THE UNIVERSITY OF CHICAGO

CONDITIONAL MARKET EXPOSURES OF THE VALUE PREMIUM

A DISSERTATION SUBMITTED TO

THE FACULTY OF THE UNIVERSITY OF CHICAGO

BOOTH SCHOOL OF BUSINESS

IN CANDIDACY FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

BY

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CHICAGO, ILLINOIS

JUNE 2016

To my parents, Shalin Qiao and Hanlian Dai, whose continuous and unconditional support gave me a chance to express my creativity.

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Acknowledgment

I am forever indebted to my advisers Toby Moskowtiz, Ľuboš Pástor, Bryan Kelly, and Pietro Veronesi for their guidance, encouragement, support, and very helpful discussions throughout the development of this dissertation. I am extremely fortunate to have had the opportunity to study under these superb mentors. Gene Fama's rigorous arguments and carefully constructed exposition greatly influenced those of my own, and Blair Hull showed me the value in considering the practitioner's perspective.

I would also like to thank Gurbinder Brar, Ken French, Christopher Grohe, Mark Huang, Michael Katz, Jack Mo, Walter Prahl, Jeff Russell, Jonathan Sage, Leah Traub, Sergiy Vestuk, Ram Yamarthy, Mark Yarger, Hanzhe Zhang, and Vladimir Zdorovtsov for helpful comments.

Special thanks to my parents, Shalin Qiao and Hanlian Dai, who always inspired me with their dedication towards excellence and provided me with constant support over the years. Without their understanding, this dissertation may never have been finished.

I acknowledge financial support from the Katherine Dusak Miller PhD Fellowship and the University of Chicago Booth School of Business. All errors are my own.

Abstract

Value strategies exhibit a large positive beta if contemporaneous market excess returns are positive, and a small beta if contemporaneous market excess returns are negative. Value also has a large positive beta after bear markets, but a small beta after bull markets. These facts hold for equity-value strategies in 21 countries, and to a lesser extent for three non-equity-value strategies. Betas conditional on contemporaneous market returns capture expected-return variation associated with the book-to-market ratio. These betas also partially capture the value premium, and are related to larger cash-flow risks of value strategies.

Key Words: value premium, beta, asymmetry, cash-flow risk, conditional beta

JEL: G10, G11, G12

1 Introduction

The value premium refers to the observation that securities with low prices relative to fundamentals have higher average returns compared to those with relatively high prices (Stattman, 1980; Rosenberg et al., 1985; Fama and French, 1992, 1993), without larger market betas to compensate for the premium. It is one of the most prominent empirical facts in finance due to its prevalence (Fama and French, 1998) and robustness (Asness et al., 2013). However, there is no consensus explanation for value. Many proposed explanations seem plausible, and are typically able to match the unconditional premium of roughly 5% per year for a long-short portfolio. I examine the conditional return distributions of value strategies to shed light on its mechanism and distinguish between different models.

I study conditional market exposures of value strategies and find two patterns. Value strategies exhibit asymmetric betas: a large and positive up-market beta when the contemporaneous market excess returns are positive, and a small or negative down-market beta when the contemporaneous market excess returns are zero or negative. Value strategies also exhibit time-varying betas: after a string of good market returns, value has a small negative bull-market beta. After a string of poor market returns, value has a large positive bear-market beta. Asymmetric betas and time-varying betas are also found for international equity-value strategies, and to a lesser extent, for three non-equity-value strategies.

I examine the asset-pricing implications of asymmetric betas and time-varying betas, and find that time-varying betas are not able to capture average returns in portfolios sorted on the book-to-market ratio. Similar to Petkova and Zhang (2005) and Lettau and Ludvigson (2001), I find a conditional CAPM that takes into account time-varying betas does not resolve the value premium. On the other hand, asymmetric betas do capture expected return variation associated with the book-to-market ratio. GMM and Fama-MacBeth tests show that the up-market beta is significantly priced in book-to-market deciles and 25 portfolios formed on size and book-to-market, as well as portfolios based on asymmetric betas and book-to-market. Removing asymmetric beta exposure from HML reduces its average monthly returns from 0.39% to 0.16%. A strategy that seeks to replicate HML's returns using asymmetric betas and a market-timing model produces an average return of 0.14% per month, which equals 40% of the unconditional value premium.

Good models should take into account the property that asymmetric betas partially capture the value premium. I examine the asymmetric beta implications of two theoretical models, both of which are able to match the 5% unconditional value premium, and find that the Lettau and Wachter (2007) model does not capture asymmetric betas, but the Santos and Veronesi (2010) model does. Through an alternative calibration of Lettau and Wachter (2007), I find cash-flow risk is linked to asymmetric betas. A decomposition of beta into its cash-flow and discount-rate components reveals asymmetric betas mostly come from cash-flow betas, consistent with the idea that value securities have higher cash-flow risk. Of the two models, Santos and Veronesi (2010) contains the cash-flow risk channel and appears to be a more plausible explanation of value.

The paper is organized as follows. Section 2 presents asymmetric betas and time-varying betas and provides some evidence of how they are linked. Section 3 examines asset-pricing implications of asymmetric betas and time-varying betas. Section 4 interprets the asset-pricing results in section 3, and discusses two theoretical models in light of my findings. Section 5 concludes.

2 Conditional Market Exposures of the Value Strategy

A long-short equity-value strategy on average earns 5% per year. Many explanations of value are able to match this unconditional premium and its CAPM beta. To distinguish between competing theories, we must look beyond the unconditional premium. I focus on characterizing the conditional distribution of value, and find HML has asymmetric betas conditional on contemporaneous market returns and time-varying betas conditional on past market returns. Asymmetric and time-varying betas are plausibly linked through market mean-reversion.

2.1 Why Conditional Betas?

Conceptually, any unconditional and conditional moments may be used to distinguish between models of value. I focus on conditional market exposures because the anatomy of value leads to intriguing beta patterns. Value is a cross-sectional contrarian strategy that takes long positions in recent losers and short positions in recent winners. Suppose market returns have been poor. A value strategy takes long positions in stocks with large positive betas that have recently performed poorly, and shorts stocks with small or negative betas that have performed relatively well. The overall market exposure for value is large and positive in such a bear market. In bull markets the reverse is true, and value has a small or negative bull-market beta. Kothari et al. (1995) and Daniel and Moskowitz (2013) have made similar points about investment strategies that depend on past returns.

Market returns have dynamics that translate these bear-market and bull-market betas into asymmetric contemporaneous betas. Low realized returns are associated with high expected returns. After a bear market, value has a large positive beta and expected returns are high. On the other hand, value has a small beta after a bull market when expected returns are low. Taken together, value returns do not appear to be linearly exposed to the same period market returns. I explore this relationship in the next section.

To understand how market-return dynamics affect conditional betas, consider the following model of expected returns $\mu_t = \mathbb{E}_t r_{m,t+1}$, and log dividend growth rate $\triangle d_{t+1}$:

$$\mu_{t+1} = \phi \mu_t + \epsilon_{\mu,t+1} \tag{1}$$

$$\triangle d_{t+1} = \epsilon_{d,t+1} \tag{2}$$

Unobserved expected returns are persistent and mean-reverting ($\phi < 1$), and expected dividend growth is assumed to be constant. This setup is a simplified version of models often found in return-predictability studies[1] to capture the essence of mean-reverting market returns. Applying the Campbell and Shiller (1988a,b) return identity gives us the following observed system:

$$r_{m,t+1} = (1 - \rho\phi)\, dp_t + \epsilon_{r,t+1}, \quad \epsilon_{r,t+1} = \epsilon_{d,t+1} - \frac{\rho}{1 - \rho\phi}\epsilon_{\mu,t+1} \tag{3}$$

$$dp_{t+1} = \phi dp_t + \epsilon_{dp,t+1}, \quad \epsilon_{dp,t+1} = \frac{\epsilon_{\mu,t+1}}{1 - \rho\phi} \tag{4}$$

Figure 1 shows the impulse response functions of the above system. This model has two parameters: ϕ and ρ. At the monthly frequency for 1926 to 2014, I estimate Equations (3) and (4) to find ϕ to be 0.998 and ρ to be 0.7234. I consider three consecutive positive shocks to expected returns of size 0.2, and trace out the responses for r_t, dp_t, and μ_t. The resulting impulse responses resemble what happens in a bear market: the returns process receives multiple negative shocks, but immediately afterward gets a positive boost from the higher expected returns and dividend-price ratio. Alternatively, consecutive unexpected return shocks result in a bear market. Unexpected return shocks are uncorrelated by definition and do not contribute to value's contemporaneous betas when combined with different bear- and bull-market betas.

Value strategies have large positive betas when realized returns are low and expected returns are high, as in periods 2, 3, and 4, and small or negative betas when realized returns are high and expected returns are low. Combining large betas with high expected returns and small betas with low expected returns suggests value is likely to have different betas depending on the same period market returns. With this example in mind, we expect value strategies to have different bear-market and bull-market betas and different up-market and down-market betas. I look for

[1] Stambaugh (1999), Cochrane (2008), Pastor and Stambaugh (2009), Binsbergen et al. (2010) are a few papers that have used a similar setup.

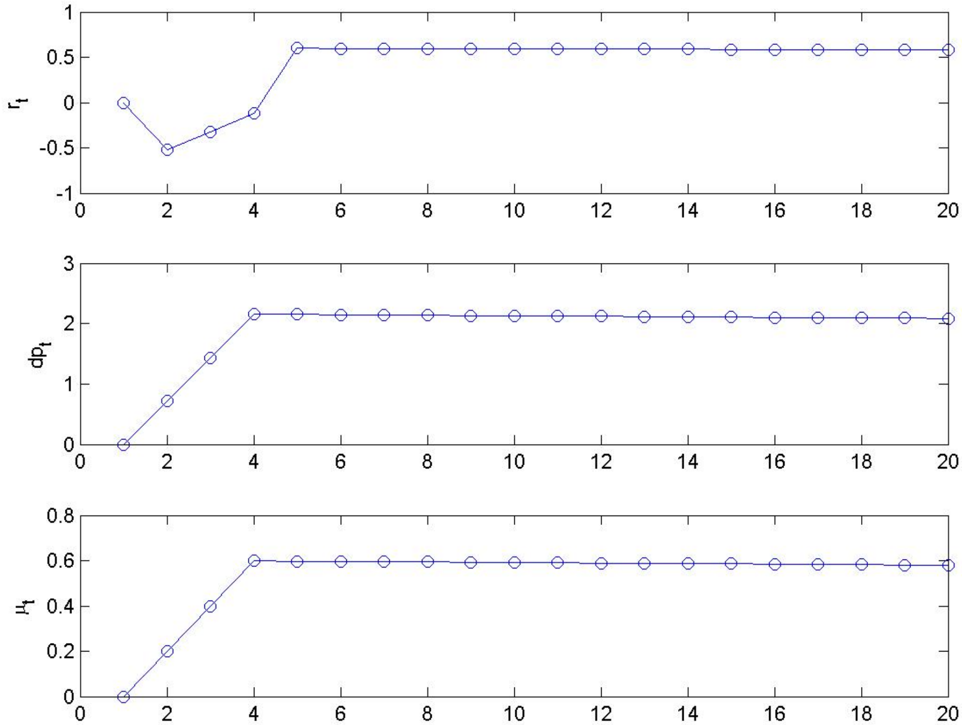

Figure 1: **Impulse Response Functions to Expected Returns Shocks.**
I plot impulse response functions for realized returns r_t, dividend yield dp_t, and expected returns μ_t to several positive expected return shocks in a row. The variables evolve as follows:
$r_{m,t+1} = (1 - \rho\phi)\, dp_t + \epsilon_{r,t+1}$
$dp_{t+1} = \phi dp_t + \epsilon_{dp,t+1}$
$\mu_{t+1} = \phi\mu_t + \epsilon_{\mu,t+1}$
$\epsilon_{\mu,2}$, $\epsilon_{\mu,3}$, and $\epsilon_{\mu,4}$ are set equal 0.2. The pattern in r_t resembles a bear market.

those patterns in the data.

2.2 Data

Monthly returns on the Fama-French value factor HML, momentum factor UMD, market excess returns, the risk-free rate, portfolios sorted on book-to-market, dividend-price ratios, and monthly returns on portfolios sorted by both size and book-to-market ratios are obtained from Kenneth French's website[1] from July 1926 through December 2014. Monthly returns of international value portfolios formed on price ratios as well as the market returns for 21 countries from January 1975

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

through December 2014 are also from French's website.

Daily and monthly individual stock return data and the value-weight market returns from January 1926 to December 2014 are from the Center for Research in Security Prices (CRSP). Balance sheet items required to form the book value of individual stocks are from COMPUSTAT. Monthly returns of value strategies in exchange rates, country bonds, and commodities of Asness et al. (2013) from January 1972 to December 2014 are obtained from Toby Moskowitz's website.[1]

2.3 Asymmetric Betas and Time-Varying Betas

Fama and French (1992, 1993) construct a long-short value strategy, HML, by taking long positions in 30% of all NYSE, AMEX, and NASDAQ securities with the highest book-to-market ratios, and short positions in 30% of the lowest book-to-market securities. I focus on HML for my analysis.

HML has asymmetric betas on the contemporaneous market: the up-market beta is large and positive, and the down-market beta is small and negative. Instead of a market model, I fit a piecewise linear model similar to Henriksson and Merton (1981) to the Fama and French's value factor HML, and find different up-market and down-market betas. The specification is as follows:

$$HML_t = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r^e_{m,t} + \epsilon_t \quad (5)$$

where $r^e_{m,t}$ is the market excess returns at time t, and $I_{up,t} = 1$ if $r^e_{m,t} > 0$ and zero otherwise. The regression fit is also improved compared to the CAPM. Table 1 presents the results.

HML's up-market and down-market betas are economically and statistically different. If market excess returns are negative, HML has a -0.03 beta with a weak t-statistic of -0.83. If market excess returns are positive, HML has a beta of 0.29, which has a large t-statistic. The regression fit for this piecewise linear model is also improved compared to the market model. If we fit a market model, the regression R^2 is 4.70%. R^2 for the piecewise linear fit is considerably higher at 7.80%. Table 1 also presents the piecewise linear regressions of the constituent portfolios H and L for HML.

