}, \hat{a}^\star_{\tau,t+1}; \theta_{\tau,t}, w_{\tau,t}\right) = 0,$$

and

$$\lim_{\theta_{\tau,t}\to 1} \hat{a}^\star_{\tau,t} = \lim_{\theta_{\tau,t}\to 1} a^\star_{\tau,t} = \frac{\hat{p}\,(\mu - \hat{p})}{\gamma\sigma^2},$$

which verifies the existence of $\lim_{\theta_{\tau,t}\to 1} \hat{a}^\star_{\tau,t}$ and completes the proof. ■