THE UNIVERSITY OF CHICAGO

ESSAYS IN MACROECONOMICS AND FINANCE

A DISSERTATION SUBMITTED TO
THE FACULTY OF THE DIVISION OF THE SOCIAL SCIENCES
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

KENNETH C. GRIFFIN DEPARTMENT OF ECONOMICS

BY
JEREMIAH BEJARANO

CHICAGO, ILLINOIS
JUNE 2021

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGMENTS

I am grateful to my advisors, Lars Hansen, Ralph Koijen, and Harald Uhlig, for all their support, guidance, and encouragement over the years. I am also grateful to the organizers and participants of the University of Chicago's Capital Theory and Economic Dynamics and Financial Markets working groups. And, finally, I owe an additional thanks to my classmates, friends, family, and my wife Marie.

ABSTRACT

In the first chapter of my dissertation, I characterize the relationship between dividend dynamics and the term structure of equity risk premia. Within a class of log-linear asset pricing models, I show that the risk exposure associated with dividend futures is equal to the impulse response function of dividends and that the average slope of the term structure depends on the relationship between the permanent and transitory components of dividends. Going beyond the class of log-linear models, I then explore the consequences of adding a transitory, mean-reverting component to dividend dynamics within several classic asset pricing models, such as the extended consumption capital asset pricing model and an external habits model. Recent empirical evidence suggests that the term structure of equity may be downward sloping on average, which is at odds with the traditional specification of many common asset pricing models. I show that this potential discrepancy can be reconciled by adjusting cash flow growth dynamics in the proposed way.

In the second chapter of my dissertation, I argue that the term structure of equity as characterized by expected holding period returns on dividend strips can be used as a diagnostic to evaluate the quantity dynamics that arise in a macroeconomic model. For instance, as shown in the first chapter, the risk exposures associated with dividend futures are equal to the impulse responses of aggregate consumption with respect to the underlying shocks. As an application, I derive the asset pricing implications of a multi-sector production network model and use this to shed light on relative importance of idiosyncratic and aggregate total factor productivity (TFP) shocks. Though aggregate TFP in the U.S. over the last 60 years has grown approximately 1.4 percent annually, these gains have been dispersed across individual sectors, with some sectors even seeing substantial declines. This dispersion is either the result of idiosyncratic sectoral shocks or aggregate shocks that shift the composition of the economy without necessarily affecting long-run aggregate output. Decomposing the contribution of each shock to this term structure of equity, I show that the shift shocks contribute to a downward sloping term structure of equity while others contribute to an

upward sloping term structure. Thus, imposing a downward sloping term structure in this model amounts to putting a lower bound on the contribution of aggregate shifts relative to other shocks.

CHAPTER 1

CHARACTERIZING THE ROLE OF DIVIDEND DYNAMICS IN THE TERM STRUCTURE OF EQUITY RISK PREMIA

1.1 Introduction

As early as Gordon (1962) and Lucas (1978), researchers have understood that the proper pricing of the individual cash flows paid by an asset, the so-called dividend strips, is crucial to understanding the pricing of the asset as a whole. Since van Binsbergen, Brandt, and Koijen (2012) provided the first direct measurement of dividend strips on a market index, there has been considerable progress made in using the strips to construct additional empirical moments to aid in distinguishing between competing asset pricing models. This approach has been especially enticing given that many leading asset pricing models, such as the external habits model of Campbell and Cochrane (1999) or the long-run risk model of Bansal and Yaron (2004), predict flat or upward sloping term structures of expected returns and volatilities on dividend strips while the empirical evidence, such as in van Binsbergen, Brandt, and Koijen (2012) or van Binsbergen et al. (2013), suggests that this term structure may be downward sloping. Historically, most of these asset pricing models were developed with consideration to other empirical features (e.g., in response to the risk-premium puzzle or the risk-free rate puzzle) and not the properties of the dividend strip returns. Thus, as argued by Belo, Collin-Dufresne, and Goldstein (2015), the properties of the dividend strip returns serve as an out-of-sample test for these competing asset pricing models.

Several recently developed models have been able to successfully generate risk premia for dividend strips that decline with maturity. A subset of these have done so by altering the technology process for consumption and dividends. Example include Nakamura et al. (2013), who leverage the partial nature of recoveries after rare disasters, and Belo, Collin-Dufresne, and Goldstein (2015), who modify dividend dynamics so as to produce stationary leverage ratios. This is in contrast to many of the leading asset pricing models, which assume that consumption and dividends follow a random walk. In this chapter, I investigate the role in which these modifications to cash flow dynamics play in determining the shape of the term structure of risk premia on dividend strips.

To understand the role of dividend dynamics in shaping the term structure of equity risk premia, I develop a series of analytical characterizations of dividend risk premia in as general a setting as possible. I begin with a broad class of reduced-form log-linear asset pricing models in which both dividend dynamics and the stochastic discount factor dynamics are flexibly specified as scalar processes driven by a Gaussian vector autoregression. In this framework, I express risk premia as a product of risk prices and risk exposures, and demonstrate a sharp link between the term structure of dividend risk exposures and the shape of the impulse response function of dividends. Analogous to the result of Alvarez and Jermann (2005), who show that the risk exposures associated with holding the limiting long-maturity bonds depend only on the transitory of component of marginal utility, I furthermore show that long-maturity dividend futures depend only on the permanent component of the dividend process.

I continue by adding more structure. Here, I consider the implications of flexible growth dynamics within an extended consumption capital asset pricing model (extended CCAPM). Supposing that a representative investor has Epstein-Zin recursive preferences (Epstein and Zin, 1989) and that dividend growth is proportional to consumption growth, I show that the average slope of the term structure of equity puts restrictions on the composition of the permanent and transitory components of growth. I express this as a bound on the conditional covariance of contemporaneous dividend growth and the transitory component of growth. In particular, if the term structure of equity is on average downward sloping, these quantities must be positively correlated—indicating that dividends must exhibit some degree of mean reversion.

I conclude by exploring the consequences of modifying dividend dynamics within the external habits model of Campbell and Cochrane (1999). An important limitation of the class of models that I examine is that risk prices are constant over time. An important question that remains is, to what degree do the results above approximate the resulting behavior in non-linear models? To explore the effects of dividend dynamics in a setting with varying risk prices, I modify dividends process in the external habits model of Campbell and Cochrane (1999) so that dividends feature both a permanent and transitory component. Using a simple process for dividend growth, I demonstrate that the slope of the term structure of equity in this model depends on the composition of permanent and transitory growth, and crucially, on whether dividends feature mean-reversion.

In the remaining portion of this section, I review the relevant literature. In the section following that, I define dividend strips and the term structure of equity risk premia. In the following section, I describe the class of reduced form asset pricing models under consideration and derive risk prices and risk exposures in terms of the dynamics of dividends. In the following section, I analyze the model under the additional assumptions of Epstein-Zin preferences and proportional dividend/consumption growth. Finally, in the next section I set up a modified version of the external habits model of Campbell and Cochrane (1999) and solve it.

1.1.1 Related Literature

The literature in asset pricing that studies this term structure of equity is a relatively new one. Lettau and Wachter (2007) and Hansen, Heaton, and Li (2008) were among the first to emphasize the importance of the term structure of risk prices and risk exposures for asset pricing. These initial studies were motivated by potential for differences in cash flow growth across firms to explain the value premium. van Binsbergen, Brandt, and Koijen (2012) and van Binsbergen et al. (2013) were the first to develop and explore empirical counterparts to the holding-period return associated with dividend strips and the associated term structure. They document that the term structure is downward sloping on average, meaning that the expected holding-period return on short-maturity equity is higher than the return on long-maturity equity. Although there is an ongoing debate regarding the robustness of these claims,[1] the slope of this term structure has emerged as a powerful way to distinguish between competing asset pricing models, since a downward sloping term structure is inconsistent with many leading asset pricing models. And even if the term structure is not downward sloping, the empirical evidence appears to contradict the prediction of a strongly upward sloping term structure as appears in leading asset pricing models, such as the long-run risk model of Bansal and Yaron (2004) the external habits model of Campbell and Cochrane (1999). Consequently, measuring the term structure and addressing this potential challenge represents an important and active area of research within finance.

Several models have had some success in explaining the average downward sloping term structure of risk premia. A few examples of models that capture this feature are Ai et al. (2012), Andries, Eisenbach, and Schmalz (2014), and Nakamura et al. (2013). Some do so by modifying the preferences of the representative agent, altering their beliefs, or considering alternative technology specifications. van Binsbergen and Koijen (2017) provide a nice overview of such models. In this paper, my focus is on understanding the underlying mechanism driving the results within the subset of such models that achieve a downward sloping term structure via alternative technology specifications. These models result in dynamics that deviate from the random walk endowment specification seen in many traditional asset pricing models. For example, Nakamura et al. (2013) specifies a model in which the economy suffers rares disasters, from which it partially recovers gradually over time. Similarly, Belo, Collin-Dufresne, and Goldstein (2015) modify dividend dynamics so as to produce stationary leverage ratios. That is, they construct a model in which shareholders are forced to follow a policy in which they divest (invest) when leverage is low (high), thus shifting risk from long- to short-horizon dividend strips. In both cases, the result is that dividends behave like a unit root process with mean reversion. In this paper, I seek to sharply characterize the

1. See, e.g., Schulz (2016), van Binsbergen and Koijen (2016), Boguth et al. (2019), or Bansal et al. (2020).

contribution of such dynamics to the term structure of equity.

1.2 The Term Structure of Equity Risk Premia

An asset with cash flows or dividend payments payed over time can be viewed as a collection of claims to the individual payments in each period, often called "dividend strips." That is, given a stochastic discount factor process, $\{S_t\}$, suppose that an asset pays a series of cash flows, $\{D_t\}$. The price of the asset, the claim to this series of cash flows, is then

$$P_t = \mathbb{E}_t \left[\sum_{\tau=1}^{\infty} \frac{S_{t+\tau}}{S_t} D_{t+\tau} \right]. \tag{1.1}$$

Likewise, the price of the τ-horizon dividend strip is

$$P_t^{\tau} = \mathbb{E}_t \left[\frac{S_{t+\tau}}{S_t} D_{t+\tau} \right], \tag{1.2}$$

implying $P_t = \sum_{\tau=1}^{\infty} P_t^{\tau}$. The one-period holding period return on these dividend strips,

$$R_{t+1}^{\tau} = \frac{P_{t+1}^{\tau-1}}{P_t^{\tau}}. \tag{1.3}$$

The properties of P_t^{τ} and R_{t+1}^{τ}, where the asset whose cash flows under consideration are the market portfolio or wealth portfolio, constitute the *term structure of equity.*[2] Decomposing a broad market index, such as the S&P 500, into these strips in this way provides a richer set of information about future cash flows and discount rates. This can be seen by comparing the price of the index, P_t, in (1.1) to the prices of the strips, $\{P_t^{\tau}\}$, in (1.2). The price of the index contains information about the present discounted sum of future payments, while the strips give more granular information about how those cash flows and discount rates behave

2. Several different aspects of this term structure of equity are studied in the literature. In this paper, I focus on the properties of the holding period returns on the dividend strips. Alternatives include "equity yields" (dividend-price ratios associated with equity securities) as well as returns on assets held over different horizons.

at different horizons.

It it worth mentioning that the term structure of equity bears analogy to the term structure of interest rates associated with risk-free bonds. A zero-coupon bond can be interpreted simply as a dividend strip whose cash flow payment is fixed. For example, if the payment were fixed at unity, the time t price of the risk-free bond maturing at time $t+\tau$ would be $B_t^\tau = \mathbb{E}_t\left[S_{t+\tau}/S_t\right]$ and the holding period return would be

$$R_{t+1}^{f,\tau} = \frac{B_{t+1}^{\tau-1}}{B_t^\tau}. \tag{1.4}$$

As I show in the following upcoming sections, the macroeconomic information contained in the term structure of equity is different but complementary to that contained in the term structure of interest rates.

Given the framing of dividend strips as a risky analog to zero-coupon bonds, it is natural to focus on the contribution of dividend risk to the pricing of the security. This, thus, motivates interest in the excess holding period return of a dividend strip in excess of the holding period return on a risk-free zero coupon bond with the same maturity,

$$\log R_{t+1}^\tau - \log R_{t+1}^{f,\tau}.$$

Note that the holding period return on the τ-horizon risk-free bond is indeed risky relative to the short-term risk-free rate. Nonetheless, this excess return captures the premium associated with the risky nature of the future dividend, $D_{t+\tau}$. For this reason, the holding period returns associated with an asset called a "dividend future" are of particular interest. A *dividend future* is a forward claim on a dividend strip. It is defined as a contract in which an investor enters into an agreement at time t in which she agrees to pay P_t at time $t+\tau$ in exchange for the cash flow $D_{t+\tau}$. That is, the price is agreed upon at time t while the money is exchanged at time $t+\tau$. By no-arbitrage, the price of a dividend future must be

$$F_t^\tau = \frac{P_t^\tau}{B_t^\tau}. \tag{1.5}$$

Thus, the log of the holding period return associated with a dividend futures is

$$\log R_{t+1}^{DF,\tau} := \log \frac{F_{t+1}^{\tau-1}}{F_t^\tau} = \log R_{t+1}^\tau - \log R_{t+1}^{f,\tau}, \tag{1.6}$$

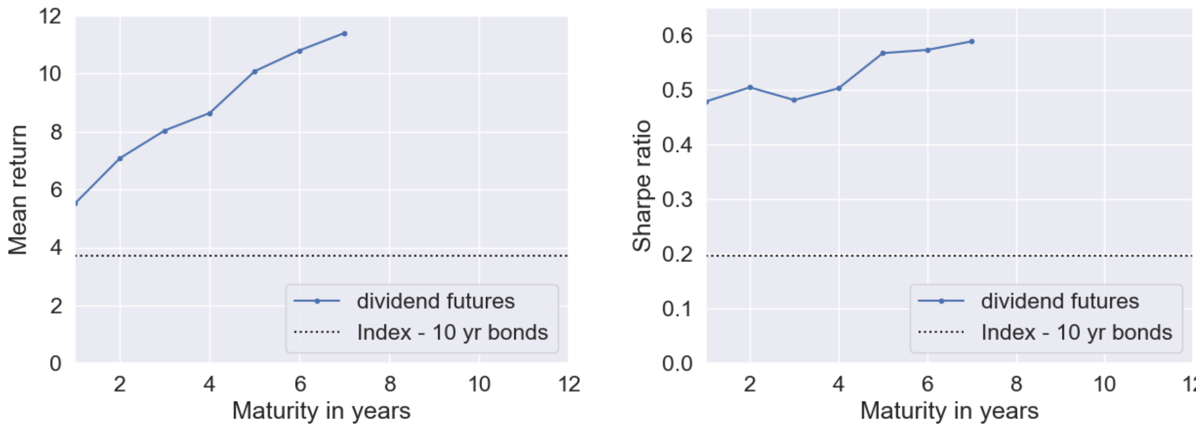
thus capturing the dividend risk premium described above.

To get a sense of the empirical facts associated with the term structure of dividend risk premia, Figure 1.1 plots summary statistics for the average returns and Sharpe ratios on dividend futures of various maturities on the dividends paid by the S&P 500 index, as reported in van Binsbergen and Koijen (2017). The figure plots the mean returns and Sharpe ratios for dividend futures with maturities from one to seven years only, since dividend futures with maturities greater than seven are not traded. The horizontal dotted line in the figure reports the average return on the S&P 500 index, in excess of the average holding period returns on 10 year Treasury bonds. Since these dividend futures are only traded for maturities of one to seven years, the right tail of the plot, the average dividend future returns at longer horizons, must be inferred by incorporating the information contained in the returns on the index. This can be done by noting that the return on the index can be expressed (up to a first-order approximation) as a weighted average of the returns on the dividend strips associated with the index. Rewritten as excess returns, van Binsbergen and Koijen (2017) derive the approximation where the excess return on the market portfolio is approximately equal to a weighted average of the dividend future returns,

$$R_{t+1}^M - R_{t+1}^{f,10} \approx \sum_{\tau=1}^{\infty} w_{t,\tau} R_{t+1}^{DF,\tau}, \tag{1.7}$$

where $w_{t,\tau} = P_t^\tau / P_t$. Using this, the key stylized fact to be seen in Figure 1.1 is that, while

Figure 1.1: Term structure of equity



Average returns and Sharpe ratios for dividend futures across maturities of one to seven years. These are dividend futures of dividends paid out by the S&P 500, over a sample of daily returns spanning Nov. 2002 - Jul. 2014, as reported in van Binsbergen and Koijen (2017). Figures reported are annualized. The dotted line represents the figures associated with the index, in excess of returns on 10 year Treasury bonds.

returns and Sharpe ratios may appear to be rising over the maturities 1–7, the lower average returns on the underlying index suggest that the term structure must be downward sloping at higher, unobserved maturities. Since the average excess return on the index is uniformly less than the average returns of the dividend futures with maturities of one to seven years, the implied average returns on hypothetical longer-term dividend futures must be lower. In this sense, the term structure of equity is downward sloping.

Loosely speaking, the fact that the term structure of equity is downward sloping means that cash flows at shorter horizons must be in some sense riskier than cash flows at longer horizons. As we will see, one way to achieve this is specify a mean-reverting process for dividends. Furthermore, the average returns on dividend futures at each tenor represents an additional moment condition that can be used to inform a model. As I will show in the next section, within a log-normal class of models, the risk exposures associated with dividend futures at each horizon are exactly equal to the impulse response function of dividends at that horizon. Thus, the empirical moments plotted in Figure 1.1 can help pin down the dynamics of dividends while the overall conclusion about the long-run slope of the term structure of equity can inform the composition of permanent and transitory components of growth.

1.3 Term structure of equity in a class of log-normal, reduced form asset pricing models

Consider a reduced form asset pricing model in which the vector of state variables x_t can be written as a linear state-space model,

$$x_{t+1} = Gx_t + Hw_{t+1}, \tag{1.8}$$

where G is an $N \times N$ matrix with spectral radius less than one, H is a $N \times M$ constant matrix, and $w_{t+1} \sim \mathcal{N}(0, I)$ is an i.i.d. random vector representing the underlying structural shocks driving the model. Suppose further that the stochastic discount factor (SDF), S_t, and a given cash flow process, D_t, can be written as

$$\log S_{t+1} - \log S_t = \mu_s + U_s' x_t + \lambda_s' w_{t+1} \tag{1.9}$$

$$\log D_{t+1} - \log D_t = \mu_d + U_d' x_t + \lambda_d' w_{t+1}, \tag{1.10}$$

where μ_s and μ_d are constants, and U_s, U_d, λ_s, and λ_d are conforming vectors.

By definition of the SDF, the gross returns over the period t to $t+1$, R_{t+1}, satisfy

$$\mathbb{E}\left[\frac{S_{t+1}}{S_t} R_{t,t+1} \mid x_t\right] = 1. \tag{1.11}$$

Supposing that we can express the log one-period returns of a given asset as

$$\log R_{t+1} = \mu_r + U_r' x_t + \lambda_r' w_{t+1}, \tag{1.12}$$

for some fixed μ_r, U_r, and λ_r, the one-period risk-premium takes on a simple form, given in Lemma 1: a product of a set of risk prices, λ_s, and a set of risk exposures, λ_r.

Lemma 1. *The risk premium, the expected returns associated with R_{t+1} in excess of the short-term risk-free rate, is equal to a product of risk prices and risk exposures:*

$$\log E[R_{t+1}] - \log[R^f_{t+1}] = - \underbrace{\lambda_s}_{\textit{risk-prices}} \cdot \underbrace{\lambda_r,}_{\textit{risk-exposures}} \tag{1.13}$$

where R^f_{t+1} is the one-period risk-free rate implied by the SDF.[3] *The vector λ_s thus represents a vector of* risk prices *associated with exposure to each source of risk in w_{t+1}. The vector λ_r represents the vector of* risk exposures*, defining the exposure of the returns R_{t+1} to each source of risk.*

The risk prices represent the marginal increase in the risk-premium associated with an additional unit of risk while the risk exposures represent the quantities of each source of risk that the asset with return R_{t+1} is exposed to.

1.3.1 Risk premia and Sharpe ratios in terms of impulse responses

Now, for the given cash flow process $\{D_t\}$ (e.g. aggregate dividends), the price of an asset that pays such cash flows is given in (1.1). The associated dividend strip prices are given in (1.2) and the holding period returns in (1.3). I now characterize the expected excess returns on these dividend strips in terms of the underlying model dynamics, as characterized by the impulse response functions. In this setting the risk premium associated with holding period returns has a simple characterization in terms of the impulse response functions of the dividends and stochastic discount factors processes, given in Proposition 3. I preface this with a definition of these impulse response functions.

3. See the Appendix, Section A.1.1 for the proof, and equation (A.1) for the expression for the one-period risk-free rate.

Definition 2. Let $\psi_d(\tau)$ be the impulse response function of the dividends process D_t at horizon τ. That is, $\psi_d(\tau)$ is defined so as to satisfy

$$\Delta\mathbb{E}_{t+1}\left[\log D_{t+\tau}\right] = \psi_d(\tau) \cdot w_{t+1}, \tag{1.14}$$

where $\Delta\mathbb{E}_{t+1}\left[\log D_{t+\tau}\right] \coloneqq \mathbb{E}_{t+1}\left[\log D_{t+\tau}\right] - \mathbb{E}_t\left[\log D_{t+\tau}\right]$. Define ψ_s similarly as the impulse response function for the log SDF, $\log S_t$.

Most of the derivations that I will present are written in terms of the impulse response functions, as they provide a convenient way to link asset pricing results with commonly studied objects in macroeconomics. Now, recall that $R^{\tau,f}_{t+1}$ is the holding period return associated with holding the zero-coupon risk-free bond with maturity τ, defined in (1.4). Define the short-term risk-free rate as $R^{f}_{t+1} = R^{f,1}_{t+1}$. This gives us the following characterization of the risk premium associated with dividend strips:

Proposition 3. *The risk premium associated with the holding-period return on the τ-horizon dividend strip is*

$$\log\mathbb{E}[R^{\tau}_{t+1}] - \log\mathbb{E}[R^{f}_{t+1}] = -\underbrace{\lambda_s}_{\textit{risk-prices}} \cdot \underbrace{(\psi_s(\tau) - \psi_s(1) + \psi_d(\tau)).}_{\textit{risk-exposures}} \tag{1.15}$$

The proof of this claim is given in the appendix, in Section A.1. To better understand this formula, first note that the impulse response functions measure the sensitivity of each process to the underlying shocks over alternative horizons. λ_s describes the prices associated with a unit of risk from each source while $(\psi_s(\tau) - \psi_s(1) + \psi_d(\tau))$ describes the quantities of risk that the holding period return is exposed to.[4] An interpretation of (1.15) is that the risk exposures associated with the holding-period return are comprised of two components:

4. In the more general framework of Borovička and Hansen (2014), these are shock-value and shock-exposure elasticities, respectively, and their product $\lambda_s \cdot (\psi_s(\tau) - \psi_s(1) + \psi_d(\tau))$, represents the shock-price elasticities at each horizon j.

a dividend-risk channel, embodied in the term $\psi_d(\tau)$, and a valuation channel, embodied in the term $\psi_s(\tau) - \psi_s(1)$. The dividend-risk channel captures the risk associated with fluctuations in the cash-flow process and the valuation channel captures the risk associated with changing prices of the claim.

With this result in hand, the derivation of the expected returns associated with dividend futures follows immediately. This, along with the expected holding period returns on τ-maturity risk-free bonds, is given in Corollary 4.

