

THE UNIVERSITY OF CHICAGO

LIQUIDITY SHARING AND FINANCIAL CONTAGION

A DISSERTATION SUBMITTED TO  
THE FACULTY OF THE UNIVERSITY OF CHICAGO  
BOOTH SCHOOL OF BUSINESS  
IN CANDIDACY FOR THE DEGREE OF  
DOCTOR OF PHILOSOPHY

BY

JOHN GERARD FRANCIS NASH

CHICAGO, ILLINOIS

JUNE 2016

Copyright © 2016 by John Gerard Francis Nash

All rights reserved

*To Mum, Dad, Theresa, James, and Maddy*

# CONTENTS

LIST OF FIGURES . . . . .	vi
LIST OF TABLES . . . . .	vii
ACKNOWLEDGMENTS . . . . .	viii
ABSTRACT . . . . .	ix
1 INTRODUCTION . . . . .	1
1.1 Literature Review . . . . .	5
2 MODEL . . . . .	8
2.1 Investment Opportunities . . . . .	8
2.2 Lending Relationships . . . . .	11
2.3 Timing . . . . .	13
3 OPTIMIZATION PROBLEM AND BASELINE . . . . .	15
3.1 Autarky . . . . .	18
4 ECONOMY WITH PERFECT INFORMATION . . . . .	21
4.1 Lending Relationship Properties . . . . .	21
4.2 Equilibrium and Discussion . . . . .	26
5 ECONOMY WITH IMPERFECT INFORMATION . . . . .	29
5.1 Equilibrium and Discussion . . . . .	32
5.2 Contagion . . . . .	35
5.3 Commitment . . . . .	43
5.4 Social Planner . . . . .	47
6 POLICY IMPLICATIONS . . . . .	52
7 CONCLUSION . . . . .	55
A PROOFS OF PROPOSITIONS . . . . .	57
A.1 Proof of Proposition 1 . . . . .	57
A.2 Proof of Proposition 2 . . . . .	58
A.3 Proof of Proposition 3 . . . . .	60
A.4 Proof of Proposition 4 . . . . .	62
A.5 Proof of Proposition 5 . . . . .	63
A.6 Proof of Proposition 6 . . . . .	68
A.7 Proof of Proposition 7 . . . . .	70

A.8	Proof of Proposition 8 . . . . .	72
A.9	Proof of Proposition 9 . . . . .	80
B	FUNCTIONAL FORMS . . . . .	84
B.1	Autarky . . . . .	84
B.2	Lending Relationships . . . . .	86
B.3	Contagion . . . . .	96
	REFERENCES . . . . .	100

## LIST OF FIGURES

2.1	Timeline . . . . .	13
5.1	Pooling and Separating Equilibria . . . . .	33
5.2	Expected Contagion . . . . .	38
5.3	Probability of $k$ Contagious Defaults . . . . .	42

## **LIST OF TABLES**

2.1	Key Parameter List . . . . .	14
-----	------------------------------	----

## ACKNOWLEDGMENTS

I am incredibly grateful to my advisor Douglas Diamond, a constant source of encouragement, feedback, and ideas. Thank you for always having time for me, and for constantly challenging and stimulating me intellectually. I thank my committee, Zhiguo He, Amit Seru, Stavros Panageas, and Lin William Cong for their invaluable time and input. This dissertation would not be possible without all of your support.

I thank Jonathan Adams, Chanont Banterghansa, Philip Barrett, Jorn Boehnke, Bruce Carlin, Mark Egan, Kinda Hachem, Yunzhi Hu, Nitish Kumar, Chattrin Laksanabunsong, Chenfei Lu, Gregor Matvos, and Wei Wu, as well as seminar participants at Chicago Booth for their thoughtful comments and feedback.

My time in Chicago has been one of the richest and most enjoyable experiences of my life. To my amazing friends, in particular, Adelle Yang, Ippei Shibata, and Philip Merwood, you guys are the best. To my awesome family, Mum, Dad, Theresa, James, and Maddy, thanks for your unwavering support and encouragement throughout. Finally, I would like to give special mention to Nana, who passed away one week prior to my departure for Chicago. You were an inspiration throughout.



