THE UNIVERSITY OF CHICAGO

SUBGAP COLLECTIVE MODES IN TWO-DIMENSIONAL SUPERFLUID NEAR THE POMERANCHUK INSTABILITIES

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TABLE OF CONTENTS

LIST OF FIGURES v

ACKNOWLEDGMENTS vi

ABSTRACT viii

1 INTRODUCTION 1
   1.1 The Statement of the Problem 1
   1.2 The Background 2
      1.2.1 The Fractional Quantum Hall Effect 2
      1.2.2 Superfluids of Paired Composite Fermions 5
      1.2.3 The Pomeranchuk Instability 9
   1.3 Main Contribution 12
   1.4 Plan of the thesis 13

2 KINETIC THEORY AND COLLECTIVE MODES 14
   2.1 Introduction 14
   2.2 General kinetic theory 14
      2.2.1 Spin-singlet pairing 19
      2.2.2 spin-triplet pairing 20
   2.3 Collective Modes 21
      2.3.1 s-wave pairing 22
      2.3.2 p-wave pairing 22
      2.3.3 d-wave pairing 26
      2.3.4 Higher L chiral ground states 26
   2.4 Conclusion 27

3 FERMI LIQUID CORRECTIONS 28
   3.1 Introduction 28
   3.2 Mass Renormalization by the Landau Parameters 28
      3.2.1 Massless modes 28
      3.2.2 Massive subgap Modes 30
   3.3 Conclusion 34

4 FIELD-THEORY MODEL FOR CHIRAL P-WAVE SUPERFLUID 35
   4.1 Introduction 35
   4.2 Method of Effective Action 36
   4.3 The Model for the p-Wave Superfluid near the Nematic Critical Point 38
   4.4 Conclusion 45

5 CONCLUSION AND OUTLOOK 47

A γ AND THE TSUNEDO FUNCTION λ 49

B FULL DYNAMICAL EQUATIONS 51
B.0.1 Singlet-pairing ground state 52
B.0.2 Triplet-pairing ground state 53

REFERENCES 57

LIST OF FIGURES

1.1 The process of flux attachment illustrated for a half-filled Landau level. 5
1.2 Decomposition of the Fermi surface deformation into different channels of angular momentum. 11

3.1 The root to (3.21) depending on the value of Landau parameter F. This figure was first published in Ref.[28] with the permission for reuse here. 33

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ABSTRACT

In this thesis, we utilize semi-classical kinetic equations to investigate the order parameter collective modes of a class of two-dimensional superfluids. Extending the known results for p-wave superfluids, we show that for any chiral ground state of angular momentum $L \geq 1$, there exists a subgap mode with a mass $\sqrt{2}\,\Delta$ in the BCS limit, where Δ is the magnitude of the ground-state gap. We determine the most significant Landau parameter that contributes to the mass renormalization and show explicitly that the renormalized modes become massless at the Pomeranchuk instability of the fermion vacuum. Particularly for $L = 1$, we propose a continuous field theory to include the Fermi liquid effect in the quadrupolar channel and produce the same result under consistent approximations. They provide potential diagnostics for distinguishing two-dimensional chiral ground states of different angular momenta with the order parameter collective modes and reveal another low-energy degree of freedom near the nematic transition. The main results were published in the earlier work [28] with the permission for reuse.

CHAPTER 1

INTRODUCTION

1.1 The Statement of the Problem

In this chapter, we will state the main problem, the motivation, and the structure of this thesis. In the studies of interacting quantum many-body systems, collective modes allow physicists to explore the correlated motions of underlying degrees of freedom. Classic instances are hydrodynamic sounds in classical fluids and zero-sound of a Fermi liquid in the normal state[6, 56]. More intriguingly, for a system in the superfluid phase, the order parameter component enriches the nature of collective excitations. Paradigmatic examples include the A and B phases of superfluid ^{3}He in (3+1)D [77], where massive sub-gap modes exist owing to the triplet pairing structure. They manifest themselves in terms of resonant signatures of transport properties when coupled to the particle-hole channel [26].

These developments permit various extensions. Natural questions include: (i) do sub-gap massive collective modes also exist in finite angular momentum pairing channels and in (2+1)D spacetime? (ii) Do these bosonic degrees of freedom reflect the underlying fermionic state or the property of the Fermi surface? Regarding (i), it is known that the two dimensional analog of B-phase hosts four modes of mass $\sqrt{2}\,\Delta$ with angular momenta $\ell = \pm 2$. Δ is the magnitude of mass of the Bogoliubov quasiparticle. Similarly, the analog of the A phase, whose fermionic spectrum is fully gapped in two dimensions, hosts six modes of mass $\sqrt{2}\,\Delta$ [8, 9, 74]. On the other hand, puzzle (ii) has been investigated in the context of (3+1)D ^{3}He superfluid with the Fermi liquid theory [77, 85, 70, 61], where the corrections to the masses of massive subgap modes and the sound speeds of the Goldstone modes can be expressed in terms of the Landau parameters. In addition, for Sr_2RuO_4 [63], it has been shown that the strong-coupling effect and gap anisotropy are able to modify the magnitude of the masses and break the spectrum degeneracy.

This work intends to address the complementary features of (i) and (ii). We specifically focus on superfluids in (2+1)D with general pairing channels of angular momenta $L = 0, 1 \cdots$. For $L = 1$, the two-dimensional analogs of the A and B phases are considered, whereas for higher L we concentrate on chiral ground states. We look for massive subgap modes, and investigate the mechanisms that may correct their masses in long wavelength limit $q = 0$.

We pay special attention to these questions mainly because of the puzzle of $\nu = \frac{5}{2}$ fractional quantum Hall state. Three of the most prominent candidates of the ground state, the Pfaffian[44], T-Pfaffian [72], and the anti-Pfaffian states[39, 38], are understood as $p+ip$, $p-ip$ and $f-if$ chiral superconductors of non-relativistic composite fermions respectively [73]. Moreover, both experimental [60, 64, 65] and numerical [37] studies have revealed the importance of nematic fluctuations and quantum criticality in the second Landau level. As a consequence, an investigation of the spectrum of chiral superfluids/superconductors including these effects is pursued to provide further insights to these problems. In the following section we will provide a review of these backgrounds before stating the contribution.

1.2 The Background

1.2.1 The Fractional Quantum Hall Effect

The quantum Hall effect concerns a two-dimensional system of fermions placed in a uniform out-of-plane magnetic field B. For simplicity, let us first consider a species of nonrelativistic fermion. In the first-quantized formulation, the Hamiltonian reads

$$H = \sum_i \frac{1}{2m}[\mathbf{p}_i - \mathbf{A}(\mathbf{x}_i)]^2 + \sum_{i<j} V(\mathbf{x}_i - \mathbf{x}_j) \tag{1.1}$$

In the single-particle picture where the interaction is omitted, the spectrum is organized into Landau levels:

$$\epsilon_n = \hbar\omega_c\Big(n + \frac{1}{2}\Big), \; n \in \mathbb{Z}, \tag{1.2}$$

where ω_c is the cyclotron frequency $\frac{B}{m}$. The magnitude of magnetic field defines a length scale $\ell = B^{-1/2}$, the magnetic length. For each n, the Landau level degeneracy per unit area is given by $\frac{B}{2\pi} = \frac{1}{2\pi\ell^2}$. At a finite fermion density ρ, the portion of filled Landau levels is given by the filling factor

$$\nu = \frac{2\pi\rho}{B} = \frac{\rho}{1/(2\pi\ell^2)}. \tag{1.3}$$

It was discovered in the transport measurement that at low temperature, the Hall conductivity σ_{xy} is quantized in unit of e^2/h

$$\sigma_{xy} = -\sigma_{yx} = \nu \tag{1.4}$$

for some special values of ν, at which plateaus form in the resistivity-magnetic field plot and the system forms an gapped and incompressible liquid. The integer quantum Hall states refer to the the states where ν's take integral values. These values can be understood as the number of filled Landau levels in the presence of strong disorder and the insulating gap is given by the cyclotron frequency.

The single-particle picture fails for the fractional quantum Hall states, those which take rational values of ν such as $\frac{1}{3}, \frac{2}{5}, \cdots$. For those states, the highest Landau level is partially filled. In the limit of ultra-high magnetic field, the effect of other Landau levels is suppressed. Within this Landau level, the kinetic energy is quenched and therefore the formation of the gap owes entirely to the interaction effect. The nature of these strongly correlated insulating states has been understood, in part, by a class of trial wavefunctions. Another popular perspective is Jain's composite fermion picture [31], in which each electron is visualized as a composite fermion and $2p$ magnetic fluxes. Owing to the attached fluxes, the composite fermions actually see a reduced magnetic field $b = B \mp 2\pi\rho \times 2p$ rather than B. We can then derive a relationship between the filling factor of electron ν and the filling factor of composite fermion ν_{CF}.

$$\nu_{\text{CF}} = \frac{2\pi\rho_{\text{CF}}}{b} = \frac{2\pi\rho}{B \mp 4\pi p\rho} = \frac{\nu}{1-2p\nu} \Rightarrow \nu = \frac{\nu_{\text{CF}}}{2p\nu_{\text{CF}} \pm 1}. \tag{1.5}$$

The simplest case has $p = 1$. In particular, when the composite fermions form an integer quantum Hall state $\nu_{\text{CF}} = n$ or $-(n+1)$, we discover the following

$$\nu = \begin{cases} \frac{n}{2n+1}, & \nu_{\text{CF}} = n \\ \frac{n+1}{2n+1}, & \nu_{\text{CF}} = -(n+1) \end{cases}, \tag{1.6}$$

which are exactly the experimentally discovered filling factors. Eq. (1.6) is the well-known Jain's sequence that enumerate potential filling factors for fractional quantum Hall states, and the two branches can map to each other via the *particle-hole conjugation* $\nu \to 1-\nu$. Physically, it refers to replacing every electron in the Landau level with a hole and vice versa. Furthermore, the paradigm of the composite fermion provides us a insightful correspondence: *A fractional quantum Hall state of ordinary electrons is equivalent to an integer state of composite fermions.*

Let us now explore various limits of Eq. (1.6). In the limit $n \to \infty$, both branches have the same limit $\nu = \frac{1}{2}$, at which the Landau level is half-filled and the physical configuration is *particle-hole symmetric*. In the language of the composite fermion, it corresponds to filling infinite number of Landau levels or equivalently the composite fermions do not see any residual background magnetic field and form a Fermi sea, a metallic state. This is illustrated in Fig.1.1. More precisely, it is a Fermi surface coupled to a fluctuation gauge boson. This state was thoroughly studied by Halperin, Lee, and Read in the seminal work Ref.[24] and

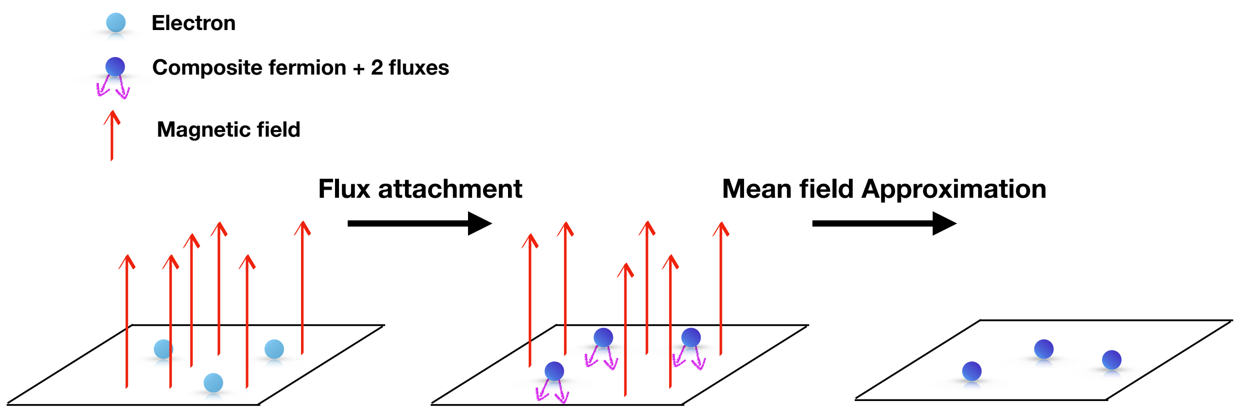

Figure 1.1: The process of flux attachment illustrated for a half-filled Landau level.

the field theory they initiated is now dubbed the HLR theory. A particle-hole symmetric alternative, where the composite fermions obeys relativistic dispersion, was proposed by Son [72]. The ultimate understanding of the $\nu = \frac{1}{2}$ state is still under active investigations. In spite of the debate on the theoretical side, the $\nu = \frac{1}{2}$ state has been observed experimentally [33] and serves as a support for the existence of composite fermions.

1.2.2 Superfluids of Paired Composite Fermions

The $\nu = \frac{1}{2}$ is not the only even-denominator fractional quantum Hall state observed in reality and may not be the most intriguing one after the discovery of the $\nu = \frac{5}{2}$ state [82]. Its configuration can be understood as a half-filled Landau level on top of two fully occupied ones. In the limit of zero Landau-level mixing, we expect that it behaves qualitatively similar to the $\nu = \frac{1}{2}$ state. However, unlike the latter being a metallic state, the $\nu = \frac{5}{2}$ state is an incompressible quantum Hall state. Various theoretical questions including the ground-state wavefunction [50] and the effective field theories arise. For the purpose of this thesis, we focus on the most promising candidate of trial wavefunction, the Moore-Read state[44]:

$$\psi_{\mathrm{MR}}(z) = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^2, \tag{1.7}$$

where Pf is the Pfaffian and $z_i = x_i - iy_i$ is the complex coordinate of the i-th particle. This state is rich in its mathematical properties. For instance, it hosts non-abelian anyonic excitations.

Below we elaborate its physical interpretation in terms of Jain's composite fermions, a $(p+ip)$-wave superfluid state of composite fermions [19, 58]. Note that the factor $\prod_{i<j}(z_i - z_j)^2$ comes from attaching two fluxes to each composite fermion. The assertion can be confirmed by providing an argument that connects the Pfaffian factor with the superfluid ground state. Let us recall the BCS theory for a species of spinless fermion $c^\dagger_{\mathbf{k}}, c_{\mathbf{k}}$. The mean-field Hamiltonian reads

$$H_{\mathrm{MF}} = \sum_{\mathbf{k}} \left[\xi_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + \frac{1}{2}(\Delta^\dagger_{\mathbf{k}} c_{-\mathbf{k}} c_{\mathbf{k}} + c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}} \Delta_{\mathbf{k}}) \right], \tag{1.8}$$

where $\xi_{\mathbf{k}}$ is the single-particle energy measured relative to the chemical potential. The expectation value of the Cooper pair amplitude $\Delta_{\mathbf{k}}$ is nonvanishing in the superfluid phase, the ground state of which is given by the celebrated BCS wavefunction [5, 67].

$$|\Omega\rangle = \prod_{\mathbf{k}}{}' (u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}})|0\rangle. \tag{1.9}$$

The prime indicates we do identify the pair $\mathbf{k}$ and $-\mathbf{k}$ to avoid double counting. Following the standard procedure, u and v are determined by a Bogoliubov transformation that diagonalizes Eq. (1.8). The eigenvalue is $\pm E_{\mathbf{k}} = \pm\sqrt{\xi^2_{\mathbf{k}} + |\Delta_{\mathbf{k}}|^2}$ and

$$\frac{v_{\mathbf{k}}}{u_{\mathbf{k}}} = -\frac{(E_{\mathbf{k}} - \xi_{\mathbf{k}})}{\Delta^\dagger_{\mathbf{k}}} = g(\mathbf{k}). \tag{1.10}$$

Given $g(\mathbf{k})$ and its inverse Fourier transform

$$g(\mathbf{r}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} g(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{1.11}$$

the BCS wave function can be written in position space as [58, 67]

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_N) \propto \sum_P \operatorname{sgn} P \prod_{i=1}^{N/2} g(\mathbf{r}_{P(2i-1)} - \mathbf{r}_{P(2i)}), \tag{1.12}$$

where P refers to all possible permutation of $(1, 2, \cdots, N)$. This sum corresponds exactly to the Pfaffian of the matrix $g(\mathbf{r}_i - \mathbf{r}_j)$. Suppose the fermions form a $p + ip$ superfluid. $g(\mathbf{k}) \sim \frac{1}{k_x - ik_y}$ in the weak-coupling limit and therefore $g(\mathbf{r}) \sim \frac{1}{z}$. Hence, the correspondence between Eq. (1.7) and Eq. (1.9) is established. In addition to this formal matching in the weak-coupling limit, it can be shown that states Eq. (1.7) and Eq. (1.9) share the same properties such as the edge modes and quasi-hole excitation.

