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DO FINANCIAL FACTORS DRIVE AGGREGATE PRODUCTIVITY? EVIDENCE FROM INDIAN MANUFACTURING ESTABLISHMENTS

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To Holly, without whom this would not have been possible.

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ABSTRACT

Numerous countries have implemented financial reforms in the past three decades, but how these reforms affect economic growth has not been established. I develop a novel dynamic equilibrium model with heterogeneous firms and endogenous leverage to isolate the effects of changing financial frictions on economic growth. Changes in financial frictions affect aggregate productivity by shifting the allocation of resources across firms. However, common shocks to productivity unrelated to finance also change the allocation of resources across firms, because more-productive firms respond to shocks by changing leverage. I show using plant-level microdata from India that although difference in difference regressions and aggregate productivity decompositions suggest that financial reforms have led to economic growth in India since 1990, I can generate the same patterns in my calibrated model with only common shocks to productivity. In addition, when I calibrate a model to match the dynamic evolution of the size-productivity distribution in India, I find that financial factors explain 71% of Indian labor productivity growth from 1990 to 1995, but only 2%–8% from 1995 to 2011. My work suggests that factors that affect productivity within firms are more important determinants of aggregate productivity growth than financial development, and might explain why developing economies lag behind the United States in growth and productivity.

CHAPTER 1

INTRODUCTION

1.1 Background

Although dozens of countries have instituted financial reforms in the past three decades, economists still debate whether and how these reforms affect economic growth. One channel through which better-functioning financial markets might increase economic growth is through a superior allocation of resources, which raises aggregate productivity. This channel represents conventional wisdom among financial economists; however, research focusing on this channel has not tied improved financial markets directly to aggregate productivity growth through this channel.

In this paper, I gauge the quantitative effect of financial development on aggregate productivity growth by separating shocks to financial development from common shocks to productivity. In chapter 2, I apply two reduced-form measures of such a separation—difference in difference regressions and a decomposition of aggregate productivity into within- and between-firm components—to establishment-level microdata from Indian manufacturing plants, and show that both suggest that reallocation of resources from less- to more-productive firms has occurred in India from 1990 to 2011. I then calibrate a dynamic equilibrium model in which financial development can drive such reallocation, and show that neither measure is a necessary or sufficient indicator of financial development when common shocks to productivity are large.

I extend this result in chapter 3 by simplifying the model and using it to match the dynamic evolution of the aggregate productivity decomposition in the data from 1990–2011. I find that financial shocks explain a substantial part of productivity growth only in the first few years of the sample, from 1990 to 1995. From 1995 to 2011, common shocks to labor productivity can explain both the within-firm and the across-firm components of aggregate labor productivity growth. Although the level of financial frictions is important in understanding how common shocks to productivity affect the allocation of resources across firms, financial frictions themselves are not changing in India after 1995. Thus, my work suggests that factors that affect productivity within firms, rather than the allocation of resources across firms, are important for understanding why India and other developing countries continue to lag behind the United States in labor productivity and economic growth.

In chapter 4 I develop intuition for my results, and tie them to previous work on the efficient allocation of resources in India, using a simple static model. I use this model to illustrate the difference between the misallocation of resources—a focus of much recent literature, not just on India, but also Mexico, China, and other developing economies—and the change in the allocation of resources over time. This conceptual difference is crucial for understanding the difference between my analysis, on other analyses of Indian manufacturing plants over this sample period.

India is an ideal laboratory for analyzing the effects of financial development, because it began a major series of financial reforms in 1991, after the start of my sample period. Although the biggest changes occurred near the beginning of my sample, reforms continued over the entire sample period. At the same time, the labor productivity of the Indian manufacturing sector has grown tremendously since 1990 (Figure 1.1). I analyze the effects of financial reforms by hypothesing that they affect the real side of the economy through the reallocation of resources across firms, which I show can affect aggregate productivity. Putting the empirical and theoretical evidence together, I find that common shocks to productivity better explain the Indian data than shocks to financial development that reallocate resources.

Empirically, standard techniques used to identify financial development separately from common shocks to productivity suggest that financial development is a driver of economic growth in the Indian manufacturing sector from 1990 to 2011. I separate firms into those that, based on standard criteria, are more or less sensitive financial development, and show that more-sensitive firms and industries exhibited greater leverage and aggregate output growth from 1990 to 2011. I also show that within industries, the covariance between size

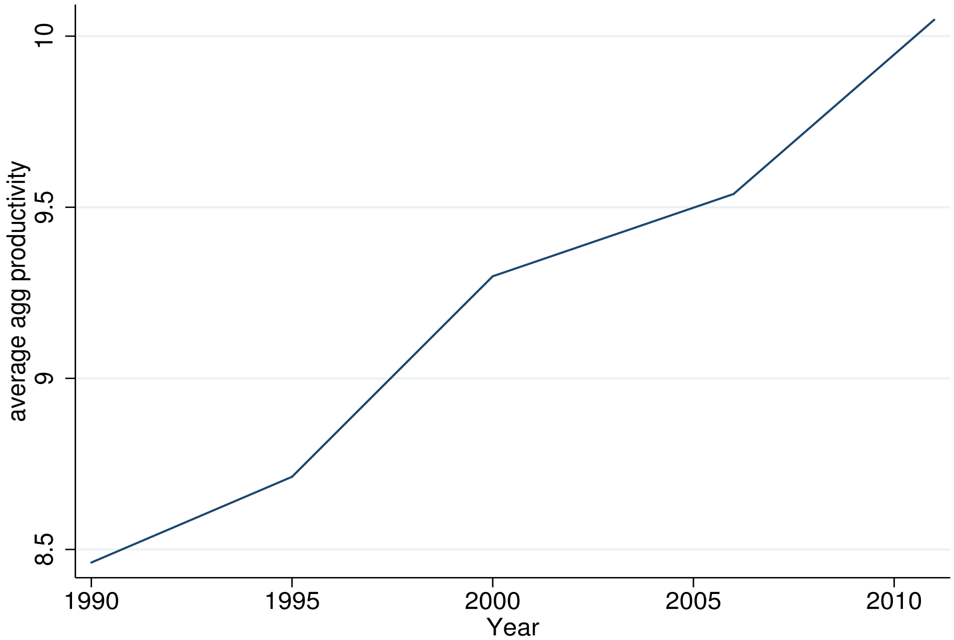


Figure 1.1. Aggregate Labor Productivity
The figure plots aggregate log labor productivity in India from 1990 to 2011. I define aggregate labor productivity as an employment-weighted average of industry-level aggregate labor productivity, where the weights are each industry's share of total employment at each date. Industry-level aggregate labor productivity is a employment-weighted average of plant-level log labor productivity, as in equation (2.2).

and productivity has been growing over time; this covariance ties directly into aggregate productivity and is a common measure of the efficiency of the allocation of resources across plants. However, interpreting these findings without the aid of an explicit model of growth and financial frictions can be misleading.

I derive a dynamic equilibrium model of borrowing, investment, and growth subject to financial frictions that can replicate the patterns in the data with only common shocks to productivity, and without any financial development. Reductions in financial frictions increase aggregate output and productivity by reducing the cost of capital for borrowing firms, causing these firms to borrow more and grow faster than other firms. In addition, reductions in financial frictions strengthen the covariance between size and productivity, because in equilibrium more-productive firms choose to borrow and are thus more sensitive to financial development. Importantly, the model also implies that common shocks to productivity can produce similar effects; such shocks do a better job of explaining the productivity growth in Figure 1.1.

Common shocks to productivity induce reallocation across firms in the model because (1) borrowers can default on their debt, and (2) their default probability depends on their productivity. Borrowers pay a premium for any leverage (liabilities over assets) beyond the collateral rate sufficient to cover the lender's losses should they default. For any level of leverage, more-productive firms represent a better credit risk, and thus enjoy a lower interest rate. An increase in productivity, even if it is common to all firms, affects borrowing firms more than others because it reduces their cost of borrowing. This implies that even a perfect separation of firms into those more- and less-affected by financial development will simulatensouly separate firms into those more- and less-affected by a common shock to productivity. Likewise, because more-productive firms borrow more in equilibrium, a common shock to productivity will also increase the covariance between size and productivity.

My paper furthers our understanding of how financial markets affect economic growth. Since the pioneering work of King and Levine (1993), a vast literature has shown that economic growth is robustly correlated with larger and more-efficient financial sectors; however, this literature struggles to impute causality. Rajan and Zingales (1998) note, "One way to make progress on causality is to focus on the details of theoretical mechanisms through which financial development affects economic growth, and document their working." Levine (2005) goes further: "If finance is to explain economic growth, we need theories that describe how financial development influences resource allocation decisions in ways that foster productivity growth." Hennessy (2013) goes even further, noting that "using a model allows us to make informed plausibility of the *magnitude* of an empirical moment [emphasis his]." In this paper, I propose a model in which financial development can affect economic growth, through a reallocation channel. I fit the model to Indian establishment-level microdata and use it to gauge the likely magnitude of financial development on aggregate productivity relative to other factors, represented by a common shock to productivity.

1.2 Literature Review

The analysis in this paper follows a series of seminal papers in finance that use structural models to interpret standard empirical methods. Gomes (2001) uses a model of capital accumulation subject to financial frictions to show that positive investment cash-flow sensitivy estimates are neither necessary nor sufficient evidence for financial frictions. Strebulaev (2007) shows that cross-sectional tests of capital structure theories applied to dynamic models based on those theories can still reject if firms adjust their capital structure infrequently. Both of these papers, like the current paper, use data simulated from a model to match empirical observations in order to cast doubt on the conventional interpretation of those observations. Finally, Hennessy and Strebulaev (2015) use a simple dynamic model to show that causal effects estimated from natural experiments can be subject to attenuation bias and sign reversals in dynamic settings. Like these other papers, I use a structural model as a lens through which to interpret two common empirical methods (difference in difference regressions and aggregate productivity decompositions).

I contribute to a literature that uses cross-country regression evidence to show that economic growth is highly correlated with the size of the financial sector. Since its beginnings, this literature has struggled to show that causality runs from finance to growth. King and Levine (1993) argued that because financial development predicts subsequent economic growth over the next thirty years, causality does not run from growth to finance. Subsequent work has deepened the basic finance-growth correlation using sophisticated panel-IV methods (Levine, Loayza and Beck 2000) and by showing that more financial intermediation is primarily associated with higher productivity, and not more savings or capital accumulation (Beck, Levine and Loayza 2000). La Porta, Lopez-De-Silanes and Shleifer (2002) show that in addition to the indicators used in Beck, Levine and Loayza (2000), higher government ownership of the banking sector is strongly associated across countries with lower aggregate productivity. This finding is important because since reforms began in 1991, policymakers in India have worked continually to reduce the proportion of the banking sector controlled by the government (Rajan et al. 2009, Rajan 2016).

My paper also contributes to a related literature that uses industry- and firm-level data, rather than country-level aggregate variables, to draw out the mechanisms whereby financial innovation affects economic growth. In a seminal paper, Rajan and Zingales (1998) argue that better-functioning financial markets should disproportionately affect industries that are more dependent on external finance, by lowering their cost of capital; they extract a measure of this external dependence from US firm-level data, and show that measures of financial development do exert a greater influence on the growth rates of externally-dependent industries across countries. Wurgler (2000) also uses industry-level data to show that the elasticity of investment growth to value-added growth is higher in more financially-developed countries, suggesting that better-functioning financial markets improve the allocation of resources. Beck, Demirgüç-Kunt and Maksimovic (2005) use cross-country firm-level survey data to show that financial factors are an important obstacle to firm growth, especially for small firms. The results of this literature suggest that improvements in financial markets remove constraints on the growth of some industries and firms, enhancing allocative efficiency and through it, aggregate productivity and economic growth. I contribute to this literature by using a calibrated model to show that, while the conventional intuition is correct in that more externally-dependent firms are disproportionately affected by financial development, they are also disproportionately affected by aggregate productivity growth. Thus the evidence used by this literature pointing to the importance of financial frictions is not sufficient to conclude that financial development is an important factor leading to economic growth.

In an influential paper, Olley and Pakes (1996) derive another empirical measure used to measure changes in allocative efficiency over time. They decompose aggregate productivity into a within-plant term and an across-plant term representing the extent to which larger plants are more productive. They then relate changes in the second term to regulatory changes that affected the telecommunications industry in the United States. Bartelsman, Haltiwanger and Scarpetta (2013) perform the same decomposition for all manufacturing industries in a number of European countries and interpret the across-plant term as an overall measure of allocative efficiency, showing that it is high for more-developed economies, such as the United States and Germany, but lower—though growing over time—for some formerly Communist countries in Eastern Europe. I derive an equilibrium model that relates the across-plant term directly to economic shocks—in particular, changes in financial frictions and common shocks to productivity. I then apply the model to another country in which the across-plant term has grown over time, and show that I can generate the same growth pattern without any re-allocative (financial) shocks. In my model, growth of the Olley-Pakes covariance over time is a natural consequence of aggregate productivity growth in the presence of (possibly constant) financial frictions.

My paper also contributes to a related literature that ties productivity differences across countries to the misallocation of resources. In a seminal paper, Hsieh and Klenow (2009) use a parsimonious model to measure how much of the productivity difference between the United States and two large developing economies is due to a poor allocation of resources. My paper is also related to recent work by Banerjee and Moll (2010) and Midrigan and Xu (2014), who propose models with financial frictions to explain this poor allocation. They find that any misallocation due to financial frictions disappears quickly over time because firms can grow their way out of binding financial constraints. I find that to match the size-productivity distribution among Indian manufacturing plants, enough firms are constrained in equilibrium that misallocation is high and shocks to financial frictions will have an impact, even though my model nests the Hsieh and Klenow (2009) framework, and has the same persistence of shocks as Midrigan and Xu (2014). Thus, financial shocks have the potential in my model to exert a strong influence on aggregate productivity. Nevertheless, I find that common shocks to productivity better explain the patterns in the Indian data from 1990 to 2011 than improvements in financial development.

My paper relates to a new wave of macroeconomic research that adds financial factors to macroeconomic models. Much of this research, including Gertler and Kiyotaki (2010), Gertler, Kiyotaki and Queralto (2012), Buera and Moll (2012), and Jermann and Quadrini (2012), assumes a maximum leverage constraint as a parsimonious way to model financial frictions. By requiring that borrowing be fully collateralized, these models assume that debt is riskless; often this assumption is justified as satisfying an incentive-compatibility constraint. My paper enriches the financial side of these models by noting that lenders are not interested in whether individual loans are incentive-compatible; in equilibrium, lenders care only about the return on their entire portfolio. Lenders, who observe borrowers' productivity and leverage choices, can earn the same expected return by charging borrowers a higher interest rate to compensate them for default risk. Borrowers, rather than facing a "hard" borrowing constraint, face an interest rate schedule that depends on their productivity and choice of leverage. In equilibrium, this modeling framework allows common shocks to productivity to induce reallocation across firms, because *ceteris paribus* an increase in productivity reduces the probability of default and thus firms' borrowing costs. This is the key ingredient in my model, absent in previous work, that allows the model to match empirical patterns commonly interpreted as indicative of financial development, without any such development at all.

Finally, I contribute to the corporate finance literature on capital structure by analyzing the leverage decisions of small, privately owned firms. Most empirical work on capital structure focuses on publicly-traded firms, for example firms in the Compustat database. This focus may be why the canonical theories of optimal capital structure in a dynamic setting, such as Leland (1994, 1998) and Hennessy and Whited (2005, 2007), focus on the trade-off between debt and external equity, usually from the standpoint of a borrower balancing the tax advantages of debt against the costs of bankruptcy and (frequently) some equity-issuance costs. This trade-off is most appropriate for large firms with access to public equity markets. Other quantitative and empirical studies of capital structure, such as Strebulaev

(2007), Bhamra, Kuehn and Strebulaev (2010), Chen (2010), Korteweg (2010), and Glover (2014) also focus on the tax treatment of debt versus equity and on firms large enough to be able to exploit it. In contrast, I analyze data on the financial decisions of firms from a representative cross-section of manufacturing firms, most of which are small and unlikely to have access to public equity markets, and how they affect aggregate outcomes. This may be why in the Indian data I find that leverage and productivity are positively correlated (a fact matched well by my model), in contrast to a canonical result of Rajan and Zingales (1995) that, among publicly-traded firms and controlling for Tobin's Q, leverage and productivity are negatively correlated.

CHAPTER 2

DO FINANCIAL FACTORS DRIVE AGGREGATE PRODUCTIVITY?

2.1 Empirical Evidence

In this section, I describe the Indian economic reforms mentioned in section 1.1, and use microdata from Indian manufacturing plants to explore two common reduced-form methods for separating reallocation effects from aggregate productivity growth: difference in difference regressions, and a decomposition of aggregate productivity into within- and between-firm components. In section 2.1.1, I describe the Indian macroeconomic and financial reforms in more detail. I describe the data in section 2.1.2. In section 2.1.3 I run a series of difference in difference regressions that, under conventional interpretations, suggest an important role for financial development in Indian economic growth. Section 2.1.4 describes and analyzes the Olley-Pakes aggregate productivity decomposition, which also suggests that financial development led to productivity growth. Sections 2.2 and 2.3 derive, calibrate, and analyze a model to interpret these empirical results.

2.1.1 Indian Economic Reforms

India instituted a number of economic reforms in 1991 after a severe balance-of-payments crisis. Many of the reforms affected the manufacturing sector directly. For instance, the goverment essentially abolished the system of industrial licensing, a major constraint on investment and output for registered manufacturing firms (Aghion et al. 2008). In addition, a large number of industries that had been reserved solely for the government were opened up to private entry (Ahluwalia 2002). Reforms in 1991 that took immediate effect also include a massive drop in tariffs, especially for capital goods, and freeing up of foreign direct investment restrictions (Joshi and Little 1996). Both of these reforms are likely to have made expanding production easier for productive firms.

In addition to reforms targeted directly at the manufacturing sector, the government also instituted various financial reforms in 1991. Chief among these were changes to the Indian banking system, which was dominated by poorly-performing government-owned banks. Accounting rules that allowed banks to hide non-performing assets were changed, and the government recapitalized public banks with negative net worth. High reserve ratios, whose main purpose was to pre-empt banking resources to finance the government deficit, fell dramatically in 1991. The government also allowed for more private entry into the banking sector, including allowing the public-sector banks to raise capital in public equity markets, diluting the amount of government ownership (Joshi and Little 1996).

Not all Indian financial reforms occurred immediately after the 1991 crisis. The Recovery of Debts Act, which established tribunals in several major cities to aid in the process of recovering bad debts, was passed in 1993 but did not become effective immediately, because of challenges in the court system (Joshi and Little 1996). Visaria (2009) analyzes the effects of these tribunals on firm borrowing and repayment decisions. The Securitisation and Reconstruction of Financial Assets and Enforcement of Security Interest Act of 2002 (known as the Sarfaesi Act) allowed for the creation of asset-reconstruction companies to which banks could auction non-performing loans (Rajan et al. 2009). However, Vig (2013) argues that the Sarfaesi Act mainly resulted in borrowers substituting away from secured debt, towards unsecured debt (which is not covered by the Act). Finally, the process of removing the government from control of the banking sector has also proceeded slowly since 1991 (Rajan et al. 2009). Rajan (2016) shows that, although the efficiency of the banking sector in India has improved tremendously, there is still a good deal of reform left to be done, especially among public-sector banks.

2.1.2 Data

I combine data from two sources, the Annual Survey of Industries (ASI) and the National Sample Survey (NSS). The ASI is a survey of manufacturing establishments in India that are registered under the 1948 Factories Act, which covers plants with more than 10 employees, or more than 20 employees if the plant doesn't use electric power. Registered plants in the ASI usually have more than 10 employees, though they remain registered even if their employment falls below 10, and include the largest factories in India. By contrast, the NSS is a survey of unregistered plants, which are typically much smaller than those in the ASI.[1] Because the NSS is administered to manufacturing plants only roughly every five years, I only have five years of combined ASI-NSS data: for 1990, 1995, 2000, 2006, and 2011 (data years refer to the Indian fiscal year, which runs from April to March).

Table 2.1 reports various summary statistics for the NSS and ASI data sets separately. Panel A reports the total number of manufacturing plants and employees, as well as the NSS shares of firms, employment, and output (real value-added in 1993–1994 rupees) over time. The Indian manufacturing sector employs between 27 million and 46 million people, and this number has been rising over time. Over 99% of the 12–17 million manufacturing plants in India are in the unregistered, informal sector covered by the NSS. These plants account for about fourth-fifths of manufacturing employment in India, but only between a sixth and a quarter of manufacturing output. Thus, NSS plants must be much smaller, and substantially less productive, than their ASI counterparts.

Panel B of Table 2.1 reports the employment distribution of plants in the NSS and ASI. NSS plants employ on average only two employees, compared to 67–135 employees on average in the ASI. Median employment in the ASI is much lower at about 20 employees, reflecting the substantial positive skewness in the plant-size distribution in the ASI. The employment

1. Although I will refer to plants in the NSS as "informal" plants or representing the "informal" sector, NSS respondents are not informal in the sense of operating illegally or evading taxes. They are merely manufacturers that are not covered by the 1948 Factories Act, and as a result are not part of the sampling frame of the ASI.

Panel A					
	Aggregate		NSS Shares		
Year	# Firms	# Employees	% Firms	% Employment	% Output
1990	12.3	26.3	99.8	86.0	31.0
1995	11.9	34.5	99.4	84.7	26.0
2000	14.2	36.5	99.2	80.5	24.4
2006	14.1	38.7	99.2	78.5	21.8
2011	17.0	45.7	99.1	74.8	18.1

Panel B: Employment					
Year	mean	s.d.	1%	50%	99%
NSS					
1990	1.8	1.0	1.0	2.0	5.0
1995	2.5	2.4	1.0	2.0	11.0
2000	2.1	2.2	1.0	2.0	10.0
2006	2.2	3.0	1.0	2.0	11.0
2011	2.0	3.2	1.0	1.0	11.0
ASI					
1990	135.4	661.6	1.0	23.1	1824.0
1995	78.8	499.3	3.0	20.0	960.0
2000	66.1	435.1	2.0	18.0	766.0
2006	71.8	367.1	3.0	20.0	834.0
2011	77.3	381.1	2.0	21.0	916.0

Panel C: Log Labor Productivity					
Year	mean	s.d.	1%	50%	99%
NSS					
1990	8.3	1.2	5.3	8.3	11.3
1995	8.2	1.2	4.9	8.3	10.7
2000	8.7	1.1	5.8	8.7	11.0
2006	8.8	1.2	5.6	8.8	11.1
2011	9.3	1.1	6.5	9.4	11.4
ASI					
1990	10.3	1.3	6.5	10.3	13.7
1995	10.4	1.2	6.9	10.4	13.6
2000	10.7	1.2	7.7	10.7	13.8
2006	10.7	1.2	7.8	10.7	13.8
2011	11.2	1.1	8.5	11.1	14.3

Table 2.1. NSS and ASI Summary Statistics.
The first two columns of Panel A report the total estimated number of firms and employees (in millions) in the combined ASI-NSS dataset over time. Columns 3 through 5 of report the aggregate shares of plants, employment, and output (real value added in 1993-1994 rupees) accounted for by respondents in the NSS data. Panel B reports the distribution of employment in the NSS and ASI, while Panel C reports the distribution of log labor productivity (real value-added per employee) in the two datasets. The first two columns in Panels B and C report the mean and standard deviation, respectively, while the last three columns report the 1st, 50th, and 99th percentiles.

measure used in this paper is the average number of employees over the course of the year, including administrative and part-time employees.

Plants in the ASI are not only larger, but also much more productive than plants in the NSS. Panel C of Table 2.1 reports the distribution of (log) labor productivity in the two data sets. Plants in the ASI are roughly 200 log-points more productive than plants in the NSS, though productivity is rising over time for both types of plants. I define productivity as the natural logarithm of real value-added, in 1993-1994 rupees, per employee. Value-added is total revenue, including revenue from non-production activites, less total costs, including materials inputs and other expenses but excluding the cost of labor. I deflate nominal value-added using the industry-level Wholesale Price Indices; see Appendix **??** for details.

I focus on revenue productivity, rather than physical productivity, in this paper in order to include the NSS plants in my analysis. Because the NSS plants are such a substantial fraction of employment and output in India, I opt to include them and focus on revenue productivity for the empirical analysis of this paper. Revenue productivity can differ from physical productivity when plants in the same industry charge different prices; see Foster, Haltiwanger and Syverson (2008) for an analysis of this phenomenon in the United States. Although the exogenous shocks in the model of section 2.2 will be to physical productivity, I construct all measures in the model (for example, of the Olley-Pakes decomposition) using endogenous revenue productivity.

Although plants in the NSS are much smaller than those in the ASI, they do have access to external sources of credit that were affected by the Indian financial reforms described in section 2.1.1. Table 2.2 reports statistics on borrowing across the two data sets. Between 6% and 10% of NSS plants have loans outstanding, though this number is decreasing over time. ASI plants are much more likely to borrow; between two-thirds and three-quarters of ASI plants report having loans. Because ASI plants are so much bigger than NSS plants, and are more likely to have a loan at all, plants in NSS account for between 3% and 7% of total debt outstanding in the combined ASI-NSS data. The tiny fraction of total borrowing

Year	% that Borrow			NSS % of Total
	All	NSS	ASI	Borrowing
1990	10.7	10.6	66.2	2.8
1995	9.2	8.9	77.8	5.0
2000	8.1	7.6	73.0	3.2
2006	9.0	8.4	74.2	6.9
2011	6.6	6.0	73.9	2.6

Table 2.2. Borrowing
The first three columns report percentages of plants that borrow in the combined ASI–NSS data. The first column reports the percentage across all plants, the second column reports the percentage of NSS plants that borrow, and the third column reports the percentage of ASI plants that borrow. The last column reports the percentage of all borrowing that is accounted for by NSS plants.

that NSS plants account for is consistent with the model developed in section 2.2, because NSS plants are so much less productive than their ASI counterparts.

Figure 2.1 plots the sources of credit for NSS plants over time.[2] About 60% of NSS loans come from commercial banks, including state-owned banks and other government sources (e.g., the Khadi & Village Industries Commission). This observation is significant because many of the financial reforms covered in section 2.1.1 were specifically targeted at making the banking sector more efficient. The remaining 40% of NSS credit is about evenly split between money-lenders, loans from friends, family, and business partners, and other sources. The NSS does not ask about loans from mciro-finance institutions until 2011; these loans represent a small fraction of total borrowing in that year.

2.1.3 Difference in Difference Regressions

In this section I assign firms to "treated" and "control" groups and run difference-in-difference regressions to show that, according to this methodology, financial development has occurred in India over this time period. I assign firms using industry-wide characteristics that are commonly used in the literature, including dependence on external finance (Rajan

2. ASI plants do not break down their sources of credit.

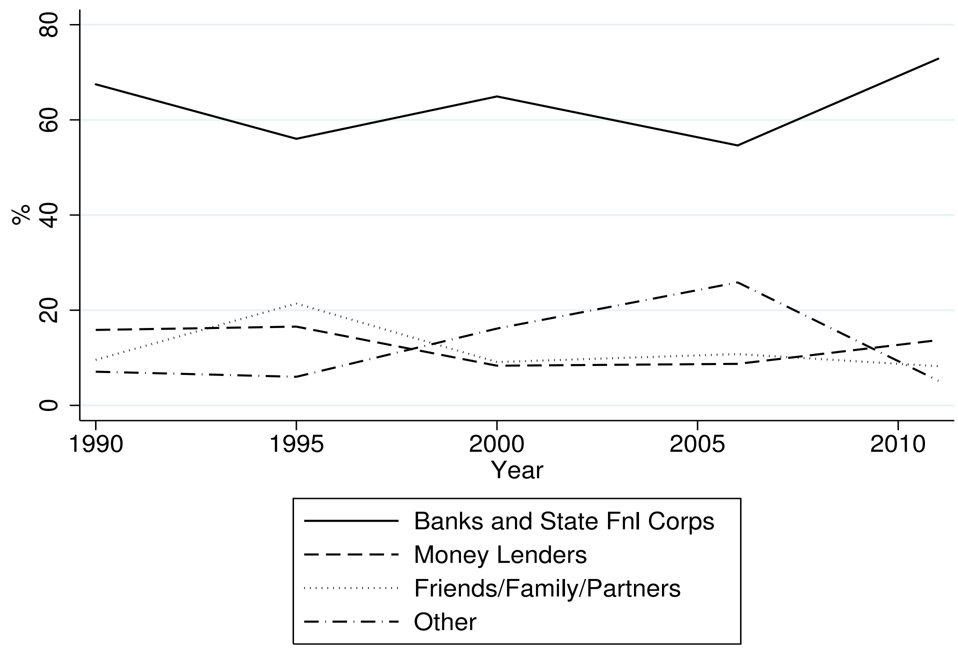


Figure 2.1. Sources of Credit
The figure plots the sources of credit (loans) reported by NSS plants over time. Plants in the ASI do not report their sources of credit. Each line plots the percentage of total credit reported from each source: banks and state financial corporations (including loans from government-owned commercial banks and Khadi & Village Industries Commission), money-lenders, friends, family, & business partners, and other.

and Zingales 1998) and share of fixed assets in total assets (Vig 2013). Both "treated" and "control" industries increase their average leverage and aggregate output over this sample period, but "treated" industries do so by more.

In addition to leverage and output growth, I also show that in most cases, "treated" industries experienced greater productivity growth. In section 2.3.2 I compute some difference-in-difference means using the calibrated model with various assumptions on financial development and aggregate productivity growth to show that a positive difference-in-difference can arise without financial development, and that a negative difference-in-difference can obtain despite financial development.

I sort firms into "treated" and "control" firms according to industry, as described in detail below. I define the second difference as time: I set the "pre" period to the observations in

1990, 1995, and 2000, and the "post" period to 2006 and 2011. This corresponds roughly to the passage of the Sarfaesi Act in 2002; therefore these regressions can be interpreted as a long-run estimate of the effect of the Sarfaesi Act on leverage and aggregate output.

Table 2.3 reports the industries I selected for this analysis. The Pharmaceuticals industry is ranked by Rajan and Zingales (1998) as the most externally-dependent manufacturing industry. It also has a low share of employment accounted for by informal plants, and a high percentage of plants that report positive leverage in the pre-period (1990–2000). In contrast, the Tobacco industry is ranked by Rajan and Zingales (1998) as the least externally-dependent industry. It has a low NSS share of employment, and a low percentage of borrowing plants in the pre period. Ex ante, it is natural to assume that Pharmaceutical plants will be more sensitive to financial development than Tobacco plants.

I also included the Motor Vehicles and Motorcycles & Parts industries in the "affected" category. Motor Vehicles have a high external-dependence measure according to Rajan and Zingales (1998), and unlike the industries that have a slightly higher ranking (such as Ships or Office & Computing), they are responsible for a non-trivial share of Indian manufacturing employment, especially after includng Motorcycles & Parts along with them. These two industries have a high percentage of borrowing plants and low NSS shares of employment, as well.

In terms of less-affected industries, I added Other Chemical Products because it is likely to be similar to Pharmaceuticals in many respects, though it ranks lower in the Rajan and Zingales (1998) external-dependence measure and has a lower percentage of firms borrowing in the pre period. It also has a relatively low share of NSS employment, though it is higher than the more-affected industries. I included Carpentry & Joinery goods because it has low percentage of borrowing plants, a high share of employment in NSS plants, and accounts for roughly the same share of aggregate employment as all the more-affected industries combined (unlike Tobacco, which accounts for much more employment).

For another pair of affected vs. not-affected industries, I chose Cement & Plaster Products

Panel A: Summary Statistics					
Industry	% of Total Employment	% that Borrow	Avg Fixed Asset %	NSS % of Employment	Log Agg Output Growth
2100 Pharmaceuticals	0.65	35.66	32.74	10.31	1.14
2910 Motor Vehicles	0.90	31.32	33.61	14.42	2.03
3091 Motorcycles & Parts	0.23	30.45	35.28	19.88	2.04
2395 Cement & Plaster Prod	0.32	18.64	33.79	68.61	1.35
1200 Tobacco	9.37	2.32	29.76	88.00	0.46
2029 Other Chemical Products	0.73	8.57	32.41	50.81	0.95
1622 Carpentry & Joinery Gds	2.04	5.80	30.61	99.47	0.42
1072 Sugar	1.66	19.55	24.88	56.80	0.58

Panel B: Average Differences						
Industry	Avg Leverage			Avg Log Labor Productivity		
	Pre 2000	Post 2000	Diff	Pre 2000	Post 2000	Diff
2100 Pharmaceuticals	9.64	10.80	1.17	9.30	10.01	0.71
2910 Motor Vehicles	8.56	10.40	1.85	10.05	10.64	0.59
3091 Motorcycles & Parts	8.00	9.39	1.39	10.08	10.43	0.35
2395 Cement & Plaster Prod	4.12	9.51	5.40	9.23	9.99	0.76
1200 Tobacco	0.25	0.26	0.00	8.05	7.97	-0.08
2029 Other Chemical Products	0.79	1.21	0.42	7.90	8.49	0.59
1622 Carpentry & Joinery Gds	1.19	1.53	0.34	9.19	9.59	0.41
1072 Sugar	3.82	4.03	0.21	7.69	9.44	1.75

Table 2.3. Industry Selection and Differences

Both panels report summary statistics for industries that are *ex ante* likely to be more or less affected by financial development than other industries. The industries in the top three rows are likely to be more affected by financial development, while the industries in the bottom four rows are likely to be less affected. The first column of Panel A reports the percentage of total employment across the ASI and NSS and all five time periods accounted for by each industry. The second column of Panel A reports the percentage of firms in 1990–2000 that have positive leverage. The third column of Panel A reports the within-industry average percentage of total assets accounted for by fixed assets; this statistic is computed among ASI plants only (NSS plants do not report fixed vs. current assets). The fourth column of Panel A reports the share of employment accounted for by NSS plants, and the last column of Panel A reports the average log productivity difference from 1990-2000 and 2006–2011. Panel B reports pre-2000 and post-2000 average differences in leverage in the first 3 columns, and differences on log aggregate output before and after 2000 in the last three columns.

and Carpentry & Joinery Goods. Both industries are in a similar sector of the economy (construction materials), and it is reasonable to assume that Cement plants are more affected by financial development because more Cement plants borrow, and the Cement industry has a smaller share of employment in the informal sector. Cement plants in the ASI also have a slightly higher average fixed-asset share than Carpentry & Joinery plants.