Different up-market and down-market betas appear to be a characteristic of value stocks. Looking at the value and growth portfolios that make up HML separately, the non-linearity comes

[1] http://faculty.chicagobooth.edu/tobias.moskowitz/research/chronology.html

Table 1: **Conditional Betas of HML and Its Two Constituent Portfolios, July 1926 to December 2014.**
The top panel presents asymmetric betas for HML and its two constituent portfolios, allowing for different betas if the contemporaneous market excess returns are positive or negative. Asymmetric betas are estimated from the following specification:
$HML_t = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r^e_{m,t} + \epsilon_t$,
where $r^e_{m,t}$ is the market excess returns at time t. $I_{up,t} = 1$ if $r^e_{m,t} > 0$ and zero otherwise. The bottom panel presents a market model regression of HML, asymmetric betas, and time-varying betas. The market model is
$HML_t = \alpha + \beta r^e_{m,t} + \epsilon_t$
Time-varying betas are estimated from the following specification:
$HML_t = \alpha + [\beta_{bull} + I_{bear,t-1}\beta_{bear-bull}]r^e_{m,t} + \epsilon_t$,
where $I_{bear,t-1} = 1$ if the cumulative market returns in the past 24 months were negative. Because the bear-market indicator builds in temporal dependence, the second row in the lower panel reports Newey and West (1987) standard errors with 23 lags.

	α	β	β_{down}	$\beta_{up-down}$	β_{bull}	$\beta_{bear-bull}$	R^2
HML	-0.32%		-0.03	0.32			7.80%
	(-2.20)		(-0.83)	(5.94)			
$Value(H)$	-0.13%		1.08	0.34			82.36%
	(-0.93)		(33.48)	(6.64)			
$Growth(L)$	0.22%		1.11	0.00			92.67%
	(2.95)		(64.35)	(0.08)			
HML	0.30%	0.140					4.70%
	(2.84)	(7.19)					
HML	0.40%				-0.069	0.438	16.24%
	(3.22)				(-1.04)	(3.88)	

entirely from the value stocks. The high book-to-market portfolio H has a down-market beta of 1.08 and an up-market beta of 1.42, whereas the low book-to-market portfolio L has the same beta for up and down markets.

Figure 2 shows a scatter plot of HML returns against contemporaneous market excess returns, with the piecewise linear model superimposed. The scatterplot reveals why the piecewise linear model provides a superior fit compared to a market model. When market returns are large and positive, HML tends to also have large and positive returns. When market returns are large and negative, HML returns are moderately negative. This asymmetric exposure to the same period market excess returns is what gives rise to HML's asymmetric betas.

HML also has different market exposures depending on past market returns. Define a bear market as an episode in which the trailing 24-month cumulative market returns have been negative, and define a bull market similarly. I estimate the following equation to find bear- and bull-market betas:

$$r_i^e = \alpha + [\beta_{bull} + I_{bear,t-1}\beta_{bear-bull}]r_{m,t}^e + \epsilon_i \tag{6}$$

where the bear-market indicator $I_{bear,t-1}$ is equal to 1 if the cumulative market returns in the past 24 months were negative, and zero otherwise. The bear-market beta is large and positive, whereas the bull-market beta is small and negative. The bottom panel of Table 1 shows HML has a bear-market beta of 0.369, and a bull-market beta of -0.069. The bear-market beta is several times larger in magnitude than the bull-market beta.

Table 2 shows the asymmetric betas and time-varying beta for the 10 book-to-market sorted portfolios. The variation across portfolios in down-market betas is small, but there is significant variation in the difference between up-market and down-market betas $\beta_{up-down}$. For the right-most column, V-G takes a long position in the extreme Value portfolio, and a short position in the extreme Growth portfolio. It has a down-market beta of 0.173, and a much larger up-market beta of 0.667. The lower panel shows bull-market betas β_{bull} and the difference between bear-market and bull-market betas $\beta_{bear-bull}$. In bull markets, the long-short portfolio V-G has a beta of 0.125, whereas in bear markets, V-G has a beta of 0.769. These results are robust to bear-market definitions, as shown in Appendix F.

Value portfolios have larger conditional betas in bear markets compared to bull markets, and growth portfolios have slightly smaller betas in bear markets relative to bull markets. These patterns suggest that if recent market returns have been poor, value stocks tend to be those with high betas and growth stocks tend to be those with low betas. Because expected returns tend to be high in bear markets, my time-varying betas result is consistent with the finding in Petkova and Zhang (2005) that value stocks have larger betas in states of the world with higher expected market-risk premium. The time variation in betas for value strategies have also been documented by Lettau and Ludvigson (2001) and Jagannathan and Wang (1996) using macroeconomic variables to capture changing betas.

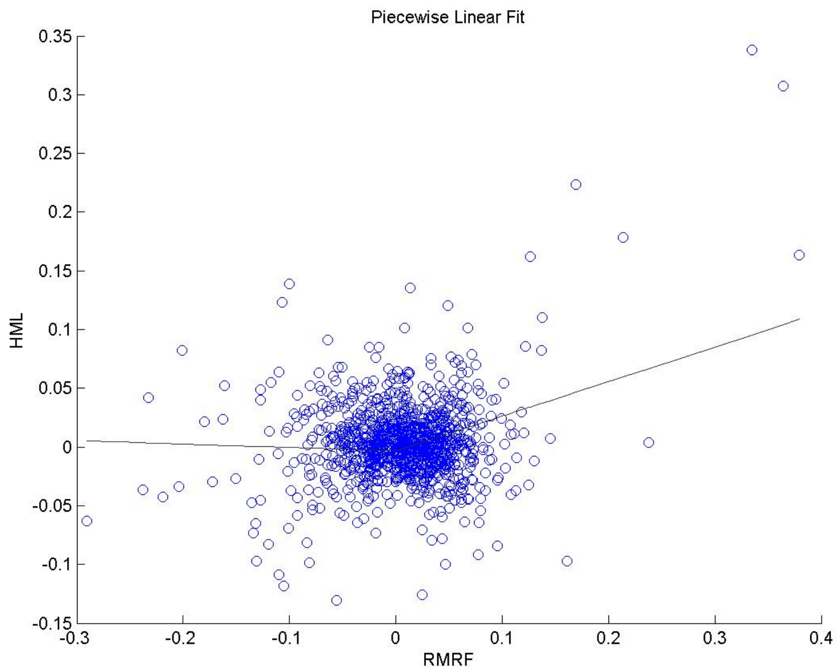

Figure 2: **Scatter Plot of HML against Market Excess Returns (RMRF), July 1926 to December 2014.**
Monthly returns on Fama and French's long-short value portfolio HML are plotted against the same-period market excess returns over the one-month T-Bill. I overlay a piecewise linear fit allowing for different up-market and down-market betas depending on the same period market excess returns:
$HML_t = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r^e_{m,t} + \epsilon_t$
where $I_{up,t} = 1$ if $r^e_{m,t} > 0$ and zero otherwise.

Table 2: **Asymmetric Betas and Time-Varying Betas in 10 Book-to-Market Portfolios, July 1926 to December 2014.**
This table presents up-market and down-market betas, and bull-market and bear-market betas for 10 portfolios formed on the book-to-market ratio. Asymmetric betas are estimated as
$r_i^e = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r_{m,t}^e + \epsilon_i$
where $I_{up,t} = 1$ if $r_{m,t}^e > 0$ and zero otherwise. Time-varying betas are estimated as
$r_i^e = \alpha + [\beta_{bull} + I_{bear,t-1}\beta_{bear-bull}]r_{m,t}^e + \epsilon_i$
where the bear-market indicator $I_{bear,t-1}$ is equal to 1 if the cumulative market returns in the past 24 months were negative, and zero otherwise. If the past 24-month market returns have been positive, the bull-market beta is β_{bull}. If the past 24-month market returns have been negative, the bear-market beta is $\beta_{bull} + \beta_{bear-bull}$. Growth is the portfolio containing the lowest book-to-market stocks; Value is the portfolio containing stocks with the highest book-to-market ratios. V-G is the long-short portfolio formed by taking the difference between Value and Growth portfolios. Newey and West (1987) standard errors with 23 lags are shown for time-varying betas in the bottom panel.

	Growth	2	3	4	5	6	7	8	9	Value	V-G
α	-0.08%	0.26%	0.03%	-0.32%	-0.20%	-0.27%	-0.46%	-0.48%	-0.61%	-0.80%	-0.73%
	(-0.95)	(3.84)	(0.52)	(-3.81)	(-2.34)	(-2.70)	(-4.11)	(-3.96)	(-4.03)	(-3.78)	(-2.80)
β_{down}	1.017	1.009	0.967	0.976	0.932	0.968	0.953	0.959	1.051	1.189	0.173
	(55.41)	(64.53)	(63.16)	(51.00)	(47.65)	(42.52)	(36.71)	(34.13)	(29.87)	(24.22)	(2.87)
$\beta_{up-down}$	-0.003	-0.105	0.006	0.157	0.139	0.173	0.265	0.340	0.420	0.491	0.494
	(-0.10)	(-4.21)	(0.23)	(5.15)	(4.44)	(4.76)	(6.40)	(7.59)	(7.49)	(6.27)	(5.14)
α	-0.12%	0.04%	0.00	0.03%	0.12%	0.13%	0.12%	0.27%	0.32%	0.27%	0.39%
	(-1.84)	(0.91)	(0.17)	(0.43)	(1.55)	(1.80)	(1.36)	(2.73)	(2.48)	(1.57)	(1.86)
β_{bull}	1.054	1.011	0.998	0.979	0.935	0.940	0.946	0.952	1.034	1.179	0.125
	(42.17)	(51.10)	(46.41)	(36.15)	(31.97)	(27.57)	(19.43)	(17.74)	(15.29)	(15.40)	(1.29)
$\beta_{bear-bull}$	-0.087	-0.117	-0.060	0.171	0.148	0.252	0.303	0.388	0.489	0.557	0.644
	(-2.87)	(-2.01)	(-2.21)	(2.01)	(2.53)	(2.27)	(3.19)	(3.98)	(3.87)	(4.33)	(4.36)

2.4 The Relationship between Asymmetric Betas and Time-Varying Betas

In this section, I show unconditional betas, asymmetric betas, and time-varying betas can be related through a simple model of market mean-reversion: mean-reverting market returns combined with bear- and bull-market betas are able to generate the unconditional betas and asymmetric betas in the data. Consider the following model for market returns $r_{m,t+1}$ and a strategy r_{t+1}^{Value} that has distinct bear- and bull-market betas:

$$r_{m,t+1} = \mu_{t+1} + \epsilon_{t+1} \tag{7}$$

$$r_{t+1}^{Value} = \beta_{t+1} r_{m,t+1} + \epsilon_{t+1}^{Value} \tag{8}$$

$$\begin{aligned} \beta_{t+1} &= \beta_{bear}, \quad if \quad I_{bear,t} = 1 \\ &= \beta_{bull}, \quad if \quad I_{bear,t} = 0 \end{aligned} \tag{9}$$

The "value" strategy has β_{bear} after bear markets, and β_{bull} after bull markets. Expected returns follow an AR(1) process as in the motivating example in Section 2.1:

$$\mu_{t+1} = \phi \mu_t + \epsilon_{\mu,t+1}, \quad \epsilon_{\mu,t+1} \sim N(0, \sigma_\mu^2) \tag{10}$$

$$\triangle d_{t+1} = \epsilon_{d,t+1} \tag{11}$$

$$r_{m,t+1} = (1 - \rho\phi)\, dp_t + \epsilon_{r,t+1} \tag{12}$$

$$dp_{t+1} = \phi dp_t + \epsilon_{dp,t+1}, \quad \epsilon_{dp,t+1} = \frac{\epsilon_{\mu,t+1}}{1 - \rho\phi} \tag{13}$$

We can investigate unconditional beta and asymmetric betas in the context of this model.

Proposition 1. The unconditional beta of a value strategy with β_{bear} and β_{bull} is given by $\beta = \beta_{bear} Pr(I_{bear,t} = 1) + \beta_{bull} Pr(I_{bear,t} = 0)$, where $Pr(I_{bear,t} = 1)$ is the unconditional probability of a bear market and $Pr(I_{bear,t} = 0)$ is the unconditional probability of a bull market.

Proposition 1 shows that given β_{bear} and β_{bull}, the fraction of bear markets pins down the unconditional beta.

Proposition 2. β_{bear}, β_{bull}, and the strength of return predictability determine the up-market and down-market betas:

$$\beta_{up,t+1} = \beta_{bear}\frac{Pr(r_{m,t+1} > 0|I_{bear,t} = 1)Pr(I_{bear,t} = 1)}{Pr(r_{m,t+1} > 0)} + \beta_{bull}\frac{Pr(r_{m,t+1} > 0|I_{bear,t} = 0)Pr(I_{bear,t} = 0)}{Pr(r_{m,t+1} > 0)} \tag{14}$$

$$\beta_{down,t+1} = \beta_{bear}\frac{Pr(r_{m,t+1} \leq 0|I_{bear,t} = 1)Pr(I_{bear,t} = 1)}{Pr(r_{m,t+1} \leq 0)} + \beta_{bull}\frac{Pr(r_{m,t+1} \leq 0|I_{bear,t} = 0)Pr(I_{bear,t} = 0)}{Pr(r_{m,t+1} \leq 0)} \tag{15}$$

where $Pr(r_{m,t+1} > 0|I_{bear,t} = 1)$, $Pr(r_{m,t+1} > 0|I_{bear,t} = 0)$, $Pr(r_{m,t+1} \leq 0|I_{bear,t} = 1)$, and $Pr(r_{m,t+1} \leq 0|I_{bear,t} = 0)$ depend on the strength of return predictability.

Proposition 2 shows up-market and down-market betas can be written as weighted averages of bear-market and bull-market betas. Given β_{bear} and β_{bull}, up-market and down-market betas depend on the strength of return predictability through the conditional probabilities in Equations (14) and (15). In a bear market, market prices become depressed and the dividend-price ratio increases. A higher dividend-price ratio is associated with higher expected returns, which increases the probability of a positive market return next month. The logic of the above thought experiment can be summarized as follows:

$$I_{bear,t} = 1 \implies dp_t \uparrow \implies \mu_t \uparrow \implies Pr(r_{m,t+1} > 0) \uparrow$$

Since $\mu_t = (1 - \rho\phi)dp_t$, for the same change in the dividend-price ratio, a larger predictive coefficient $(1 - \rho\phi)$ leads to a larger increase in expected returns μ_t and an increased probability for a positive market return next period, holding volatility fixed. Proposition 2 shows that holding β_{bear} and β_{bull} constant, a stronger predictive coefficient on the dividend-price ratio would lead to an up-market beta closer in value to β_{bear}, and a down-market beta closer in value to β_{bull}. Empirically, β_{bear} is large and positive, and β_{bull} is small. Therefore, stronger return predictability leads to a large and positive up-market beta, and a small down-market beta.