Corollary 4. *Since the zero-coupon bond with maturity τ can be interpreted as a dividend strip with a fixed cash flow, the risk premium associated with holding this bond for a period is easily calculated to be*

$$\log \mathbb{E}\left[R^{f,\tau}_{t+1}\right] - \log \mathbb{E}\left[R^{f}_{t+1}\right] = - \underbrace{\lambda_s}_{\text{risk prices}} \cdot \underbrace{(\psi_s(\tau) - \psi_s(1)).}_{\text{risk exposures}} \tag{1.16}$$

Thus, expected return on dividend strip in excess of the expected holding period return on a risk-free bond with the same maturity is

$$\log \mathbb{E}\left[R^{\tau}_{t+1}\right] - \log \mathbb{E}\left[R^{f,\tau}_{t+1}\right] = - \underbrace{\lambda_s}_{\text{risk prices}} \cdot \underbrace{\psi_d(\tau).}_{\text{risk exposures}} \tag{1.17}$$

This derivation can be interpreted as the expected return on a dividend future with maturity τ, less a one-half variance term.

The expression (1.17) is particularly useful because, it demonstrates that we can control for the valuation channel in this setting by simply netting out the returns associated with holding a risk free bond with the same maturity as the dividend strip. Thus, the risk exposures in this expression are simply equal to the impulse responses of dividends ψ_d. Furthermore, this expression can conveniently be interpreted as the return on a dividend future. As defined in (1.6), the log return on a dividend future with maturity τ is equal to the difference of the log returns of the τ-horizon dividend strip return and the τ-horizon risk-free bond return. However, note that the expected value of this return differs from the expression appearing in (1.17). Arising from Jensen's inequality, the log of the expectation is not necessarily equal to the expectation of the log. Thus, the expressions differ by a half-variance term. This, however, can be corrected for in empirical work. The full characterization of the the expected holding period returns on dividend futures is given in the appendix. In this paper, however, for simplicity I will use (1.17).

Definition 5 (Dividend Future Premium)**.** Define DF_{τ} as the expected return on a dividend strip in excess of the expected return on a risk-free bond with the same maturity,

$$DF_{\tau} \coloneqq \log \mathbb{E}\left[R^{\tau}_{t+1}\right] - \log \mathbb{E}\left[R^{f,\tau}_{t+1}\right]. \tag{1.18}$$

I refer to this as the "dividend future premium." It is characterized in (1.17). It differs from the expected holding period return on a dividend future by a half-variance term.

I use the expression for the dividend future premium, DF_{τ}, over all horizons $\tau = 1, 2, 3, \ldots$ to represent the *term structure of equity risk premia.* Thus, as demonstrated in Corollary 4, the average returns in Figure 1.1 constitute moment conditions that put restrictions on the dynamics of aggregate dividends, as characterized by the impulse responses $\psi_d(\tau)$, and the risk prices λ_s.

An expression for the Sharpe ratios, also given in Figure 1.1, follow similarly. These are given in the follow proposition.

Proposition 6. *From the definition of the impulse response functions and the derivation of the dividend future premium in Proposition 3 and Corollary 1.17,*

$$Cov(\Delta \log S_{t+1}, \Delta \mathbb{E}_{t+1}\left[\log D_{t+\tau}\right]) = \psi_s(1) \cdot \psi_d(\tau)$$

and

$$Var(\log R^{\tau}_{t+1} - \log R^{f,\tau}_{t+1}) = \|\psi_d(\tau)\|^2.$$

It then follows that the ratio of this risk premium to the standard deviation of the excess returns is

$$SR_\tau := \frac{\log \mathbb{E}[R^{\tau}_{t+1}] - \log \mathbb{E}[R^{f,\tau}_{t+1}]}{\sqrt{Var(\log R^{\tau}_{t+1} - \log R^{f,\tau}_{t+1})}} = -\psi_s(1) \cdot \frac{\psi_d(\tau)}{\|\psi_d(\tau)\|}. \tag{1.19}$$

For ease of discourse, the definition I use for the Sharpe ratio is SR_τ. Again, discrepancies can be accounted for in empirical work.

As a final note, see that we can decompose the term structure into the contributions of each of the individual shocks in the economy. The shocks, w_{t+1}, are an i.i.d. random, normally distributed vector with mean $\vec{0}$ and variance-covariance matrix I. So, decomposing the contribution of each component in the vector w_{t+1} is a trivial Corollary of the expression in (1.17).

Corollary 7. *Since the dividend future premium is a dot product of a vector of risk prices and risk exposures associated with the individual components of w_{t+1}, expressed as $DF_\tau = -\lambda_s \cdot \psi_d(\tau)$, the contribution of the i'th shock to the dividend future premium DF_τ is given by*

$$DF_{\tau,i} := -\lambda_{s,i}\ \psi_{d,i}(\tau),$$

where $\lambda_s = [\lambda_{s,i}]$, $\psi_d(\tau) = [\psi_{d,i}(\tau)]$, and

$$DF_\tau = \sum_i DF_{\tau,i}.$$

This decomposition can be used to examine how each shock contributes to the term structure to identify which shocks might be most important for achieving an adequate fit of the model-implied term structure to the empirically-observed term structure.

1.3.2 Risk premia in terms of permanent and transitory components of the dividend process

Here I discuss the information contained by the extremes of the term structure of equity—the short-term compared to the very long-term. Investigation of these extremes can be aided by characterizing the risk premia in terms of the permanent and transitory components of dividend variation. As demonstrated in (1.17), the risk exposures associated with the dividend futures are equal to the impulse response functions of dividends. The long-term limit of impulse responses, $\psi_d(\infty) = \lim_{\tau \to \infty} \psi_d(\tau)$, has a useful interpretation as the permanent component of D_t. This helps us understand the information content of term structure of equity.

To see this, consider the following decomposition.

Lemma 8. *Given an arbitrary log linear process of the form* $\log Y_{t+1} - \log Y_t = \mu_y + U_y' x_t + \lambda_y' w_{t+1}$ *with* $x_{t+1} = G x_t + H w_{t+1}$*, the process can be decomposed into a deterministic trend, permanent, and transitory component. That is,*

$$\log Y_t = \underbrace{t\,\mu_y}_{\textit{det. trend}} + \underbrace{\sum_{k=1}^{t} M_y w_k}_{\textit{permanent component}} + \underbrace{F_y x_t}_{\textit{stationary component}} + \underbrace{\log Y_0 - F_y x_0}_{\textit{initial conds.}},$$

where

$$\begin{aligned} F_y &:= -U_y'(I-G)^{-1} \\ M_y &:= \lambda_y' + U_y'(I-G)^{-1} H. \end{aligned} \tag{1.20}$$

Furthermore, given the impulse response function of Y*,* ψ_y*, note that*

$$\psi_y'(\infty) := \lim_{\tau \to \infty} \psi_y'(\tau) = M_y \tag{1.21}$$

and

$$\psi_y'(1) - \psi_y'(\infty) = F_y H. \tag{1.22}$$

Note that, in this decomposition, the permanent and transitory components are generally not uncorrelated.

Applying Lemma 8 to the results in Corollary 1.17, we see that the risk exposures in the extreme long-run depend exclusively on the permanent component of dividend growth while those associated with the short-term asset depend on a combination of the permanent and transitory components. This is written out in the following proposition,

Proposition 9. *The risk exposures associated with the dividend future premium at the extreme long horizon depend only on the permanent component of the dividend process,*

$$DF_\infty = -\lambda_s \cdot \underbrace{M_d,}_{\text{risk exposures}} \tag{1.23}$$

and the exposures associated with the short horizon futures depend on the sum of the permanent and transitory components,

$$DF_1 = -\lambda_s \cdot \underbrace{(F_d H + M_d)}_{\text{risk exposures}} \tag{1.24}$$

where F_d and M_d are defined as in (1.20).

Thus, the term structure of equity can give us information regarding the permanent and transitory variation of the cash flow process, D_t. Note that, in contrast, from (1.16) we can see that the risk exposures associated with the holding period returns on long-term bonds, $\psi_s(\infty) - \psi_s(1)$, depends exclusively on the transitory component of the SDF, S_t. This mirrors the result of Alvarez and Jermann (2005). Thus, in this sense the risk exposures associated with the long-horizon risk-free bonds contain different but complementary information to the long-horizon dividend futures.

1.4 The Role of Dividend Dynamics in the Extended Consumption CAPM

Up until this point, I have only considered a reduced form representation of the stochastic discount factor, S_t, and computed prices and expected returns associated with a generic cash flow process D_t. If we now put some additional structure on the economy, we can derive the restrictions that the term structure imposes on the economy. This is important because, if dividends and consumption share similar dynamics, the properties of dividend/consumption dynamics may affect the shape of the term structure through risk prices as well as risk exposures. Here, I do so by investigating the term structure of equity in an extended consumption CAPM model in which consumption and dividends are given the same flexible specification as in the previous section., That is, I assume an endowment economy in which a representative household has Epstein-Zin preferences, aggregate dividends are defined in terms of aggregate consumption, and aggregate consumption is driven by a state vector that evolves according to a vector autoregression.

As is commonly done in the asset pricing literature, assume that log aggregate dividend, $\log D_t$, are proportional to log aggregate consumption:

Assumption 10. *Let the aggregate dividends process be equal to a levered index of consumption,*

$$\log D_t = \eta \log \mathcal{C}_t, \tag{1.25}$$

where $\eta \geq 1$ is the leverage factor.

Now, suppose also that consumption $\mathcal{C}_t$ is defined exogenously by

$$\log \mathcal{C}_{t+1} - \log \mathcal{C}_t = \mu_{\mathcal{C}} + U_{\mathcal{C}}' x_t + \lambda_{\mathcal{C}}' w_{t+1}, \tag{1.26}$$

given constant conforming scalars and vectors, $\mu_{\mathcal{C}}$, $U_{\mathcal{C}}'$, and $\lambda_{\mathcal{C}}$. Suppose further that that a representative household has Epstein-Zin preferences given by the recursion

$$V_t = \{(1-\beta)(\mathcal{C}_t)^{1-\rho} + \beta[\mathcal{R}_t(V_{t+1})]^{1-\rho}\}^{1/(1-\rho)} \tag{1.27}$$

where $\mathcal{R}_t$ is the certainty equivalent operator defined by

$$\mathcal{R}_t(V_{t+1}) \equiv \mathbb{E}_t[(V_{t+1})^{1-\gamma}]^{1/(1-\gamma)},$$

ρ^{-1} is the elasticity of intertemporal substitution, and γ is the risk-aversion parameter.

When $\rho = 1$ with $\gamma > 1$, the agent still exhibits a concern for long-run risk. However, the form of the SDF simplifies and can be written in the previously described form. This characterization was first used in Hansen, Heaton, and Li (2008). This result is important because it indicates that in this version of the extended consumption CAPM, the same results from the previous section all hold.

Lemma 11. *Given the dynamics of consumption in (1.26) and the assumption of Epstein-Zin utility with $\gamma > \rho = 1$, the stochastic discount factor in equilibrium is*

$$\frac{S_{t+1}}{S_t} = \beta \left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-1} \frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]}.$$

and can be written as

$$\log S_{t+1} - \log S_t = \mu_s + U_s' x_t + \lambda_s' w_{t+1},$$

where constants μ_s, U_s, and λ_s are expressed as functions of the model parameters G, H, $\mu_{\mathcal{C}}$, $U_{\mathcal{C}}$, $\lambda_{\mathcal{C}}$. The expressions are given in the appendix.

Now, with this additional structure in place, suppose we now impose a restriction on the sign of slope of the term structure of equity. Supposing that over the extremes the term structure is downward sloping amounts to imposing $DF_\infty < DF_1$. Given our previous results, this amounts to putting restrictions on the relationship between the permanent and transitory components of consumption growth. This is expressed as follows.

Proposition 12. *Given the dynamics of consumption in (1.26) and the assumption of Epstein-Zin utility with* $\gamma > \rho = 1$*, imposing that the term structure of equity is downward sloping,* $DF_\infty < DF_1$*, amounts to imposing bounds on the permanent and transitory components of consumption,*

$$0 > \gamma \underbrace{\lambda_{\mathcal{C}} \cdot (U_{\mathcal{C}}'(I-G)^{-1}H)}_{\text{Cov. of contemp. } \mathcal{C}_t \text{ and transitory}} + (\gamma - 1) \underbrace{\beta U_{\mathcal{C}}'(I-\beta G)^{-1}HH'(I-G')^{-1}U_{\mathcal{C}}}_{\approx \text{ Var. of transitory}}. \tag{1.28}$$

When the subjective discount factor approaches unity, $\beta \to 1$*, this takes on a simpler form. Applying this approximation and using the notation of Lemma 8, this becomes*

$$Cov_t(\log \mathcal{C}_{t+1} - \log \mathcal{C}_t, F_{\mathcal{C}} x_{t+1}) > \frac{\gamma - 1}{\gamma} \, Var_t(F_{\mathcal{C}} x_{t+1}). \tag{1.29}$$

Since the variance term must be nonnegative and since $\gamma > 1$*, a downward sloping term structure imposes the restriction that the covariance of conditional consumption growth must be positively correlated with the transitory component of consumption growth. In other words, the consumption growth process must exhibit some degree of mean-reversion.*

Thus, we have shown here that the term structure of equity can inform us about the dynamics of consumption and dividend growth. Observation of the average returns on dividend futures at each horizon amount to an extra moment condition that can inform us of the impulse response function governing macroeconomic growth (Corollary 1.16). Furthermore, in the context of a macroeconomic model where dividends are taken to be levered consumption, the slope of the term structure of equity puts bounds on the relationship between the permanent and transitory components of consumption growth (Proposition 12). This information can be used to discriminate between various models that aim to describe the important sources of aggregate variation.

1.5 The Role of Dividend Dynamics in an External Habits Model

In this section, I move beyond the log-normal models considered in the previous sections and analyze the consequences of adding a transitory component to dividend growth in the external habits model of Campbell and Cochrane (1999). Most leading asset pricing models will not fit neatly into the log-linear, normal framework considered in Section 1.3. One major limitation of that framework, for instance, is that risk prices are constant. In this section, I consider a commonly used asset pricing model to get an idea of the degree to which the results derived above might approximate the results within a higher order model.

I begin by setting up the preferences of the model. Suppose that a representative agent has a utility function defined by

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{(\mathcal{C}_{t+j} - X_{t_j})^{1-\gamma} - 1}{1-\gamma}, \tag{1.30}$$

where β is the subjective discount factor, γ controls the curvature of the utility function, and X_t is the level of habit. Define surplus consumption by

$$\widetilde{S}_t \coloneqq \frac{\mathcal{C}_t - X_t}{\mathcal{C}_t}. \tag{1.31}$$

Let $\widetilde{s}_t = \log \widetilde{S}_t$. Suppose that log surplus consumption evolves according to an AR(1) process,

$$\widetilde{s}_t = (1-\phi)\bar{\widetilde{s}} + \phi_s \widetilde{s}_t + \lambda(\widetilde{s}_t)\epsilon_{c,t+1}, \tag{1.32}$$

where $\bar{\widetilde{s}}$ defines the unconditional average value of the log surplus ratio, ϕ_s governs its persistence, $\epsilon_{c,t}$ is an i.i.d. innovation to consumption growth, and $\lambda(\widetilde{s}_t)$ is a function that governs the sensitivity of this surplus to the innovations in consumption growth.

Now, to define the remaining objects in the model. Let log consumption growth follow a random walk,

$$\log \mathcal{C}_{t+1} - \log \mathcal{C}_t = \mu_c + \epsilon_{c,t+1}, \tag{1.33}$$

where $\epsilon_{c,t+1}$ is an i.i.d., mean zero, Normally distributed random variable, with volatility σ_c. As in Campbell and Cochrane (1999), let the sensitivity function take on the form,

$$\lambda(\tilde{s}_t) = \frac{1}{\exp\{\bar{\tilde{s}}\}}\sqrt{1 - 2(\tilde{s}_t - \bar{\tilde{s}})} - 1, \tag{1.34}$$

with $\bar{\tilde{s}} = \log\left(\sigma_c\sqrt{\frac{\gamma}{1-\phi_s}}\right)$. Now, for this exercise, I will consider a simplified process for dividends, relative to those considered in previous sections. Here I assume that aggregate dividends grow according to the process,

$$\log D_{t+1} - \log D_t = \mu_c + (1-\phi_z)(z_t - \bar{z}) + \epsilon_{c,t+1} + \epsilon_{z,t+1} \tag{1.35}$$

$$z_{t+1} = (1-\phi_z)\bar{z} + \phi_z z_t + \epsilon_{z,t+1}, \tag{1.36}$$

where ϕ_z governs the persistent of the slow-moving component of dividend growth, z_t, and

$$\begin{bmatrix} \epsilon_{c,t+1} \\ \epsilon_{z,t+1} \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(0, \begin{bmatrix} \sigma_c^2 & \rho\sigma_c\sigma_z \\ \rho\sigma_c\sigma_z & \sigma_z^2 \end{bmatrix}\right). \tag{1.37}$$

Under this process, consumption and dividend are cointegrated, as

$$z_t = \log \mathcal{C}_t - \log D_t \tag{1.38}$$

is stationary.

Now, I will solve the model. I use the solution method of Wachter (2005) and simulate the returns on dividend futures. Note that under this specification, mean reversion in dividends occurs only when $\epsilon_{c,t}$ and $\epsilon_{z,t}$ are negatively correlated. Thus, the results of Corollary 4 and Proposition 1.29 would suggest that the slope of the term structure of equity would depend crucially on the sign of ρ and the relative magnitudes of σ_c and σ_z. I test this by solving the model and simulating the dividend future returns under two separate specifications. In the first, I use the original specification of Campbell and Cochrane (1999), under which $\bar{z} = 0$

Table 1.1: Parameter choices

	CC1999 Values	Extended Model
Mean consumption growth, μ_c	1.89	1.89
Consumption growth volatility, σ_c	1.50	1.50
Utility curvature	2.00	2.00
Habit persistence	0.87	0.87
Discount rate	0.90	0.90
Conditional z_t volatility, σ_z	NA	20
Correlation of shocks, ρ	NA	-.8
Persistence of z_t	NA	0.95
Mean log ratio of consumption and dividends, $\bar{z}$	NA	0

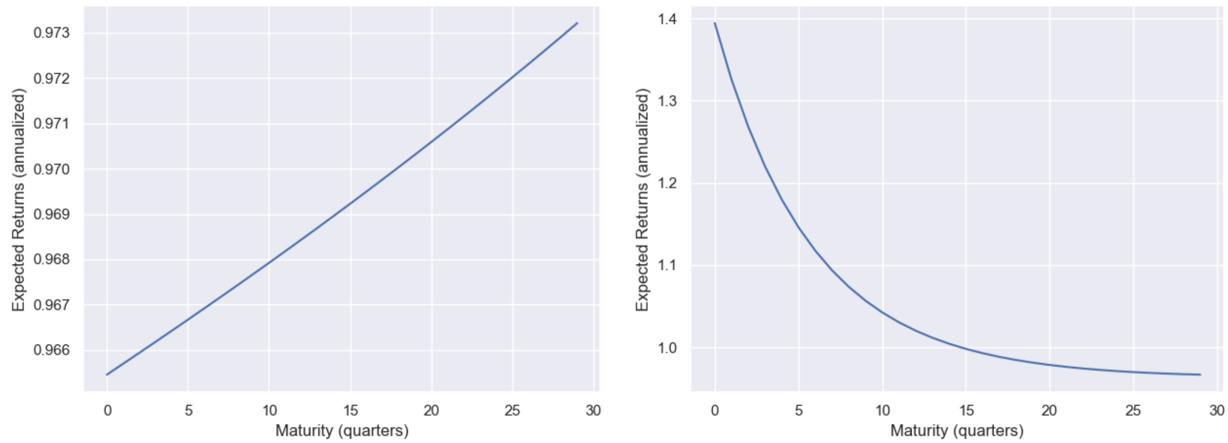
This table reports the two parameterizations which are compared when solving the model. The first parameterization uses the calibration of Campbell and Cochrane (1999) and the second modifies this to utilize the added transitory dynamics. The second parameterization is specified so have an exaggerated transitory component that creates mean reversion in dividends. Parameters are annualized, though the model is solved at a monthly frequency.

and $\sigma_z = 0$, and a second in which this slow moving component induces significant transitory variation and $\rho < 0$, creating mean reversion in dividends.

Table 1.1 reports the two parameterizations for which I solve the model. The first parameterization uses the calibration of Campbell and Cochrane (1999) and the second modifies this to utilize the added transitory dynamics. The second parameterization is specified so have an exaggerated transitory component that creates mean reversion in dividends. This is done to emphasize the effects of the modification to dividend dynamics. In each case, the model is solved on a grid of 1000 points and the 100,000 years of data are simulated to construct moments.

Figure 1.2 compares the resulting term structure of equity risk premia resulting from each parameterization. It plots the average log dividend future return over tenors of 1 to 30 quarters. As we can see, the slope of the term structure of equity in the classic Campbell-Cochrane calibration is upward sloping. In contrast, the addition of the mean-reverting component in dividends produces a downward sloping term structure of equity. Thus, these types of modifications to dividend dynamics can drastically change the shape of the term

Figure 1.2: Comparing term structure of equity risk premia in two different parameterizations



Traditional CC1999 calibration on the left. Calibration with mean-reversion on the right. DF_τ over various horizons, $\tau = 1, \ldots, 30$.

structure even in this kind of nonlinear model.

1.6 Conclusion

In this chapter, I have presented a series of characterizations of the term structure of equity risk premia in a variety of setting. I characterize the relationship between dividend dynamics and the term structure of equity risk premia within a class of log-linear asset pricing models, showing that the risk exposure associated with dividend futures is equal to the impulse response function of dividends and that the average slope of the term structure depends on the relationship between the permanent and transitory components of dividends. Going beyond the class of log-linear models, I then explore the consequences of adding a transitory, mean-reverting component to dividend dynamics within two different asset pricing models: the extended consumption capital asset pricing model and an external habits model of Campbell and Cochrane (1999). I demonstrate that a downward sloping term structure of equity depends the presence of mean-reversion in dividends. Even in a non-linear model, such as the external habits model considered here, the downward sloping term structure can be recovered by altering dividend dynamics in this way.

CHAPTER 2

SECTORAL SHIFTS, PRODUCTION NETWORKS, AND THE TERM STRUCTURE OF EQUITY

2.1 Introduction

This paper has two objectives. The first is to show how the term structure of equity can be used to inform quantity dynamics in a generic macroeconomics model. The term structure of equity has found use in the asset pricing literature as a way to discriminate between competing asset pricing models. I argue that it can also be used to discriminate between competing macroeconomic models. The second objective is to demonstrate an application of using this set of asset pricing facts within a multi-sector production network model. Output dynamics in this model are driven by idiosyncratic, sector-specific shocks and aggregate shocks to TFP. I show that we can derive the model-implied term structure of equity within this model and compare it against the empirically observed term structure. Furthermore, we can decompose this term structure to observe how each source of uncertainty—each shock—contributes to this term structure. Of these, I show that only the aggregate shocks that shift TFP between sectors without increasing long-run growth can potentially contribute to a downward sloping term structure. Thus, if we were to impose a downward sloping term structure on this model, we must impose a restriction that these shift shocks are large relative to the other shocks in the model. I describe these two objectives in greater detail here.

This first objective amounts to demonstrating how asset pricing data, specifically that associated with the term structure of equity, can be used to inform a macroeconomic model. Though most macroeconomic models of consumption and investment are not used to examine this term structure, they nonetheless contain theoretical predictions for it (Borovička and Hansen, 2014) and can be used to evaluate the empirical plausibility of various models. To build the intuition for this, I start be defining this term structure. This is also defined in Chapter 1 of this disseration. However, I repeat some expressions and derivations (but not all), to preserve a moderate amount of self-containment.