While much attention has been paid to the topological properties of the Moore-Read Pfaffian state, its spectrum of collective excitation is not yet as throughly investigated. We have learnt the existence of bosonic collective excitation in the fractional quantum Hall states, the magneto-roton, ever since the seminal work by Girvin, MacDonald and Platzman[16, 17] (GMP). The superfluid picture of the Moore-Read state predicts another neutral-fermionic excitation by breaking a Cooper pair of the composite fermions [19]. This mode has been discovered in numerical recent studies [87, 43, 7] and inspire the application of methods from supersymmetric theories [87, 21].

We commented in the previous section that a half-filled Landau level is symmetric under the particle-hole conjugation $\nu \to 1 - \nu$. If a state is an energy eigenstate of the half-filled Landau level, and so should be its particle-hole conjugate. It is realized in Ref. [39, 38] the particle-hole conjugate of the Moore-Read Pfaffian state (1.7), dubbed the anti-Pfaffian state, belongs a different topological class, and hence is a distinct candidate for the ground state of the $\nu = \frac{5}{2}$ state. In the picture of paired composite fermions, the composite fermions in the anti-Pfaffian state pair in the $(p_x - ip_y)^3$ or $f - if$ channel. Beside these two Pfaffian-like states, there is another relative which is the particle-hole conjugate of itself, called the PH-Pfaffian state[72], and can be visualized as composite fermions paired in $p_x - ip_y$ channel. It became appealing recently because it is particle-hole symmetric, being consistent with the idealized half-filled Landau level.

We will not provide a derivation for the $f - if$ and $p - ip$ pairing structures of the anti-Pfaffian and PH-Pfaffian states in terms of wavefunctions. Instead, we deliver a simpler and more transparent argument in terms of the pairing of *Dirac composite fermions* [72]. At the level of mean-field theory, it only requires the knowledge of BCS theory for relativistic fermions and that the particle-hole conjugation on electrons is the time-reversal operation on Dirac composite fermions. The strategy goes as follows: we first construct a Cooper pair $\Delta_{\mathbf{p}}$ using a pair of Dirac fermions and express it using the fermions close to the Fermi sea and deep in the Fermi sea. In the limit of deep Fermi sea, only the former remains and we then can infer the pairing structure of the corresponding nonrelativistic fermions. Consider the Hamiltonian of a species of Dirac fermions $H_D = \sum_{a,b=1}^{2} \psi_a^\dagger v_F (\mathbf{p}\cdot\boldsymbol{\sigma})_{ab}\psi_b - \mu \sum_{a=1}^{2} \psi_a^\dagger \psi_a$. It can be diagonalized by a unitary transformation $H_D = (v_F|\mathbf{p}| - \mu)\psi_p^\dagger\psi_p - (v_F|\mathbf{p}| + \mu)\psi_h^\dagger\psi_h$. Under the same transformation, the Cooper pair bilinear

$$\int d^2\mathbf{x}\, e^{-i\mathbf{p}\cdot\mathbf{x}} \langle \epsilon^{ab}\psi_a(\mathbf{x})\psi_b(\mathbf{x})\rangle \approx \langle |\mathbf{p}|^{-1}(p_x - ip_y)[\psi_p(-\mathbf{p})\psi_p(\mathbf{p}) + \cdots]\rangle. \tag{1.13}$$

As a consequence, a Dirac Cooper pair in s-channel corresponds to a nonrelativistic Cooper pair in $(p_x - ip_y)$ channel. The s-wave Dirac superfluid is time-reversal symmetric, implying the particle-hole symmetric superfluid of nonrelativistic composite fermion should pair in $p_x - ip_y$ channel. Let us now apply this argument to the Pfaffian state. In order to have a $(p_x + ip_y)$ structure in the right-hand side of Eq. (1.13), the Dirac fermions must pair in the $(p_x + ip_y)^2$ or the $(d + id)$ channel. Taking the time-reversal operation upon the Dirac fermions, the resulting pair condenses in $(p_x - ip_y)^2$ channel. Consequently, the anti-Pfaffian state is a $(f - if)$ superfluid of nonrelativistic composite fermions.

Which Pfaffian state is preferred by nature is not yet revealed. These three states share several topological properties including the fractionalized charge of the quasiparticle and the existence of non-abelian anyon. One of the popular experimental probes is transport measurement on the edge, which is able to distinguish these states by the value of thermal conductivity. Even though Pfaffian or anti-Pfaffian is favored in numerical studies [51, 39, 29, 76, 83, 39], the transport measurements seem to be consistent with the prediction of the PH-Pfaffian state [41, 3, 88, 4]. Following these discoveries, efforts have been devoted to stabilization of the PH-Pfaffian state by disorders and Landau level mixing [92, 71, 46, 80].

1.2.3 The Pomeranchuk Instability

We now change the gear from the exotic quantum Hall effect to a more conventional topic, the Pomeranchuk instability of a Fermi surface in the Fermi liquid theory [57, 10]. The Fermi liquid theory is a low-energy effective theory describing a system of possibly strongly correlated fermions. There exist well-defined quasiparticles, each of which is adiabatically connected with, and carries the same set of quantum numbers such spin as a free fermion in the interaction-free limit. The energy of the state is a functional of the distribution function $n_{\mathbf{p}}$ of the quasiparticles. $\mathbf{p}$ is the momentum of the quasiparticle. At zero temperature, the ground state is a Fermi sea enclosed by the Fermi surface. $n_{\mathbf{p}} = 1$ if $|\mathbf{p}| < p_F$, the Fermi momentum, and $n_{\mathbf{p}} = 0$ otherwise. The variation of distribution $\delta n_{\mathbf{p}}$ is associated with the deformation of the Fermi surface δp_F. At low energy, $\delta n_{\mathbf{p}}$ is nonvanishing only close to the Fermi surface. The variation of the energy of the system is given by the following expression.

$$\delta E[n_{\mathbf{p}}] = \sum_{\mathbf{p}} \varepsilon^0_{\mathbf{p}} \delta n_{\mathbf{p}} + \frac{1}{2V} \sum_{\mathbf{p},\mathbf{p}'} f_{\mathbf{p}\mathbf{p}'} \delta n_{\mathbf{p}}\, \delta n_{\mathbf{p}'}, \tag{1.14}$$

where $\varepsilon^0_{\mathbf{p}}$ is the single-particle energy for the free fermion, which is linearized as $\mu + v_F(|\mathbf{p}| - p_F)$ in the vicinity of the Fermi surface. v_F is the Fermi velocity treated as a parameter in this setting. $f_{\mathbf{pp'}}$ is the phenomenological function, known as the Landau function, characterizing the effect of interaction. In practice, $f_{\mathbf{pp'}}$ is often decomposed in partial-wave expansions to clarify scatterings in different angular-momentum channel. An instability refers to the situation where $\delta E < 0$. The Pomeranchuk instability is a type of Fermi surface instability led by an extremely negative scattering amplitude in an angular-momentum channel.

Here we derive the condition for the Pomeranchuk instability in a two-dimensional Fermi liquid. In two dimensions, the direction of momenta can be parametrized by the polar angle θ. In particular, the Landau function $f_{\mathbf{p'p}} = f(\theta_{\mathbf{p}} - \theta_{\mathbf{p'}})$. and the variation of Fermi surface $\delta p_F = \delta p_F(\theta)$. The variation $\delta n_{\mathbf{p}}$ can be expressed in terms of δp_F.

$$\delta n_{\mathbf{p}} = \delta p_F\, \delta(p_F - |\mathbf{p}|) - \frac{1}{2}(\delta p_F)^2 \frac{\partial}{\partial |\mathbf{p}|} \delta(p_F - |\mathbf{p}|). \tag{1.15}$$

Plugging $\delta n_{\mathbf{p}}$ into Eq. (1.14),

$$\delta E = \frac{p_F^2}{2} V \int \frac{d\theta\, d\theta'}{(2\pi)^4}\, f(\theta - \theta')\delta p_F(\theta)\, \delta p_F(\theta'). \tag{1.16}$$

Next let us introduce the Fourier components of the angular variables

$$\delta p_F = \sum_{n=-\infty}^{\infty} u_n e^{-in\theta}, \tag{1.17}$$

$$f(\theta - \theta') = \sum_{n=-\infty}^{\infty} \frac{F_n}{\nu_{2\mathrm{D}}} e^{-in(\theta - \theta')}, \tag{1.18}$$

where $\nu_{2\mathrm{D}}$ is the density of state at Fermi surface. In terms of u_n and F_n, the variation of energy reduces to

$$\delta E = \frac{v_F p_F}{4\pi} \sum_{n=-\infty}^{\infty} (1 + F_n) u_{-n} u_n. \tag{1.19}$$

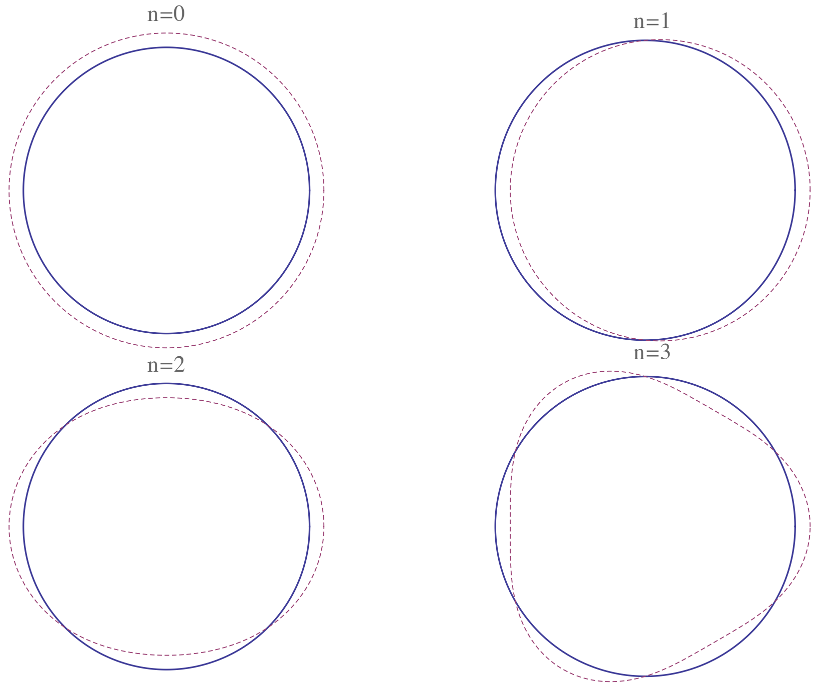

Figure 1.2: Decomposition of the Fermi surface deformation into different channels of angular momentum.

It shows that the Fermi surface is unstable as $F_n \leq -1$.

For $n = \pm 2$, the instability is related to the nematic critical point, which separates an isotropic Fermi surface from an elliptical one. To bridge with the last section, such a phase transition has been reported in several experimental investigation of $\nu = \frac{5}{2}$ fractional quantum Hall state [65, 60, 64] under isotropic experimental setting, and is considered interaction driven [65, 66]. On the other hand, numerical computation of F_ℓ [37] of the composite fermions indicates the onsite of Pomeranchuk instability in the nematic channel as the thickness of the Hall bar goes beneath a critical value. Hence, understanding the spectrum of the Moore-Read Pfaffian state amounts to a thorough investigation of chiral superfluids in the presence of strong nematic fluctuation characterized by the Landau parameters.

To conclude, we reviewed the physics of fractional quantum Hall effect, the Moore-Read Pfaffian state, and the Pomeranchuk instability in the Fermi liquid theory. By rephrasing them in terms of the nonrelativistic composite fermions, we narrow down the big puzzle to a simpler problem of the collective spectra of chiral superfluids in the presence of the Landau parameters, and reinforce the motivation for the bulk of this work.

1.3 Main Contribution

Starting from superfluids pairing in angular channel L, we find that in the limit with weak-coupling and exact particle-hole symmetry, there is at least one pair of bosonic modes of universal mass $\sqrt{2}\,\Delta$ for all $L \geq 1$. We investigate corrections to these degenerate modes owing to fermionic vacuum in a phenomenological manner and determine the angular momentum channels substantial for mass renormalization. For a given chiral ground states of angular momentum L, the order parameter fluctuations longitudinal to the ground state are renormalized by the Landau parameter in the angular momentum channel $2L$, F_{2L}, and thus correspond to a type of spin-$2L$ mode.

The Fermi liquid correction is especially intriguing in (2+1)D. As we will show shortly in chapter 3, it implies the subgap modes soften when F_{2L} is negative. Explicitly, as $F_{2L} \to -1$, the mass of the collective modes vanishes as

$$\sqrt{\frac{12(1+F_{2L})}{6+F_{2L}}}\,\Delta \to 0. \tag{1.20}$$

In particular, taking $L = 1$, the limit $F_2 \to -1$ serves as one of the mechanisms behind nematic electronic phases [15, 49]. On top of previous studies on unconventional superconductors [36] and quantum Hall nematic phases [60, 64, 65], this is another example where the Pomeranchuk instability in quadrupolar channel influences the nature of a paired phase [1] and it allows us to probe the high frequency spin-2 mode omitted in most literatures. We thereby propose a toy model and compute its effective action in Gaussian approximation and show the kinetic result can be captured after implementing exact particle-hole symmetry.

1. We note that similar indication is also found in 3 dimensional p-wave superfluid [61], but in general nematic instability is easier triggered in pure 2 spatial dimensions. An example of composite fermions is studied in a recent work Ref. [37]

In this thesis, (i) we generalize the known high-frequency subgap modes in p-wave superfluids in (2+1)D to higher angular momentum channels and compute their mass renormalizations in terms of the Landau parameters. (ii) Moreover, for p-wave chiral superfluids, we propose a continuous field-theory model to include the Landau parameter effect in the quadrupole channel. In addition to confirming the kinetic theory result, this model could easily be generalized when loosening particle-hole symmetry and provides an understanding of the underlying nature of spin-2 modes and nematic fluctuations.

1.4 Plan of the thesis

This thesis is organized as follows. In chapter 2, we review the semiclassical equation approach for the computation of collective excitations. The equations derived are used in section 2.3 to compute collective excitations for various ground states. In chapter 3, we calculate the Fermi liquid ground-state effect upon the bare bosonic spectra. Finally, chapter 4 presents a field-theory model for a p-wave chiral superfluid with a continuous quadrupole interaction. We demonstrate that the results in chapter 2.3 and 3 can be produced in the limit of the exact particle-hole symmetry. Finally, a summary and several open directions are composed. The full solutions to the kinetic equation (2.1) without assuming $q = 0$ and $\Delta \in \mathbb{R}$, and the computational method for the effective field theory are present along with the method in the appendixes.

CHAPTER 2

KINETIC THEORY AND COLLECTIVE MODES

2.1 Introduction

In this chapter we review the computation of collective modes using the semiclassical theory. Bosonic collective modes in superfluids or superconductors [1] can be computed with various approaches. We start off with the time-dependent mean field approximation in order to include the Fermi liquid corrections with the Landau parameters. This approach can be formulated in terms of generalized Landau-Boltzmann kinetic equations [77, 84, 85], or the linearized non-equilibrium Eilenberger equation [70, 42, 12]. The semiclassical theories for normal fluids and superfluids are well-studied and the derivations are documents in enormous classic textbooks [35, 32, 34]. We will not repeat the derivations of the formalism, which we refer the readers to Ref.[70, 42, 12] for approaches akin to this thesis. Nevertheless, we will give a complete elaboration of the workflow for computation of collective modes.