## **ABSTRACT**

This dissertation studies multi-lateral incentive provision and contagion in the financial system. I develop a model of the financial system with liquidity shocks, moral hazard, and asymmetric information. Banks share liquidity by forming lending relationships with and without commitment. The decision to make lending relationships committed or uncommitted involves a trade-off between liquidity provision and moral hazard. I highlight how uncommitted lending relationships, such as the credit lines between banks in the federal funds market, expose banks to potential liquidity shortages. However, liquidity shortages, which induce early default, can also align banks' screening incentives. If banks collectively use uncommitted lending relationships, incentive alignment can be multi-lateral. Liquidity shortages can exacerbate or ameliorate contagion, depending on information quality. These effects result from informed banks exerting externalities on uninformed banks. The model suggests the “tightness” of connections in the financial system should be an active consideration for policies targeting systemic risk-mitigation.

# CHAPTER 1

## INTRODUCTION

The recent financial crisis has, once again, underlined the importance of a functioning financial system to a healthy economy. At the core of the financial system, is the prevalent use of debt. Although a large body of research exists on how debt contracts can provide incentives, reallocate funds, and hedge risks, the literature focuses on contracts written in a bilateral setting.<sup>1</sup> However, the financial system is itself a multi-lateral environment consisting of a collection of bilateral contracts. This paper argues that a full understanding of debt contracts within the financial system, and thus their importance to the economy, requires a careful examination of the characteristics of debt in a multi-lateral environment.

I develop a model of the financial system with liquidity shocks, moral hazard, and asymmetric information. In the model, banks allocate their funds between a liquid asset that is held as insurance against liquidity shocks, and a profitable but risky collection of illiquid assets. At the same time, banks decide whether to reduce the riskiness of their illiquid assets through costly screening. In the absence of a liquidity-sharing mechanism, I assume banks elect to self-insure against liquidity shocks and forgo screening. However, because liquidity shocks hit only a subset of banks, and bank liabilities are fixed ex-ante, both self-insurance and a lack of screening are socially suboptimal.<sup>2</sup>

To allow these decisions to become relevant, and to study them in a setting that mimics the real world, I allow banks to form bilateral lending relationships. A prominent example of bilateral lending relationships within the financial sector is the direct credit lines

---

1. For example, see Calomiris and Kahn (1991), Flannery (1994), and Holmstrom and Tirole (1998).

2. Self-insurance by all banks is socially suboptimal because it implies excess liquidity in the economy. A standard assumption in the literature is that screening is socially optimal.

between banks within the federal funds market. Banks with liquidity shortages regularly access these credit lines to meet their overnight liquidity needs. Similar to the credit lines in the federal funds market, lending relationships facilitate liquidity sharing. In equilibrium, banks hit by liquidity shocks borrow the required liquidity from banks with liquidity surpluses. Consequently, banks are able to ex-ante tilt their portfolios towards the relatively profitable illiquid assets.

I consider committed and uncommitted lending relationships. Commitment is a key component of any lending relationship.<sup>3</sup> The decision to make lending relationships committed or uncommitted involves a trade-off between liquidity provision and moral hazard. Banks with uncommitted lending relationships can face liquidity shortages if a sufficient number of lenders refuse to provide liquidity, especially if lenders perceive the borrower as particularly risky. Liquidity shortages are costly because they lead to the premature default of banks and the inefficient liquidation of bank assets. However, because liquidity shortages are more common for risky banks, and a failure to screen induces additional risk, uncommitted lending relationships can provide banks with incentives to screen, and thus allay moral hazard. Moreover, because ex-ante, banks do not know which among them will require liquidity, the collective use of uncommitted lending relationships can provide incentives in a multi-lateral manner. That is, each bank is both assisting in incentive provision to all other banks, and simultaneously experiencing the alignment of its own incentives by other banks within the financial sector. By contrast, committed lending relationships ensure liquidity provision, but exacerbate moral hazard.

I then study the consequences of inter-linkages that arise from lending relationships in such a multi-lateral environment. Lending relationships in such an environment provide a channel through which individual bank distress can spread across the financial system. That

---

3. Empirical evidence suggests the majority of lending relationships lack full commitment. For example, Sufi (2009) shows that almost all syndicated loans contain material adverse change clauses.

is, the formation of lending relationships opens up banks to the possibility of contagion. I define a contagious default to be lender default induced by borrower default. I define information quality to be the probability that a lender becomes informed about a risky borrower. I show that the amount of contagion depends on both the choice of lending relationships, and information quality. In particular, for uncommitted lending relationships, the relationship between information quality and contagion is non-monotone, with intermediate levels of information causing the greatest amount of contagion.