Finally, I included Sugar as a less-affected industry because among ASI plants, the Sugar industry has a low average fixed-asset share. Vig (2013) uses the share of fixed assets in total assets as a measure of asset tangibility, and argues that firms with low asset tangibility are less affected by the Sarfaesi Act than firms with higher asset tangibility. Indeed, Vig uses the fixed-asset share to assign firms to "treatment" and "control" groups in his difference-in-difference regressions Although I can only compute the fixed-asset share for plants in the ASI, I assume that NSS plants in the same industry are likely to have a similar-enough fixed-asset share that I can classify an industry as high or low tangibility based on the average ASI values. According to this scheme, Sugar plants on average tend to have lower asset tangibility than the three more-affected industries in Table 2.3.

Panel B of Table 2.3 reports differences in mean leverage and log labor productivity for the selected industries. In general, the more-affected industries have higher pre-period leverage, and more leverage growth, between 1990—2000 and 2006–2011. In addition, with the exception of Tobacco plants, the less-affected industries have positive leverage growth; this growth is smaller on average than for the more-affected industries. This difference-in-difference result is consistent with financial development occurring over the 1990–2011 period; this development seems to have exerted a greater influence on industries we expect to be more affected. The final column of Panel A of Table 2.3 shows that, wth the exception of the Sugar industry, the more-affected industries also had greater aggregate output growth over this period than the less affected industries. This industry-level measure is used by Rajan and Zingales (1998) in cross-country regressions to gauge the extent of financial development.

Tables 2.4 and 2.5 present the evidence from the first three columns of Panel B of Table 2.3

in regression form. I run the following regression;

$$\text{lev}_{i,t} = \alpha_s + \beta_0 \mathbb{1}\{i \in \text{treat}\} + \beta_1 \mathbb{1}\{t > 2000\} + \beta_3 \mathbb{1}\{t > 2000\} \times \mathbb{1}\{i \in \text{treat}\} + \beta_4 \text{log labor prod}_{i,t} + \varepsilon_{i,t}, \tag{2.1}$$

where i indexes plant, t indexes time, α_s is a state fixed-effect, $\mathbb{1}\{i \in \text{treat}\}$ is a dummy equal to 1 if a firm is in a treated industry, and $\mathbb{1}\{t > 2000\}$ is a dummy equal to 1 for years after 2000. The estimated coefficient β_3 represents the extent to which treated industries increased their leverage by more than non-treated industries. The different columns of Tables 2.4 and 2.5 define treatment and control industries in different ways.

The first column of Table 2.4 defines Pharmaceutical plants as "treated" and Tobacco plants as "control." On average, Pharmaceutical plants increased their leverage ratios by 1.16 percentage points more than Tobacco plants. Comparing Pharmaceuticals to Other Chemicals plants (column 2), the difference-in-difference estimate is slightly smaller at 74 bps. The third column of Table 2.4 assigns "treatment" status to Cement & Plaster Products plants, and "control" status to Carpentry & Joinery plants. These plants have much closer leverage ratios in the pre-period, though average leverage at Cement plants increases by much more. The fourth column of Table 2.4 compares the four more-affected industries with the Sugar industry, which has a lower average fixed-asset share. Finally, the last column of Table 2.4 defines all four more-affected industries as treatment, and all four less-affected industries as control. The results are similar in all three cases: the coefficient on the interaction term is positive and large, suggesting an increase in financial development that led more-affected plants to increase their leverage ratios by more than other plants.

However, the final three columns in Panel B of Table 2.3 show that, in addition to higher average leverage and aggregate output, the more-affected industries also had higher average labor productivity and higher labor productivity growth. The final three columns in Panel B of Table 2.3 report pre-2000 and post-2000 average log labor productivities in each

	(1)	(2)	(3)	(4)	(5)
Treatment Industry	Pharmaceuticals	Pharmaceuticals	Cement	All 4 in Table 2.3	All 4
Control Industry	Tobacco	Other Chemical	Carpentry	Sugar	All 4 in Table 2.3
Treatment X Post	1.162***	0.744***	5.056***	1.363***	1.635***
	(5.340)	(3.405)	(47.71)	(10.96)	(14.83)
Treatment	9.384***	8.847***	2.926***	5.000***	8.223***
	(67.27)	(63.14)	(40.74)	(60.21)	(105.3)
Post	0.00492**	0.422***	0.339***	0.215***	-0.0570***
	(2.163)	(19.85)	(30.34)	(3.724)	(-18.83)
Constant	0.252***	0.789***	1.189***	3.820***	0.597***
	(175.1)	(59.64)	(185.9)	(135.0)	(290.1)
State Fixed Effects	NO	NO	NO	NO	NO
Observations	9,570,289	401,673	2,542,429	426,312	12,773,207
Obs unweighted	38,235	8,981	19,027	14,375	71,234
R-squared	0.024	0.086	0.023	0.027	0.022

Table 2.4. Leverage by Industry Regressions

The table reports results from estimating equation (2.1) on the combined ASI-NSS data. The dependent variable is leverage, defined as total liabilities divded by total assets times 100%. Each column includes only the industries indicated in the top two rows, and defines the "treated" industry as the industry listed in the top row. The dependent variable is leverage, defined as total liabilities divded by total assets times 100%. "Post" is a dummy variable equal to 1 for years 2006 and 2011. These regressions do not include log labor productivity or state fixed-effects as independent variables. All regressions use the ASI and NSS sample weights; the number of unweighted sample observations (survey responses) are listed below the number of weighted observations. t-statistics are reported in parentheses below each coefficient.

	(1)	(2)	(3)	(4)	(5)
Treatment Industry	Pharmaceuticals	Pharmaceuticals	Cement	All 4 in Table 2.3	All 4
Control Industry	Tobacco	Other Chemical	Carpentry	Sugar	All 4 in Table 2.3
Treatment X Post	0.476**	-0.567***	4.481***	1.971***	0.653***
	(2.171)	(-2.586)	(41.74)	(15.11)	(5.998)
Treatment	8.383***	4.435***	2.453***	2.415***	7.569***
	(58.95)	(27.74)	(33.66)	(22.12)	(94.61)
Post	0.0365***	-0.438***	0.323***	-2.898***	-0.0219***
	(14.15)	(-18.74)	(25.71)	(-40.18)	(-6.672)
Log Labor Prod	0.443***	1.678***	0.535***	1.742***	0.673***
	(110.8)	(71.22)	(46.02)	(85.12)	(199.5)
State Fixed Effects	YES	YES	YES	YES	YES
Observations	9,570,289	401,673	2,542,429	426,312	12,773,207
Obs unweighted	38,235	8,981	19,027	14,375	71,234
R-squared	0.046	0.178	0.036	0.079	0.043

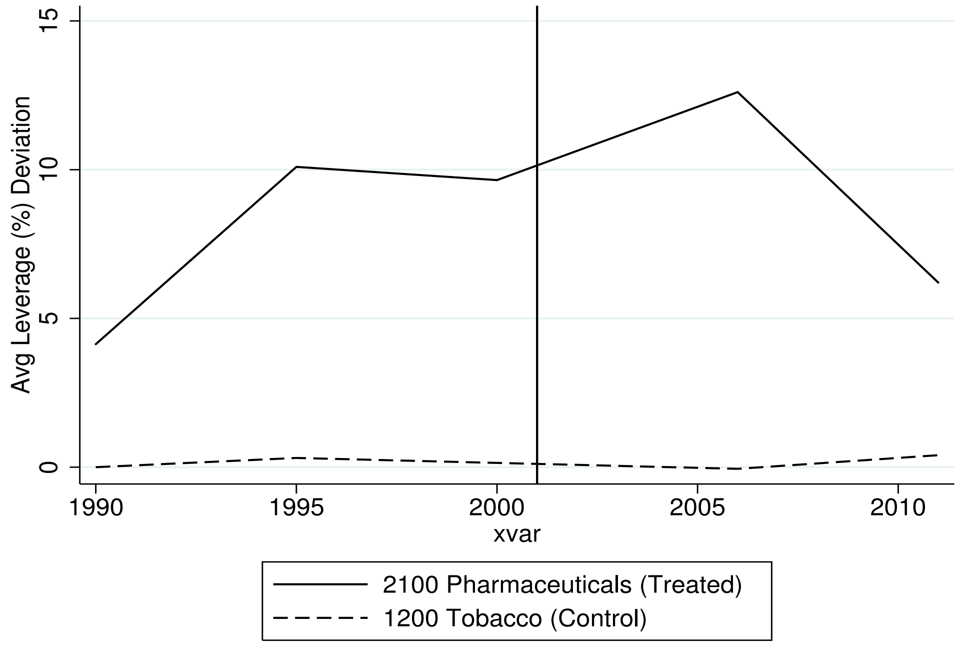
Table 2.5. Leverage by Industry Regressions with Productivity
The table reports results from estimating equation (2.1) on the combined ASI-NSS data. The dependent variable is leverage, defined as total liabilities divded by total assets times 100%. Each column includes only the industries indicated in the top two rows, and defines the "treated" industry as the industry listed in the top row. The dependent variable is leverage, defined as total liabilities divded by total assets times 100%. "Post" is a dummy variable equal to 1 for years 2006 and 2011. All regressions use the ASI and NSS sample weights; the number of unweighted sample observations (survey responses) are listed below the number of weighted observations. t-statistics are reported in parentheses below each coefficient.

industry. Labor productivity is higher on average for treated firms, and with the exception of Sugar plants, labor productivity growth was also higher among treated plants than control plants.

Table 2.5 examines the role of labor productivity in driving these results by including firm-level labor productivity as an explanatory variable. I also include state fixed effects. Many of the results from Table 2.4 are robust to including these extra controls. In particular, in all but the second column the interaction term is positive and large. In addition, the coefficient on leverage is also positive, suggesting that more-productive firms borrow more on average than less-productive ones. This result is consistent with the model of section 2.2, in which agents borrow to move wealth from expected future productivity to the present.

An important identifying assumption when running difference in difference regressions is that both treatment and control groups exhibit parallel trends; that is, were the treatment group to not be treated, the change in its explanatory variable would be the same as the control group. This assumption cannot be tested directly. However, one way to argue for the parallel trends assumption is to interact the treatment variable with multiple time periods, and show that the positive estimated effects only obtain for periods after the change.

Figure 2.2 shows that while some of the regressions reported in Table 2.5 appear to violate the parallel trends assumption, others do not. Figure 2.2 plots coefficient estimates from running regression (2.1) augmented to include time dummies and interaction terms for each time period in the data. Although average Pharmaceutical leverage is higher in the last two periods than the first three (top panel of Figure 2.2), consistent with the results reported in Table 2.5, the top panel of Figure 2.2 is not encouraging in terms of the parallel trends assumption: Pharmaceutical leverage seems to jump the most between 1990 and 1995, and actually drops a good deal from 2006 to 2011. The first effect might be explained as a result of the major economic reforms of 1991, which included financial reforms (see section 2.1.1). The bottom panel of Figure 2.2 looks better. Average leverage in Carpentry & Joinery plants seems to be dropping slightly before 2000, but this change is very small relative to the large



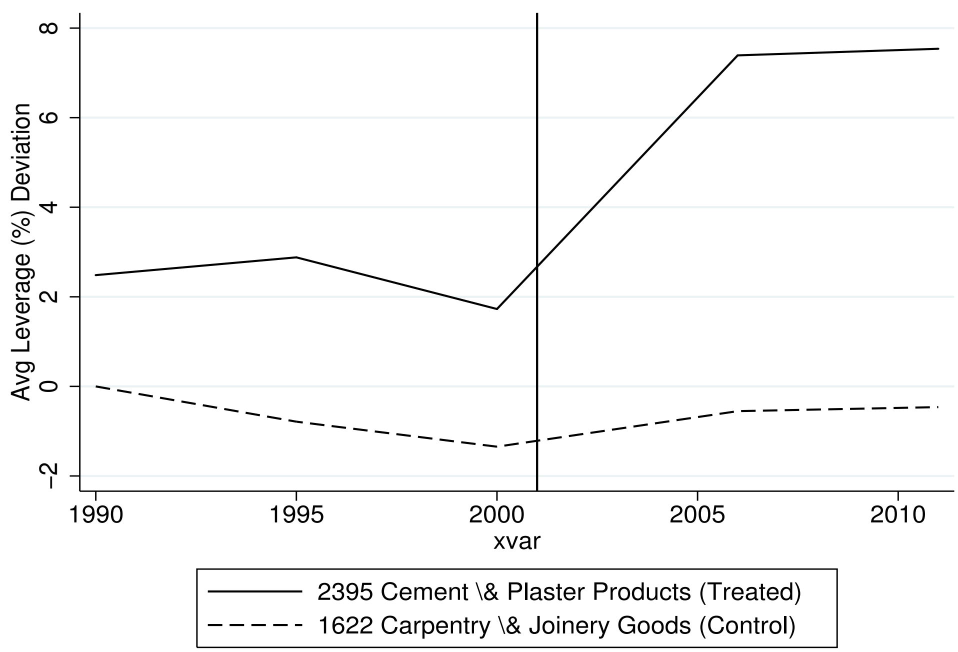


Figure 2.2. Interaction Effects over Time

Both figures plot regression coefficients from estimating equation (2.1), augmented to include time dummies and treatment interactions for each year in the data. The omitted category is control firms in 1990, so each point represents the difference in average leverage in percentage points from this value. Regressions include state fixed-effects and log labor productivity. The top panel plots coefficients from defining Pharmaceutical plants as the treatment group and Tobacco plants as the control, and the bottom panel from defining Cement & Plaster Products plants as the treatment group and Carpentry & Joinery Goods as the control.

jump in average leverage at Cement & Plaster plants after 2001.

Overall, the results in Tables 2.4 and 2.5 and Figure 2.2 suggest that financial development did play a role in Indian economic growth from 1990 to 2011. The model in section 2.2, calibrated and analyzed in section 2.3, will show that these patterns in the data can be driven by common shocks to productivity alone, without any financial development at all.

2.1.4 Aggregate Productivity

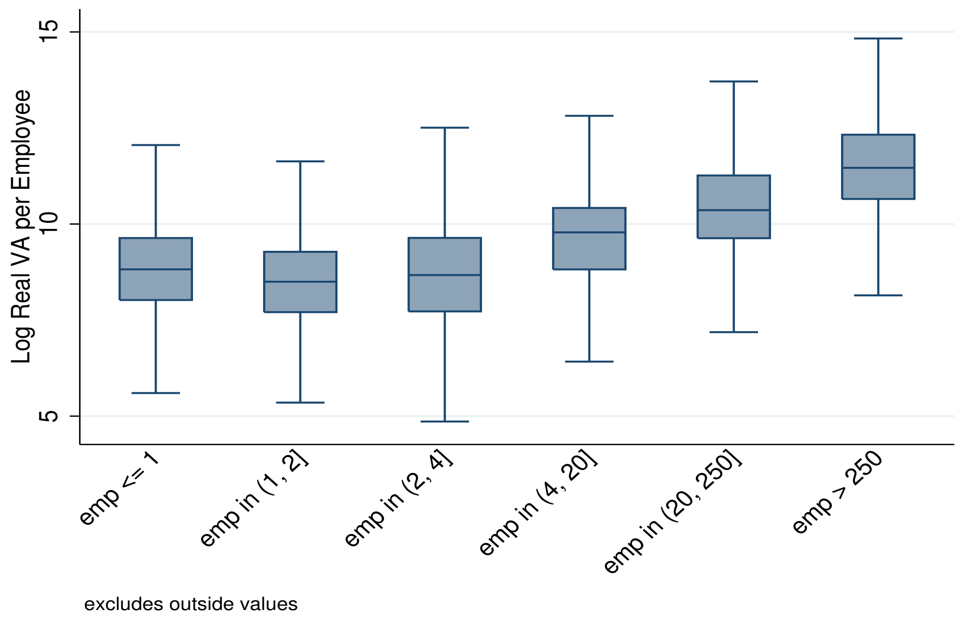
In this section, I document the strong correlation between plant size and labor productivity in India. I then quantify the effect of this covariance on aggregate productivity using a simple decomposition due to Olley and Pakes (1996).

In India, larger plants are vastly more productive than smaller plants, as shown in Figure 2.3. The top panel plots the distribution of real log labor productivity across plants in six employment categories, where I chose the categories to account for roughly the same share of total employment.[3] Although there is substantial dispersion in productivity across plants within any size category, the entire distribution of productivity is shifted upwards for plants with more than five employees. The bottom panel of Figure 2.3 reports the shares of aggregate employment accounted for by each size bin, for reference. Each bin accounts for a large share of aggregate employment, although over 60% of Indian manufacturing employees (leftmost three categories) work in low-productivity establishments with less than five employees.

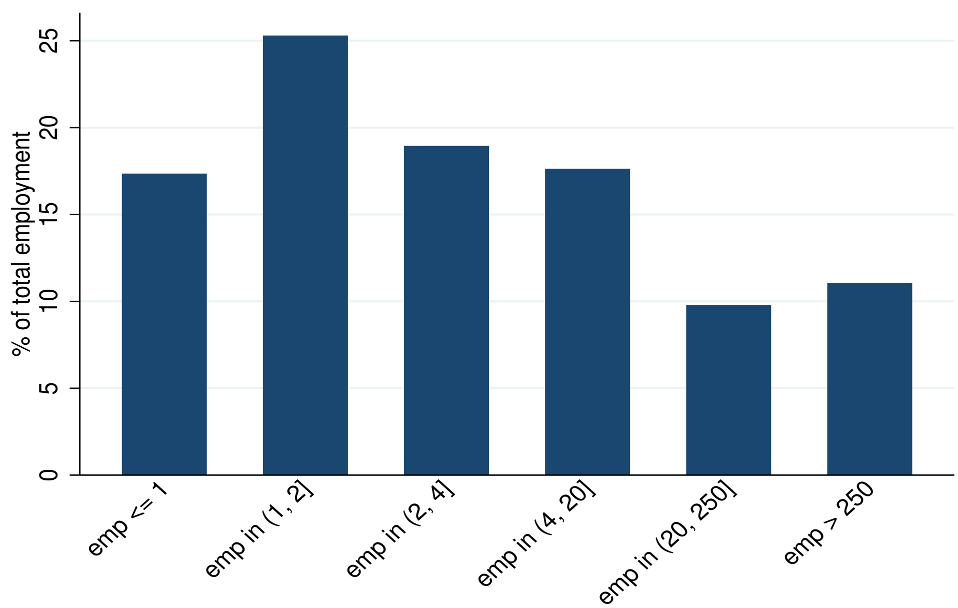
I quantify the impact of the covariance between size and productivity using a simple decomposition of aggregate productivity derived by Olley and Pakes (1996). If aggregate productivity is defined as a weighted average of plant-level productivities,[4] then it can be written as the sum of a within-firm (average) productivity term and an across-firm covariance

3. The figure pools across both industries and time. Removing industry means changes the picture imperceptibly; the entire picture is shifted upwards over time, in all six categories.

4. This can be justified as a first-order approximation to the natural logarithm of total output divided by total employment, around any point where all plants have the same productivity.



(a) Log Labor Productivity



(b) Shares of Total Employment

Figure 2.3. Productivity and Employment Shares by Size
The top panel plots the distribution of log labor productivity, in 1993-1994 rupees of value-added per employee, across six size categories: plants with 1 employee, 2 employees, 3–4 employees, 5–20 employees, 21–250 employees, and more than 250 employees. The edges of each box represent the 25th and 75th percentiles, while the middle line inside each box is the median. The bars represent the 1st and 99th percentiles. The data are pooled across industries and dates, and I use the sample weights. The bottom panel plots the percentages of total employment accounted for by each size category.

term:

$$\text{Aggregate Productivity} \equiv \sum_{i=1}^{N} w_i z_i$$
$$= \underbrace{\frac{1}{N}\sum_{i=1}^{N} z_i}_{\text{average productivity } \mathcal{Z}} + \underbrace{\sum_{i=1}^{N} (z_i - \mathcal{Z})(w_i - \overline{w})}_{\text{OP covariance } \mathcal{C}}, \qquad (2.2)$$

where z_i is log labor productivity at plant i, w_i is plant i's share of total industry employment, and $\overline{w}$ is the unweighted average of w_i. I define aggregate productivity in this identity as a weighted average of plant-level productivities, which, because the weights $\{w_i\}$ sum to 1, can be decomposed into an unweighted average $\mathcal{Z}$ across plants plus a term $\mathcal{C}$ representing the covariance between the weights (plant size) and productivity. The level of $\mathcal{C}$ indicates how much aggregate productivity would drop if, holding all plant productivities fixed, employment were re-allocated uniformly across plants. In this sense, $\mathcal{C}$ is a measure of allocative efficiency.

The two panels of Figure 2.4 show that the Olley-Pakes covariance $\mathcal{C}$ accounts for a substantial portion of the aggregate productivity growth in India from 1990 to 2011. Figure 2.4 plots values of the two terms on the right-hand side of equation (2.2) over time in India according to the combined ASI-NSS data as solid black lines. The top panel plots the average log labor productivity, while the bottom panel plots the Olley-Pakes covariance term. Both terms are computed within each of 100 different manufacturing industries, and then averaged across industries using each industry's share of aggregate employment in each year.

The rise in $\mathcal{C}$ over time is mainly due to changes within industries, and not to changes in the industry composition. The two dotted lines in Figure 2.4 plot the values of $\mathcal{Z}$ and $\mathcal{C}$, respectively, using constant aggregation weights across industries, computed either at the beginning of the sample (bottom dotted line), or at the end (top dotted line). Some of the change in average $\mathcal{C}$ is due to a shift over time toward industries with a higher value of $\mathcal{C}$, because the 2011-weights line is higher than the other two, but all three lines feature a dramatic increase in $\mathcal{C}$ over the sample period.

Changes in $\mathcal{C}$ within a country over time suggest changes in allocative efficiency, which can be driven by financial development. Bartelsman, Haltiwanger and Scarpetta (2013) show that $\mathcal{C}$ is increasing over time in all the economies they study, and it is increasing more rapidly in the developing economies of astern Europe. In the next section, I derive a structural model of firm investment and growth in which financial frictions have a direct impact on allocative efficiency, and this impact feeds through to $\mathcal{C}$. However, I show in section 2.3 using a calibrated version of the model that common shocks to productivity also increase $\mathcal{C}$, and that in fact I can match the time-series evolution of $\mathcal{C}$ in the bottom panel of Figure 2.4 without any financial development at all.

2.2 Model

In this section, I derive a model in which entrepreneurs borrow to smooth consumption in the face of persistent idiosyncratic productivity shocks, but cannot commit to repaying their debt. I then put many such entrepreneurs together in industry equilibrium and compute the endogenous size-productivity distribution, enabling the model to replicate both the difference in difference regressions of section 2.1.3, and the Olley-Pakes covariance of section 2.1.4, as endogenous responses to exogenous shocks to aggregate TFP and financial frictions. In the model, reductions in financial frictions do lead to positive difference in difference coefficients and a rising Olley-Pakes covariance, but so do common shocks to productivity. This section derives the model and characterizes its equilibrium, while in section 2.3 I calibrate the model to match some features of the ASI-NSS data and show that, quantitatively, common shocks to productivity alone are more than sufficient to drive the empirical results of section 2.1.

The heart of the model derived in this section is a borrowing decision by agents in the face of a financial friction: agents may borrow as much as they wish, but they cannot commit to paying their debt back, and lenders price debt according to the probability of default. Because defaulting entails a loss of output, an increase in productivity (all else equal) increases the cost of default, lowering its probability, resulting in higher bond prices. In equilibrium, an increase in productivity thus mimics a reduction in the financial friction itself.

To be consistent with previous literature, and to closely match the empirical measure of productivity I use in section 2.1, I layer the financial friction on top of an extended version of the static model of Hsieh and Klenow (2009). My model features monopolistically-competitive firms combining labor and capital in a Cobb-Douglas production function, allowing the model to endogenously generate differences in physical vs. revenue productivity, and total-factor vs. labor productivity. In both the data and the model, I observe only revenue labor productivity, although the exogenous shocks I apply to the model are to physical total-factor productivity.

Layering the financial friction on top of a model with decreasing returns to scale in revenue works against the conclusions of the paper for two reasons: first, in the first-best allocation of both Hsieh and Klenow (2009) and the model in this paper, the Olley-Pakes covariance between the average revenue product of labor and firm employment is zero. Second, firms have an optimal scale of production which, if it can be reached in equilibrium, will take the bite out of financial frictions entirely. I discuss each of these challenges in turn before proceeding to the details of the model.

First, the production function used in this model implies a zero Olley-Pakes covariance in the first-best allocation. This is because in the optimal allocation firms equalize their marginal revenue products of labor and capital, and the production function is such that average products are proportional to marginal products, and thus also equalized. This implies that a positive and increasing Olley-Pakes covariance, such as that seen in India or in all the countries analyzed by Bartelsman, Haltiwanger and Scarpetta (2013), reflects increasing misallocation over time, and not (as argued here) potential financial development. Instead, I show using the model derived in this section that financial frictions naturally imply a positive Olley-Pakes covariance that grows over time in response to positive shocks to financial development and productivity.

Second, in this model each firm has an optimal scale of production, so that in equilibrium firms might accumulate enough assets that they are effectively unconstrained. This is precisely the mechanism behind the result of Midrigan and Xu (2014) that financial frictions are not important: they find that firms can "grow their way out of" finacial constraints. I find that for the calibration presented in section 2.3, enough firms are financially constrained in equilibrium that financial development can still have big effects on the allocation of employment across firms.

The rest of this section is organized as follows: in section 2.2.1 I describe the agents in the model, their preferences, their constraints, and their potential actions. In section 2.2.2 I define and characterize equilibrium in this model.

2.2.1 Environment

Time is countable and there is a single good that can be consumed or used for investment. There is a continuum of agents, each of whom has log utility and discounts the future at a rate $\beta \leq 1$. Agents do not live forever, but die each period with iid probability π. Each agent i has idiosyncratic productivity $z_{i,t}$, which evolves according to the AR(1) process

$$z_{i,t+1} = \rho z_{i,t} + \sigma \varepsilon_{i,t+1}, \tag{2.3}$$

where $\varepsilon_{i,t+1}$ is a standard normal random variable that is independent across agents and time. Physical production combines labor l and capital k according to the constant returns to scale production function

$$y_{i,t} = A e^{z_{i,t}} k_{i,t}^{\alpha} l_{i,t}^{1-\alpha}. \tag{2.4}$$

Agents can freely convert the consumption good into units of capital, and vice versa, at a relative price of 1; there are no capital adjustment costs. Capital does not depreciate and must be non-negative. Producers are monopolistically competitive and face individual demand curves of the form

$$p_{i,t} = y_{i,t}^{-\frac{1}{\gamma}}, \tag{2.5}$$

so that equations (2.4) and (4.2) together imply a decreasing returns to scale revenue function, as in Hsieh and Klenow (2009).

Agents solve a two-stage optimization problem: the first stage is an investment stage in which agents choose how much to consume, borrow, and invest for the future, before learning their own idiosyncratic shock ε. The second stage is a production stage, after idiosyncratic uncertainty has been resolved, in which agents choose how to allocate resources between labor and capital, given resources saved from the previous period.

In the production stage, given productivity z' and total resources \$, agents choose capital and labor to maximize revenue and undepreciated capital, subject to their budget:

$$\begin{aligned} y^*\left(\widetilde{A}, \$\right) = \max_{k,m} \quad & \underbrace{\left[\widetilde{A} m^{1-\alpha} k\right]^{\frac{\gamma-1}{\gamma}}}_{py} + k \\ \text{s.t.} \quad & \\ & (1 + wm)\, k \leq \$ + w\underline{L} \\ & mk \geq \underline{L}, \end{aligned} \tag{2.6}$$

where w is the wage, m denotes the labor-capital ratio, and $\widetilde{A} \equiv Ae^{z'}$ depends on the current level of idiosyncratic productivity z' as well as any aggregate shock to productivity A. y^* denotes the solution to equation (2.6); it represents the total resources available to the agent in the next period, apart from any debt repayment.

The second-stage problem (2.6) has two constraints: a working-capital constraint and an overhead-labor constraint. The first constraint means that agents must invest in physical capital k, and pay the wage bill wkm, before output is produced. Agents may borrow against future earnings in order to raise working capital, but to make their problem economically interesting I assume that borrowing must occur in the first stage, before idiosyncratic risk is realized.[5]

The second constraint in problem (2.6) is that agents must use at least $\underline{L}$ units of labor in production. Agents are endowed with $\underline{L}$ units of their own labor, but they must use all of this labor on their own project; thus $w\underline{L}$ appears on the right-hand side of the budget constraint and agents only pay wages for labor they hire beyond $\underline{L}$. In equilibrium, the solution to problem (2.6) is a labor-capital ratio m that is increasing in productivty. In the limit as productivity goes to $-\infty$, absent the overhead labor constraint, agents choose (effectively) zero labor. This means these agents make no contribution to aggregate labor productivity. In the data (see Figure 2.3), there is a massive number of very-small plants with very low labor productivity. Allowing plants in the model to "exit" by setting their labor-capital ratio very close to zero precludes matching this feature of the data, and I get around this problem by assuming that each agent must invest her endowment of labor in her own project.

In the first stage, before the idiosyncratic shock is realized, agents choose consumption c, savings \$, and borrowing b to maximize the present expected discounted value of their stream of utility, knowing that their future wealth depends on the outcome of solving problem (2.6). Financial markets are incomplete, and the only asset that agents may trade are one-period zero-coupon bonds. To match the dispersion in productivity in the combined ASI-NSS data, I assume that agents cannot lend to each other; lenders in this economy are agents outside this production sector. Let q denote the price of one unit of face value of the agent's debt, and the agent's chosen face value b. $b > 0$ denotes that the agent is borrowing funds.

Agents are free to borrow as much as they wish, but they cannot commit to repaying

5. A simpler model, in which agents invest capital k before uncertainty is realized but hire labor in frictionless markets afterwards, using part of the proceeds from production to pay the wage bill, would lead to an equilibrium where agents equalize labor productivity ex post regardless of their size or TFP. In fact in India labor productivity varies widely across plants, as I show in section 2.1.

their debt in the future. The timing of the default decision is as follows: at the beginning of each period, the agent learns her own productivity value $z_{i,t}$. At that moment, she decides whether she will pay back her borrowing (if $b > 0$) or default. If she defaults, she retains a fraction $1 - \theta_{i,t}$ of her own savings \$, may not produce using the solution to problem (2.6), and sets $b = 0$. The lender recovers a total amount $\theta_{i,t}\$$, that must be distributed among a total face value b, so that per bond, the lender recovers

$$\begin{aligned} \chi &\equiv \min\left\{1, \theta_{i,t}\frac{\$}{b}\right\} \\ &= \min\left\{1, \frac{\theta_{i,t}}{\ell}\right\}, \end{aligned} \tag{2.7}$$

where the last line defines the agent's leverage as $\ell \equiv \frac{b}{\$}$. The min ensures that lenders do not recover more than they were owed; if agents default with enough assets such that lenders would recover more than they were owed, the extra resources are lost. This kind of deadweight loss will not occur in equilibrium. Agents that default do not exit the economy, but start a new firm next period. They do not suffer any exclusion from financial markets; the only penalty for defaulting is the loss of all potential output this period and a fraction θ of their capital. Notice also that the value of θ can vary across agents i and over time t.

Lenders are perfectly competitive and require an exogenous expected return of r on their portfolios. Lenders understand that borrowers may default and that if they do, the lenders only recover $\chi \leq 1$ per bond, and they incorporate this default risk into the zero-coupon bond price q. For their part, borrowers understand that the bond price they pay per unit of face value depends on the current state and their choices for investment and borrowing.

In equilibrium, the realized return to each lender's portfolio of bonds is riskless at an exogenous level r, because lenders can lend to a sufficiently diverse cross-section of borrowers such that a law of large numbers applies and their total return across all bonds is riskless. A key tractability assumption behind this result is that $A_{i,t}$ and $\theta_{i,t}$ change after default decisions are made and production has occurred, but before new borrowing, investment, and consumption decisions are made. This assumption means that when agents borrow, the only unknowns are the realizations of the idiosyncratic shocks $\varepsilon_{i,t+1}$, which are diversifiable by lenders. The bond price q charged to any individual borrower can perfectly offset that borrower's default risk, so that the lender's expected return is constant across loans and this expected return is equal to the realized return on their entire portfolio.

Each period, agents choose consumption c, capital next period k', and financial assets b' (or equivalently, leverage ℓ'). The budget constraint for an individual agent is then

$$c - q\left(\ell', \$', z; A_{i,t}, \theta_{i,t}\right)\ell'\$' + \$' = \begin{cases} y^* - \ell\$ & \text{if debt is repaid} \\ \left(1-\theta_{i,t}\right)\$ & \text{if defaulted on debt} \end{cases}$$

where the bond price q depends on the agent's choices for investment and leverage, their productivity z, aggregate productivity $A_{i,t}$, and the aggregate recovery rate $\theta_{i,t}$.

Figure 3.1 illustrates the timing assumptions of the model. For technical reasons, I assume that agents learn early in the period whether they will exit, immediately after their productivity z is realized. This timing assumption, which is also used by Gilchrist, Sim and Zakrajšek (2014) and Khan, Senga and Thomas (2014), ensures that agents cannot exit the model exogenously with outstanding debts. Otherwise, the exogenous death risk would enter the bond-price equation, where it would complicate the model solution without adding anything substantive.