2.2 General kinetic theory

In the semiclassical limit, physical quasiparticle distribution is related to the Keldysh Green's function $\widehat{g}(\varepsilon, \hat{\mathbf{p}}; \omega, \mathbf{q})$. In our computation, it is a 4×4 matrix function. We use two sets of Pauli matrices $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ to span the particle-hole and spin spaces, respectively. In its argument $(\varepsilon, \mathbf{p} = p_F \hat{\mathbf{p}})$ are the Fourier transformed variables of the fast coordinates, where $(\omega, \mathbf{q})$ are the Fourier. transformed variables of the coordinates of the center of mass[2]. p_F is the magnitude of the Fermi momentum and v_F is the Fermi velocity. In clean limit, the linear response of a nonrelativistic fermion without spin-orbital coupling

1. In this work, we turn off the U(1) gauge field and therefore do not strictly distinguish these 2 terminologies.

2. The fast and the slow coordinates refer to the relative coordinates and the center of mass coordinates in the non-equilibrium Green's function.

is given by the following kinetic equation:

$$\varepsilon_+\tau_3\delta\widehat{g} - \delta\widehat{g}\tau_3\varepsilon_- - v_F\hat{\mathbf{p}}\cdot\mathbf{q}\delta\widehat{g} - [\widehat{\sigma}_0, \delta\widehat{g}] = \delta\widehat{\sigma}\,\widehat{g}_0(\varepsilon_-) - \widehat{g}_0(\varepsilon_+)\,\delta\widehat{\sigma}, \tag{2.1}$$

where $\varepsilon_\pm$ denotes $\varepsilon \pm \omega/2$. The operator $\widehat{\sigma}_0(\hat{\mathbf{p}})$ is the molecular mean field or the self-energy at equilibrium, while $\delta\widehat{\sigma}(\omega, \mathbf{q})$ is the linear perturbation of $\widehat{\sigma}_0$. Similarly, $\widehat{g}_0(\varepsilon, \hat{\mathbf{p}})$ represents the Keldysh Green's function at equilibrium. It is related to the retarded and advanced Green's functions via $\widehat{g}_0 = (g_0^R - g_0^A)\tanh(\varepsilon/2T)$, which yields [42]

$$\widehat{g}_0 = \frac{-2\pi i(\tau_3\varepsilon - \widehat{\Delta})}{\sqrt{\varepsilon^2 - |\Delta|^2}}\Theta(\varepsilon^2 - |\Delta|^2)\mathrm{sgn}(\varepsilon)\tanh\frac{\varepsilon}{2T}. \tag{2.2}$$

The low energy fluctuation of quasi-particles and the deduced physical quantities are given by the ε-integrated $\widehat{g}$ $\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi i}\widehat{g}(\varepsilon, \hat{\mathbf{p}}; \omega, \mathbf{q})$. In particular, the perturbation $\delta\widehat{\sigma}$ is self-consistently determined by the convolution of the interparticle potentials and $\delta\widehat{g}$.

To further elaborate, we note that $\delta\widehat{g}$ has a general structure in particle-hole space

$$\delta\widehat{g} = \begin{pmatrix} \delta g + \delta\mathbf{g}\cdot\boldsymbol{\sigma} & (\delta f + \delta\mathbf{f}\cdot\boldsymbol{\sigma})i\sigma_2 \\ i\sigma_2(\delta f' + \delta\mathbf{f}'\cdot\boldsymbol{\sigma}) & \delta g' + \delta\mathbf{g}'\cdot\boldsymbol{\sigma}^t \end{pmatrix}, \tag{2.3}$$

and accordingly so does $\delta\widehat{\sigma}$

$$\delta\widehat{\sigma} = \begin{pmatrix} \delta\varepsilon + \delta\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma} & (d + \mathbf{d}\cdot\boldsymbol{\sigma})i\sigma_2 \\ i\sigma_2(d' + \mathbf{d}'\cdot\boldsymbol{\sigma}) & \delta\varepsilon' + \delta\boldsymbol{\varepsilon}'\cdot\boldsymbol{\sigma}^t \end{pmatrix}, \tag{2.4}$$

where the primed variables are

$$\delta g'(\hat{\mathbf{p}};\omega,\mathbf{q}) = \delta g(-\hat{\mathbf{p}};\omega,\mathbf{q}) \tag{2.5a}$$

$$\delta\varepsilon'(\hat{\mathbf{p}};\omega,\mathbf{q}) = \delta\varepsilon(-\hat{\mathbf{p}};\omega,\mathbf{q}) \tag{2.5b}$$

$$\delta f'(\hat{\mathbf{p}};\omega,\mathbf{q}) = \delta f^*(\hat{\mathbf{p}};-\omega,-\mathbf{q}) \tag{2.5c}$$

$$d'(\hat{\mathbf{p}};\omega,\mathbf{q}) = d^*(\hat{\mathbf{p}};-\omega,-\mathbf{q}). \tag{2.5d}$$

We would like to explain the notations here before moving forward. The diagonal parts of $\delta\widehat{g}$ refer to the normal, or particle-hole, correlation functions $\langle\psi\psi^\dagger\rangle$ and $\langle\psi^\dagger\psi\rangle$, while the off-diagonal parts denote the anomalous or particle-particle correlation functions $\langle\psi\psi\rangle$ and $\langle\psi^\dagger\psi^\dagger\rangle$. $\boldsymbol{\sigma}$ denotes the Pauli matrices $(\sigma_1,\sigma_2,\sigma_3)$ in spin space as defined earlier and $\boldsymbol{\sigma}^t$ denotes the transposed Pauli matrices. Looking at the Green's function $\delta\widehat{g}$, in the particle-hole channel, δg and $\delta\mathbf{g}$ denote spin-independent and spin-dependent correlations respectively. In the particle-particle channel, δf represent the spin-singlet pairing amplitude and $\delta\mathbf{f}$ the spin-triplet one. Correspondingly, the diagonal part of the self-energy $\delta\widehat{\sigma}$ is the particle-hole self-energy, including the spin-independent $\delta\varepsilon$ and spin-dependent part $\delta\boldsymbol{\varepsilon}$. The off-diagonal part of the self-energy is the superfluid gap induced by anomalous correlations. d is the spin-singlet gap and $\mathbf{d}$ denotes the spin-triplet gap.

Note that physical observables are usually expressed in terms of the symmetric and anti-symmetric combination of δg, δf, and their primed partners. In this work, we define $(+)$ and $(-)$ combinations of a function f as

$$f^{(\pm)} = f \pm f'. \tag{2.6}$$

The eigenvalues (± 1) represent the parity under charge conjugation. As we will see, the charge density and energy stress tensor correspond to the scalar and quadrupole modes of $\delta g^{(+)}$, respectively, whereas the current density is proportional to the vector mode of $\delta g^{(-)}$.

Similarly, $\delta f^{(+)}$ and $\delta f^{(-)}$ stand for the amplitude and phase fluctuations of the anomalous correlation functions.

To complete the equations, the correction to the self-energy is determined by the two-body vertex. Evaluating the internal momentum integral over the Fermi surface, we have, in the particle-hole channel [6, 1, 35, 61],

$$\delta\varepsilon(\hat{\mathbf{p}};\omega,\mathbf{q}) = \delta\varepsilon_{\text{ext}}(\hat{\mathbf{p}};\omega,\mathbf{q}) + \int \frac{d\theta'}{2\pi} A^s(\theta,\theta') \int \frac{d\varepsilon'}{4\pi i}\delta g(\varepsilon',\hat{\mathbf{p}}';\omega,\mathbf{q}), \tag{2.7a}$$

$$\delta\boldsymbol{\varepsilon}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \delta\boldsymbol{\varepsilon}_{\text{ext}}(\hat{\mathbf{p}};\omega,\mathbf{q}) + \int \frac{d\theta'}{2\pi} A^a(\theta,\theta') \int \frac{d\varepsilon'}{4\pi i}\delta\mathbf{g}(\varepsilon',\hat{\mathbf{p}}';\omega,\mathbf{q}). \tag{2.7b}$$

These two equations state that at one-loop, the particle-hole self-energy consists of an external perturbation $\delta\varepsilon_{\text{ext}}$ or $\delta\boldsymbol{\varepsilon}_{\text{ext}}$ and a fermion loop closed by a two-body interaction vertex. An example of $\delta\varepsilon_{\text{ext}}$ is a background inhomogeneous chemical potential, whereas an example of $\delta\boldsymbol{\varepsilon}_{\text{ext}}$ could be a weak external magnetic field. A^s (A^a) is the spin-independent (exchange) forward scattering amplitude, which can be rewritten in terms of the Landau parameters F via the relation [6, 1, 35]

$$A(\theta,\theta') = F(\theta,\theta') - \int \frac{d\theta''}{2\pi} F(\theta,\theta'')A(\theta'',\theta'). \tag{2.8}$$

Similar expressions arise in the particle-particle channel. The relations Eq. (2.7a), and Eq. (2.7b) can be regarded as the alternative representations of Eq. (1.14) in terms of the microscopic variables δg and $\delta\mathbf{g}$.

Since the fluctuations of the superfluid gaps directly come from the anomalous correlation functions, the off-diagonal components are related by the linearized gap equations.

$$d(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\text{e}}(\theta,\theta') \int \frac{d\varepsilon'}{4\pi i}\delta f(\varepsilon',\hat{\mathbf{p}}';\omega,\mathbf{q}) \tag{2.9a}$$

$$\mathbf{d}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\rm o}(\theta,\theta') \int \frac{d\varepsilon'}{4\pi i} \delta\mathbf{f}(\varepsilon',\hat{\mathbf{p}}';\omega,\mathbf{q}), \tag{2.9b}$$

where $V_{\rm e}$ ($V_{\rm o}$) is the pairing potentials in even (odd) angular momentum channel.

In two dimensions, the scattering amplitudes and pairing potentials yield the approximate angular expansions

$$A = \sum_{\ell=-\infty}^{\infty} A_\ell e^{-i\ell(\theta-\theta')},\ A_\ell = A_{-\ell}, \tag{2.10a}$$

$$V_{\rm e} = \sum_{\ell\in\{\text{even}\}} V_\ell [e^{-i\ell(\theta-\theta')} + \text{h.c.}] \tag{2.10b}$$

$$V_{\rm o} = \sum_{\ell\in\{\text{odd}\}} V_\ell [e^{-i\ell(\theta-\theta')} + \text{h.c.}]. \tag{2.10c}$$

from which and (2.8) we can derive $A_\ell = \frac{F_\ell}{1+F_\ell}$, where F_ℓ is the conventional dimensionless Landau parameter of angular momentum channel ℓ defined in Eq. (1.18). Notation-wise, for other functions $f(\hat{\mathbf{p}})$ evaluated at a point on the Fermi surface $\hat{\mathbf{p}}$, the angular decomposition is defined as

$$f = \sum_{\ell=-\infty}^{\infty} e^{-i\ell\theta} f_\ell. \tag{2.11}$$

We can then provide a recipe for the computation. We first invert (2.1) to obtain the perturbed Green's function $\delta\widehat{g}$ as a function of the equilibrium Green's function $\widehat{g}_0$, equilibrium self-energy $\widehat{\sigma}_0$ and perturbed self-energy $\delta\widehat{\sigma}$. Taking the convolution as in (2.7a), (2.7b), (2.9a), and (2.9b) establishes integral equations for $\delta\widehat{\sigma}$. Projecting equations (2.9a) and (2.9b) to different angular modes ℓ gives us the coupled equations of d_ℓ, $\mathbf{d}_\ell$, $\delta\varepsilon$, and $\delta\boldsymbol{\varepsilon}$. The bare bosonic collective modes are given by the normal modes of the homogeneous part of the equations. To include the Fermi liquid corrections, we project (2.7a), and (2.7b) to their ℓth angular modes as well and solve $\delta\varepsilon_\ell$ and $\delta\boldsymbol{\varepsilon}_\ell$ in terms of $\delta\varepsilon_{\rm ext}$, $\delta\boldsymbol{\varepsilon}_{\rm ext}$ d and $\mathbf{d}$. Plugging the results back into the equations for d_ℓ and $\mathbf{d}_\ell$ yields inhomogeneous equations sourced solely by external fields. The renormalized mass spectrum is solved as the poles of the solution kernels.

In the rest of this section, we use the above formulation to derive the integral equation for two-dimensional spin-singlet and spin-triplet superfluids and compute the collective modes and Fermi liquid corrections in the sections following. While in the main text only the equations in long-wavelength limit are presented, the complete set of dynamical equations are given in Appendix B.

2.2.1 Spin-singlet pairing

In a spin-singlet pairing channel, the equilibrium self-energy is characterized by a complex gap field Δ.

$$\widehat{\sigma}_0 = \widehat{\Delta} = \begin{pmatrix} 0 & \Delta i\sigma_2 \\ \Delta^* i\sigma_2 & 0 \end{pmatrix}. \tag{2.12}$$

The fluctuation of the spin-singlet order parameter can be parametrized by a complex number d. It transforms as a scalar under spin rotation $\mathrm{SO}_S(3)$ and can have internal structures, i.e., tensor indices under orbital rotation $\mathrm{SO}_L(2)$ depending on pairing symmetries. In the absence of magnetic field, the spin-triplet fluctuations $\mathbf{d}$ are decoupled from d. Hence we consider them separately in the present work.

Plugging (2.12) into (2.1), inverting it using the variables defined in (2.3) and (2.4), and taking the convolution as in (2.9a) and (2.9b) give us, in the long-wavelength limit, the off-diagonal components of the molecular fields

$$d(\hat{\mathbf{p}};\omega) = \int \frac{d\theta'}{2\pi} V_{\mathrm{e}}(\theta,\theta')\Big[\Big(\gamma + \frac{1}{4}\bar{\lambda}[\omega^2 - 2|\Delta|^2]\Big)d - \frac{\bar{\lambda}}{2}\Delta^2 d' - \frac{\omega}{4}\bar{\lambda}\Delta\delta\varepsilon^{(+)}\Big], \tag{2.13a}$$

$$d'(\hat{\mathbf{p}};\omega) = \int \frac{d\theta'}{2\pi} V_{\rm e}(\theta,\theta')\Big[\Big(\gamma + \frac{1}{4}\bar{\lambda}[\omega^2 - 2|\Delta|^2]\Big)d' - \frac{\bar{\lambda}}{2}(\Delta^*)^2 d + \frac{\omega\bar{\lambda}}{4}\Delta^*\delta\varepsilon^{(+)}\Big]. \tag{2.13b}$$

γ is the BCS logarithm given explicitly in Appendix (A.1). The function λ is often called the Tsunedo function, whose complete form is given in appendix A. In $q \to 0$ limit,

$$\bar{\lambda} = \frac{\lambda(\hat{\mathbf{p}};\omega)}{|\Delta|^2} = \int_{|\Delta|}^{\infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2 - |\Delta|^2}} \frac{\tanh\frac{\varepsilon}{2T}}{\varepsilon^2 - \omega^2/4}. \tag{2.14}$$

There could be angular dependence through the anisotropy in $|\Delta|^2$ even in the long wavelength limit. Suppose only a single pairing channel L is significant, i.e., that $V = V_L(e^{-iL(\theta-\theta')} +$ h.c.). Taking $\int \frac{d\theta}{2\pi} e^{iL\theta}$ on both sides of (2.13a) and (2.13b) eliminates γs. The dynamical equations of motion are then obtained

$$\Big\langle e^{iL\theta}\bar{\lambda}\Big([\omega^2 - 2|\Delta|^2]d - 2\Delta^2 d' - \omega\Delta\delta\varepsilon^{(+)}\Big)\Big\rangle = 0 \tag{2.15a}$$

$$\Big\langle e^{iL\theta}\bar{\lambda}\Big([\omega^2 - 2|\Delta|^2]d' - 2(\Delta^*)^2 d + \omega\Delta^*\delta\varepsilon^{(+)}\Big)\Big\rangle = 0, \tag{2.15b}$$

where we use the angle bracket $\langle\cdots\rangle$ to denote the angular average $\int_{-\pi}^{\pi}\frac{d\theta}{2\pi}\cdots$.