An asset with cash flows or dividend payments payed over time can be viewed as a collection of claims to the individual payments in each period, often called "dividend strips." Recall in Chapter 1 that, when the price of a claim to the cash flows is $\{D_t\}$ for a given stochastic discount factor process $\{S_t\}$, the price of the claim is

$$P_t = \mathbb{E}_t\left[\sum_{\tau=1}^{\infty} \frac{S_{t+\tau}}{S_t} D_{t+\tau}\right], \tag{2.1}$$

and the price of the τ-horizon dividend strip is

$$P_t^{\tau} = \mathbb{E}_t\left[\frac{S_{t+\tau}}{S_t} D_{t+\tau}\right], \tag{2.2}$$

where $P_t = \sum_{\tau=1}^{\infty} P_t^{\tau}$. The one-period holding period return on these dividend strips,

$$R_{t+1}^{\tau} = \frac{P_{t+1}^{\tau-1}}{P_t^{\tau}}, \tag{2.3}$$

and, as defined in equations (1.5) and (1.6) in Chapter 1, a dividend future captures a notion of the risk premium arising from dividend risk at each horizon τ,

$$\log R_{t+1}^{DF,\tau} \coloneqq \log \frac{F_{t+1}^{\tau-1}}{F_t^{\tau}} = \log R_{t+1}^{\tau} - \log R_{t+1}^{f,\tau}.$$

The literature in finance that analyzes this term structure of equity risk premia seeks to decompose broad market indexes, such as the S&P 500, into these strips and to measure the associated returns, since doing so gives a richer set of information about future cash flows and discount rates. Historically, the risk premia associated with returns on the market portfolio (where, for example, the S&P 500 is often used as a proxy) has been used to discipline an asset pricing or macroeconomic model. When decomposed into dividend strips and dividend futures at multiple horizons, a researcher can gain a more granular view of the macroeconomic information embedded in the market portfolio. This can be seen by comparing the price of the index, P_t, in (1.1) to the price of the strip, P_t^{τ}, in (1.2). The price of the index contains information about the present discounted sum of future payments, while the strips given information about how those cash flows and discount rates behave at different horizons. This illustrates the intuition behind using this set of facts to inform quantity dynamics in a macroeconomic model.

Furthermore, within this literature there is some evidence that suggests that this term structure is downward sloping, in the sense that the average returns on longer horizon strips is lower than shorter horizon strips (see, e.g., van Binsbergen, Brandt, and Koijen 2012; van Binsbergen et al. 2013; Gormsen 2018). In this paper, I argue that this information about the slope can put restrictions on the contributions of various shocks to short-run and long-run growth. Thus, together with the previous intuition, we can see that the information contained in the dividend strip prices and their associated returns represent a rich avenue of opportunities to explore the ability of asset prices to inform macroeconomic models.

In Figure 1.1 in Chapter 1, I plot summary statistics for the average returns and Sharpe ratios on dividend futures based on the dividends paid by the S&P 500 index, as reported in van Binsbergen and Koijen (2017). As described in Chapter 1, using the approximation in (1.7), we can infer that the term structure of equity, as captured by the expected returns on dividend futures at various horizons τ, must be downward sloping at the longer, unobserved horizons. As shown in Chapter 1, this means that aggregate cash flows at shorter horizons must be in some sense riskier than cash flows at longer horizons and that the set of collection of empirical average dividend future returns observed over these seven tenors constitute a set of moment conditions that put restrictions of the dynamics of dividends in the model. As is common within the macro-finance literature, if we take the S&P 500 as a proxy for the market portfolio and assume that dividends are related to aggregate consumption, then the properties associated with the returns on dividend futures should tell us something about the dynamics of aggregate consumption and output dynamics.

The second objective of this paper is an application of the results of the first. As an application of using the term structure of equity to evaluate a macroeconomic model, I examine a simple multi-sector production networks model through the lens of asset pricing. In this model, the source of uncertainty that I consider are shocks to total factor productivity (TFP). This model features idiosyncratic shocks specific to individual sectors as well as common, aggregate shocks. I consider an aggregate shock that moves all sectors up and down, as well as shocks that shift TFP between sectors without necessarily affecting output in the long-run (shift shocks). I decompose the contribution of each of these shocks to show how each shock contributes to the shape of the term structure of equity. I show that the shift shocks increase the risk premia associated with short maturity dividend strips without having much of an effect of longer maturity dividend strips. Thus, these shift shocks contribute to a downward sloping term structure of equity while all other shocks in this model contribute to an upward sloping term structure. Thus, I demonstrate that if we were to impose a downward sloping term structure, as some of the empirical asset pricing literature suggests, then this model amounts to putting a lower bound on the contribution of aggregate shift shocks relative to other sources. I argue that, given that at the heart of this model is series of real investment decisions, comparing the model's implicit asset pricing predictions against those observed in the data is a reasonable exercise. Furthermore, analyzing the term structure implied by the model serves as a convenient and readily interpretable way to evaluate the contributions of each shocks since the contributions are measured in terms of returns on traded assets.

2.1.1 Related Literature

The literature in asset pricing that studies this term structure of equity is a relatively new one. van Binsbergen, Brandt, and Koijen (2012) and van Binsbergen et al. (2013) were among the first to develop and explore empirical counterparts to the holding-period return associated with dividend strips and the associated term structure. They document that the term structure is downward sloping on average, meaning that the expected holding-period return on short-maturity equity is higher than the return on long-maturity equity. Although there is an ongoing debate regarding the measurement of this term structure, the slope of this term structure has emerged as a powerful way to distinguish between competing asset pricing models, since a downward sloping term structure is inconsistent with many traditional asset pricing models, such as the long-run risk model of Bansal and Yaron (2004) the external habits model of Campbell and Cochrane (1999). Consequently, measuring the term structure and addressing this potential challenge represents an important and active area of research within finance.

Lettau and Wachter (2007) and Hansen, Heaton, and Li (2008) were among the first to emphasize the importance of the term structure of risk prices and risk exposures for asset pricing. The model of Lettau and Wachter (2007) exhibits a downward-sloping term structure, as it is designed to. However, the stochastic discount factor analyzed is specified exogenously and does not represent a fully-fledged model of equilibrium. A logical next step would be to explore micro foundations that could give risk to such a model. What is more, though standard consumption-based asset pricing models already consider the joint modeling of asset prices and aggregate consumption, an equally important endeavor is to is to consider the joint restrictions between these series and other components within general equilibrium, such as aggregate output and investment. The literature that studies asset prices in full general equilibrium models with production does exactly this. As emphasized by Borovička and Hansen (2014), a fully specified dynamic stochastic general equilibrium model will have predictions for asset prices, including the term structure of risk premia. Thus, evidence regarding the term structure of equity represents a rich opportunity to "examine macroeconomic models through the lens of asset pricing."[1]

Several models have had some success in explaining the average downward sloping term

1. Hansen, Heaton, and Li (2008), Hansen and Scheinkman (2012), Borovicka et al. (2011), and Borovička and Hansen (2014) are examples of papers that develop tools and methods to examine macroeconomic models in this way.

structure of risk premia. A few examples of models that capture this feature are Ai et al. (2012), Andries, Eisenbach, and Schmalz (2014), and Nakamura et al. (2013). Some do so by modifying the preferences of the representative agent, altering their beliefs, or considering alternative technology specifications. van Binsbergen and Koijen (2017) provide a nice overview of such models. In this paper, I focus on the asset pricing outcomes within a relatively standard, frictionless production network economy. I use the model-implied term structure of equity in this setting to demonstrate how one can use the term structure to inform the relative importance of different shocks within this model and how the term structure of equity can be used to evaluate the empirical plausibility of the model's dynamics.

With regard to studying the asset pricing implications within a production network model, there are a few relative studies. For example, Herskovic (2018) explores a related question. In that paper, he decomposes consumption growth into three factors, where one is an aggregation of TFP shocks and the other two are related to innovations in the shape of the production networks. TFP growth, along with changes in the network structure over time, proxy for consumption growth and shocks to any of these factors should be priced in equilibrium. In contrast, the shape of the network in my model is constant over time. Furthermore, I distinguish between the network for intermediate goods and the network for investment goods and emphasize the ability of the investment network to propagate shocks over time. It is this gradual shock propagation that helps to inform the shape of the term structure of equity.

In a similar vein, Richmond (2019) explores centrality in global trade networks and shows a strong relationship between centrality and both interest rates and currency risk premia. The main mechanism in this paper is that central countries' consumption growth is more exposed to global consumption growth shocks via the trade network. This exposure is a contemporaneous exposure and it explains differences in the currency risk premia. In contrast, cross-sectional differences in risk-premia in my paper may arise due to differences in exposures across time. Though I also explore the effects of centrality on risk, the mechanism that I explore leads to a different measure of centrality depending on the horizon analyzed. Risk exposures in the short-run depend more on the intermediate goods network, while risk exposures in the long run tilt more towards the investment network.

Also, the main mechanism that drives the results in this production networks model relies on the hypothesis that shocks propagate through the network over time. Several recent papers provide evidence that these effects exist and are strong. Barrot and Sauvagnat (2016) and Carvalho et al. (2016) use natural disasters as a source of exogenous variation to identify firm- or industry-level idiosyncratic shocks. Acemoglu, Akcigit, and Kerr (2015) explore a variety of instruments and show that supply shocks transmit from supplier to customer and that shocks resembling demand shocks transmit from customer to supplier. An important paper related to the mechanism explored in my model is vom Lehn and Winberry (2019). While other papers have also explored the consequences of the shape of production networks, this paper emphasizes that the implications of the shape of the intermediate goods network are different from the investment network. They document that the investment network is dominated by a few "investment hubs" and that this structure is important for understanding the business cycle and the nature of sectoral comovement. Shocks to investment hubs have larger and more persistent effects on aggregate GDP and employment and these shocks lead many of these effects. In this paper, I adopt their framework and study the asset pricing implications of the model. However, it's important to note that I consider a different source of variation. While they de-trend the data to study how the model propagates transitory shocks, I consider the variation stemming from the stochastic trends in sectoral TFP growth. I do this because, in order to produce a realistic model of asset prices, I must model the sources of non-stationarity in the model. While the transitory variation in TFP growth is certainly very important, modeling it adds additional complexity to the model that I avoid for now. As an extension, I will later consider the case in which TFP is modeled as a unit root process.

2.2 Term structure equity in log-linear economies

As in Chapter 1, I focus on asset pricing in log-linear state space economies. That is, I again consider macroeconomic models that can be expressed by a series of quantities that are driven by a state vector that evolves according to a vector autoregression. Consider a macroeconomic model in which the vector of state variables x_t can be written as a linear state-space model,

$$x_{t+1} = Gx_t + Hw_{t+1}, \tag{2.4}$$

where G is an $N \times N$ matrix with spectral radius less than one, H is a $N \times M$ constant matrix, and $w_{t+1} \sim \mathcal{N}(0, I)$ is an i.i.d. random vector representing the underlying structure shocks of the model. Suppose further that the model-implied stochastic discount factor (SDF) and a given cash flow process (e.g., aggregate dividends) can be written as

$$\log S_{t+1} - \log S_t = \mu_s + U_s' x_t + \lambda_s' w_{t+1} \tag{2.5}$$

$$\log D_{t+1} - \log D_t = \mu_d + U_d' x_t + \lambda_d' w_{t+1}, \tag{2.6}$$

where μ_s and μ_d are constants, and U_s, U_d, λ_s, and λ_d are conforming vectors. Doing so allows me to leverage, in particular, the results in Corollary 1.17 and Corollary 7. Furthermore, I will explore a macroeconomic model in which there is a representative agent with Epstein-Zin preferences with an intertemporal elasticity of substitution equal to 1 and dividends are proportional to consumption, which will allow me to leverage these results in conjunction with Lemma 11.

The main result from Chapter 1 which I will use in this Chapter is the observation that the term structure of equity can inform us about the dynamics of consumption and output growth. Observation of the average returns on dividend futures at each horizon amount to an extra moment condition that can inform us of the impulse response function governing dividend growth (Corollary 1.16). Furthermore, in the context of a macroeconomic model where dividends are taken to be levered consumption, the slope of the term structure of equity puts bounds on the relationship between the permanent and transitory components of consumption growth (Proposition 12). It is my position that this information can be used to discriminate between various models that aim to describe the important sources of aggregate variation.

With this in mind, I now proceed to explore how the term structure of equity can inform a particular macroeconomic model.

2.3 A multi-sector production network model

In this section, I analyze the dynamics of a multi-sector production network model through the lens of the term structure of equity. Recent evidence regarding the shape of this term structure have proven to be useful for discriminating between competing asset pricing models. My position is that this information can similarly be used to discriminate between various models of the sources of macroeconomic variation. I focus here on a production network model only as an example. As discussed in the previous section, this information applies generally.

I use a standard multi-sector production model, amended so that households have recursive preferences of the Epstein-Zin variety (Epstein and Zin, 1989). The production and investment technology specification is otherwise standard, as in Foerster, Sarte, and Watson (2011). In this model, aggregate consumption growth follows from growth in sectoral TFP. Transitory variation at business cycle frequencies arises due to the interaction of the shape of the production networks with the covariance structure of sectoral TFP growth. I demonstrate the use of the term structure of equity to evaluate the magnitudes and relationship of these sources of variation in consumption growth. This section therefore proceeds as follows:

1. Describe the multisector production network model and derive its solution.

2. Show that the solution can be written in the form studied in the previous section,

described in equations (1.8), (1.9), and (1.10).

3. Describe how each source of uncertainty in the model contributes to the model-implied shape of the term structure. In this case, only one shock—sectoral shift shocks—contributes to a downward sloping term structure. Thus, I'll show that if we wish to replicate this fact, then the shift shock must be sufficiently large relative to the other shocks.

2.3.1 Model Description

Consider an economy with n distinct industries, indexed $i = 1, \ldots, n$. Each industry produces a quantity Q_{it} of a distinct good. Industries have Cobb-Douglas production technologies with constant returns to scale, transforming intermediate goods, capital, and labor into a new product. The gross output of good i is

$$Q_{i,t} = \exp(\xi_{i,t}) K_{it}^{a_i^k} L_{it}^{a_i^\ell} M_{it}^{a_i^m}, \quad i = 1, \ldots, n, \tag{2.7}$$

where $\xi_t = (\xi_{1,t}, \ldots, \xi_{n,t})'$ is the vector of log total factor productivity associated with each sector, with capital K_{it}, labor L_{it}, and M_{it} a bundle of intermediate goods used in the production of good i at time t. a_i^k, a_i^ℓ, and a_i^m are fixed parameters and, since the production function features constant returns to scale, $a_i^k + a_i^\ell + a_i^m = 1$.

In each sector i, the capital stock follows the law of motion,

$$K_{i,t+1} = I_{it} + (1 - \delta) K_{it},$$

where I_{it} is a bundle of investment goods used in sector i and δ is the depreciation rate common to all sectors.

The bundle of intermediates goods used by i is a an aggregation of goods produced by other industries,

$$M_{it} = \prod_{j=1}^{n} M_{ijt}^{a_{ij}}.$$

When a_{ij} is higher, it means that good j is more important in producing good i. With respect to intermediate goods, a_{ij} characterizes the input-output linkages between sectors. With respect to intermediate goods, I summarize the input-output linkages between sectors with the matrix $\boldsymbol{A} = [a_i^m a_{ij}]$, which I refer to as the intermediate goods network input-output matrix, or just the *input-output matrix*.

The bundle of investment goods used in sector i is formed according to the constant returns to scale technology

$$I_{it} = \prod_{j=1}^{n} I_{ijt}^{\theta_{ij}},$$

with $\sum_{j=1}^{n} \theta_{ij} = 1$ and I_{ijt} as the quantity of good j used to produce investment in sector i at time t. I summarize these linkages between sectors with the matrix $\boldsymbol{\Theta} = [a_i^k \theta_{ij}]$, which is referred to as the *investment network matrix*.

The goods produced in each sector can be used as intermediate goods applied to the production to other goods, can be used towards the capital investments in a particular sector, or can be consumed. Thus, each sector is subject to the resource constraint,

$$C_{jt} + \sum_{i=1}^{n} M_{ijt} + \sum_{i=1}^{n} I_{ijt} = Q_{jt},$$

where C_{jt} denotes the quantity of good j that is consumed at time t by a representative household.

I assume that the economy has a representative household. This household divides time between labor allocated to the various industries, L_{it} for $i = 1, \ldots, n$, and leisure $\mathcal{L}_t$. The household consumes the n different goods C_{it}, which it aggregates with a Cobb-Douglas aggregator,

$$C_t = \prod_{i=1}^{n} C_{i,t}^{\alpha_i}, \tag{2.8}$$

with $\alpha = (\alpha_1, \ldots, \alpha_n)'$ and $1 = \sum_i \alpha_i$. This household has Epstein-Zin utility, given by the recursion

$$V_t = \{(1-\beta)(\mathcal{C}_t)^{1-\rho} + \beta[\mathcal{R}_t(V_{t+1})]^{1-\rho}\}^{1/(1-\rho)} \tag{2.9}$$

where $\mathcal{C}_t = \mathcal{L}_t^{1-s_c} C_t^{s_c}$ is a measure of per-period utility, $s_c \in [0,1]$ controls preferences for consumption relative to leisure, $\mathcal{R}_t$ is the certainty equivalent operator defined by

$$\mathcal{R}_t(V_{t+1}) \equiv \mathbb{E}[(V_{t+1})^{1-\gamma} \mid \mathcal{F}_t]^{1/(1-\gamma)},$$

ρ^{-1} is the elasticity of intertemporal substitution, and γ is the risk-aversion parameter. The households is endowed with H units of labor/leisure, so that is

$$H = \mathcal{L}_t + \sum_{i=1}^{n} L_{it}.$$

Finally, let sectoral TFP growth be distributed as a factor model,

$$\Delta\xi_{i,t+1} = \mu_{\xi,i} + \beta_{ai}\varepsilon_{a,t+1} + \beta_{bi}\varepsilon_{b,t+1} + \varepsilon_{i,t+1} \tag{2.10}$$

where $\varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{n,t}, \varepsilon_{a,t}, \varepsilon_{b,t})' \overset{\text{iid}}{\sim} \mathcal{N}(0, \Sigma)$ and $\mu_{\xi,i}$ is a set of constants. Here, I have specified a two-factor model. Though we could include more common factors, the presentation is simpler this way and the data suggest that two is appropriate. As a factor model, let $\varepsilon_{a,t}$ and $\varepsilon_{b,t}$ be aggregate (common) shocks, whereas $\varepsilon_{i,t}$ are idiosyncratic, industry-specific shocks. That is, assume $\varepsilon_{i,t}$ is uncorrelated with $\varepsilon_{j,t}$ for all $i \neq j$ for $i, j \in 1, 2, \ldots, n$ and that $\varepsilon_{i,t}$ is uncorrelated with $\varepsilon_{x,t}$, for $x = a, b$. While many of the derivations presented in this paper hold with a more general model for TFP, I assume this form in order illustrate properties of the model in a simpler setting. Furthermore, I will later put some extra structure of these common factors by specifying properties of the factor loadings β_{ai} and β_{bi}. I will assume that a positive shock to ε_{at} increases long-run aggregate consumption while a positive shock to ε_{bt} has a net-zero long-run effect on aggregate consumption. In this sense, ε_{bt} has the interpretation as a sectoral shift shock that only shifts the composition of the economy.

Note that one of the costs of modeling asset prices in a macroeconomic model is that, in order to get realistic asset price behavior, the economist must model the sources of non-stationarity. To keep things simple, I have assumed that sectoral TFP grows according to this factor model. Although I allow TFP growth across sectors to be correlated via latent common factors, this model implies that TFP in each sector, when viewed individually, follows a random walk. Thus, any transitory variation must arise from the mechanisms within the model. As shown in the previous section, this transitory variation is important for determining the shape of the term structure of equity.

The competitive equilibrium of this economy is defined in the usual way. That is, a competitive equilibrium is a set of prices and quantities such that the representative household maximizes her utility while taking prices and the wage as given, the representative firm in each sector maximizes its profits while taking prices and the wage as given, and all markets clear. Since I assume no frictions, I solve this model via the social planner's problem.

In the following section, I define the risk prices and risk premia associated with aggregate vs idiosyncratic shocks. In Section 2.3.2, I examine the competitive equilibrium dynamics and risk prices via a first-order approximation around the non-stochastic balanced growth path. This approximated solution will be especially useful as its simple form will make econometric evaluation simpler. In Section 2.3.3, I will explore some qualitative features of the equilibrium in a simplified versions of the model. This more stylized environment will help to better understand the model's underlying mechanisms.

2.3.2 *Equilibrium solution via log-linearization*

Here I derive a solution to the model and demonstrate that it can be written in the form described in equations (1.8), (1.9), and (1.10). To simplify the analysis of the model, assume that the elasticity of substitution $\rho^{-1} = 1$ and that $\gamma > 1$. Under this assumption, utility as characterized in equation (2.9) simplifies to

$$\log V_t = (1 - \beta) \log \mathcal{C}_t + \beta \log \mathcal{R}_t(V_{t+1}).$$

Though preferences under these assumptions simplify greatly, note that the assumption that $\gamma \neq \rho$ ensures that households still exhibit a concern for long-run risk.[2] I then examine equilibrium dynamics and asset prices by analyzing a first-order approximation around a balanced growth path. Though certainty equivalence applies to quantity dynamics under this approximation, assets still exhibit positive risk premia and equilibrium still imposes joint restrictions on quantity dynamics and the term structure of risk premia. Furthermore, since my analysis revolves primarily around output dynamics and consumption, I will abstract away from labor supply decisions by letting labor be supplied inelastically $L_{it} = L_i$.

Under these assumptions, the non-stochastic balanced growth path of the model is analytically tractable and a linear approximation of the first-order conditions and the resource constraints around this path yields a vector ARMA(1,1) model for sectoral output growth,

$$\Delta q_{t+1} = \Phi \Delta q_t + \Pi_0 \Delta \xi_{t+1} + \Pi_1 \Delta \xi_t, \tag{2.11}$$

where $q_{it} = \log Q_{it}$ is log output in sector i, $q_t = [q_{it}]$ is the vector of output in each sector, and $\Delta q_{t+1} = q_{t+1} - q_t$ is the vector of log output growth. Φ, Π_0, and Π_1 are $N \times N$ matrices that depends only on the model parameters a^k, a^ℓ, a^m, A, Θ, δ, α, β, s_c, and γ.

2. Note that when $\rho = \gamma$, preferences collapse to CRRA utility and that when $\rho = \gamma = 1$, preferences become log-utility. However, here I assume that $\rho = 1$ and I will typically assume that $\gamma > 1$. Under this assumption, the result is still a non-time-additive von Neumann-Morgenstern utility function.

Furthermore,

$$\Delta \log \mathcal{C}_{t+1} = s_c \alpha' \Delta q_{t+1}, \tag{2.12}$$

where $\alpha = [\alpha_i]$.

The balanced growth path and approximation derivation is given in the appendix, in Section A.2.1. Note that the shocks w_{t+1} in (1.8) are orthogonal, with $w_{t+1} \sim \mathcal{N}(0, I)$. Since the shocks in $\varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma)$ are not, the final step here is to choose an orthogonalization P, where

$$\varepsilon_{t+1} = P w_{t+1}. \tag{2.13}$$

Thus, by stacking and applying this orthogonalization, we can write the equilibrium solution as a linear state space model of the form (1.8).