In the investment stage, before the idiosyncratic shock is realized, agents choose consumption, savings, and leverage to maximize the present discounted value of their stream of utility, leading to the following recursive representation:

$$
\begin{aligned}
V_{i,t}(x,z) = \max_{\$,\ell\geq 0} \quad & \log(c) + \beta\left[(1-\pi)E\left\{V_{i,t+1}\left(x',z'\right)\right\} + \pi E\left\{\log\left(x'\right)\right\}\right] \\
\text{s.t.} \quad & \\
& c \equiv x - (1-q\ell)\$ \\
& q \equiv q\left(\ell,\$,z;A_{i,t},\theta_{i,t}\right) \\
& z' = \rho z + \sigma\varepsilon \\
& x' = \max\left\{y^*\left(A_{i,t}e^{z'},\$\right) - \ell\$, \left(1-\theta_{i,t}\right)\$\right\},
\end{aligned}
\tag{2.8}
$$

where $\ell = \frac{b}{\$}$ is leverage, and $q(\ell,\$,z;A,\theta)$ is the bond price lenders charge to ensure that their expected return is r. The value function $V(x,z)$ is indexed by (i,t) because the values of A and θ may vary across agents or over time; these changes are deterministic and foreseen by agents.

2.2.2 Equilibrium

The key endogenous object that allows the model to replicate the difference in difference regressions of section 2.1.3 and the aggregate productivity decomposition of section 2.1.4 is the cross-sectional distribution of size and productivity. This distribution depends on the equilibrium policy functions that arise from solving problems (2.6) and (2.8). In this section I define the equilibrium and characterize it.

To ensure a stationary equilibrium, agents enter the model exogenously to “replace” those that exit due to the iid probability π of dying. Assume a continuum of agents and normalize its measure to 1. Because a measure π of agents will exit the model each period, a measure π of agents must enter the model each period to maintain stationarity. Let these new agents have net wealth x drawn from the lognormal distribution $\log x \sim \mathcal{N}\left(0,\sigma^2_{x_0}\right)$, and their independent idiosyncratic productivity z is drawn as $z \sim \mathcal{N}\left(0,\sigma^2_{z_0}\right)$.

I characterize the cross-sectional joint distribution of net wealth x and productivity z for agents of type i at time t as a cumulative distribution function $F_{i,t}(\log x^*, z^*)$. This function is the probability that a randomly-drawn firm of type i at time t has net wealth $x < x^*$ and idiosyncratic productivity $z < z^*$. Agent types are permanent, so I can compute the CDFs for each type of agent separately and later aggregate across them according to the measure of each type.

I parameterize F in terms of $\log x$ instead of x because in equilibrium this parameterization will behave better computationally. For fixed values of $A_{i,t}$, $\theta_{i,t}$ and the function $F_{i,t}$, the function $F_{i,t+1}$ is given by

$$F_{i,t+1}(\log x^*, z^*) = (1-\pi) \int \Pi_{i,t} \left\{ \begin{pmatrix} x^* \\ z^* \end{pmatrix}, \begin{pmatrix} x \\ z \end{pmatrix}; A_{i,t+1}, \theta_{i,t} \right\} dF_{i,t}(\log x, z) + \pi F^e(\log x^*, z^*), \tag{2.9}$$

where $F^e(\cdot,\cdot)$ is the CDF of $(\log x, z)$ for the agents that enter exogenously each period, described above, and the function $\Pi_{i,t}$ is the conditional transition CDF for agents of type i at time t, which depends on the decision rules of individual agents, the law of motion of the idiosyncratic productivity z, and the paths of the exogenous aggregate variables $A_{i,t}$ and $\theta_{i,t}$ (see Appendix B.3 for details). I use equation (3.11) to compute the evolution over time of the size-productivity distribution, and also to compute the steady-state distribution where $F_{i,t+1} = F_{i,t}$.

I define equilibrium in this model as a collection of distribution functions $F_{i,t}(\log x^*, z^*)$, policy functions $\$_{i,t}(x,z)$, $\ell_{i,t}(x,z)$, $m\left(\widetilde{A}, \$\right)$, and $k\left(\widetilde{A}, \$\right)$, and a bond-price function $q(\ell, \$, z; A, \theta)$ that satisfies the following conditions, given the sequence of exogenous aggregate states $A_{i,t}$ and $\theta_{i,t}$:

1. The functions $m\left(\widetilde{A}, \$\right)$ and $k\left(\widetilde{A}, \$\right)$ solve equation (2.6), given productivity $\widetilde{A} \equiv Ae^{z'}$ and cash on hand $\$$. Denote the solution to equation (2.6) as $y^*\left(\widetilde{A}, \$\right)$.

2. The function $q(\ell, \$, z; A, \theta)$ ensures that the expected return to lenders over the idiosyncratic shock ε is r, given the recovery rate equation (2.7) and that borrowers default whenever $y^*\left(Ae^{z'}, \$\right) - \ell\$ < (1-\theta)\$$.

3. The functions $\$_{i,t}(x,z)$ and $\ell_{i,t}(x,z)$ solve equation (2.8), given $y^*\left(\widetilde{A}, \$\right)$, the current values of x and z, the function $q(\ell, \$, z; A, \theta)$, and the entire time-series paths of $A_{i,t}$ and $\theta_{i,t}$.

4. The distribution functions $F_{i,t}(\log x^*, z^*)$ satisfy equation (3.11), given the time-series evolution of $A_{i,t}$ and $\theta_{i,t}$, where the transition functions $\Pi_{i,t}$ are consistent with the policy functions $\$_{i,t}(x,z)$ and $\ell_{i,t}(x,z)$.

The solution to the model consist of four pieces: first, I solve the static problem (2.6). I use this solution to derive the equilibrium default policy, which determines the bond-price function $q(\cdot)$. Given both of these elements, I solve problem (2.8) numerically using value function iteration. This solution then determines the transition CDF in equation (3.11), allowing me to characterize the endogenous distribution of net wealth x and productivity z and compute moments of interest.

The solution to the static second-stage problem in equation (2.6) is given in the following proposition:

Proposition 1. *The solution (k^*, m^*) to problem (2.6) is given by*

$$k^* = \frac{\$ + w\underline{L}}{1 + wm^*},$$

where m^* is either the value of m that solves

$$m^{1-\zeta} = A^*\left(1 + wm\right)^{\frac{1}{\gamma}}\left(\frac{1-\alpha}{w} - \alpha m\right), \tag{2.10}$$

with

$$A^* \equiv \frac{\gamma-1}{\gamma}\left(Ae^{z'}\right)^{\frac{\gamma-1}{\gamma}}\left(\$+w\underline{L}\right)^{-\frac{1}{\gamma}} \tag{2.11}$$

and $\zeta \equiv (1-\alpha)\frac{\gamma-1}{\gamma}$, *or*

$$m = \frac{\underline{L}}{\$},$$

whichever is larger.

So long as $\frac{1-\alpha}{\alpha} < \gamma$, *there is a unique solution* $m^*\left(z', \$\right)$ *to equation (2.10) that is weakly increasing in* z' *and decreasing in* \$.

Proof. See Appendix A. □

Apart from the overhead labor constraint, the optimal labor-capital ratio m^* that solves (2.6) is increasing in productivity z', and decreasing in total resources \$, because the agent retains capital k after production occurs. Intuition for this result is clearer for the negative case: as revenue productivity goes to zero (either because $z' \to -\infty$ or because $\$ \to \infty$), the agent derives little benefit from production and consequently hires the minimum amount of labor, setting $k = \$$ and $mk = \underline{L}$. Increasing revenue productivity raises the benefit to production relative to undepreciated capital and induces some resources to be spent on labor, raising m.

Given the solution to equation (2.6), agents will default whenever their resources from production and repayment, $y^* - \ell\$$, are less than the resources they derive from defaulting, $(1-\theta)\,\$$. The following proposition characterizes their optimal default behavior:

Proposition 2. *Suppose* $\theta < 1$, *and* $\alpha < \min\left(\frac{1}{2}, \frac{1}{\gamma-1}\right)$. *Fix* A, z, ℓ, *and* \$. *The default threshold equation*

$$y^*\left(Ae^{\rho z+\sigma\underline{\varepsilon}}, \$\right) - \ell\$ = (1-\theta)\,\$ \tag{2.12}$$

either has a unique solution $\underline{\varepsilon}$, or has no solution. Agents default if and only if $\varepsilon < \underline{\varepsilon}(\ell, \$, z)$ when equation (2.12) has a unique solution, or they never default. The $\underline{\varepsilon}$ that solve equation (2.12) are decreasing in z and increasing in \$. If $\ell \leq \theta$, the probability of default is zero ($\underline{\varepsilon} = -\infty$).

Proof. See Appendix A. ☐

The proof of Proposition 2 does more than establish the result, which is straightforward from the fact that y^* is increasing in z': it gives a formula for $\underline{\varepsilon}$ in closed form, which allows the bond price, given by

$$q = \frac{1}{1+r}\left[1 - \left(1 - \min\left\{1, \frac{\theta}{\ell}\right\}\right)\Phi\left\{\underline{\varepsilon}\left(\ell, \$, z; A, \theta\right)\right\}\right], \tag{2.13}$$

to also be derived in closed form, irrespective of the \$ or ℓ policies or the value function of the agent. This allows for tractable numerical solutions to problem (2.8) and equation (3.11), which I describe in the next section.

2.3 Results

In this section, I use a calibrated version of the model derived in section 2.2 to understand how financial development (changes in θ) and common shocks to productivity (changes in A) affect the difference in difference regressions and the aggregate productivity decomposition of section 2.1. I show that while changes in θ do generate positive diff-in-diff coefficients and increase the Olley-Pakes covariance, consistent with conventional intuition, I can also generate the same effects with a constant θ and an increase in aggregate TFP A alone, suggesting that these reduced-form measures are not sufficient indicators of financial development. In chapter 3 I use a simpler version of the model in section 2.2 to map both terms in the Olley-Pakes decomposition to changes in A and θ; the results ot aht exercise are similar to what is presented here.

The results in this section are based on a numerical approximation to the model's solution conditional on a set of parameters. In section 2.3.1 I calibrate the parameters of the model to match some key moments o the ASI-NSS data. Section 2.3.2 describes the difference in difference exercise I perform in the model, for various aggregate shocks, and section 2.3.3 analyzes the determinants of the Olley-Pakes decomposition in the model.

2.3.1 Calibration

In order to match the size and productivity differences across ASI and NSS plants in the data, in all the exercises performed in this section I assume two types of agents who differ in their average physical productivity, given by the parameter A. NSS or informal firms have a much lower level of A than ASI or formal firms, so that in equilibrium the agents with lower physical productivity will be smaller and, because their overhead-labor constraint is more likely to bind, they will also have lower revenue labor productivity. Both types of agents may borrow, although those with very low values of A are much less likely to do so in equilibrium. Common shocks to productivity increase the A values of formal and informal firms by the same percentage.

The parameters that apply across all exercises in this section are listed in Table 2.6. I group parameters into two categories: Panel A lists the parameters that are standard and that I do not use to calibrate the model. These include the riskless interest rate r, the rate of time preference β, the demand elasticity parameter γ, the capital share α, the persistence of the productivity shock ρ, and the share of low-TFP plants μ. The model requires persistent productivity shocks because without them, agents would not respond to the current value of their productivity. I do not calibrate ρ because I do not observe plants over time in the combined ASI-NSS data. I set the measure of low-productivity plants to a constant 0.99, even though the share of NSS plants is changing over time (see Panel A of Table 2.1), for simplicity. Results of this section are similar for extended versions of the model where I vary the entry share each period (but hold the exit probability fixed) to match the changing NSS

Panel A: Set Parameters		
Parameter	Value	Description
r	1%	Riskless Interest Rate
β	0.99	Time Discount
γ	3	Demand Elasticity
α	0.3	Capital Share
ρ	0.95	TFP Persistence
μ	0.99	Measure of Low-A Plants

Panel B: Calibrated Parameters					
Parameter	Value	Description	Target Moment	Data	Model
π	0.083	Exit Probability	Average Age	12	11.95
w	0.2	Wage	NSS % of Labor	86%	86%
A	0.34	Average Gross TFP	NSS % of Output	31%	43%
σ	0.45	Std Dev Shock	Labor Prod. Disp.	1.07	1.07
$\Delta_i \log A$	-3.7	log TFP Diff.	ASI-NSS Prod. Diff	-2.0	-2.2

Table 2.6. Model Parameters
The table reports the parameter values of the dynamic model described in section 2.2, and analyzed in section 2.3. Panel A reports the fixed parameters that are standard and not used in calibration. Panel B reports the fixed parameters that are used to match the indicated data moments.

share over time.

Panel B of Table 2.6 reports the parameters of the model I use to calibrate to the data. All the moments listed in the table are determined by all the parameters jointly, but I group data moments with the parameters that most affect them. I set the exogenous exit rate π to 1/12 to match the average age of plants in the data of 12 years. The model-implied average age of 11.95 years is less than 12 because I assume that defaulting firms have their age reset to 1; default rates in equilibrium are relatively low, so this correction has little effect on average firm age. I set the standard deviation of the exogenous idiosyncratic productivity shock σ to 0.45 to match the high standard deviation of labor productivity in the data; the value of 1.07 is the standard deviation of labor productivity in the data after removing industry means.

I set the wage rate w, the base rate of productivity A, and the productivity difference across agent types $\Delta_i \log A$ to match the NSS shares of aggregate labor and revenue, and the labor productivity spread between ASI and NSS plants. The decreasing returns to scale in revenue imply that the cross-sectional labor-productivity distribution will be a compressed version of the TFP distribution. Thus to ensure a log labor productivity difference of about 2 between formal and informal firms, the model requires an average log TFP difference of 3.7. The current calibration matches the share of economic activity accounted for by low-TFP firms relatively well, but does so with too great a spread between their labor productivity and the labor productivity of high-TFP firms. Matching this moment more tightly is a priority for future work.

2.3.2 Difference in Difference Regressions

In this section I use the model described in section 2.2 to perform idealized difference-in-difference regressions, and show that when there are aggregate shocks to productivity, a positive difference-in-difference in either leverage or aggregate output is neither necessary nor sufficient evidence for financial development. I define financial development as a change in the collateral rate θ, which (as described above) in equilibrium reduces the cost of borrowing for firms, without affecting their exogenous total-factor productivity.

In a perfect difference-in-difference scenario, only one set of agents receives the treatement, and this set of agents is chosen randomly. In order to use a difference-in-difference design to understand the effects of an aggregate shock, however, treatment must be defined in terms of sensitivity. In principle, financial development is a treatment that applies to all firms in an economy; the identification argument in this case is instead that some firms are more or less sensitive to the same shock. For example, firms in industries that (for whatever reason) tend to borrow more, or have more collateralizable assets, are presumably more affected by a change in financial development than other firms. In this section I show using the calibrated model that such firms may also be more sensitive to other aggregate shocks, confounding identification of the effect of interest even when firms are perfectly separated into "more sensitive" and "less sensitive" groups.

To match the empirical results of section 2.1.3, in the exercises in this section I compute aggregate statistics over twenty-two time periods (years). I assume the economy is in stochastic steady-state from period 1 to period 11. In period 12, agents learn that the aggregate values of total-factor productivity (A) and/or financial development (θ) will change in period 13, to values where they will remain forever. This means that the value and policy functions from period 12 onwards are constant; however, aggregate moments can still move because the endogenous joint distribution of (x, z), which is evolving according to equation (3.11), may not settle down to its stationary value immediately.

Table 2.7 reports model estimates for four difference-in-difference exercises. In each exercise, I assume four types of firms, two with a high level of θ (the treatment group) and two with a low level of θ (the control group). Within each type for θ are high- and low-TFP firms, where the spread in $\log A$ is reported in Table 2.6. These are the only dimensions along which firms vary ex ante (firms within each group vary in terms of idiosyncratic total-factor productivity z, but the distribution of z is identical across treatment and control types); in this sense, identification in the model is exact and along the same lines used by Vig (2013), who argues that high-tangibility firms are more affected by the Sarfaesi Act than low-tangibility firms. The identification used here is also similar to that in Rajan and Zingales (1998): firms with higher values of θ face higher bond prices at all values of leverage and productivity, and so in equilibrium will finance a greater share of investment externally; thus the Rajan & Zingales method likely would classify high-θ firms as more "exernally dependent."

The first set of rows in Table 2.7 reports results from an idealized difference-in-difference scenario: both types of firms are identical in every respect other than the level of θ, which at $t = 13$ increases by 10%. Columns four through six of Table 2.7 show that as a result of this shock, both types increase leverage on average, but that the high types do indeed increase their leverage by about 30 bps more on average. The last column of Table 2.7 shows that aggregate output in the treated industry rises almost six percentage points more than the un-treated industry.

The second set of rows in Table 2.7 add a common shock to productivity in addition to the shock to θ: both types on average become 10% more productive at $t = 13$. A difference-in-difference methdology still "works" in this case: leverage and output rise on average for both types of firms, but by more for the "treated," high-θ firms. Together, the first two groups of rows in Table 2.7 suggest that difference-in-dffierence regressions of the kind analyzed in section 2.1.3 will produce positive coefficients when financial develoment occurs, even if there is also productivity growth.

However, the third and fourth exercises in Table 2.7 show that such a positive difference-in-difference result is neither necessary nor sufficient to identify financial development. In the third exercise, the two types differ in θ as before but θ is constant over time. Both types receive, as in the second exercise, a 10% productivity shock. In this case, the productivity shock lowers the cost of borrowing, but it does so by more for the high-θ firms. This results in the high-θ types increasing leverage by 35 bps more than the low-θ types. It also results

Exogenous			Endogenous			
θ_0	θ_1	$\%\Delta A$	$E\{\ell\}_0$	$E\{\ell\}_1$	diff	$\%\Delta \log Y$
0.6	0.66	0	35.7	37.9	2.2	6.57
0.4	0.44	0	26.9	28.8	1.91	0.78
					0.292	5.79
0.6	0.66	10	35.7	39.4	3.73	18.9
0.4	0.44	10	26.9	29.9	3	12.3
					0.726	6.65
0.6	0.6	10	35.7	36.9	1.27	12
0.4	0.4	10	26.9	27.8	0.924	11.4
					0.35	0.624
0.6	0.66	5	35.7	38.4	2.77	12.9
0.4	0.44	20	26.9	30.8	3.91	23
					-1.13	-10.1

Table 2.7. Difference-in-Difference in the Model
Each panel reports model results from a difference-in-difference computation. The first three columns report the exogenous aggregate variables for each exercise. The first column reports the initial (pre) value for θ, the second column the post value of θ, and the third column reports the percentage change in total-factor productivity. The rightmost five columns report endogenous model moments from each exercise: the fourth column reports average leverage in the pre-period, the fifth column average leverage in the post-period, and the sixth column the difference between the two. The last column reports the log change in output in percent. The third row in each section reports the difference in difference (top minus bottom row) for leverage, output, and labor productivity. Each computation lasts for twenty-two periods (years); agents learn in period 12 that total-factor productivity and/or output will change to their post values at period 13, where they will remain. The pre-period is periods 1–12 and the post-period is period 13–22.

in output growing by about 62 bps more for the high-θ types.

The third exercise in Table 2.7 features no financial development, but a positive difference-in-difference result; the final exercise in Table 2.7 has financial development, but a negative difference-in-difference result. Here I assume that θ grows by 10% for both types as before, but that the low-θ types have faster average total-factor productivity growth than the high-θ types. In this exercise, the difference in productivity growth is more important than the shock to θ, so that while both types experience output growth, and increase their leverage ratios on average, the low-θ type increases everything by more.

None of the exogenous shocks examined in Table 2.7 are large when compared to variation in the data; if anything, they are too small. Leverage differences in the data between types range from 5 to 9 percentages points (see Table 2.4), and the assumed differences in θ lead to a roughly 8 percentage point difference in Table 2.7. Average output growth over the entire sample period, reported in the last column of Panel A of Table 2.3, is between 0.4 and 2.0 in log-points, compared to at most 20% in Table 2.7. Likewise, labor productivity growth in the data (apart from Tobacco plants) is significantly larger than that reported in the last column of Table 2.7. I conjecture that larger productivity shocks would lead to even bigger movements in leverage and output, and further confound attempts to use difference-in-difference methodologies to identify financial development.

As an additional robustness check, and in parallel to the regressions in Table 2.5, in Table 2.8 I run difference in difference regressions in the model controlling for plant-level labor productivity. Unlike the results reported in table 2.7 which are differences in means (integrals) that can be computed directly, controlling for labor productivity means simulating data from the model and running regressions on the simulated data. Details on these computations are in Appendix C.

The results reported in Table 2.8 show that, even controlling for firm-level labor productivity, aggregate productivity growth can induce a positive difference in difference coefficient even in the absence of financial development. As in Table 2.7, the first two blocks of rows

Exogenous			Coefficients			
θ_0	θ_1	$\%\Delta A$	Treat x Post	Treat	Post	Prod
0.6 0.4	0.66 0.44	0 0	0.245	8.79	1.72	13.2
0.6 0.4	0.66 0.44	10 10	0.661	8.78	2.01	13.4
0.6 0.4	0.6 0.4	10 10	0.363	8.77	0.155	12.8
0.6 0.4	0.66 0.44	5 20	-0.0622	8.79	2.2	13.4

Table 2.8. Difference in Difference in the Model, Controlling for Productivity Each panel reports model results from a difference-in-difference regression computed on simulated data. I simulate 5 million firms of each type (high and low θ); see Appendix C for details on the simulation procedure. The first three columns report the exogenous aggregate variables for each exercise. The first column reports the initial (pre) value for θ, the second column the post value of θ, and the third column reports the percentage change in total-factor productivity. The rightmost four columns report OLS coefficient estimates for the following variables: a dummy variable equal to 1 for observations of the high-θ type in the post period, a dummy equal to 1 for observations of the high-θ type, a dummy equal to 1 for observations in the post period, and each firm's realized log revenue labor productivity. Each computation lasts for twenty-two periods (years); agents learn in period 12 that total-factor productivity and/or output will change to their post values at period 13, where they will remain. The pre-period is periods 1–12 and the post-period is period 13–22.

show that financial development does induce a positive diff-in-diff coefficient. However, the third block also features a positive coefficient, without any financial development at all; moreover the coefficient is not appreciably different from the corresponding coefficient in Table 2.7. Finally, higher productivity growth amongst the low-θ firms can induce a negative diff in diff coefficient, even when financial development did occur. However, controlling for labor productivity reduces this effect considerably relative to the results in the last block of rows in Table 2.7. Finally, the coefficient on leverage in Table 2.8 is positive, as in the regressions results in Table 2.5 and in contrast to a canonical finding of Rajan and Zingales (1995). I attribute the difference in the model to a different assumed rationale for borrowing (consumption smoothing rather than tax evasion), and in the data to a focus on small, privately-owned firms.

2.3.3 Aggregate Productivity

In this section I use the calibrated model to analyze the determinants of the Olley-Pakes covariance, plotted in the bottom panel of Figure 2.4. I perform two exercises: first, I demonstrate that when comparing stochastic steady-states across economies, a higher level of financial development θ or higher aggregate total-factor productivity A increases the Olley-Pakes covariance. Second, I compute a dynamic equilibrium in which aggregate total-factor productivity is growing over time. I calibrate the path of aggregate TFP to closely match the observed Olley-Pakes covariance in the data, holding the level of financial development fixed.

Both results suggest that while financial development does improve the allocation of labor as measured by the Olley-Pakes covariance, so does aggregate productivity growth in the presence of (constant) financial frictions. There are two features of the model that work against this conclusion. The first is that, because firms have an optimal scale of production, in principle they have the ability to accumulate enough capital to "grow out of" their financial constraints. If this were the case, increasing θ would have little to no real effects in the aggregate. The second is that, in the first-best allocation, labor productivity depends only the wage w (and other parameters). Thus increasing aggregate total-factor productivity should only affect total output and the size of firms, but not labor productivity. In equilibrium, enough firms are constrained that these effects are smaller than the real effects of changes in θ and A on the allocation of resources.

Figure 2.6 examines these questions directly by plotting four comparative statics of this model, for $\theta \in \{0.4, 0.8\}$ and $A \in \{0.34, 0.425\}$. Each bar represents an aggregate value at the ergodic steady-state size-productivity distribution for the indicated parameter values. The top panel reports the Olley-Pakes covariance, and the bottom panel plots the average log labor productivity, in each of these four economies. Formulas and other calculation details can be found in Appendix C. The top panel of Figure 2.6 shows that the Olley-Pakes covariance increases with both financial development θ and with aggregate TFP A. The bottom panel shows that, although the wage is fixed, average labor productivity responds to productivity shocks in equilibrium.

The results reported in Figure 2.6 compare distinct economies with different values of aggregate productivity and financial development, once the firm-size distribution has converged to a stochastic steady-state: individual firms still experience idiosyncratic shocks, but aggregate values are constant. Figure 2.7 instead explores how aggregate values in a single economy change over time, as aggregate productivity increases. I start the economy at a stochastic steady-state size-productivity distribution, hit it with a path of aggregate productivity growth, and then trace out the implied Olley-Pakes covariance over time.

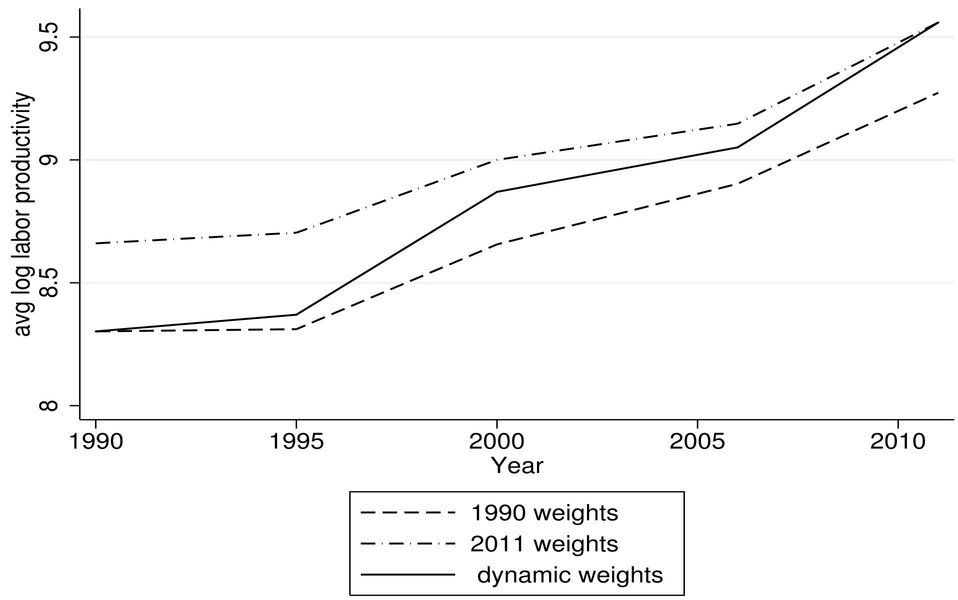
To solve the model, I start at the final period where the exogenous variables are no longer moving. I solve for the steady-state value and policy functions at this date according to equation (2.8), and then step backwards one period at a time, solving for value and policy functions in equation (2.8) using next period's value function as the continuation value. This gives me the sequence of policy functions at each date. I then solve for the $t = 0$ endogenous distribution of (x, z) in equation (3.11) by solving the steady-state problem for the $t = 1$

aggregate variables, and iterate this distribution forward using equation (3.11) and the policy functions at each date.

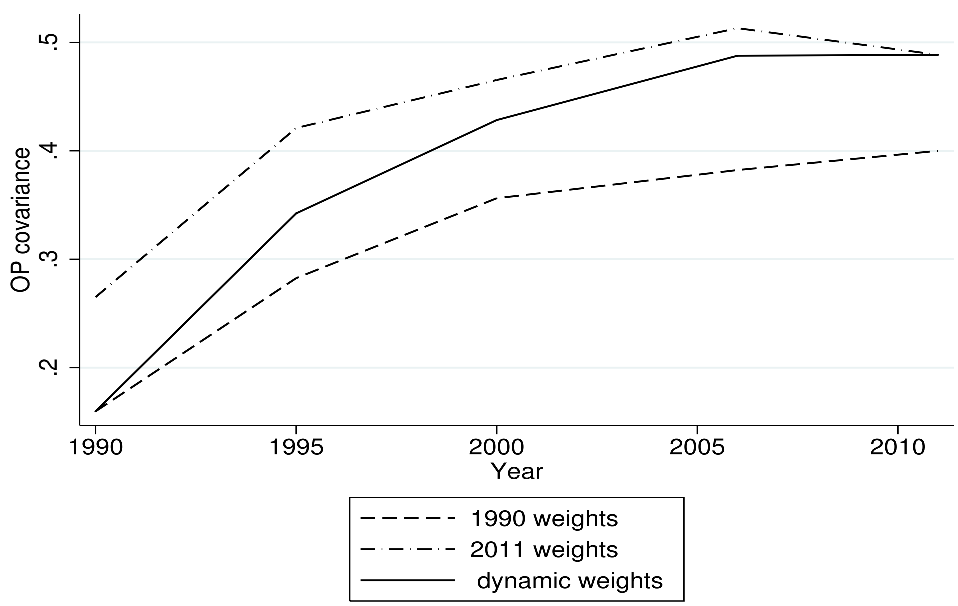
Figure 2.7 plots the exogenous paths of aggregate productivity A (top panel), and the implied path of the Olley-Pakes covariance (bottom panel). I hold the level of financial development θ constant at 0.4, and chose the precise path of A_t in order to match the observed path of the Olley-Pakes covariance in the bottom panel of Figure 2.4. For reference, I then repeat the exercise for parallel productivity paths that are 10% higher and lower, to further illustrate the sensitivity of the Olley-Pakes covariance to productivity growth.

The bottom panel of Figure 2.7 shows that the Olley-Pakes covariance is responsive to aggregate productivity growth, and in fact that modest amounts of productivity growth can match the rise in the Olley-Pakes covariance over time in India.[6] The reason is that, as discussed above in section 2.2, for every value of leverage productivity growth reduces firms' default probabilities. This in turn raises their bond prices, allowing them to borrow more in equilibrium and grow faster. This effect is most pronounced for firms that are already borrowing; less productive firms remain non-borrowers and gain only from the direct impact of the extra productivity.

6. In chapter 3 I derive a simpler version of the model to match both terms of the Olley-Pakes decomposition with changes in both A and θ. The result is that changes in A alone can explain both terms in the decomposition since 1995; from 1990 to 1995 A is roughly constant and θ increases dramatically.



(a) Average Labor Productivity $\mathcal{Z}$



(b) OP Covariance $\mathcal{C}$

Figure 2.4. OP Decomposition over Time
The top panel plots the average productivity $\mathcal{Z}$ term from equation (2.2) over the five years in the combined ASI-NSS data. I compute $\mathcal{Z}$ within each industry and then average across industries using each industry's share of total employment at each date. The upper and lower dotted lines represent averages using constant industry employment shares in 2011 and 1990, respectively. The bottom panel is identical except that it plots the OP covariance term $\mathcal{C}$ for the three sets of weights.

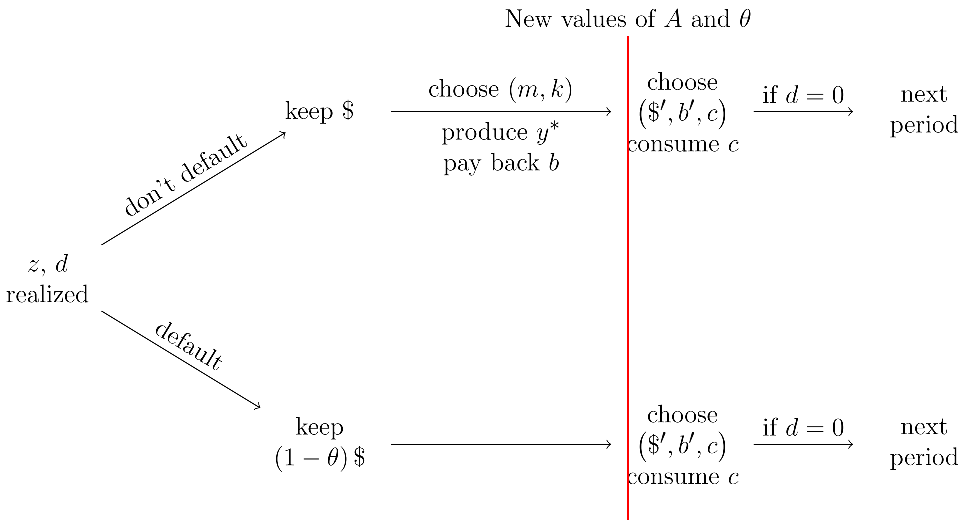
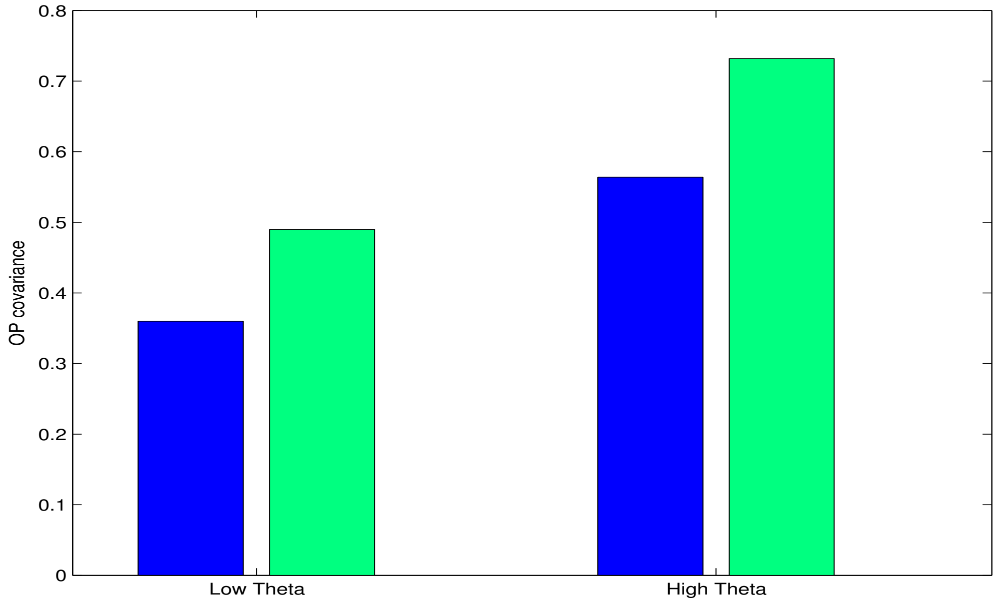
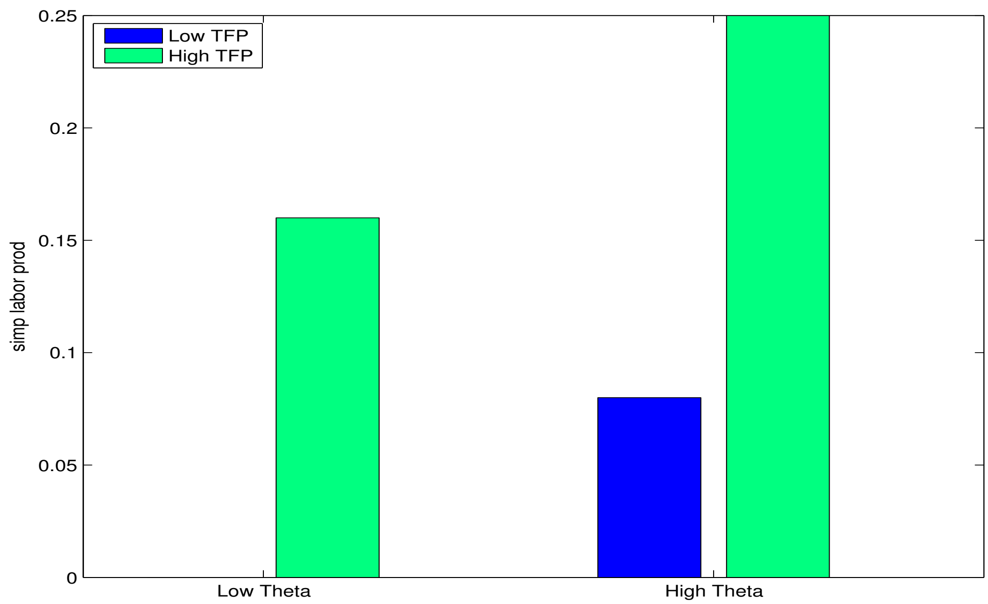


Figure 2.5. Timing
The figure plots the timing of shocks and decisions within the period. Agents realize their idiosyncratic productivity and exit shocks at the start of the period, and must immediately decide whether to default on their outstanding debt. Firms that default keep a fraction $1 - \theta$ of their capital stock and cannot produce, but set $b = 0$. Firms that repay their debt may produce but must pay back their debt. After production occurs, but before investment and borrowing decisions are made, agents learn the values of A and θ that will obtain for future loans. After choosing consumption, borrowing, and investment, agents consume and continue to the next period if they did not receive the exit shock.