Propositions 1 and 2 qualitatively link asymmetric betas with time-varying bear- and bull-market betas through mean-reverting market returns. Is this effect quantitatively strong enough to produce the patterns we see in the data? To answer this question, I simulate the system above, setting β_{bear} and β_{bull} equal to the estimated values for HML. Converting annual values in Cochrane (2011) to monthly values, $Var(\mu) = 0.01576$, which implies $\sigma_\mu = \sqrt{(1-\phi^2)Var(\mu)} = 0.001$. For the aggregate dividend volatility, I use the NIPA consumption growth volatility of 2% per year, which translates to a monthly volatility of 0.167%. At the monthly frequency, I estimate ϕ to be 0.998, in line with the annual value used in Cochrane (2011). I estimate the forecasting regression in Equation (12), and use the estimate for $(1-\rho\phi)$ along with ϕ to back out ρ.

For 1926 to 2014, $\hat{\beta}_{bull} = -0.069$ and $\hat{\beta}_{bear} = 0.369$, and the predictive coefficient on the dividend-price ratio for next-month returns is 0.278. I simulate the model 2,000 times using these parameters, each with 1000 months, and report the median values of unconditional betas and asymmetric betas in the top panel of Table 3. The median value for the unconditional beta in the simulation is 0.190. Its estimate in the data (see Table 1) is 0.140. Looking at the simulated distribution of unconditional betas, a value of 0.140 would have a p-value of 0.262, which suggests this model is able to capture the unconditional beta. The median down-market beta and up-market beta are -0.021 and 0.262, respectively. The estimated values for those in the data are -0.028 and 0.297 and have p-values 0.424 and 0.453 in the simulations. The simulated distribution of unconditional betas and asymmetric betas are shown in Appendix B.

Past research has shown HML has had different market betas over time (Ang and Chen, 2007). I break the full sample into two equal halves and examine asymmetric betas and time-varying betas in Table 4. Time-varying betas are strong in both subsamples. In the second half, the bull-market beta is large and negative. The point estimates of asymmetric betas were strong prior to 1970, but weak after 1970. I show through simulations that a combination of smaller bear-market beta and weak predictability accounts for the weak asymmetric betas after 1970.

I investigate if time-varying betas combined with return predictability generate asymmetric betas in the two subsamples. As for the full sample, I take bear- and bull-market betas, and the monthly predictive coefficient on DP as given for each of the subsamples, simulate my model, and check for model implications for unconditional and asymmetric betas. The results are presented in the bottom panel of Table 3. In the pre-1970 sample, both bear- and bull-market betas are positive, and predictability was strong. $\beta_{bull} = 0.106$, $\beta_{bear} = 0.531$, and the predictive coefficient on the dividend-price ratio is 0.478. Using these parameters, the median simulated value for unconditional betas is 0.343, and the data estimate of 0.329 lies in the middle of the simulated

Table 3: **Simulation of Asymmetric Betas.**
I simulate market returns and a strategy with bear-market and bull-market betas set equal to those of HML. The simulated model is the following:

$r_{m,t+1} = \mu_{t+1} + \epsilon_{t+1}$
$r^{Value}_{t+1} = \beta_{t+1} r_{m,t+1} + \epsilon^{Value}_{t+1}$
$\beta_{t+1} = \beta_{bear}, \quad if \quad I_{bear,t} = 1$
$= \beta_{bull}, \quad if \quad I_{bear,t} = 0$
$\mu_{t+1} = \phi\mu_t + \epsilon_{\mu,t+1}, \quad \epsilon_{\mu,t+1} \sim N(0, \sigma^2_\mu)$
$\triangle d_{t+1} = \epsilon_{d,t+1}$
$r_{m,t+1} = (1 - \rho\phi)\, dp_t + \epsilon_{r,t+1}$
$dp_{t+1} = \phi dp_t + \epsilon_{dp,t+1}, \quad \epsilon_{dp,t+1} = \frac{\epsilon_{\mu,t+1}}{1-\rho\phi}$

Given the simulated market and strategy returns, I estimate market models and asymmetric betas from the simulated data:

$r^{Value}_{t+1} = \alpha + \beta r_{m,t} + \epsilon_t$
$r^{Value}_{t+1} = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}] r_{m,t} + \epsilon_t$

I report the median values from 2,000 simulations of 1,000 time periods each. I take the distribution of parameters in the 2000 simulations, compute p-values for the estimated values from data in the simulated distribution, and report them underneath the median simulated values.

	1926 – 2014	
	(1)	(2)
β	0.190	
P-Value	0.262	
β_{down}		-0.021
P-Value		0.424
$\beta_{up-down}$		0.283
P-Value		0.453
DP Coeff	0.278	

	1926 – 1970		1971 – 2014	
	(1)	(2)	(3)	(4)
β	0.343		-0.141	
P-Value	0.396		0.085	
β_{down}		0.155		-0.186
P-Value		0.262		0.259
$\beta_{up-down}$		0.283		0.105
P-Value		0.104		0.016
DP Coeff	0.478		0.197	

Table 4: **Subsamples of HML's Conditional Market Exposures.**
I split the full sample of 1926-2014 into two equal halves and estimate asymmetric betas and time-varying betas on each half. The first half goes from July 1926 to December 1970; the second half goes from January 1971 to December 2014. The regressions are the following:
$HML_t = \alpha + \beta r^e_{m,t} + \epsilon_t$
$HML_t = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r^e_{m,t} + \epsilon_t$
$HML_t = \alpha + [\beta_{bull} + I_{bear,t-1}\beta_{bear-bull}]r^e_{m,t} + \epsilon_t$
I also include the predictive coefficient for the one-month-ahead returns from the dividend-price ratio for the two subsamples.

	1926-1970			1971-2014		
	(1)	(2)	(3)	(4)	(5)	(6)
α	0.18%	-0.73%	0.35%	0.46%	0.66%	0.40%
	(1.21)	(-3.75)	(2.42)	(3.74)	(3.40)	(2.56)
β	0.329			-0.203		
	(13.65)			(-7.52)		
β_{down}		0.094			-0.149	
		(2.29)			(-3.03)	
β_{bull}			0.106			-0.293
			(1.75)			(-4.32)
$\beta_{up-down}$		0.436			-0.112	
		(6.88)			(-1.33)	
$\beta_{bear-bull}$			0.425			0.404
			(6.47)			(3.08)
DP Coeff	0.478			0.197		

distribution as indicated by the p-value. The median value for simulated down-market beta is 0.155, and 0.438 for the up-market beta. The estimates from the data again fall within the 95% confidence interval.

In the post-1970 sample, $\beta_{bull} = -0.293$ is large and negative, $\beta_{bear} = 0.111$, and the predictive coefficient on the dividend-price ratio is relatively weaker at 0.197. The model is able to capture the unconditional and down-market betas, but not the up-market beta. The median value for simulated unconditional beta is -0.141, and the data estimate of -0.203 has a p-value of 8.5% in the simulations. The median value for simulated down-market beta is -0.186 which has a p-value of 25.9%. The model predicts the difference between up-market and down-market betas is 0.105, whereas the estimated value is -0.112. The 1.6% p-value indicates -0.112 is in the tail of the simulated distribution.

The simulation provides evidence of a link between time-varying betas and asymmetric betas. The simple specification of AR(1) expected returns with a constant dividend growth rate, combined with exogenous bear- and bull-market betas, qualitatively and quantitatively captures asymmetric betas. Taking the estimated values of time-varying betas and predictive coefficients as given, this model matches eight of the nine parameters in Table 3, although it does have some difficulty matching the up-market beta in the post-1970 sample.

2.5 International Evidence

Asymmetric betas and time-varying betas are not limited to the U.S. equities market; international equities exhibit similar patterns. For 21 countries,[1] equity-value strategies based on the book-to-market ratio exhibit asymmetric and time-varying betas. Because data are available only starting in 1975, and some series start even later, I run pooled regressions to increase statistical power. The results are shown in Table 5.

The top panel shows results using all available countries. Column (1) contains a pooled cross-sectional time-series regressions with no controls or fixed effects. The down-market beta is -0.013 and the up-market beta is a magnitude larger at 0.107. In column (2), with country fixed effects, the coefficients are estimated purely from time-series variation within countries and are weighted

[1] Austria, Australia, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Malaysia, Netherlands, New Zealand, Norway, Singapore, Spain, Sweden, Switzerland, and United Kingdom.

Table 5: **Asymmetric and Time-Varying Betas of International BE/ME Strategies, January 1975 to December 2014.**

I estimate up-market betas ($\beta_{down} + \beta_{up-down}$), down-market betas ($\beta_{down}$), bear-market betas ($\beta_{bull} + \beta_{bear-bull}$), and bull-market betas (β_{bull}) for international value stratgies based on the book-to-market ratio:

$Value_{i,t} = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r_{i,t}^{m,e} + \epsilon_t$

$Value_{i,t} = \alpha + [\beta_{bull} + I_{bear,t-1}\beta_{bear-bull}]r_{i,t}^{m,e} + \nu_t$

where $Value_{i,t}$ and $r_{i,t}^{m,e}$ are the value returns and market excess returns for country i. The coefficient estimates are pooled, with and without country fixed effects. Standard errors are clustered by time.

	All Countries			
	(1)	(2)	(3)	(4)
β_{down}	-0.013	-0.013		
	(-0.49)	(-0.48)		
β_{bull}			-0.062	-0.062
			(-2.74)	(-2.72)
$\beta_{up-down}$	0.120	0.119		
	(2.15)	(2.19)		
$\beta_{bear-bull}$			0.318	0.318
			(5.69)	(5.71)
FE	None	Country	None	Country

	Countries with Data Available in 1975			
	(1)	(2)	(3)	(4)
β_{down}	0.020	0.022		
	(0.75)	(0.83)		
β_{bull}			-0.010	-0.009
			(-0.48)	(-0.45)
$\beta_{up-down}$	0.128	0.126		
	(2.18)	(2.21)		
$\beta_{bear-bull}$			0.291	0.290
			(5.41)	(5.46)
FE	None	Country	None	Country

averages of individual country time-series regression coefficients. The results for country fixed effects are unchanged compared to those without fixed effects, indicating the variation entirely comes from the time series dimension. The size of up-market betas in column (2) is smaller compared to that of the United States (0.29), but the up-market beta is still a magnitude larger than the down-market beta, just like for the United States. Columns (3) and (4) show the bear-market beta is positive and large, and the bull-market beta is small and negative, again just like the case for the United States.

Of the 21 countries, 14 of them have data available since 1975, and seven have data that start later. The bottom panel of Table 5 shows results for 14 countries that have data available since 1975, and exclude those that start later. the results are similar to those in the top panel: the up-market beta is statistically significant and a magnitude larger compared to the down-market betas. The bear-market beta is large and positive; the bull-market beta is small and negative. In country-specific time-series regressions, value strategies in five of the 21 countries have positive and statistically significant up-market betas, and none have large negative up-market betas. Eight of the 21 countries have large positive bear- market betas that are a magnitude larger than their bull-market betas.

2.6 Commodities, Country Bonds, and Exchange Rates

I examine value strategies in non-equity assets and find they also exhibit asymmetric betas and time-varying betas, although the patterns are not as strong as for the equity-value strategies. Asness et al. (2013) investigate the behavior of value in several asset classes in addition to equities. These include currency exchange rates, country bonds, and commodities. I use their long-short value portfolios, along with the appropriate market returns, to look for asymmetric betas and time-varying betas. Table 6 presents the results.

Columns (1) and (2) show that with or without asset fixed effects, the down-market beta is large and negative, and the up-market beta is less negative. We still see asymmetry in the sense that the contemporaneous market exposure is convex. The large and negative down-market beta may be related to the large negative bull-market betas in columns (3) and (4). The bull-market betas are still large and positive.

Table 6: **Asymmetric Betas and Time-Varying Betas of Non-Equity Value Strategies in Commodities, Country Bonds, and Exchange Rates from January 1972 to December 2014.**
I estimate up-market betas ($\beta_{down} + \beta_{up-down}$), down-market betas ($\beta_{down}$), bear-market betas ($\beta_{bull} + \beta_{bear-bull}$), and bull-market betas (β_{bull}) for long-short value strategies in exchange rates, country bonds, and commodities in Asness et al. (2013):
$Value_{i,t} = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r_{i,t}^{m,e} + \epsilon_t$
$Value_{i,t} = \alpha + [\beta_{bull} + I_{bear,t-1}\beta_{bear-bull}]r_{i,t}^{m,e} + \nu_t$
where $Value_{i,t}$ and $r_{i,t}^{m,e}$ are the value and market excess returns for asset i = commodity, country bonds, and exchange rates. The coefficient estimates are pooled, with and without country fixed effects. Standard errors are clustered by time.

	Non-Equity Value Strategies			
	(1)	(2)	(3)	(4)
β_{down}	-0.316	-0.289		
	(-3.18)	(-2.72)		
β_{bull}			-0.359	-0.360
			(-4.77)	(-4.77)
$\beta_{up-down}$	0.167	0.117		
	(1.19)	(0.77)		
$\beta_{bear-bull}$			0.538	0.540
			(4.02)	(4.03)
FE	None	Asset	None	Asset

Asymmetric betas and time-varying betas for non-equity-value strategies show qualitatively the same patterns as for equity-value strategies. The main quantitative difference is that the down-market and bull-market betas are large and negative for non-equity assets. This difference may be due to the different definitions of value used in these markets. Unlike equities, fundamental values are not well-defined for these assets. In lieu of a standard definition of value, Asness et al. (2013) use the negative of the five-year return for commodities and exchange rates, and the yield on the MSCI 10-year government bond yield minus the inflation forecast for country bonds. These different definitions make direct comparisons to equity-value strategies difficult. However, it is encouraging to see that despite the different definitions, we do still see similar conditional betas for these assets compared to equities.

3 Asset Pricing Implications of Conditional Betas

Although asymmetric and time-varying betas illustrate intriguing conditional distributions of value strategies, a key question remains: do these conditional market exposures help explain the value premium? To answer this question, I examine the asset pricing implications of time-varying betas and asymmetric betas.

3.1 Time-Varying Betas

A conditional CAPM using the bear-market indicator as the conditioning variable is not able to resolve the value premium. In earlier sections when I estimated different bear- and bull-market betas, I used a bear-market indicator that was known one period in advance. We could interpret this time-series regression as a conditional CAPM that uses the bear-market indicator as the conditioning variable. The lower panel of Table 1 presents the comparison between an unconditional CAPM and this conditional CAPM. The unconditional CAPM has a monthly alpha of 0.30%, whereas the conditional CAPM allowing for different bear- and bull-market betas has an even larger monthly alpha of 0.40%.