2.2.2 spin-triplet pairing

In a spin-triplet pairing channel, the ground state self-energy is characterized by the vector-valued gap function $\mathbf{\Delta}$

$$\widehat{\Delta} = \begin{pmatrix} 0 & \mathbf{\Delta}\cdot i\boldsymbol{\sigma}\sigma_2 \\ \mathbf{\Delta}^*\cdot i\sigma_2\boldsymbol{\sigma} & 0 \end{pmatrix}. \tag{2.16}$$

The fluctuation is encoded in the dynamics of the $\mathbf{d}$ vector, which transforms as a vector under $\mathrm{SO}_S(3)$, and could contain internal structure depending on pairing symmetry as well. Taking two-dimensional p-wave superfluids for example, it can be expanded as $d_\mu(\hat{\mathbf{p}}) = d_{\mu i}\hat{p}_i$, where $i = x, y$. Inverting the kinetic equations, the dynamical equations for $\mathbf{d}$ in $q \to 0$ limit are

$$
\begin{aligned}
\mathbf{d} = \int \frac{d\theta'}{2\pi} V_{\text{o}}(\theta,\theta')\Big[\Big(\gamma + \frac{1}{4}\bar{\lambda}(\omega^2 - 2|\Delta|^2)\Big)\mathbf{d} \\
+ \frac{\bar{\lambda}}{2}[(\boldsymbol{\Delta}\cdot\boldsymbol{\Delta})\mathbf{d}' - 2(\boldsymbol{\Delta}\cdot\mathbf{d}')\boldsymbol{\Delta}] - \frac{\omega\bar{\lambda}}{4}(\boldsymbol{\Delta}\delta\varepsilon^{(+)} - i\boldsymbol{\Delta}\times\delta\boldsymbol{\varepsilon}^{(+)})\Big].
\end{aligned} \tag{2.17a}
$$

$$
\begin{aligned}
\mathbf{d}' = \int \frac{d\theta'}{2\pi} V_{\text{o}}(\theta,\theta')\Big[\Big(\gamma + \frac{1}{4}\bar{\lambda}(\omega^2 - 2|\Delta|^2)\Big)\mathbf{d}' \\
+ \frac{\bar{\lambda}}{2}[(\boldsymbol{\Delta}^*\cdot\boldsymbol{\Delta}^*)\mathbf{d} - 2(\boldsymbol{\Delta}^*\cdot\mathbf{d})\boldsymbol{\Delta}^*] + \frac{\omega\bar{\lambda}}{4}(\boldsymbol{\Delta}^*\delta\varepsilon^{(+)} + i\boldsymbol{\Delta}^*\times\delta\boldsymbol{\varepsilon}^{(+)}).
\end{aligned} \tag{2.17b}
$$

Again multiplying (2.17a) and (2.17b) by $V = V_L[e^{-iL(\theta-\theta')} + e^{iL(\theta-\theta')}]$ and integrating over θ will give us

$$
\begin{aligned}
&\Big\langle e^{iL\theta}\bar{\lambda}\Big([\omega^2 - 2|\Delta|^2]\mathbf{d} + 2(\boldsymbol{\Delta}\cdot\boldsymbol{\Delta})\mathbf{d}' - 4(\boldsymbol{\Delta}\cdot\mathbf{d}')\boldsymbol{\Delta}\Big)\Big\rangle \\
=&\omega\Big\langle e^{iL\theta}\bar{\lambda}(\boldsymbol{\Delta}\delta\varepsilon^{(+)} - i\boldsymbol{\Delta}\times\delta\boldsymbol{\varepsilon}^{(+)})\Big\rangle.
\end{aligned} \tag{2.18a}
$$

$$
\begin{aligned}
&\Big\langle e^{iL\theta}\bar{\lambda}\Big([\omega^2 - 2|\Delta|^2]\mathbf{d}' + 2(\boldsymbol{\Delta}^*\cdot\boldsymbol{\Delta}^*)\mathbf{d} - 4(\boldsymbol{\Delta}^*\cdot\mathbf{d})\boldsymbol{\Delta}^*\Big)\Big\rangle \\
=&-\omega\Big\langle e^{iL\theta}\bar{\lambda}(\boldsymbol{\Delta}^*\delta\varepsilon^{(+)} + i\boldsymbol{\Delta}^*\times\delta\boldsymbol{\varepsilon}^{(+)})\Big\rangle.
\end{aligned} \tag{2.18b}
$$

In the following section, we will solve (2.15a), (2.15b), (2.18a), and (2.18b) for the ground states of different pairing channels and symmetries.

2.3 Collective Modes

In this section, we utilize the equations derived in the last section to compute the bare bosonic spectra for various superconducting ground states. We focus on the chiral ground states of angular momentum $L \neq 0$, in which the massive collective modes are interpreted as spin-$2L$ modes. The masses of order parameter collective modes appear as normal modes of the homogeneous part in (2.15a), (2.15b), (2.18a), and (2.18b). The self-energy $\delta\varepsilon$ and $\delta\boldsymbol{\varepsilon}$ in Landau channel are treated as external sources at the zeroth order, and they will *renormalize* the above bare masses in the next section as we conclude Fermi liquid effects.

2.3.1 s-wave pairing

For s-wave pairing, it is possible to choose a gauge such that $\Delta \in \mathbb{R}$. In such a limit the amplitude mode $d^{(+)}$ and phase mode $d^{(-)}$ decouple. The superscripts $(+)$ and $(-)$ are defined according to (2.6). The bosonic field has no internal structure and is simply a complex scalar. Two order parameter collective modes thus exist and obey the equations

$$(\omega^2 - 4\Delta^2)d^{(+)} = 0, \tag{2.19a}$$

$$\omega^2 d^{(-)} = 2\omega\Delta\delta\varepsilon_0^{(+)}, \tag{2.19b}$$

where the zero-angular momentum quasiparticle energy $\delta\varepsilon_0^{(+)}$ is obtained under the projection (2.11). The normal modes have masses 2Δ and 0 corresponding to the simplest example of *Higgs* and *Goldstone* bosons respectively. Note that if we compute (2.19b) to the leading nonvanishing order in q^2, we would have obtain $(\omega^2 - \frac{1}{2}(v_F q)^2)d^{(-)}$, entailing the Goldstone boson moves at the speed $v_F/\sqrt{2}$. Another observation is that the Higgs mode receives no external force and consequently it would not be renormalized by particle-hole self-energy. On the other hand, the Goldstone boson is sourced by the density mode $\delta\varepsilon_0^{(+)}$, which would trigger the Higgs mechanism in the presence of the Coulomb interaction.

2.3.2 p-wave pairing

Owing to the triplet pairing and orbital structure, the p-wave pairing states have more degrees of freedom and thus more collective modes. In two dimensions, the fluctuation of p-wave superconductors can be represented by the complex tensor $d_{\mu i}$, which contains 3×2

complex degrees of freedom, leading to 12 collective modes in total. The number of the massless modes N_G, as we will see shortly, can be determined by ground-state symmetry breaking pattern. The rest $(6 - N_G) \times 2$ is number of subgap collective modes.

B phase. We first consider the 2-dimensional analog of ^{3}He B-phase, where the gap function assumes the form

$$\mathbf{\Delta} = \frac{\Delta}{p_F}(\hat{\mathbf{x}}p_x + \hat{\mathbf{y}}p_y), \ \ \Delta \in \mathbb{R}. \tag{2.20}$$

In this phase, the global symmetry breaks following the pattern $\mathrm{SO}_S(3)\otimes\mathrm{SO}_L(2)\otimes\mathrm{U}(1)\to$ SO(2), which immediately indicates the existence of four Goldstone modes. Besides, the residual symmetry is SO(2) rotation and we expect the fluctuations can be characterized by total angular momentum J. Owing to this fact, it is convenient to first decompose d_μ into different angular momentum channels $d_\mu = \sum_{m=\pm 1} d_{\mu m} e^{-im\theta}$, where θ is the polar angle of $\hat{\mathbf{p}}$, and take the linear combinations as follows:

$$D_{\pm m} = d_{xm} \pm i d_{ym}, \tag{2.21}$$

$$D_{0m} = d_{zm}. \tag{2.22}$$

These $D_{\sigma\sigma'}$'s form a nice basis in which the dynamical equations can be solved. Moreover, as the gap function is real, modes transforming differently under charge conjugation again decouple. That is to say, we can further separate $d^{(\pm)} = d \pm d'$ degrees of freedom. We first look at the $\mathbf{d}^{(-)}$ modes governed by the equation

$$(\omega^2 - 4\Delta^2)\mathbf{d}^{(-)} + 4(\mathbf{\Delta}\cdot\mathbf{d}^{(-)})\mathbf{\Delta} = 2\omega\mathbf{\Delta}\delta\varepsilon^{(+)}. \tag{2.23}$$

Organizing the dynamical equations using the basis $D_{\sigma\sigma'}$, we could find

$$(\omega^2 - 4\Delta^2)D_{0\pm}^{(-)} = 0, \tag{2.24a}$$

$$(\omega^2 - 2\Delta^2)D_{\pm\pm}^{(-)} = 2\omega\Delta\delta\varepsilon_{\pm 2}^{(+)}, \tag{2.24b}$$

$$(\omega^2 - 4\Delta^2)(D_{+-}^{(-)} - D_{+-}^{(-)}) = 0, \tag{2.24c}$$

$$\omega^2(D_{+-}^{(-)} + D_{-+}^{(-)}) = 4\omega\Delta\delta\varepsilon_{0}^{(+)}. \tag{2.24d}$$

Consequently, $d^{(-)}$ has two subgap massive modes $J = \pm 2$ of the same mass $\sqrt{2}\,\Delta$, sourced by the spin-independent quadrupolar molecular field $\delta\varepsilon_{\pm 2}^{(+)}$.

Next we look at $d^{(+)}$, which obeys

$$[\omega^2\mathbf{d}^{(+)} - 4\boldsymbol{\Delta}(\boldsymbol{\Delta}\cdot\mathbf{d}^{(+)})] = -2i\omega\boldsymbol{\Delta}\times\delta\boldsymbol{\varepsilon}^{(+)}. \tag{2.25}$$

Following the same procedure to project each component to different J sectors, we would obtain

$$\omega^2 d_0^{(+)} = -2i\omega(\boldsymbol{\Delta}\times\delta\boldsymbol{\varepsilon}^{(+)})\cdot\hat{\mathbf{z}}, \tag{2.26a}$$

$$(\omega^2 - 2\Delta^2)D_{\pm\pm}^{(+)} = \mp 2\omega\Delta\delta\boldsymbol{\varepsilon}_{\pm 2}^{(+)}\cdot\hat{\mathbf{z}}, \tag{2.26b}$$

$$\omega^2(D_{-+}^{(+)} - D_{+-}^{(+)}) = 4\Delta\omega\delta\boldsymbol{\varepsilon}_{0}^{(+)}\cdot\hat{\mathbf{z}}, \tag{2.26c}$$

$$(\omega^2 - 4\Delta^2)(D_{+-}^{(+)} + D_{+-}^{(+)}) = 0. \tag{2.26d}$$

Again modes $D_{\pm\pm}^{(+)}$ have the rest mass $\sqrt{2}\,\Delta$ and they are driven by the z component of the spin-dependent quadrupolar fields $\delta\boldsymbol{\varepsilon}_{\pm 2}^{(+)}\cdot\hat{\mathbf{z}}$.

A-phase. Considering only the continuous symmetry, the two-dimensional A phase has a different symmetry breaking pattern $\mathrm{SO}_S(3)\otimes\mathrm{SO}_L(2)\otimes\mathrm{U}(1)\rightarrow\ \mathrm{U}_{L-N/2}(1)\otimes\mathrm{U}_{S_z}(1)$. The residual symmetry contains two parts. $\mathrm{U}_{L-N/2}(1)$ refers to the combination of orbital and phase rotation. The order parameter is symmetric when an orbital rotation of angle α is followed by a phase rotation $-\alpha/2$. The $\mathrm{U}_{S_z}(1)$ is the residual spin rotation about the direction of the ground state $\mathbf{\Delta}$.

Let us now consider a $p + ip$ ground state described by

$$\mathbf{\Delta} = \frac{p_x + ip_y}{p_F}\Delta\hat{\mathbf{z}} = e^{i\theta}\Delta\hat{\mathbf{z}},\ \Delta \in \mathbb{R}. \tag{2.27}$$

The dynamic equations for $\mathbf{d}_\ell$ and $\mathbf{d}'_\ell$ ($\ell = \pm 1$) are now coupled and given as follows:

$$\begin{aligned}&\langle e^{i\ell\theta}[(\omega^2 - 2\Delta^2)\mathbf{d} + 2\Delta^2\mathbf{d}' e^{2i\theta} - 4\Delta^2\hat{\mathbf{z}}(\hat{\mathbf{z}}\cdot\mathbf{d}')e^{i2\theta}]\rangle\\ =&\omega\Delta\langle e^{i\ell\theta}e^{i\theta}[\hat{\mathbf{z}}\delta\varepsilon^{(+)} - i\hat{\mathbf{z}}\times\delta\boldsymbol{\varepsilon}^{(+)}]\rangle,\end{aligned} \tag{2.28a}$$

$$\begin{aligned}&\langle e^{i\ell\theta}[(\omega^2 - 2\Delta^2)\mathbf{d}' + 2\Delta^2\mathbf{d} e^{-2i\theta} - 4\Delta^2\hat{\mathbf{z}}(\hat{\mathbf{z}}\cdot\mathbf{d})e^{-i2\theta}]\rangle\\ =&-\omega\Delta\langle e^{i\ell\theta}e^{-i\theta}[\hat{\mathbf{z}}\delta\varepsilon^{(+)} + i\hat{\mathbf{z}}\times\delta\boldsymbol{\varepsilon}^{(+)}]\rangle.\end{aligned} \tag{2.28b}$$

We first look at the angular modes $\mathbf{d}_{\ell=1}$ and $\mathbf{d}'_{\ell=-1}$. They obey the equations

$$(\omega^2 - 2\Delta^2)\mathbf{d}_1 = \omega\Delta\langle\hat{\mathbf{z}}\delta\varepsilon_2^{(+)} - i\hat{\mathbf{z}}\times\delta\boldsymbol{\varepsilon}_2^{(+)}\rangle, \tag{2.29a}$$

$$(\omega^2 - 2\Delta^2)\mathbf{d}'_{-1} = -\omega\Delta\langle\hat{\mathbf{z}}\delta\varepsilon_{-2}^{(+)} + i\hat{\mathbf{z}}\times\delta\boldsymbol{\varepsilon}_{-2}^{(+)}\rangle, \tag{2.29b}$$

and have the same mass $\sqrt{2}\,\Delta$. The external forces consist of both spin-dependent and spin-independent molecular fields, both of which are projected to quadrupolar channels. On the other hand, equations for $\mathbf{d}_{\ell=-1}$ and $\mathbf{d}'_{\ell=1}$ are coupled. Solving these equations, one can find three massless modes and three modes with the mass 2Δ. The external forces on the right-hand sides of (2.29a) and (2.29b) consist of both spin-dependent and spin-independent molecular fields, both of which are projected to the quadrupolar channels.

2.3.3 d-wave pairing

The d-wave gap fluctuation is captured by the complex field $d_{ij}\hat{p}_i\hat{p}_j$ with the irreducible complex degrees of freedom 1×2, represented by the modes $d_{\pm 2}e^{\mp i2\theta}$. In this work, we consider the chiral ground state

$$\frac{\Delta}{p_F^2}(p_x + ip_y)^2 = \Delta e^{i2\theta},\ \Delta \in \mathbb{R}. \tag{2.30}$$

Equations (2.15a), and (2.15b) then become

$$(\omega^2 - 2\Delta^2)d_2 = \omega\Delta\delta\varepsilon_4^{(+)}, \tag{2.31a}$$

$$(\omega^2 - 2\Delta^2)d'_{-2} = -\omega\Delta\varepsilon_{-4}^{(+)}, \tag{2.31b}$$

$$(\omega^2 - 4\Delta^2)(d_{-2} + d'_2) = 0, \tag{2.31c}$$

$$\omega^2(d_{-2} - d'_2) = 2\omega\Delta\delta\varepsilon_0^{(+)}. \tag{2.31d}$$

Clearly d_2 and d'_{-2} have masses $\sqrt{2}\,\Delta$ and the external driving forces have angular momenta ± 4.