The intuition for the non-monotonicity is as follows. If information quality is high, a higher fraction of lenders are likely to be informed, and thus fewer uninformed lenders exist to provide liquidity to the borrower. On the one hand, the presence of fewer uninformed lenders increases the probability that the borrower will experience a liquidity shortage and thus a premature default. Premature default by a borrower is costly to the uninformed lenders because it wipes out any possibility of repayment, and thus increases their own probability of default. On the other hand, the presence of fewer lenders implies fewer banks that become connected to the borrower, and thus fewer banks that can default contagiously. Taken together, contagion is maximized for intermediate levels of information quality.

I highlight two important policy implications. First, policy makers must understand that commitment and liquidity provision in the interbank market are intertwined with incentive provision. Thus, any policy aimed at either restricting or encouraging bank-lending activity in interbank markets, or alternatively, targeted to induce (or relax) commitment, must take into account the effect it will have on banks' screening incentives. For example, a policy structured to reduce contagion by facilitating the provision of liquidity to a distressed bank, is likely to reduce bank incentives to screen, and thus increase the probability of future bank distress. Note that in principle, such a policy does not necessarily constitute a bailout. Rather, organizing liquidity provision to a distressed bank is akin to the action taken by the Federal Reserve in 1998 when the hedge fund giant Long-Term Capital Man-

agement (LTCM) was near collapse. In the case of LTCM, the intervention of the Federal Reserve, which induced lending, was arguably collectively beneficial to LTCM's lenders. Therefore, perhaps unsurprisingly, many of the banks that participated in the LTCM liquidity negotiations found themselves in a similar situation merely 10 years later.<sup>4</sup>

Second, from a network perspective, recent work has emphasized the importance of both the size of financial interdependencies and the density of the financial network as key determinants of systemic risk in the economy. To the extent that the information environment encapsulates the amount of liquidity provision, it also reflects the "tightness" of connections within the financial system. A rich information environment gives rise to connections that break apart frequently when borrowers become distressed, whereas conversely, a poor information environment is consistent with connections that are less likely to rupture. Taken together, this result implies contagion is a function of how tight connections are, and how easily the financial system breaks apart. As such, any policy designed to mitigate systemic risk generated within the financial system ex-ante should take into account both the current and future "tightness" of financial connections, instead of only the size and density of the connections the financial network comprises.

Finally, the model assumes no pre-existing connections within the financial system. Alternative connections, such as long-term debt and cross-holdings of equity, introduce additional channels through which distress may spread. Moreover, because I do not allow banks to earn rents from liquidity provision, the economy lacks precautionary liquidity. As such, incentive provision, which relies on the combination of liquidity shortages at distressed banks and the willingness of lending banks to allow distressed banks to fail, may be diluted. However, I also focus on a symmetric equilibrium, and thus, if the model is

---

4. In fact, of the banks at the negotiating table with the Federal Reserve, only Lehman Brothers and Bear Stearns refused to provide liquidity. See Lowenstein (2000).

extended to incorporate some asymmetry, either in information arrival or bank connections, incentives may in fact be enhanced. I leave these questions to future research.

## 1.1 Literature Review

This paper is closely related to multiple strands of economic literature. First, as a model of multi-lateral incentive provision within the banking sector, the paper has ties to the literature on the incentive provision through debt. In early work, Calomiris and Kahn (1991) illustrate the role of demandable (short-term) debt in aligning bank-risk taking incentives through forced liquidation. I build on Calomiris and Kahn by considering the problem in a multi-lateral setting where banks simultaneously provide and receive incentive alignment. I argue the mechanism through which incentives are aligned, liquidity sharing without commitment, is a double-edged sword in that it can exacerbate contagion ex-post. In related work, Rajan and Winton (1995) emphasize the role of covenants in incentivizing banks to monitor loan contracts. I deviate from Rajan and Winton by focusing on the importance of a lack of commitment in providing incentives for potential borrowers. That is, whereas Rajan and Winton argues covenants are necessary for inducing monitoring by lenders, I argue that a lack of commitment, similar to the material adverse change clause covenants, is crucial for the borrower's screening incentives.