Several other papers have found conditional CAPMs are not able to explain the value premium. Petkova and Zhang (2005) use dividend yield, default spread, term spread, and the short-term Treasury rate as conditioning variables, and show conditional betas go in the right direction but cannot quantitatively explain the value premium. Lettau and Ludvigson (2001) use a cointegrating residual between log consumption, asset wealth, and labor income (*cay*) in a conditional CAPM, and find the value premium is virtually unchanged. Of course, the performance of conditional CAPMs depends on the conditioning variables. I use a bear-market indicator because it is a simple way to capture large time variation in conditional betas.

3.2 Asymmetric Betas

Several papers (Petkova and Zhang, 2005; Lettau and Ludvigson, 2001; Jagannathan and Wang, 1996) have tried to explain the value premium using conditional CAPMs, but relatively little work exists relating the value premium to asymmetric betas. Ang et al. (2006) and Lettau et al. (2014) are two papers relating the value premium to asymmetric betas, but they both focus on downside betas. I find up-market betas are significantly priced in portfolios sorted on the book-to-market ratio, and partially captures the value premium.

3.2.1 GMM Tests

I examine the pricing of asymmetric exposures to the market in portfolios sorted on the book-to-market ratio through GMM tests of linear-factor models in the following form:

$$M_t = a - \delta_m RMRF_t - \delta_{up} RMRF_t I_{RMRF_t>0} - \delta_{SMB} SMB_t - \delta_{HML} HML_t - \delta_{RMW} RMW_t - \delta_{CMA} CMA_t \quad (16)$$

Cochrane (2009) provides extensive discussion of GMM tests of asset pricing models. I follow the approach in Cochrane (2009) and estimate SDF loadings using efficient GMM.

The stochastic discount factor M_t is a linear function of long-short portfolios. RMRF is market excess returns. SMB and HML are the size and value factors of Fama and French (1992, 1993). RMW and CMA are profitability and investment factors of Fama and French (2015). $I_{RMRF_t>0}$ is equal to 1 if $RMRF_t > 0$. I choose the above factors because they have been demonstrated to capture cross-sectional variation in average returns, and the SDF specification nests the CAPM, the Fama and French (1992, 1993) three-factor model, and the Fama and French (2015) five-factor model. The novel element here comes from the up-market beta, $RMRF_t I_{RMRF_t>0}$, and the goal of the exercise is to see if this quantity is priced in portfolios formed on the book-to-market ratio.

If up-market exposure is important for pricing, we would expect to see a significant estimate of δ_{up}. For excess returns $r^e_{i,t}$, the expected discounted value should be zero. The moment conditions are

$$\mathbb{E}[M_t r^e_{i,t}] = 0 \quad (17)$$

My test assets are 25 size-value portfolios, 10 book-to-market portfolios, and 25 portfolios formed on up-market betas and BE/ME. Using excess returns only allows the parameters to be identified up to a scale. Therefore, I set $a = 1$ and estimate the remaining parameters using two-stage GMM. I report Newey and West (1987) standard errors, with the number of lags set to the cube root of the number of observations. The results for 25 size and value portfolios and BE/ME deciles are in Table 7.

The top panel of Table 7 contains estimates of SDF loadings for 25 size and value portfolios. Each row is a subset specification of the full model including all five Fama and French (2015) factors and the up-market beta. Row (2) shows that on its own, δ_{up} gets a significant and positive price of risk. Row (3) allows for both market returns and its up-market portion. δ_m is negative, whereas δ_{up} gets a positive price of risk. Rows (4) and (5) compare the Fama and French (1992, 1993) model with and without the up-market beta. We see the significant positive price of risk on HML is driven out by the up-market beta in (5). Row (6) shows when δ_{up} is included, HML does not help to price the portfolios. Rows (7) and (8) compare the Fama and French (2015) model with and without the up-market exposure. δ_{up} improves the model fit even when added to the Fama and French (2015) five-factor model. Formal model comparisons on the right in rows (3), (5), and (8) show, in each case, adding δ_{up} significantly improves the fit, and row (6) shows that in the presence of δ_{up}, HML does not further help price these portfolios.

The bottom panel of Table 7 shows estimates of SDF loadings for 10 portfolios formed on the book-to-market ratio. Up-market exposure δ_{up} again carries a positive and significant price of risk. In row (3), as is the case for the 25 size and value portfolios, δ_{up} drives out δ_m. Interestingly, with HML in the model, δ_{up} does not help price BE/ME deciles, but with the five-factor model, δ_{up} is priced significantly when included in the model. Model comparisons show δ_{up} improves the fit of the CAPM and the Fama and French (2015) five-factor model, but does not help improve the Fama and French (1992, 1993) three-factor model.

Why might up-market exposure be priced? For market excess returns RMRF_t, we expect a positive price of risk because if a security co-moves with market returns, it pays off in good states of the world in which marginal utility is low, and does not pay off in high marginal utility states. Investors are not willing to hold such a security, so it must have higher expected returns in equilibrium.

Two competing effects are at work for the up-market exposure. First, investors still require a premium for covariance with the market. Conditional on positive market returns, positive covariance with market returns still implies the security will pay off in relatively low marginal utility states. Second, all else equal, investors would want to hold a security that covaries more with the market when market returns are positive. Such a security effectively times the market, and should

Table 7: **GMM Tests of Linear-Factor Models on 25 Size and Value Portfolios and BE/ME Deciles, July 1926 to December 2014.**

I estimate the prices of risk in linear-factor models using two-stage GMM. The SDF is

$M_t = a - \delta_m RMRF_t - \delta_{up} RMRF_t I_{RMRF_t>0} - \delta_{SMB} SMB_t - \delta_{HML} HML_t - \delta_{RMW} RMW_t - \delta_{CMA} CMA_t$

I use excess returns in my tests, and coefficients are only identified up to a scale. I fix the constant in the SDF to be 1, and estimate the remaining parameters. Newey and West (1987) t-stats with the number of lags set equal to the cube root of the number of observations are in parentheses. The top panel contains estimates for the 25 size and value portfolios; the bottom panel contains results for BE/ME deciles. I test the fit between two models in the right column. $\triangle J$ is the test statistic, distributed as χ^2_1 for one parameter restriction. The p-values are shown below the test statistics.

25 Size and Value Portfolios

	a	δ_m	δ_{up}	δ_{SMB}	δ_{HML}	δ_{RMW}	δ_{CMA}		Model Test	
(1)	1.00	2.34								
		(3.20)								
(2)	1.00		4.05							
			(3.62)							
(3)	1.00	-11.64	23.88					(1) vs (3)	$\triangle J$	10.81
		(-4.50)	(4.24)						pval	0.00
(4)	1.00	1.51		1.33	1.87					
		(2.66)		(1.55)	(2.20)					
(5)	1.00	-12.97	27.50	-1.01	-1.47			(4) vs (5)	$\triangle J$	16.17
		(-4.12)	(4.54)	(-1.24)	(-0.73)				pval	0.00
(6)	1.00	-12.39	25.80		-1.31			(3) vs (6)	$\triangle J$	2.15
		(4.23)	(4.43)		(-0.72)				pval	0.14
(7)	1.00	5.50		6.34	4.20	17.79	13.45			
		(4.76)		(4.50)	(0.90)	(5.20)	(1.57)			
(8)	1.00	-17.63	38.55	4.00	1.05	3.03	4.79	(7) vs (8)	$\triangle J$	12.15
		(-5.39)	(8.32)	(3.65)	(0.31)	(1.19)	(0.66)		pval	0.00

BE/ME Deciles

	a	δ_m	δ_{up}	δ_{SMB}	δ_{HML}	δ_{RMW}	δ_{CMA}		Model Test	
(1)	1.00	2.43								
		(3.42)								
(2)	1.00		4.12							
			(3.75)							
(3)	1.00	-3.78	11.97					(1) vs (3)	$\triangle J$	4.59
		(-1.24)	(2.14)						pval	0.03
(4)	1.00	2.88		-2.44	2.15					
		(3.29)		(-0.82)	(1.68)					
(5)	1.00	5.33	-4.14	-2.92	3.11			(4) vs (5)	$\triangle J$	0.26
		(0.90)	(-0.40)	(-1.05)	(1.47)				pval	0.61
(6)	1.00	2.97	-0.82		1.86			(3) vs (6)	$\triangle J$	1.81
		(0.55)	(-0.08)		(1.09)				pval	0.18
(7)	1.00	3.45		0.89	6.80	-0.54	-3.70			
		(2.44)		(0.16)	(1.10)	(-0.07)	(-0.32)			
(8)	1.00	-10.79	24.95	3.52	2.43	-1.34	0.92	(7) vs (8)	$\triangle J$	8.23
		(-1.93)	(2.45)	(0.96)	(0.65)	(-0.26)	(0.13)		pval	0.00

receive a negative premium from this behavior. These are offsetting effects. Empirically, the fact that up-market betas carry a positive price of risk implies the first effect dominates.

As an alternative to up-market beta, I consider down-market beta in the spirit of Ang et al. (2006) and Lettau et al. (2014). I find the down-market exposure carries a negative price of risk in all my test portfolios. Using a different definition, Ang et al. (2006) also find their downside beta measure carries a negative price of risk in the 25 size and value portfolios. This finding is puzzling. A large down-market beta should receive a positive premium for co-movement with market returns, and possibly an additional premium for larger co-movement when market returns are low. These effects go in the same direction, and overall the down-market beta should carry a positive premium. In comparison, a positive premium on the up-market beta is more plausible. Because my focus is not on down-market betas, I leave this puzzling finding for future research.

For additional asset pricing tests, I form double-sorted portfolios on up-market betas and the book-to-market ratio, and examine the price of risk of up-market betas in these portfolios. I estimate the up-market betas of individual stocks using daily returns for the past six months, and BE/ME is lagged at least six months as in Fama and French (1992, 1993). I examine two ways of forming the double-sorted portfolios. First, I form five portfolios independently on the two characteristics, and cross them to form independently sorted 25 portfolios. Second, I first form five portfolios based on the up-market betas, and then within each portfolio, I form five portfolios based on BE/ME. I call these conditionally sorted 25 portfolios.

The top panel of Table 8 presents results for independently double-sorted portfolios on β_{up} and BE/ME. δ_{up} consistently carries a positive and significant price of risk. Model tests in rows (3), (5), and (8) show δ_{up} significantly improves the model fit when it is added to the CAPM, the Fama and French (1992, 1993) three-factor model, and the Fama and French (2015) five-factor model. Row (6) shows HML still carries a significant price of risk when δ_{up} is included in the model. Significant prices of risk on both the up-market beta and HML indicate that the expected return variation in these portfolios cannot be captured by either characteristic alone. Perhaps significant and positive δ_{up} and δ_{HML} are not surprising given the portfolios are sorted by β_{up} and BE/ME. It is surprising to see δ_{up} drive out δ_{HML} in the 25 size and value portfolios.

I examine conditional double sorts by first forming portfolios based on β_{up}, and then within each portfolio, form quintiles based on BE/ME. The results are in the bottom panel of Table 8.

Table 8: **GMM Tests of Linear-Factor Models on 25 Portfolios Based on Up-Market Betas and BE/ME, January 1963 to December 2014.**

I estimate the prices of risk in linear-factor models using two-stage GMM. The SDF is

$M_t = a - \delta_m RMRF_t - \delta_{up} RMRF_t I_{RMRF_t>0} - \delta_{SMB} SMB_t - \delta_{HML} HML_t - \delta_{RMW} RMW_t - \delta_{CMA} CMA_t$

I use excess returns, and coefficients are identified only up to a scale. I fix the constant in the SDF to be 1, and estimate the remaining parameters. The top panel shows independent sorts based on the up-market beta and the book-to-market ratio. The bottom panel shows portfolios first formed on β_{up}, and then on BE/ME within each β_{up} quintile. Portfolio returns are value weighted. Newey and West (1987) t-stats are in parentheses. I test the fit between two models in the right column. $\triangle J$ is the test statistic, distributed as χ^2_1 for one parameter restriction. The p-values are shown below the test statistics.

25 β_{up} and BE/ME Portfolios, Independent Sort										
	a	δ_m	δ_{up}	δ_{SMB}	δ_{HML}	δ_{RMW}	δ_{CMA}			Model Test
(1)	1.00	3.12								
		(2.74)								
(2)	1.00		5.82							
			(3.56)							
(3)	1.00	-22.19	46.31					(1) vs (3)	$\triangle J$	12.86
		(-5.42)	(6.60)						pval	0.00
(4)	1.00	2.99		2.50	7.68					
		(2.16)		(1.13)	(4.13)					
(5)	1.00	-16.72	35.75	1.94	2.84			(4) vs (5)	$\triangle J$	15.02
		(-5.01)	(6.80)	(1.27)	(2.01)				pval	0.00
(6)	1.00	-16.50	36.22		2.80			(3) vs (6)	$\triangle J$	4.98
		(-4.86)	(6.71)		(2.01)				pval	0.03
(7)	1.00	4.87		8.97	0.93	13.48	21.98			
		(4.20)		(2.45)	(0.30)	(2.12)	(3.72)			
(8)	1.00	-11.74	27.32	5.32	2.92	7.10	3.76	(7) vs (8)	$\triangle J$	11.26
		(-5.01)	(7.90)	(2.49)	(1.29)	(2.18)	(0.87)		pval	0.00
25 β_{up} and BE/ME Portfolios, Conditional Sort										
	a	δ_m	δ_{up}	δ_{SMB}	δ_{HML}	δ_{RMW}	δ_{CMA}			Model Test
(1)	1.00	2.69								
		(2.37)								
(2)	1.00		5.17							
			(2.99)							
(3)	1.00	-24.62	51.46					(1) vs (3)	$\triangle J$	11.84
		(-4.77)	(5.32)						pval	0.00
(4)	1.00	1.88		5.29	6.78					
		(1.30)		(2.41)	(3.72)					
(5)	1.00	-17.85	37.78	2.80	2.73			(4) vs (5)	$\triangle J$	7.71
		(-5.18)	(6.49)	(1.65)	(1.88)				pval	0.01
(6)	1.00	-18.79	40.86		3.19			(3) vs (6)	$\triangle J$	10.33
		(-5.11)	(6.57)		(2.63)				pval	0.00
(7)	1.00	3.96		13.87	0.21	16.25	15.27			
		(3.15)		(3.92)	(0.05)	(2.66)	(2.13)			
(8)	1.00	-16.82	35.18	7.19	3.25	7.56	4.23	(7) vs (8)	$\triangle J$	9.36
		(-6.67)	(9.17)	(3.41)	(1.24)	(2.38)	(0.83)		pval	0.00

These results look almost identical to the top panel for independently-sorted portfolios. Again, δ_{up} estimates are positive and statistically large, for every specification. Model comparisons indicate up-market exposure improves the SDF for the CAPM, the Fama and French (1992, 1993) three-factor model, and the Fama and French (2015) five-factor model. Both HML and the up-market beta are priced simultaneously.