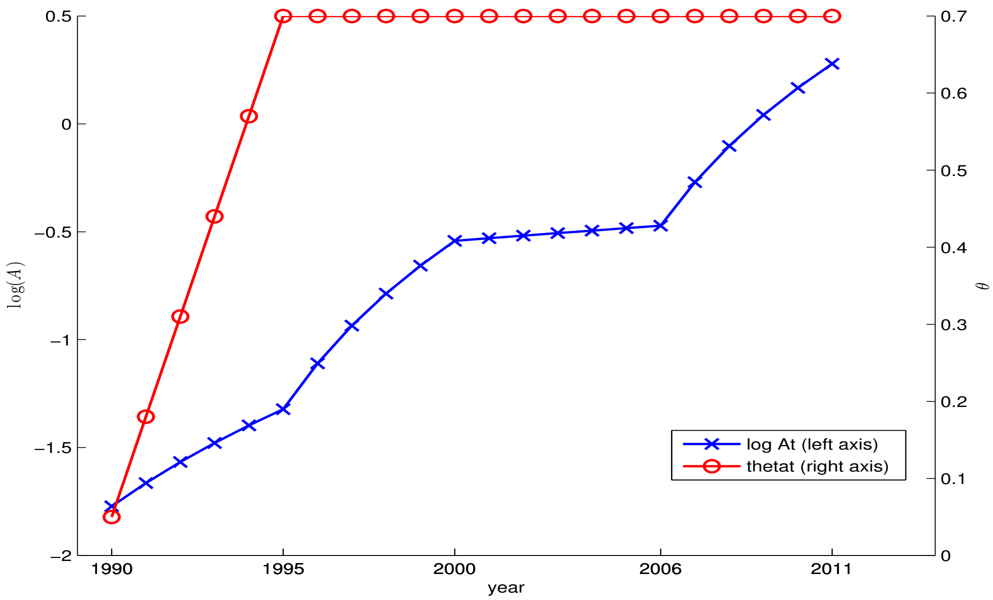


(a) OP Covariance

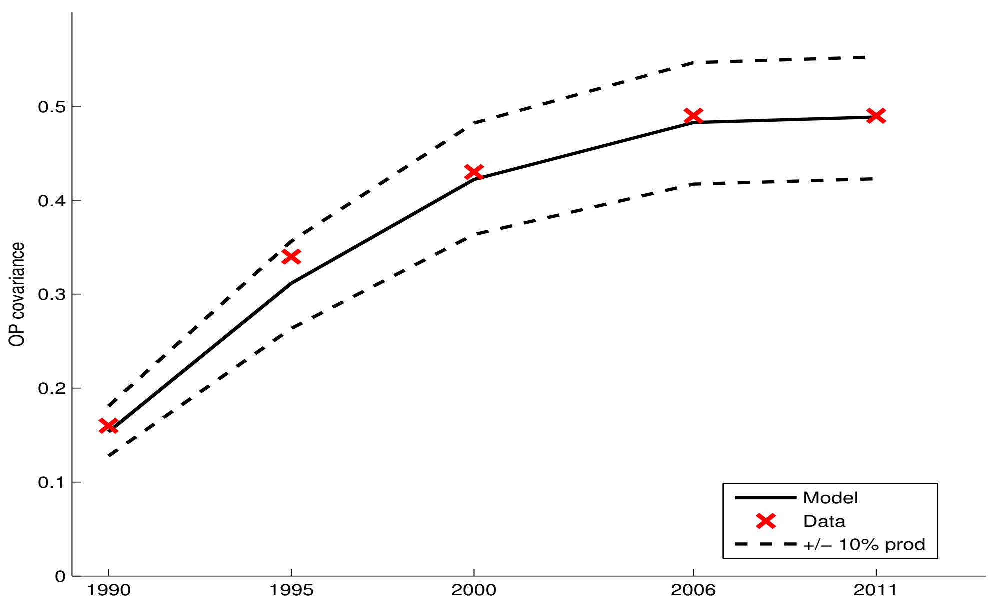


(b) Log Average Labor Productivity

Figure 2.6. Comparative Statics
Each figure plots a comparative static for a different model-implied value, across $\theta \in \{0.4, 0.8\}$ and $A \in \{0.34, 0.425\}$. Other parameters are listed in Table 2.6. The dark blue bars represent values for the low values of A, and the light green bars represent values for the high values of A. The left pair of bars represent values for the low value of θ, and the right pair of bars represent values for the high value of θ. The top panel plots the model-implied Olley-Pakes covariance, and the two panel plots average log labor productivity. The bar in the bottom two panel are normalized to the low-θ, low-A value. Formulas for these calculations are in Appendix C.



(a) Exogenous Productivity Path



(b) Endogenous Olley-Pakes Covariance

Figure 2.7. Dynamic Model
The top panel plots the path of A_t for the high-A types over time as a solid line. Low-A types have a proportional path, where the constant of proportionality is given in Table 2.6 (in logs). The dotted lines in the top panel plot additional paths of A_t, shifted up and down by 10%. The bottom panel plots the implied values of the Olley-Pakes covariance over time; the solid lines correspond to the solid-line path for A_t in the top panel, and the top and bottom dotted lines correspond to the top and bottom dotted lines plotted in the top panel. The red X's in the bottom panel report the values of the Olley-Pakes covariance in the data.

CHAPTER 3

SIMPLER DYNAMIC MODEL

In this chapter, I derive a model that simplifies many of the elements of the model in the chapter 2. In particular, the model in this section uses only capital in production, and firms are perfectly competitive. These assumptions combined with a constant returns to scale production function and log utility greatly simplify the aggregate dynamics of the economy, without sacrificing the model's ability to match the two terms of the Olley-Pakes decomposition. All model intuitions remain the same.

The main purpose of this model, other than to illustrate the robustness of the quantitative results of the model in chapter 2, is to match both terms in the Olley-Pakes decomposition over time with shocks to both aggregate productivity, and financial development. I find that since 1995, shocks to financial development explain only between 2 and 8 percent of aggregate productivity growth.

Sections 3.1.1, 3.1.2, and 3.1.3 derive the model, solve it, and characterize the equilibrium. In section 3.1.3, I assume, for tractability, that agents have log utility; this assumption, along with a constant-returns-to-scale production technology, ensures that the endogenous joint distribution of wealth and productivity does not enter as a state variable in individual agents' maximization problems, as tends to happen in models with heterogeneous agents and aggregate shocks, such as Krusell and Smith (1998). Section 3.1.4 derives the law of motion of the joint distribution of size and productivity, and describes the calculation of aggregate capital and the model-implied values of $\mathcal{Z}$ and $\mathcal{C}$. For details on the formulas, see Appendix C. Section 3.2.1 reports the calibration and implied equiliubrium policy functions, and in section 3.2.2 I find aggregate productivity and financial shocks to match the observed Olley-Pakes decomposition in the data.

3.1 Model

3.1.1 Environment

The environment closely follows that derived in section 2.2, with two exceptions: first, the production function is a simple Ak technology, replacing the two-stage process described section 2.2. Production now only involves physical capital, and no labor. Second, there is an aggregate demand curve and an aggregate output price P that add an element of decreasing returns to scale in the aggregate.

First, production: instead of the decreasing returns to scale Cobb-Douglas revenue function in equations (4) and (5), and the ex-post problem (6), agents produce according to

$$y_{i,t} = Ae^{z_{i,t}+\overline{z}_i+Z_t}k_{i,t}. \tag{3.1}$$

where in this model I index each agent type's permanent productivity level with $\overline{z}_i$, and the common shock to productivity Z_t (A is now a constant parameter). The financial development shock is still θ_t.

Agents face an exogenous demand curve for their output good, so that the output price P satisfies

$$\log P = -\eta \log Y + D, \tag{3.2}$$

where η and D are parameters, and Y in equation (3.2) is aggregate output from all agents. Agents take the equilibrium output price P as given when making their investment and financing decisions.

As in the model of section 2.2, agents can borrow but cannot commit to repay. Each period, agents choose consumption c, capital next period k', and financial assets b'. The budget constraint for an individual agent is then

$$c - q\left(k', b', z; Z, \theta, P\right) b' + k' = \begin{cases} Py + k - b & \text{if debt is repaid} \\ \left(1 - \theta_t\right) k & \text{if defaulted on debt,} \end{cases}$$

where the bond price q depends on the agent's choices for capital and savings, their productivity z, aggregate productivity Z_t, the aggregate recovery rate θ_t, and the equilibrium output price P.

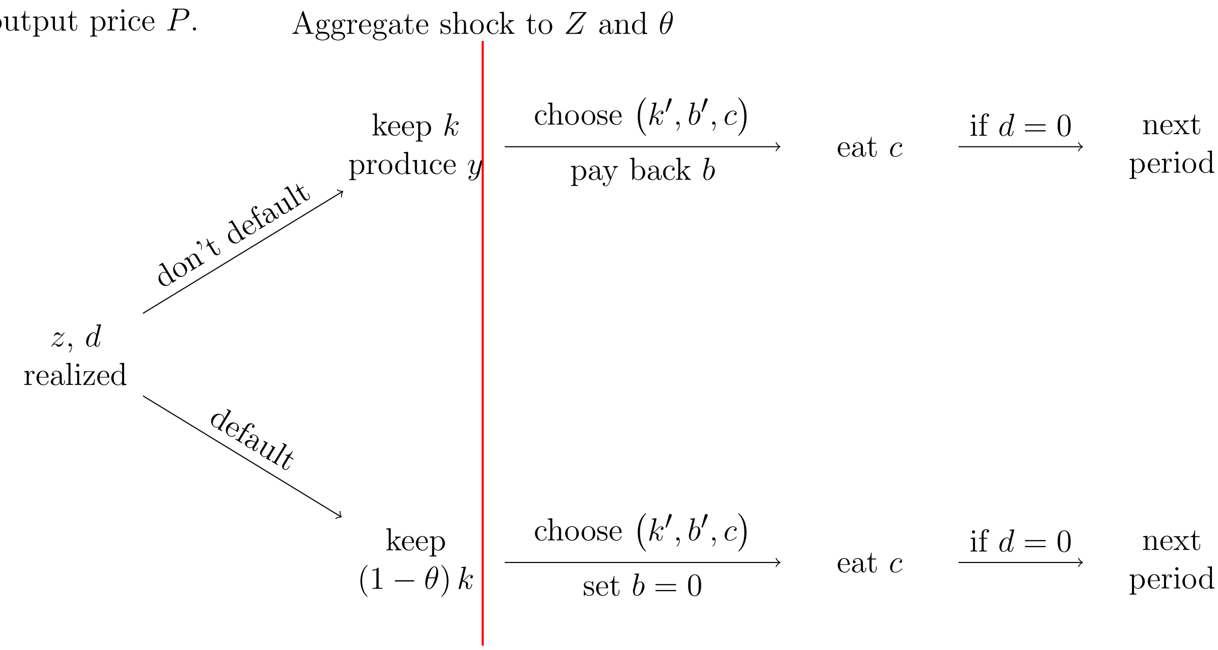


Figure 3.1. Timing
The figure plots the timing of shocks and decisions within the period. Agents realize their idiosyncratic productivity and exit shocks at the start of the period, and must immediately decide whether to default on their outstanding debt. Firms that default keep a fraction $1 - \theta$ of their capital stock and cannot produce, but set $b = 0$. Firms that repay their debt may produce but must pay back their debt. After production occurs, but before investment and borrowing decisions are made, agents learn the values of θ that will obtain for future loans (the aggregate financial shock, if there is one) as well as the expected value of their productivity next period (the aggregate productivity shock, if there is one) and the equilibrium price of output P. After choosing consumption, borrowing, and investment, agents consume and continue to the next period if they did not receive the exit shock.

To ensure a stationary distribution across agents, I assume that they exit the economy exogenously with probability π each period.[1] This assumption is necessary for modeling an equilibrium with many agents, because each agent's log wealth will be a random walk with drift, so that if agents live forever the log difference in wealth between any two agents will also be a random walk and one agent will quickly make up 100% of the economy.

Figure 3.1 illustrates all of the timing assumptions of the model.

3.1.2 Individual Agent's Problem

Agents choose consumption, investment, and borrowing to maximize the expected discounted flow of utility. The description above leads to the following recursive representation of a single agent's problem:

$$\begin{aligned} V\left(k, b, z; Z, \theta, P, \widetilde{F}\right) = \max_{k', b' \geq 0} \quad & u(c) + \beta \Big[(1-\pi) E\left\{ V\left(k', b', z'; Z', \theta', P', \widetilde{F}'\right)\right\} + \\ & \pi E\left\{ V^d\left(k', b', z'; Z', \theta', P', \widetilde{F}'\right)\right\}\Big] \\ \text{s.t.} \quad & \\ & c \equiv (PAe^z + 1)\, k - b + q\left(k', b', z; Z, \theta, P\right) b' - k', \end{aligned} \tag{3.3}$$

where u is the agent's utility function, Z is the current level of aggregate productivity, $\widetilde{F}$ is the joint distribution function of (k, b, z) across all agents, V is the agent's value function conditional on surviving to the next period, V^d is their value when they receive the exogenous exit shock at the beginning of the period, and the laws of motion for the agent's indiosyncratic productivity z is given in equation (2.3) . P and the wealth distribution function $\widetilde{F}$ are endogenous and potentially important state variables to individuals; I describe their calculation and law of motion below. The value function in equation (3.3) is the agent's value after the default decision and the aggregate shock have been realized, right before deciding how much to invest, borrow, and consume (just to the right of the large red line in

1. As in the main model, agents learn whether they will exit at the beginning of the period.

Figure 3.1). Thus the expectation in equation (3.3) is taken with respect to the idiosyncratic shock ε_i alone, and (Z, P) refers to the aggregate productivity and output price that will apply to production *next* period.

The basic recursive relation (3.3) has several simplifications that can be performed at this point. The value function V^d is straightforward: because she will not continue to the next period, the agent has no desire to save any resources, and lenders know that she cannot pay back any debt were she to borrow. Therefore, she optimally consumes all her resources. The dependence of V on k and b separately in equation (3.3) is not necessary; agents need only know their net wealth $x \equiv [PAe^z + 1]\,k - b$ (or $(1-\theta)\,k$ if they had chosen to default), not how it separates into real and financial assets, which means that the endogenous distribution of interest is no longer $\widetilde{F}(k, b, z)$, but $F(x, z)$. At the same time, the bond price does not depend on k' and b' separately, but only on their ratio $\ell = \frac{b}{k}$.

Finally, the bond price can be derived in closed-form as follows: suppose the agent's productivity in the next period (inclusive of aggregate productivity) is z'. If she does not default, her net wealth will be $x' = \left(PAe^{z'} + 1\right) k - b$, whereas if she defaults, her net wealth will be $x' = (1-\theta)\,k$. Equating these two values and plugging in the law of motion for z in equation (2.3) leads to a cutoff rule for default: the agent will default on her debt if her productivity shock is less than or equal to $\underline{\varepsilon}$ that satisfies

$$\underline{\varepsilon} = \begin{cases} \frac{1}{\sigma}\left[\log(\ell-\theta) - \log(A) - \widetilde{z}\right] & \ell > \theta \\ -\infty & \text{otherwise} \end{cases}, \tag{3.4}$$

where $\widetilde{z} \equiv \rho z + Z + \log P$ is the agent's forward-looking productivity.

From equation (3.4), it is straightforward to show that lenders earn an expected return of $1 + r$ if the bond price satisfies

$$q\left(k', b', \widetilde{z}; \theta\right) = q\left(\ell, \widetilde{z}; \theta\right) = \frac{1}{1+r}\left[1 - (1 - \chi(\ell, \theta))\,\Phi\left\{\underline{\varepsilon}\right\}\right], \tag{3.5}$$

where $\chi(\ell, \theta)$ is given by equation (2.7), and $\Phi\{\cdot\}$ is the CDF of the standard normal distribution.

These changes lead to a simplified recursive representation:

$$V(x, z; Z, \theta, P, F) = \max_{k', b' \geq 0} \; u(c) + \beta\left[(1-\pi) E\left\{V\left(x', z'; Z', \theta', P', F'\right)\right\} + \pi E\left\{u\left(x'\right)\right\}\right]$$

$$\text{s.t.} \tag{3.6}$$

$$c \equiv x + qb' - k'$$

$$q \equiv q(\ell, \widetilde{z}; \theta)$$

$$\widetilde{z} \equiv \rho z + Z + \log P$$

$$x' = \max\left\{\left[Ae^{\widetilde{z}+\sigma\varepsilon} + 1\right] k' - b', (1-\theta) k'\right\},$$

where F now denotes the joint distribution across all agents of (x, z).

3.1.3 Equilibrium

As in the main model of the paper, a measure π of agents enter each period to replace those that exit exogenously. Let these new agents have net wealth x drawn from the lognormal distribution $\log x \sim \mathcal{N}\left(0, \sigma_{x_0}^2\right)$, and their independent idiosyncratic productivity z is drawn as $z \sim \mathcal{N}\left(0, \sigma_{z_0}^2\right)$. Fixing the cross-sectional entry distribution for x and assuming $\pi > 0$ ensures a stationary distribution for x across agents even though each agent's $\log x$ will be a random walk with drift in equilibrium.

Agents take the output price P as given when making their borrowing and investment decisions, but in equilibrium, P will be determined according to the demand curve (3.2). Define equilibrium as

1. Agents of type i chooses $k' = k_i(x, z; Z, \theta, P, F)$ and $b' = b_i(x, z; Z, \theta, P, F)$ to solve their individual problem (3.6), appropriately extended to include the fixed productivities $\overline{z}_i$. All agents take P and the function $q(\cdot, \widetilde{z}; \theta)$ as given (the latter depends on $\overline{z}_i$

as well).

2. Aggregate output (next period) is

$$Y = \sum_i \mu_i \int A \exp\{\rho z + \overline{z}_i + Z + \sigma\varepsilon\} k_i(x, z; Z, \theta, P, F) \phi(\varepsilon) d\varepsilon dF_i(x, z), \quad (3.7)$$

where $F_i(X, z)$ is the joint distribution function of wealth and productivity for agents with average idiosyncratic productivity $\overline{z}_i$.

3. The output price P satisfies

$$\log P = -\eta \log Y + D.$$

4. Bond prices satisfy equation (3.5).

Aggregate output next period Y (and the output price P) are predetermined because the only uncertainty is the idiosyncratic shock ε.

To further simplify the solution of the model, assume that the agent's utility function is $u(c) = \log(c)$. This leads to the following proposition:

Proposition 3. *Suppose* $u(c) = \log(c)$. *Then* $V(x, z; Z, \theta, P, F) = a_0 + a_1 \log x + f(z; Z, \theta, P, F)$ *solves equation (3.6) for constants* a_0 *and* a_1 *and some function* $f(\cdot)$, *and the optimal policies* (c, k', b') *satisfy*

$$k' - q(\ell, \widetilde{z}; \theta) b' = \beta^* x \quad (3.8)$$

$$c = (1 - \beta^*) x$$

for some $\beta^* \leq \beta$.

Proof. See Appendix A. □

Proposition 3 greatly simplifies the analysis of the effects of aggregate shocks, because it means that optimal policies can be solved "state by state" rather than all at once, and removes any dependence of policies on the complicated state variable F, which is a distribution function. The simplification occurs because optimal consumption under log utility is independent of everything except net wealth x, thus ensuring that agents are "optimally myopic" and have no hedging motives; that is, the current state affects their decisions only through its effect on current variables, not through what information (if any) the current state implies about future variables. In other words, the law of motion of the aggregate state (including the joint distribution of size and productivity) does not enter agents' decision rules. Without log utility and Proposition 3, I would have to solve the individual agent's problem given an assumed law of motion for the endogenous aggregate state, and then use the implied policy functions to check whether the assumed laws of motion are correct.

The next proposition characterizes the investment and financing decision of agents in more detail.

Proposition 4. *The agent's optimal policy satisfies a threshold rule in* $\widetilde{z} \equiv \rho z + \overline{z}_i + Z + \log P$*:*

- *If* $\widetilde{z} \leq \overline{z}(r)$*, then* $b' = 0$ *and* $k' = \beta^*$*.*
- *If* $\widetilde{z} > \overline{z}(r)$*, then* $k' = \frac{\beta^*}{1-q\ell}$ *,* $b' = k'\ell$*, and* $\ell > 0$ *solves*

$$\frac{q + \frac{\partial q}{\partial \ell}\ell}{1 - q\ell} = \int_{\underline{\varepsilon}(\ell)}^{\infty} \frac{\phi(\varepsilon)\, d\varepsilon}{Ae^{\widetilde{z}+\sigma\varepsilon} + 1 - \ell}, \tag{3.9}$$

- $\overline{z}(r)$ *is the value of* $\widetilde{z}$ *that solves*

$$1 = (1 + r) \int_{-\infty}^{\infty} \frac{\phi(\varepsilon)\, d\varepsilon}{Ae^{\widetilde{z}+\sigma\varepsilon} + 1}. \tag{3.10}$$

Proof. See Appendix A. □

Proposition 4 characterizes the investment and borrowing decisions of all agents in the economy as a function of their forward-looking productivity $\widetilde{z}$. The left-hand side of equation (3.9) represents the marginal benefit of an additional unit of leverage, which depends not only on the bond price q, but also on how quickly that price is changing as the agent increases leverage. Agents optimally equate this marginal benefit with the marginal cost of an additional unit of leverage, given by the right-hand side of equation (3.9). The marginal cost integrates over the idiosyncratic shock ε, but is only paid if agents do not default on their debt, so the lower limit of integration is $\underline{\varepsilon}$.

Equation (3.10) is equation (3.9) with $\ell = 0$; the level of $\widetilde{z}$ for which this equation is satisfied is the cutoff value for forward productivity at which agents are just indifferent between borrowing and not. For productivity just below this value, they do not borrow, and set $k' = \beta^*$.

3.1.4 Aggregation

In this section, I close the model by describing the time-series evolution of the endogenous wealth-productivity distribution and the calculation of the various aggregates used in section 2.3. These aggregates include quantities used in the calibration, such as aggregate output and capital for agents of each type i, as well as the cross-sectional standard deviation of productivity. I also compute the OP decomposition terms $\mathcal{Z}$ and $\mathcal{C}$ in this section. For detailed formulas, see Appendix C.

I characterize the cross-sectional joint distribution of size and productivity for agents of type i at time t as a cumulative distribution function $F_{i,t}(\log x^*, z^*)$, where i indexes the fixed-effect in idiosyncratic productivity $\overline{z}_i$. This function is the probability that a randomly-drawn firm at time t with fixed effect $\overline{z}_i$ has net wealth $x < x^*$ and idiosyncratic productivity $z < z^*$. Because agents never change their value of $\overline{z}_i$, I can compute the CDFs for each type of agent separately and later aggregate across them according to the measure μ.

I parameterize F in terms of $\log x$ instead of x because in equilibrium this parameterization will behave better computationally. For fixed values of P_{t+1}, θ_t, Z_{t+1}, and the function

$F_{i,t}$, the function $F_{i,t+1}$ is given by

$$F_{i,t+1}\left(\log x^*, z^*\right) = (1-\pi) \int \Phi \left\{ \mathcal{E}_i \left(\frac{x^*}{x}, z^*, z; Z_{t+1}, \theta_t, P_{t+1} \right) \right\} dF_{i,t}\left(\log x, z\right) + \pi F^e \left(\log x^*, z^*\right), \tag{3.11}$$

where $F^e(\cdot,\cdot)$ is the CDF of $(\log x, z)$ for the agents that enter exogenously each period, and the function $\mathcal{E}_i$ is the conditional transition CDF for agents of type i, which depends on the decision rules of individual agents and the law of motion of the idiosyncratic productivity z (see Appendix C for details). I use equation (3.11) to compute the evolution over time of the size-productivity distribution, and also to compute the steady-state distribution where $F_{i,t+1} = F_{i,t}$.

Given a distribution F_t, I compute aggregate output Y and capital K as integrals over $dF_{i,t}$, the idiosyncratic productivity shocks ε, and the agent-type distribution. The aggregate capital stock and output at $t+1$, K_{t+1} and Y_{t+1}, are given by

$$\begin{aligned} K_{t+1} &= \sum_i \mu_i \int_{(\log x, z)} k\left(\widetilde{z};\theta_t\right) x \left[1 - \Phi\left\{\underline{\varepsilon}\left(\widetilde{z},\theta_t\right)\right\}\right] dF_{i,t}\left(\log x, z\right) \\ Y_{t+1} &= \sum_i \mu_i \int_{(\log x, z)} A \exp\left\{\widetilde{z} + \tfrac{1}{2}\sigma^2 + \log x\right\} k\left(\widetilde{z};\theta_t\right) \left[1 - \Phi\left\{\underline{\varepsilon}\left(\widetilde{z},\theta_t\right) - \sigma\right\}\right] dF_{i,t}\left(\log x, z\right) \\ \widetilde{z} &\equiv \rho z + \overline{z}_i + Z_{t+1} + \log P_{t+1}. \end{aligned} \tag{3.12}$$

The capital that will be used in production at $t+1$ is chosen by all firms at time t, so that K_{t+1} is time-t measureable, even though I don't know at t which firms will default; by the law of large numbers, I know the measure of firms that will default. The $1 - \Phi\{\cdot\}$ term in equation (3.12) takes care of the defaulting agents. Total output Y_{t+1}, and thus the output price P_{t+1}, are also time-t measureable, because the only unknown at time t after investment occurs is the idiosyncratic shock ε (the next aggregate productivity shock occurs after production takes place; see Figure 3.1). From equation (3.12), it is straightforward to calculate the shares of agents of type i in aggregate output and capital.

To compute the cross-sectional dispersion in productivity and the OP decomposition terms $\mathcal{Z}$ and $\mathcal{C}$ in equation (2.2), I define the log-productivity of an agent of type i with idiosyncratic productivity z as

$$\log \frac{y}{k} = \log A + Z + \overline{z}_i + \rho z + \sigma \varepsilon. \tag{3.13}$$

I compute the unweighted average productivity $\mathcal{Z}$ by integrating equation (3.13) over the densities $F_{i,t}$, the distribution of ε, and the distribution of agent types; aggregate productivity is similar except that I weight each agent by her share of aggregate capital, $k\left(\widetilde{z};\theta_t\right) x/K_{t+1}$. Then $\mathcal{C}$ is the difference between aggregate productivity and $\mathcal{Z}$. To compute the standard deviation of log productivity across agents, I square equation (3.13) and integrate to get the uncentered second moment of log productivity, from which the standard deviation is straightforward. For details, see Appendix C.

3.2 Results

This section describes the results of calibrating and applying the model derived in section 3.1. In section 3.2.1 I calibrate the model, and in 3.2.2 I use the calibrated model to back out the time-series paths of aggregate productivity and financial development to match the observed evolution of the two terms of the Olley-Pakes decomposition in India from 1990 to 2011.

3.2.1 Calibration

Table 3.1 reports the parameters for the quantitatve application of the model. Some parameters can be set to match empirical targets without solving the model. I set $N = 2$ as the number of agent-types in the model; $i = 1$ corresponds to "informal" agents with $\overline{z}_i = -2$. A measure 0.99 of agents in the economy are of this type, whereas the remaining 1% of agents have $\overline{z}_i = 0$. This not only matches thes shares of informal and formal firms in the NSS and ASI data, but allows the model to match the unconditional standard deviation of

Parameter	Value	Target Moment	Data	Model
π	0.1	Average Age	12	10
r	1%	Percentage with Leverage = 0	91%	84%
β	0.97	NSS Share of Output	31%	25%
A	0.024	Median Leverage | Borrow	24%	39%
ρ	0.95	Midrigan and Xu (2014) Value	–	–
σ	0.34	Std Dev of Log Productivity Shock	1.1	1.1
μ	0.99	NSS Share of Plants	99%	99%
$\overline{z}_1$	-2	ASI–NSS Log Productivity Difference	-2	-2
η	0.47	Change in NSS Share of Inputs	-11.2%	-11.1%
D	-1.52	$P = 1$ in Steady-State	–	–

Table 3.1. Calibration
The table reports the parameter values of the model solved in section 3.1. The first two columns report the parameter and its value. The third column reports the target moment used to set the given parameter, and the last two columns report the value of the target moment in the data and in the model, respectively.

productivity in the data without setting ρ or σ too high.

I set the exogenous exit probability π to 0.1, which means that the average plant age in the model is 10 years, which is a bit lower than the average age of 12 years in the data. Lowering π to match the average age in the data, however, leads to instability. In particular, if π is too low, plants can grow on average faster than the exit rate, leading to a stationary firm-size distribution with infinite output. In the context of Gabaix (2011), these densities would resemble a power law with tail parameter less than 1. Other models with heavy right tails in the firm-size distribution, for example Buera, Kaboski and Shin (2011), avoid this problem by imposing tails greater than 1 in calibration. They do so with ease because they have closed-form solutons for the tail parameter. Lacking this luxury, I instead set the exit rate such that the average age of plants in the model is somewhat lower than what is observed in the data. This higher exit rate is sufficient to keep output finite in steady state, and I verify in unrepoerted results that this moment is not crucial for any of my results.

I set the persistence of productivity shocks to be $\rho = 0.95$ following Midrigan and Xu (2014). Given this value and the calibrated values for the share and productivity differences between the formal and informal sectors, I set the idiosyncratic productivity volatility σ to match the cross-sectional standard deviation of log productivity. A key assumption to match this moment is that agents are restricted from lending, so that in particular, the informal agents with $\overline{z}_i = -2$ must produce with their low-productivity technology. Absent the lending restriction, most of these agents would choose to lend to more-productive agents, lowering the standard deviation of productivity (and raising the interest rate, which would have to be endogenous). Raising σ to combat this selection effect mainly serves to increase the average growth rate of producing firms, which leads to explosive behavior as described above. Such a model also seems at odds with the data, which feature a massive number of manufacturers with very low productivity. Surely such agents would be better off investing (or working!) in higher-productivity plants, but understanding why they do not do so is beyind the scope of the present paper.

I choose the riskless interest rate r, subjective discount rate β, and the (gross) level of productivity A to match the share of plants reporting zero leverage, the share of informal firms in total output, and the median leverage ratio of plants that do borrow, respectively. Of course, each of these parameters affects all three moments, but I have chosen the mapping between parameters and moments in Table 3.1 to loosely convey which moment is most affected by which parameter. The fit is not perfect, particularly the median leverage ratio, but I have verified that the qualitative results of the paper are not sensitive to this parameter.

Finally, I set the parameters of the exogenous demand curve to normalize the steady-state price to 1 and to match the evolution of the informal share of inputs. The demand-curve intercept D is not separately identified without price information, so I set it to ensure that the steady-state price of output is 1. I then set the price elasticity of demand η to match the change in the NSS share of employment, which drops by roughly 11 percentage points from 1989–1990 to 2010–2011. This moment is highly sensitive to changes in η, which regulates the extent to which agents respond to productivity shocks. For example, if $\eta = 1$, all increases in productivity are exactly offset by reductions in P, so that plants do not respond to common productivity shocks (all else equal). In this case, the NSS share of plants would drop by less than 11 percentage points, because ASI plants would invest less over time as their productivity increases. On the other hand, a value of η much lower than the value in Table 3.1 would make investment more responsive to changes in physical productivity, because the price would drop less, so that the NSS share would drop by more than 11 percentage points.