2.3.4 Higher L chiral ground states

Extending the analyses for p- and d-channels, we could actually consider a more general ground state

$$\text{singlet} : \Delta e^{iL_s\theta},\ L_s = \text{even}, \tag{2.32a}$$

$$\text{triplet} : \hat{\mathbf{z}}\Delta e^{iL_t\theta},\ L_t = \text{odd}. \tag{2.32b}$$

Modes d_{L_s}, d'_{-L_s}, $\mathbf{d}_{L_t}$, and $\mathbf{d}'_{-L_t}$ would automatically satisfy

$$(\omega^2 - 2\Delta^2) d_{L_s} = \omega\Delta\delta\varepsilon^{(+)}_{2L_s}, \tag{2.33a}$$

$$(\omega^2 - 2\Delta^2) d_{-L_s} = -\omega\Delta\delta\varepsilon^{(+)}_{-2L_s}, \tag{2.33b}$$

$$(\omega^2 - 2\Delta^2)\hat{\mathbf{z}} \cdot \mathbf{d}_{L_t} = \omega\Delta\delta\varepsilon^{(+)}_{2L_t}, \tag{2.33c}$$

$$(\omega^2 - 2\Delta^2)\hat{\mathbf{z}} \cdot \mathbf{d}'_{-L_t} = -\omega\Delta\delta\varepsilon^{(+)}_{-2L_t}. \tag{2.33d}$$

In this sense, $\sqrt{2}\Delta$ is a universal order parameter collective mode for any chiral ground state of the angular momentum L, each of which is sourced by quasiparticle self-energy $\delta\varepsilon_{2L}$. Since the right-hand sides belong to specific angular momentum channels, the collective modes could be regarded as generalized spin-$2L$ modes.

2.4 Conclusion

In this chapter, we have reviewed the method of computing bosonic collective modes with the kinetic theory, or the linearized Eilenberger equation. We applied the machinery to various superfluids in two dimensions including the parings in the s-wave, chiral d-wave, and the A and B phases of the p-wave channels. We showed that in all of the above models, there exists a mode with the mass $\sqrt{2}\,\Delta$, where Δ is the mass of the Bogoliubov quasiparticle. It is massive but still below the gap for pair breaking. We further proved that such a mode with mass $\sqrt{2}\,\Delta$ exists in all chiral superfluids paired in arbitrary angular momentum channel and therefore is universal.

CHAPTER 3

FERMI LIQUID CORRECTIONS

3.1 Introduction

In the previous chapter, we found that for chiral ground states of given L, spin-$2L$ bosonic modes d_L and d'_{-L} have the finite mass $\sqrt{2}\Delta$. In this chapter, we compute the Fermi liquid corrections to the mass spectra. In other words, the masses of collective modes will be functions of the Landau parameters $\{F_\ell\}$ and the results in the previous chapter correspond to the limit of $F_\ell = 0$. Conceptually, this is a phenomenological way of *mass renormalization.* In particular, we utilize and generalize the methods introduced in Ref. [62, 61] to include the Landau parameters in the computation for superfluids in the long-wavelength limit. Before presenting quantitative details, we enumerate some general features. Those modes with the mass 2Δ, e.g., Eq. (2.19a), in general are not sourced by fermionic self-energy, and consequently these modes are not renormalized. On the other hand, for those massless modes, e.g., Eq. (2.19b), short-range fermionic self-energy can at most renormalize the sound speed and the magnitude of external source fields instead of generating a gap.

We will therefore focus on the spin-$2L$ modes of mass $\sqrt{2}\,\Delta$, and show that given L, only the Landau parameters $F_{\pm 2L}$ renormalize the mass. Furthermore, the algebraic equation that determines the value of renormalized mass is universal for any L.

3.2 Mass Renormalization by the Landau Parameters

3.2.1 Massless modes

Let us first look at the massless modes in the s-wave channel (2.19b). The right-hand side $\delta\varepsilon_0^{(+)}$ consists of pure external perturbation and the renormalization coming form the integral part of (2.7a). Since we have rewritten Eq. (2.7a) and (2.7b) using $F(\theta,\theta')$ instead of $A(\theta,\theta')$, we substitute the properly normalized external perturbations $\delta\varepsilon_{\text{ext}}$ and $\delta\boldsymbol{\varepsilon}_{\text{ext}}$

with new symbols δu and $\delta\mathbf{u}$. In the long-wavelength limit,

$$\delta\varepsilon^{(+)}(\theta) = \delta u^{(+)} + \int \frac{d\theta'}{2\pi} F^s(\theta,\theta')[-\lambda\delta\varepsilon^{(+)} + \frac{\omega\lambda}{2\Delta} d^{(-)}]. \tag{3.1}$$

Projecting out $\ell = 0$ component, we obtain

$$(1 + \lambda(\omega)F_0^s)\delta\varepsilon_0^{(+)} = \delta u_0^{(+)} + \frac{\omega\lambda}{2\Delta} F_0 d^{(-)}, \tag{3.2}$$

plugging which back into (2.19b) yields

$$\omega^2 d^{(-)} = 2\omega\Delta\delta u_0. \tag{3.3}$$

It entails that $d^{(-)}$ remains massless. To demonstrate a triplet-pairing example, we look at B-phase (2.20) and (2.24c). For triplet-pairing states, the diagonal term of (2.7a) reads

$$\delta\varepsilon^{(+)}(\theta) = \delta u^{(+)} + \int \frac{d\theta'}{2\pi} F^s(\theta,\theta')\Big[-\lambda(\omega)\delta\varepsilon^{(+)} + \frac{1}{2}\omega\bar{\lambda}\mathbf{\Delta}\cdot\mathbf{d}^{(-)}\Big], \tag{3.4}$$

whose projection to ℓth mode is

$$\delta\varepsilon_\ell^{(+)} = \frac{\delta u_\ell^{(+)} + \frac{1}{2}\bar{\lambda}\omega F_\ell^s(\mathbf{\Delta}\cdot\mathbf{d}^{(-)})_\ell}{1 + \lambda(\omega)F_\ell^s}. \tag{3.5}$$

For $\ell = 0$,

$$\mathrm{B}: \delta\varepsilon_0^{(+)} = \frac{\delta u_0 + \frac{\lambda\omega}{4\Delta} F_0^s(D_{+-}^{(-)} + D_{-+}^{(-)})}{1 + \lambda F_0^s} \tag{3.6}$$

and we again find

$$\omega^2(D_{+-}^{(-)} + D_{-+}^{(-)}) = 4\omega\Delta\delta u_0. \tag{3.7}$$

The dynamical equations for d_0 and $D_{\{+-\}}$ are not modified by F_0^s, which implies that short-range interactions are not capable of gapping the Goldstone mode.

3.2.2 Massive subgap Modes

Let us continue to examine how Landau parameters renormalize massive modes. We start with the B-phase (2.24b). Take $\ell = \pm 2$ component of (3.5).

$$\mathrm{B} : \delta\varepsilon_{\pm 2}^{(+)} = \frac{\delta u_{\pm 2} + \frac{\omega\lambda}{4\Delta} F_2^s D_{\pm\pm}^{(-)}}{1 + \lambda F_2^s}. \tag{3.8}$$

Plugging this back into (2.24b) renormalizes the solutions as

$$D_{\pm\pm}^{(-)} = \frac{2\omega\Delta\delta u_{\pm 2}}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F_2^s(\omega^2 - 4\Delta^2)}. \tag{3.9}$$

The new mass is given by the zero of the denominator. In the limit $|F_2^s| \ll 1$,

$$\omega^2 \simeq 2\Delta^2(1 + \tfrac{1}{2}\lambda F_2^s). \tag{3.10}$$

λ is a positive number of order 1. We can see modes get heavier for repulsive interactions $F_2^s > 0$ and soften for attractive interactions $F_2^s < 0$.

Next let us look at the mode in the A-phase (2.26b) sourced by spin-dependent quasi-particle energy.

$$\delta\varepsilon_z^{(+)}(\theta) = \delta u_z + \int \frac{d\theta'}{2\pi} F^a(\theta, \theta') \Big[-\lambda\delta\varepsilon_z^{(+)} - \frac{i\omega}{2}\bar{\lambda}(\mathbf{\Delta} \times \mathbf{d}^{(+)})_z \Big] \tag{3.11}$$

with $\delta u_z = \delta\mathbf{u} \cdot \hat{\mathbf{z}}$. Projecting it to $\ell = \pm 2$ modes,

$$\delta\varepsilon_{z,\pm 2}^{(+)} = \frac{\delta u_{z,\pm 2} \mp F_2^a \frac{\omega\lambda}{4\Delta} D_{\pm\pm}^{(+)}}{1 + \lambda F_2^a}. \tag{3.12}$$

Substituting this back into (2.26b) yields

$$D_{\pm\pm}^{(+)} = \frac{\pm 2\omega\Delta\delta u_{z,\pm 2}}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F_2^a(\omega^2 - 4\Delta^2)}. \tag{3.13}$$

Therefore, the mass correction is given by the same transcendental equation with the replacement $F_2^s \to F_2^a$.

We are now ready to repeat the above computation for general chiral ground states. As it can be inferred from the previous analyses, the equations for singlet-pairing states are identical to ones for the longitudinal components $(\mathbf{d}\cdot\mathbf{\Delta})$ of the triplet-pairing states. Moreover, higher L states also have the same algebraic forms. Hence, we will concentrate on triplet-pairing states and take $L = 1$ without loss of generality.

The main difference between the preceding analyses and the one for general chiral states is that the gap function can no longer be chosen real and $d^{(\pm)}$ are no longer a good basis. Consequently, the scalar self-energy would satisfy the equation

$$\delta\varepsilon^{(+)} = \delta u^+ + \int \frac{d\theta'}{2\pi} F(\theta,\theta')\Big[-\lambda\delta\varepsilon^{(+)} + \frac{1}{2}\omega\bar{\lambda}(\mathbf{\Delta}\cdot\mathbf{d} - \mathbf{\Delta}\cdot\mathbf{d}')\Big]. \tag{3.14}$$

Let us take the z component of (2.29a) and (2.29b)

$$(\omega^2 - 2\Delta^2)d_{1z} = \omega\Delta\delta\varepsilon_2^{(+)}, \tag{3.15}$$

$$(\omega^2 - 2\Delta^2)d'_{-1z} = -\omega\Delta\delta\varepsilon_{-2}^{(+)}. \tag{3.16}$$

Renormalizing $\delta\varepsilon_{\pm 2}^{(+)}$ with (3.14), we find

$$d_{1z} = \frac{\omega\Delta\delta u_2^{(+)}}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F_2^s(\omega^2 - 4\Delta^2)}, \tag{3.17a}$$

$$d'_{-1z} = \frac{\omega\Delta\delta u_{-2}^{(+)}}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F_2^s(\omega^2 - 4\Delta^2)}. \tag{3.17b}$$

Finally we look at the transverse fluctuation by looking at the x component:

$$(\omega^2 - 2\Delta^2)d_{1x} = -i\omega\Delta(\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}_2^{(+)})_x, \tag{3.18a}$$

$$(\omega^2 - 2\Delta^2)d'_{-1x} = -i\omega\Delta(\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}_{-2}^{(+)})_x. \tag{3.18b}$$

The spin-dependent self-energy now takes the form

$$\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)} = \hat{\mathbf{z}}\times\delta\mathbf{u}^{(+)} + \int \frac{d\theta'}{2\pi} F^a(\theta,\theta')\Big[-\lambda\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)} - i\frac{\omega}{2}\bar{\lambda}\hat{\mathbf{z}} \times [(\boldsymbol{\Delta}^* \times \mathbf{d}) + (\boldsymbol{\Delta} \times \mathbf{d}')]\Big]. \tag{3.19}$$

Projecting it to $\ell = \pm 2$ allows to solve

$$d_{1x} = \frac{-i\omega\Delta(\hat{\mathbf{z}} \times \delta\mathbf{u}^{(+)})_2}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F_2^a(\omega^2 - 4\Delta^2)}. \tag{3.20}$$

To sum up, the analyses in this section have shown the following. (i) The Goldstone modes are not gapped by short-range interaction parametrized by Landau parameters. (ii) For p-wave superconductors in both B and A phases, the subgap modes $\sqrt{2}\,\Delta$ receive renormalization from quadrupolar Landau parameters F_2^s or F_2^a. (iii) For all chiral ground states of finite orbital momenta L, the subgap modes parallel to their ground states receive mass renormalization from the channel F_{2L}^s. The mass corrections referred to in (ii) and (iii) are all determined by the following equation:

$$(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda(\omega)F(\omega^2 - 4\Delta^2) = 0. \tag{3.21}$$

In Fig. 3.1, we plot the numerical solution to (3.21) as a function of F, which stands for the Landau parameter of the channel of interest. In accord with the intuition we acquired from the small F expansion, a strong repulsive interaction in particle-hole channel increases the magnitude of the gap, which asymptotically approaches the pair-breaking threshold 2Δ. On

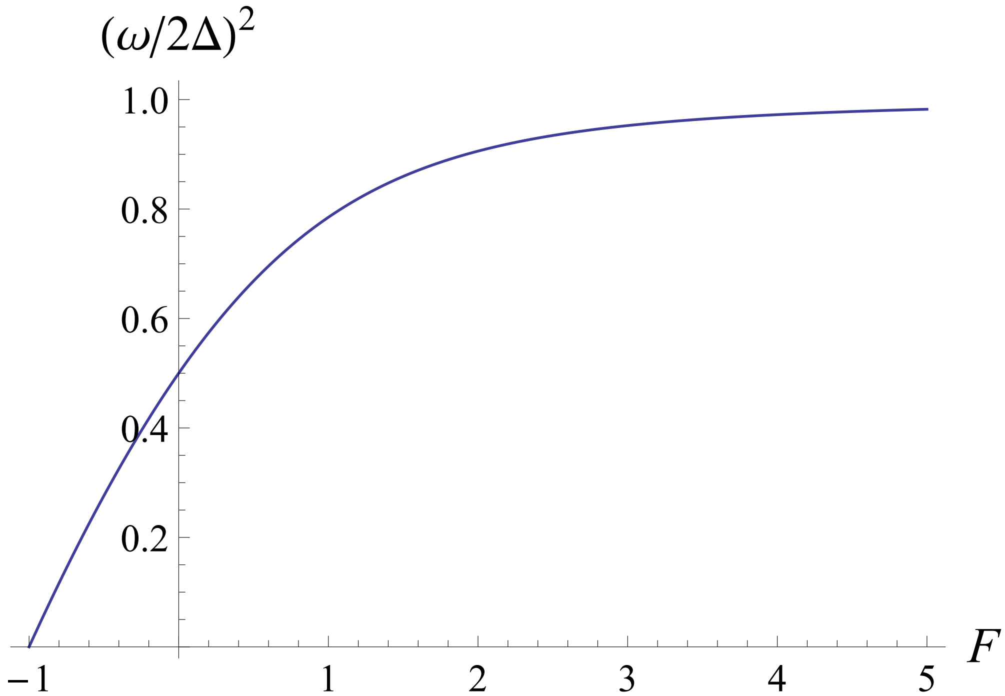

Figure 3.1: The root to (3.21) depending on the value of Landau parameter F. This figure was first published in Ref.[28] with the permission for reuse here.

the other hand, an attractive interaction softens the mass of order parameter. In particular, we see the mode would become massless as $F = -1$, at which the Pomeranchuk instability of two-dimensional Fermi liquid is triggered.

We can then look at the region $F = -1 + \epsilon$ with $\epsilon \ll 1$. At $T = 0$, we can expand Eq. (3.21) around $\omega^2 \approx 0$ and extract its dependence on F near the instability. Using the closed form (A.5a), we can deduce that Eq. (3.21) has the zero at

$$\omega^2 = \frac{3(1+F)}{6+F} \times 4\Delta^2 \approx \frac{12}{5}\epsilon\Delta^2. \tag{3.22}$$

This expression allows us to study how this mode becomes massless as we approached the instability.

3.3 Conclusion

We derived the corrections to the bosonic collective modes for chiral superfluids pairing in all angular momentum channel L, and explicitly showed only the Landau parameters $F_{\pm 2L}$ are involved in renormalizing the mass of the subgap mode. We also demonstrated the transcendental equation that determines the mass renormalization is independent of the spin dependency of the Landau parameters and the angular momentum quantum number L. In the vicinity of the Pomeranchuk instability in the channel of angular momentum $2L$, the subgap mode becomes soft according to Eq. (3.22). This indicates in a system susceptible to the quadrupolar Pomeranchuk instability, only the superfluids pairing in the p-wave could not possess a high frequency subgap mode in the spectrum.