3.2.2 Fama-MacBeth Tests

An alternative to the GMM test of linear-factor models is the Fama-MacBeth procedure (Fama and MacBeth, 1973). In Fama-MacBeth, first-stage time-series regressions determine loadings on asset pricing factors that the SDF loads on in the GMM tests. In the second stage, cross-sectional regressions are run at each point in time using the first-stage loadings. Table 9 reports the results.

The top panel contains results for the 25 size and value portfolios. Column (1) shows the up-market beta on its own carries a significant monthly premium of 0.61%. For comparison, column (2) shows the market beta is not priced in these portfolios, corresponding to the classic Fama and French (1992) results. Columns (3) and (5) show the loadings on HML or the book-to-market ratio are able to capture some expected return variation, because they both carry large prices of risk and reduce the pricing errors α. When up-market betas and HML loadings are jointly tested in column (4), β_{up} receives a positive premium of 0.61% per month, whereas β_{HML} is driven out and receives a zero premium, just like the GMM tests. In column (6), β_{up} carries a significant premium when paired with BE/ME, whereas the premium on BE/ME is reduced compared to column (5).

The bottom panel of Table 9 contains results for 10 portfolios formed on the book-to-market ratio. For these portfolios, β_{up} gets a price of risk of 0.66% per month, whereas the market beta carries an insignificant 0.38% per month. β_{HML} and BE/ME have significant prices of risk, in columns (3) and (5). In columns (4) and (6), the premiums on the value measures β_{HML} and BE/ME are reduced when we add β_{up} to the mix. In column (4), although the price of risk estimate on β_{up} is not significant, the premium on β_{HML} is reduced by about a third compared to the specification without the up-market beta in column (3). Similarly, in column (6), the price of risk on β_{up} is marginally significant, but the premium on BE/ME is reduced by a third. Overall, the up-market beta does appear to capture expected return variation associated with the book-to-market

Table 9: **Fama-MacBeth Tests of 25 Size and Value Portfolios and BE/ME Deciles, July 1926 to December 2014.**
I run Fama-MacBeth regressions on 25 size and value portfolios and 10 portfolios formed on the book-to-market ratio. The right-hand variables are time-series betas from first-stage regressions (β, β_{up}, and β_{HML}) or the book-to-market ratios (BE/ME) of each portfolio. The top panel shows results for 25 size and value portfolios; the bottom panel shows 10 portfolios formed on the book-to-market ratio. $|\alpha|$ is the average absolute alphas in the cross-sectional regressions. $\alpha'\Sigma^{-1}\alpha$ is the test statistic for the null hypothesis that all of the alphas are jointly zero. 95% CV displays the critical value for rejecting the null that all alphas are jointly zero at a 5% level.

25 Size and Value Portfolios						
	(1)	(2)	(3)	(4)	(5)	(6)
$\|\alpha\|$	0.18%	0.22%	0.19%	0.18%	0.16%	0.13%
$\alpha'\Sigma^{-1}\alpha$	80.12	80.06	69.08	66.49	68.91	53.20
95% CV	36.42	36.42	36.42	36.42	36.42	36.42
β_{up}	0.61%			0.61%		0.62%
	(2.41)			(1.68)		(2.26)
β		0.36%				
		(1.00)				
β_{HML}			0.23%	0.00%		
			(1.81)	(0.03)		
BE/ME					0.22%	0.20%
					(4.15)	(3.51)
BE/ME Deciles						
	(1)	(2)	(3)	(4)	(5)	(6)
$\|\alpha\|$	0.05%	0.05%	0.04%	0.04%	0.05%	0.04%
$\alpha'\Sigma^{-1}\alpha$	11.63	11.91	7.56	7.53	10.81	9.49
95% CV	16.92	16.92	16.92	16.92	16.92	16.92
β_{up}	0.66%			0.21%		0.32%
	(2.67)			(0.57)		(1.97)
β		0.38%				
		(1.25)				
β_{HML}			0.32%	0.23%		
			(2.70)	(1.26)		
BE/ME					0.12%	0.08%
					(2.12)	(0.72)

ratio.

The price of risk on the up-market beta is always positive in Fama-MacBeth tests, but it is not always statistically significant. In comparison, for GMM tests, the stochastic discount factor always loads significantly on the up-market exposure. The difference in results between the Fama-MacBeth procedure and GMM may be due to the difference in the statistical power of these methods. Fama-MacBeth estimates first-stage betas, and uses those estimated coefficients in second-stage cross-sectional regressions. In contrast, GMM uses all of the data to estimate only one parameter – the loading on the SDF – associated with the up market. Despite the difference in methodology, both Fama-MacBeth and GMM indicate up-market beta is able to partially resolve the value premium.

3.2.3 How Much of HML Is Accounted for by Asymmetric Betas?

The previous section demonstrates that up-market betas capture expected return variation in portfolios sorted by the book-to-market ratio. How much of the value premium can its up-market exposure explain? I address this question in two ways. First, less than half of HML's average return remains after removing asymmetric beta exposures. Second, 40% of HML's average returns can be replicated using a market-timing strategy based on its asymmetric betas. Table 10 presents the results.

HML's raw returns from 1926-2014 are reported in the top panel. In this period, HML earned an average of 39 basis points per month. In the middle panel, I use 60 months to estimate market betas and asymmetric betas, and use these estimates to remove market or asymmetric beta exposures for the following month. Removing market exposure reduces HML average returns by six basis points to 33 basis points per month. Hedging out market returns is similar to a conditional CAPM, and the result is similar to previous attempts to use conditional CAPMs to explain the value premium. Petkova and Zhang (2005) and Lettau and Ludvigson (2001) also find conditional CAPMs somewhat reduce the value premium, but the magnitude of the effect is small. In comparison, removing HML's asymmetric beta exposure greatly reduces the value premium by 23 basis points to 16 basis points per month, less than half of the raw HML returns. The reduction in average return is almost four times that of hedging out market betas. Evidently, asymmetric betas

Table 10: **Fraction of the Value Premium coming from Asymmetric Betas.**
The top panel shows average monthly returns of HML, 1926-2014. The middle panel shows the average monthly returns of HML after taking out its market exposure or asymmetric betas. I use the previous 60 months to estimate market betas or asymmetric betas, and use the estimated coefficients to remove those exposures for the following month. The bottom panel shows the average monthly returns of two market-timing strategies that try to replicate HML returns using asymmetric betas. DP market timing uses the past 60 months to estimate a predictive regression for the dividend-yield, and uses it to predict the sign of next month's returns. Based on the predicted sign, the strategy takes positions in the market according to the asymmetric betas of HML. If the forecast is positive, the investor takes a 30% long position in the market. If the forecast is negative, the investor stays out. Perfect foresight assumes knowledge of the sign of next month's returns, and uses HML's asymmetric betas to take positions.

	Monthly Returns
HML Raw Returns	0.39%
Hedging Out the Market	
Hedged with Market Exposure	0.33%
Hedged with Asymmetric Betas	0.16%
Replicating HML	
DP Market Timing	0.14%
Perfect Foresight	0.72%

account for a portion of the value premium.

The bottom panel of Table 10 reports market-timing strategies replicating HML returns. The idea is to use HML's asymmetric betas along with market forecasts to imitate HML returns. Suppose an investor knew with certainty the sign of market returns next month, and takes positions in the market according to asymmetric betas. If the next month market return is positive, he takes a 30% long position in the market, and he stays out if the next month market return is negative. Such a strategy, which I'm calling "Perfect Foresight", would earn 72 basis points per month. This strategy yields higher average returns compared to HML, because I made the infeasible assumption that the investor knows with certainty the sign of market returns.

More realistically, the investor could use 60 months to estimate predictive regressions of the sign of market returns on the dividend-price ratio, and use this model to forecast the sign of market returns in the following month. If the forecast is positive, take a 30% long position (the up-market beta for HML is about 0.3) in the market, and if the forecast is negative, stay out (the down-market beta for HML is roughly zero). This strategy produces an average monthly return of 14 basis points, or almost 40% of the raw HML average returns.

The results from GMM, Fama-MacBeth regressions, removal of asymmetric betas from HML, and replication strategies all point in the same direction: asymmetric betas explain a fraction of the average returns associated with the book-to-market ratio and partially captures the long-short value premium. I look at theoretical models in the next section to better understand this result.

4 Explanations of Value

The value strategy exhibits asymmetric and time-varying betas, and asymmetric betas partially capture the value premium. In light of my empirical findings, I consider theoretical explanations of value to better understand the link between asymmetric betas and the value premium. I investigate two theoretical models: the cash-flow duration model of Lettau and Wachter (2007), and the habit model of Santos and Veronesi (2010) with heterogeneous cash flows. I find that cash-flow risk of value securities are related to asymmetric betas.

4.1 Duration Risk

Lettau and Wachter (2007) investigate the effect of cash-flow duration on the cross-section of expected returns. In this model, assets with low duration – those that pay dividends earlier – have low price ratios and high expected returns; these are value stocks. The authors assume variation in the discount rate is unpriced, and shocks to the aggregate dividends are priced. All assets have identical, and positive, cash-flow risk, but some pay dividends earlier than others. These assumptions have two implications. First, the dividends that are paid earlier lower the price-dividend ratio because they lower expected dividend growth. Second, dividends that are paid earlier make the asset riskier because they make up a larger portion of current consumption. Hence, low-duration assets have low price ratios, are riskier, and have higher premiums.

I simulate the Lettau and Wachter (2007) model using their baseline parameters, and investigate the model implications for asymmetric betas. I simulate 5,000 quarters of data for 200 firms, and form 10 portfolios based on their measure of value, the price-dividend ratio. Every year, the portfolios are reassigned. Using the simulated data, I estimate the CAPM betas and asymmetric betas. Because the simulation is repeated many times, I take the parameter estimates to be population moments and do not report standard errors.

Table 11 presents the results. The top panel illustrates that the model can generate a value premium of 5.13% per year. The CAPM betas are larger for growth portfolios compared to value portfolios. The down-market betas are similar for all 10 portfolios, and the difference between up-market and down-market betas is small and positive for the growth portfolio and small and negative for the value portfolio. The pattern in asymmetric betas contradicts my finding of larger cross-

Table 11: **Asymmetric Betas of Lettau and Wachter (2007).** I examine the Lettau and Wachter (2007) model using their baseline parameters. I simulate 5,000 quarters of market returns and 200 securities, and form deciles based on the price-dividend ratios, the measure of value in this model. The unconditional CAPM is $r_i^e = \alpha + \beta r_{m,t}^e + \epsilon_i$
For asymmetric betas, the regression is $r_i^e = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r_{m,t}^e + \epsilon_i$, in which $I_{up,t}$ is 1 if $r_{m,t}^e > 0$ and zero otherwise. I take the parameter estimates to be population moments and do not report standard errors. Average returns in the top panel are annualized. α's are converted into monthly values for easy comparison with my empirical results.

	Growth	2	3	4	5	6	7	8	9	Value	V-G
$\bar{r}_i^e$	6.10%	6.44%	6.99%	7.76%	8.71%	9.15%	9.35%	9.69%	10.14%	11.23%	5.13%
					CAPM						
α	-0.25%	-0.24%	-0.21%	-0.15%	-0.08%	-0.02%	0.04%	0.10%	0.15%	0.22%	0.48%
β	1.094	1.106	1.109	1.099	1.067	1.013	0.941	0.872	0.830	0.827	-0.267
					Asymmetric Betas						
α	-0.31%	-0.29%	-0.26%	-0.20%	-0.11%	-0.03%	0.05%	0.13%	0.19%	0.27%	0.58%
β_{down}	1.061	1.072	1.075	1.068	1.045	1.003	0.946	0.891	0.858	0.860	-0.201
$\beta_{up-down}$	0.054	0.055	0.054	0.049	0.037	0.017	-0.008	-0.030	-0.045	-0.053	-0.107

sectional variation in up-market betas and large differences in up-market versus down-market betas in value portfolios. Empirically, the difference between up-market and down-market betas is small and negative for growth portfolios, and large and positive for value portfolios. The long-short portfolio should have a large and positive up-market beta, and a small or negative down-market beta several times smaller in magnitude.

Although Lettau and Wachter (2007) are able to match the unconditional value premium, they do not capture value's asymmetric betas. Because asymmetric betas matter for pricing, it is an important characteristic of the value strategy for theorists to consider. A strong explanation of the value premium should take into account the asymmetric betas of value.

4.2 Habit Formation

Another popular model of value is the Santos and Veronesi (2010) model. They show in non-linear external-habit models, cross-sectional heterogeneity in firms' cash-flow risk is necessary to generate a value premium. In their model, firms differ not just in expected dividend growth, but also in the covariance of their cash flows with consumption growth. Value stocks are riskier because they have low cash flows when aggregate consumption falls. I simulate the model of Santos and Veronesi (2010), using their parameters, and find their model is able to capture asymmetric betas in addition to the unconditional value premium. Table 12 presents the simulation results.

I simulate 5,000 quarters of 200 firms using the baseline parameters in Santos and Veronesi (2010). The top panel shows a value premium of 4.36% per year among portfolios sorted on their price-dividend ratios. CAPM betas are larger for the value portfolio (1.411) compared to the growth portfolio (0.821). Most of this difference comes from value's large up-market beta. The variation in down-market betas is small compared to the variation in the difference between up- and down-market betas. That difference is small and negative for growth portfolios, and large and positive for value portfolios, just as observed in the data.

The value strategy, V-G, has an up-market beta of 0.729 and a down-market beta of -0.401. The up-market beta is large and positive, and almost twice as large as the down-market beta. This model produces exaggerated asymmetric betas compared to the data – empirically the up-market beta is 0.3 and down-market beta is roughly zero. Although the pattern is exaggerated, Santos

Table 12: **Asymmetric Betas of Santos and Veronesi (2010).** I examine the Santos and Veronesi (2010) model using their baseline parameters. I simulate 5,000 quarters of market returns and 200 securities, and form deciles sorted on the price-dividend ratios, their measure of value. I report average returns, the unconditional betas, and asymmetric betas. The unconditional CAPM is $r_i^e = \alpha + \beta r_{m,t}^e + \epsilon_i$

For asymmetric betas, the regression is $r_i^e = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r_{m,t}^e + \epsilon_i$, in which $I_{up,t}$ is 1 if $r_{m,t}^e > 0$ and zero otherwise. Like Santos and Veronesi (2010), I take the parameter estimates to be population moments and do not report standard errors. Average returns in the top panel are annualized. α's are converted into monthly values for easy comparison with my empirical results.