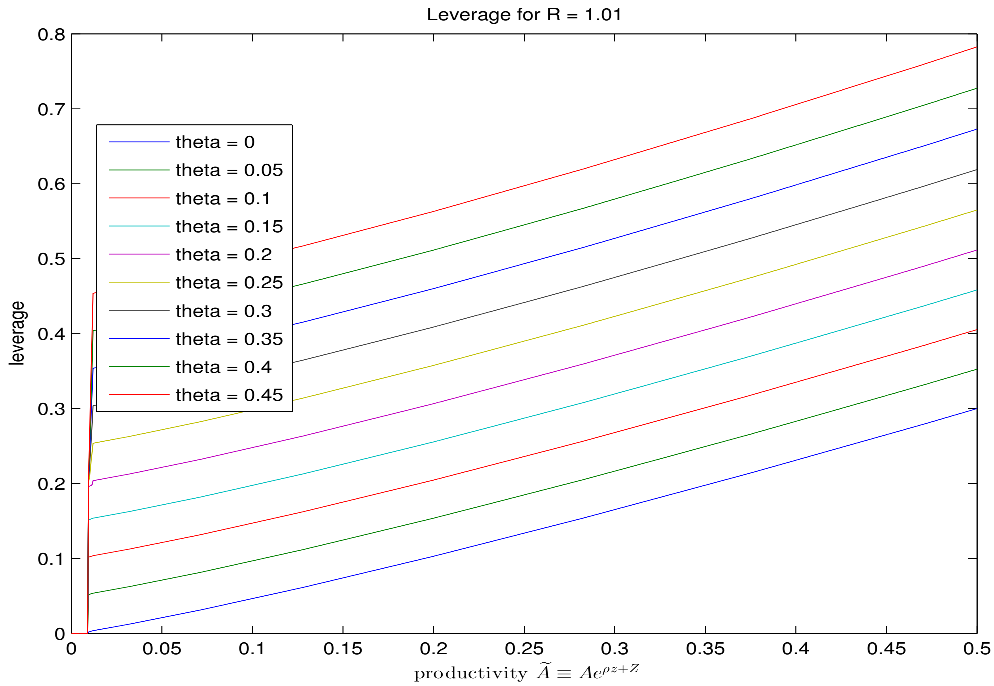
To solve the model, I solve equation (3.10) given the value of r, and equation (3.9) on a 15×15 grid of values for $\{z_i, \theta_j\}$. I linearly interpolate the policy functions between grid points, setting $\ell = 0$ whenever $z < \overline{z}(r)$. Figure 3.2 plots the agent's decision rules as a function of future productivity $Ae^{\widetilde{z}}$ for various values of θ.

Optimal leverage is increasing in productivity, as can be seen in the top right panel of Figure 3.2. The graph plots optimal leverage $\ell = \frac{b'}{k'}$ as a function of $Ae^{\widetilde{z}}$ for $r = 1\%$ and θ ranging from 0 to 0.35, whenever $\ell > 0$. Higher leverage goes hand in hand with higher investment, as can be seen in the bottom-left panel of Figure 3.2. Thus, more-productive firms will grow faster than less-productive ones, generating a positive correlation between size and productivity in equilibrium that depends strongly on the value of θ.

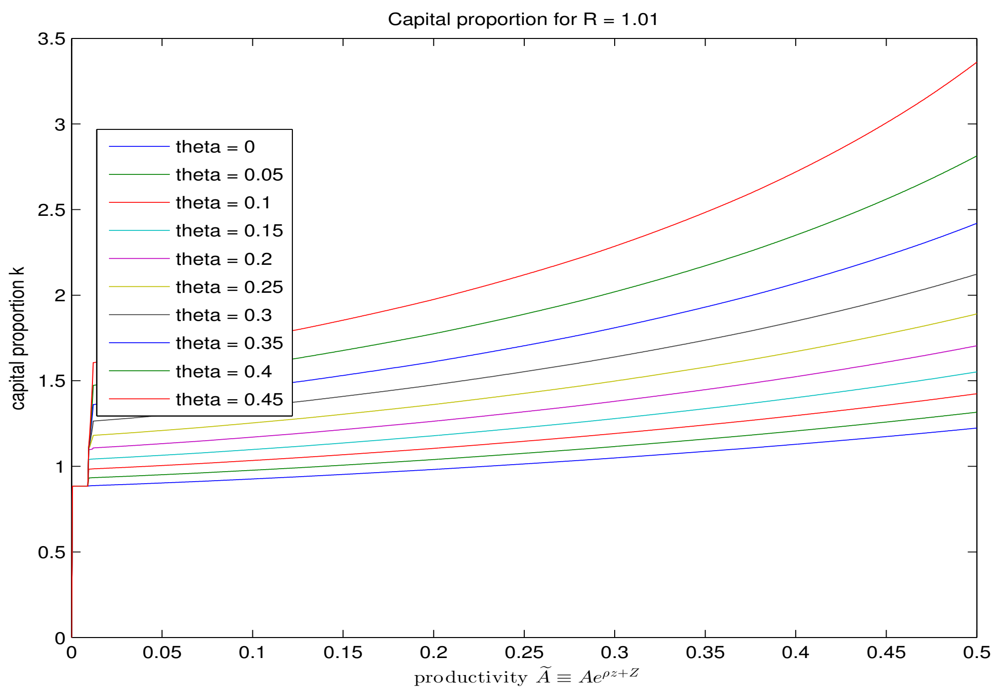
3.2.2 *Steady-State and Aggregate Productivity Growth*

In this section, I use the calibrated model to determine the aggregate shocks that drive the observed aggregate productivity decomposition in Figure 2.4. First, I compute the steady-state size-productivity distribution implied by the parameters in Table 3.1, setting θ to yield an OP covariance term $\mathcal{C} = 0.16$, the value in Indian in 1989–1990. For this date, I normalize $Z = 0$. Then, for each subsequent year to 2010–2011, I compute new values of (Z, θ) so that $\mathcal{C}$ in the model economy matches the value in Figure 5, and the change in $\mathcal{Z}$ matches the change in the value of $\mathcal{Z}$ in Figure 5. I linearly interpolate the target values of $(\mathcal{Z}, \mathcal{C})$ between the five-year observation years.

Given the policy functions illustrated in Figure 3.2, I compute the model's implied steady-



(a) Leverage ℓ



(b) Real Assets k'

Figure 3.2. Policy Functions
The top panel plots the optimal leverage policy $\ell \equiv \frac{b'}{k'}$, and the bottom panel plots the optimal investment policy k', both as a function of productivity $Ae^{\widetilde{z}}$, for $r = 1\%$ and values of θ ranging from 0 to 0.45. Each line represents a different value of θ. The bottom panel plots the optimal investment policy k'.

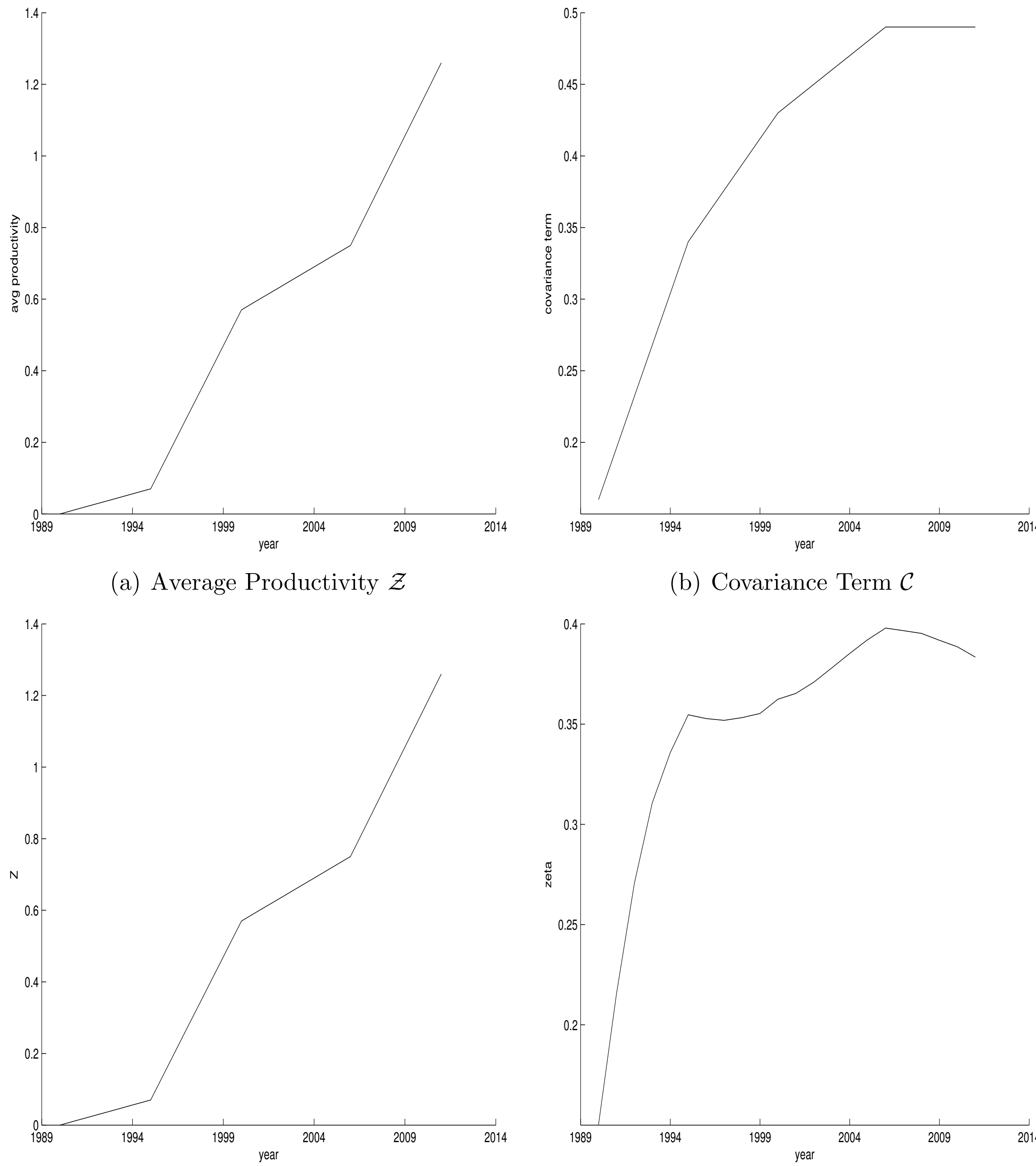
state distributions of size and productivity $F_i(\log X, z)$ according to equation (3.11) for a given value of θ, assuming that $F_{i,t+1} = F_{i,t}$ and $Z_{t+1} = 0$ for all i, t. I then search over values of θ to match the value of $\mathcal{C} = 0.16$ using equation (C.5). I choose the intercept in the demand curve c so that the market-clearing price in this year is $P = 1$, according to equation (3.2). See Appendix C for details on how I represent and solve for the endogenous density, and how I approximate integrals numerically.

For each subsequent year after 1989–1990, I solve for the implied values of (P, Z, θ) that match the observed values of $(\mathcal{Z}, \mathcal{C})$ and that clear the goods market according to equation (3.2). The values of (P, Z, θ) determine the investment and borrowing decisions of agents in the current period, which affect the future joint distribution of size and productivity according to equation (3.11). This distribution, along with the values of (Z, θ), determine aggregate output Y and the OP decomposition terms $(\mathcal{Z}, \mathcal{C})$. I match the levels of $\mathcal{C}$ to those in Figure 2.4, but I normalize $\mathcal{Z}$ to zero in 1989–1990, so that the model values equal the data values in log differences. Finally, the price P must be consistent with aggregate output Y and the aggregate demand curve.

Figures 3.3 and 3.4 plot the results of this exercise. Figure 3.3 plots the observed OP decomposition terms as well as the implied values of the fundamental shocks over this time period. The top two panels of Figure 3.3 plot the observed values of $\mathcal{Z}$ and $\mathcal{C}$ from Figure 2.4, normalizing the initial value of $\mathcal{Z}$ to 0. The bottom two panels plot the values of (Z, θ) that generate the observed OP decomposition according to the calibrated model.

The bottom two panels of Figure 3.3 show that the history of manufacturing productivity growth in India can be divided into two periods. In the first period, from 1989–1990 to 1994–1995, changes in financial frictions θ drove labor productivity growth, and common productivity shocks were small or even negative. The model identifies this period by the very high change in the OP covariance term, relative to almost no change in unweighted average productivity; the model matches both features of the OP terms by drastically increasing θ.

Figure 3.4 converts the values plotted in the bottom panels of Figure 3.3 into contribu-



(a) Average Productivity $\mathcal{Z}$ (b) Covariance Term $\mathcal{C}$

(c) Common Productivity Shock Z (d) Recovery Rate θ

Figure 3.3. Aggregate Productivity Decomposition
The top two panels plot the values of $\mathcal{Z}$ and $\mathcal{C}$ from Figure 2.4. The bottom two panels plot the implied fundamental aggregate shocks Z and θ that, when combined with the parameters in Table 3.1 and assuming the economy is in steady-state in 1989–1990, lead to the observed values of $(\mathcal{Z}, \mathcal{C})$.

tions to aggregate productivity growth. Each bar represents the contribution to aggregate productivity growth from each shock over the indicated time span. I compute these contributions as counterfactuals: in each time span, I compute average productivity growth holding one shock constant at its beginning-of-period value, and allowing the other shock to change as in the bottom panels of Figure 3.3. Because the effect of shocks to θ depend on the level of Z, and vice versa, the sum of the two contributions may not equal realized productivity growth. I plot the difference in Figure 3.4 as the "joint" contribution.

For the first five years of the sample, improvements in financial conditions (the black bars in Figure 3.4) contributed almost four percentage points per year to labor productivity growth; this is 71% of total labor productivity growth in that time frame. In other years, common shocks to productivity explain nearly all the change in aggregate productivity; financial shocks were even a slight drag on productivity growth in 2006–2011. The average contribution of financial shocks to total productivity growth from 1995 to 2011 is between 2.4% and 7.5%, depending on whether one includes the joint term.

Although I find that *changes* in financial frictions are not important drivers of aggregate productivity since 1994–1995, the *level* of financial frictions from 1994–1995 onward is important for allowing common productivity shocks Z to affect the covariance term $\mathcal{C}$. In a simplified version of the model without borrowing, common shocks to productivity lead to uniform changes in growth rates across the tproductivity distribution: all firms grow faster, by the same amount, leaving the covariance term unchanged.[2] But when firms can borrow, their leverage depends on the level of productivity. Thus, larger, more-productive firms respond to the common shock to productivity by choosing higher leverage ratios, and grow faster than smaller, less-productive firms. In equilibrium, this mechanism increases the

2. In fact the covariance term is slightly affected by common shocks to productivity, because the amount of undepreciated capital that firms carry over into the next period is unaffected by their productivity. The future wealth of firms with very low levels of productivity comes entirely from undepreciated capital, and a productivity shock does not affect their growth. The reverse is true for extremely productive firms, for whom left over capital is a small share of their future wealth. Thus, productivity shocks will have a bigger effect on the growth rates of more-productive firms, and the covariance term $\mathcal{C}$, even in a model without financial frictions. However, this effect is quantitatively negligible.

η	Terminal θ	Percent of Growth from θ 1990–1995	1995–2011	Δ% NSS Share Model	Data
0.3	0.29	70%	-8.4–10.0%	-14.0%	-11.2%
0.4	0.33	71%	-3.5–8.6%	-12.3%	-11.2%
0.5	0.38	71%	2.4–7.5%	-11.1%	-11.2%
0.6	0.45	71%	6.6–9.9%	-10.0%	-11.2%

Table 3.2. Robustness to Price Elasticity of Demand η
The table reports model results from changing the parameter η, holding all other parameters fixed at their values given in Table 3.1. The first column reports the new value of η; the second column reports the implied value of θ in 2010–2011 to match the time-series evolution of the OP terms plotted in Figure 2.4. The third and fourth column report the percentage of aggregate productivity growth accounted for by θ shocks, from 1990–1995 and 1995–2011, respectively, as in Figure 3.4. The range of values given in column 4 reflect different ways of handling the "joint" term. The fifth and sixth columns report the change in the NSS share of plants in the model and data, respectively, in percentage points.

covariance between size and productivity without any changes in financial frictions.

A crucial parameter driving the constancy of θ after 1994–1995 is the price elasticity of demand, η. This parameter controls the degree to which shocks to physical productivity affect the investment decisions of firms. In the corner case where $\eta = 1$, the price of output adjusts to automatically cancel any changes in physical productivity, all else equal, so agents do not respond to common shocks to productivity (although aggregate output does). For $\eta < 1$, the price of output does not adjust fully to shocks to Z, so agents respond to positive aggregate productivity shocks by increasing investment.

Changing η from its value in Table 3.1 has two effects on the model results. First, increasing η makes agents less responsive to common shocks to productivity, which means that to match the observed values of the OP covariance $\mathcal{C}$, θ must increase by more, increasing the contribution of financial shocks to aggregate productivity growth. Second, at these higher values of η, the more-productive plants with $\bar{z}_i = 0$ grow more slowly after common productivity shocks than otherwise, and become a smaller share of total inputs by the end of the sample. I use the second effect to discipline my choice of η, as described in section 3.2.1.

To illustrate the sensitivity of model results to changes in η, Table 3.2 reports model output for three different values of η, holding all other parameters fixed at their values in Table 3.1. When $\eta = 0.3$, the terminal value of θ at t =2010–2011 is 0.29, compared to the benchmark value of 0.38. At this lower value of η, the percentage of aggregate productivity growth accounted for by financial shocks changes from 2.4%–7.5% to -8.4%–10%. On the other hand, increasing η to 0.6 makes agents less responsive to productivity shocks, so that the terminal value of θ is higher at 0.45, and the share of productivity growth accounted for by financial shocks increases to 6.6%–9.9%. However, in both cases, the drop in the NSS share of firms differs from that observed in the data, by about a percentage point on either side (second to last column of Table 3.2). Setting $\eta = 0.47$ matches this data moment.

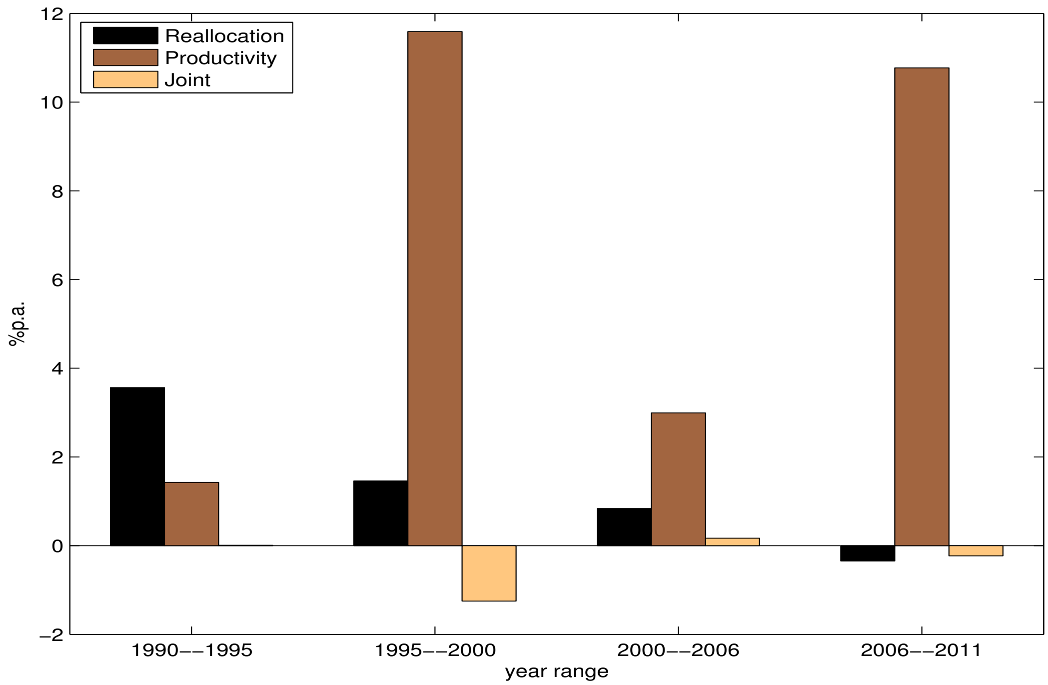


Figure 3.4. Contributions to Aggregate Productivity Growth
The figure plots the average contributions to aggregate productivity growth, in annualized percentage points, coming from changes in the two fundamental shocks Z and θ for the four time intervals in the sample period. The black bars represent the contributions to productivity growth coming from shocks to financial frictions θ, and the dark brown bars the contribution from common shocks to productivity Z. Each bar represents an average value over the indicated years. I compute contributions from each source by solving the model over each time frame, evolving the indicated shock as in the bottom panels of Figure 3.3, and holding the other shock fixed at its beginning of period value. Then each bar is the average growth in aggregate productivity from the appropriate counterfactual. The light brown bars labeled "joint" are the difference between actual aggregate productivity growth, and the sum of the two counterfactual values.

CHAPTER 4

STATIC MODEL

In this chapter, I derive a simple static model similar to that used in Hsieh and Klenow (2009). The main difference is that I model the friction that keeps the economy away from the first-best explicitly, and show how changes in that friction and in aggregate productivity affect the equilibrium allocation andmeasured misallocation.

I use this simple model to illustrate the substantive difference between my analysis and that of the seminal work of Hsieh and Klenow (2009), who use nearly the same dataset as I do and appear to come to the opposite conclusion regarding the time-series behavior of Indian allocative efficiency. In particular, Hsieh and Klenow (2009) find that the misallocation among Indian manufacturing plants got worse from 1987 to 1994, roughly the same time that I argue that the allocation improved (1990 to 1995) in chapter 3.

The analysis of this section shows that Hsieh and Klenow (2009) and I appear to disagree about the 1987-1995 period because we are measuring different things: their work measures the productivity difference between the current allocation of resources and a model-implied first-best allocation, while my analysis uses the changes in the allocation over time to infer what shocks are important drivers of productivity growth.

The distinction is important: I show that aggregate productivity growth in a financial-friction extension of the Hsieh and Klenow (2009) will cause misallocation to grow, even though everyone is better off. The reason is that first-best productivity responds more to productivity growth than the current allocation. This explains why there is no contradiction in their finding that *mis*-allocation grew from 1987 to 1994, and my finding that productivity growth in this period was driven by shocks that improved the allocation of resources. I also show that even though the Olley-Pakes covariance is zero in the first-best allocation, it might still respond positively to aggregate shocks in the actual allocation.

4.1 Model

There are two periods and no uncertainty. Each agent has physical productivity z_i and operates a constant returns to scale production function in capital k and labor L:

$$Y_i = Ae^{Z+z_i} k^{\alpha} L^{1-\alpha}, \tag{4.1}$$

where the capital share α and average productivity A are parameters, and Z represents a common factor in productivity across agents. Aggregate output Y is a constant elasticity of substitution aggregate in each plant's output:

$$Y \equiv \left[\int Y_i^{\frac{\gamma-1}{\gamma}} di \right]^{\frac{\gamma}{\gamma-1}},$$

where γ is a parameter. It is easy to show that a representative consumer minimizing cost to achieve a given level of output, or maximizing output subject to a budget constraint, will imply plant-specific demand curves of the form

$$P_i = Y_i^{-\frac{1}{\gamma}}, \tag{4.2}$$

so that the parameter γ represents the price elasticity of demand for each good i. When $\gamma < \infty$ it will be optimal for plant i to produce, even if it is less productive than another plant j, because their outputs are not perfect substitutes.

Each agent has wealth x and is subject to a working-capital constraint: they must hire labor and capital before producing output. If they desire inputs greater than their wealth x, they can borrow, but only in imperfect credit markets. Specifically, outside creditors require a rate of return r on their lending and will not lend to any agent a leverage ratio (face value of bonds b per unit of capital) greater than θ.

Agents internalize their own demand curves (4.2), and so their profit-maximization problem is given by

$$\max_{k,m,\ell} \quad \underbrace{\left[Ae^{Z+z_i}m^{1-\alpha}k\right]^{\frac{\gamma-1}{\gamma}}}_{PY} + (1-\ell)\,k$$

$$\text{s.t.} \tag{4.3}$$

$$(1+wm-q\ell)\,k \leq x$$

$$\ell \leq \theta,$$

where $q = \frac{1}{1+r}$ is the bond price, $m = \frac{L}{k}$ denotes the labor-capital ratio, and $\ell = \frac{b}{k}$ is leverage. Profits are revenue, PY, plus undepreciated capital k, less the costs of financing $b = \ell k$.

Proposition 5. *There exists a level of productivity $\overline{z}$ such that*

- *If $Z + z_i \leq \overline{z}$, the agent is unconstrained and chooses*

$$\begin{aligned} m &= m^* \equiv \frac{r}{1+r}\frac{1}{w}\frac{1-\alpha}{\alpha} \\ k &= k^*\left(Z+z_i\right) \equiv \left[\frac{\gamma-1}{\gamma}\frac{\alpha}{r}\right]^{\gamma}\left[Am^{*(1-\alpha)}\right]^{\gamma-1}\exp\left\{(\gamma-1)\left(Z+z_i\right)\right\} \\ \ell &= \ell^*\left(Z+z_i\right) \equiv (1+r)\left(1+wm^* - \frac{x}{k^*\left(Z+z_i\right)}\right) \end{aligned} \tag{4.4}$$

- *If $Z + z_i > \overline{z}$, the agent is constrained and chooses m that solves*

$$m^{1-\zeta}\left(\frac{a_0}{m+a_1}\right)^{\frac{1}{\gamma}} = a_2 - a_3 m \tag{4.5}$$

for $\zeta \in (0,1)$ and positive constants a_0, a_1, a_2, a_3 that depend on parameters, Z, and

θ. Equation (4.5) has a unique solution for m. Given m, k and ℓ are then given by

$$k = \frac{x}{1 + wm - q\theta}$$
$$\ell = \theta.$$

Proof. See Appendix A. □

Proposition 5 characterizes the decisions of agents in this model. There is an unconstrained optimal scale, given by equation (4.4), which agents choose so long as the leverage required to reach that scale is less than θ. If it is not, then agents lever up to θ and are smaller than optimal.

Log aggregate total-factor productivity as a function of the individual policies k_i and m_i is given by

$$\log(\text{aggTFP}) \equiv \frac{\gamma}{\gamma - 1} \log \int (P_i Y_i)^{\frac{\gamma-1}{\gamma}} di - \alpha \log \int k_i di - (1-\alpha) \log \int m_i k_i di, \qquad (4.6)$$

which I compute numerically using an interpolated approximation to equation (4.5) for constrained agents, and an assumed distribution for the individual productivities z_i.

Corollary. *Suppose the z_i are normally distributed with mean zero and variance σ_z^2. Then if the leverage constraint binds for no agent, log aggregate total-factor productivity is given by*

$$\log(\text{aggTFP}_{\text{FB}}) = Z + \log A + \frac{1}{2}(\gamma - 1)\sigma_z^2. \qquad (4.7)$$

Proof. Insert equations (4.1) and (4.4) into equation (4.6) and simplify. □

Equation (4.7) characterizes the first-best aggregate total-factor productivity in the absence of frictions. Notice that it is the same as Hsieh and Klenow (2009) equation (16) when $\text{var}\{\log \text{TFPR}_i\} = 0$ (no distortions). I then define the percentage productivity gains from

Parameter	Value	Description
γ	3	Price Elasticity of Demand
x	1	Agent Wealth
A	0.024	Average Productivity
σ_z	1.281	Productivity Dispersion
α	0.3	Capital Share
w	0.01	Wage
r	0.01	Interest Rate

Table 4.1. Static Model Parameters
The table reports the parameter values for the static model analyzed in section 4.

reallocation as

$$\text{Gains} \equiv 100\% \times [\log(\text{aggTFP}_{\text{FB}}) - \log(\text{aggTFP})]$$

4.2 Results

In this section I compute actual and first-best aggregate productivity over a range of values for common productivity Z and financial frictions θ, to show that improvements in common productivity can increase the gains from reallocation. The reason is that first-best aggregate productivity responds more to changes in Z than the actual allocation, because of the constraints. On the other hand, financial frictions have no effect on first-best productivity so reductions in financial frictions always reduce the gains from reallocation.

Table 4.1 reports the parameters I use for this exercise. The most important parameter is γ, which I set to 3, the same value as the benchmark model in Hsieh and Klenow (2009) (they call it σ). I chose other parameters in Table 4.1 to more closely match the model in section 2.3. The results of this section are robust to other values.

Figure 4.1 plots aggregate productivity and the gains from reallocation for different values of Z and θ. The top-right panel of Figure 4.1 shows that, as expected, increasing the common factor in agents' productivity raises aggregate productivity. The top-left panel of Figure 4.1

shows that relaxing the financial constraint also raises aggregate productivity. The reason is that loosening the leverage constraint allows more-productive firms to grow larger. This shifts the allocation of employees towards more-productive firms, raising aggregate productivity. This is substantially the same mechanism as the model in section 2.3.

The bottom panels of Figure 4.1 plots the gains from reallocation for the same values of Z and θ. Loosening the financial constraint θ raises aggregate productivity without affecting first-best productivity, reducing the gains from reallocation.

However, the bottom-right panel of Figure 4.1 shows that increasing every agent's productivity through Z actually *increases* the gains from reallocation. Although aggregate productivity increases (top-right panel), first-best productivity increases by even more, so that the gains from reallocation go up. Thus, economic growth through productivity growth in Z can actually increase misallocation, even though every agent is strictly better off and aggregate productivity has increased.

This effect of productivity growth on the Hsieh-Klenow misallocation measure is not merely of theoretical interest: empirically, it explains Hsieh & Klenow's surprising result (see page 1424) that misallocation is increasing in India from 1987–1994, a period which includes India's major economic reforms. Increased misallocation is exactly what we would expect to see if financial frictions are an important factor in firms' decisions, and their productivity is rising.

To explore this effect, the top panel of Figure 4.2 replicates the calculations of Table IV of Hsieh and Klenow (2009). Following Hsieh and Klenow (2009), I include only ASI plants in this exercise; this allows me to include more years of data than the analysis in chapters 2 and 3, which is restricted to years of joint ASI-NSS data. Although I do not have the same years of data as Hsieh and Klenow (2009), the top panel of Figure 4.2 comes close to replicating their reported numbers. In particular, misallocation is rising in the early part of the sample, during India's major economic reforms. Extending their calculations beyond 1995, misallocation improves up to 2005 and then deteriorates from 2005 to the present.

The bottom panel of Figure 4.2 suggests that the increases in misallocation plotted in the top panel are not the result of anything getting worse in India over this period. The top panel of Figure 4.2 plots the difference between first-best and actual aggregate TFP over time; in the bottom panel I plot the two series separately. Aggregate productivity in India is rising rapidly over time for the majority of the sample, so that periods of increased misallocation are merely times when first-best productivity rises more rapidly than actual productivity.

Finally, I close this section by showing that even though the Olley-Pakes covariance is zero in the frictionless first-best allocation in this model, it can increase in response to positive aggregate shocks. Following section 2.3, I define the Olley-Pakes covariance in this model using the average revenue product of labor, which is given by

$$\begin{aligned}\log \mathrm{ARPL}_i &\equiv \log \frac{P_i Y_i}{L_i} \\ &= \frac{\gamma-1}{\gamma}\left[\log A + Z + z_i\right] + \left[(1-\alpha)\frac{\gamma-1}{\gamma} - 1\right]\log m_i - \frac{1}{\gamma}\log k_i.\end{aligned}$$

The Olley-Pakes covariance is then

$$\mathcal{C} \equiv \int w_i \log \mathrm{ARPL}_i di - \int \log \mathrm{ARPL}_i di \qquad (4.8)$$

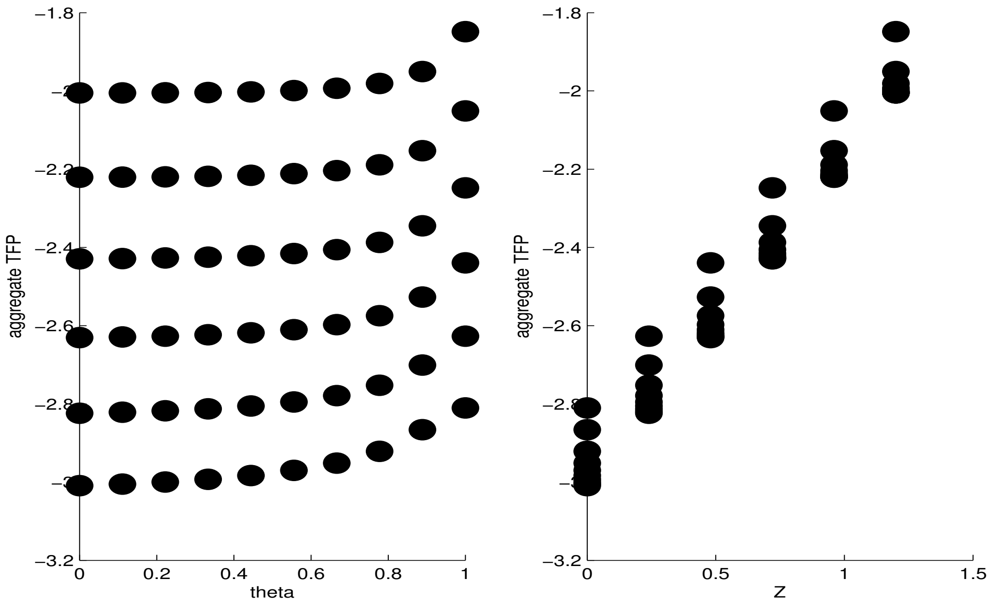
where w_i is agent i's share of aggregate employment:

$$w_i \equiv \frac{m_i k_i}{\int m_j k_j dj}.$$

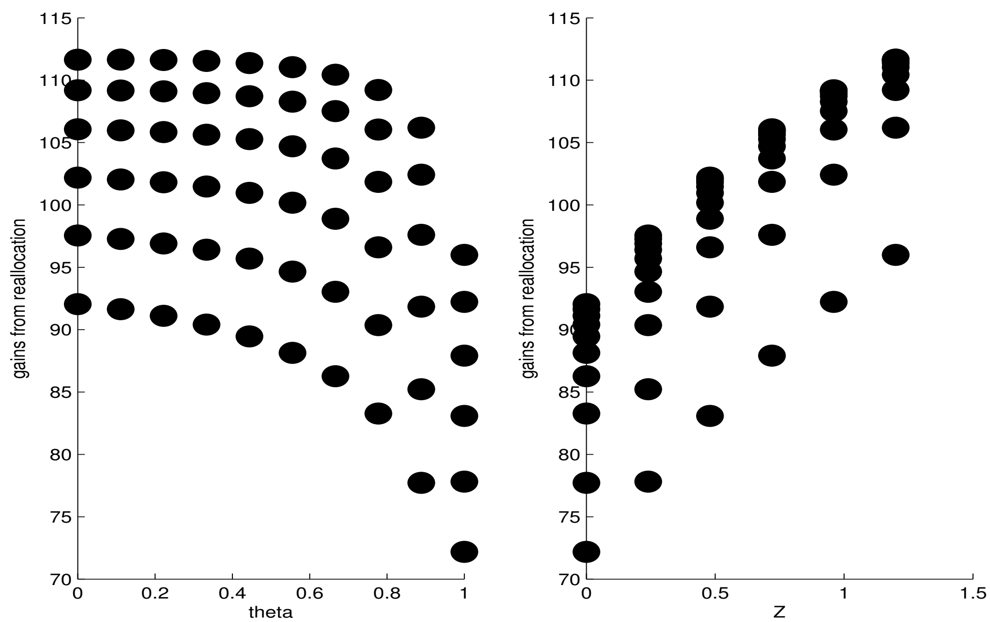
It is easy to show that without financial frictions, agents equalize their revenue products of labor, so that $\log \mathrm{ARPL}_i$ is constant across i and the Olley-Pakes covariance is zero. However, because financial frictions bind for more-productive firms, outside of the first-best allocation revenue productivity is positively correlated with size. Thus the Olley-Pakes covariance is positive in equilibrium with financial frictions, as it is in India and in the Western economies analyzed by Bartelsman, Haltiwanger and Scarpetta (2013).