The stochastic discount factor I now discuss the derivation of the stochastic discount factor (SDF). In equilibrium with $\rho = 1$, the SDF is

$$\frac{S_{t+1}}{S_t} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left[\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]} \right]$$

and can be written as $\log S_{t+1} - \log S_t = \mu_s + U_s' x_t + \lambda_s' w_{t+1}$, where μ_s, U_s, and λ_s are constants that depend on the model parameters. The proof of this is given in the appendix, in Section A.1.9. Also note that risk prices λ_s can be decomposed into two parts—a part capturing investor concern for long-run risk $\lambda_{s,LR}$ and myopic risk-prices $\lambda_{s,SR}$,

$$\lambda_s = \lambda_{s,SR} + (\gamma - 1)\lambda_{s,LR}. \tag{2.14}$$

This interpretation comes from the fact that when $\gamma = \rho = 1$, the Epstein-Zin utility functions collapses into log-utility and investors no longer exhibit a concern for long-run risk.

Thus, have shown that SDF takes on the form in (1.9) and that the state variables follow dynamics of the form in (1.8).

Impulse Responses, Risk Exposures, and Risk Prices

We have now specified a cash flow process that satisfies (1.10). We have previously shown that SDF takes on the form in (1.9) and that the state variables follow dynamics of the form in (1.8). If we now define a dividends process that takes the form (1.10), the formulas for the term structure of holding period returns described in Proposition 3 apply to this solution of the model. As discussed in Section 1.3, the risk exposures associated with the dividend futures are equal to the impulse responses of the dividends process. Here I defined aggregated dividends and derive the impulse responses.

As discussed previously, the wealth portfolio is a claim to all future consumption and leisure and I will use the market portfolio as a proxy for this wealth portfolio. Thus, I adopt Assumption 10 and set the aggregate dividends equal to the levered consumption index,

$$\log D_t = \eta \log \mathcal{C}_t, \tag{2.15}$$

where $\eta \geq 1$ is the leverage factor. Given this assumption, $\psi_d(\tau) = \eta\, \psi_{\mathcal{C}}(\tau)$. Thus, I now only need to derive the impulse responses of the consumption index, $\psi_{\mathcal{C}}$.

Risk Exposures of Dividend Futures I now focus on the dividend futures premium defined in Definition 5. Recalling that the risk-exposures associated the dividend futures are equal to impulse response function of the dividend process, to proceed I simply need compute the impulse response functions for the dividends process. With the assumption of dividends being equal to levered consumption, this is just the impulse response function of the aggregate consumption index $\mathcal{C}_t$.

As a preface, define the impulse response function of TFP as the matrix function Ψ_ξ such that

$$\Delta \mathbb{E}_{t+1}[\xi_{t+\tau}] = \Psi_\xi(\tau)\, w_{t+1}. \tag{2.16}$$

Under this definition, entry (i,j) of $\Psi_\xi(\tau)$ represents the impulse response of TFP of sector i to a unit impulse to the j'th shock at horizon τ. Since sectoral TFP growth is i.i.d., this is constant over τ, with

$$\Psi_\xi(\tau) = \bar{\Psi}_\xi = \left[\underbrace{\text{diag}\begin{pmatrix}\sigma_1 \\ \vdots \\ \sigma_n\end{pmatrix}}_{\varepsilon_{it} \text{ for all } i} \quad \underbrace{\sigma_a \begin{bmatrix}\beta_{a1} \\ \vdots \\ \beta_{an}\end{bmatrix}}_{\varepsilon_{at}} \quad \underbrace{\sigma_b \begin{bmatrix}\beta_{b1} \\ \vdots \\ \beta_{bn}\end{bmatrix}}_{\varepsilon_{bt}} \right] \tag{2.17}$$

for all τ, where σ_i for $i = 1, \ldots, n$ is the volatility of the idiosyncratic shocks, ε_{it}, σ_a is the volatility of the first common factor ε_{at}, and σ_b is the volatility of the second, ε_{bt}, defined in the TFP growth process in (2.10).

Using this, the impulse responses are given as follows.

Proposition 13. *The short-term impulse response, the short-term risk exposure, is*

$$\psi_d'(1) = \eta\, s_c \underbrace{\alpha' \Pi_0}_{\substack{\textit{sector}\\ \textit{weights}}} \underbrace{\Psi_\xi(1)}_{\substack{\textit{TFP}\\ \textit{IRFs}}}. \tag{2.18}$$

while the long-term impulse response is

$$\psi_d'(\infty) = \eta\, s_c \underbrace{\alpha'(I - \Phi)^{-1}\Pi_0}_{\substack{\textit{sector}\\ \textit{weights}}} \underbrace{\Psi_\xi(\infty)}_{\substack{\textit{TFP}\\ \textit{IRFs}}}. \tag{2.19}$$

Given that sectoral TFP growth is i.i.d., $\Psi_\xi(\infty) = \Psi_\xi(1) = \bar{\Psi}_\xi$, *as defined in (2.17).*

From this derivation, note that $\alpha'\Pi_0$ is a vector and that η and s_c are scalars. From this we can see that the one-period risk exposures are a set of weighted sums, where the risk exposure associated with each shock is a weighted sum of the impulse response functions of TFP, weighted by the vector $\alpha'\Pi_0$. Substituting $\Psi_\xi(1) = \bar{\Psi}_\xi$ we see that the risk exposure associated with the idiosyncratic shocks is just σ_i multiplied by the sector's weight given in the vector $\eta\, s_c \alpha' \Pi_0$. The risk exposures associated with the common, aggregate shocks is a weighted sum of the factor loadings and the weights $\alpha' \Pi_0$ and scaled by scalars η and s_c. E.g., for the shock ε_{at}, this is a weighted sum of the β_{at}, multiplied by the volatility σ_a. Similarly, the long-term risk exposures are the impulse responses of sectoral TFP weighted by the vector $\alpha'(I - \Phi)^{-1}\Pi_0$ rather than $\alpha'\Pi_0$. I will argue later that these weights can be interpreted as a measure of sectoral centrality within the production networks and that the short-term weights are governed largely by centrality within the intermediate goods network and that the long-term weights are tilted towards centrality in the investment network. Note that these weights do not sum to one. Though I will later discuss the interpretation of these weights as a measure of centrality, I define them now as such.

Definition 14. I call the weights vector $\alpha'\Pi_0$ in the short-term case the *short-term centrality* vector and the weights vector $\alpha'(I - \Phi)^{-1}\Pi_0$ in the long-term case the *long-term centrality* vector.

The weights in the intermediate term can be interpreted as interpolating these two extremes. Mathematically, these intermediate term weights take on a more complicated form which I will skip for now.

Note that, though the equilibrium solution given in equation (2.11) admits a VARMA(1,1), the parameters of the model are not easily interpretable in terms of the model primitives. In the Section 2.3.3, I consider a simpler case of the model that produces analytical expressions Φ, Π_0, and Π_1 that will shed additional light on the nature of the sectoral weights discussed here. However, before moving to this simpler case, I proceed with a discussion of the risk prices.

Risk prices I now turn my attention to deriving risk prices in equilibrium. These also take on a simple form in terms of the impulse responses. I will derive the two components discussed in (2.14).

Proposition 15. *Use the notation set forth in (2.14) and the derivation from Lemma 11. Then, the expression for the myopic risk prices,* $\lambda_{s,SR}$*, share proportionally the same form as the short-term risk exposures,*

$$\lambda_{s,SR}' = -s_c \alpha' \Pi_0 \Psi_\xi(1) = -\psi_{\mathcal{C}}'(1). \tag{2.20}$$

The risk prices arising from a concern for long-run risk are approximately equal to the long-run risk exposures. If we suppose $\Delta\xi_t$ *is i.i.d. as we did before, then*

$$\lambda_{s,LR}' = -s_c \alpha' (I - \beta\Phi)^{-1} \Psi_\xi(\infty), \tag{2.21}$$

with

$$\lambda_s = \lambda_{s,SR} + (\gamma - 1)\lambda_{s,LR}.$$

Note that as $\beta \to 1$ *we see that*

$$\lambda_{s,LR} \approx -\psi_{\mathcal{C}}(\infty). \tag{2.22}$$

Note that in calculating the risk premium associated with dividend futures, the risk prices do not change with the horizon. Rather, the risk prices λ_s as expression in (2.14) are a linear combination of $\lambda_{s,SR}$ and $\lambda_{s,LR}$. When γ is larger, the investor puts more weight into the long-term risk prices. As we see in (1.19), a changing Sharpe ratio over the term structure is due to risk exposures changing, tilting towards sources of risk with a higher or lower risk price. With a derivation of the risk exposures and risk prices in place, we have everything we need to compute the dividend future premiums in Definition 5. However, to better understand the model outcomes, I first consider a simpler case of the model.

I now turn to a special case of equilibrium that will help to interpret these expressions.

2.3.3 The special case of full depreciation

In this section, I explore the special case that admits a closed form solution that will allow for a clearer characterization of equilibrium prices and quantities. In addition to assuming that $\rho = 1$, as we did in the previous section, I will assume that capital depreciates fully after one period, $\delta = 1$. Note that in this case, I do not need to assume that labor is inelastically supplied. The absence of capital ensures that labor supply is constant. I will discuss the dynamics of sectoral output in this case as well as the resulting asset prices, including the term structure of equity. This setting will allow me to cleanly characterize short-run and long-run centrality in terms of the intermediate goods and investment networks.

Equilibrium in this special case is described in Proposition 16, the proof of which is given in the appendix, Section A.2.3.

Proposition 16. *Suppose $\rho = 1$ and $\delta = 1$. Let q_t be the vector of log-output at time t such that $q_{it} = \log Q_{it}$ and let ξ_t be the vector of log TFP shocks, $\xi_{it} = \log \Xi_{it}$. Let Δ be the difference operator, so that $\Delta q_{t+1} = q_{t+1} - q_t$. Then, output growth in equilibrium must satisfy*

$$\Delta q_{t+1} = (I - A)^{-1}\Theta\Delta q_t + (I - A)^{-1}\Delta\xi_{t+1}, \tag{2.23}$$

where $A = [a_i^m a_{ij}]$ and $\Theta = [a_i^k \theta_{ij}]$. Equilibrium leisure and labor is constant and, furthermore,

$$\begin{aligned} \Delta \log \mathcal{C}_{t+1} &= s_c \alpha' \Delta q_{t+1} \\ &= s_c \alpha' (I - A)^{-1}\Theta\Delta q_t + s_c \alpha' (I - A)^{-1}\Delta\xi_{t+1}, \end{aligned} \tag{2.24}$$

where $\alpha = [\alpha_i]$.

As can be seen, the resulting dynamics can be considered a special case of form of the dynamics seen previously in equation (2.11), with $\Phi = (I - A)^{-1}\Theta$, $\Pi_0 = (I - A)^{-1}$, and

$\Pi_1 = 0$. In this special case, we can see that the investment network's role in determining the relationship between output growth today and output growth tomorrow. When capital plays no role in production, $a_i^k = 0$, this intertemporal relationship disappears, with $\Theta = \mathbf{0}$. Under that assumption, output growth in previous periods would not predict future output growth and propagation through the intermediate goods network happens instantaneously,

$$\Delta q_{t+1} = (I - A)^{-1} \Delta \xi_{t+1}. \tag{2.25}$$

Evidence of this instantaneous propagation can be seen in the multiplier on log TFP growth, as

$$(I - A)^{-1} = I + A + A^2 + A^3 + \dots,$$

where the term A represents the effect after one degree of separation, A^2 represents the effect after two degrees of separation, etc. All of these effects occur simultaneously, leading to (2.25). In contrast, shocks propagating through the investment network propagate gradually over time. Similarly, if we to omit the intermediate goods network with $A = \mathbf{0}$, then we can simplify (2.23) so that gradual propagation through the investment network would manifest as

$$\Delta q_{t+1} = (I - \Theta L)^{-1} \Delta \xi_{t+1}, \tag{2.26}$$

where L here is the lag operator. Evident from,

$$(I - \Theta L)^{-1} = I + \Theta L + \Theta^2 L^2 + \Theta^3 L^3 + \dots,$$

the term representing the effects of after one degree of separation, Θ, occurs only after one period has passed and the term representing the effects after two degrees, Θ^2, occurs occurs after two periods has passed, etc. This difference in the role of each network in the propagation of shocks results in different implications for asset prices and the term structure of risk premia.

Risk Prices and Risk Exposures

Given that this simplified version of the model is a special case, I can leverage the derivations from Section 2.3.2. I present the risk exposures in this simplified case as Corollary to Proposition 13. This Corollary follows from the derivation of the simplified equilibrium as well as the derivation of the risk prices in the generalized setting described in Lemma 11 in Chapter 1.

Corollary 17. *When $\delta = 1$, the risk exposures for the short-horizon dividend futures are given by*

$$\psi_d'(1) = \eta\, s_c \alpha'(I - A)^{-1}\Psi_\xi(1), \tag{2.27}$$

where the sector weights $\alpha'(I - A)^{-1}$ depend on centrality in the intermediate goods network. This is the short-run centrality *vector. The risk exposures for the long-horizon risk exposures are*

$$\psi_d'(\infty) = \eta\, s_c \alpha'(I - (A + \Theta))^{-1}\Psi_\xi(\infty), \tag{2.28}$$

where the sector weights $\alpha'(I - (A + \Theta))^{-1}$ depend on the sum of the intermediate goods network and investment network matrices. This is the long-run centrality *vector.*

Centrality Interpretation Within the network theory literature, *alpha centrality* is a measure of a node's centrality within a network. It determines the centrality of a node by computing a weighted sum of the centrality of its neighbors, weighted by the strength of the connection between its neighbors, and then adding some baseline level of "centrality" to each node. For example, let $c(\alpha, A)$ be the alpha centrality of the intermediate goods network as represented by the matrix A where α is the vector of baseline centrality given to each industry. Then, by definition, the alpha centrality is characterized as the vector c that solves

$$\alpha' + c'A = c'.$$

Since

$$c(\alpha, A) = \alpha'(I - A)^{-1}$$

solves this equation, $\alpha'(I - A)^{-1}$ is the alpha centrality of the network A. Recall that α is the vector of Cobb-Douglas shares in the consumption aggregator, so this expression measures the centrality of industries within the intermediate goods network, weighted by the importance of each industry within the consumption aggregator. With this in mind, we also see that long-run centrality in this case

$$c(\alpha, A + \Theta) = \alpha'(I - (A + \Theta))^{-1}$$

measures centrality within a network formed by summing the shares of the intermediate goods network A and the investment network Θ. Thus, the centrality measure in the long-run is tilted towards giving more weight to industries that are central within the investment network.

Risk exposures in the intermediate term Given the VARMA form for output growth relative to TFP, the risk exposures of the dividend process take on a somewhat complicated form in the intermediate term. However, since in this special case $\Pi_1 = \mathbf{0}$, this is simplified here. In this case, the risk exposures in the intermediate term are

$$\psi_d'(\tau) = \eta\, s_c \alpha'(I - \Phi)^{-1}(I - \Phi^\tau)\Pi_0 \Psi_\xi(\tau) \tag{2.29}$$

where $\Phi = (I - A)^{-1}\Theta$ and $\Pi_0 = (I - A)^{-1}$. This also has an interpretation similar to alpha centrality. Alpha centrality has a recursive definition but can be interpreted as counted walks that are discounted by distance. Since

$$(I - \Phi)^{-1}(I - \Phi^\tau) = I + A^2 + \ldots + A^{\tau-1},$$

the intermediate term risk exposures can be though of as a truncated form of alpha centrality which counts walks with of length $\tau - 1$ or shorter.

Risk Prices As demonstrated in equations (2.20) and (2.22), the short-run and long-run risk exposures can also be used to describe equilibrium risk prices. Substituting the expressions derived in Proposition 16 and Corollary 17 into Proposition 15 gives the risk prices. Again, these depend on the impulse responses of TFP, weighted by short-run and long-run centrality. When $\gamma = 1$ and the utility function collapses into log utility, the risk prices depend only on short-run centrality. As γ becomes larger, risk prices depend more on long-run centrality.

The composition of these risk prices and risk exposures determine the risk premia that make up the term structure of equity. In the following section I present a quantitative example to demonstrate the dividend future premia in relationship to the model primitives.

2.3.4 Simple two-sector example

In this section I consider the implications of the model for the term structure of equity. I now consider the conditions we must impose on the distribution of TFP in (2.10) in order to obtain a downward sloping term structure over the long-run. For simplicity and for the sake of clear illustration, I will use the full-depreciation version of the model.

We will see that, as specified, if this model has only a single sector then a downward sloping term structure is impossible. When multiple sectors are present, a downward sloping term structure can also be achieved by including a shock that shifts TFP between sectors. Intuitively, this effect will be largest if this shock imposes a negative correlation between the TFP growth of intermediate goods hubs and that of investment hubs. I will illustrate this in a simple, two-sector model.

Consider a two-sector example where 1 sector is a clear intermediates goods hub and the other is an investment hub, with the intermediate goods network and investment network

	a_{ij}		θ_{ij}	
Sector i, j	1	2	1	2
1	.9	.1	.1	.9
2	.9	.1	.1	.9

Table 2.1: Intermediate goods network and investment network for two-sector example. Let the cost shares in the intermediate goods network, a_{ij}, be defined so that industry 1 is an intermediate goods hub and industry 2 is an investment hub. Let $a_i^m = .4$, $a_i^k = .4$, $a_i^\ell = .2$.

Sector i	$\mu_{\xi,i}$	β_{ai}	std($\varepsilon_{i,t}$)	α_i
1	0.005	.02	0.01	0.5
2	0.005	.02	0.01	0.5

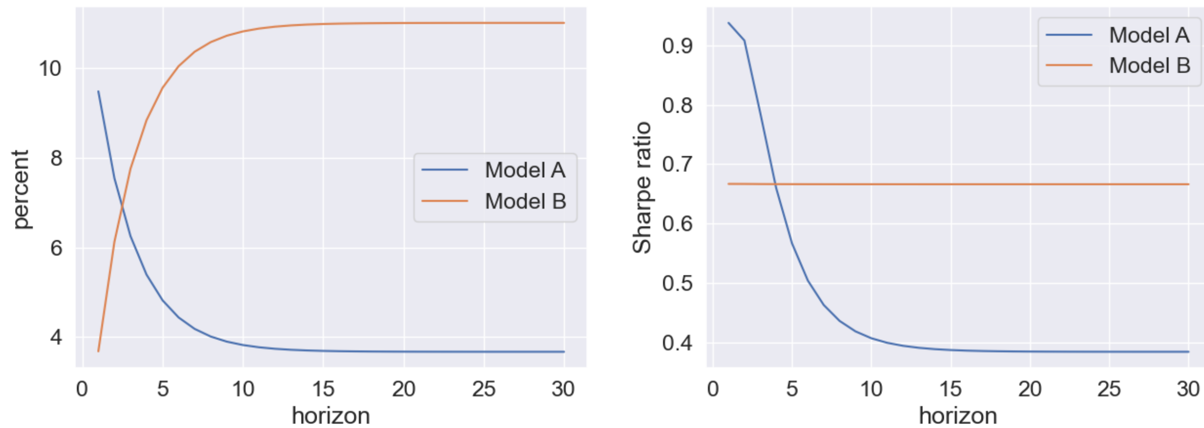
Table 2.2: Model parameters of two-sector example. I will examine two versions. In model A, the factor loadings β_{bi} will be different than those in model B. Let $\text{Var}(\varepsilon_{at}) = \text{Var}(\varepsilon_{bt}) = 1$.

defined in Table 2.1. In the following examples, let the model have the parameters defined in Table 2.2, with $\gamma = 10$. Consider two alternatives, Model A and Model B:

- Model A: Negatively correlated sectors, $\beta_{b1} = 1$, $\beta_{b2} = -1$,

- Model B: Positively correlated sectors, $\beta_{b1} = 0.3$, $\beta_{b2} = 0.3$,

Model A features a negative correlation between the investment hub and the intermediate goods hub and, thus, exhibits a downward sloping term structure, illustrated in Figure 2.1. This downward slope appears both in the dividend future premium (Panel (a)) and in the Sharpe ratios (Panel (b)). Here, I have specified the common shock ε_{at} so that both sectors have positive associated loadings. This shock represents an aggregate shock that moves up the TFP of all industries. The second aggregate shock, ε_{bt}, shifts TFP between the two sectors. This shock delivers a downward sloping term structure because it delivers sufficient transitory variation to satisfy (1.29) in Proposition 12. A similar exercise will show also that idiosyncratic shocks alone cannot generate a downward slope.



(a) Expected annual returns on dividends futures.

(b) Sharpe ratio on dividends futures

Figure 2.1: A model with negative correlation between intermediate goods hubs and investment hubs delivers a downward sloping term structure of equity.

2.4 Empirical Application

In this section, I take the the benchmark model of Section 2.3.2, as well as the simplified, full depreciation model of Section 2.3.3, to the data. That is, I estimate sectoral and aggregate shocks that drive growth in the model and derive the model-implied term structure of equity. I then compare this to the observed term structure of equity, as illustrated in Figure 1.1.

As I demonstrated previously in Proposition 9, the slope of the term structure of equity depends on both stationary and non-stationary components of growth. Therefore, this empirical exercise must model and estimate the sources of non-stationarity. For this reason I model and estimate TFP growth as described in (2.10). That is, although I assume sectoral TFP individually follows a random walk, I assume that there exist some latent, common factors that induce some amount correlation among sectoral TFP growth shocks. Once we are able to identify sectoral TFP growth $\Delta\xi_t$, a factor analysis can tell us how much each latent factor contributes to the covariation in sectoral growth. I argue that we can then use these estimates to derive the model-implied term structure of equity and identify how each type of shock contributes to this term structure. This then serves as a diagnostic to evaluate the fit of the model.

In the following exercise, I will demonstrate that we can separate the common factors into two types: an aggregate shock that moves long-term aggregate output up and down and another that doesn't affect long-term output, but causes short-term disturbances by shifting TFP between sectors. As demonstrated in Section 2.3.4, these shift shocks contribute to a downward sloping term structure. I'll show that this shift shock can account for as much as 40% of the covariation of TFP growth among sectors, but that its aggregate effect are not large enough to make the model-implied term structure downward sloping. This indicates to us that we need to search for another source of short-term variation or that we need to introduce frictions into the model to amplify the aggregate effects of these shift shocks.

To summarize, the empirical exercise takes on the following steps:

1. **Model Filter:** Estimate sectoral TFP growth from the implied model filter, following the procedure developed by Foerster, Sarte, and Watson (2011). For example, in the full depreciation case this involves solving $\Delta\xi_t$ in terms of lags of Δq_t in (2.23).

2. **Fit Factor Model:** With sectoral TFP growth $\Delta\xi_t$ recovered, estimate the panel of sectoral TFP as a linear factor model with latent common factors. This estimates the degree to which comovement in TFP is driven by common, "aggregate" shocks, relative to idiosyncratic movements.

3. **Factor Loadings Rotation:** The factor loadings recovered in the previous step are only identified up to an orthogonal rotation. That is, we may choose an alternative rotation of the factor loadings to help us to interpret the fit of the model. Conveniently, a two-factor model appears to provide the best fit in both cases. I therefore choose a rotation so that one factor has no long-run impact on aggregate output. This factor will be called the "shift shock." Almost surely, there are two such rotations to satisfy this restriction. I therefore choose the rotation that sets the other factor has a positive long-run impact on aggregate output.