CHAPTER 4

FIELD-THEORY MODEL FOR CHIRAL P-WAVE SUPERFLUID

4.1 Introduction

A complete kinetic theory treatment for both spin-singlet and spin-triplet chiral superfluids in the previous section has been conducted. However, it is still tempting to acquire an effective theory formulation, which would allow us to investigate the problem with techniques and insights across communities. Here, we propose a toy microscopic model for $L = 1$ p-wave chiral superfluid at $T = 0$. This channel is particularly interesting since it can be potentially probed in a thin film of superfluid ^{3}He and A phase or the $\nu = \frac{5}{2}$ fractional quantum Hall state as we pointed out in chapter 1.2.2. The corresponding Pomeranchuk instability in $2L = 2$ channel triggers the charge nematic order. In the approximation consistent with the kinetic theory approach, it reproduces exactly the same result, and moreover reveals the spin-2 nature of the subgap modes of interest.

To this end, we will first review the method of effective action [81, 68, 55, 90] for a general quantum field theory with the modification for the superfluid phase [2, 11], and discuss the meaning of collective modes in this language. We then propose the minimal model of a p-wave superfluid near the nematic critical point for a species of spinless nonrelativistic fermion in two dimensions, and present the Feynman rules required for the computation of effective action. The explicit effective action at one-loop follows immediately, and we show the previous results can be derived in the *particle-hole-symmetric* limit. We note that the *particle-hole symmetry* in this chapter differs from that defined in chapter 1. Here, a hole refers to an unoccupied state below the Fermi level, whereas a particle means an excited and occupied orbital above the Fermi sea. In the context of chapter 1, the particles and holes refer to occupied and unoccupied orbitals in a Landau level. The particle-hole symmetry is exact on the Fermi surface [54] and the correction due to its violation will be discussed.

4.2 Method of Effective Action

The method of effective action is a useful technique for quantum field theories whose field contents have separations of energy scales. Let us consider a schematic action functional $\mathcal{S}[\{\phi_s\}, \{\phi_f\}]$, where $\{\phi_s\}$ and $\{\phi_f\}$ are the sets of slow fields and fast fields. By *fastness* we mean the energy scales of $\{\phi_f\}$ are much higher than those of $\{\phi_s\}$. The effective field theory relevant at large time and length scales can be defined via the path integral

$$\int \mathscr{D}\phi_s \, \exp\left(i\mathcal{S}_{\text{eff}}[\{\phi_s\}]\right) = \int \mathscr{D}\phi_s \, \mathscr{D}\phi_f \, \exp(i\mathcal{S}[\{\phi_s\}, \{\phi_f\}]), \tag{4.1}$$

where the fast fields $\{\phi_f\}$ are said to be integrated out in the path integral.

Classic examples include the theory of weak interactions in the standard model of particle physics [68, 55]. The starting action is the involved model of weak interactions $\mathcal{S} = S_{\text{weak}}$. After Higgs mechanism, the W and Z bosons acquire masses of values m_W and m_Z. For scattering processes with momenta much smaller than m_W, we can integrate out W and Z bosons to get an easier theory for computation. The resulting theory is Fermi's theory of beta decay [14], that is $\mathcal{S}_{\text{eff}} = \mathcal{S}_{\text{Fermi}}$.

Another celebrated example concerns quantum electrodynamics (QED) in four dimensions. The effective action of the electromagnetic field can be derived by integrating out the massive electron field.

$$\exp(i\mathcal{S}_{\text{eff}}[A_\mu]) = \int \mathscr{D}\bar{\psi}\mathscr{D}\psi \, \exp\left[i\int d^4x \, \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi\right)\right]. \tag{4.2}$$

When the A_μ profile produces a constant electromagnetic field, the resulting effective action is the powerful Euler-Heisenberg effective action [25, 69], from which the QED beta function, chiral anomaly, and the decay of the vacuum can be derived.

On some occasions, the slow field may not exist in the theory to start with. It is very typical in field theories with two-fermion interactions such as Gross-Neveu, Thirring theories, as well as the microscopic theories for the spin-polarized p-wave chiral superfluid [23, 75, 78]. An standard trick is to decouple these interacting terms by introducing a dynamical auxiliary field via Hubbard-Stratonovich transformations. These auxiliary fields are the conjugates of the fermion bilinears under Legendre transformations. We will denote the auxiliary fields as ϕ_I in the following.

Let us now consider a generic microscopic Hamiltonian of a species of nonrelativistic fermions.

$$\mathcal{S} = \int d^3x\, \psi^\dagger\, (i\partial_t - \varepsilon)\, \psi + \mathcal{S}_{\text{int}}, \tag{4.3}$$

where ε is the single-particle energy and $\mathcal{S}_{\text{int}}$ involves the bilinear forms of the fermion field $\psi^\dagger$ and ψ. Without loss of generality, the fermionic part of the action after possibly multiple Hubbard-Stratonovich transformations can be written as $\mathcal{S} = \int (\mathrm{d}x)\, \Psi^\dagger i D^{-1} \Psi$, where Ψ is the Nambu spinor $\Psi^T = (\psi, \psi^\dagger)$. The partition function of the fermion sector is then

$$\int \mathscr{D}\Psi^\dagger \mathscr{D}\Psi\, \exp\Bigg(- \int (\mathrm{d}x) \Psi^\dagger D^{-1} \Psi \Bigg) := \exp\Bigg(i\mathcal{S}_{\text{eff}} \Bigg). \tag{4.4}$$

Since we are only considering a single species of fermions, the effective action of auxiliary fields $\{\phi_I\}$ reads

$$\mathcal{S}_{\text{eff}} = -\frac{i}{2} \mathrm{Tr} \ln D^{-1}. \tag{4.5}$$

The factor of $\frac{1}{2}$ comes from duplicating the degrees of freedom as we construct the kernel iD^{-1}. Formally, we can expand the action with respect to a classical solution $\phi^0_J(k) = \langle \phi^0_J \rangle \times (2\pi)^3 \delta(k)$. In terms of the deviation $\delta\phi_J = \phi_J - \phi^0_J$

$$\begin{aligned} \mathcal{S} =& \mathcal{S}_{\text{eff}} + \mathcal{S}_{\text{aux}}[\phi_I] \\ =& S[\phi^0_I] + \int (\mathrm{d}q)\, \frac{\delta S[\phi^0]}{\delta \phi_J(q)} \delta\phi_J(q) + \frac{1}{2} \int (\mathrm{d}q)(\mathrm{d}q')\, \delta\phi_I(q) \frac{\delta^2 S[\phi^0]}{\delta\phi_I(q) \delta\phi_J(q')} \delta\phi_J(q') + \cdots \end{aligned} \tag{4.6}$$

The first-order condition

$$\frac{\delta S}{\delta \phi_I(k)}[\phi^0] = 0 \tag{4.7}$$

is often used to specify the information of a certain uniform ground state $\phi_I^0(2\pi)^3\delta(k)$. The key ingredient of the Gaussian effective theory is the second derivative of the action evaluated with respect to the ground state. To extract the contribution from S_{eff}, we have to compute

$$\begin{aligned}
&\frac{\delta^2}{\delta\phi_I(q)\delta\phi_J(q')}S_{\text{eff}} = -\frac{i}{2}\frac{\delta^2}{\delta\phi_I(q)\delta\phi_J(q')}\text{Tr}\ln D^{-1} \\
=& -\frac{i}{2}\frac{\delta}{\delta\phi_I(q)}\text{Tr}D\frac{\delta D^{-1}}{\delta\phi_J(q')} = -\frac{i}{2}\text{Tr}\frac{\delta D}{\delta\phi_I(q)}\frac{\delta D^{-1}}{\delta\phi_J(q')} - \frac{i}{2}\text{Tr}D\frac{\delta^2 D^{-1}}{\delta\phi_I(q)\delta\phi_J(q')} \\
=&\frac{i}{2}\text{Tr}[D\frac{\delta D^{-1}}{\delta\phi_I(q)}D\frac{\delta D^{-1}}{\delta\phi_J(q')}] - \frac{i}{2}\text{Tr}[D\frac{\delta^2 D^{-1}}{\delta\phi_I(q)\delta\phi_J(q')}]
\end{aligned} \tag{4.8}$$

Thus the 2-point function of our interest is then

$$M^{IJ}(q,q') = \frac{i}{2}\text{Tr}[D\frac{\delta D^{-1}}{\delta\phi_I(q)}D\frac{\delta D^{-1}}{\delta\phi_J(q')}] - \frac{i}{2}\text{Tr}[D\frac{\delta^2 D^{-1}}{\delta\phi_I(q)\delta\phi_J(q')}]\Big|_{\phi=\phi^0} \tag{4.9}$$

The second term in (4.9) is usually referred to as the contact term. Example includes the diamagnetic current term of electromagnetic response. For the model concerning us in this thesis, we only have to focus on the first term in (4.9).

4.3 The Model for the p-Wave Superfluid near the Nematic Critical Point

Let us consider a two-dimensional spin-polarized nonrelativistic fermion ψ with the kinetic term $H_{\text{F}}[\psi]$, and a contact-pairing potential in $L = 1$ channel H_V [78] responsible for inducing BCS mechanism. Inclusion of the Landau parameters is not as straightforward because strictly speaking they are defined phenomenologically in terms of scattering amplitudes instead of parameters in a specific microscopic model. In the Hatree-Fock approximation, the

Landau function $f_{\mathbf{kk}'}$ can be written in terms of the interaction potential $V(|\mathbf{k}-\mathbf{k}'|)$

$$f_{\mathbf{kk}'} = V(0) - V(|\mathbf{k}-\mathbf{k}'|), \tag{4.10}$$

and the coefficients F_ℓ can be further projected by expanding $|\mathbf{k}-\mathbf{k}'|$ with the Legendre polynomials $P_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')$. We adapt a natural quadrupolar-density interaction H_Q devised in Ref. [49] for a normal Fermi liquid in the vicinity of nematic critical point. It was later applied to the problem of the nematic phase fractional quantum Hall regime [89]. Note that the spin degree of freedom is frozen in this regard, and therefore we no longer have the $\mathrm{SO}_S(3)$ symmetry to start with. The model thus describes a minimalistic p-wave superfluid. The system presented in the previous section in this sense is considered to be three copies of the model here. Nonetheless, these ingredients suffice to produce the subgap modes and their renormalization. The exact form of the models read

$$H_{\mathrm{F}} = \int \frac{\mathrm{d}^2k}{(2\pi)^2}\,\psi^\dagger_{\mathbf{k}}\left(\frac{\mathbf{k}^2}{2m} - \epsilon_F\right)\psi_{\mathbf{k}} := \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\psi^\dagger_{\mathbf{k}}\xi_{\mathbf{k}}\psi_{\mathbf{k}} \tag{4.11a}$$

$$H_V = -\frac{1}{p_F^2}\int \frac{\mathrm{d}^2k}{(2\pi)^2}\frac{\mathrm{d}^2k'}{(2\pi)^2}V_1\psi^\dagger_{-\mathbf{k}}\psi^\dagger_{\mathbf{k}}\mathbf{k}\cdot\mathbf{k}'\psi_{\mathbf{k}'}\psi_{-\mathbf{k}'} \tag{4.11b}$$

$$H_Q = \frac{1}{2}\int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}f_2(\mathbf{q})\mathbf{M}(-\mathbf{q})\cdot\mathbf{M}(\mathbf{q}), \tag{4.11c}$$

where $\mathbf{M} = (M_1, M_2)^T$ is defined by the quadrupole moment of particle density

$$\begin{pmatrix} M_1 & M_2 \\ M_2 & -M_1 \end{pmatrix} = -\frac{1}{p_F^2}\psi^\dagger \begin{pmatrix} k_x^2 - k_y^2 & 2k_xk_y \\ 2k_xk_y & k_y^2 - k_x^2 \end{pmatrix}\psi \tag{4.12}$$

and

$$f_2(\mathbf{q}) = \frac{f_2}{1+\kappa\mathbf{q}^2} = \nu_{\mathrm{2D}}^{-1}\frac{F_2}{1+\kappa\mathbf{q}^2}. \tag{4.13}$$

$\nu_{2\mathrm{D}} = \frac{m}{2\pi}$, denoting the density of state of 2D electron gas, and F_2 is the conventional Landau parameter defined in Eq. (1.18). Owing to the frozen spin-degree of freedom, the F_2 here corresponds to the spin-independent Landau parameter, F_2^s in chpater 1. κ characterizes the interaction range and is irrelevant in the long-wavelength properties explored below. We introduce Hubbard-Stratonovich fields $(\phi, \phi^\dagger)$, $(\Phi, \Phi^\dagger)$, and $(\bar{Q}, Q)$ to decouple the two-body terms and rewrite the full action as

$$\mathcal{S} = \int \mathrm{d}^3x\, \Psi^\dagger (i\partial_t - H)_{\mathrm{BdG}} \Psi - \frac{1}{2V_1} \int \mathrm{d}^3x\, (\phi^\dagger \phi + \Phi^\dagger \Phi) + \int \mathrm{d}^3x \bar{Q}\, f_2^{-1}(-i\nabla) Q, \quad (4.14)$$

where $\Psi = \frac{1}{\sqrt{2}}(\psi, \psi^\dagger)^T$ and

$$\begin{aligned}(i\partial_t - H)_{\mathrm{BdG}} = &\begin{pmatrix} i\partial_t - \xi_{(-i\nabla)} & 0 \\ 0 & i\partial_t + \xi_{(-i\nabla)} \end{pmatrix} \\ &+ p_F^{-2} \begin{pmatrix} Q(\partial_x - i\partial_y)^2 + \bar{Q}(\partial_x + i\partial_y)^2 & 0 \\ 0 & -[(\partial_x - i\partial_y)^2 Q + (\partial_x + i\partial_y)^2 \bar{Q}] \end{pmatrix} \\ &- \frac{i}{p_F} \begin{pmatrix} 0 & \phi(\partial_x - i\partial_y) + \Phi(\partial_x + i\partial_y) \\ \phi^\dagger(\partial_x + i\partial_y) + \Phi^\dagger(\partial_x - i\partial_y) & 0 \end{pmatrix}. \end{aligned} \quad (4.15)$$

The derivatives are understood to act on all quantities on their right. From the structure of the action, we see ϕ and Φ represent the $p - ip$ and $p + ip$ pairing amplitudes, respectively. In the mean-field limit,

$$p - ip : \langle \phi \rangle = \Delta, \langle \Phi \rangle = 0, \quad (4.16a)$$

$$p + ip : \langle \phi \rangle = 0, \langle \Phi \rangle = \Delta. \quad (4.16b)$$

On the other hand, Q and $\bar{Q}$ represent the nematic order parameters.

To proceed, we consider a ground state with a gapped fermion spectrum from either of the above choices, integrate the fermion sector, and compute the bosonic Gaussian fluctuation.