Not only is the Olley-Pakes covariance positive in equilibrium, but it is positively correlated with positive aggregate shocks, as in the model of chapters 2 and 3. The top panels of Figure 4.3 plot $\mathcal{C}$ from equation (4.8) over the same values of Z and θ as in Figure 4.1. Increasing each agent's productivity, or loosening financial constraints, induces $\mathcal{C}$ to rise, even though in the first-best allocation $\mathcal{C} = 0$. This is true even when $\theta = 1$ (full collateralization of capital).

A related concern is that, because the wage rate w is fixed in these exercises, the first-best revenue product of labor is also constant: all plants should grow until their revenue product of labor is equal to w. The bottom panels of Figure 4.3 show that this intuition does not extend to the second-best allocation with frictions: increasing all agents' TFP through Z, or changing the collateral rate θ, both induce higher aggregate labor productivity. The reason for Z result is that constrained firms, which have higher TFP than unoncstrained firms, are only more constrained when Z rises. On the margin they choose a slightly higher labor-capital ratio than other firms as Z rises, but this does not completely offset the oncreased productivity. Likewise, increasing θ allows larger, more-productive firms to be even bigger; this lowers their labor productivity somewhat, but not enough to offset their higher share of economic activity, with the net result in the aggregate being a rise in aggregate labor productivity.

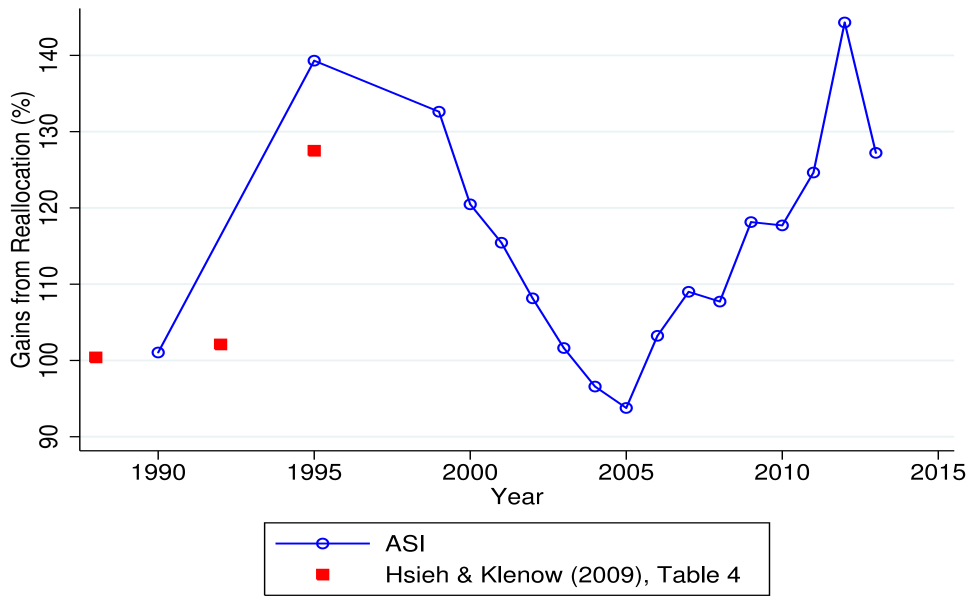


(a) Log Aggregate TFP

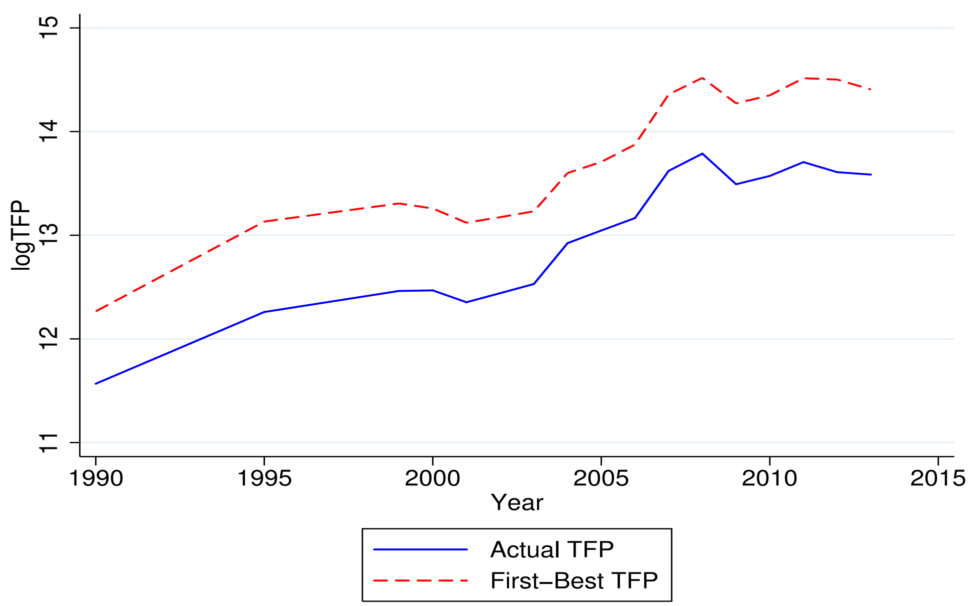


(b) % Gains from Reallocation to First-Best

Figure 4.1. Aggregate TFP and Gains from Reallocation
The top panel plots comparative statics in log aggregate physical total-factor productivity (TFP) as the financial friction θ and common productivity Z vary in the Hsieh-Klenow model with financial frictions. Each dot represents an equilibrium for a different value of (Z, θ). The bottom panel plots instead the percent difference between aggregate TFP and the first-best allocation. The left panels plot comparative statics in θ, and the right panels plot comparative statics in Z. Multiple dots for each value of θ in the left panels represent different values of Z, and multiple dots for each value of Z in the right panels represent different values of θ.

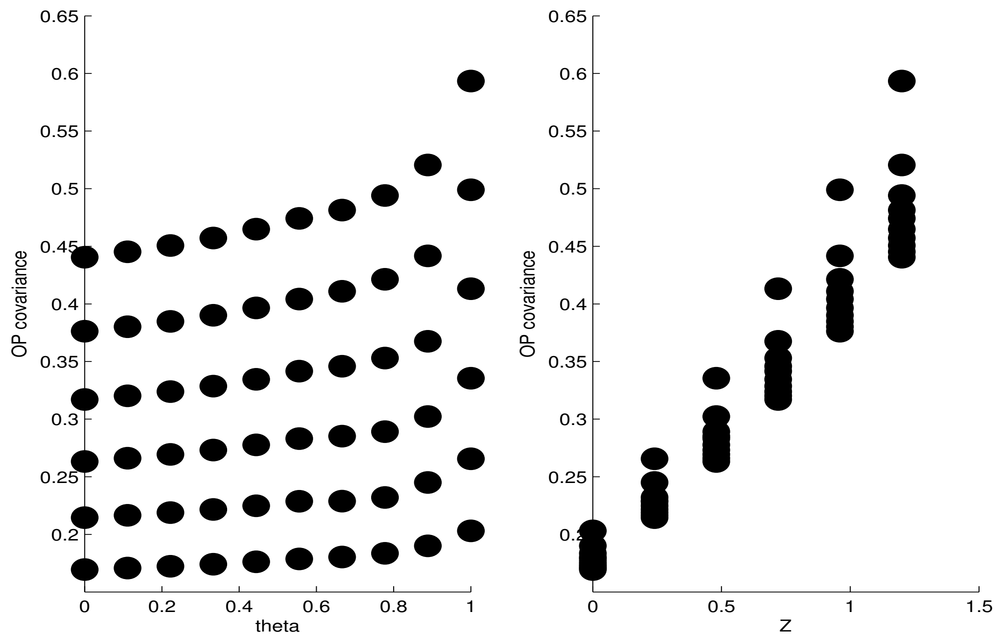


(a) % Gains from Reallocation to First-Best

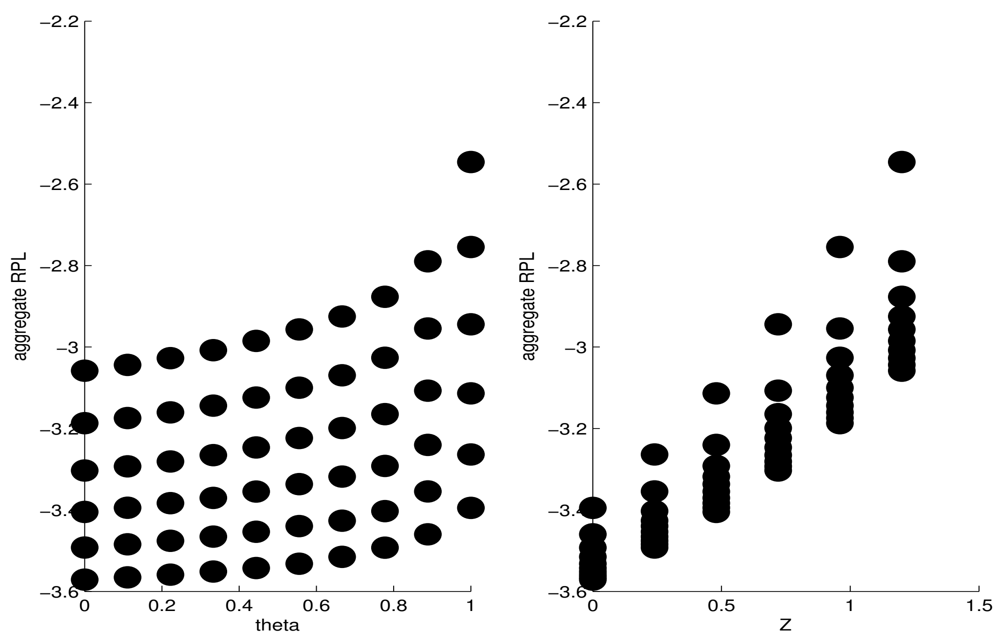


(b) Log Aggregate TFP, Actual and First-Best

Figure 4.2. Aggregate TFP and Gains from Reallocation over Time
The figures plot calculations from the Annual Survey of Industries designed to replicate the main results of Hsieh and Klenow (2009). The top panel plots the percentage productivity gains from reallocation from the observed allocation to the first-best allocation, $100\% \times \left(1 - \frac{Y}{Y_{\text{eff}}}\right)$. The red squares are the values reported by Hsieh and Klenow (2009) in Table 4, while the blue dots and lines are my own calculations from the Annual Survey of Industries. The bottom panel plots the time-series of $\log Y$ and $\log Y_{\text{eff}}$ separately.



(a) OP Covariance



(b) Log Aggregate Revenue Product of Labor

Figure 4.3. Olley-Pakes Covariance in the Static Model

The figure plots comparative statics in the Olley-Pakes covariance and aggregate revenue labor productivity as the financial friction θ and common productivity Z vary in the Hsieh-Klenow model with financial frictions. Each dot represents an equilibrium for a different value of (Z, θ). The top panels plot the Olley-Pakes covariance, and the bottom panels plot the aggregate revenue productivity of labor. The left panels plot comparative statics in θ, and the right panels plot comparative statics in Z. Multiple dots for each value of θ in the left panels represent different values of Z, and multiple dots for each value of Z in the right panels represent different values of θ.

CHAPTER 5
CONCLUSION

In this paper I test the hypothesis that financial development has led to productivity growth in the Indian manufacturing sector since 1990. I show that standard techniques to separate reallocation from growth in aggregate productivity suggest a role for financial development. I then derive and calibrate a model in which these techniques will produce this result even in the absence of financial development, when productivity is rising in all firms. I show using a second, simpler model, that if financial development had a non-negligible impact on productivity growth, it was only in 1990–1995; productivity growth since 1995 has been consistent with large common shocks to productivity, and small or negative shocks to financial development. Finally, I use a simple static model closely related to the seminal framework of Hsieh and Klenow (2009) to show that the inuition of the two dynamic models carries over to their setting, and that there is no contradiction between their results on the growth of misallocation over time in India, and my results on the improvement of the allocation in India.

Based on the analysis of the plant-level microdata from India and the three models meant to interpret it, I conclude that there is no evidence that financial development has affected economic growth in India. I am not suggesting that financial frictions are not important in determining real outcomes; indeed, the results of the paper depend on the existence of financial frictions. Rather, I argue that financial frictions do not appear to be *changing* over time, at least in their effects on leverage, output, and productivity growth. A common increase in total-factor productivity explains the rise in labor productivity evident in Figure 1.1, the increased leverage and output growth in the more-sensitive industries analyzed in section 2.1.3, and the increase in the Olley-Pakes covariance analyzed in section 2.1.4.

Overall, I argue that factors other than financial frictions are important in explaining Indian economic growth. One such factor might be frictions in adopting modern management practices. Bloom et al. (2013) show that management practices are important factors driving productivity in large Indian textile firms. They argue that informational barriers, rather than financial frictions, are what prevent firms from adopting efficiency-enhancing management practices. Brooks, Donovan and Johnson (2016) illuminate a particular informational barrier in small Kenyan retail establishments, as well as how eliminating that barrier with inter-firm training and mentorhsip programs can have a large impact on productivity.

Alternatively, financial frictions that primarily affect within-firm productivity could also be driving my results. For example, it may be that adopting a higher-productivity-growth technology or entering into a more-productive sector requires large fixed costs that are difficult to finance, as in the models of Buera, Kaboski and Shin (2011) and Cole, Greenwood and Sanchez (2016). Although both papers focus primarily on differences across countries, it may be that changing financial frictions over time within a country affect average productivity through this channel. Uncovering the exact underlying factors behind the common productivity growth identified by my model is an avenue that I hope to explore in future research.

REFERENCES

Aghion, Philippe, Robin Burgess, Stephen J. Redding, and Fabrizio Zilibotti. 2008. "The Unequal Effects Of Liberalization: Evidence From Dismantling The License Raj In India." *American Economic Review*, 98(4): 1397–1412.

Ahluwalia, Montek S. 2002. "Economic Reforms In India Since 1991: Has Gradualism Worked?" *The Journal of Economic Perspectives*, 16(3): 67–88.

Banerjee, Abhijit V., and Benjamin Moll. 2010. "Why Does Misallocation Persist?" *American Economic Journal: Macroeconomics*, 2(1): 189–206.

Bartelsman, Eric, John Haltiwanger, and Stefano Scarpetta. 2013. "Cross-Country Differences In Productivity: The Role Of Allocation And Selection." *American Economic Review*, 103(1): 305–34.

Beck, Thorsten, Asli Demirgüç-Kunt, and Vojislav Maksimovic. 2005. "Financial and Legal Constraints to Growth: Does Firm Size Matter?" *The Journal of Finance*, 60(1): 137–177.

Beck, Thorsten, Ross Levine, and Norman Loayza. 2000. "Finance And The Sources Of Growth." *Journal of Financial Economics*, 58(1G2): 261–300. Special Issue on International Corporate Governance.

Bhamra, Harjoat S., Lars-Alexander Kuehn, and Ilya A. Strebulaev. 2010. "The Aggregate Dynamics Of Capital Structure And Macroeconomic Risk." *Review of Financial Studies*, 23(12): 4187–4241.

Bloom, Nicholas, Benn Eifert, Aprajit Mahajan, David Mckenzie, and John Roberts. 2013. "Does Management Matter? Evidence From India." *The Quarterly Journal of Economics*, 128(1): 1–51.

Brooks, Wyatt, Kevin Donovan, and Terence R. Johnson. 2016. "The Dynamics of Inter-Firm Skill Transmission." Working paper.

Buera, Francisco J., and Benjamin Moll. 2012. "Aggregate Implications Of A Credit Crunch." National Bureau of Economic Research Working Paper 17775.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin. 2011. "Finance And Development: A Tale Of Two Sectors." *American Economic Review*, 101(5): 1964–2002.

Chen, Hui. 2010. "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure." *The Journal of Finance*, 65(6): 2171–2212.

Cole, Harold L., Jeremy Greenwood, and Juan M. Sanchez. 2016. "Why Doesn't Technology Flow From Rich To Poor Countries?" *Econometrica*, 84(4): 1477–1521.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2008. "Reallocation, Firm Turnover, And Efficiency: Selection On Productivity Or Profitability?" *The American Economic Review*, 98(1): 394–425.

Gabaix, Xavier. 2011. "The Granular Origins Of Aggregate Fluctuations." *Econometrica*, 79(3): 733–772.

Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial Intermediation And Credit Policy In Business Cycle Analysis." In . Vol. 3 of *Handbook of Monetary Economics*, , ed. Benjamin M. Friedman and Michael Woodford, 547–599. Elsevier.

Gertler, Mark, Nobuhiro Kiyotaki, and Albert Queralto. 2012. "Financial Crises, Bank Risk Exposure And Government Financial Policy." *Journal of Monetary Economics*, 59, Supplement: 0–0. Supplement issue:October 15-16 2010 Research Conference on 'Directions for Macroeconomics: What did we Learn from the Economic Crises' Sponsored by the Swiss National Bank (http://www.snb.ch).

Gilchrist, Simon, Jae W. Sim, and Egon Zakrajšek. 2014. "Uncertainty, Financial Frictions, And Investment Dynamics." National Bureau of Economic Research Working Paper 20038.

Glover, Brent. 2014. "The Expected Cost of Default." Working paper.

Gomes, Joao F. 2001. "Financing Investment." *The American Economic Review*, 91(5): 1263–1285.

Hennessy, Christopher A. 2013. "Model Before Measurement." *Critical Finance Review*, 2: 193–215.

Hennessy, Christopher A., and Ilya A. Strebulaev. 2015. "Beyond Random Assignment: Credible Inference Of Causal Effects In Dynamic Economies." National Bureau of Economic Research Working Paper 20978.

Hennessy, Christopher A., and Toni M. Whited. 2005. "Debt Dynamics." *The Journal of Finance*, 60(3): 1129–1165.

Hennessy, Christopher A., and Toni M. Whited. 2007. "How Costly Is External Financing? Evidence from a Structural Estimation." *The Journal of Finance*, 62(4): 1705–1745.

Hsieh, Chang-Tai, and Peter J. Klenow. 2009. "Misallocation And Manufacturing Tfp In China And India." *The Quarterly Journal of Economics*, 124(4): 1403–1448.

Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects Of Financial Shocks." *American Economic Review*, 102(1): 238–271.

Joshi, Vijay, and I.M.D. Little. 1996. *India's Economic Reforms 1991-2001.* Clarendon Press, Oxford.

Khan, Aubhik, Tatsuro Senga, and Julia K. Thomas. 2014. "Credit Shocks in an Economy with Heterogeneous Firms and Default." Working paper.

King, Robert G., and Ross Levine. 1993. "Finance And Growth: Schumpeter Might Be Right." *The Quarterly Journal of Economics*, 108(3): 717–737.

Korteweg, Arthur. 2010. "The Net Benefits To Leverage." *The Journal of Finance*, 65(6): 2137–2170.

Krusell, Per, and Anthony A. Smith, Jr. 1998. "Income And Wealth Heterogeneity In The Macroeconomy." *The Journal of Political Economy*, 106(5): 867–896.

La Porta, Rafael, Florencio Lopez-De-Silanes, and Andrei Shleifer. 2002. "Government Ownership Of Banks." *The Journal of Finance*, 57(1): 265–301.

Leland, Hayne E. 1994. "Corporate Debt Value, Bond Covenants, And Optimal Capital Structure." *The Journal of Finance*, 49(4): 1213–1252.

Leland, Hayne E. 1998. "Agency Costs, Risk Management, And Capital Structure." *The Journal of Finance*, 53(4): 1213–1243.

Levine, Ross. 2005. "Finance And Growth: Theory And Evidence." In . Vol. 1, Part A of *Handbook of Economic Growth*, , ed. Philippe Aghion and Steven N. Durlauf, 865–934. Elsevier.

Levine, Ross, Norman Loayza, and Thorsten Beck. 2000. "Financial Intermediation And Growth: Causality And Causes." *Journal of Monetary Economics*, 46(1): 31–77.

Midrigan, Virgiliu, and Daniel Yi Xu. 2014. "Finance And Misallocation: Evidence From Plant-Level Data." *American Economic Review*, 104(2): 422–58.

Olley, G. Steven, and Ariel Pakes. 1996. "The Dynamics Of Productivity In The Telecommunications Equipment Industry." *Econometrica*, 64(6): 1263–1297.

Rajan, Raghuram G. 2016. "Policy and Evidence." Inaugural Address given at the 10th Statistics Day Conference.

Rajan, Raghuram G., and Luigi Zingales. 1995. "What Do We Know About Capital Structure? Some Evidence From International Data." *The Journal of Finance*, 50(5): 1421–1460.

Rajan, Raghuram G., and Luigi Zingales. 1998. "Financial Dependence and Growth." *The American Economic Review*, 88(3): 559–586.

Rajan, Raghuram G., Suman Bery, Uday Kotak, Rajiv Lall, Vijay Mahajan, O.P. Bhatt, K.V. Kamath, Chitra Ramakrishna, R. Ravimohan, J.R. Varma, and R. Sridharan. 2009. "A Hundred Small Steps: Report of the Committee on Financial Sector Reforms." Government of India Planning Commission.

Rouwenhorst, K. Geert. 1995. "Asset Pricing Implications of Real Business Cycle Models." In *Frontiers of Business Cycle Research.* , ed. Thomas F. Cooley, 294–330. Princeton University Press.

Strebulaev, Ilya A. 2007. "Do Tests Of Capital Structure Theory Mean What They Say?" *The Journal of Finance*, 62(4): 1747–1787.

Vig, Vikrant. 2013. "Access To Collateral And Corporate Debt Structure: Evidence From A Natural Experiment." *The Journal of Finance*, 68(3): 881–928.

Visaria, Sujata. 2009. "Legal Reform And Loan Repayment: The Microeconomic Impact Of Debt Recovery Tribunals In India." *American Economic Journal: Applied Economics*, 1(3): 59–81.

Wurgler, Jeffrey. 2000. "Financial Markets And The Allocation Of Capital." *Journal of Financial Economics*, 58(1G2): 187–214. Special Issue on International Corporate Governance.

APPENDIX A

PROOFS

Proof of Proposition 1

Proof. Optimal capital k^* follows directly from the budget and the fact that revenue is strictly increasing in k, so that the budget holds with equality. Plugging this in to equation (2.6), the first-order condition for m becomes

$$w\frac{\$}{(1+wm)^2} = \frac{\gamma-1}{\gamma}\left(Ae^{z'}\right)^{\frac{\gamma-1}{\gamma}}\left(m^{1-\alpha}k^*\right)^{-\frac{1}{\gamma}}\left[(1-\alpha)\,m^{-\alpha}k^* - wm^{1-\alpha}\frac{\$}{(1+wm)^2}\right],$$

which can be simplified to

$$1 = \frac{\gamma-1}{\gamma}\frac{PY}{mk^*}\left[\frac{1-\alpha}{w} - \alpha m\right],$$

which upon further simplification yields equation (2.10).

To show that the solution to equation (2.10) is unique, consider that the left-hand-side of equation (2.10) is strictly increasing in m, and equals 0 at $m = 0$. The right-hand-side of equation (2.10) is strictly positive so long as $m \in \left(0, \frac{1}{w}\frac{1-\alpha}{\alpha}\right)$, thus the solution m^* must also lie in this interval. The derivative of the right-hand-side of equation (2.10) with respect to m is given by

$$A^*\left(1+wm\right)^{\frac{1}{\gamma}}\left[\frac{w}{\gamma}\left(\frac{1-\alpha}{w} - \alpha m\right) - \alpha\right]$$

which has the same sign as the term in square braces. This term is linear and decreasing in m; at m's supremum value of $\frac{1}{w}\frac{1-\alpha}{\alpha}$, it is negative. At m's minimum value of zero, it is

$$\frac{1-\alpha}{\gamma} - \alpha,$$

which is negative if and only if $\frac{1-\alpha}{\alpha} < \gamma$. Thus, this restriction on parameters ensures that the right-hand-side of equation (2.10) is decreasing for $m \in \left(0, \frac{1}{w}\frac{1-\alpha}{\alpha}\right)$; because it is a continuous function of m, and is strictly greater than the left-hand side for $m = 0$ and strictly smaller than the left-hand side for $m = \frac{1}{w}\frac{1-\alpha}{\alpha}$, there must be a unique solution where the two sides are equal.

To show that m^* is increasing in z' and decreasing in \$, it suffices to show that it is increasing in A^* defined in equation (2.11). This follows directly from the fact that the right-hand side of equation (2.10) is increasing in A^*, while the left-hand side is increasing in m^* but constant with respect to A^*.

The proof up to this point has ignored the overhead labor constraint $mk \geq \underline{L}$. If $m^*k^* > \underline{L}$, then the constraint doesn't bind and is irrelevant. If instead $m^*k^* < \underline{L}$, the constraint binds and the agent sets $k = \$$ and $m = \frac{\underline{L}}{\$}$, which satisfies both the budget and the overhead labor constraint.

□

Proof of Proposition 2

Proof. Fix values for A, \$, and z. There are two cases to consider, whether the overhead-labor constraint binds or not. This depends on ε; in fact, it is straightforward to see that agents with $\varepsilon < \varepsilon^*$ choose $mk = \underline{L}$, where

$$\varepsilon^* \equiv \frac{1}{\sigma}\left[\frac{\gamma}{\gamma-1}\left((1-\zeta)\log\frac{\underline{L}}{\$} - \log\left(\frac{1-\alpha}{w} - \alpha\frac{\underline{L}}{\$}\right) - \frac{1}{\gamma}\log\left(1 + w\frac{\underline{L}}{\$}\right) - \log\frac{\gamma-1}{\gamma}\right)\right.$$
$$\left. + \frac{1}{\gamma-1}\log\left(\$ + w\underline{L}\right) - \log A - \rho z\right], \tag{A.1}$$

$\zeta \equiv (1-\alpha)\frac{\gamma-1}{\gamma}$, and $\varepsilon^* = -\infty$ if the second term in logarithms is negative. If $\varepsilon > \varepsilon^*$ the overhead labor constraint does not bind.

Solving for the default threshold involves solving equation (2.12) for both the case where the overhead labor constraint binds ($\underline{\varepsilon}_1$) and where it does not ($\underline{\varepsilon}_2$). Then $\underline{\varepsilon} = \underline{\varepsilon}_2$ if $\underline{\varepsilon}_2 > \varepsilon^*$, and $\underline{\varepsilon} = \underline{\varepsilon}_1$ otherwise.

Computing $\underline{\varepsilon}_1$ is straightforward: because the labor constraint binds, $mk = \underline{L}$ and ε only appears once in equation (2.12). Inverting this equation for ε yields

$$\underline{\varepsilon}_1 = \begin{cases} \frac{1}{\sigma}\left[\frac{\gamma}{\gamma-1}\log(\ell-\theta) - \log A - \rho z - (1-\alpha)\log\frac{\underline{L}}{\$} + \frac{1}{\gamma-1}\log\$\right] & \text{if } \ell > \theta \\ -\infty & \text{otherwise} \end{cases}$$

Labor-constrained agents who borrow $\ell < \theta$ will never default, since regardless of the shock they always retain some output, ensuring their post-repayment value is higher than $(1-\theta)$.

Solving for $\underline{\varepsilon}_2$ requires a bit more work, since agents for whom the overhead labor constraint does not bind have ε appearing in multiple places in equation (2.12). Rewrite the threshold equation (2.12) as

$$(1+\ell-\theta)\frac{\$}{\$+w\underline{L}} = \left(\frac{Ae^{z'}m^{1-\alpha}}{1+wm}\right)^{\frac{\gamma-1}{\gamma}}(\$+w\underline{L})^{-\frac{1}{\gamma}} + \frac{1}{1+wm},$$

which, after plugging in the optimal m from equation (2.10) and rearranging, yields

$$(1+wm)(1+\ell-\theta)\frac{\$}{\$+w\underline{L}} = 1 + \frac{\frac{\gamma}{\gamma-1}m}{\frac{1-\alpha}{w} - \alpha m}. \tag{A.2}$$

Equation (A.2) is a quadratic equation in m, $0 = am^2 + bm + c$, with coefficients given by

$$\begin{aligned} a &= -\alpha w(1+\ell-\theta)\frac{\$}{\$+w\underline{L}} \\ b &= -\frac{\gamma}{\gamma-1} + \alpha + \frac{\$}{\$+w\underline{L}}(1-2\alpha)(1+\ell-\theta) \\ c &= \frac{1-\alpha}{w}\left[(1+\ell-\theta)\frac{\$}{\$+w\underline{L}} - 1\right]. \end{aligned} \tag{A.3}$$

The rest of the proof consists of showing that this equation has a unique positive root $\underline{m}$, if and only if $\ell > \theta + \frac{\underline{L}}{\$}$. Given such a root, A, \$, and z, the corresponding value for $\underline{\varepsilon}_2$ can be found using the solution to equation (2.10).

Suppose $\ell > \theta + \frac{w\underline{L}}{\$}$. Then $c > 0$, and because $a < 0$ it follows that $b^2 - 4ac > b^2$. Then regardless of the sign of b, one of the roots is positive and one is negative.

Suppose $\ell = \theta + \frac{w\underline{L}}{\$}$. Then $c = 0$ and $b = \alpha - \frac{\gamma}{\gamma - 1} < 0$, so that either $m = 0$ or $m = -\frac{b}{a} < 0$.

Suppose $\ell < \theta + \frac{w\underline{L}}{\$}$. Then $c < 0$ and $b^2 - 4ac < b^2$, so it is sufficient (using the quadratic formula and the fact that $a < 0$) to show that $b < 0$. Suppose it does not; then

$$\frac{\$}{\$ + w\underline{L}} (1 + \ell - \theta)(1 - 2\alpha) \geq \frac{\gamma}{\gamma - 1} - \alpha,$$

so that

$$\begin{aligned} \frac{\$}{\$ + w\underline{L}} (1 + \ell - \theta) &\geq \frac{\frac{\gamma}{\gamma - 1} - \alpha}{1 - 2\alpha} \\ &\geq 1, \end{aligned} \tag{A.4}$$

where the second line follows because the numerator is greater than 1 (since $\alpha < \frac{1}{\gamma - 1}$), and $\alpha < \frac{1}{2}$ so the denominator is less than 1 but positive. But equation (A.4) is a contradiction, since it implies that $c \geq 0$ which was assumed false at the start. Thus $b < 0$ and there are no positive roots for this case.

The solution $\underline{m}$ to equation (A.2) is the optimal labor-capital ratio, given A, z, and \$, at which agents are indifferent between defaulting and repaying. The fact that the optimal labor-capital ratio is strictly increasing in z', means that there is a one-to-one mapping between $\underline{m}$ and $\underline{\varepsilon}$, given A, z, and \$; moreover equations (2.10) and (2.11) give this mapping in closed form (even though m^* itself as a function of A^* must be computed numerically). That $\underline{\varepsilon}$ is decreasing in z and increasing in \$ then follows directly from the same facts (reversed) for m^*. □

Proof of Proposition 3

Proof. For ease of exposition, suppose that $\overline{z}_i = 0$.

The proof consists of guessing that $V(x, z; Z, \theta, P, F)$ of the form $a_0 + a_1 \log(X) + f(z, Z; \theta, P, F)$ solves equation (3.6), and then verifying that a_0, a_1, and $f(\cdot)$ exist. The value of a_1 will also be important for characterizing the decision rules.