	Growth	2	3	4	5	6	7	8	9	Value	V-G
$\bar{r}_i$	5.54%	6.03%	6.81%	7.27%	7.91%	8.08%	8.80%	8.92%	9.67%	9.92%	4.36%
CAPM											
α	-0.01%	0.00%	0.07%	0.15%	0.18%	0.16%	0.25%	0.24%	0.26%	0.07%	0.08%
β	0.821	0.886	0.961	0.978	1.055	1.091	1.147	1.167	1.265	1.411	0.592
Asymmetric Betas											
α	0.20%	0.23%	0.17%	0.16%	0.10%	0.06%	0.06%	0.01%	-0.08%	-0.88%	-1.08%
β_{down}	1.001	1.084	1.046	0.986	0.982	1.005	0.992	0.967	0.977	0.602	-0.401
$\beta_{up-down}$	-0.205	-0.226	-0.098	-0.010	0.083	0.098	0.177	0.227	0.327	0.920	1.130

and Veronesi (2010) do capture the general pattern of asymmetric betas in a large and positive up-market beta, and a relatively smaller down-market beta.

4.3 Cross-Sectional Cash-Flow Risk

The Santos and Veronesi (2010) model is able to capture the asymmetric betas of value, but the Lettau and Wachter (2007) model is not. I investigate the difference between the two models to shed light on the economic mechanism producing asymmetric betas, and find heterogeneous cash-flow risk is a probable driver. Furthermore, I decompose asymmetric betas into cash-flow and discount-rate components, and find cash-flow betas contribute more to up-market betas compared to discount-rate betas.

It is difficult to directly compare the baseline models of Lettau and Wachter (2007) and Santos and Veronesi (2010), because they differ on two main aspects. First, the specifications of the stochastic discount factor are different. In the baseline calibration of Lettau and Wachter (2007), the correlation between dividend-growth innovations and discount-rate innovations, ρ_{dx}, is zero. For these parameters, the model does not produce asymmetric betas. In habit models, this correlation is negative. In fact, these quantities are perfectly negatively correlated in Campbell and Cochrane (1999) and Menzly et al. (2004). In Lettau and Wachter (2007), holding other parameters unchanged, if ρ_{dx} is set to a moderate negative value such as -0.5, the term structure of risk premiums becomes upward sloping for all maturities instead of downward sloping, and a growth premium results. This result occurs because if ρ_{dx} isn't zero, shocks to discount rates are priced and long-duration (growth) assets are more sensitive to discount-rate shocks.

I examine an alternative calibration of Lettau and Wachter (2007) to allow for a negative ρ_{dx} while maintaining a value premium. The parameters are largely based on Picca (2015), who matches the key time-series properties of market returns in Lettau and Wachter (2007) with a negative ρ_{dx}. In particular, this correlation between dividend-growth innovations and discount-rate innovations is set to -0.63. The correlation between dividend-growth innovations and expected-dividend-growth innovations is kept at its baseline value of -0.83. The average level of the discount-rate process is changed from 0.625 to 0.36, and the persistence of the discount-rate process is reduced from 0.87 to 0.4. These parameters are set such that the time-series properties of market

Table 13: **Alternative Calibration of Lettau and Wachter (2007).** I simulate the Lettau and Wachter (2007) model with 5,000 quarters and 200 securities using the parameters of Picca (2015) and form deciles sorted on the price-dividend ratios. The correlation between dividend shocks and discount-rate shocks is -0.63, and the correlation between expected-dividend-growth shocks and discount-rate shocks is 0.10. I report average returns, the unconditional CAPM, and asymmetric betas. The unconditional CAPM is $r_i^e = \alpha + \beta r_{m,t}^e + \epsilon_i$ For asymmetric betas, the regression is $r_i^e = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r_{m,t}^e + \epsilon_i$, in which $I_{up,t}$ is 1 if $r_{m,t}^e > 0$ and zero otherwise. I take the parameter estimates to be population moments and do not report standard errors. Average returns in the top panel are annualized. α's are converted into monthly values for easy comparison to my empirical results.

	Growth	2	3	4	5	6	7	8	9	Value	V-G
$\bar{r}_i^e$	5.65%	5.93%	6.38%	7.00%	7.77%	8.74%	10.04%	12.35%	13.45%	11.58%	5.94%
						CAPM					
α	0.09%	0.09%	0.10%	0.12%	0.14%	0.19%	0.28%	0.47%	0.53%	0.30%	0.21%
β	0.761	0.790	0.830	0.877	0.929	0.973	1.002	1.011	1.048	1.156	0.394
						Asymmetric Betas					
α	0.16%	0.15%	0.15%	0.16%	0.16%	0.19%	0.28%	0.46%	0.50%	0.24%	0.08%
β_{down}	0.812	0.835	0.865	0.904	0.943	0.975	0.999	1.003	1.028	1.110	0.298
$\beta_{up-down}$	-0.079	-0.070	-0.055	-0.043	-0.022	-0.002	0.005	0.012	0.031	0.070	0.149

returns in Lettau and Wachter (2007) are preserved.

A less persistent discount-rate process reduces the price of risk on long-duration assets, such that the term structure of risk premiums is first increasing and then decreasing[1]. Table 13 contains the unconditional betas and asymmetric betas of the alternative calibration of Lettau and Wachter (2007). In the top panel, this calibration is still able to generate a reasonable unconditional value premium of 5.94% per year. The CAPM betas are larger for value portfolios compared to growth portfolios, and the long-short portfolio has a CAPM beta of 0.394. The bottom panel shows some variation in the difference between up-market and down-market betas. The long-short portfolio V-G has a down-market beta of 0.298 and an up-market beta of 0.447. Compared to the estimated values of 0.125 and 0.769 for book-to-market deciles, the alternative calibration is able to produce the correct signs on asymmetric betas, but the magnitude of up-market betas relative to down-market betas is too small.

Asymmetric betas may be a result of the cash-flow risk of value stocks. Both the alternative calibration of Lettau and Wachter (2007) and Santos and Veronesi (2010) feature habit-style SDFs. The remaining key difference between the two models is the role of cross-sectional cash-flow risk. In Lettau and Wachter (2007), cash-flow risk is identical for all securities, whereas Santos and Veronesi (2010) contain heterogeneous cash-flow risk in the cross section. The fact that Santos and Veronesi (2010) generates asymmetric betas and the alternative calibration of Lettau and Wachter (2007) produces the right sign but incorrect magnitude points to the possibility that asymmetric betas are produced by cross-sectional cash-flow risk.

I further investigate the linkage between cash-flow risk and asymmetric betas by looking at a decomposition of market betas into cash-flow and discount-rate betas as in Campbell and Vuolteenaho (2004). First, I decompose innovations to market returns into a cash-flow component and a discount-rate component as in Campbell (1991) and Campbell and Vuolteenaho (2004):

$$r_{m,t+1} - \mathbb{E}_t r_{m,t+1} = N_{CF,t+1} - N_{DR,t+1} \tag{18}$$

I estimate cash-flow news, $N_{CF,t+1}$, and discount-rate news, $-N_{DR,t+1}$ by assuming state vari-

[1]For the full term structure, please refer to Appendix C

ables z_t follow a first-order vector autoregression (VAR):

$$z_{t+1} = a + \Gamma z_t + u_{t+1} \tag{19}$$
$$z_t = \begin{Bmatrix} r^e_{m,t} \\ DP_t \end{Bmatrix}$$

This specification is quite general, as we can always expand the state vector. The first element of z_t is market excess returns. I use the dividend-yield, DP_t, to capture conditional expected returns. Cash-flow and discount-rate news $N_{CF,t+1}$ and $-N_{DR,t+1}$ can be calculated from the VAR residuals u_t (see Appendix D for details). To confirm that using only DP_t captures sufficient expected-return variation, I verify this system produces similar patterns of cash-flow and discount-rate betas as in Campbell and Vuolteenaho (2004) for the 25 size and value portfolios.

I quantify the specific contribution of cash-flow and discount-rate betas to asymmetric betas by decomposing up-market and down-market betas into their cash-flow and discount-rate components. For example, the up-market beta is the sum of two parts:

$$\beta_{Up,CF} = \frac{Cov(r^e_{i,t}, N_{CF,t}|r^e_{m,t} > 0)}{Var(N_{CF,t} - N_{DR,t}|r^e_{m,t} > 0)}, \quad \beta_{Up,DR} = \frac{Cov(r^e_{i,t}, -N_{DR,t}|r^e_{m,t} > 0)}{Var(N_{CF,t} - N_{DR,t}|r^e_{m,t} > 0)}$$

And the down-market beta is decomposed similarly. In Table 14, summing up the "Up" columns gives the up-market betas, and summing up the "Down" columns gives the down-market betas, as estimated using market return innovations. For 1926-2014, HML's down-market beta is small. Correspondingly, both the down-market cash-flow and discount-rate parts are small. The up-market beta is large and positive, and comes almost entirely from the cash-flow component. The up-market and down-market discount-rate betas are similar and do not contribute to the asymmetry.

From 1926 to 1970, the point estimates for the up-market beta is strong, and the asymmetry is more pronounced compared to the full sample. Cash-flow betas are larger for both up and down markets, and discount-rate betas also exhibit some asymmetry in up versus down markets. In terms of magnitude, cash-flow betas dominate in its contribution to a large up-market beta. For 1971 to 2014, the point estimate for the up-market beta is weak. The up-market cash-flow

Table 14: **Decomposition of HML's Asymmetric Betas Into Cash-Flow and Discount-Rate Components.** I separate market return innovations into its cash-flow and discount-rate components as in Campbell and Vuolteenaho (2004). With the news components, I decompose up-market and down-market betas into their cash-flow and discount-rate counterparts, for a total of four betas: Up-market cash-flow, up-market discount-rate, down-market cash-flow, and down-market discount-rate.

	1925-2014		1925-1970		1971-2014	
	Up	Down	Up	Down	Up	Down
CF	0.256	0.064	0.439	0.154	-0.187	-0.120
DR	0.084	0.067	0.118	0.055	-0.050	-0.020

and discount-rate betas are both negative. Summing down columns reveals a more negative up-market beta compared to the down-market beta, consistent with Table 4.

Cash-flow risks are linked to asymmetric betas. The up- and down-market discount-rate betas are generally close to zero and do not contribute to the asymmetry. The cash-flow betas are what drive the difference between the first half and second half of the sample. Large positive up-market cash-flow betas in the first half turn negative for the second half, leading to smaller up-market betas. Through comparing Lettau and Wachter (2007) with Santos and Veronesi (2010) and decomposing beta into cash-flow and discount-rate components, it is evident asymmetric betas are related to cash-flow risks of value securities. A good model should be able to capture the cash-flow sensitivity and asymmetric betas of value.

5 Conclusion

In this paper, I document conditional market exposures of value that shed light on its theoretical explanations. I find asymmetric betas and time-varying betas for value strategies in U.S. equities, international equities, and three non-equity asset classes. Asymmetric betas and time-varying betas are linked through a simple model of mean-reverting market returns. Asymmetric betas are able to partially capture the expected return variation associated with the book-to-market ratio, and are related to the cash-flow risk of value securities.

Asymmetric betas come entirely from the value end. The up-market beta for value stocks are economically and statistically larger compared to the down-market beta. The up-market and down-market betas for growth stocks are identical, and their bear and bull-market betas are similar. Further research to understand the option-like behavior of value stocks, as well as the different conditional distributions for value and growth securities may prove to be fruitful.

If value were one side of a coin, momentum would be the other side. Both strategies depend on past returns but in opposite ways. Value and momentum have been shown to be negatively correlated, and this negative correlation may vary through time. I have some preliminary results that show the correlation is larger conditional on positive market returns: a fraction of their negative correlation may come from their opposing conditional betas. Further investigation into the connection between value and momentum returns may be a worthwhile research direction.

Although I showed asymmetric betas could result as a combination of time-varying betas and market mean-reversion, I did not provide an explanation for time-varying betas. An ideal explanation of value should be able to match both asymmetric and time-varying betas. One possible channel for time-varying betas comes from the changing correlations between individual stocks and the market. Empirically, stock returns are more correlated with market returns in downturns compared to booms. These asymmetric correlations may generate time-varying betas. Furthermore, the state variable that affects the comovement in stocks may also affect the value premium. The identity of such a variable may be important in understanding the time-varying risk of value.

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Appendix A Proof of Propositions

The model is

$$
\begin{aligned}
r_{m,t+1} &= \mu_{t+1} + \epsilon_{t+1} \\
r_{t+1}^{Value} &= \beta_{t+1} r_{m,t+1} + \epsilon_{t+1}^{Value} \\
\beta_{t+1} &= \beta_{bear}, \quad if \quad I_{bear,t} = 1 \\
&= \beta_{bull}, \quad if \quad I_{bear,t} = 0
\end{aligned}
$$

Proof of Proposition 1:

Proof.

$$
\begin{aligned}
& Cov(r_{m,t+1}, r_{t+1}^{Value}) = Cov(r_{m,t+1}, \beta_{t+1} r_{m,t+1} + \epsilon_{t+1}^{Value}) \\
& = Cov(r_{m,t+1}, \beta_{bear} r_{m,t+1} + \epsilon_{t+1}^{Value} | I_{bear,t} = 1) Pr(I_{bear,t} = 1) \\
& + Cov(r_{m,t+1}, \beta_{bull} r_{m,t+1} + \epsilon_{t+1}^{Value} | I_{bear,t} = 0) Pr(I_{bear,t} = 0) \\
& = \beta_{bear} Var(r_{m,t+1}) Pr(I_{bear,t} = 1) + \beta_{bull} Var(r_{m,t+1}) Pr(I_{bear,t} = 0)
\end{aligned}
$$

$$
\beta_{t+1} = \frac{Cov(r_{m,t+1}, r_{t+1}^{Value})}{Var(r_{m,t+1})} = \beta_{bear} Pr(I_{bear,t} = 1) + \beta_{bull} Pr(I_{bear,t} = 0)
$$

□

Proof of Proposition 2:

Proof. For the up-market beta,

$$
\beta_{up,t+1} = \frac{Cov(r_{m,t+1}, r_{t+1}^{Value} | r_{m,t+1} > 0)}{Var(r_{m,t+1} | r_{m,t+1} > 0)}
$$

$$Cov(r_{m,t+1}, r^{Value}_{t+1}|r_{m,t+1} > 0) = Cov(r_{m,t+1}, \beta_{t+1} r_{m,t+1} + \epsilon^{Value}_{t+1}|r_{m,t+1} > 0)$$

$$= Cov(r_{m,t+1}, \beta_{bear} r_{m,t+1} + \epsilon^{Value}_{t+1}|r_{m,t+1} > 0, I_{bear,t} = 1) Pr(I_{bear,t} = 1|r_{m,t+1} > 0)$$

$$+ Cov(r_{m,t+1}, \beta_{bull} r_{m,t+1} + \epsilon^{Value}_{t+1}|r_{m,t+1} > 0, I_{bear,t} = 0) Pr(I_{bear,t} = 0|r_{m,t+1} > 0)$$

$$= \beta_{bear} Var(r_{m,t+1}|r_{m,t+1} > 0) Pr(I_{bear,t} = 1|r_{m,t+1} > 0)$$

$$+ \beta_{bull} Var(r_{m,t+1}|r_{m,t+1} > 0) Pr(I_{bear,t} = 0|r_{m,t+1} > 0)$$

$$\beta_{up,t+1} = \beta_{bear} Pr(I_{bear,t} = 1|r_{m,t+1} > 0) + \beta_{bull} Pr(I_{bear,t} = 0|r_{m,t+1} > 0)$$
$$= \beta_{bear} \frac{Pr(r_{m,t+1} > 0|I_{bear,t} = 1) Pr(I_{bear,t} = 1)}{Pr(r_{m,t+1} > 0)} + \beta_{bull} \frac{Pr(r_{m,t+1} > 0|I_{bear,t} = 0) Pr(I_{bear,t} = 0)}{Pr(r_{m,t+1} > 0)}$$

Similarly, for the down-market beta,

$$\beta_{down,t+1} = \frac{Cov(r_{m,t+1}, r^{Value}_{t+1}|r_{m,t+1} \leq 0)}{Var(r_{m,t+1}|r_{m,t+1} \leq 0)}$$
$$= \beta_{bear} Pr(I_{bear,t} = 1|r_{m,t+1} \leq 0) + \beta_{bull} Pr(I_{bear,t} = 0|r_{m,t+1} \leq 0)$$
$$= \beta_{bear} \frac{Pr(r_{m,t+1} \leq 0|I_{bear,t} = 1) Pr(I_{bear,t} = 1)}{Pr(r_{m,t+1} \leq 0)} + \beta_{bull} \frac{Pr(r_{m,t+1} \leq 0|I_{bear,t} = 0) Pr(I_{bear,t} = 0)}{Pr(r_{m,t+1} \leq 0)}$$

□

Appendix B Simulated Distribution of Unconditional and Asymmetric Betas

In section 3.4, I showed time-varying betas and a mean-reverting market produce the sort of asymmetric betas we observe in the data. In this appendix, I plot the simulated distributions of unconditional and asymmetric betas from section 3.4.

Figures A1, A2, and A3 illustrate the unconditional, down-market, and the difference between up-market and down-market betas for the full sample, 1926-2014. The simulation clearly captures the estimated unconditional beta and asymmetric betas in Table 1. Figures A4, A5, and A6 illustrate the simulated beta distributions for the pre-1970 sample. Figures A7, A8, and A9 illustrate the simulated beta distributions for the post-1970 sample.

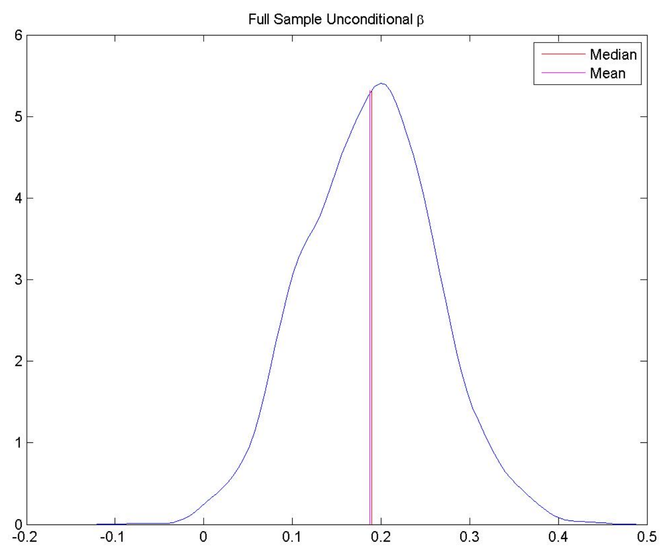

Figure A1: **Distribution of Simulated Unconditional Betas, Full Sample.**
Kernel density of 2,000 simulated unconditional betas based on the system in section 3.5 is shown. $\beta_{bull} = -0.069$, $\beta_{bear} = 0.369$, and the predictive coefficient $(1 - \rho\phi) = 0.278$. The red line marks the median value, and the magenta line marks the mean.

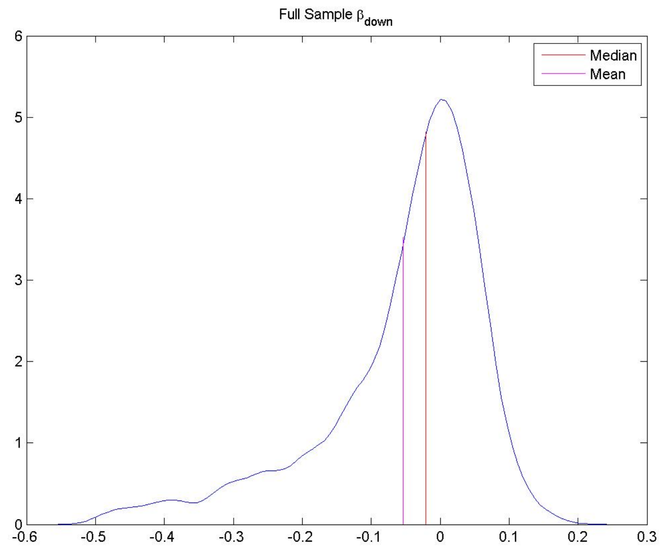

Figure A2: **Distribution of Simulated Down-Market Betas, Full Sample.**
Kernel density of 2,000 simulated down-market betas based on the system in section 3.5 is shown. $\beta_{bull} = -0.069$, $\beta_{bear} = 0.369$, and the predictive coefficient $(1 - \rho\phi) = 0.278$. The red line marks the median value, and the magenta line marks the mean.

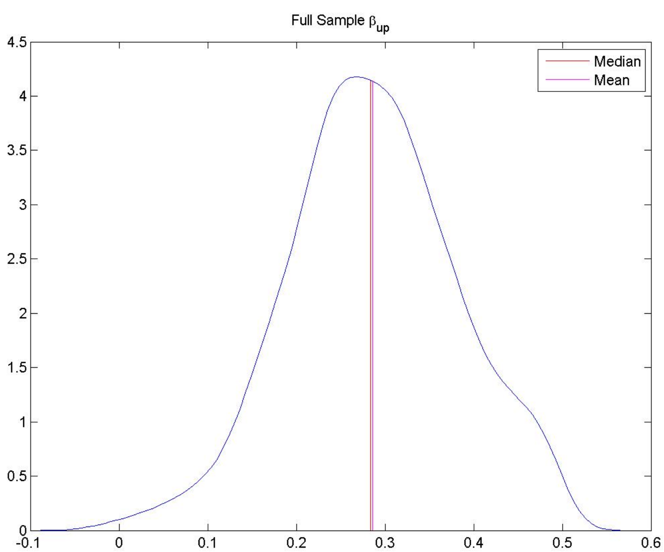

Figure A3: **Distribution of Simulated Difference between Up-Market and Down-Market Betas, Full Sample.**
Kernel density of 2,000 simulated $\beta_{up-down}$ based on the system in section 3.5 is shown. $\beta_{bull} = -0.069$, $\beta_{bear} = 0.369$, and the predictive coefficient $(1 - \rho\phi) = 0.278$. The red line marks the median value, and the magenta line marks the mean.

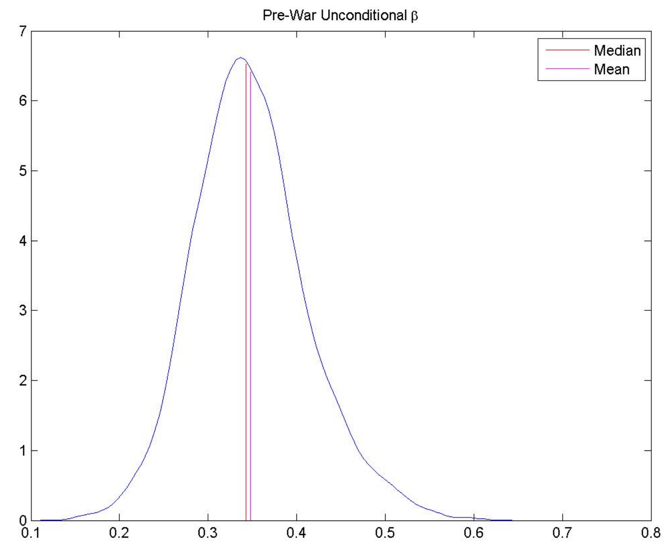

Figure A4: **Distribution of Simulated Unconditional Betas, Pre-1970 Sample.**
Kernel density of 2,000 simulated unconditional betas based on the system in section 3.5 is shown. $\beta_{bull} = 0.106$, $\beta_{bear} = 0.531$, and the predictive coefficient $(1 - \rho\phi) = 0.478$. The red line marks the median value, and the magenta line marks the mean.

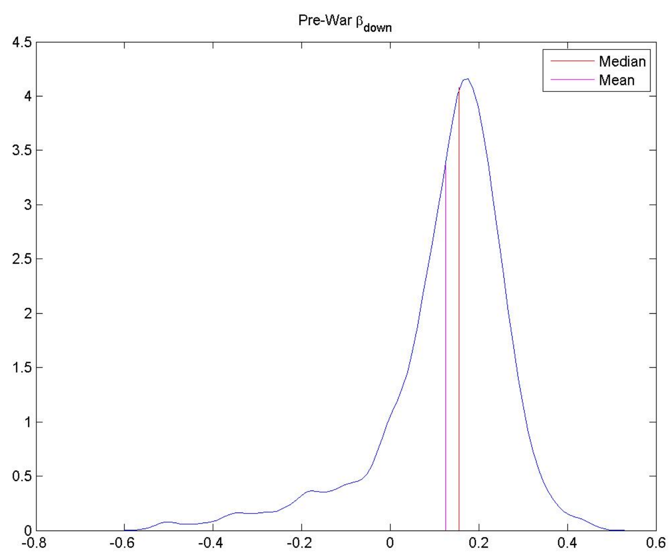

Figure A5: **Distribution of Simulated Down-Market Betas, Pre-1970 Sample.**
Kernel density of 2,000 simulated down-market betas based on the system in section 3.5 is shown. $\beta_{bull} = 0.106$, $\beta_{bear} = 0.531$, and the predictive coefficient $(1 - \rho\phi) = 0.478$. The red line marks the median value, and the magenta line marks the mean.

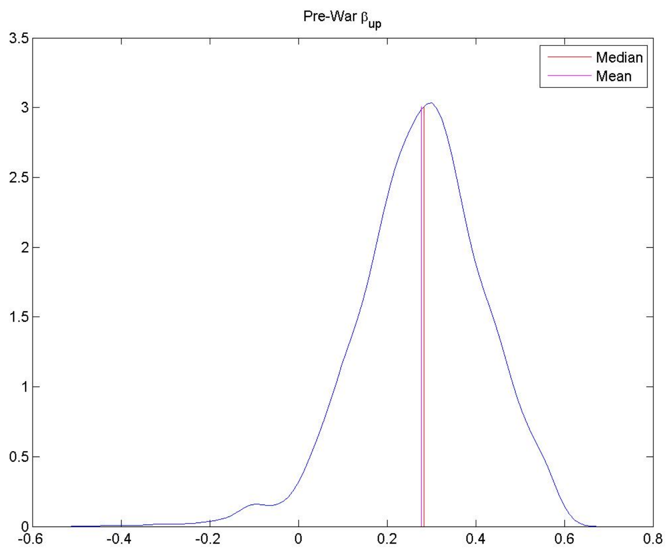

Figure A6: **Distribution of Simulated Difference between Up-Market and Down-Market Betas, Pre-1970 Sample.**
Kernel density of 2,000 simulated $\beta_{up-down}$ betas based on the system in section 3.5 is shown. $\beta_{bull} = 0.106$, $\beta_{bear} = 0.531$, and the predictive coefficient $(1 - \rho\phi) = 0.478$. The red line marks the median value, and the magenta line marks the mean.

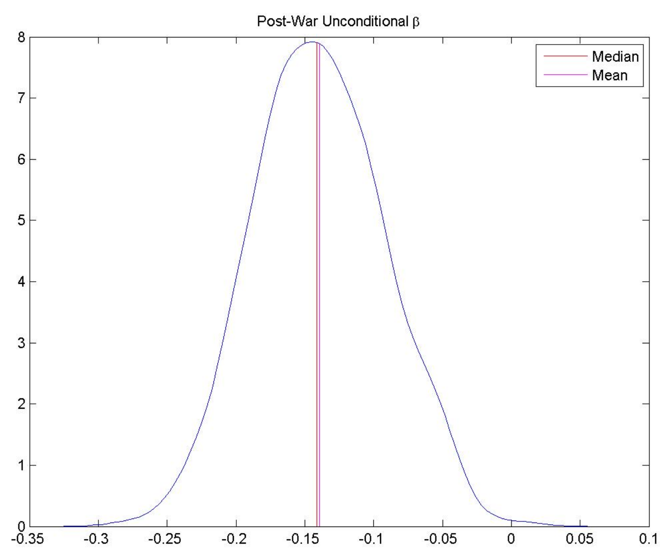

Figure A7: **Distribution of Simulated Unconditional Betas, Post-1970 Sample.**
Kernel density of 2,000 simulated unconditional betas based on the system in section 3.5 is shown. $\beta_{bull} = -0.293$, $\beta_{bear} = 0.111$, and the predictive coefficient $(1 - \rho\phi) = 0.197$. The red line marks the median value, and the magenta line marks the mean.