In the explicit computation following, we choose the $p-ip$ ground state (4.16a) and shift $\phi \to \Delta + \phi$ so that ϕ presents purely the fluctuation. In this scenario, ϕ and Φ would be playing the role of $\mathbf{d}\cdot\hat{\mathbf{z}}$ in our kinetic approach. Note that the particle-hole symmetry is usually assumed in kinetic theory, whereas it is exact only on the Fermi surface. The effective field theory respecting this symmetry would acquire an emergent relativistic covariant form [54], even though the microscopic origin might rather respect Galilean symmetry. In the long-wavelength limit $\mathbf{q} \to 0$, the particle-hole symmetry can be implemented by evaluating loop momentum on the Fermi surface and extending the depth of the Fermi sea to infinity. After specifying the ground state, we can now read off the Feynman rules from the form of the action $(i\partial_t - H)_{\text{BdG}}$. The kernel D and propagator are

$$iD_0^{-1}(p,p') = (2\pi)^3\delta(p-p')\begin{pmatrix} p_0 - \xi_{\mathbf{p}} & \Delta p_F^{-1}(p_x - ip_y) \\ \Delta p_F^{-1}(p_x + ip_y) & p_0 + \xi_{\mathbf{p}} \end{pmatrix}, \tag{4.17a}$$

$$D_0(p,p') = (2\pi)^3\delta(p-p')\frac{i}{p_0^2 - E_{\mathbf{p}}^2 + i\eta}\begin{pmatrix} p_0 + \xi_{\mathbf{p}} & -\Delta p_F^{-1}(p_x - ip_y) \\ -\Delta p_F^{-1}(p_x + ip_y) & p_0 - \xi_{\mathbf{p}} \end{pmatrix} \tag{4.17b}$$

with $E_{\mathbf{p}}^2 = \xi_{\mathbf{p}}^2 + p_F^{-2}\Delta^2\mathbf{p}^2$. The vertices are the variation of iD^{-1} with respect to fields $\bar{Q}, Q, \Phi^\dagger$ and Φ.

$$\frac{\delta iD^{-1}(p,p')}{\delta Q(q)} = \frac{1}{p_F^2}\begin{pmatrix} -(p'_x - ip'_y)^2 & 0 \\ 0 & (p_x - ip_y)^2 \end{pmatrix}(2\pi)^3\delta(p - p' - q), \tag{4.18a}$$

$$\frac{\delta iD^{-1}(p,p')}{\delta \bar{Q}(q)} = \frac{1}{p_F^2}\begin{pmatrix} -(p'_x + ip'_y)^2 & 0 \\ 0 & (p_x + ip_y)^2 \end{pmatrix}(2\pi)^3\delta(p' - p - q), \tag{4.18b}$$

$$\frac{\delta iD^{-1}(p,p')}{\delta \Phi(q)} = (2\pi)^3\delta(p - p' - q)\begin{pmatrix} 0 & p_F^{-1}(p'_x + ip'_y) \\ 0 & 0 \end{pmatrix}, \tag{4.18c}$$

$$\frac{\delta iD^{-1}(p,p')}{\delta \Phi^\dagger(q)} = (2\pi)^3\delta(p' - p - q)\begin{pmatrix} 0 & 0 \\ p_F^{-1}(p'_x - ip'_y) & 0 \end{pmatrix}. \tag{4.18d}$$

Using the method presented earlier and the Feynman rules, the resulting effective action has the form:

$$\mathcal{S}_{\text{eff}} = \mathcal{S}_0[\Delta_0, \phi, \phi^\dagger] + \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \bigg(\bar{Q}(q) M_{\bar{Q}Q}(q) Q(q) + \Phi^\dagger(q) M_{\Phi^\dagger\Phi}(q)\Phi(q) \\ + M_{Q\Phi}(q) Q(-q)\Phi(q) + M_{\bar{Q}\Phi^\dagger}(q)\bar{Q}(-q)\Phi^\dagger(q)\bigg). \tag{4.19}$$

The leading part $\mathcal{S}_0$ contains the mean field free energy, Goldstone fluctuations $(\phi^\dagger - \phi)/i$ and the amplitude mode $\phi^\dagger + \phi$. The bare masses of Q and Φ are given by the zeros of $M_{\bar{Q}Q}$ and $M_{\Phi^\dagger\Phi}$, whose explicit forms are given by

$$M_{\bar{Q}Q} = \nu_{2\text{D}}\left(\lambda(\omega) + \frac{1}{F_2}\right) \tag{4.20}$$

$$M_{\Phi^\dagger\Phi} = \frac{\nu_{2\text{D}}\lambda(\omega)}{8\Delta^2}(\omega^2 - 2\Delta^2). \tag{4.21}$$

λ is again the Tsunedo function (A.5a). Hence, the bare mass of Q depends on the parameter F_2 and becomes soft as $F_2 \to -1$. On the other hand, the mass of Φ is shown to be $m_\Phi = \sqrt{2}\,\Delta$, in agreement with the result (2.29a) and (2.29b). The Fermi-liquid correction can now be understood in terms of the coupling $Q\Phi$ and $\bar{Q}\Phi^\dagger$

$$M_{Q\Phi} = M_{\bar{Q}\Phi^\dagger} = -\frac{\nu_{2\text{D}}\omega\lambda(\omega)}{4\Delta}, \tag{4.22a}$$

indicating Φ and Q are actually not independent modes. As $F_2 \neq -1$ and Q has a finite bare mass, we are able to integrate out Q to obtain a more compact effective theory.

$$\mathcal{S}_{\text{eff}} = \mathcal{S}_0 + \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\nu_{2\text{D}}\bar{\lambda}(\omega)}{8(1 + F_2\lambda(\omega))}\Phi^\dagger(\omega)\Big[(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda(\omega)F_2(\omega^2 - 4\Delta^2)\Big]\Phi(\omega), \tag{4.23}$$

The mass of the collective mode $\Phi(\omega)$ is given by the zero of the Lagrangian. The condition for the integrand in Eq. (4.23) to be zero reproduces explicitly the result (3.21). Alternatively, one could integrate out Φ to derive an effective theory of Q.

$$\mathcal{S}_{\text{eff}} = \mathcal{S}_0 + \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \nu_{2\mathrm{D}} \bar{Q}(\omega) \left[\frac{\omega^2 - 4\Delta^2}{2(\omega^2 - 2\Delta^2)} \lambda(\omega) + \frac{1}{F_2} \right] Q(\omega). \tag{4.24}$$

Straightforward investigation shows the renormalized mass of Q in the above action is still given by Eq. (3.21).

In addition to reproducing the known result, the field theory approach already offers some implications beyond the semi-classical kinetic theory approach:

1. The exact value $\sqrt{2}\,\Delta$ is closely related to the assumption of particle-hole symmetry, or the approximate relativistic nature of the fermionic superfluid on the Fermi surface. Loosening this approximation allows corrections of order Δ/ϵ_F. Moreover, a term $\Phi^\dagger i\partial_t \Phi$ appears in the action if we breaks particle-hole symmetry during computation, which in turn modifies the value of the bare mass m_Φ as well. In this computation we impose the particle-hole symmetry in order to be consistent with the assumptions of the kinetic theory. While one could compute non-universal corrections to the value of m_Φ by breaking the particle-hole symmetry, we comment that in the weak-coupling computation terms odd in frequency merely change the mass slightly and the magnitude of m_Φ would remain $\mathcal{O}(\Delta)$. The qualitative fact that this mass is reduced by negative F_2 is not affected.

2. In the presence of a condensate, operators are classified by the residual symmetry respected by the ground state. Taking the $p - ip$ ground state, for instance,

$$\Delta_{\mathbf{p}} \sim (p_x - ip_y)\langle \psi(-\mathbf{p})\psi(\mathbf{p})\rangle. \tag{4.25}$$

$\Delta_{\mathbf{p}}$ is symmetric under a combination of U(1) charge transformation and orbital rotation

$$\psi \to e^{i\alpha/2}\psi \tag{4.26a}$$

$$\begin{pmatrix} p_x \\ p_y \end{pmatrix} \to \begin{pmatrix} p_x \cos\alpha - p_y \sin\alpha \\ p_y \cos\alpha + p_x \sin\alpha \end{pmatrix}. \tag{4.26b}$$

In this example, the operators are classified using the *angular momentum* ℓ defined by this combined transformation $\mathcal{O} \to e^{i\ell\alpha}\mathcal{O}$. In particular, the fluctuation of $p + ip$ condensate transforms as

$$(p_x + ip_y)\langle\psi(-\mathbf{p})\psi(\mathbf{p})\rangle \to e^{2i\alpha}(p_x + ip_y)\langle\psi(-\mathbf{p})\psi(\mathbf{p})\rangle \tag{4.27}$$

and has angular momentum ℓ =2. Similarly, the nematic order parameter transforms as

$$(p_x + ip_y)^2\langle\psi^\dagger(\mathbf{p})\psi(\mathbf{p})\rangle \to e^{2i\alpha}(p_x + ip_y)^2\langle\psi^\dagger(\mathbf{p})\psi(\mathbf{p})\rangle, \tag{4.28}$$

indicating both operators possess the *spin-2* nature under the residual symmetry group. The *spin-2L* states can also be understood from this perspective. Besides, that Φ and Q are not independently fluctuating can be explained in terms of the notion of emergent geometry [79, 20, 18]. The nematic order parameters $Q := (Q_1 + iQ_2)/2$ and $\bar{Q} := (Q_1 - iQ_2)/2$, under a proper normalization [1], also parametrize an emergent unimodular metric $\mathfrak{g}_{ij}$ via

$$\mathfrak{g} := \exp\begin{pmatrix} Q_1 & Q_2 \\ Q_2 & -Q_1 \end{pmatrix}. \tag{4.29}$$

Similarly, the order parameters of a p-wave superfluid Δ^i also define an emergent

1. In the present work, we have to replace $Q \to -\epsilon_F Q$.

geometric degree of freedom $\mathfrak{G}^{ij} \sim \Delta^{*i}\Delta^{j}$. The subgap modes, in this language, correspond to the spin-2 sector of $\mathfrak{G}$. Placed on a flat space and close to equilibrium, both $\mathfrak{g}$ and $\mathfrak{G}$ favor the Euclidean metric δ_{ij}. Hence, their fluctuations, both being spin-2, are indistinguishable from this geometric perspective.

3. In previous studies, these modes are usually overlooked regarding low energy physics [27], even though they are responsible for electromagnetic response at high frequency [26]. That the spin-2 mode becomes soft as $F_2 = -1$ suggests that an effective low-energy theory different from one in Ref. [27] should be formulated to incorporate a spin-2 mode close to a nematic critical point. While different microscopic models could produce different results depending on model dependent parameters, symmetry principle together with the above geometric picture suggests an effective action that replaces the background geometry with the internal fluctuating geometry. A proper microscopic model is then responsible for correctly producing effects such as the analogous Hall viscosity, which comes from a $\bar{Q}\partial_t Q$ term in effective action. The existence of this term breaks time reversal symmetry and distinguish a $p - ip$ ground state from a $p + ip$ one. This issue is beyond the scope of current work and will be addressed in detail separately in the future.

4.4 Conclusion

We proposed a microscopic model of a species of nonrelativistic fermions in two dimensions to capture the features of the p-wave superfluids and the Landau-parameter effect in the nematic channel. The exact form of the model is motivated by earlier studies on superfluid ^{3}He, nematic Fermi liquid, and the nematic phase in fractional quantum Hall states [78, 49, 89]. After computing the one-loop effective action to the quadratic order in the long-wavelength limit, we realized the results of the subgap collective mode and the Fermi liquid correction from the kinetic theory can be produced by evaluating the loop momentum on the Fermi surface, which is equivalent to imposing the particle-hole symmetry in the superfluid. The potential correction from loosening this symmetry was discussed. Furthermore, we explained the meaning of the spin-2 mode by examining how an operator transforms under the residual symmetry of the superfluid ground state and connect the subgap mode to the geometric developments in other works on superfluids.

CHAPTER 5

CONCLUSION AND OUTLOOK

In conclusion, we started from puzzles and phenomena in the fractional quantum Hall systems, and argued one of the keys to the ultimate resolution is a thorough understanding of the spectra of the chiral superfluids in two dimensions in the presence of the density fluctuations in the nematic channel. Bearing with this idea, we revisit a class of two-dimensional superfluids. Using the semiclassical kinetic equation, we compute the order parameter collective modes for the two-dimensional B phase and general chiral ground states of angular momenta $L \geq 0$. Extending the known results for $L = 1$, we show that the sub-gap modes of the universal mass value $\sqrt{2}\,\Delta$ exist for all chiral ground states $L \geq 1$ in the limit with exact particle-hole symmetry. By renormalizing the fermionic self-energy, we calculate the correction of these subgap modes from Fermi-liquid corrections and discover those sub-gap modes, sourced by F_{2L}, could be regarded as spin-$2L$ modes, where L is the angular momentum of their underlying ground state. The masses increase for repulsive fermionic interactions and soften for attractive ones. Remarkably, renormalized subgap modes become gapless when the Pomeranchuk instability in the corresponding channel is triggered.

Moreover, we proposed a toy model for the case $L = 1$, whose effective bosonic action is able to reproduce the kinetic result under the consistent approximations. This model could describe a p-wave chiral superfluid near a nematic critical point, and furthermore allows us to loosen the common assumptions made in semiclassical approaches and utilize the insights from field-theory communities to understand the nature of the subgap modes. We hope the approaches and conclusions drawn from this work could provide the studies of quantum Hall nematic physics and nematic unconventional superfluid a complementary perspective and new insights.

In the regard of observability, at this time we have not been able to compute the dispersion of the subgap modes and therefore cannot identify them with the magneto-roton in the fractional quantum Hall regime. A reasonable pursuit at the current stage is to look for these modes in the systems hosting well-recognized chiral superfluids. The system of ultracold ^{3}He thin film is a good candidate because of the confirmation of the chiral-superfluid phase in the limit of zero thickness [40, 30, 91].

The order-parameter collective modes in the three-dimensional superfluid ^{3}He have been theoretically investigated in a variety of contexts [85, 47, 77, 61]. In the B phase, there are analogous subgap modes with the values of mass $\Delta\sqrt{12/5}$ and $\Delta\sqrt{8/5}$. Although they do not appear directly as hydrodynamic modes, they couple to Goldstone modes and therefore can be detected indirectly with sound propagation experiments. Both subgap mode contribute to the sound attenuation and the shift of sound velocity [84] and manifest themselves as resonance peaks in labs [52, 59]. In the A phase, the subgap collective modes do not have concise solutions owing to the anisotropy and nodes in the gap. Nevertheless, analogous results apply [53, 86]. Surprisingly, the Landau parameter in the nematic channel F_2^s can be determined [13] by the shift of sound velocity as function of pressure, and it exhibit a negative value of F_2^s at low pressure. At the best of our knowledge, there is no purely two-dimensional counterparts of the above studies, and we look forward to future developments following the increasing accessibility of pure two-dimensional superfluids.

Besides, stemming from the idea of emergent geometry, it is known that the nematic fluctuations close to the Pomeranchuk instability permit a bimetric description [20, 48, 22]. We expect a similar formulation will extend the current understanding of the p-wave chiral superfluids [27, 45] on the edge of quantum phase transition.

APPENDIX A

γ AND THE TSUNEDO FUNCTION λ

The integral γ is

$$\int_{|\Delta|}^{\infty} \frac{d\varepsilon}{2\pi i} \frac{1}{\sqrt{\varepsilon^2 - |\Delta|^2}} \tanh \frac{\varepsilon}{2T} = \gamma. \tag{A.1}$$

It is formally divergent, but can be regularized and identified with $1/V_\ell$ [or $1/(2V_0)$] using linearized gap equation.

The function λ was first introduced by Tsunedo as a kind of Cooper pair susceptibility.

$$\begin{aligned} \bar{\lambda}(\hat{\mathbf{p}};\omega,q) &= \frac{\lambda}{|\Delta(\hat{\mathbf{p}})|^2} \\ &= \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi i} \frac{\mathsf{n}(\varepsilon_-)(2\varepsilon\omega - \eta^2) - \mathsf{n}(\varepsilon_+)(2\varepsilon\omega + \eta^2)}{(4\varepsilon^2 - \eta^2)(\omega^2 - \eta^2) + 4|\Delta|^2\eta^2}, \end{aligned} \tag{A.2}$$

where $\eta = v_F \mathbf{q} \cdot \hat{\mathbf{p}}$ and

$$\mathsf{n}(\varepsilon) = -\frac{2\pi i \operatorname{sgn}(\varepsilon)}{\sqrt{\varepsilon^2 - |\Delta|^2}} \Theta(\varepsilon^2 - |\Delta|^2) \tanh \frac{\varepsilon}{2T}. \tag{A.3}$$

In $q \to 0$ limit, the integral reduces to

$$\lambda = |\Delta|^2 \int_{|\Delta|}^{\infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2 - |\Delta|^2}} \frac{\tanh \frac{\varepsilon}{2T}}{\varepsilon^2 - \omega^2/4}. \tag{A.4}$$

These expressions can be used in numerical evaluation. This function actually has an analytic closed form in the limit $T \to 0$. Writing $x = \omega/(2|\Delta|)$,

$$\lambda(\omega) = \frac{\sin^{-1} x}{x\sqrt{1 - x^2}}, \ \ |x| < 1, \tag{A.5a}$$

where as for $|x| > 1$,

$$\lambda(\omega) = \frac{1}{2x\sqrt{x^2-1}} \left[\ln \left| \frac{\sqrt{x^2-1}-x}{\sqrt{x^2-1}+x} \right| + i\pi \mathrm{sgn}(x) \right]. \tag{A.5b}$$

APPENDIX B

FULL DYNAMICAL EQUATIONS

In this section we sketch the steps for inverting the kinetic equation and give the full dynamical equations at finite wavelength. Expanding (2.1) with respect to the ground state of interest, we could found components of the Keldysh Green's function satisfy a general equation

$$\boldsymbol{\Omega}|\widehat{g}\rangle = \mathsf{M}|\widehat{\sigma}\rangle. \tag{B.1}$$

The quasi-classical Green's functions can thus be obtained via

$$\int \frac{d\varepsilon}{2\pi i}|\widehat{g}\rangle = \int \frac{d\varepsilon}{2\pi i}\boldsymbol{\Omega}^{-1}\mathsf{M}|\widehat{\sigma}\rangle. \tag{B.2}$$

We note that when performing ε integral in this work, the particle-hole symmetry $\varepsilon \leftrightarrow -\varepsilon$ is assumed.