Thus, suppose $V(x', z'; Z', \theta', P', F') = a_0 + a_1 \log(x') + f(z'; Z', \theta', P', F')$. Plugging into equation (3.6) and rearranging yields

$$\begin{aligned} V(x, z; Z, \theta, P, F) = \max_{k', b' \geq 0} \; & \log(x + qb' - k') \\ & + \beta\left[(1-\pi)\left(a_0 + E\left\{f(z'; Z', \theta', P', F')\right\}\right) + ((1-\pi)a_1 + \pi) E\left\{\log(x')\right\}\right]. \end{aligned} \tag{A.5}$$

Because agents are restricted from lending, it must be the case thet $k' > 0$ strictly, otherwise net wealth next period would be negative. Thus I can rewrite the original problem in terms of choosing $k' > 0$ and $\ell = \frac{b'}{k'} \geq 0$. Net wealth becomes $x' = k' \max\left\{PAe^{z'} + 1 - \ell, 1 - \theta\right\}$, and equation (A.5) becomes

$$\begin{aligned} V(x, z; Z, \theta, P, F) = \max_{k' \geq 0, \ell \geq 0} \; & \log\left[x - (1 - q\ell)k'\right] + \beta\left[(1-\pi)\left(a_0 + E\left\{f(z'; Z', \theta', P', F')\right\}\right)\right. \\ & \left. + ((1-\pi)a_1 + \pi)\left(\log(k') + E\left\{\log\max\left\{PAe^{z'} + 1 - \ell, 1 - \theta\right\}\right\}\right)\right]. \end{aligned} \tag{A.6}$$

The first-order condition for k' is

$$\frac{1 - q\ell}{x - (1 - q\ell)k'} = \beta\frac{(1-\pi)a_1 + \pi}{k'},$$

which I rewrite as

$$
\begin{aligned}
(1 - q\ell)\, k' &= \frac{\widetilde{\beta}}{1+\widetilde{\beta}} x \\
&\equiv \beta^* x,
\end{aligned}
\tag{A.7}
$$

where $\widetilde{\beta} \equiv \beta\left[(1-\pi)\, a_1 + \pi\right]$ and the second line defines β^*. Plugging equation (A.7) back into the definitions for consumption and ℓ then yields that $c = (1-\beta^*)\, x$ and $k' - qb' = \beta^* x$.

It remains to verify the guess. The first-order condition for leverage ℓ is given by

$$
\frac{\left(q + \frac{\partial q}{\partial \ell}\ell\right) k'}{x - (1-q\ell)\, k'} = \widetilde{\beta} \int_{\underline{\varepsilon}}^{\infty} \frac{\phi(\varepsilon)\, d\varepsilon}{Ae^{\widetilde{z}+\sigma\varepsilon} + 1 - \ell},
$$

which can be written using equation (A.7) as

$$
\frac{q + \frac{\partial q}{\partial \ell}\ell}{1 - q\ell} = \int_{\underline{\varepsilon}}^{\infty} \frac{\phi(\varepsilon)\, d\varepsilon}{Ae^{\widetilde{z}+\sigma\varepsilon} + 1 - \ell},
\tag{A.8}
$$

Equations (A.7) and (A.8) imply that optimal capital k' and consumption c are proportional to wealth x, while optimal leverage ℓ is independent of x. Plugging into equation (A.6) yields

$$
\begin{aligned}
V(x, z; Z, \theta, P, F) &= \log\left[(1-\beta^*)\, x\right] + \beta(1-\pi)\left(a_0 + E\left\{f\left(z'; Z', \theta', P', F'\right)\right\}\right) \\
&\quad + \widetilde{\beta}\log(\beta^* x) - \widetilde{\beta}\log(1-q\ell) \\
&\quad + \widetilde{\beta} E\left\{\log\max\left\{Ae^{\widetilde{z}+\sigma\varepsilon} + 1 - \ell, 1 - \theta\right\}\right\} \\
&= \underbrace{\log(1-\beta^*) + \beta(1-\pi)\, a_0 + \widetilde{\beta}\log(\beta^*)}_{\text{constant}} + \underbrace{\left(1+\widetilde{\beta}\right)\log(x)}_{\text{depends on } \log x} \\
&\quad - \underbrace{\widetilde{\beta}\log(1-q\ell) + \beta E\left\{f\left(z'; Z', \theta', P', F'\right)\right\}}_{\text{depends on } z, Z, \theta, P} \\
&\quad + \underbrace{\widetilde{\beta} E\left\{\log\max\left\{Ae^{\widetilde{z}+\sigma\varepsilon} + 1 - \ell, 1 - \theta\right\}\right\}}_{\text{depends on } z, Z, \theta, P}.
\end{aligned}
\tag{A.9}
$$

Equating terms when the left-hand-side of equation (A.9) equals

$$a_0 + a_1 \log(x) + f(z; Z, \theta, P, F)$$

yields

$$\begin{aligned} a_1 &= 1 + \beta\left[(1-\pi)\, a_1 + \pi\right] \\ &= \frac{1+\beta\pi}{1-\beta(1-\pi)}, \end{aligned}$$

from which $\widetilde{\beta}$ and β^* can be calculated. □

Proof of Proposition 4

Proof. Suppose that z is large enough that $\ell > 0$. Then by Proposition 3, k' and ℓ satisfy

$$k' = \frac{\beta^*}{1 - q\ell}$$

and

$$\frac{q + \frac{\partial q}{\partial \ell}\ell}{1 - q\ell} = \int_{\underline{\varepsilon}}^{\infty} \frac{\phi(\varepsilon)\, d\varepsilon}{Ae^{\widetilde{z}+\sigma\varepsilon} + 1 - \ell},$$

where $\widetilde{z} \equiv \rho z + \overline{z}_i + Z + \log P$. It is straightforward to show that the optimal policy for ℓ is continuous in z. Now suppose I lower $\widetilde{z}$ until $\ell = 0$ is optimal. At this point $\ell < \theta$ so $q = \frac{1}{1+r}$ and $\frac{\partial q}{\partial \ell} = 0$, so equation (A.8) becomes

$$\frac{1}{1+r} = \int_{-\infty}^{\infty} \frac{\phi(\varepsilon)\, d\varepsilon}{Ae^{\widetilde{z}+\sigma\varepsilon} + 1},$$

as was to be shown. □

Proof of Proposition 5

Proof. For ease of notation and without loss of generality, let $Z = 0$ and $z_i = z$. The agent's problem is

$$\max_{k,m,\ell} \underbrace{\left[Ae^z m^{1-\alpha} k\right]^{\frac{\gamma-1}{\gamma}}}_{PY} + (1-\ell)\,k$$

s.t.

$$(1 + wm - q\ell)\,k \leq x$$

$$\ell \leq \theta.$$

Let λ_0 and λ_1 be the Lagrange multiplier on the budget and leverage constraints, respectively. The first-order conditions for k, m, and ℓ are given by

$$\frac{\gamma-1}{\gamma}\frac{PY}{k} + 1 - \ell = \lambda_0\,(1 + wm - q\ell) + \lambda_1 m$$

$$-k = -\lambda_0 kq + \lambda_2$$

$$(1-\alpha)\,\frac{\gamma-1}{\gamma}\frac{PY}{m} = \lambda_0 kw + \lambda_1 k,$$

where $PY \equiv \left[Ae^z m^{1-\alpha} k\right]^{\frac{\gamma-1}{\gamma}}$ is optimal revenue.

When $\lambda_2 = 0$, so the leverage constraint does not bind, the first-order condition for ℓ implies that $\lambda_0 = 1/q = 1 + r$. In this case equation (4.4) follows after inserting the first-order condition for m into that for k and simplifying. This solution is feasible only if implied leverage ℓ^* in equation (4.4) is less than θ, confirming that $\lambda_2 = 0$. Because optimal capital

$k^*(z)$ in equation (4.4) is increasing in z, $\lambda_2 = 0$ only if $z < \overline{z}$ that satisfies

$$\ell^* \equiv (1+r)\left(1 + wm^* - \frac{x}{k^*(\overline{z})}\right) = \theta$$

$$\Leftrightarrow$$

$$\overline{z} = \frac{1}{\gamma - 1}\left[\log x - \log\left(1 + wm^* - \frac{\theta}{1+r}\right) - \log\left(\left[\frac{\gamma-1}{\gamma}\frac{\alpha}{r}\right]^{\gamma}\left[Am^{*(1-\alpha)}\right]^{\gamma-1}\right)\right].$$

Any agent with $z > \overline{z}$ will be constrained, so that $\lambda_2 \neq 0$. For these agents, $\ell = \theta$ and the value of λ_2 is irrelevant because it appears in no other equation. The first-order condition for m then implies that

$$\lambda_0 = (1-\alpha)\frac{\gamma-1}{\gamma}\frac{PY}{wmk},$$

which plugging into the first-order condition for k and simplifying yields

$$1 - \theta = \frac{\gamma-1}{\gamma}\frac{PY}{k}\left[\frac{(1-\alpha)\left(1 - \frac{\theta}{1+r}\right)}{wm} - \alpha\right]. \tag{A.10}$$

If $\theta = 1$, then the term in square brackets in equation (A.10) must equal zero, which implies that

$$m = \frac{1-\alpha}{\alpha}\frac{1}{w}\left(1 - \frac{1}{1+r}\right),$$

which is in fact the optimal labor-capital ratio in equation (4.4). This will turn out to be a special case of the general $\theta < 1$ solution.

If $\theta < 1$, then dividing equation (A.10) by $1 - \theta$ and solving for k using the definition of $\frac{PY}{k}$ yields

$$k = \left[\frac{\gamma-1}{\gamma}\frac{1}{1-\theta}\left(\frac{(1-\alpha)\left(1 - \frac{\theta}{1+r}\right)}{wm} - \alpha\right)\right]^{\gamma}\left[Ae^{z}m^{1-\alpha}\right]^{\gamma-1}. \tag{A.11}$$

Rearranging the budget equation, inserting equation (A.11), and simplifying gives

$$\left[\frac{x}{w}\left(\frac{\gamma-1}{\gamma\left(1-\theta\right)}\right)^{-\gamma}\left(Ae^{z}\right)^{1-\gamma}\frac{1}{m+\frac{1-\frac{\theta}{1+r}}{w}}\right]^{\frac{1}{\gamma}}=\left[\frac{\left(1-\alpha\right)\left(1-\frac{\theta}{1+r}\right)}{w}-\alpha m\right]m^{\left(1-\alpha\right)\frac{\gamma-1}{\gamma}-1},$$

which implies equation (4.5) when

$$\begin{aligned}
a_0 &\equiv \frac{x}{w}\left[\frac{\gamma-1}{\gamma\left(1-\theta\right)}\right]^{-\gamma}\left(Ae^{z}\right)^{1-\gamma}\\
a_1 &\equiv \frac{1-\frac{\theta}{1+r}}{w}\\
a_2 &\equiv \left(1-\alpha\right)\frac{1-\frac{\theta}{1+r}}{w}\\
a_3 &\equiv \alpha\\
\zeta &\equiv \left(1-\alpha\right)\frac{\gamma-1}{\gamma}.
\end{aligned}$$

If $\theta=1$, then $a_0=0$ and $m=\frac{a_2}{a_3}=m^*$ in equation (4.4).

To show that equation (4.5) has a unique solution in the general case of $\theta<1$, it suffices to show that the left-hand side is increasing in m, because it is zero when $m=0$, and the right-hand side is decreasing and linear in m, and strictly positive when $m=0$. The derivative of the left hand side of equation (4.5) is given by

$$\frac{\partial}{\partial m}\text{LHS}\left(m\right)=\text{LHS}\left(m\right)\left[\frac{1-\zeta}{m}-\frac{\frac{1}{\gamma}}{m+a_1}\right]. \tag{A.12}$$

Then using the definition of ζ, we have that

$$\begin{aligned}
1-\zeta &= \frac{1+\alpha\gamma-\alpha}{\gamma}\\
&= \frac{1}{\gamma}+\alpha\left(\gamma-1\right)\\
&> \frac{1}{\gamma},
\end{aligned}$$

and since $a_1 > 0$ we also have that $a_1 + m > m$. Thus the left-hand term in square brackets in equation (A.12) has a larger numerator and a smaller denominator than the right-hand term, so that their difference must be positive. Then because $\text{LHS}(m)$ is also positive for all m, the left-hand side of equation (4.5) is increasing and equation (4.5) has a unique solution for m. $\square$

APPENDIX B

MODEL SOLUTION APPROXIMATION

In this section I describe the numerical approximations I use to solve the model described in section 2.2. The solution consists of three parts: solving the static production problem (2.6), using that solution to solve the dynamic investment problem (2.8), and using the implied policy functions from the dynamic problem to solve the Fredholm equation (3.11).

B.1 Approximating Equation (2.10)

I approximate the solution m^* to equation (2.10) as a univariate function in A^*, defined in equation (2.11), by inversion: that is, I guess a range of values for m^*, compute the implied A^* for a large number of m^* in that range, linearly interpolate the resulting function, and invert the linear interpolation at the appropriate value of A^*.

More specifically: define the function $A^*(m)$ implicitly from equation (2.10). Given a list $\vec{A}$ of values of A^* at which to evaluate m^*, I find m_0 sufficiently close to zero and m_1 sufficiently close to $\frac{1-\alpha}{\alpha}\frac{1}{w}$ such that $A^*(m_0) < \min\left\{\vec{A}\right\}$ and $A^*(m_1) > \max\left\{\vec{A}\right\}$. Because the true function $m^*(A^*)$ is monotonically increasing, the desired values $m^*\left(\vec{A}\right)$ lie in the interval (m_0, m_1).

The function $A^*(m)$ implictily defined by equation (2.10) is in closed form; I thus compute a large number of values of A^* on a uniform grid for m in the interval (m_0, m_1). These values of A^*, and the grid for m, define a piecewise linear function $\widetilde{A}^*(m)$, which I then invert and evaluate at the desired values $\vec{A}$ of A^* to approximate the true m^*.

The above procedure yields a univariate function $m^*(A^*)$; the bivariate function $m\left(Ae^{z'}, \$\right)$ can then be computed using the definition of A^* in equation (2.11).

B.2 Approximating the Value and Policy Functions

I compute the value and policy functions $V(x,z)$, $\$(x,z)$, and $\ell(x,z)$ by approximating the continuous state-space (x,z) with a set of discrete points. The grid of points is a tensor product of grids in x and z. I specify the x grid as 150 evenly-spaced points, in logs, from -6 to 11 for the high-A types and from -6 to 1 for the low-A types. I verify ex post that these bounds on $\log x$ are not binding for either type; that is, a negligible mass of firms lies at either boundary point. Because the size distribution has fat tails in equilibrium, I also check that the mass at each boundary, multiplied by the level value $\exp\{\log x\}$, is also a negligible value of the total expected value of x. I specify the z grid as 10 points using the Rouwenhorst (1995) method. I then organize the $N = 150 \times 10 = 1,500$ state-space points in a list, so that the first 10 points are the first $\log x$ point and all 10 z points, the next 20 points are the second $\log x$ point and all 10 z points, and so on (this accounting, while tedious, matters for the formulas below).

The value and policy functions in this approximation are represented by vectors of values at each gridpoint. Given a value function, at each gridpoint I search over policies $\log \$$ and ℓ to maximize current-period utility plus the continuation value. To do so, I specify a policy grid for $\log \$$ on the interval $[\log x - 5, \log x + 5]$ and a policy grid for leverage on the interval $[0, 1.3]$. I verify ex post that in equilibrium only a negligible measure of the stationary distribution of $(\log x, z)$ chooses these extreme values. These grids have $M = 150$ points each, so that there are $M^2 = 22,500$ potential actions at each state-space point.

The approximate solution to system (2.8) induces an $N \times N$ transition matrix Π, whose (i, i') element represents the probability of transitioning to state i' from state i. This matrix is the key component in approximating the equilibrium size-productivity distribution function $F(\log x, z)$ in equation (3.11), which I also approximate as a $N \times 1$ vector of values. The rest of this subsection describes the computation of Π in more detail.

First, fix the value function vector $\vec{V}$. The following procedure will then produce a new value of $\vec{V}$, and iterate to convergence. Next, fix a state-space point $i = (\log x, z)$. Everything beyond this point is then repeated N times, once for each point. Equation (2.8) at this point becomes

$$v_i = \max u^* + \beta \vec{P} \left[(1 - \pi) \vec{V} + \pi \log \vec{x} \right], \tag{B.1}$$

where u^* is the $M^2 \times 1$ vector of implied period-utility values for each policy choice from point i, $\vec{x}$ is the $N \times 1$ vector of the $\log x$ values of the state (i.e., 10 copies of the first $\log x$, followed by 10 copies of the next, etc), and $\vec{P}$ is the $M^2 \times N$ matrix of transition implied by each policy choice from point i. The max is taken over the M^2 discrete policy choices, as described below. Denote the index of the optimal policy by j^*; then the ith row of Π is given by the j^*th row of $\vec{P}$. Solving equation (B.1) at each point i then fills out the entire Π matrix.

To compute element (j, i') of the matrix $\vec{P}$ I calculate the implied transition probabilities from point i to point i', for each i' from 1 to N. To do so, I first compute the $M^2 \times N$ transition CDF matrix $\widetilde{P}$, whose (j, i') element is the probability of moving to a $\log x$ point below that in state i' and a z point below that in state i' when the policy is j. I convert $\widetilde{P}$ to the matrix of transition probabilities $\vec{P}$ using

$$\vec{P} = \widetilde{P} T^{-1'}, \tag{B.2}$$

where $T \equiv t_{150} \otimes t_{10}$ is the matrix that converts a discrete pdf to a CDF through appropriate additions (so that T^{-1} converts a CDF vector into a pdf vector). The matrices t_n are the $n \times n$ lower-triangular matrices of 1s.

Let p_1 be the matrix of probabilities of moving from i to a $\log x$ point lower than i', and p_2, the probability of moving from i to a z point lower than i', given the current policy choice j. Then $\widetilde{P} = \min \{p_1, p_2\}$, where the minimum is element-wise.

Transition probabilties p_1 in the $\log x$ dimension depend on the agent's policy choice for $\log \$$ and ℓ, but transition probabilities p_2 in the exogenous z dimension do not. They are therefore simpler to compute. Let P_z be the 10×10 matrix of transition probabilities implied by the Rouwenhorst (1995) method, and let $\widetilde{i} \in \{1, 2, ...10\}$ denote the z index of the current point i. Likewise let $\widetilde{i'}$ be the z-point of the next-period point i' under consideration. Then the 10 unique values over $\widetilde{i'}$ in p_2 are given by

$$p_2\left(\widetilde{i'}\right) = \left[P_z t'_{10}\right]_{\widetilde{i},\widetilde{i'}}$$

where the subscript denotes the given element of the matrix in brackets. The above values of p_2 then need to be copied appropriately so that they correspond to the $M^2 \times N$ values of the policies and state-space. The role of the matrix t'_{10} is to convert the transition probabilities in P_z into a transition CDF; this conversion is later reversed, after being combined with the $\log x$ transition CDF described below, in equation (B.2).

I compute p_1 by finding the values e of ε low enough to push x' below the value x^* implied by point i', using the current policy choice j. It is possible that $e = -\infty$, for example if $x^* < (1 - \theta)\,\$$; agents are guaranteed at least this value of x' by their default option. Solving for e at each policy choice j is complicated by the fact that the overhead-labor constraint in equation (2.6) may or may not bind. I handle this issue by finding the value of ε where the overhead labor constraint binds, and solving for e in each region of x^* space separately.

Fix the current policy $j = (\$, \ell)$. Define

$$\overline{x} \equiv y^*\left(Ae^{\rho z + \sigma\varepsilon^*}, \$\right) - \ell\$,$$

where ε^* is defined in equation (A.1). For $x^* \leq \overline{x}$, the overhead-labor constraint binds, and for $x^* > \overline{x}$, it does not. In what follows, I fix x^* (from next period's state i', i.e. column i' of $\widetilde{P}$) and consider each case in turn. Before I do so, it is important to note that I sample the x^* points not from the uniform $\log x$ grid used everywhere else in the paper, but from the midpoints of that grid, plus an additional point just past the maximum value of that grid (such that the resulting $\log x^*$ grid is uniform). The reason is that the computations below are for a CDF, i.e. probabilities that $\log x$ is less than $\log x^*$. It is important that each $\log x^*$ point be at the right end-point of each interval defined by the $\log x$ grid, as a simple application of the discretization to distributions with a known solutions (an AR(1), for example) shows. Using the same grids for the $\log x$ at which policies and other functions are evaluated, as for the CDF computationm leads to bias.

If $x^* \leq \overline{x}$, then the overhead-labor constraint binds, ε appears only once in the definition of y^* and e is given by

$$e = \frac{1}{\sigma}\left[\frac{\gamma}{\gamma-1}\log\left[x^* - (1-\ell)\,\$\right] - \log A - \rho z - (1-\alpha)\log \underline{L} - \alpha \log \$\right]. \tag{B.3}$$

If instead $x^* > \overline{x}$, then ε appears in the definition of y^* in multiple places, because the labor-capital ratio m depends on it. Nevertheless, there is a unique value of ε that moves to $x' = x^*$; the proof and construction of such an ε is entirely analogous to Proposition 2, with $(1-\theta)\,\$$ replaced by x^*.

The details are as follows: following the algebra in Proposition 2, the value of m that sets $y^* = x^*$ is given by the quadratic equation

$$0 = am^2 + bm + c, \tag{B.4}$$

where

$$\begin{aligned} a &= -\alpha w\left(\frac{x^*}{\$} + \ell\right)\frac{\$}{\$ + w\underline{L}} \\ b &= -\frac{\gamma}{\gamma-1} + \alpha + \frac{\$}{\$ + w\underline{L}}(1-2\alpha)\left(\frac{x^*}{\$} + \ell\right) \\ c &= \frac{1-\alpha}{w}\left[\left(\frac{x^*}{\$} + \ell\right)\frac{\$}{\$ + w\underline{L}} - 1\right]. \end{aligned}$$

Denote the solution to equation (B.4) by m^*. It is straightforward to show that equation (B.4) has a unique solution for m^* if and only if $x^* > (1-\theta)\,\$$. Then e can be calculated from m^* by computing the implied value of A^* using equation (2.10), and converting this to ε using the definition of A^* in equation (2.11).

Putting all three pieces together, $e = -\infty$ if $x^* < (1-\theta)\,\$$. If $x^* > \overline{x}$, I compute e using the solution m^* to equation (B.4). Then, if $\overline{x} > (1-\theta)\,\$$,and x^* lies in it, I compute e from equation (B.3). I repeat this computation for each policy j and each future $\log x$ value in i' to completely fill the $M^2 \times N$ matrix $\vec{e}$, after which $p_1 = \Phi\left\{\vec{e}\right\}$.

The above procedure gives me the matrix $\vec{P}$ of transition probabilities from equation (B.2), which I compute at each state-space point i, and take the max according to equation (B.1). I stack the corresponding values of u^* into an $N \times 1$ vector $\vec{u}^*$, and the corresponding rows of each $\vec{P}$ into the $N \times N$ matrix Π, and then compute a new value vector $\vec{V}$ according to

$$\vec{V} = \left[I - \beta\left(1-\pi\right)\Pi\right]^{-1}\left(\vec{u}^* + \beta\pi\Pi\log\vec{x}\right), \tag{B.5}$$

which approximately solves system (2.8). I then use this new value of $\vec{V}$ in the next iteration, repeat the entire procedure, and iterate until the policy functions converge (at which point the value function will also converge).

To speed up the above procedure, I ignore policy choices with negative consumption or that have ℓ beyond the Laffer peak; that is, leverage points (for each value of $\log\$$) in which the default probability is so high that increasing leverage actually reduces $q\ell$. Rational agents would not choose such leverage values, since they result in less resources today for a higher future liability, than other choices. Wherever possible, I vectorize calculations or compute values once and then distribute them to appropriately-sized vectors or matrices; this speeds up calculation but means that the actual computations differ somewhat from the heuristic description above. Many of these calculations are embarrassingly parallel, and I take advantage of this fact where possible.

Finally, everything described in this section is for a single value of the parameters and a single time period t—equation (B.5) in particular applies only for a steady-state value function. Equilibria for agents with different values of A or θ involve repeating all the steps described here for each agent type. Dynamics imply taking a future value function vector $\vec{V}$ as given, computing the resulting policy functions for period t, and generating a new value function (to be used as period $t-1$'s continutaiton value) as

$$\vec{V}_t = \vec{u}^* + \beta\Pi\left[(1-\pi)\,\vec{V}_{t+1} + \pi\log\vec{x}\right].$$

B.3 Approximating the Size-Productivity Distribution

The model implies an equilibrium distribution across log net wealth $\log x$ and productivity z. To compute aggregste quantities such as total output and capital, or the excess demand for savings, I need to integrate over this distribution. In this section I describe how I represent the endogenous distribution of (x, z) that solves equation (3.11).

As in Appendix B.2, I approximate the equilibrium distribution of $(\log x, z)$ as a vector of values on a grid of point for $\log x$ and z. In fact, I describe most of the work involved in the approximation in Appendix B.2, since the key ingredient is the equilibrium state-transition matrix $\Pi_{i,t}$. Given this matrix, which varies across agents of type i (which indexes their value of A and θ) and over time, the vector $f_{i,t+1}$ of the measure of agents in each state-space point in the grid that approximate equation (3.11) is

$$f_{i,t+1} = \pi h + (1-\pi)\,\Pi_{i,t} f_{i,t}, \tag{B.6}$$

where the vector h contains the measure of agents in each state that enter exogenously each period. I describe the construction of this vector below. I solve equation (B.6) for steady-state values when $f_{i,t+1} = f_{i,t}$, or for a dynamic endogenous distributon whe A or θ are changing over time. Integrals over the endogenous distribution of $(\log x, z)$ are then summations over the $f_{i,t}$ vector.

I compute the vector h (which does not vary across i or t, though this extension is minor) as follows. I approximate the entry measure over $\log x$ as

$$h_1 \equiv t_{150}^{-1} \left(\Phi \left\{ \frac{\log x^*}{\sigma_{x_0}} \right\} \otimes \vec{1}_{10} \right), \tag{B.7}$$

where $\log x^*$ is the 150×1 vector of CDF $\log x$ points, $\Phi\{\cdot\}$ is the standard normal CDF, $\vec{1}_{10}$ is the 10×1 vector of 1s, and the matrix t_{150} is defined just below equation (B.2). The Kronecker product in equation (B.7) expands the 150×1 CDF to the 1500×1 state-space grid (10 points for each z point). The pre-multiplication by t_{150}^{-1} converts the CDF into a pdf (measure).

I compute the ergodic density of z as the unit eigenvector of the P_z transition matrix from the Rouwenhorst (1995) method. I tile this 10×1 vector 150 times to make it 1500×1; denote this vector as h_2. Then the h vector is given by

$$h = h_1 \odot h_2$$

where $\odot$ represents an element-wise product.

APPENDIX C

FORMULAS

This section derives the formulas used to compute aggregates and other moments of interest in both the models of chapter 2 and 3.

C.1 Main Model

In this section I describe how I combine the policy functions computed in Appendix B.2 and the cross-sectional distribution(s) of $(\log x, z)$ computed in Appendix B.3 to approximate aggregate quantities of interest, such as aggregate aggregate output, average leverage, and the Olley-Pakes covariance. Most of the formulas in this section apply to a single type of agent, i.e. one value of A and θ. At the end of the section I describe how I aggregate across agent types.

Given a size-productivity distribution vector f^*, I approximate integrals of functions of $\log x$ and z as f^*-weighted averages of the functions evaluated at each of the 1,500 points in the $(\log x, z)$ grid. Average leverage is then given by

$$E\{\ell\} = \frac{1}{\sum_i (1 - \Phi\{\underline{\varepsilon}_i\}) f_i^*} \sum_i \ell_i (1 - \Phi\{\underline{\varepsilon}_i\}) f_i^* \tag{C.1}$$

where i indexes each $(\log x, z)$ point, ℓ_i is the optimal leverage choice at point i, $\underline{\varepsilon}_i$ is the implied default probability at point i implied by ℓ_i and the investment choice $\$_i$, and $\Phi\{\cdot\}$ is the standard normal CDF. The $1 - \Phi$ terms in equation (C.1) condition the expectation on not defaulting.

To compute average firm age in the model, I need to correct for firm default, the probability of which depends on the current state i. I assume that when an agent defaults, he starts a brand-new firm, and from the point of view of the econometrician this firm has just been born. This will reduce the average age of firms below $\frac{1}{\pi}$. The vector of average age in each state point $\overrightarrow{a}$ satisfies

$$\overrightarrow{a} = (1-\pi)\,\Pi'\left(I - \text{diag}\left\{\overrightarrow{d}\right\}\right)\left(\overrightarrow{a} + \overrightarrow{1}\right) + \left(\pi I + \Pi' \,\text{diag}\left\{\overrightarrow{d}\right\}\right)\overrightarrow{1} \tag{C.2}$$

where $\overrightarrow{d} \equiv \Phi\left\{\overline{\varepsilon}\right\}$ is the default probability in each state, $\Phi\left\{\cdot\right\}$ is the standard normal CDF, and Π is the transition matrix computed in section B.2. The first term on the right-hand side of equation (C.2) handles the firms who do not exit exogenously and do not default. These firms get one year older, and transition to a new state through the matrix Π'. The second term handles the firms who will be viewed as one year-olds: these include a measure π of new entrants, plus agents who have defaulted in a previous state, who transition through the matrix Π' as well. Solving equation (C.2) for $\overrightarrow{a}$ yields

$$\overrightarrow{a} = \left[I - (1-\pi)\,\Pi'\left(I - \text{diag}\left\{\overrightarrow{d}\right\}\right)\right]^{-1}\left(\overrightarrow{1} + \pi\Pi'\overrightarrow{d}\right)$$

so that average age is given by

$$E\left\{a\right\} = \overrightarrow{a}'f^{*}. \tag{C.3}$$

The expressions in equations (C.1) and (C.3) are straightforward to compute from the policy and density vectors. However, most aggregates in the model require an additional integration, because they depend not just on $\log x$ and z but also on the realization of ε, primarily through the dependence of the labor-capital ratio m. To overcome this issue, at each point in the 150×10 integration grid I use Matlab's `integral` numeric integration routine to compute the appropriate integral over ε. In addition, whether or not the $mk \geq \underline{L}$ constraint binds also depends on ε, and this needs to be taken into account in the integrals. When the overhead-labor constraint binds, I compute integrals over ε inc closed-form rather than numerically.

With this in mind, the appropriate formulas for revenue py_i and labor l_i at a given point

$i = (\log x, z)$ with optimal policy $(\$, \ell)$ are given by

$$py_i = \left(e^{\frac{1}{2}\sigma^2\left(\frac{\gamma-1}{\gamma}\right)^2} \left[\Phi\left\{ \overline{\varepsilon} - \sigma\frac{\gamma-1}{\gamma} \right\} - \Phi\left\{ \underline{\varepsilon} - \sigma\frac{\gamma-1}{\gamma} \right\} \right] \left[\left(\frac{\underline{L}}{\$} \right)^{1-\alpha} / \left(1 + w\frac{\underline{L}}{\$} \right) \right]^{\frac{\gamma-1}{\gamma}} \right.$$
$$\left. + \int_{\overline{\varepsilon}}^{\infty} \left[\frac{e^{\sigma\varepsilon} m(\varepsilon)^{1-\alpha}}{1 + wm(\varepsilon)} \right]^{\frac{\gamma-1}{\gamma}} \phi(\varepsilon)\, d\varepsilon \right) \left[Ae^{\rho z}\$ \right]^{\frac{\gamma-1}{\gamma}}$$
$$l_i = \underline{L}\left(\Phi\{\overline{\varepsilon}\} - \Phi\{\underline{\varepsilon}\} \right) + \$ \int_{\overline{\varepsilon}}^{\infty} \frac{m(\varepsilon)}{1 + wm(\varepsilon)} \phi(\varepsilon)\, d\varepsilon$$

where $\$$ is the optimal savings at this particular value of $i = (\log x, z)$, $m(\varepsilon) = m^*\left(Ae^{\rho z + \sigma\varepsilon}, \$\right)$ from Proposition 1 when the labor constraint does not bind, and $\overline{\varepsilon} \equiv \max(\underline{\varepsilon}, \varepsilon^*)$. ε^* is the shock at which the labor constraint binds, given in equation (A.1), so the definition of $\overline{\varepsilon}$ ensures that I include, or do not include, labor-constrained agents as appropriate.

Aggregates revenue and labor are then given by the appropriate summations across i, using the computed density vector f^*:

$$PY = \sum_i py_i f_i^*$$
$$L = \sum_i l_i f_i^*. \tag{C.4}$$

To compute the Olley-Pakes covariance, I compute log revenue labor productivity for plants $(\log x, z, \varepsilon)$, and then integrate this quantity over ε:

$$\log \text{ARPL}_i = (1 - \Phi\{\underline{\varepsilon}\}) \left(\frac{\gamma-1}{\gamma} (\log A + \rho z) - \frac{1}{\gamma} \log(\$ + w\underline{L}) \right)$$
$$+ \sigma\frac{\gamma-1}{\gamma} \left[\phi(\underline{\varepsilon}) - \phi(\overline{\varepsilon}) \right]$$
$$- \left(\Phi\{\overline{\varepsilon}\} - \Phi\{\underline{\varepsilon}\} \right) \left((1-\zeta) \log \frac{\underline{L}}{\$} - \frac{1}{\gamma} \log \left[1 + w\frac{\underline{L}}{\$} \right] \right)$$
$$+ \int_{\overline{\varepsilon}}^{\infty} \left[(1-\zeta) \log m(\varepsilon) - \frac{1}{\gamma} \log\left[1 + wm(\varepsilon)\right] - \sigma\frac{\gamma-1}{\gamma}\varepsilon \right] \phi(\varepsilon)\, d\varepsilon,$$

where the first line reflects the common expected components of revene labor productivity across agents in cell i, the second line reflects the idiosyncratic shock ε in output, and the last two lines integrate over the choice of labor-capital ratio m for agents who are constrained to set $mk = \underline{L}$, and those who are not.