4. **Implied Term Structure (decomposed by shock):** After choosing a particular

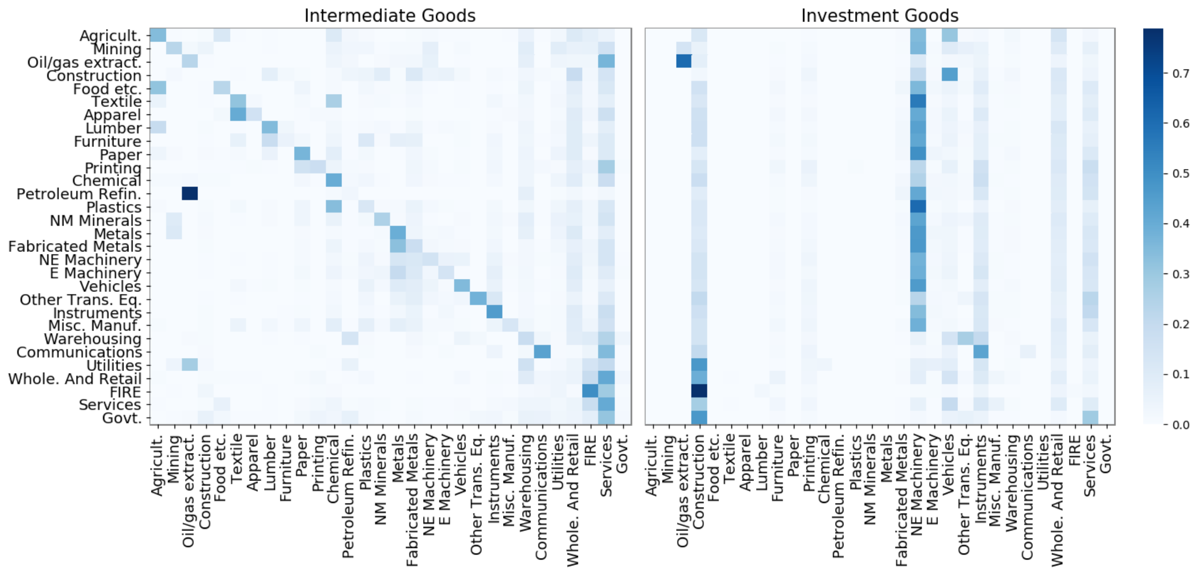
rotation of the factor loadings, I can then measure the risk exposures and, thus, the risk premia associated with each shock at each point in time. This allows me to express how much each shock contributes to the term structure of equity at each horizon in terms of financial returns. We will see that the shift shock contributes a downward sloping term structure. All other shocks tend to contribute to an upward sloping term structure. The size of the shift-shock is no large enough to imply that the aggregate term structure is downward sloping.

2.4.1 Data Description

In my main analysis, I use data from the BEA Input-Output Tables and BEA Capital Flow tables, the BEA Industry Accounts, and Dale Jorgenson's KLEMS data set. Following the same or similar procedures as used elsewhere in the production networks literature, such as in Atalay (2017) and vom Lehn and Winberry (2019), I use these to measure the empirical intermediate goods network and investment network; labor, investment, and intermediate goods shares; and consumption shares. The BEA IO Tables and the Capital Flows tables are used to construct the networks and the factor shares in production. I use Dale Jorgenson's 35-sector KLEMS data set and the BEA Industry Accounts, following the procedure outlined in Atalay (2017), to produce a measure of sectoral output growth over the years 1960–2013. I describe this data in more detail in the Appendix, in section A.3.

As described previously, my model follows vom Lehn and Winberry (2019) and distinguishes between network of intermediate goods and the network of investment goods—goods that are used in the production of new capital. Since I have assumed Cobb-Douglas technologies throughout, the empirical counterparts to these networks can be represented conveniently as a matrix of cost shares derived from the BEA Input-Output and Capital Flows tables. Using the 1997 tables (these are the most recent figures that also include the Capital Flows table), these cost shares as illustrated in Figure 2.2. Since each entry represents a share of expenditures that the row industry spends on the output of the column industry,

Figure 2.2: Empirical Production Networks



Heatmaps of empirical production network for intermediate goods and for investment goods. In the intermediate goods plot, each entry depicts the total expenditures of the row industry on the intermediate goods produced by the column industry, divided by the column industry's total expenditures on intermediate goods. The investment network plot is similar, but for expenditures on investment goods. Calculated from 1997 BEA Input-Output tables and Capital Flows table.

darker columns represent industries that play an important role producing goods used by many other industries. In this sense, these industries are more central within their respective networks. As argued in the previous section, the intermediate goods network contributes to short-term centrality and the investment goods network contributes mainly to long-term centrality. If there are significant differences between the intermediate goods network and the investment network, as there appear to be in Figure 2.2, then the measures of short-term and long-term centrality will tend to be quite different. As argued previously, this is important for determining the impulse responses of aggregate output to the shocks, and thus important for determining the term structure of equity.

2.4.2 Model Filter

In order to estimate sectoral TFP, I follow the procedure of Foerster, Sarte, and Watson (2011). In the case of the full-depreciation model, I solve (2.23) for TFP growth, giving

$$\Delta\xi_{t+1} = (I - A)\Delta q_{t+1} - \Theta\Delta q_t.$$

Since output growth, Δq_t, is observed and the intermediate goods network, A, and the investment good network, Θ, can be estimated from the BEA Input-Output and Capital Flows tables, we have everything we need to solve for $\Delta\xi_t$. The process is similar for the benchmark model, except that the coefficients on output growth are a function of other model parameters and are a product of the log-linearization.

This step is important because, as emphasized by Foerster, Sarte, and Watson (2011), a factor analysis of Δq_{t+1} would overestimate the importance of common factors because of the way that shocks are propagated through the production networks. For a more detailed explanation of why this step is important for estimating the aggregate versus idiosyncratic shocks, see the appendix, Section A.2.4.

After performing this step, I now have a measure of TFP growth across sectors. Table 2.3 reports the average annual output and TFP growth among these sectors. This table shows annual growth rates for TFP and value added across 30 sectors comprising the US economy over 1960-2013. As we can see, while aggregate TFP growth over this time period has grown about 1.4 percent per year, this growth has not been shared equally among sectors. While TFP in sectors such as Communications and Services have grown more than 1.4 percent per year over this period, TFP in sectors such as Metals, Non-electric Machinery, or Apparel have seen substantial declines. This dispersion may represent, for example, differences in technological trends across industries or losses in efficiency due to decreases in economies of scale (say, resulting from shifts due to globalization). While I don't take a stand on the economic origins of this dispersion, I do measure the degree to which TFP growth across

Table 2.3: Sectoral Growth in the US (1960-2013)

Industry Name	TFP	Value Added	Industry Name	TFP	Value Added
Agricult.	0.46	1.53	Metals	-1.89	0.65
Mining	0.16	1.39	Fabricated Metals	0.21	1.50
Oil/gas extract.	-0.35	1.00	NE Machinery	-1.53	4.53
Construction	1.50	0.72	E Machinery	2.62	4.30
Food etc.	0.07	1.50	Vehicles	0.97	2.75
Textile	-0.77	0.26	Other Trans. Eq.	1.55	1.94
Apparel	-1.44	-1.29	Instruments	1.69	4.40
Lumber	0.45	1.41	Misc. Manuf.	0.72	2.25
Furniture	0.14	1.66	Warehousing	1.15	2.66
Paper	-0.04	1.37	Communications	3.57	5.35
Printing	0.31	1.94	Utilities	0.39	1.54
Chemical	1.12	2.39	Whole. And Retail	1.85	3.32
Petroleum Refin.	-0.04	1.41	FIRE	1.38	3.81
Plastics	1.69	3.31	Services	2.11	3.57
NM Minerals	0.31	0.85	Govt.	1.69	2.32
			Aggregate	1.40	2.92

Here I show real annual growth rates for a decomposition of the US economy into 30 different sectors, given in percentage points. The row labeled "Aggregate" is a share-weighted average of the 30 sectors. The data comes from Dale Jorgensen's KLEMS data set and the BEA Industry Accounts data. TFP is estimated using the procedure of Foerster, Sarte, and Watson (2011), as discussed in Section 2.4.

industries can be explained latent common factors relative to idiosyncratic, sector-specific shocks.

2.4.3 Factor analysis of TFP growth

I now estimate a statistical factor model of the panel of TFP growth. Throughout, I will estimate this factor model using maximum likelihood. In Table 2.4 I report the proportion of sample variance explained by each factor when a 6-factor model is used. I do this for the sample of 1960–2013 using TFP measured in the benchmark case. The results using the full-depreciation case are similar. As we can see, a great majority of the variation is explained by the first two factors. For this reason, I proceed with the assumption of a two-factor model,

Table 2.4: Factor Analysis of Sectoral TFP Growth and Output Growth

Factor num.	1	2	3	4	5	6	total
$\Delta\xi_{it}$	0.31	0.22	0.04	0.04	0.03	0.03	0.67
Δq_{it}	0.58	0.06	0.04	0.03	0.02	0.02	0.76

The proportion of total sample variance explained by the k-th factor, R^2, for the series $\Delta\xi_{it}$ (TFP growth) and Δq_{it} (output). This uses output data over the sample of 1960–2013 for the benchmark model.

as in

$$\Delta\xi_{i,t+1} = \mu_{\xi,i} + \beta_{ai}\varepsilon_{a,t+1} + \beta_{bi}\varepsilon_{b,t+1} + \varepsilon_{i,t+1}, \tag{2.30}$$

with mean zero, normally distributed shocks $\varepsilon_{i,t}$ and $\varepsilon_{x,t}$ that are all mutually uncorrelated for $x = a, b$ and all $i = 1, \ldots, n$.

2.4.4 Factor Loading Rotation

Again, since the factor loadings or only identified up to an orthogonal rotation, I can choose the orthogonal rotation of the loadings that gives the factors the most natural interpretation. For this purpose, I consider a rotation of the factor loadings such that the shock ε_{at} has a positive long-run effect of aggregate output while ε_{bt} has a zero long-run effect. Almost surely, there is only one such rotation that has this property. Solving for this rotation is easy since the long-run impact can be determined by the weighted sum of the factor loadings, weighted by the long-run centrality scores described in Section 2.3.2.

After solving for this particular rotation, I summarize new configuration in Table 2.5. Each shock is assumed to have unit standard deviation. For each shock, I report the the proportion of total TFP variation that can be explained by the shock and the short-run (one-period) and long-run impact on consumption. In the full-depreciation case, the shift shocks, ε_{bt}, account for 8% of the variation in TFP but has almost no short-run or long-run

Table 2.5: Summary of Common Shocks

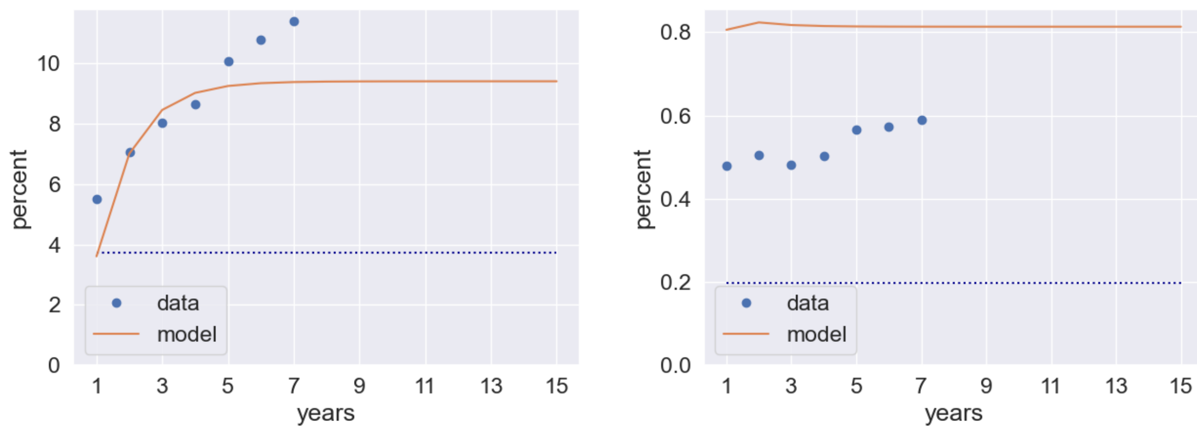
Model	Common Factor	Proportion of total variance (R^2)	Short-run Impact	Long-run Impact
benchmark	$\varepsilon_{a,t}$	0.15	-0.02	0.69
	$\varepsilon_{b,t}$	0.39	0.02	0.00
full depreciation	$\varepsilon_{a,t}$	0.30	0.02	0.05
	$\varepsilon_{b,t}$	0.08	0.00	0.00

Here I summarize the analysis of the factor loadings.

impact. In the benchmark case, the shift shocks appear to explain nearly 40% of the total variance of TFP growth. A shift has a small short-run impact of consumption. The common growth shock, ε_{at}, accounts for about 15% of TFP growth and has a large long-run impact on consumption.

Implied Term Structure of Equity Here I report the model's implied term structure of equity. Table 2.3 reports the model's (full-depreciation version) predicted returns and Sharpe ratios for $\gamma = 25$, $\eta = 2$, $s_c = 0.9$, and $\beta = 0.95$.

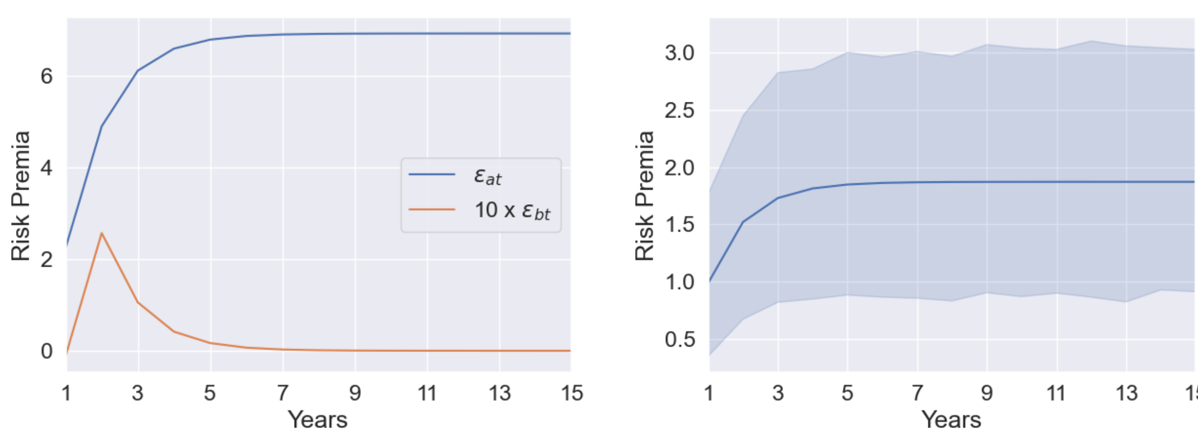
In Table 2.4, I decompose the model-implied expected returns into the contributions from each shock. I do this by utilizing Corollary 7. In Panel (a), the blue line is the contribution of the shock ε_{at}. The orange line is the contribution of the shift shock, ε_{bt}. However, I have multiplied the effect by 10 so that it is more visible. Otherwise, as we can see, the effect is small. In Panel (b), I report the average contributions of the 30 sectoral shocks. I've multiplied the effect by 30, so that the blue solid line reflects the total contribution of the idiosyncratic sectoral shocks. As we can see, the sectoral shocks contribute to an upward sloping term structure. It contributes more to the returns on longer maturity dividend futures than to shorter maturity dividend futures. Thus, only the shift shock potentially contributes to a downward sloping term structure.



(a) Expected annual returns on dividends futures.

(b) Sharpe ratio on dividends futures

Figure 2.3: This plots the model's implied term structure of equity for $\gamma = 25$, $\eta = 2$, $s_c = 0.9$, and $\beta = 0.95$. For this parameterization, the model appears to match the rising expected returns in the short term. However, The term structure does not bend back down as we would hope. Sharpe ratios, also, are mostly flat.



(a) Contribution of common shocks ε_{at} and ε_{bt} to the term structure of equity returns.

(b) Contribution of the idiosyncratic sectoral shocks to the term structure of equity returns.

Figure 2.4: These figures decompose the expected returns on dividend futures at each horizon into the contributions coming from each source of uncertainty. As we can see, only the shift shock, ε_{bt}, appears to contribute to a negatively sloped term structure. However, it is not nearly large enough to overcome the effects of the other shocks.

2.5 Conclusion

I conclude by recapping the main contributions of this paper. These can be divided into two parts. The first is to establish the information content of the term structure of equity with regard to quantity dynamics in a general class of macroeconomic models. The second is to apply this to specific model. I use a multi-sector production model, and demonstrate that the term structure of equity can be used as a diagnostic to evaluate the relative importance of various shocks within the model.

Within this first part, the main contribution is to demonstrate the term structure can be used to inform quantity dynamics in a macroeconomic model. Specifically, I showed in Corollary 4 that the risk exposures associated with the dividend futures premium, seen in equation (1.17), are equal to the impulse response function of the cash flows with respect to the underlying shocks. When these cash flows are taken to be proportional to aggregate consumption, the term structure of equity then provides information about the dynamics of aggregate consumption. I then showed in Corollary 7 that we can decompose the contributions of each shock to the dividend futures premia to analyze the importance of each shock in contributing to the shape of the term structure.

Furthermore, in Proposition 12, I demonstrated that if we take aggregate dividends to be proportional to aggregate consumption, then the slope of the term structure is determined by the relationship between the permanent and transitory components of consumption growth. I showed that, if the term structure is to be downward the consumption process must feature some degree on mean-reversion.

In this second part, the application of these results to a multi-sector production networks model, I first characterize the dynamics of the model with respect to a set of shocks to sectoral TFP. In Proposition 16 and in Corollary 17, I show that the production networks and covariance structure of TFP growth in the various sectors play important roles in the short-run and long-run effects of shocks to various sectors. I then demonstrate in Section 2.3.4 how the covariance structure of TFP shocks can crucially determine the slope of the term structure of equity. I then take this model to the data. In Section 2.4, I estimate the panel of TFP growth across sectors and use this to derive the model implied term structure of equity. I use factor analysis to separate the aggregate shocks into two sources of uncertainty: a shock that increases or decreases long-term output and a shock that shifts TFP between sectors. In Section 2.4.4, I use the decomposition from Corollary 7 to analyze how each shock contributes to the term structure. I show that only the shift-shock appears to potentially contribute to a downward sloping term structure, but is not large enough relative to the other shocks to impose a downward sloping term structure. Thus, if the model is to exhibit a downward sloping term structure, we must search for another source of transitory variation to add to the model or search for some kind of friction to add to the model that would amplify the effects of these shift shocks.

REFERENCES

Acemoglu, Daron, Ufuk Akcigit, and William R. Kerr. 2015. "Networks and the Macroeconomy: An Empirical Exploration." *SSRN Electronic Journal* .

Ahern, Kenneth R. and Jarrad Harford. 2014. "The importance of industry links in merger waves." *Journal of Finance* 69 (2):527–576.

Ai, Hengjie, Mariano M Croce, Anthony M Diercks, Kai Li, Andrew Abel, Ravi Bansal, Joao Gomes, Amir Yaron, Nick Roussanov, and Lukas Schmid. 2012. "Production-Based Term Structure of Equity Returns."

Alvarez, Fernando and Urban J. Jermann. 2005. "Using asset prices to measure the persistence of the marginal utility of wealth." *Econometrica* 73 (6):1977–2016.

Andries, Marianne, Thomas Eisenbach, and Martin Schmalz. 2014. "Horizon-Dependent Risk Aversion and the Timing and Pricing of Uncertainty." Tech. rep., Federal Reserve Bank of New York.

Atalay, Enghin. 2017. "How Important Are Sectoral Shocks?" *American Economic Journal: Macroeconomics* 9 (4):254–80.

Bansal, Ravi and Amir Yaron. 2004. "Risks for the long run: A potential resolution of asset pricing puzzles."

Bansal, Ravi, Amir Yaron, Shane Miller, and Dongho Song. 2020. "The Term Structure of Equity Risk Premia." *SSRN Electronic Journal* .

Barrot, Jean and Julien Sauvagnat. 2016. "Input specificity and the propagation of idiosyncratic shocks in production networks." *Quarterly Journal of Economics* 131 (3):1543–1592.

Belo, Frederico, Pierre Collin-Dufresne, and Robert S. Goldstein. 2015. "Dividend Dynamics and the Term Structure of Dividend Strips." *Journal of Finance* 70 (3):1115–1160.

Boguth, Oliver, Murray D. Carlson, Adlai J. Fisher, and Mikhail Simutin. 2019. "Levered Noise and the Limits of Arbitrage Pricing: Implications for the Term Structure of Equity Risk Premia." *SSRN Electronic Journal* .

Bonacich, Phillip. 2002. "Power and Centrality: A Family of Measures." *American Journal of Sociology* 92 (5):1170–1182.

Borovicka, J., L. P. Hansen, M. Hendricks, and J. A. Scheinkman. 2011. "Risk-Price Dynamics." *Journal of Financial Econometrics* 9 (1):3–65.

Borovička, Jaroslav and Lars Peter Hansen. 2014. "Examining macroeconomic models through the lens of asset pricing." *Journal of Econometrics* 183 (1):67–90.

Campbell, John Y. and John H. Cochrane. 1999. "By force of habit: A consumption-based explanation of aggregate stock market behavior." *Journal of Political Economy* 107 (2):205–251.

Carvalho, Vasco M., Makoto Nirei, Yukiko U. Saito, and Alireza Tahbaz-Salehi. 2016. "Supply Chain Disruptions: Evidence from the Great East Japan Earthquake." *SSRN Electronic Journal* .

Epstein, Larry G. and Stanley E. Zin. 1989. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica* 57 (4):937.

Foerster, Andrew T., Pierre-Daniel G. Sarte, and Mark W. Watson. 2011. "Sectoral versus Aggregate Shocks: A Structural Factor Analysis of Industrial Production." *Journal of Political Economy* 119 (1):1–38.

Gordon, Myron J. 1962. *The Investment, Financing, And Valuation Of The Corporation.* R.D. Irwin.

Gormsen, Niels Joachim. 2018. “Time Variation of the Equity Term Structure.” *SSRN Electronic Journal* .

Hansen, Lars Peter, John C. Heaton, and Nan Li. 2008. “Consumption Strikes Back? Measuring Long-Run Risk.” *Journal of Political Economy* 116 (2):260–302.

Hansen, Lars Peter and José A. Scheinkman. 2012. “Pricing growth-rate risk.” *Finance and Stochastics* 16 (1):1–15.

Herskovic, Bernard. 2018. “Networks in Production: Asset Pricing Implications.” *Journal of Finance* 73 (4):1785–1818.

Horowitz, Karen J. and Mark A. Planting. 2009. “Concepts and Methods of the U.S. Input-Output Accounts.” Tech. rep., Bureau of Economic Analysis.

Jackson, Matthew O. 2010. *Social and Economic Networks*. Princeton University Press, 1st ed.

Lettau, Martin and Jessica Wachter. 2007. “Why is Long Horizon Equity Less Risky? A Duration Based Explanation of the Value Premium.” *The Journal of Finance* 62 (1):55–92.

Lucas, Robert E. Jr. 1978. “Asset Prices in an Exchange Economy.” *Econometrica* 46 (6):1429–1445.

Nakamura, Emi, Jón Steinsson, Robert Barro, and José Ursúa. 2013. “Crises and recoveries in an empirical model of consumption disasters.” *American Economic Journal: Macroeconomics* 5 (3):35–74.

Newman, Mark. 2018. *Networks: An Introduction*. Oxford University Press, second ed.

Richmond, Robert J. 2019. “Trade Network Centrality and Currency Risk Premia.” *The Journal of Finance* 74 (3):1315–1361.