More generally, my paper is related to the vast literature on the role of banks as intermediaries and the optimality of debt.<sup>5</sup> I contribute to this literature by modeling the interbank liquidity-provision role of banks in the presence of moral hazard, and the cor-

---

5. Seminal papers in this literature include Diamond (1984), Rochet and Tirole (1996), Kiyotaki and Moore (1997), Bolton and Scharfstein (1996), Diamond and Rajan (2005), Hart and Moore (1994), Flannery (1994), and Calomiris and Kahn (1991).

responding implications for liquidity sharing, incentives, the distribution of bank defaults, and contagion.

Second, my paper is related to the growing literature on network failures, contagion, and systemic risk in financial markets.<sup>6</sup> In their seminal work, Allen and Gale (2000) showed how regional shocks can propagate through an interbank network to produce aggregate fragility. Subsequent work by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015a, 2015b) characterizes the relationship between the architecture of the financial system and the likelihood of systemic failures due to contagion. However, to my knowledge, this literature does not allow for ex-post responses from financial institutions to shocks to members of the network. Although my model is not a network model in the traditional sense, I add to this literature by emphasizing the importance of looking beyond current characterizations of the financial system architecture to consider the types of connections within the financial system. In particular, my paper highlights how the tightness of financial connections is a key determinant of systemic risk, because tightness influences both the avenues through which financial distress can spread, as well as the level of distress at a given source.

Finally, my paper has ties to the literature on credit lines. From a theoretical perspective, Homlstrom and Tirole (1998) examine the role of committed credit lines in providing liquidity to the real sector. Empirically, the work of Sufi (2009) has documented the prevalence of material adverse change clauses in syndicated loans.

Section 2 outlines the key features of the model, and section 3 describes the optimization problem of banks and the economy under autarky. Section 4 presents a simple economy with perfect information in order to explain the intuition behind some of the key forces of the model. Section 5 describes the equilibrium and provides the results for the general

---

6. See Allen and Gale (2000), Freixas, Parigi, and Rochet (2000), Cifuentes, Ferucci, and Shin (2005), Allen, Babus, and Carletti (2012), Zawadowski (2013), Caballero and Simsek (2013), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015a, 2015b), Garleanu, Panageas, and Yu (2015), and Elliot, Golub, and Jackson (2014).

model. Section 6 discusses some applications and some potential extensions, and section 7 concludes.



## CHAPTER 2

### MODEL

The model has three periods,  $t = \{0, 1, 2\}$ , and  $N$  risk-neutral banks indexed by  $i = \{1, 2, \dots, N\}$ . Each bank maximizes its expected profits and has a charter value  $V_i = V$  that is lost if it goes bankrupt. The financial system consists of banks, their investments, and a set of bilateral exposures that represent lending and borrowing relationships between banks.

Each bank is endowed at  $t = 0$  with one unit of funds from a mass of depositors,  $D = 1$ .<sup>1</sup> Bank funding is subject to liquidity shocks, or the early withdrawal of funds by depositors. At  $t = 1$ , up to one random bank receives a liquidity shock that requires it to raise  $\phi$  units of liquidity. If a bank fails to raise  $\phi$  units of liquidity, it defaults and loses its charter value  $V$ . A bank that raises the required  $\phi$  at  $t = 1$ , survives until  $t = 2$  and has remaining deposits of  $D - \phi$ .

### 2.1 Investment Opportunities

Each bank has two investment decisions to make at  $t = 0$ . First, banks must decide how to allocate their funds between two potential investment opportunities that I will call “liquid” and “illiquid” assets. Liquid assets can be thought of as securities that can be easily traded for cash, such as government bonds, whereas illiquid assets represent more complex and long term investments with an uncertain return, such as commercial and industrial

---

1. I do not model depositors, and thus  $D$  provides a default boundary in the absence of interbank lending.

loans. Let  $\alpha_i$  denote the fraction of liquid assets bank  $i$  chooses in its portfolio, and thus  $1 - \alpha_i$  is the fraction of illiquid assets bank  $i$  holds in its portfolio.