Then the simple-average and weighted-average labor productivities are given by

$$\mathcal{Z} = \sum_i \log \text{ARPL}_i f_i^*$$
$$\mathcal{Z}_{\text{wt}} = \sum_i \log \text{ARPL}_i w_i f_i^*,$$

where

$$w_i \equiv \frac{L_i}{L}$$

is cell i's share of aggregate labor. The Olley-Pakes covariance is then given by

$$\mathcal{C} = \mathcal{Z}_{\text{wt}} - \mathcal{Z}. \tag{C.5}$$

Finally, to compute the dispersion in labor productivity, I compute the second moment of log labor productivity $\hat{z}$ for plants $(\log x, z, \varepsilon)$ as

$$\hat{z}_i^2 = \left[\underbrace{\frac{\gamma - 1}{\gamma} (\log A + \rho z) - \frac{1}{\gamma} \log (\$ + w\underline{L})}_{h} \underbrace{- (1 - \zeta) \log m + \frac{1}{\gamma} \log (1 + wm)}_{g} + \frac{\gamma - 1}{\gamma} \sigma \varepsilon \right]^2 . \tag{C.6}$$

I first integrate equation (C.6) over ε, at each state-space point, and then sum over f^*

to get the implied second moment of log labor productivity:

$$E\left\{\hat{z}\right\} = \sum_i f_i^* \int_{\underline{\varepsilon}_i}^{\infty} \hat{z}_i^2 \phi\left(\varepsilon\right) d\varepsilon$$

Then the dispersion in log labor productivity is given by

$$\sigma_{\log \text{ARPL}} = \sqrt{E\left\{\hat{z}^2\right\} - \mathcal{Z}^2}.$$

I integrate equation (C.6) over ε in two steps: first for $\varepsilon \in (\underline{\varepsilon}, \overline{\varepsilon})$, where the overhead-labor constraint binds. In this case, both the h and g terms in equation (C.6) are independent of ε, since $m = \frac{L}{\$}$, so the integral is given by

$$\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \hat{z}_i^2 \phi\left(\varepsilon\right) d\varepsilon = \left(h+g\right)^2 \left(\Phi\left\{\overline{\varepsilon}\right\} - \Phi\left\{\underline{\varepsilon}\right\}\right) + 2\left(h+g\right) \frac{\gamma-1}{\gamma} \sigma \left[\phi\left(\underline{\varepsilon}\right) - \phi\left(\overline{\varepsilon}\right)\right] + \left(\frac{\gamma-1}{\gamma}\sigma\right)^2 \left[\Phi\left\{\overline{\varepsilon}\right\} - \overline{\varepsilon}\phi\left(\overline{\varepsilon}\right) - \Phi\left\{\underline{\varepsilon}\right\} + \underline{\varepsilon}\phi\left(\underline{\varepsilon}\right)\right],$$

which exploits standard closed-form expressions for truncated integrals over normal distributions.

The second step involves integrating equation (C.6) for $\varepsilon \in (\overline{\varepsilon}, \overline{\infty})$, i.e. where the overhead labor constraint does not bind. This is more complicated, because in this region m, and therefore the expression marked g in equation (C.6), now depends on ε:

$$\int_{\overline{\varepsilon}}^{\infty} \hat{z}_i^2 \phi\left(\varepsilon\right) d\varepsilon = \left(1 - \Phi\left\{\overline{\varepsilon}\right\}\right) h^2 + \left(\frac{\gamma-1}{\gamma}\sigma\right)^2 \left[1 - \Phi\left\{\overline{\varepsilon}\right\} + \overline{\varepsilon}\phi\left(\overline{\varepsilon}\right)\right] + 2\frac{\gamma-1}{\gamma}\sigma h \phi\left(\overline{\varepsilon}\right) + \int_{\overline{\varepsilon}}^{\infty} \left(g\left(\varepsilon\right)^2 + 2\frac{\gamma-1}{\gamma}\sigma\varepsilon g\left(\varepsilon\right)\right) \phi\left(\varepsilon\right) d\varepsilon + 2h \int_{\overline{\varepsilon}}^{\infty} g\left(\varepsilon\right)\phi\left(\varepsilon\right) d\varepsilon,$$

where I approximate the integrals in the second line using Matlab's `integral` function.

The above formulas are valid for a single type of agent, i.e. a single value of A and θ. Adding an additional type of agent only complicates aggregation slightly, as follows. Denote the measure of each type of agent j by μ_j. Then aggregate revenue and labor, and average leverage and age, across all agents are given by

$$\begin{aligned} PY &= \sum_j \mu_j PY_j \\ L &= \sum_j \mu_j L_j \\ E\{\ell\} &= \sum_j \mu_j E\{\ell\}_j \\ E\{a\} &= \sum_j \mu_j E\{a\}_j \end{aligned}$$

where with a slight abuse of notation I have indexed aggregates on the left-hand side of equation (C.4) for type j with a j subscript.

The simple average of the the Olley-Pakes decomposition across types is just a weighted average of the type-specific values,

$$\mathcal{Z} = \sum_j \mu_j \mathcal{Z}_j.$$

The weighted-average term of the Olley-Pakes decomposition can be computed as a weighted averaged across types, where the weights depend on each type's share of aggregate labor:

$$\mathcal{Z}_{\text{wt}} = \sum_j \mu_j \frac{L_j}{L} \mathcal{Z}_{\text{wt},j}.$$

Finally, I aggregate the dispersion in labor productivity by aggregating over the second moment, subtracting the squared first moment, and taking a square root:

$$\sigma_{\log \text{ARPL}} = \sqrt{\sum_j \mu_j \left(\sigma^2_{\log \text{ARPL},j} + \mathcal{Z}_j^2\right) - \mathcal{Z}^2}.$$

In this final paragraph I describe the simulations used in Table 2.8. For each type of firm and at each time period, I draw 5 million times from the 1500 state-space points with probability proportional to the density vector f^*. For each of these draws, I compute leverage and log revenue labor productivity, and assign appropriate values to dummy variables equal to 1 for treatment firms, 1 for post-shock time periods, and the product of the two. I then regress leverage on the dummies and log labor productivity, and report the coefficients in Table 2.8.

C.2 Simpler Model

In this section I derive the formulas for the aggregate variables and the endogenous size-productivity transition density described in section 3.1.4.

C.2.1 Policy Rules and Aggregates

Aggregate capital K_{t+1} and output Y_{t+1} are integrals over the densities $dF_{i,t}$ and the distribution of ε, omitting the agents who default by receiving a low value of ε. The formula for K_{t+1} in equation (3.12) follows immediately, but total output Y_{t+1} is given by

$$\begin{aligned} Y_{t+1} &= \sum_i \mu_i \int_{(\log x, z)} \int_{\underline{\varepsilon}(\widetilde{z},\theta_t)}^{\infty} A e^{\widetilde{z}+\sigma\varepsilon} k(\widetilde{z};\theta_t)\, x \phi(\varepsilon)\, d\varepsilon dF_{i,t}(\log x, z) \\ &= \sum_i \mu_i \int_{(\log x, z)} A e^{\widetilde{z}} k(\widetilde{z};\theta_t)\, x \int_{\underline{\varepsilon}(\widetilde{z},\theta_t)}^{\infty} e^{\sigma\varepsilon} \phi(\varepsilon)\, d\varepsilon dF_{i,t}(\log x, z) \\ &= \sum_i \mu_i \int_{(\log x, z)} A \exp\left\{\widetilde{z} + \tfrac{1}{2}\sigma^2 + \log x\right\} k(\widetilde{z};\theta_t) \left[1 - \Phi\left\{\underline{\varepsilon}(\widetilde{z},\theta_t) - \sigma\right\}\right] dF_{i,t}(\log x, z) \end{aligned}$$

where $\widetilde{z}$ is defined in equation (3.12), the limit of integration over ε in the first line reflects the assumption that defaulting firms do not produce, the function $\underline{\varepsilon}(\widetilde{z},\theta)$ is given by equation (3.4) after plugging in optimal leverage $\ell(\widetilde{z},\theta)$, and the third line applies the identity $\int_a^b e^{\sigma\varepsilon}\phi(\varepsilon)\, d\varepsilon = \exp\left\{\frac{1}{2}\sigma^2\right\}\left[\Phi\{b-\sigma\} - \Phi\{a-\sigma\}\right]$.

To compute the distribution function of leverage conditional on borrowing, I change variables from $\log x$ to $y \equiv \log k = \log k(\widetilde{z};\theta_t) + \log x$. With this change of variables, the distribution function for leverage conditional on borrowing is

$$F_{t+1}^{\ell}\left(\bar{\ell}\right)\propto\int_{-\infty}^{\infty}\int_{\overline{z}(r)}^{\ell^{-1}(\bar{\ell};\theta_t)}\left[1-\Phi\left\{\underline{\varepsilon}\left(\widetilde{z};\theta_t\right)\right\}\right]dF_t\left(y-\log k\left(\widetilde{z};\theta_t\right),z\right),\tag{C.7}$$

where $\overline{z}\left(r\right)$ given in equation (3.10) is the cutoff in forward-looking productivity where agents begin to borrow, and where $\ell^{-1}\left(\bar{\ell};\theta_t\right)$ returns the level of productivity $\widetilde{z}$ such that leverage is $\bar{\ell}$; this function exists because the leverage function is strictly increasing in $\widetilde{z}$ for $\widetilde{z}>\overline{z}\left(r\right)$. The "$\propto$" symbol indicates that I divide the function in equation (C.7) by its value at ∞ so that it ranges from 0 to 1, as a distribution function should. Inverting F^{ℓ} yields the quantile function for leverage, from which I compute its median.

In the rest of this section I derive the formulas for the first and second weighted and unweighted moments of log-productivity, which I need for the two terms in equation (2.2) and the cross-sectional disperion of log productivity. Averaging the log productivity of an agent of type i with idiosyncratic productivity z in equation (3.13) over $\log x$, z, and ε yields

$$\mathcal{Z}_i=\log A+Z+\overline{z}_i+\rho\int_{(\log x,z)}z dF_i\left(\log x,z\right)+\sigma\int_{(\log x,z)}\theta\left(\underline{\varepsilon}\right)dF_i\left(\log x,z\right)\tag{C.8}$$

where $\underline{\varepsilon}$ is defined in equation (3.4), and $\theta\left(\cdot\right)$ is defined as

$$\theta\left(e\right)\equiv\begin{cases}\frac{\phi(e)}{1-\Phi\{e\}} & e>-\infty\\ 0 & \text{otherwise}\end{cases}.$$

The penultimate term in equation (C.8) is zero for all types, because the marginal distribution of idiosyncratic productivity is mean-zero. The last term in equation (C.8) corrects for the bias coming from agents with low draws for ε who default and thus do not produce; it depends on the agent's type i as well as the current value of θ. This means that changes in θ will affect unweighted-average productivity, though this effect is quantitatively small relative to the effect of changes in Z for the parameters given in Table 3.1. Aggregate unweighted log productivity then just averages over the different agent types:

$$\mathcal{Z} \equiv \sum_i \mu_i \mathcal{Z}_i. \tag{C.9}$$

Equation (C.9) gives the first moment of aggregate log productivity: to compute its standard deviation, I need its second moment. Squared log productivity is given by

$$\begin{aligned}\left[\log \frac{y}{k}\right]^2 &= \left[\log A + Z + \rho z + \overline{z}_i + \sigma\varepsilon\right]^2 \\ &= \left[\log A + Z\right]^2 + \rho^2 z^2 + \overline{z}_i^2 + \sigma^2\varepsilon^2 \\ &\quad + 2\left(\log A + Z\right)\rho z + 2\left(\log A + Z\right)\overline{z}_i + 2\left(\log A + Z\right)\sigma\varepsilon + 2\rho z\overline{z}_i + 2\rho z\sigma\varepsilon + 2\overline{z}_i\sigma\varepsilon.\end{aligned}$$

Taking the integral over the distributions of ε, $\overline{z}_i$, and $(\log x, z)$ and simplifying gives

$$\begin{aligned}E\left\{\left[\log \frac{y}{k}\right]^2\right\} &= \left(\log A + Z\right)^2 + \sigma^2 + \sum_i \mu_i \left[\overline{z}_i^2 + 2\left(\log A + Z\right)\overline{z}_i\right] \\ &\quad + \sum_i \mu_i \int_{(\log x, z)} \Big[\rho^2 z^2 + 2\left(\log A + Z\right)\rho z + 2\rho z \overline{z}_i + \\ &\quad\quad \theta\left(\underline{\varepsilon}\right)\left(2\sigma\left(\log A + Z + \rho z + \overline{z}_i\right) + \sigma^2\underline{\varepsilon}\right)\Big] dF_i\left(\log x, z\right).\end{aligned}$$

Then the standard deviation of log productivity is given by

$$\sigma\left\{\mathcal{Z}\right\} = \sqrt{E\left\{\left[\log \frac{y}{k}\right]^2\right\} - \mathcal{Z}^2}.$$

Finally, to compute the model-implied value of the OP covariance term $\mathcal{C}$, I compute weighted-average productivity as

$$\mathcal{Z}_{i,\mathrm{wt}} = \log A + Z + \overline{z}_i + \rho \int_{(\log x, z)} z \frac{k\left(\widetilde{z}; \theta\right) x}{K} dF_i\left(\log x, z\right) + \sigma \int_{(\log x, z)} \theta\left(\underline{\varepsilon}\right) \frac{k\left(\widetilde{z}; \theta\right) x}{K} dF_i\left(\log x, z\right)$$

where K is aggregate capital across all agent types from equation (3.12). Aggregate productivity is then given by

$$\mathcal{Z}_{\mathrm{wt}} = \sum_i \mu_i \mathcal{Z}_{i,\mathrm{wt}}$$

so that the OP covariance $\mathcal{C}$ is again given by equation (C.5).

C.2.2 Endogenous Distribution Approximation

The model implies an equilibrium distribution across log net wealth $\log x$ and productivity z. To compute aggregste quantities such as total output and capital, or the excess demand for savings, I need to integrate over this distribution. In this section I describe how I represent the endogenous distribution of (x, z) and solve for it in steady-state equilibrium, as well as how it varies over time according to the aggregate shocks to Z and θ.

The conditional transition CDF for agents of type i in equation (3.11) is given by

$$\begin{aligned}
&\mathcal{E}_i\left(\frac{x^*}{x}, z^*, z, Z, \theta_t, P_{t+1}\right) \equiv \min\left\{\mathcal{E}_1\left(\frac{x^*}{x}, \rho z + \overline{z}_i + Z_{t+1} + \log P_{t+1}; \theta_t\right), \mathcal{E}_2\left(z^*, z\right)\right\} \\
&\mathcal{E}_1\left(\frac{x^*}{x}, \widetilde{z}, \theta_t\right) \equiv \begin{cases} \frac{1}{\sigma}\left(\log\left[\frac{\frac{x^*}{x} + b(\widetilde{z}, \theta_t)}{k(\widetilde{z}, \theta_t)} - 1\right] - \log A - \widetilde{z}\right) & \text{if } \frac{x^*}{x} > (1-\theta_t)\, k\left(\widetilde{z}, \theta_t\right) \\ -\infty & \text{otherwise} \end{cases} \\
&\mathcal{E}_2\left(z^*, z\right) \equiv \frac{z^* - \rho z}{\sigma},
\end{aligned} \tag{C.10}$$

where $\widetilde{z}$ is defined in equation (3.12) and depends in part on the productivity fixed-effect $\overline{z}_i$. The functions $k(\cdot)$ and $b(\cdot)$ in equation (C.10) are the investment and borrowing decisions from Proposition 4, equation (3.9), expressed as shares of net wealth x. Heuristically, $\mathcal{E}_1$ inverts the policy function for net wealth x and $\mathcal{E}_2$ inverts the law of motion for z, so that $\Phi\{\mathcal{E}\}$ returns the probability of receiving a shock ε sufficiently large to move from $(\log x, z)$ to a value lower than $(\log x^*, z^*)$. Such an ε may not exist, as the default option puts a lower bound on the value of x in the next period. Thus the policy functions are not strictly invertible, though the transition distribution function is still well defined.

To solve for the distribution F and its density $dF = f$ in equation (3.11), I define grids for $\log x$ and z and represent F and f as vectors with values at a discrete number of points. This reduces equation (3.11) to a set of linear equations, as follows: let $\vec{F}$, $\vec{f}$, and $\vec{F}^e$ be vectors containing the values of the CDF, pdf, and exogenous-entry CDF at the N^2 gridpoints (the tensor product of N points for $\log x$ and N points for z), respectively. Then equation (3.11) becomes, for a single grid value $\vec{F}_i$,

$$\begin{aligned}\vec{F}_i &= \Pr\left\{\log x < \log x_i, z < z_i\right\} \\ &= (1-\pi)\sum_{j=1}^{N^2} g_{ij}\left(Z,\theta,P\right)\vec{f}_j + \pi\vec{F}^e_i,\end{aligned} \tag{C.11}$$

where

$$g_{ij}\left(Z,\theta,P\right) \equiv \Phi\left\{\mathcal{E}\left(\frac{x_i}{x_j}, z_i, z_j, Z, \theta, P\right)\right\}$$

is the discretized version of the transition CDF $\Phi\left\{\mathcal{E}\right\}$ from equation (C.10). Stacking all N^2 equations (C.11) yields the system of linear equations

$$\vec{F}_{t+1} = (1-\pi)\, G\left(Z_{t+1}, \theta_t, P_{t+1}\right)\vec{f}_t + \pi\vec{F}^e. \tag{C.12}$$

There are two complications that arise when solving equation (C.12). First, the left-hand side is the CDF, while the right-hand side contains the pdf. To correct for this, define

$$T = t \otimes t,$$

where t is a lower-triangular $N \times N$ matrix of 1's, as the matrix that transforms a vector pdf into a vector CDF. The matrix t transforms a univariate pdf into a CDF by summing across values; T extends this matrix to the two-dimensional case. Then T^{-1} is the matrix that converts a CDF into a pdf.

The second complication when using equation (C.12) is that care must be taken to choose at what points to evaluate functions. When defining the gridpoints, the CDF returns the probability of being less than a given gridpoint; it is evaluated at the right boundary of each interval. However, evaluating the pdf at the right endpoints leads to bias that significantly distorts the solution to equation (3.11), as can be verified using simple examples with known solutions, such as a normal AR(1). To get around this problem, I evaluate all pdfs at the midpoint of each interval; this means losing one point in each grid, so that N^2 CDF points become $(N-1)^2$ pdf points. If the grid is wide enough so that the CDF is effectively zero at all the dropped points, this truncation is computationally negligible. Define

$$\begin{aligned} S &\equiv s \otimes s \\ s &\equiv \begin{bmatrix} \vec{0}_{1,N} \\ I_{N-1} \end{bmatrix}, \end{aligned}$$

so that $S\vec{f}$ effectively chops off the bottom-right series of values of the 2-dimensional grid. Although S is not invertible (it is not square), notice that $SS' = I$. The matrix S chops off the bottom values of the grid, and the matrix S' adds back a row and column of zeroes to make things conformable again. With the two matrices T and S and their inverses in hand, equation (C.12) becomes

$$\begin{aligned} \vec{f}_{t+1} &= ST^{-1}\vec{F} \\ &= (1-\pi)\, ST^{-1}G\left(Z_{t+1}, \theta_t, P_{t+1}\right) S'\vec{f}_t + \pi ST^{-1}\vec{F}^e \end{aligned} \tag{C.13}$$

whose solution for the steady-state density $\vec{f}$ is (recall that the parameter D in equation (3.2) is chosen so that $P = 1$ in steady-state)

$$\vec{f} = \pi \left[I_{N-1} - (1-\pi) ST^{-1} G(0, \theta, 1) S' \right]^{-1} ST^{-1} \vec{F}^{e}. \tag{C.14}$$

The $(N-1)^2$ values in $\vec{f}$ are density values at the midpoints of each square in the grid, not the upper-right corners at which the CDFs G and $\vec{F}$ are evaluated.

I apply equation (C.14) on grids of size $N = 64$ for $\log x$ and z. The marginal distribution of z for incumbent agents is normal with mean 0 and variance $\sigma_z^2 = \frac{\sigma^2}{1-\rho^2}$; I define the z-grid to be the 63 evenly-spaced points on $[-7\sigma_z, 7\sigma_z]$, plus the hurdle point $\overline{z}(r)$, where I convert from $\widetilde{A}$ to z according to $\widetilde{A} \equiv Ae^{\rho z}$. For the $\log X$-grid I choose the 64 evenly-spaced points on $[-12, 12]$, and verify ex-post that this maximum value for $\log x$ is not binding.

To solve for the steady-state $\vec{f}$, I solve equation (C.14) for $\theta = \bar{\theta}$ and an aribtrary value of P, and compute the implied aggregate output Y. I then search over P until I find a value such that $\log P = -\eta \log Y + D$.

APPENDIX D

DATA CONSTRUCTION

Table D.1 lists the 100 four-digit NIC 2008 industries used in the paper, in decreasing order of average (over time) employment share. I compute all aggregates in the paper within each of these industries in each year, and then report the overall aggregate as a weighted average of the industry values, using each industry's employment share at time t as the weights.

All aggregate statistics use the reported sampling weights in both the ASI and NSS; I verify that these aggregates match the publicly-available aggregates for each dataset when possible (for example, for employment, gross value added, number of firms, etc).

I define productivity as annual real gross value added per employee, where value added is gross of depreciation and defined as the ex-factory value of goods produced less cost of material inputs, plus other income (from services, goods sold in same condition as purchased, value of own construction, and rent received for fixed assets) less other expenses (work done by others, repair and maintenance of machinery and equipment, operating expenses, and insurance, interest, and rent expenses) . In the 2006 and 2011 NSS many gross value added figures are reported at monthly (rather than annual) rates. The 2006 NSS reports individual-observation reference periods; I infer reference period in the 2011 NSS by whether the respondent used a reliable book of accounts to answer the survey, as described in the document "Key Results of the Survey on Unincorporated Non-Agricultural Enterprises (Excluding Construction) in India" for the NSS 67th round. The other NSS years report annualized figures for all observations. I verify these data manipulations by comparing aggregate values with the publicly-available aggregates for the NSS.

I deflate value-added using the Indian Whole Price Index (WPI) for each industry. To do so, I manually link the NIC-2008 industries (for the 2011 data) with the NIC-2004 industries (for the 2006 data), the NIC-1998 industries (for the 2000 data), and the NIC-1987 industries (for the 1990 and 1995 data). To make these links I have to merge some NIC-2008 industries together, for example Fertilizers with Agrochemical Products, because they are not separated

Industry	% of Total Employment	Log Labor Productivity	Covariance Term $\mathcal{C}$	NSS % of Employment	NSS % of Output	Fixed Asset % of Total
1311 Weaving & Fiber	10.30	9.72	0.68	77.08	30.07	47.70
1200 Tobacco	9.37	8.24	0.09	88.00	39.38	23.40
1410 Apparel	8.59	9.56	0.26	85.76	57.59	24.47
1061 Grain Milling	6.65	8.94	0.01	91.13	59.25	25.48
1623 Wooden Container	5.15	8.08	-0.03	99.65	96.04	24.89
1399 Other Textiles	4.19	8.79	0.22	98.58	83.33	42.75
1629 Other Wood Products	3.50	8.16	-0.10	99.56	93.28	26.87
3211 Jewelry	3.20	9.73	0.15	95.78	77.65	12.52
1075 Prepared Meals	3.07	8.98	0.46	69.69	25.43	40.83
2393 Other Ceramic Products	3.03	8.54	-0.02	96.47	69.98	46.62
3100 Furniture	2.67	9.49	0.01	97.24	84.58	29.25
3240 Toys & Other Articles	2.52	8.24	0.19	94.60	58.69	28.30
2391 Brick & Ceramic	2.42	9.27	0.30	77.10	47.96	46.11
1622 Carpentry & Joinery	2.04	9.48	-0.01	99.47	97.70	31.19
1072 Sugar	1.66	9.50	0.97	56.80	10.33	32.20
1394 Thread and Rope	1.54	8.33	0.35	91.66	58.33	42.99
2593 Tools and Hswres	1.45	9.32	0.24	89.48	49.29	29.27
2410 Basic Iron & Steel	1.25	11.65	1.74	7.59	1.24	57.36
1073 Candy	1.11	9.62	0.09	96.37	73.38	51.11
1520 Footwear	1.08	9.67	0.33	74.82	43.23	28.52
2511 Str Metal Products	0.97	10.35	0.24	79.35	50.77	24.07
2220 Other Plastic Products	0.95	10.26	0.73	64.95	25.95	41.00
1313 Textile Finishing	0.95	10.06	0.94	62.97	31.07	43.09
2599 Fabr Metal Prod nec	0.92	10.26	0.33	79.82	45.73	34.46
2910 Motor Vehicles	0.90	11.47	1.12	14.42	2.41	44.81
1820 Recorded Media	0.87	9.81	0.28	85.92	55.03	38.91
2023 Soap	0.83	8.81	0.81	79.59	12.47	31.35
1392 Non-Apparel Textile	0.80	9.25	0.20	90.60	54.51	31.24
1050 Dairy Products	0.73	9.57	0.51	75.77	22.30	35.63
2029 Other Chemical Products	0.73	9.31	1.09	50.81	7.92	38.76
1071 Bakery Products	0.69	9.77	0.18	86.28	48.53	39.07
2100 Pharmaceuticals	0.65	11.17	1.51	10.31	1.51	36.53
2396 Stone Finishing	0.59	9.32	0.31	74.18	43.41	41.87
1010 Meat Processing	0.59	9.84	-0.03	97.54	87.25	36.97
1610 Wood Milling	0.54	9.98	0.01	94.40	94.35	18.94
2710 Elc Mtrs & Distrb	0.51	11.26	0.80	30.25	9.23	20.25
2592 Metal Trtmt & Coating	0.50	10.08	0.08	90.84	76.55	39.52

Table D.1. Industry Detail
The table lists the NIC-2008 4-digit manufacturing industries used in the paper in decreasing order of average employment share. The first column reports the average percentage of total employment, across both the NSS and ASI, accounted for by the indicated industry. The second column reports weighted-average log labor productivity. The third column reports the average OP covariance term $\mathcal{C}$. The fourth and fifth columns report the average percentage of employment and output accounted for by NSS firms, respectively. The last column reports the average percentage of assets accounted for by fixed assets. Averages are equal-weighted across the five data years from 1990 to 2011.

Industry	% of Total Employment	Log Labor Productivity	Covariance Term $\mathcal{C}$	NSS % of Employment	NSS % of Output	Fixed Asset % of Total
2819 Machinery nec	0.48	11.11	0.57	42.06	18.87	25.01
1702 Cardboard Products	0.47	9.36	0.66	77.01	42.92	40.20
1040 Food Oil Mfg	0.47	10.12	0.70	56.26	12.47	28.73
1101 Liquor	0.46	9.15	0.46	83.06	14.48	37.37
1104 Non-Alcoholic Beverages	0.40	9.39	0.42	83.01	34.86	50.67
1391 Knitted Goods	0.39	9.72	0.85	62.64	34.86	28.39
2821 Ag & Forest Machinery	0.39	9.82	0.79	74.87	21.95	30.37
2813 Mechanical Equipment	0.39	11.21	0.75	27.34	8.64	29.62
1811 Printing	0.37	10.45	0.36	60.19	37.77	33.52
2420 Non-Ferrous Metals	0.35	11.13	1.50	44.35	4.65	53.12
2310 Glass Products	0.34	9.86	0.64	62.59	16.91	55.14
1512 Luggage	0.33	9.56	0.23	86.27	61.08	20.28
2395 Cement & Plaster Products	0.32	10.08	0.52	68.61	25.01	39.92
2394 Cement & Plaster	0.31	11.31	2.24	26.43	0.97	70.70
1701 Primary Paper Materials	0.28	11.48	1.85	13.16	2.86	61.13
1030 Fruit Processing	0.28	9.26	0.44	75.58	37.84	42.09
2011 Basic Chemical Compounds	0.27	11.79	1.54	13.37	1.13	54.31
1709 Other Paper Products	0.26	9.37	0.47	81.15	35.00	46.77
2219 Other Rubber Products	0.25	10.43	1.08	48.30	22.16	34.48
2432 Non-Fe Casting	0.24	10.71	0.90	22.31	7.59	39.53
2512 Metal Containers	0.24	10.50	0.55	52.84	25.99	26.93
3091 Motorcycles & Parts	0.23	11.45	1.03	19.88	7.64	43.29
2750 Appliances & Elctrncs	0.22	11.25	0.83	48.36	19.76	30.84
1103 Beer	0.21	8.91	0.71	77.39	6.58	41.93

Table D.1. Industry Detail, continued
The table lists the NIC-2008 4-digit manufacturing industries used in the paper in decreasing order of average employment share. The first column reports the average percentage of total employment, across both the NSS and ASI, accounted for by the indicated industry. The second column reports weighted-average log labor productivity. The third column reports the average OP covariance term $\mathcal{C}$. The fourth and fifth columns report the average percentage of employment and output accounted for by NSS firms, respectively. The last column reports the average percentage of assets accounted for by fixed assets. Averages are equal-weighted across the five data years from 1990 to 2011.

Industry	% of Total Employment	Log Labor Productivity	Covariance Term $\mathcal{C}$	NSS % of Employment	NSS % of Output	Fixed Asset % of Total
2012 Fertilizers	0.20	12.12	2.62	9.17	0.28	49.19
2825 Food & Txtl Machinery	0.20	10.97	0.63	34.88	13.06	26.99
2211 Tires & Tubes	0.20	11.50	1.29	22.08	4.45	47.97
2822 Mtl-Frm Machinery	0.18	10.78	0.32	49.12	25.91	30.15
2630 Communications Equipment	0.18	12.13	1.17	18.67	3.81	28.40
2399 Non-Mtl Minl Prod nec	0.17	10.27	1.00	42.77	10.58	36.17
1511 Leather and Related	0.17	9.73	0.76	51.35	14.99	22.17
2513 Engines and Steam Generators	0.17	11.47	1.25	14.23	1.76	18.11
1020 Fish Processing	0.17	9.39	1.00	57.52	18.29	29.64
1621 Plywood Products	0.16	9.54	0.70	50.16	21.38	31.96
3020 Railroad Stock & Parts	0.16	11.17	0.52	9.45	2.76	32.17
1812 Printing nec	0.15	11.04	0.86	36.81	17.35	36.61
3092 Bicycles	0.15	10.51	0.50	39.92	12.19	21.81
2591 Forging	0.14	10.87	0.67	47.89	17.12	38.21
2022 Paint and Caulk	0.14	11.39	1.05	32.99	5.72	30.42
2610 Electronic Components	0.13	11.40	0.90	22.71	6.79	32.20
2740 Elec Lighting	0.13	11.00	0.88	40.64	10.47	45.86
2732 Wires & Cables	0.12	11.38	0.63	18.94	4.61	25.32
2520 Weapons and Ammunition	0.12	10.14	0.37	47.11	29.18	26.12
1920 Other Petroleum Products	0.11	12.52	2.31	13.76	0.33	42.97
2651 Mm & Control Eqp	0.11	11.65	0.77	26.17	12.70	28.60
2790 Elec Eqp nec	0.11	11.10	0.87	40.43	17.96	31.04
2720 Batteries	0.10	10.88	0.98	53.12	9.29	37.17
1910 Coke Products	0.10	10.72	1.24	30.32	3.87	35.71
2013 Plastic and Rubber	0.09	12.73	1.77	15.06	2.24	62.83
1080 Animal Feed	0.09	10.54	0.86	41.49	10.24	27.81
2824 Mining Machinery	0.08	11.23	0.78	25.14	6.56	22.74
3230 Sports Goods	0.07	9.59	0.58	78.54	48.00	18.83
3099 Trspt Eqp nec	0.07	10.30	0.46	66.70	35.23	36.02
2030 Synthetic Fiber	0.06	12.45	2.43	4.15	0.26	66.42
1420 Fur Products	0.06	9.37	0.20	96.19	87.14	32.24
3011 Boats	0.06	11.11	1.15	21.96	6.80	25.41
2817 Office Eqp	0.06	12.07	1.05	27.60	12.80	27.10
1062 Starch Products	0.06	10.54	1.04	21.22	5.83	44.44
2652 Watches & Clocks	0.04	11.53	1.11	27.50	11.58	27.20
2680 Magnetic Media	0.03	10.54	0.30	79.17	29.88	33.81
3220 Musical Instruments	0.03	9.38	0.16	95.78	77.24	23.84
1102 Wine	0.03	9.35	1.60	45.04	1.24	28.64
3030 Aircraft & Spacecraft	0.01	12.13	1.01	10.09	6.21	41.45

Table D.1. Industry Detail, continued
The table lists the NIC-2008 4-digit manufacturing industries used in the paper in decreasing order of average employment share. The first column reports the average percentage of total employment, across both the NSS and ASI, accounted for by the indicated industry. The second column reports weighted-average log labor productivity. The third column reports the average OP covariance term $\mathcal{C}$. The fourth and fifth columns report the average percentage of employment and output accounted for by NSS firms, respectively. The last column reports the average percentage of assets accounted for by fixed assets. Averages are equal-weighted across the five data years from 1990 to 2011.

in some years. I then link these merged industry codes with the manufacutring commodities in the 1982, 1993-1994, and 2004-2005 WPI series. I construct a single price index for each industry by extending the 1993-1994 WPI series before its initial year (1993-1994) and beyond its final year (2009-2010) using the appropriate growth rates from the 1982 and 2004-2005 WPI, respectively, so that real value added in the paper is at constant 1993-1994 prices. The 1982 WPI is monthly, which I convert to the annual frequency by taking the average value of the index over each year (April–March).

I drop from the sample any observations from non-manufacturing industries, including recycling plants and repair shops, and any observation missing industry information. I also drop any observation missing employment information or all components of gross value-added, because I cannot infer productivity for these observations and including them would invalidate the identity in equation (2.2). I drop one firm in the 2006 NSS that reports 8,000 employees; this is almost twenty times as large as the next-largest plant, though his inclusion chiefly affects the value of the standard deviation of log employment in that year (raising it to 6 from 3.0). My final sample consists of 612,907 (unweighted) plant-year observations over the five years 1990, 1995, 2000, 2006, and 2011.

I do not include the 2000-2001 NSS and ASI because the former appears to be based on a different Indian Economic Census, as described on page 18–19 of NSS Report 477 "NSS 56th Round Key Results." The problem with including this year of data is that the 2000–2001 NSS reports 17 million plants and over 37 million employees (compared to 14 million plants the year before *and* five years later), which if included would make for a sharp drop in aggregate productivity and the covariance term in that year, only one year away from the 2000 NSS and ASI. I believe these values reflect differences in the Economic Census sampling frames, as suggested in Report 477, and not drastic changes in real Indian manufacturing plants or employment.