Schulz, Florian. 2016. "On the timing and pricing of dividends: Comment." *American Economic Review* 106 (10):3185–3223.

van Binsbergen, Jules, Michael Brandt, and Ralph Koijen. 2012. "On the Timing and Pricing of Dividends On the Timing and Pricing of Dividends." *American Economic Review* 102 (1024):1596–1618.

van Binsbergen, Jules, Wouter Hueskes, Ralph Koijen, and Evert Vrugt. 2013. "Equity yields." *Journal of Financial Economics* 110 (3):503–519.

van Binsbergen, Jules H. and Ralph S.J. Koijen. 2016. "On the timing and pricing of dividends: Reply." *American Economic Review* 106 (10):3224–3237.

———. 2017. "The term structure of returns: Facts and theory." *Journal of Financial Economics* 124 (1):1–21.

vom Lehn, Christian and Thomas Winberry. 2019. "The Investment Network, Sectoral Comovement, and the Changing U.S. Business Cycle." *SSRN Electronic Journal* .

Wachter, Jessica A. 2005. "Solving models with external habit." *Finance Research Letters* 2 (4):210–226.

APPENDIX:
PROOFS AND DATA

A.1 Proofs from Chapter 1

This section provides the proofs to the lemmas, propositions, and corollaries that appear in the first chapter of this dissertation, "Characterizing the Role of Dividend Dynamics in the Term Structure of Equity Risk Premia."

A.1.1 Proof of Lemma 1

Proof. I begin by deriving the expression for the one-period risk-free rate. Using the assumed expression for the SDF, (1.9), the one-period conditional risk-free rate is given by

$$\log R^f_{t,t+1} = -\mu_s - U_s' x_t - \frac{1}{2}\|\lambda_s\|^2. \tag{A.1}$$

This is derived by

$$\begin{aligned} \log R^f_{t,t+1} &= -\log \mathbb{E}_t\left[\frac{S_{t+1}}{S_t}\right] \\ &= -\log \mathbb{E}_t\left[\exp\left\{\mu_s + U_s' x_t + \lambda_s \cdot w_{t+1}\right\}\right]. \end{aligned}$$

Proceeding, from the pricing equation (1.11) we have

$$\begin{aligned} 1 &= \mathbb{E}_t\left[\frac{S_{t+1}}{S_t} R_{t,t+1}\right] \\ &= \mathbb{E}_t[\exp\{s_{t+1} - s_t + \log R_{t,t+1}\}] \\ 0 &= \mu_s + \mu_r + (U_s + U_r)' x_t + \frac{1}{2}\|\lambda_s + \lambda_r\|^2 \\ 0 &= \mu_s + \mu_r + (U_s + U_r)' x_t + \frac{1}{2}\|\lambda_s\|^2 + \frac{1}{2}\|\lambda_r\|^2 + \lambda_s \cdot \lambda_r. \end{aligned}$$

Substituting into the definition for the returns process (1.12),

$$\begin{aligned}\log \mathbb{E}_t[R_{t,t+1}] &= \mu_r + U_t' x_t + \frac{1}{2}\|\lambda_r\|^2 \\ &= -\mu_s - U_s' x_t - \frac{1}{2}\|\lambda_s\|^2 - \lambda_s \cdot \lambda_r.\end{aligned}$$

Subtracting the expression for the one-period risk-free rate, the one-period conditional risk-premium is

$$\log E_t[R_{t,t+1}] - \log R^f_{t,t+1} = -\lambda_s \cdot \lambda_r.$$

□

A.1.2 Lemma A1: Impulse Response Functions

Before computing risk prices and risk-premia, I begin with a preliminary calculation. Here I derive an expression for the impulse response functions defined in (1.14). Given the assumed dynamics for the state, x_t, in (1.8) and dividends, D_t, in (1.10), the impulse response function for dividends is

$$\psi_d'(k) = \lambda_d' + U_d'(I - G)^{-1}(I - G^{k-1})H \qquad \text{for } k = 1, .., \tau, \tag{A.2}$$

such that

$$\log D_{t+\tau} = \log D_t + \tau\mu_d + U_d'(I - G)^{-1}(I - G^{\tau})x_t + \sum_{k=1}^{\tau} \psi_d(k) \cdot w_{t+1+\tau-k}. \tag{A.3}$$

The analogous expression holds for the SDF, S_t,

$$\psi_s'(k) = \lambda_s' + U_s'(I - G)^{-1}(I - G^{k-1})H$$

Proof. Consider the expression for dividend growth over τ periods. Using

$$x_{t+k} = Gx_{t+k-1} + Hw_{t+k} = G^k x_t + \sum_{i=0}^{k-1} G^i H w_{t+k-i}$$

together with (1.10),

$$\begin{aligned}
\log D_{t+\tau} - \log D_t &= \sum_{k=0}^{\tau-1} \log D_{t+k+1} - \log D_{t+k} \\
&= \sum_{k=0}^{\tau-1} \mu_d + U_d' x_{t+k} + \lambda_d' w_{t+k+1} \\
&= \tau\mu_d + U_d' \sum_{k=0}^{\tau-1} G^k x_t + U_d' \sum_{k=0}^{\tau-1}\sum_{i=0}^{k-1} G^i H w_{t+k-i} + \sum_{k=0}^{\tau-1} \lambda_d' w_{t+k+1}.
\end{aligned}$$

Gathering terms and simplifying,

$$\begin{aligned}
\sum_{k=0}^{\tau-1}\sum_{i=0}^{k-1} G^i H w_{t+k-i} &= \sum_{k=1}^{\tau-1}\sum_{i=0}^{k-1} G^i H w_{t+k-i} \\
&= \sum_{k=1}^{\tau-1}\sum_{i=0}^{\tau-k-1} G^i H w_{t+k} \\
&= \sum_{k=1}^{\tau-1} (I-G)^{-1}(I-G^{\tau-k}) H w_{t+k},
\end{aligned}$$

where the last equality applies since $I - G$ is invertible, following from the fact that G has a spectral radius less than one. Continuing, we arrive at

$$\log D_{t+\tau} - \log D_t = \tau\mu_d + U_d'(I-G)^{-1}(I-G^\tau)x_t + \sum_{k=1}^{\tau} \psi_d'(k) w_{t+1+\tau-k}, \qquad \text{(A.4)}$$

where

$$\psi_d'(k) \coloneqq \lambda_d' + U_d'(I-G)^{-1}(I-G^{k-1})H.$$

With this expression in hand, we can compute the expectations defining the impulse response.

Substitute (A.4) into $\Delta\mathbb{E}_{t+1}\left[\log D_{t+\tau}\right] := \mathbb{E}_{t+1}\left[\log D_{t+\tau}\right] - \mathbb{E}_t\left[\log D_{t+\tau}\right]$ to arrive at

$$\Delta\mathbb{E}_{t+1}\left[\log D_{t+\tau}\right] = \psi_d(\tau) \cdot w_{t+1}.$$

Given the definition of the impulse response function described in (1.14), this verifies that ψ_d is the impulse response function of dividends with respect to the structural shocks w_{t+1}. The derivation for S_t proceeds in the same way.

□

A.1.3 Proof of Proposition 3

Proof. The one-period holding period return on a dividend strip with maturity τ, given in (1.3), can be rewritten as

$$R_{t+1}^{\tau} = \frac{P_{t+1}^{\tau-1}}{P_t^{\tau}} = \frac{P_{t+1}^{\tau-1}/D_{t+1}}{P_t^{\tau}/D_t}\frac{D_{t+1}}{D_t}.$$

Now, substituting (1.9) and (1.10) into (1.2) and using the expression for the impulse responses derived in Section A.1.2 allows us to arrive at the following expression for the price of the dividend strip,

$$\begin{aligned} P_t^{\tau} &= D_t \mathbb{E}_t\left[\frac{S_{t+\tau}}{S_t}\frac{D_{t+\tau}}{D_t}\right] \\ &= D_t \exp\left\{\tau(\mu_s + \mu_d) + (U_s + U_d)'(I-G)^{-1}(I-G^{\tau})x_t + \frac{1}{2}\sum_{k=1}^{\tau}\|\psi_s(k) + \psi_d(k)\|^2\right\}. \end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{P_{t+1}^{\tau-1}/D_{t+1}}{P_t^{\tau}/D_t} &= \exp\Bigg\{-(\mu_s+\mu_d)+(U_s+U_d)'(I-G)^{-1}\Big[(I-G^{\tau-1})x_{t+1}-(I-G^{\tau})x_t\Big] \\
&\qquad -\frac{1}{2}\|\psi_d(\tau)+\psi_d(\tau)\|^2\Bigg\} \\
&= \exp\Bigg\{-(\mu_s+\mu_d)+(U_s+U_d)'(I-G)^{-1}(I-G^{\tau-1})Hw_{t+1}-(U_s+U_d)'x_t \\
&\qquad -\frac{1}{2}\|\psi_d(\tau)+\psi_d(\tau)\|^2\Bigg\} \\
&= \exp\Bigg\{-(\mu_s+\mu_d)+(\psi_d(\tau)-\psi_d(1)+\psi_s(\tau)-\psi_s(1))'w_{t+1}-(U_s+U_d)'x_t \\
&\qquad -\frac{1}{2}\|\psi_s(\tau)+\psi_d(\tau)\|^2\Bigg\},
\end{aligned}$$

where we have applied the expressions for the impulse response functions from Section A.1.2,

$$\begin{aligned}
\psi_d'(\tau) &= \lambda_d'+U_d'(I-G)^{-1}(I-G^{\tau})H \\
\psi_s'(\tau)-\psi_s'(1) &= U_s'(I-G)^{-1}(I-G^{\tau})H.
\end{aligned}$$

Substituting all this into the log holding period return gives us

$$\begin{aligned}
\log R_{t,t+1}^{\tau} &= \log\left(\frac{P_{t+1}^{\tau-1}/D_{t+1}}{P_t^{\tau}/D_t}\frac{D_{t+1}}{D_t}\right) \\
&= \log\left(\frac{P_{t+1}^{\tau-1}/D_{t+1}}{P_t^{\tau}/D_t}\right)+\log\left(\frac{D_{t+1}}{D_t}\right) \\
&= \log\left(\frac{P_{t+1}^{\tau-1}/D_{t+1}}{P_t^{\tau}/D_t}\right)+\Big(\mu_d+U_d'x_t+\lambda_d'w_{t+1}\Big) \\
&= -\mu_s-\frac{1}{2}\|\psi_s(\tau)+\psi_d(\tau)\|^2-U_s'x_t+(\psi_d(\tau)+\psi_s(\tau)-\psi_s(1))'w_{t+1}.
\end{aligned}$$

Thus,

$$\log R_{t,t+1}^{\tau} = \mu_\tau+U_\tau'x_t+\lambda_\tau'w_{t+1}, \tag{A.5}$$

where

$$\mu_\tau = -\mu_s - \frac{1}{2}\|\psi_s(\tau) + \psi_d(\tau)\|^2$$
$$U_\tau = -U_s$$
$$\lambda_\tau = \psi_s(\tau) - \psi_s(1) + \psi_d(\tau).$$

Finally, applying Lemma 1, we arrive at

$$\log \mathbb{E}[R^\tau_{t+1}] - \log \mathbb{E}[R^f_{t+1}] = -\lambda_s \cdot (\psi_s(\tau) - \psi_s(1) + \psi_d(\tau)). \tag{A.6}$$

□

As an aside, expressing the return as

$$R^\tau_{t+1} = \underbrace{\frac{P^{\tau-1}_{t+1}/D_{t+1}}{P^\tau_t/D_t}}_{\text{valuation risk}} \underbrace{\frac{D_{t+1}}{D_t}}_{\text{cash flow risk}} .$$

can serve as a helpful way to distinguish two potential sources of risk—valuation risk coming from variation in the asset's price-dividend ratio and dividend risk coming from variation in the level of dividend payouts. In this scenario, the valuation risk arises from the asset's sensitivity to changes in the state, x_{t+1}, and cash flow risk comes from exposure to changes in dividends. When dividends are fixed, $\psi_d(\tau)$ is equal to a vector of zeros. Under such a situation, we see that the premium associated with valuation risk is captured by the terms $\psi_s(\tau) - \psi_s(1)$ and dividend risk is captured by the term $\psi_d(\tau)$.

A.1.4 Proof of Corollary 4

Proof. The two claims in this statement, embodied in (1.16) and (1.17), follow directly from Proposition 3.

- The proof of this corollary follows almost immediately. Equation (1.16) comes from the fact that the risk premium associated with the holding period return on a τ-horizon bond is a special case of the same associated with a τ-horizon dividend strip. If the dividend payment in the dividend strip is fixed and riskless, then $\psi_d(\tau) = 0$.
- The second expression, (1.17), then follows from differencing the first expression from the expression in Proposition 3.

□

A.1.5 Proof of Proposition 6

Proof. This proof relies on the derivation of the impulse response function in Section A.1.2 and Corollary 1.17.

- The expression

$$\mathrm{Cov}(\Delta \log S_{t+1}, \Delta \mathbb{E}_{t+1} [\log D_{t+\tau}]) = \psi_s(1) \cdot \psi_d(\tau)$$

  follows from the definition of the impulse response function,

$$\Delta \mathbb{E}_{t+1} [\log D_{t+\tau}] = \psi_d(\tau) \cdot w_{t+1}$$

  and the definition

$$\Delta \log S_{t+1} = \log S_{t+1} - \log S_t = \mu_s + U_s' x_t + \lambda_s' w_{t+1}.$$

- The derivation of the expression

$$\mathrm{Var}(\log R_{t+1}^{\tau} - \log R_{t+1}^{f,\tau}) = \|\psi_d(\tau)\|^2,$$

follows from computing the variance of the expression

$$\log R^{\tau}_{t,t+1} - \log R^{f}_{t,t+1} = -\frac{1}{2}\|\psi_s(\tau) - \psi_s(1) + \psi_d(\tau)\|^2 - (\psi_s(\tau) - \psi_s(1) + \psi_d(\tau)) \cdot \psi_s(1) + (\psi_s(\tau) - \psi_s(1) + \psi_d(\tau)) \cdot w_{t+1}. \tag{A.7}$$

This expression can be derived from taking (A.5) and substracting the expression for the log short-term risk-free rate, (A.1).

- The expression for the Sharpe ratio follows from Corollary 4 and dividing by the standard

□

A.1.6 Proof of Corollary 7

This derivation follows trivially from the definition of the dot product.

A.1.7 Proof of Lemma 8

Proof.

- This first claim can be easily verified by substituting the expression for F_y and M_y into

$$\log Y_t = t\mu_y + \sum_{k=1}^{t} M_y w_k + F_y x_t + \log Y_0 - F_y x_0$$

and computing

$$\log Y_{t+1} - \log Y_t.$$

- The claim that

$$\psi'_y(\infty) \coloneqq \lim_{\tau \to \infty} \psi'_y(\tau) = M_y$$

follows from the derivation of the impulse response function, given in (A.2). That is,

$$\psi_y'(k) = \lambda_y' + U_y'(I - G)^{-1}(I - G^{k-1})H.$$

- The expression

$$\psi_y'(1) - \psi_y'(\infty) = F_y H$$

follows because $\psi_y(1) = \lambda_y$.

□

A.1.8 Proof of Proposition 9

Proof. This follows from applying Lemma 8 to the expression for the dividend future premium, (1.17), derived in Corollary 4.

□

A.1.9 Proof of Lemma 11

Proof. In this section, I derive risk prices in an economy with Epstein-Zin preferences that is governed by a VAR(1) state space. I derive the solution where the risk aversion parameter is different from the reciprocal of the elasticity of intertemporal substitution, $\gamma \neq \rho$ and the elasticity of intertemporal substitution is one, $1/\rho = 1$. Under such an assumption, the household still has concern for long-run risk, but the solution admits an analytic solution. Assuming that the elasticity of intertemporal substitution is one, I show that the SDF in this case takes on the form $\log S_{t+1} - \log S_t = \mu_s + U_s' x_t + \lambda_s' w_{t+1}$. I derive the parameters in terms of the primitives defining consumption dynamics. Note that the derivations here closely follow those within Hansen, Heaton, and Li (2008).

Solving for stochastic discount factor In this section, I solve the utility recursion resulting from the assumed Epstein-Zin preferences and the given stochastic process governing the dynamics of the state of the economy. This will give us a characterization of the stochastic discount factor in terms of the underlying state dynamics and will allow a clean expression for risk-prices and risk-premia over arbitrary horizons.[1]

Let a representative agent have Epstein-Zin preferences, defined by the recursion

$$V_t = \{(1-\beta)(C_t)^{1-\rho} + \beta[\mathcal{R}_t(V_{t+1})]^{1-\rho}\}^{1/(1-\rho)},$$

where

$$\mathcal{R}_t(V_{t+1}) \equiv \mathbb{E}[(V_{t+1})^{1-\gamma} \mid \mathcal{F}_t]^{1/(1-\gamma)}.$$

Using the framework laid out in the previous section, again let x_t be a $N \times 1$ state vector capturing the state of the economy. Assume that it follows the VAR(1) process defined in equation (2.4), with w_{t+1} as a $M \times 1$ vector of i.i.d. shocks, $w_{t+1} \sim \mathcal{N}(0, \mathcal{I})$ and G and H

1. When dealing with Epstein-Zin preferences, there are generally two approaches in the literature to taking the SDF to the data. On approach is to rewrite the continuation value term in terms of the return on the wealth portfolio,

$$R^W_{t+1} = \frac{W_{t+1}}{W_t - C_t}. \tag{A.8}$$

This involves an application of Euler's theorem and results in

$$\frac{S_{t+1}}{S_t} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\theta\rho} (R^w_{t+1})^{\theta-1}, \tag{A.9}$$

where $\theta = \frac{1-\gamma}{1-\rho}$. While some papers use the returns on broad market indices (e.g., the S&P 500 index) as a proxy for R^W_t, in general this return in unobservable. The Roll critique applies here. The aggregate value of stock market indices make up only a small portion of total wealth in the economy. Importantly, it is missing the value of human capital. Furthermore, the returns on these proxies may not adequately capture the long-run effects of certain shocks to the macroeconomy. In models such as those of Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008), the risk associated with these long-run shocks is crucial. To alleviate the shortcomings of using such a proxy, the second approach is to solve for the continuation value directly in terms of stochastic process governing the state of the economy, including consumption growth.

as conforming matrices. Let log consumption growth be given by

$$c_{t+1} - c_t = \mu_c + U_c \cdot x_t + \lambda_c \cdot w_{t+1}. \tag{A.10}$$

The assumed preferences imply the stochastic discount factor (SDF),

$$\begin{aligned}\frac{S_{t+1}}{S_t} &= (MV_{t+1})\frac{MC_{t+1}}{MC_t} \\ &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right)^{\rho-\gamma} \\ &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left[\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]}\right]^{\frac{\rho-\gamma}{1-\gamma}}.\end{aligned}$$

To characterize this SDF in terms of the assumed dynamics for the state (2.4) and consumption growth (A.10), we need to solve for the continuation values V_t in terms of the same. The utility recursion that characterizes Epstein-Zin preferences, paired with the assumed state and consumption growth dynamics, results in a functional equation that we may solve. Rearranging terms,

$$\begin{aligned}V_t &= \{(1-\beta)(C_t)^{1-\rho} + \beta[\mathcal{R}_t(V_{t+1})]^{1-\rho}\}^{1/(1-\rho)} \\ \frac{V_t}{C_t} &= \left\{(1-\beta) + \beta\left[\mathcal{R}_t\left(\frac{V_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)\right]^{1-\rho}\right\}^{1/(1-\rho)},\end{aligned}$$

so we need only solve $v_t - c_t \equiv \log\frac{V_t}{C_t}$ as a function of the state x_t.

Adding the assumption that the elasticity of intertemporal substitution is one Here I derive the solution for risk prices in terms of the underlying state dynamics, in the case that $\rho = 1$. This requires us to solve for the SDF, and thus the continuation values of utility. If we assume that $\rho = 1$, then we may obtain an analytic solution. Otherwise, we must approximate the solution. Note that when $\rho = \gamma$, the assumed preferences collapse into CRRA preferences. Further, when $\rho = \gamma = 1$, they become log-utility. When $\rho = 1$

and $\gamma > 1$, the agent's preferences are not of the time-additive von Neumann–Morgenstern expected utility variety and the agent still exhibits a concern for long-run risk.

Solving for the continuation value. Calculating the limit of the utility recursion as $\rho \to 1$ (applying l'Hopital's rule), we get

$$\begin{aligned} v_t - c_t &= \frac{\beta}{1-\gamma} \ln \left(\mathbb{E}\left[\left(\frac{V_{t+1}}{C_{t+1}} \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \middle| \mathcal{F}_t \right] \right) \\ &= \frac{\beta}{1-\gamma} \ln \left(\mathbb{E}_t \left[\exp \left\{ (1-\gamma)(v_{t+1} - c_{t+1} + c_{t+1} - c_t) \right\} \right] \right) \\ &= \frac{\beta}{1-\gamma} \ln \left(\mathbb{E}_t \left[\exp \left\{ (1-\gamma)(v_{t+1} - c_{t+1} + \mu_c + U_c' x_t + \lambda_c \cdot w_{t+1}) \right\} \right] \right). \end{aligned}$$

The end result here is a difference equation in $v_t - c_t$. In order to solve this difference equation, I proceed by the method of undetermined coefficients. Let us postulate that $v_t - c_t = \phi(x_t) = \mu_v + U_v \cdot x_t$ for some presently unknown coefficients μ_v and U_v. Substituting,

$$\begin{aligned} \mu_v + U_v' x_t &= \frac{\beta}{1-\gamma} \ln \left(\mathbb{E}_t \left[\exp \left\{ (1-\gamma)(\mu_v + U_v'(G x_t + H w_{t+1}) + \mu_c + U_c' x_t + \lambda_c \cdot w_{t+1}) \right\} \right] \right) \\ &= \beta \left(\mu_v + \mu_c + (U_v' G + U_c') x_t + \frac{1}{2}(1-\gamma)^2 \| U_v' H + \lambda_c' \|^2 \right). \end{aligned}$$

Then, matching terms, we see that we must have

$$\begin{aligned} U_v' &= \beta U_c' (I - \beta G)^{-1} \\ \mu_v &= \frac{\beta}{1-\beta} \left(\mu_c + (1-\gamma)^2 \frac{1}{2} \| U_v' H + \lambda_c' \|^2 \right). \end{aligned}$$

Note that these all conform, since if x_t is $N \times 1$ and w_{t+1} is $M \times 1$, then U_v and U_c are both $1 \times N$, G is $N \times N$, H is $N \times M$, and λ_c is $1 \times M$.