Second, providing  $\alpha_i < 1$ , banks must also decide whether to screen their illiquid assets at  $t = 0$ . The decision to screen can be thought of as a tradeoff between the risk of the investments and the private benefits captured by the bank shareholders. Let the binary variable  $e_i = \{0, 1\}$  denote bank  $i$ 's screening decision, with  $e_i = 1$  indicating that bank  $i$  opts to screen its illiquid assets.

Liquid assets are the numeraire. They have a per-unit cost of 1 at  $t = \{0, 1\}$ , provide a return of 1 at  $t = \{1, 2\}$ , and their primary purpose is to deal with liquidity shocks. Illiquid assets also have a per-unit cost of 1 at  $t = 0$  but provide an uncertain return to bank  $i$  of  $\tilde{R}_i$  at  $t = 2$ . Returns are independent and identically distributed across banks. I denote the probability density function (pdf) for illiquid asset returns at  $t = 0$  by  $g(\tilde{R}_i)$  and assume illiquid assets are more profitable investments than liquid assets ( $\mathbb{E}(\tilde{R}_i) > 1$ ). Illiquid assets are perfectly illiquid at  $t = 1$ , and thus premature liquidation returns zero funds.

Illiquid assets are also subject to solvency shocks. Throughout, I will refer to banks that have received solvency shocks as “distressed” and banks that have not as “healthy.” Let  $l_i(s_i)$  be the indicator variable that is equal to 1 if bank  $i$  receives a liquidity (solvency) shock and zero otherwise. At  $t = 1$ , bank  $i$ 's illiquid assets receive a solvency shock with probability  $p_s(e_i, l_i)$ . I denote  $g_1(\tilde{R}_i) = g(\tilde{R}_i | s_i = 1)$  as the conditional pdf of bank  $i$ 's illiquid asset returns if bank  $i$  is distressed, and  $g_0(\tilde{R}_i) = g(\tilde{R}_i | s_i = 0)$  as the conditional pdf of bank  $i$ 's illiquid asset returns if bank  $i$  is healthy.<sup>2</sup> I assume  $g_0(\tilde{R}_i) \sim \text{Unif}[\underline{R}, \bar{R}]$ , with

---

2. I do not model the underlying reason for the occurrence of solvency shocks, because I am mostly interested in what happens following the solvency shock. The economy contains many potential candidates for a micro-foundation. For example, a negative shock to house prices for a bank that is heavily invested in real estate would cause a significant reduction in the value of the bank's portfolio of assets.

$\underline{R} = 1$ ,<sup>3</sup> and  $g_0(R_i - \Delta) = g_1(R_i)$ . That is, the illiquid assets of a distressed bank have a future returns distribution that is a leftward shift of the healthy banks' illiquid assets future returns distribution. Thus, we can view  $g(\tilde{R}_i)$  as a mixture of the distributions  $g_1(\tilde{R}_i)$  and  $g_0(\tilde{R}_i)$  such that:

$$g(\tilde{R}_i) = \begin{cases} g_0(\tilde{R}_i) \sim \text{Unif}[\underline{R}, \bar{R}] & \text{with probability } 1 - p_s(e_i, l_i) \\ g_1(\tilde{R}_i) \sim \text{Unif}[\underline{R} - \Delta, \bar{R} - \Delta] & \text{with probability } p_s(e_i, l_i) \end{cases}$$

Bank  $i$ 's screening choice is unobservable and unverifiable, and presents bank  $i$  with a trade-off between private benefits and risk. Whereas screening reduces the probability bank  $i$  receives a solvency shock, opting not to screen provides bank  $i$  with private benefits that creditors cannot appropriate.<sup>4</sup>

More formally, I assume the probability bank  $i$  is hit by a solvency shock  $p_s(e_i, l_i)$  takes the form  $p_s(e_i, l_i) = p_s^{e_i} \times l_i$ , where  $p_s^{e_i} = \{p_s^0, p_s^1\}$ . In words, if bank  $i$  does not receive a liquidity shock ( $l_i = 0$ ), it cannot receive a solvency shock. However, if bank  $i$  is hit by a liquidity shock ( $l_i = 1$ ), it has probability  $p_s^{e_i}$  of also receiving a solvency shock.<sup>5</sup> Because screening reduces bank  $i$ 's probability of distress, it follows that  $0 < p_s^1 < p_s^0$ . I define  $P(\alpha_i, e_i) = (1 - \alpha_i)P^{e_i}$  to be the private benefits bank  $i$  receives given liquid assets

---

3.  $\underline{R} = 1$  implies that banks in autarky with  $\alpha_i \geq \phi$  require a solvency shock to default.