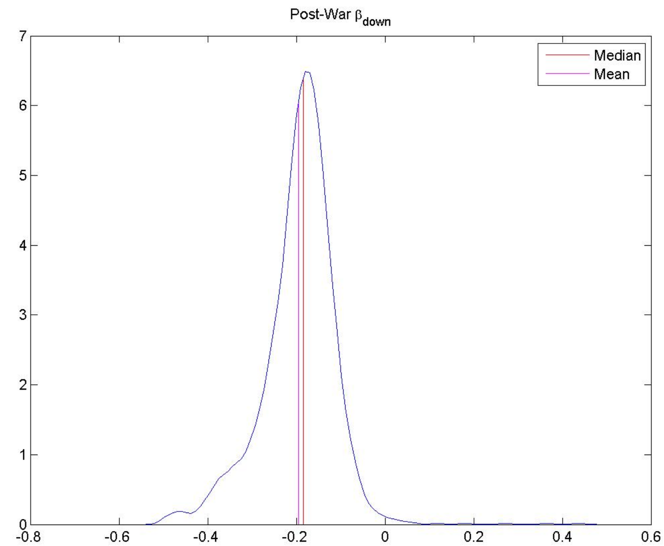

Figure A8: **Distribution of Simulated Down-Market Betas, Post-1970 Sample.**
Kernel density of 2,000 simulated down-market betas based on the system in section 3.5 is shown. $\beta_{bull} = -0.293$, $\beta_{bear} = 0.111$, and the predictive coefficient $(1 - \rho\phi) = 0.197$. The red line marks the median value, and the magenta line marks the mean.

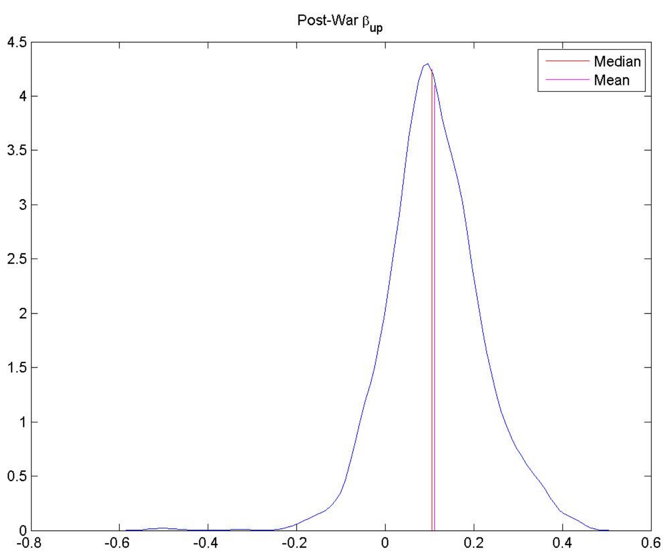

Figure A9: **Distribution of Simulated Difference between Up-Market and Down-Market Betas, Post-1970 Sample.**
Kernel density of 2,000 simulated $\beta_{up-down}$ betas based on the system in section 3.5 is shown $\beta_{bull} = -0.293$, $\beta_{bear} = 0.111$, and the predictive coefficient $(1 - \rho\phi) = 0.197$. The red line marks the median value, and the magenta line marks the mean.

Appendix C Risk Premiums under Alternative Calibration of Lettau and Wachter (2007)

In the baseline calibration of Lettau and Wachter (2007), the correlation matrix among dividend-growth shocks, expected-dividend-growth shocks, and discount-rate shocks is the following:

$$\begin{bmatrix} 1.00 & -0.83 & 0 \\ -0.83 & 1.00 & 0 \\ 0 & 0 & 1.00 \end{bmatrix}$$

I use the parameters in Picca (2015) to simulate the model of Lettau and Wachter (2007). In his calibration, Picca (2015) uses the following correlation matrix:

$$\begin{bmatrix} 1.00 & -0.83 & -0.63 \\ -0.83 & 1.00 & 0.10 \\ -0.63 & 0.10 & 1.00 \end{bmatrix}$$

By setting $\rho_{dx} = -0.63$, discount-rate shocks are priced and the SDF is reminiscent of a habits model such as Campbell and Cochrane (1999). The long-run level of the discount rate is changed from 0.625 to 0.36, and the persistence of the discount-rate process is reduced from 0.87 to 0.4. I simulate the model using these parameters. Picca (2015) shows his parameter choices produces aggregate moments similar to the baseline calibration of Lettau and Wachter (2007).

I simulate the model using these parameters and look at the cross section. For maturities short than one year, the risk premiums are increasing in maturity. Although the discount rate is priced, due to its lower persistence, at longer horizons, the negative correlation between dividend growth and expected dividend growth dominates. Therefore, at longer horizons, the risk premiums are lower.

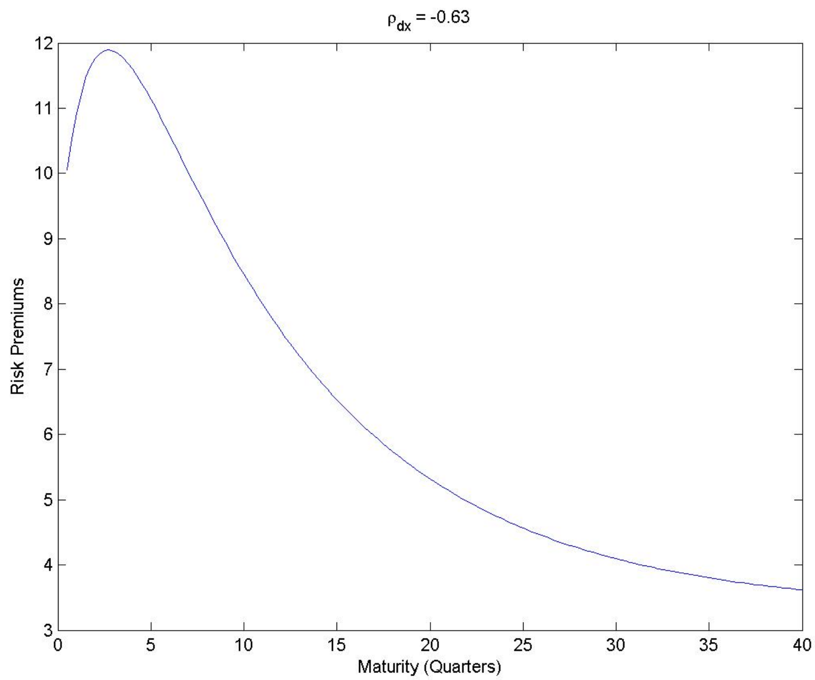

Figure A10: **Term Structure of Risk Premiums of Lettau and Wachter (2007) under Alternative Calibration.**
I plot the risk prices at different horizons for the Lettau and Wachter (2007) model under the calibration of Picca (2015). The correlation between dividend growth shocks and discount rate shocks is -0.63.

Appendix D Decomposing Market Betas into Cash-Flow and Discount-Rate Components

I follow Campbell and Vuolteenaho (2004) in decomposing market betas into its cash-flow and discount-rate components. From the Campbell and Shiller (1988a,b) return identity, unexpected returns either comes from news about cash-flows (CF) or discount-rates (DR):

$$r_{m,t+1} - \mathbb{E}_t r_{m,t+1} = \triangle\mathbb{E}_{t+1} \sum_{j=0}^{\infty} \rho^j \triangle d_{t+1+j} - \triangle\mathbb{E}_{t+1} \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j}$$

The first term on the right side is cash-flow news $N_{CF,t+1}$ and the second term is discount-rate news $N_{DR,t+1}$. To estimate these two components, the underlying data-generating process is assumed to follow a vector autoregression:

$$z_{t+1} = a + \Gamma z_t + u_{t+1}$$

where z_{t+1} is a vector of m state variables. Its first element is $r_{m,t+1}$, the market excess returns. I use the dividend-yield as the return predictor:

$$z_{t+1} = \begin{Bmatrix} r_{m,t+1} \\ DP_{t+1} \end{Bmatrix}$$

The dividend-price ratio captures sufficient expected return variation to match the Campbell and Vuolteenaho (2004) CF and DR beta results for the Fama-French 25 size and value portfolios. I estimate the VAR system, and construct CF and DR news from the residuals:

$$N_{CF,t+1} = (e1' + e1'\lambda)u_{t+1}$$
$$N_{DR,t+1} = e1'\lambda u_{t+1}$$
$$\lambda = \rho\Gamma(I - \rho\Gamma)^{-1}$$

where $e1$ is a vector with 1 as its first element, and zero everywhere else. ρ is set to be $0.95^{1/12}$ in Campbell and Vuolteenaho (2004). Whereas Campbell and Vuolteenaho (2004) calculate unconditional cash-flow and discount-rate betas, I calculate CF and DR betas conditional on up and down markets. The up-market beta is the sum of the two components:

$$\beta_{Up,CF} = \frac{Cov(r^e_{i,t}, N_{CF,t} | r^e_{m,t} > 0)}{Var(N_{CF,t} - N_{DR,t} | r^e_{m,t} > 0)} \quad \beta_{Up,DR} = \frac{Cov(r^e_{i,t}, -N_{DR,t} | r^e_{m,t} > 0)}{Var(N_{CF,t} - N_{DR,t} | r^e_{m,t} > 0)}$$

And the down-market beta is the sum of the following:

$$\beta_{Down,CF} = \frac{Cov(r^e_{i,t}, N_{CF,t} | r^e_{m,t} \leq 0)}{Var(N_{CF,t} - N_{DR,t} | r^e_{m,t} \leq 0)} \quad \beta_{Down,DR} = \frac{Cov(r^e_{i,t}, -N_{DR,t} | r^e_{m,t} \leq 0)}{Var(N_{CF,t} - N_{DR,t} | r^e_{m,t} \leq 0)}$$

Appendix E Definition of Value Strategy

We have seen asymmetric betas and time-varying betas are not limited to HML; they are also present in international equity-value strategies and value strategies in commodities, exchange rates, and country bonds. In the section below, I examine asymmetric betas and time-varying betas of value strategies based on the dividend-yield.

The dividend yield is another variable for which a value strategy can be constructed. I find that a value strategy constructed using the dividend-yield gives similar results as the ones for the book-to-market ratio. Table A1 shows the results. I include three different value strategies based on the dividend yield – terciles, quintiles, and deciles. Each value strategy is a portfolio that goes long the portfolio with the highest DP, and shorts the portfolio with the lowest DP. The piecewise linear specifications in (2), (5), and (8) show improvement compared to the market model in (1), (4), and (7). The R^2 are considerably bigger, and both estimates of up-market and down-market betas are highly significant. The up-market betas are all positive just like for the book-to-market ratio, but the down-market betas are now large and negative. This result is likely related to the bull-market betas, which are also large and negative, as in columns (3), (6), and (9). Allowing for time-varying betas in bear and bull markets further improves the model fit, just like the case for the book-to-market ratio.

Table A1: **Asymmetric Betas of Dividend Yield Portfolios, July 1927 to December 2014.**
I estimate up-market betas ($\beta_{down} + \beta_{up-down}$), down-market betas ($\beta_{down}$), bear-market betas ($\beta_{bull} + \beta_{bear-bull}$), and bull-market betas (β_{bull}).
$Value_{DP,t} = \alpha + [\beta_{down} + I_{up,t}\beta_{up-down}]r^e_{m,t} + \epsilon_t$
$Value_{DP,t} = \alpha + [\beta_{bull} + I_{bear,t-1}\beta_{bear-bull}]r^e_{m,t} + \nu_t$
The three groups of DP strategies are based on the number of portfolios formed. The left set of results is for a zero-cost portfolio long the highest DP tercile, and short the lowest DP tercile. The middle set of results is for a portfolio long the highest DP quintile, and short the lowest DP quintile. The right set of results is for a portfolio that's long the highest DP decile, and short the lowest DP decile.

	DP - Terciles			DP - Quintiles			DP - Deciles		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
α	0.19%	-0.39%	0.30%	0.19%	-0.46%	0.31%	0.22%	-0.68%	0.40%
	(1.67)	(-2.42)	(2.87)	(1.41)	(-2.47)	(2.58)	(1.33)	(-2.95)	(2.66)
$\beta / \beta_{down} / \beta_{bull}$	-0.060	-0.217	-0.332	-0.082	-0.256	-0.398	-0.122	-0.363	-0.531
	(-2.84)	(-5.81)	(-12.43)	(-3.35)	(-5.94)	(-12.96)	(-4.02)	(-6.82)	(-14.14)
$\beta_{up-down}$		0.302			0.335			0.465	
		(5.07)			(4.88)			(5.47)	
$\beta_{bear-bull}$			0.571			0.663			0.862
			(14.80)			(14.94)			(15.88)
R^2	0.76%	3.15%	18.09%	1.06%	3.26%	18.62%	1.52%	4.26%	20.80%

Appendix F Bear Market Definition

My results on time-varying betas are robust to different bear market definitions. For the bulk of the paper, I have used cumulative returns in the past 24 months to define a bull market, as in Daniel and Moskowitz (2013). Table A2 presents the results of time-varying betas on HML if different bear-market definitions are used. In particular, column (6) contains the benchmark results when using the 24-month definition I have used throughout the paper. The general pattern of the table is that in bear markets, beta on value is large and positive. In bull markets, value betas range from small and positive to small and negative, but are all generally much smaller in magnitude compared to the bull-market betas.

Table A2: **Time-Varying HML Betas under Different Bear Market Definitions, July 1926 to December 2014.**
I estimate market models allowing for different HML betas in bull markets ($\beta_{bull} + \beta_{bear-bull,i}$) and bear markets ($\beta_{bull}$).
$HML_t = \alpha + [\beta_{bull} + I_{bear,t-1}\beta_{bear-bull,i}]r^e_{m,t} + \nu_t$
Where i is the number of previous month used. For example, $\beta_{bull} + \beta_{bear-bull,12}$ is the bear-market beta on HML, with the bear market defined as the cumulative return over the past 12 months being negative. Newey and West (1987) t-statistics are shown in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	0.30%	0.34%	0.33%	0.38%	0.38%	0.40%	0.39%	0.40%	0.30%	0.35%
	(2.84)	(2.84)	(2.65)	(2.99)	(2.98)	(3.22)	(3.00)	(3.54)	(2.83)	(2.92)
β_{bull}	0.140	0.060	0.091	0.008	-0.010	-0.069	-0.087	-0.088	-0.030	-0.017
	(7.19)	(1.21)	(1.24)	(0.12)	(-0.14)	(-1.04)	(-1.30)	(-1.31)	(-0.34)	(-0.19)
$\beta_{bear-bull,6}$		0.145								
		(1.37)								
$\beta_{bear-bull,9}$			0.103							
			(0.75)							
$\beta_{bear-bull,12}$				0.257						
				(2.14)						
$\beta_{bear-bull,18}$					0.299					
					(2.23)					
$\beta_{bear-bull,24}$						0.438				
						(3.88)				
$\beta_{bear-bull,30}$							0.482			
							(4.28)			
$\beta_{bear-bull,36}$								0.521		
								(4.97)		
$\beta_{bear-bull,48}$									0.473	
									(4.02)	
$\beta_{bear-bull,60}$										0.467
										(3.41)