In summary, we get

$$v_t - c_t = \mu_v + U_v' x_t,$$

with

$$\mu_v = \frac{\beta}{1-\beta}\left(\mu_c + (1-\gamma)\frac{1}{2}\|U_v' H + \lambda_c\|^2\right)$$
$$U_v' = \beta U_c'(I - \beta G)^{-1}.$$

Characterizing the SDF Now that we have solved for the continuation value in terms of the state dynamics, we can solve for the stochastic discount factor in terms of the state dynamics. In the limit as $\rho \to 1$,

$$\begin{aligned}\frac{S_{t+1}}{S_t} &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left[\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]}\right] \\ &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-1}\frac{\left(\frac{V_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}}{\mathbb{E}_t\left[\left(\frac{V_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]}.\end{aligned} \tag{A.11}$$

Computing,

$$\begin{aligned}\mathbb{E}_t\left[\left(\frac{V_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right] &= \mathbb{E}_t[\exp\{(1-\gamma)((v_{t+1}-c_{t+1}) + (c_{t+1}-c_t))\}] \\ &= \exp\left[(1-\gamma)(\mu_v + \mu_c + (U_v' G + U_c')x_t) + \frac{1}{2}(1-\gamma)^2\|U_v' H + \lambda_c'\|^2\right]\end{aligned}$$

Then, defining $s_{t+1} \equiv \ln S_{t+1}$, substituting in the definition for consumption dynamics, and substituting in our solution for the continuation value, we calculate

$$\begin{aligned}s_{t+1} - s_t &= \ln\beta - (c_{t+1} - c_t) + (1-\gamma)((v_{t+1} - c_{t+1}) + (c_{t+1} - c_t)) \\ &\quad - \left[(1-\gamma)(\mu_v + \mu_c + (U_v' G + U_c')x_t) + \frac{1}{2}(1-\gamma)^2\|U_v' H + \lambda_c'\|^2\right] \\ &= \ln\beta - \gamma(\mu_c + U_c' x_t + \lambda_c \cdot w_{t+1}) + (1-\gamma)U_v' H w_{t+1} \\ &\quad - \left[(1-\gamma)(\mu_c + U_c' x_t) + \frac{1}{2}(1-\gamma)^2\|U_v' H + \lambda_c'\|^2\right] \\ s_{t+1} - s_t &= \mu_s + U_s' x_t + \lambda_s \cdot w_{t+1},\end{aligned}$$

where

$$\mu_s = \ln\beta - \mu_c - \frac{1}{2}(1-\gamma)^2 \|U_v' H + \lambda_c'\|^2$$
$$U_s = -U_c$$
$$\lambda_s' = -(\gamma-1)(\lambda_c' + U_v' H) - \lambda_c'.$$

Summary Here I summarize the assumptions and results. To summarize, x_t is an $N \times 1$ state vector capturing the state of the economy,

$$x_{t+1} = Gx_t + Hw_{t+1},$$

with w_{t+1} as a $M \times 1$ vector of i.i.d. shocks, $w_{t+1} \sim \mathcal{N}(0, \mathcal{I})$ and G and H as conforming matrices. Log consumption growth is

$$c_{t+1} - c_t = \mu_c + U_c \cdot x_t + \lambda_c \cdot w_{t+1}.$$

Solving the utility recursion for the case when $\rho = 1$, we get

$$v_t - c_t = \mu_v + U_v' x_t, \tag{A.12}$$

with

$$\begin{aligned} \mu_v &= \frac{\beta}{1-\beta}\left(\mu_c + (1-\gamma)\frac{1}{2}\|U_v' H + \lambda_c\|^2\right) \\ U_v' &= \beta U_c'(I - \beta G)^{-1}. \end{aligned} \tag{A.13}$$

The log SDF is

$$s_{t+1} - s_t = \mu_s + U_s' x_t + \lambda_s \cdot w_{t+1} \tag{A.14}$$

with

$$\mu_s = \ln\beta - \mu_c - \frac{1}{2}(1-\gamma)^2 \|U_v' H + \lambda_c'\|^2$$
$$U_s = -U_c \tag{A.15}$$
$$\lambda_s' = -(\gamma-1)(\lambda_c' + U_v' H) - \lambda_c'.$$

□

Note that when $\gamma = 1$, preferences collapse to the log-normal case. Thus, let

$$\lambda_{s,SR} = -\lambda_c$$
$$\lambda_{s,LR} = -(\lambda_c' + U_v' H)', \tag{A.16}$$

so that

$$\lambda_s = \lambda_{s,SR} + (\gamma-1)\lambda_{s,LR}.$$

These are defined with the interpretation that $\lambda_{s,LR}$ is the portion of risk prices that are due to concern for long-run risk. When $\gamma = 1$, we have $\gamma = \rho$, and preferences thus collapse into the expected utility case—the log-utility case, in particular—and $\lambda_{s,LR} = 0$. Therefore, we can interpret $\lambda_{s,LR}$ as the component of risk prices due to a concern for long-run risk, under the assumption that $\rho = 1$.

□

A.1.10 Proof of Proposition 12

Proof. This proof proceeds as follows. Suppose that $DF_\infty < DF_1$. Substituting, this is

$$\begin{aligned} 0 &> DF_\infty - DF_1 \\ &= -\lambda_s \cdot (\psi_d(\infty) - \psi_d(1)) \\ &= -\eta\,\lambda_s \cdot (\psi_{\mathcal{C}}(\infty) - \psi_{\mathcal{C}}(1)). \end{aligned}$$

Since

$$\psi'_{\mathcal{C}}(k) = \lambda'_{\mathcal{C}} + U'_{\mathcal{C}}(I-G)^{-1}(I-G^{k-1})H,$$

we can substitute to get

$$0 > \gamma\lambda_{\mathcal{C}} \cdot (U'_{\mathcal{C}}(I-G)^{-1}H) + (\gamma-1)\beta U'_{\mathcal{C}}(I-\beta G)^{-1}HH'(I-G')^{-1}U_{\mathcal{C}}.$$

Applying the limit $\beta \to 1$ gives

$$0 > \gamma\lambda_{\mathcal{C}} \cdot (U'_{\mathcal{C}}(I-G)^{-1}H) + (\gamma-1)U'_{\mathcal{C}}(I-G)^{-1}HH'(I-G')^{-1}U_{\mathcal{C}}.$$

Lemma 8 tells us that

$$\begin{aligned} F_{\mathcal{C}} &= -U'_{\mathcal{C}}(I-G)^{-1} \\ M_{\mathcal{C}} &= \lambda'_{\mathcal{C}} + U'_{\mathcal{C}}(I-G)^{-1}H. \end{aligned}$$

From this, plus the expressions

$$\begin{aligned} \log \mathcal{C}_{t+1} - \log \mathcal{C}_t &= \mu_{\mathcal{C}} + U'_{\mathcal{C}}x_t + \lambda'_{\mathcal{C}}w_{t+1} \\ F_{\mathcal{C}}x_{t+1} &= F_{\mathcal{C}}Gx_t + F_{\mathcal{C}}Hw_{t+1}, \end{aligned}$$

we can derive

$$\mathrm{Cov}_t(\log \mathcal{C}_{t+1} - \log \mathcal{C}_t, F_{\mathcal{C}}x_{t+1}) > \frac{\gamma-1}{\gamma}\mathrm{Var}_t(F_{\mathcal{C}}x_{t+1}).$$

□

A.2 Proofs and Derivations from Chapter 2

In this section of the appendix, I provide the relevant proofs and derivations from Chapter 2 of the paper.

A.2.1 Production networks benchmark model solution

Since the welfare theorems apply, competitive equilibrium can be obtained by solving the social planner's problem. A series of monotonic transformations of the utility recursion simplifies the expression of the objective function so that the social planner solves

$$\begin{aligned} V^{1-\rho}(\{K_{it},\Xi_{it}\}) = & \max_{\{I_{ijt}\},\{M_{ijt}\},\{K_{i,t+1}\},\{L_{it}\}} (1-\beta)\mathcal{C}^{1-\rho} + \beta\left(\mathcal{R}_t(V_{t+1})\right)^{1-\rho} \\ & \text{subject to} \quad K_{i,t+1} = I_{it} + (1-\delta_j)K_{it} \end{aligned} \tag{A.17}$$

and, substituting where applicable,

$$\mathcal{C}_t = \mathcal{L}_t^{1-s_c} C_t^{s_c} \tag{A.18}$$

$$C_t = \prod_{i=1}^{n} C_{it}^{\alpha_i} \tag{A.19}$$

$$I_{it} = \prod_{j=1}^{n} I_{ijt}^{\theta_{ij}} \tag{A.20}$$

$$M_{it} = \prod_{j=1}^{n} M_{ijt}^{a_{ij}} \tag{A.21}$$

$$Q_{it} = \Xi_{it} K_{it}^{a_i^k} L_{it}^{a_i^\ell} M_{it}^{a_i^m} \tag{A.22}$$

$$C_{jt} = Q_{jt} - \sum_{i=1}^{n} M_{ijt} - \sum_{i=1}^{n} I_{ijt} \tag{A.23}$$

$$\mathcal{L}_t = H - \sum_{i=1}^{n} L_{it} \tag{A.24}$$

$$\mathcal{R}_t(V_{t+1}) = \mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \tag{A.25}$$

with $a_i^k + a_i^\ell + a_i^m = 1$ for all i.

First-order conditions Let λ_{it} be the Lagrange multiplier associated with the constraints on Capital dynamics. Then, the first order conditions are as follows.

- Those associated with I_{ijt} are

$$(1-\beta)(1-\rho)s_c\frac{\mathcal{C}^{1-\rho}}{C_t}\alpha\frac{C_t}{C_{jt}}(-1)+\lambda_{it}\left(-\theta_{ij}\frac{I_t}{I_{ijt}}\right)=0 \tag{A.26}$$

- and with $K_{i,t+1}$ are

$$\begin{aligned}
&\beta(1-\rho)\mathcal{R}_t(V_{t+1})^{-\rho}\frac{\partial R_t}{\partial K_{i,t+1}}+\lambda_{it}=0\\
&\beta(1-\rho)\mathcal{R}_t(V_{t+1})^{-\rho}\mathcal{R}_t^{\gamma}\mathbb{E}_t\left[V_{t+1}^{-\gamma}\frac{\partial V_{t+1}}{\partial K_{i,t+1}}\right]=-\lambda_{it},
\end{aligned} \tag{A.27}$$

with $\mathcal{R}_t=\mathcal{R}_t(V_{t+1})$. Combining these two gives

$$(1-\beta)s_c\frac{\mathcal{C}_t^{1-\rho}}{C_{jt}}\alpha_j=\beta\mathcal{R}_t^{\gamma-\rho}\mathbb{E}_t\left[V_{t+1}^{-\gamma}\frac{\partial V_{t+1}}{\partial K_{i,t+1}}\right]\theta_{ij}\frac{I_{it}}{I_{ijt}}. \tag{A.28}$$

- The envelope theorem implies

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}K_{it}}V_t^{1-\rho}&=(1-\rho)V_t^{-\rho}\frac{\mathrm{d}V_t}{\mathrm{d}K_{it}}\\
&=(1-\beta)(1-\rho)s_c\alpha_i\frac{\mathcal{C}_t^{1-\rho}}{C_{it}}a_i^k\frac{Q_{it}}{K_{it}}-\lambda_{it}(1-\delta).
\end{aligned}$$

Combining this with the first-order conditions for investment and capital gives

$$\begin{aligned}
&\alpha_j\frac{\mathcal{C}_t^{1-\rho}}{C_{jt}}=\\
&\qquad\mathbb{E}_t\left[\beta\left(\frac{V_{t+1}}{\mathcal{R}_t}\right)^{\rho-\gamma}\theta_{ij}\frac{I_{it}}{I_{ijt}}\left(\alpha_i\frac{\mathcal{C}_{t+1}^{1-\rho}}{C_{i,t+1}}a_i^k\frac{Q_{i,t+1}}{K_{i,t+1}}+\alpha_j\frac{\mathcal{C}_{t+1}^{1-\rho}}{C_{j,t+1}}\frac{I_{ij,t+1}}{I_{i,t+1}}\frac{1}{\theta_{ijt}}(1-\delta)\right)\right].
\end{aligned} \tag{A.29}$$

The pieces of this equation can be interpreted as follows. The term,

$$\underbrace{\alpha_j \frac{\mathcal{C}_t^{1-\rho}}{C_{jt}}}_{\substack{\sim\text{marginal utility}\\ \text{of good } j}},$$

is proportional to the marginal utility of good j. The conditional expectations contains a risk adjustment, the marginal increase in the investment in good i with respect to the contribution of the investment good j, and the marginal utility associated with an increase in the capital of type i,

$$\mathbb{E}_t\left[\beta \underbrace{\left(\frac{V_{t+1}}{\mathcal{R}_t}\right)^{\rho-\gamma}}_{\substack{\text{risk adjustment of}\\ \text{next period utility}}} \underbrace{\theta_{ij}\frac{I_{it}}{I_{ijt}}}_{\substack{\text{marginal transform. of}\\ \text{good } j \text{ into inv. } i.}} \underbrace{(\ldots)}_{\substack{\text{marginal utility per}\\ \text{unit of capital type } i}}\right].$$

This final marginal utility term can be broken down as follows. It is the marginal utility associated with an increase in capital of type i, with one part coming from the marginal product of that capital and the part coming from the increase in the capital stock remaining after depreciation in the following period:

$$\left(\underbrace{\alpha_i \frac{\mathcal{C}_{t+1}^{1-\rho}}{C_{i,t+1}}}_{\substack{\sim\text{marginal utility}\\ \text{of good } i}} \underbrace{a_i^k \frac{Q_{i,t+1}}{K_{i,t+1}}}_{\substack{\text{marginal product}\\ \text{of capital in}\\ \text{sector } i}} + \underbrace{\alpha_j \frac{\mathcal{C}_{t+1}^{1-\rho}}{C_{j,t+1}}}_{\substack{\sim\text{marginal utility}\\ \text{of good } j}} \underbrace{\frac{I_{ij,t+1}}{I_{i,t+1}}\frac{1}{\theta_{ijt}}}_{\substack{\text{marginal}\\ \text{transformation}\\ \text{into good } j\\ \text{per unit of capital } i}} \underbrace{(1-\delta)}_{\substack{\text{capital remaining}\\ \text{after depreciation}}}\right).$$

Note that the risk adjustment term $\beta\left(\frac{V_{t+1}}{\mathcal{R}_t}\right)^{\rho-\gamma}$ arising from the assumption of non-expected utility, via Epstein-Zin recursive utility. A expected, when $\gamma = \rho$, this term becomes unity.

- Also, note that when we assume that the consumption bundle $\mathcal{C}_t$ is the numeraire with price normalized to one, $P_t = 1$. The price index is then

$$P_t = \prod_{i=1}^{n} \left(\frac{P_{it}}{\alpha_i} \right)^{\alpha_i},$$

so that the price of a unit of good i satisfies

$$P_{it} = \alpha_i \frac{\mathcal{C}_t}{C_{it}}.$$

The stochastic discount factor (SDF) is then

$$\frac{S_{t+1}}{S_t} = \beta \left(\frac{V_{t+1}}{\mathcal{R}_t} \right)^{\gamma-\rho} \left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t} \right)^{-\rho}. \tag{A.30}$$

We can then write the first-order conditions for investment and capital as

$$1 = \mathbb{E}_t \left[\frac{S_{t+1}}{S_t} R_{ij,t+1} \right], \tag{A.31}$$

where $R_{ij,t+1}$ is the return associated with investing a unit of the consumption good j into capital of type i,

$$R_{ij,t+1} = \theta_{ij} \frac{I_{it}}{I_{ijt}} \frac{1}{P_{jt}} \left(P_{i,t+1} a_i^k \frac{Q_{i,t+1}}{K_{i,t+1}} + P_{j,t+1} \frac{I_{ij,t+1}}{I_{i,t+1}} \frac{1}{\theta_{ij}} (1-\delta) \right). \tag{A.32}$$

- The first-order conditions with respect to the intermediate goods M_{ijt} are

$$\frac{\alpha_j}{C_{jt}} = \frac{\alpha_i}{C_{it}} a_i^m Q_{it} M_{ijt} a_{ij}. \tag{A.33}$$

- The first-order conditions of the household's problem with respect to L_{it} are

$$\frac{1-s_C}{\mathcal{L}_t} = s_C \frac{\alpha_i}{C_{it}} a_i^{\ell} Q_{it} L_{it}. \tag{A.34}$$

A.2.2 Derivation of Proposition 13: Risk prices and risk exposures in the full depreciation case

The proof of Proposition 13 follows simply from rewriting (via stacking) the state vector equation, (2.11), as a VAR(1) and substituting the values into the expression for the impulse response function derived in Lemma A1, found in Section A.1.2. Corollary 17 follows almost immediately.

A.2.3 Derivation of Proposition 16: Case with Full Depreciation

Here I use the first-order conditions derived from the full model, as derived in the previous section, Section A.2.1. When $\delta = 1$ and as ρ approaches 1 in the limit, the program (A.17) becomes

$$\begin{aligned} \log V(\{K_{it}, \Xi_{it}\}) = \max_{\{I_{ijt}\},\{M_{ijt}\},\{K_{i,t+1}\},\{L_{it}\}} \quad & (1-\beta)\log \mathcal{C}_t + \beta \log\left(\mathcal{R}_t(V_{t+1})\right) \\ \text{subject to} \quad K_{i,t+1} = & \prod_{j=1}^{n} I_{ijt}^{\theta_{ij}} \end{aligned} \tag{A.35}$$

and subject to the constraints (A.18) through (A.25). The first-order conditions derived in the previous section will hold, after substituting $\rho = 1$ and $\delta = 1$.

To solve for the optimal policy functions and the value function, I proceed by using the method of undetermined coefficients. I make a direct guess as to the functional form of V_t. From this, I derive the optimal policy functions and determine the value of V_{t+1}. The first-order conditions along with market clearing would then give us the restrictions needed to determine these unknown coefficients. I then verify that this guess indeed solves the functional equation that characterizes equilibrium.

Guess Value Function and Evaluate First-Order Conditions

- Guess that

$$V(\{K_{it}\},\{\xi_{it}\}) = \prod_{i}^{n} K_{it}^{s_c(1-\beta)a_i^k\nu_i} \exp\{J(\xi_t)\} V_0, \tag{A.36}$$

  for some unknown constants ν_i, for $i = 1, \ldots, n$, an unknown $\mathcal{F}_t$-measurable function J, and a constant V_0.

- Under this assumption,

$$\frac{\partial V_t}{\partial K_{it}} = s_c(1-\beta)a_i^k\nu_i\frac{V_t}{K_{it}}.$$

  Substituting this into the first-order conditions for investment and capital, (A.28), implies that

$$\begin{aligned}\frac{\alpha_i}{C_{jt}} &= \beta a_i^k \nu_i \theta_{ij}\frac{I_{it}}{I_{ijt}}\mathbb{E}_t\left[\left(\frac{V_{t+1}}{\mathcal{R}_t}\right)^{1-\gamma}\frac{1}{K_{i,t+1}}\right] \\ \frac{\alpha_j}{C_{jt}} &= \beta a_i^k \nu_i \theta_{ij}\frac{1}{I_{ijt}},\end{aligned} \tag{A.37}$$

  where the second line follows from

$$K_{i,t+1} = I_{it}$$

  and

$$\mathbb{E}_t\left[\left(\frac{V_{t+1}}{\mathcal{R}_t}\right)^{1-\gamma}\right] = \mathbb{E}_t\left[\frac{V_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]}\right] = 1.$$

- For convenience, define $\widetilde{\Theta} = [\theta_{ij}]$ and $\widetilde{A} = [a_{ij}]$. Since these are the Cobb-Douglas aggregator shares that sum to one, $\widetilde{\Theta}\mathbb{1} = \widetilde{A}\mathbb{1} = \mathbb{1}$, where $\mathbb{1}$ is a vector of ones.

- Let $d_{jt} = \alpha_j\frac{\sum_{i=1}^n I_{ijt}}{C_{jt}}$, with the vector $d_t = [d_{jt}]$. Then, continuing with equation (A.37), this implies that

$$d' \equiv d_t' = \beta\nu'\Theta,$$

where $\Theta = \text{diag}(a^k)\widetilde{\Theta} = [a_i^k \theta_{ij}]$ and, since d_t is constant, I've defined $d = d_t$.

- From the first-order conditions for M_{ijt}, (A.33) combined with the market clearing conditions for goods,

$$\begin{aligned} Q_{jt} - \sum_{i=1}^{n} I_{ijt} - C_{jt} &= \sum_{i=1}^{n} M_{ijt} = \sum_{i=1}^{n} \frac{\alpha_i}{\alpha_j} \frac{C_{jt}}{C_{it}} a_i^m Q_{it} a_{ij} \\ \frac{Q_{jt}}{C_{jt}} - \frac{\sum_{i=1}^{n} I_{ijt}}{C_{jt}} &= 1 + \sum_{i=1}^{n} \frac{\alpha_i}{\alpha_j} a_i^m a_{ij} \frac{Q_{it}}{C_{it}}. \end{aligned} \tag{A.38}$$

Let $b_{jt} = \alpha_j \frac{Q_{jt}}{C_{jt}}$, with the vector $b_t = [b_{jt}]$. With $A = \text{diag}(a^m)\widetilde{A} = [a_i^m a_{ij}]$, (A.38) implies

$$b_t' - d' = \alpha' + b_t' A$$

with the solution

$$\begin{aligned} b' \equiv b_t' &= (\alpha + d)' (I - A)^{-1} \\ b' &= (\alpha' + \beta \nu' \Theta)(I - A)^{-1}, \end{aligned} \tag{A.39}$$

where I've defined $b = b_t$, since b_t is constant. Also, since we assume $0 \leq a_i^m < 1$, $(I - A)$ is invertible.

- From the first-order conditions for L_{it}, we have

$$L_{it} = \frac{s_c}{1 - s_c} b_i a_i^\ell \mathcal{L}_t.$$

Market clearing for hours worked implies that

$$\mathcal{L}_t = H \left(1 + \frac{s_c}{1 - s_c} \sum_{i=1}^{n} a_i^\ell b_i \right)^{-1}.$$

Evidently, $L_{it} = L_i$ and $\mathcal{L}_t = \mathcal{L}$ are constant.

- By definition of b,

$$C_{it} = \frac{\alpha_i}{b_i} Q_{it}.$$

From eq. (A.33),

$$M_{ijt} = \frac{b_i}{b_j} a_i^m a_{ij} Q_{jt}.$$

From eq. (A.37),

$$I_{ijt} = \beta \frac{\nu_i}{b_j} a_i^k \theta_{ij} Q_{jt}.$$

- To compute the dynamics of output, take logs of the production function and substitute the optimal values of M_{ijt} and L_{it}. This gives

$$\log Q_{it} = \log \Xi_{it} + a_i^k \log K_{it} + a_i^m \sum_{j=1}^{n} a_{ij} \left(\log Q_{jt} + \left(\frac{b_i}{b_j} a_i^m a_{ij} \right) \right) + a_i^\ell \log L_i.$$

In matrix-vector form, this is

$$q_t = \xi_t + \text{diag}(a^k) k_t + \text{diag}(a^m) \widetilde{A} q_t + g^y,$$

where k_t is a vector of log capital $k_{it} = \log K_{it}$, diag is the operator that creates a matrix with the argument vector on the diagonal and g^y is a vector of constants based on the model parameters. This implies

$$(I - A) q_t = \xi_t + \text{diag}(a^k) k_t + g^y. \tag{A.40}$$

Since $K_{i,t+1} = I_{it} = \prod_{j=1}^{n} I_{ijt}^{\theta_{ij}}$, and substituting the first-order conditions for I_{ijt} and $K_{i,t+1}$, we have

$$\log K_{i,t+1} = \sum_{j=1}^{n} \theta_{ij} \log \left(\log Q_{jt} + \left(\beta \nu_i \theta_{ij} \frac{1}{b_j} \right) \right).$$

Thus,

$$k_{t+1} = \widetilde{\Theta} q_t + \widetilde{\Theta} g^k, \tag{A.41}$$

where g^k is a vector of constants based on the model parameters. Thus,

$$q_{t+1} = (I - A)^{-1} \Theta q_t + (I - A)^{-1} \xi_{t+1} + (I - A)^{-1}(g^y + \Theta g^k).$$

and

$$\Delta q_{t+1} = (I - A)^{-1} \Theta \Delta q_t + (I - A)^{-1} \Delta \xi_{t+1}$$

- The dynamics of capital can also be computed. Using equations (A.40) and (A.41),

$$k_{t+1} = \widetilde{\Theta}(I - A)^{-1} \text{diag}(a^k) k_t + \widetilde{\Theta}(I - A)^{-1} \xi_t + g^{kd} \tag{A.42}$$

$$\text{diag}(a^k) k_{t+1} = \Theta(I - A)^{-1} \text{diag}(a^k) k_t + \Theta(I - A)^{-1} \xi_t + \text{diag}(a^k) g^{kd},$$

where

$$g^{kd} = \widetilde{\Theta}(I - A)^{-1}(g^y + (I - A) g^k).$$

An interpretation of the effects here is as follows:

$$k_{t+1} = \underbrace{\widetilde{\Theta}}_{\substack{\text{investment network,} \\ \text{bundle investments}}} \quad \underbrace{(I - A)^{-1}}_{\substack{\text{instantaneous} \\ \text{effect of} \\ \text{intermediate goods} \\ \text{network}}} \quad \underbrace{\text{diag}(a^k)}_{\substack{\text{relative importance} \\ \text{of capital in} \\ \text{production}}} \quad k_t + \widetilde{\Theta}(I - A)^{-1} \xi_t + g^{kd} \tag{A.43}$$

Verify Guess of Value Function I will now verify that the guess of the value function is a solution to the value function recursion.