The defined in (B.1) the matrices are

$$\boldsymbol{\Omega} = \begin{bmatrix} -\eta & \omega & 2i\Delta_I & -2\Delta_R \\ \omega & -\eta & 0 & 0 \\ 2i\Delta_I & 0 & -\eta & 2\varepsilon \\ 2\Delta_R & 0 & 2\varepsilon & -\eta \end{bmatrix} \tag{B.3}$$

and

$$\mathsf{M} = \begin{bmatrix} 0 & -\mathsf{m}_a & -i\mathsf{n}_s\Delta_I & \mathsf{n}_s\Delta_R \\ -\mathsf{m}_a & 0 & -\mathsf{n}_a\Delta_R & i\mathsf{n}_a\Delta_I \\ -i\Delta_I\mathsf{n}_s & \Delta_R\mathsf{n}_a & 0 & -\mathsf{m}_s \\ -\mathsf{n}_s\Delta_R & i\mathsf{n}_a\Delta_I & -\mathsf{m}_s & 0 \end{bmatrix}. \tag{B.4}$$

Δ_R and Δ_I are the real and imaginary parts of the gap function. In terms of the n defined by (A.3), the elements in M are

$$\mathsf{n}_s = \mathsf{n}(\varepsilon_+) + \mathsf{n}(\varepsilon_-), \tag{B.5a}$$

$$\mathsf{n}_a = \mathsf{n}(\varepsilon_+) - \mathsf{n}(\varepsilon_-), \tag{B.5b}$$

$$\mathsf{m} = \varepsilon \mathsf{n}(\varepsilon), \tag{B.5c}$$

$$\mathsf{m}_s = \mathsf{m}(\varepsilon_+) + \mathsf{m}(\varepsilon_-), \tag{B.5d}$$

$$\mathsf{m}_a = \mathsf{m}(\varepsilon_+) - \mathsf{m}(\varepsilon_-). \tag{B.5e}$$

In the rest of the section we give the proper combinations $|\widehat{g}\rangle$ and $|\widehat{\sigma}\rangle$ and the complete dynamical equations.

B.0.1 Singlet-pairing ground state

For a singlet-pairing state, the bosonic fluctuation couples only to spin independent fermionic self-energies, and the relevant equations are those which δg, $\delta g'$, d and d' obey. These equations can be easily solved by taking

$$|\widehat{g}\rangle = \begin{pmatrix} \delta g^{(-)} \\ \delta g^{(+)} \\ \delta f^{(+)} \\ \delta f^{(-)} \end{pmatrix}, |\widehat{\sigma}\rangle = \begin{pmatrix} \delta \varepsilon^{(-)} \\ \delta \varepsilon^{(+)} \\ d^{(+)} \\ d^{(-)} \end{pmatrix}. \tag{B.6}$$

Expressing $|\widehat{g}\rangle$ in terms of $|\widehat{\sigma}\rangle$ and performing convolutions with suitable potentials would imply the following equations.

$$\delta\varepsilon^{(-)}(\hat{\mathbf{p}};\omega,\mathbf{q}) - \delta\varepsilon^{(+)}_{\text{ext}} = \int \frac{d\theta'}{2\pi} A^s(\theta,\theta')\Big\{\Big(1 + (1-\lambda(\hat{\mathbf{p}}'))\frac{\eta'^2}{\omega^2-\eta'^2}\Big)\delta\varepsilon^{(-)}(\hat{\mathbf{p}}')$$
$$+ \frac{\omega\eta'}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}') + \frac{\bar{\lambda}(\hat{\mathbf{p}}')\eta'}{2}[d(\hat{\mathbf{p}}')\Delta^*(\hat{\mathbf{p}}') - \Delta(\hat{\mathbf{p}}')d(\hat{\mathbf{p}}')]\Big\}. \tag{B.7a}$$

$$\delta\varepsilon^{(+)}(\hat{\mathbf{p}};\omega,\mathbf{q}) - \delta\varepsilon^{(+)}_{\text{ext}} = \int \frac{d\theta'}{2\pi} A^s(\theta,\theta')\Big\{\frac{\omega\eta'}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(-)}(\hat{\mathbf{p}}')$$
$$+ \frac{\omega^2}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}') - \frac{1}{2}\omega\bar{\lambda}(\hat{\mathbf{p}}')[d'(\hat{\mathbf{p}}')\Delta(\hat{\mathbf{p}}') - d(\hat{\mathbf{p}}')\Delta^*(\hat{\mathbf{p}}')]\Big\}. \tag{B.7b}$$

$$d(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\text{e}}(\theta,\theta')\Big\{\Big(\gamma + \frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')(\omega^2 - \eta'^2 - 2|\Delta(\hat{\mathbf{p}}')|^2)\Big)d(\hat{\mathbf{p}}')$$
$$- \frac{\bar{\lambda}(\hat{\mathbf{p}}')}{2}\Delta^2(\hat{\mathbf{p}}')d'(\hat{\mathbf{p}}') - \frac{\Delta(\hat{\mathbf{p}}')\bar{\lambda}(\hat{\mathbf{p}}')}{4}(\eta'\delta\varepsilon^{(-)}(\hat{\mathbf{p}'}) + \omega\delta\varepsilon^{(+)}(\hat{\mathbf{p}'})]\Big\}. \tag{B.7c}$$

$$d'(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\text{e}}(\theta,\theta')\Big\{\Big(\gamma + \frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')(\omega^2 - \eta'^2 - 2|\Delta(\hat{\mathbf{p}}')|^2)\Big)d'(\hat{\mathbf{p}}')$$
$$- \frac{\bar{\lambda}(\hat{\mathbf{p}}')}{2}(\Delta^*(\hat{\mathbf{p}}'))^2 d(\hat{\mathbf{p}}') + \frac{\Delta^*(\hat{\mathbf{p}}')\bar{\lambda}(\hat{\mathbf{p}}')}{4}[\eta'\delta\varepsilon^{(-)}(\hat{\mathbf{p}}') + \omega\delta\varepsilon^{(+)}(\hat{\mathbf{p}}')]\Big\}. \tag{B.7d}$$

B.0.2 Triplet-pairing ground state

For a triplet-pairing ground state, the vectors $\mathbf{d}$ and $\mathbf{d}'$ couple to both spin-dependent and independent self-energies. We denote the direction of the ground-state condensate $\boldsymbol{\Delta}$ as $\hat{\mathbf{n}}$. To solve vector quantities $\mathbf{d}$ and $\boldsymbol{\varepsilon}$, we decompose them into the longitudinal L and transverse T components with respect to $\hat{\mathbf{n}}$, that is, a vector $\mathbf{v}$ is decomposed as $\mathbf{v} = \mathbf{v}_L + \mathbf{v}_T$, where $\mathbf{v}_L = \hat{\mathbf{n}}(\mathbf{v}\cdot\hat{\mathbf{n}})$. The complete set of equations can be solved by considering the following combinations of $\{|\widehat{g}\rangle, |\widehat{\sigma}\rangle\}$. The part coupled with spin-independent δg is the longitudinal modes

$$\left\{ \begin{pmatrix} \delta g^{(-)} \\ \delta g^{(+)} \\ \delta \mathbf{f}_L^{(+)} \\ \delta \mathbf{f}_L^{(-)} \end{pmatrix}, \begin{pmatrix} \delta \varepsilon^{(-)} \\ \delta \varepsilon^{(+)} \\ \mathbf{d}_L^{(+)} \\ \mathbf{d}_L^{(-)} \end{pmatrix} \right\}. \tag{B.8a}$$

The part coupled with the spin-dependent $\delta\mathbf{g}$, on the other hand, includes the transverse and binormal parts of the anomalous Green's function.

$$\left\{ \begin{pmatrix} \hat{\mathbf{n}} \times \delta \mathbf{g}^{(-)} \\ \hat{\mathbf{n}} \times \delta \mathbf{g}^{(+)} \\ i\delta \mathbf{f}_T^{(-)} \\ i\delta \mathbf{f}_T^{(+)} \end{pmatrix}, \begin{pmatrix} \hat{\mathbf{n}} \times \delta \boldsymbol{\varepsilon}^{(-)} \\ \hat{\mathbf{n}} \times \delta \boldsymbol{\varepsilon}^{(+)} \\ i\mathbf{d}_T^{(-)} \\ i\mathbf{d}_T^{(+)} \end{pmatrix} \right\}. \tag{B.8b}$$

$$\left\{ \begin{pmatrix} \delta \mathbf{g}_T^{(-)} \\ \delta \mathbf{g}_T^{(+)} \\ i\delta \mathbf{f}^{(-)} \times \hat{\mathbf{n}} \\ i\delta \mathbf{f}^{(+)} \times \hat{\mathbf{n}} \end{pmatrix}, \begin{pmatrix} \delta \boldsymbol{\varepsilon}_T^{(-)} \\ \delta \boldsymbol{\varepsilon}_T^{(+)} \\ i\mathbf{d}^{(-)} \times \hat{\mathbf{n}} \\ i\mathbf{d}^{(+)} \times \hat{\mathbf{n}} \end{pmatrix} \right\}. \tag{B.8c}$$

The above two sets of vectors give only the binormal and transverse information about $\delta\mathbf{g}$. It turns out the spin-singlet components δf and d are required to access the longitudinal information of $\delta\mathbf{g}$ using the combination below.

$$\left\{ \begin{pmatrix} \delta \mathbf{g}_L^{(+)} \\ \delta \mathbf{g}_L^{(-)} \\ \delta f^{(+)} \\ \delta f^{(-)} \end{pmatrix}, \begin{pmatrix} \delta \boldsymbol{\varepsilon}_L^{(+)} \\ \delta \boldsymbol{\varepsilon}_L^{(-)} \\ d^{(+)} \\ d^{(-)} \end{pmatrix} \right\}. \tag{B.8d}$$

These spin-singlet degrees of freedom δf and d are treated as external sources and turned off at the end of computation. After solving all above $|\widehat{g}\rangle$ in terms of $|\widehat{\sigma}\rangle$, we could again make use of (2.7a), (2.7b), (2.9a), and (2.9b) to obtain the following equations.

$$\begin{aligned} \delta\varepsilon^{(-)}(\hat{\mathbf{p}};\omega,\mathbf{q}) - \delta\varepsilon^{(-)}_{\text{ext}} = \int \frac{d\theta}{2\pi} A^s(\theta,\theta') \Big\{ \Big(1 + (1-\lambda(\hat{\mathbf{p}}')) \frac{\eta'^2}{\omega^2 - \eta'^2} \Big) \delta\varepsilon^{(-)}(\hat{\mathbf{p}}') \\ + \frac{\omega\eta'}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}') + \frac{1}{2}\eta'\bar{\lambda}(\hat{\mathbf{p}}')[\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\cdot\mathbf{d}(\hat{\mathbf{p}}') - \boldsymbol{\Delta}(\hat{\mathbf{p}}')\cdot\mathbf{d}'(\hat{\mathbf{p}}')] \Big\}. \end{aligned} \tag{B.9a}$$

$$\begin{aligned} \delta\varepsilon^{(+)}(\hat{\mathbf{p}};\omega,\mathbf{q}) - \delta\varepsilon^{(+)}_{\text{ext}} = \int \frac{d\theta'}{2\pi} A^s(\theta,\theta') \Big\{ \frac{\omega\eta'}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(-)}(\hat{\mathbf{p}}') \\ + \frac{\omega^2}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}') + \frac{1}{2}\omega\bar{\lambda}(\hat{\mathbf{p}}')[\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\cdot\mathbf{d}(\hat{\mathbf{p}}') - \boldsymbol{\Delta}(\hat{\mathbf{p}}')\cdot\mathbf{d}'(\hat{\mathbf{p}}')] \Big\}. \end{aligned} \tag{B.9b}$$

$$\begin{aligned} \delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} A^a(\theta,\theta') \Big\{ \Big(1 + (1-\lambda(\hat{\mathbf{p}}')) \frac{\eta'^2}{\omega^2 - \eta'^2} \Big) \delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}') \\ + \frac{\omega\eta'}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}') - \lambda(\hat{\mathbf{p}}')[\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}')\cdot\hat{\mathbf{n}}(\hat{\mathbf{p}}')]\hat{\mathbf{n}}(\hat{\mathbf{p}}') \\ - \frac{i\eta'\bar{\lambda}(\hat{\mathbf{p}}')}{2}[\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\times\mathbf{d}(\hat{\mathbf{p}}') + \boldsymbol{\Delta}(\hat{\mathbf{p}}')\times\mathbf{d}'(\hat{\mathbf{p}}')] \Big\}. \end{aligned} \tag{B.9c}$$

$$\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} A^a(\theta,\theta')\Big\{\frac{\omega^2}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}'}))\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}}')$$
$$+\frac{\eta'\omega}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}')+\lambda(\hat{\mathbf{p}}')[\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}}')\cdot\hat{\mathbf{n}}(\hat{\mathbf{p}}')]\hat{\mathbf{n}}(\hat{\mathbf{p}}')$$
$$-\frac{i\omega}{2}\bar{\lambda}(\hat{\mathbf{p}}')[\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\times\mathbf{d}(\hat{\mathbf{p}}')+\boldsymbol{\Delta}(\hat{\mathbf{p}}')\times\mathbf{d}'(\hat{\mathbf{p}}')]\Big\}. \tag{B.9d}$$

$$\mathbf{d}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\rm o}(\theta,\theta')\Big\{\Big(\gamma+\frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')(\omega^2-\eta'^2-2|\Delta(\hat{\mathbf{p}}')|^2)\Big)\mathbf{d}(\hat{\mathbf{p}}')$$
$$-\frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')\boldsymbol{\Delta}(\hat{\mathbf{p}}')[\eta'\delta\varepsilon^{(-)}(\hat{\mathbf{p}}')+\omega\delta\varepsilon^{(+)}(\hat{\mathbf{p}}')]$$
$$+\frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')i\boldsymbol{\Delta}(\hat{\mathbf{p}}')\times(\eta'\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}')+\omega\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}}'))$$
$$+\frac{1}{2}\bar{\lambda}(\hat{\mathbf{p}}')[(\boldsymbol{\Delta}(\hat{\mathbf{p}}')\cdot\boldsymbol{\Delta}(\hat{\mathbf{p}}'))\mathbf{d}'(\hat{\mathbf{p}}')-2(\boldsymbol{\Delta}(\hat{\mathbf{p}}')\cdot\mathbf{d}'(\hat{\mathbf{p}}'))\boldsymbol{\Delta}(\hat{\mathbf{p}}')]\Big\}. \tag{B.9e}$$

$$\mathbf{d}'(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\rm o}(\theta,\theta')\Big\{\Big(\gamma+\frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')(\omega^2-\eta'^2-2|\Delta(\hat{\mathbf{p}}')|^2)\Big)\mathbf{d}'(\hat{\mathbf{p}}')$$
$$+\frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')[\eta'\delta\varepsilon^{(-)}(\hat{\mathbf{p}}')+\omega\delta\varepsilon^{(+)}(\hat{\mathbf{p}}')]$$
$$+\frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')i\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\times(\eta'\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}')+\omega\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}}'))$$
$$+\frac{1}{2}\bar{\lambda}(\hat{\mathbf{p}}')[(\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\cdot\boldsymbol{\Delta}^*(\hat{\mathbf{p}}'))\mathbf{d}(\hat{\mathbf{p}}')-2(\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\cdot\mathbf{d}(\hat{\mathbf{p}}'))\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')]\Big\}. \tag{B.9f}$$

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