- Since $\rho = 1$,

$$\log V_t = (1-\beta)\log \mathcal{C}_t + \beta \log \mathcal{R}_t(V_{t+1}), \tag{A.44}$$

where the optimal policy functions have been substituted in.

- Begin by evaluating $\mathcal{R}_t(V_{t+1})$.

$$\begin{aligned}
&\mathbb{E}_t\Big[\exp\Big\{(1-\gamma)\log V_{t+1}\Big\}\Big] = \\
&= \mathbb{E}_t\Big[\exp\Big\{(1-\gamma)s_c(1-\beta)\sum_{i=1}^{n} a_i^k \nu_i \log K_{i,t+1} \\
&\qquad + (1-\gamma)\log V_0 + (1-\gamma)J(W_{t+1})\Big\}\Big] \\
&= \mathbb{E}_t\Big[\exp\Big\{(1-\gamma)s_c(1-\beta)\nu'\Big(\Theta(I-A)^{-1}\xi_{t+1} \\
&\qquad\qquad + \Theta(I-A)^{-1}\mathrm{diag}(a^k)k_t + \mathrm{diag}(a^k)g^{kd}\Big) \\
&\qquad + (1-\gamma)\log V_0 + (1-\gamma)J(W_{t+1})\Big\}\Big] \\
&= \exp\Big\{(1-\gamma)s_c(1-\beta)\nu'\Big[\Theta(I-A)^{-1}\mathrm{diag}(a^k)k_t + \mathrm{diag}(a^k)g^{kd}\Big] \\
&\qquad + (1-\gamma)\log V_0\Big\} \\
&\quad \times \mathbb{E}_t\Big[\exp\Big\{(1-\gamma)s_c(1-\beta)\nu'\Theta(I-A)^{-1}\xi_{t+1} + (1-\gamma)J(W_{t+1})\Big\}\Big]
\end{aligned}$$

Then,

$$\begin{aligned}
\log \mathcal{R}_t(V_{t+1}) = s_c(1-\beta)\nu'\Big(&\Theta(I-A)^{-1}\mathrm{diag}(a^k)k_t + \mathrm{diag}(a^k)g^{kd}\Big) \\
&+ (1-\gamma)\log V_0 \\
&+ \frac{1}{1-\gamma}\log \mathbb{E}_t\Big[\exp\Big\{(1-\gamma)s_c(1-\beta)\nu'\Theta(I-A)^{-1}\xi_{t+1} \\
&\qquad + (1-\gamma)J(\xi_{t+1})\Big\}\Big]
\end{aligned}$$

.

- Now, evaluating $\mathcal{C}_t$,

$$\begin{aligned}\log \mathcal{C}_t &= (1-s_c)\log \mathcal{L} + s_c(g^c + \alpha'(I-A)^{-1}g^y \\ &\quad + \alpha'(I-A)^{-1}\xi_t + \alpha'(I-A)^{-1}\text{diag}(a^k)k_t) \\ &= s_c\alpha'(I-A)^{-1}\text{diag}(a^k)k_t + s_c\alpha'(I-A)^{-1}\xi_t + g^{c*},\end{aligned}$$

where g^{c*} is a constant that is a function of the model parameters.

- Now, we can use these to substitute into the value function recursion (A.44). Since (A.44) must hold for all values of the states K_{it}. The left-hand side of this recursion, in terms of the state variables, is given by the guess of the value function in (A.36):

$$\log V(\{\xi_{it}\},\{K_{it}\}) = \sum_i^n s_c(1-\beta)a_i^k\nu_i \log K_{it} + J(\xi_t) + \log V_0.$$

The right-hand side is given by the derivations of $\log \mathcal{C}_t$ and $\log \mathcal{R}_t(V_{t+1})$. Analyzing the coefficients associated with k_t on the left-hand side and right-hand side, we see that ν must satisfy

$$\begin{aligned}s_c(1-\beta)\nu'\text{diag}(a^k) &= (1-\beta)s_c\alpha'(I-A)^{-1}\text{diag}(a^k) \\ &\quad + \beta s_c(1-\beta)\nu'\theta(I-A)^{-1}\text{diag}(a^k),\end{aligned}$$

which implies that

$$\begin{aligned}\nu' &= \alpha'(I-A)^{-1}(I-\beta\Theta(I-A)^{-1})^{-1} \\ &= \alpha'(I-(A+\beta\Theta))^{-1}.\end{aligned}$$

- Notice in the second expression that this resembles a measure of centrality called alpha centrality, with $A+\beta\Theta$ as the adjacency matrix. This is discussed in Section A.2.5.

- Note that

$$\nu_j = b_j = \alpha_j \frac{Q_{jt}}{C_{jt}}. \tag{A.45}$$

From the definition in (A.39),

$$\begin{aligned} b' &= (\alpha' + \beta\nu'\Theta)(I-A)^{-1} \\ &= \alpha'(I-A)^{-1}(I + (I - \beta\Theta(I-A)^{-1})^{-1}\beta\Theta(I - A^{-1})) \\ &= \alpha'(I-A)^{-1}(I - \beta\Theta(I-A)^{-1})^{-1} \\ &= \nu', \end{aligned}$$

where the second-to-last equality can be seen by letting $C = \beta\Theta(I-A)^{-1}$ and calculating

$$\begin{aligned} (I + (I - \beta\Theta(I-A)^{-1})^{-1}\beta\Theta(I - A^{-1})) &= (I + (I-C)^{-1}C) \\ &= (I-C)^{-1}((I-C)+C) \\ &= (I-C)^{-1}. \end{aligned}$$

- Furthermore, isolating the terms on each side that depend on the shocks, we have that the function J is characterized by

$$J(\xi_t) = \beta\frac{1}{1-\gamma}\log\mathbb{E}_t\Big[\exp\Big\{(1-\gamma)s_c(1-\beta)\nu'\Theta(I-A)^{-1}\xi_{t+1} + (1-\gamma)J(\xi_{t+1})\Big\}\Big] \tag{A.46}$$

and V_0 is defined by the remaining constants. Note that when ξ_{t+1} has a joint normal conditional distribution, conditional on information at time t, this is

$$\begin{aligned} J(\xi_t) = \beta\mathbb{E}_t &\left[s_c(1-\beta)\nu'\Theta(I-A)^{-1}\xi_{t+1} + J(\xi_{t+1})\right] \\ &+ \beta\frac{1}{2}(1-\gamma)\mathrm{Var}_t\left(s_c(1-\beta)\nu'\Theta(I-A)^{-1}\xi_{t+1} + J(\xi_{t+1})\right). \end{aligned}$$

- This concludes the verification that our guessed value function solves the recursion and satisfies the first-order conditions.

□

A.2.4 Using the model filter to decompose TFP shocks into aggregate and idiosyncratic components

Here I review in more detail the argument laid out in Foerster, Sarte, and Watson (2011) regarding the procedure for disentangling common, aggregate shocks from idiosyncratic shocks propagated through the input-output network.

Suppose that output growth follows the equilibrium process in equation (2.11). Suppose again that innovations to productivity have a common, aggregate component, and an idiosyncratic component. That is, let

$$\varepsilon_t = \Lambda_a \nu_t^a + \nu_t^s,$$

where ν_t^a is a $K \times 1$ vector and common to all industries and ν_t^s is an $n \times 1$ vector of idiosyncratic shocks. Λ_a is an $n \times K$ matrix of coefficients reflecting each industry's exposure to the K common shocks. Assume that (ν_t^a, ν_t^s) are serially uncorrelated, and that ν_t^a and ν_t^s are mutually uncorrelated. Assume further that the idiosyncratic shocks are uncorrelated, so that $\Sigma_{\nu\nu} = \mathbb{E}[\nu_t^s \nu_t^s]$ is a diagonal matrix.

Under these assumptions, industry output growth can be written as a dynamic factor model

$$\Delta q_t = \Lambda(L) F_t + u_t, \tag{A.47}$$

where

$$\Lambda(L) = (I - \Phi L)^{-1}(\Pi_0 + \Pi_1 L)\Lambda_a,$$

$F_t = \nu_t^a$, and

$$u_t = (I - \Phi L)^{-1}(\Pi_0 + \Pi_1 L)\nu_t^s.$$

Importantly, the elements of u_t are a linear combination of of ν_t^s. While the elements of ν_t^s are uncorrelated, the elements of u_t need not be. The matrix $(I - \Phi L)^{-1}(\Pi_0 + \Pi_1 L)$ embodies the effects of network connections in the model and a reduced form factor analysis of industry growth rates may overestimate the importance of aggregate shocks.

To fix this, we can construct a filter based on a calibration of the structural general equilibrium model. Using (2.11) we see that

$$\varepsilon_t = (\Pi_0 + \Pi_1 L)^{-1}(I - \Phi L)\Delta q_t. \tag{A.48}$$

Since we have estimates of Π_0, Π_1, and Φ from the calibration, we can construct the right-hand side. We can then use factor analytic methods on the filtered industry growth data to estimate the contributions of the aggregate shocks ν_t^a and ν_t^s.

A.2.5 Network theory and measures of centrality

In graph theory (network theory), a graph (network) is made up of nodes (vertices) and edges (the connections between vertices). In an unweighted graph, edges between nodes either exist or they don't and are indicated with one or zero. A weighted graph is a generalization in which each edge has a numerical weight associated with it. A directed graph is a graph in which the connections between nodes have a direction associated with them.

Define a graph as an ordered pair $(\mathcal{N}, A)$, where $\mathcal{N} = \{1, 2, ..., n\}$ is a set of nodes and $A = [A_{ij}]$ is a matrix representing a possibly weighted and directed graph. This matrix A is called an adjacency matrix. In an unweighted, undirected graph, $A_{ij} = 1$ indicates an undirected connection between nodes i and j. Otherwise, the elements are zero. When the graph is undirected, A is symmetric. When the graph is directed, $A_{ij} = 1$ indicates a connection *from* node j to node i. Otherwise, the element is equal to zero. In the case of a weighted graph, the elements of A are real numbers and, in most applications, are non-negative.[2]

One exercise of interest in the study of networks is to measure the importance or influence of each node within a graph based on the node's positioning within the graph. Such measures are called measures of centrality. One of the simplest such measures is called *Degree centrality* . Degree centrality of a node i measures the number of edges that are connected to node i. In the case of a weighted graph, this measures the sum of the weights of the connected edges. This is expressed,

$$\vec{C}_{\text{degree}} \coloneqq \sum_{j=1}^{n} A_{ji} = A'\mathbb{1},$$

where $\mathbb{1}$ is a vector of ones.

Another measure of centrality is *Eigenvector centrality*, sometimes referred to as Bonacich centrality.[3] This measure is defined recursively. It captures the idea that a node has higher eigenvector centrality if it connected to another node that itself has high eigenvector centrality. $\vec{C}_{EV} = x$ such that x solves

$$x_i = \frac{1}{\lambda} \sum_{j=1}^{n} A_{ij} x_j,$$

or in matrix form,

$$Ax = \lambda x,$$

where λ is the principal eigenvalue of the adjacency matrix A and x is the associated normalized eigenvector.[4]

A third measure is *Katz centrality*, which measures the number of connections between other nodes, including connections that run through other nodes in the network. Each such

2. For more details, good references include Jackson (2010) and Newman (2018).

3. See Bonacich (2002).

4. As a note, one drawback of Eigenvector centrality is that, when the graph is acyclic, all nodes will have centrality zero. There are variants of Eigenvector centrality that address this problem. Katz centrality can be thought of as one of these.

path is weighted by an attenuation factor $\beta \in (0, 1)$ so that links to distant nodes are given less weight. This measure can be expressed as[5]

$$\vec{C}_{\text{Katz}} \coloneqq \sum_{k=0}^{\infty} \beta^k (A^k)' \mathbb{1} = \sum_{k=0}^{\infty} \sum_{j=1}^{n} \beta^k (A^k)_{ji}.$$

This definition of Katz centrality can be interpreted by noting that if A represents a non-weighted, undirected network, then the element at location (i, j) of A^k reflects the total number of k degree connections between nodes i and j. Assuming that $\beta < 1/\lambda$, where λ is the principal eigenvalue of A, Katz centrality can be expressed as

$$\vec{C}_{\text{Katz}} = (I - \beta A')^{-1} \mathbb{1},$$

where I is a conforming identity matrix and $\mathbb{1}$ is a vector of ones. Katz centrality, like Eigenvector centrality, can be interpreted as a recursive measure of influence. In Eigenvector centrality, the centrality of a node i is a linear function of the centrality of the nodes that node i is connected to. In Katz centrality, the centrality of node i is an affine function of the centrality of the nodes that it is connected to:

$$x_i = \beta \sum_{j=1}^{n} A_{ij} x_j + 1. \tag{A.49}$$

That is, each node is endowed with some small amount of centrality "for free." In matrix terms, this is

$$x = \beta A x + \mathbb{1}$$

and thus $x = (I - \beta A)^{-1} \mathbb{1}$. Katz centrality can be thought of as a generalization of degree centrality and Eigenvector centrality because when β is small, most weight is placed on first-order connections. In light of eq. (A.49), when β is larger, more weight is placed on the

5. There are different conventions for Katz centrality. In some cases, the infinite sum starts at $k = 1$, thus not including the initial term I. In Newman (2018), the sum starts at $k = 0$, as it does here.

recursive term rather than the constant term. When the attenuation factor approaches $1/\lambda$ from below, where λ is the principal eigenvalue of A, Katz centrality converges to Eigenvector centrality,

$$\lim_{\beta \nearrow \frac{1}{\lambda}} \vec{C}_{\text{Katz}} = \vec{C}_{EV}.$$

For a proof, see section A.2.5 below.

Weighted Katz Centrality For our purposes, it will be useful to consider a slight generalization of Katz centrality called *weighted Katz centrality.* This allows us to endow each node with a prior sense of centrality, apart from the centrality dictated by the network connections. Recall that Katz centrality introduces a forcing term that ensures that nodes have a positive level of centrality so that the centrality of each node is an affine function of the centrality of the nodes that its connected to. In a sense, each term is given a unit amount of centrality "for free." Weighted Katz centrality allows this forcing terms (or starting amount) to vary across nodes. Let α by an $n \times 1$ vector of positive real numbers. Then weighted Katz centrality solves the equation

$$x = \beta A x + \alpha,$$

so that

$$\vec{C}_{\text{WKatz}} \coloneqq (I - \beta A)^{-1} \alpha.$$

This measure is sometimes called *Alpha centrality.*[6]

Proof: Eigenvector centrality as the limiting case of Katz centrality

Eigenvector centrality (sometimes called Bonacich centrality) is defined as the vector x that solves

$$Ax = \kappa x,$$

6. https://en.wikipedia.org/wiki/Alpha_centrality

where κ is the principal (largest, most positive) eigenvalue of the adjacency matrix A. Katz centrality is defined as the vector y that solves

$$y = \alpha A y + \mathbf{1},$$

where $0 \leq \alpha < \kappa^{-1}$ and $\mathbf{1}$ is a conforming vector of ones. That is, $y = (\mathbf{I} - \alpha A)^{-1}\mathbf{1}$.

As $\alpha \nearrow \kappa^{-1}$, Katz centrality converges to eigenvector centrality.

Proof. Consider the definition of Eigenvector centrality given above where $\mathcal{C}^e = \frac{x}{\|x\|}$ and the definition of Katz centrality, where $\mathcal{C}^k(\alpha) = \frac{(\mathbf{I}-\alpha A)^{-1}\mathbf{1}}{\|(\mathbf{I}-\alpha A)^{-1}\mathbf{1}\|}$. When $\alpha < \kappa^{-1}$, recall that we can write $y = (\mathbf{I} - \alpha A)^{-1}\mathbf{1} = \mathbf{1} + \alpha A\mathbf{1} + \alpha^2 A^2 \mathbf{1} + ...$

Define $a_n(\alpha) = \mathbf{1} + \alpha A\mathbf{1} + \alpha^2 A\mathbf{1} + ... + \alpha^n A^n \mathbf{1}$, let $a_0 = \mathbf{1}$. Define $b_n(\alpha) = \|a_n(\alpha)\|$.

Given these definitions, we would like to calculate the limit

$$\lim_{\alpha \nearrow \kappa^{-1}} \mathcal{C}^k(\alpha) = \lim_{\alpha \nearrow \kappa^{-1}} \lim_{n \to \infty} \frac{a_n(\alpha)}{b_n(\alpha)},$$

where κ is the principal eigenvalue of A.

Note that $\lim_{n\to\infty} a_n/b_n$ is defined for all values $\alpha \in [0, \kappa^{-1})$. Also, a_n and b_n are finite for $n < \infty$. We can thus switch the order of the limits so that

$$\lim_{\alpha \nearrow \kappa^{-1}} \mathcal{C}^k(\alpha) = \lim_{n \to \infty} \lim_{\alpha \nearrow \kappa^{-1}} \frac{a_n(\alpha)}{b_n(\alpha)} = \lim_{n \to \infty} \frac{a_n(\kappa^{-1})}{b_n(\kappa^{-1})}$$

Define

$$x_{n+1} = \frac{A^{n+1}x_0}{\|A^{n+1}x_0\|},$$

with $x_0 = \mathbf{1}$. Assuming the needed conditions for the power iteration algorithm, $x_{n+1} \to x$, where x is an eigenvector associated with principal eigenvalue of A.

Since $\det\left(\mathbf{I} - \frac{1}{\kappa}A\right) = 0$, we know that a_n and b_n diverge when $\alpha = \kappa^{-1}$. Assume that A is nonnegative (network edge weights are nonnegative). Then b_n is also strictly monotonic.

This allows us to use the Stolz–Cesàro theorem

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n}.$$

Calculating,

$$\frac{a_{n+1} - a_n}{b_{n+1} - b_n} = \frac{\alpha^{n+1} A^{n+1}\mathbf{1}}{\|a_{n+1}\| - \|a_n\|} = \frac{\kappa^{-n-1}\|A^{n+1}\mathbf{1}\|\, x_{n+1}}{\|a_{n+1}\| - \|a_n\|} \geq \frac{\kappa^{-n-1}\|A^{n+1}\mathbf{1}\|\, x_{n+1}}{\|\kappa^{-n-1}A^{n+1}\mathbf{1}\|} = x_{n+1},$$

where the inequality follows from the reverse triangle inequality. Since $\|a_n(\kappa^{-1})\|$ diverges and is monotonically increasing, the inequality holds with equality in the limit. Thus,

$$\lim_{\alpha \nearrow \kappa^{-1}} \mathcal{C}^k(\alpha) = \lim_{n\to\infty} \frac{a_n(\kappa^{-1})}{b_n(\kappa^{-1})} = \lim_{n\to\infty} x_n = x = \mathcal{C}^e.$$

□

A.3 Data

A.3.1 Input-Output Accounts Data

I begin by described the procedure I use to measure the network of intersectoral trade. To construct a measure of the flow of dollars between producers and purchasers within the U.S. economy, I use the Input-Output accounts from the Bureau of Economic Analysis (BEA). These data cover all industrial sectors as well as household production and government entities. The core of the Input-Output accounts consists of two basic national-accounting tables: the "make" table, which records the production of commodities by industries, and the "supply" table, which records the uses of commodities by intermediate and final users. Also, the BEA publishes these tables at various levels of granularity. The most detailed tables are the "Benchmark Input-Output Data", which are coded at a 6-digit level and comprise between 405-544 industries, depending on the year. These tables are published every five years, each edition published with a five-year lag, starting with the 1982 tables up until the most recently published 2012 tables. I use these tables to construct a measure of cash flows between industries, with which I estimate the production technologies featured in my model. The high level of granularity available in the benchmark tables allows me to explore the heterogeneity in the network properties of various industries with the greatest precision possible.

Constructing the Inter-Industry Cash Flow Matrix As noted, I use the "make" and "use" tables from the BEA's Input-Output accounts to construct a measure of cash flows between industries within the U.S. economy. In general, the make table is an $I \times C$ matrix where the entry MAKE_{ic} records the amount of commodity c in dollars that is produced by industry i and the subset of the use table that records the purchases of intermediate users is a $C \times I$ matrix where USE_{ci} is the dollar amount of commodity c used by industry i. Following the procedure outlined in Ahern and Harford (2014), I construct a matrix of cash flows by first constructing the "share" matrix,

$$\text{SHARE}_{ic} = \frac{\text{MAKE}_{ic}}{\sum_{c'=1}^{C} \text{MAKE}_{ic'}},$$

that records the percentage of commodity c produced by industry i and then the cash flow matrix,

$$\text{FLOW}_{ij} = \sum_{c=1}^{C} \text{USE}_{ci} \cdot \text{SHARE}_{jc},$$

that records the dollar value of the products flowing from industry j to industry i.

Note that the BEA also publishes Input-Output requirements tables, such as industry-by-industry or commodity-by-commodity total requirements tables. The industry-by-industry total requirements table, for example, shows the production required, both directly and indirectly, from each industry j per dollar of delivery to final use of each industry i. The use of these tables are inappropriate for the purposes of this paper, however, since these measures include indirect requirements. That is, these measures of the requirements for the output of industry i include the inputs from its direct suppliers (those industries directly supplying industry i) as well as its indirect suppliers (the suppliers of the suppliers to industry i, etc.).

The SUPP matrix normalizes by summing across suppliers s:

$$\text{SUPP}_{ij} = \frac{\text{FLOW}_{ij}}{\sum_{s=1}^{I} \text{FLOW}_{is}}$$

Redefinitions The BEA IO make and use tables are published in two varieties: the standard tables and the supplementary tables. The standard tables are constructed before "redefinitions" of selected secondary products and the supplementary tables are constructed after. These redefinitions are one of several methods for handling the accounting of secondary products. In the tables after redefinition, the make and use tables are modified so as to better conform to a "commodity-technology" assumption. Under this assumption, it is assumed that the production of a given commodity requires a unique set of inputs, regardless of which industry produces that commodity. Under these redefinitions, the secondary products and their associated inputs are excluded from the industry that produced them and are included in the industry in which they are primary.[7] These reallocations are only made in cases where the production process for the secondary product is very dissimilar to that for the industry's primary product. [8] The result of these redefinitions is a set of tables that represent a more homogeneous relationship between input structure and products and, as such, comprise a more useful tool for analyzing the relationships between industries (Horowitz and Planting, 2009). For this reason, I use the supplementary tables (those constructed after redefinition) in my analysis.

7. Note that redefinitions do not affect the definition of the commodity or the measurement of the total output of the commodity. However, redefinitions do affect the measure of industry output.

8. Horowitz and Planting (2009) give the following example. "The production production process for restaurant services provided in hotels is very different from that of lodging services. Therefore, for the supplementary tables, the output and inputs for these restaurant services are moved or redefined from the hotel industry to the restaurant industry."