4. In this sense, screening can be thought of as bank  $i$  doing its due diligence, which reduces the probability of investing in a set of bad illiquid assets.

5. Defining the probability of a solvency shock in this way allows me to focus on the response of lending banks to solvency shocks that hit borrowing banks. An interesting extension of the model is to allow solvency shocks to hit banks that did not receive a liquidity shock. This extension introduces the possibility of lender-side distress. I discuss this possibility in section 6.

$\alpha_i$  and screening choice  $e_i$ , where  $P^{e_i} = \{P^0, P^1\}$ . I assume  $P^0 > P^1$ , and without loss of generality set  $P^0 = P$  and  $P^1 = 0$ .

Finally, I assume that screening is socially optimal. That is, the private benefits accruing from not screening are less than the increase in the sum of the expected losses from solvency shocks and the additional expected default costs. Mathematically, social optimality is equivalent to  $P < p_l \left( p_s^0 - p_s^1 \right) \frac{\Delta}{\bar{R} - \underline{R}} \left( \bar{R} - \underline{R} + \frac{1}{(1-\phi)} V \right)$ .<sup>6</sup>

## 2.2 Lending Relationships

Banks also face a decision over their lending relationships at  $t = 0$ . Lending relationships are in essence agreements to share liquidity at  $t = 1$ , and can be thought of either as lines of credit with material adverse change clauses or implicit promises between banks. A natural example of these lending relationships in the financial sector is the direct lines of credit between banks that exist in the fed funds market alongside the deposits kept at the federal reserve.

I allow each bank  $i$  to form lending relationships with other banks  $j$  at  $t = 0$ . Lending relationships are costless to form,<sup>7</sup> and are characterized by a borrower  $i$ , a lender  $j$ , and an amount of liquidity  $B_{ij}$  that the lender has the option to provide upon the borrower's request. I assume all lending relationships are implemented using standard debt contracts. Repayments occur at  $t = 2$  and are junior to the payments made to depositors.

---

6. This condition is taken under autarky with  $\alpha_i = \phi$ .

7. One possible extension is to introduce a cost to forming lending relationships. This cost would cause banks to trade-off over the number of relationships with the amount of liquidity held in equilibrium but would not change the main results of the model.

Consistent with empirical evidence, I assume lending relationships lack commitment. For example, Sufi (2009) shows the majority of syndicated loans contain material adverse change clauses that enable lenders to refuse funding when a borrower becomes distressed.<sup>8</sup> Later, I will relax this assumption and show banks can endogenously opt for a lack of commitment in their lending relationships. Lending relationships without commitment imply potential lenders cannot be compelled to provide liquidity. That is, although a relationship is formed at  $t = 0$ , the lender is not required to honor its promise at  $t = 1$ . As such, liquidity provision at  $t = 1$  depends on the borrower's ability to supply the lender with a normal rate of return.<sup>9</sup> To avoid any hold-up problems potentially associated with a lack of commitment, I assume borrowers make take-it-or-leave-it offers to lenders.

Lending relationships can provide the lender with information about the borrower. If bank  $i$  forms a lending relationship with bank  $j$  at  $t = 0$ , I assume bank  $j$  becomes informed about bank  $i$ 's status (healthy or distressed) with probability  $\theta$ . Conversely, with probability  $1 - \theta$ , bank  $j$  will remain uninformed. Importantly, I assume learning (becoming informed) is independent across banks. Thus, imperfect information creates the possibility of a heterogeneously informed group of potential lenders at  $t = 1$ . To prevent the uninformed from learning from the informed, I assume information about the borrower is private and unverifiable, and that liquidity provision from lender  $j$  to borrower  $i$  is unobservable to lender  $k$ .

Finally, I place two restrictions on borrower behavior at  $t = 1$  to ensure tractability. First, I assume borrowers always use the same set of lending relationships when borrowing,

---

8. Moreover, within the financial sector, few contracts entail full commitment. Perhaps the best examples are letters of credit or loan guarantees.

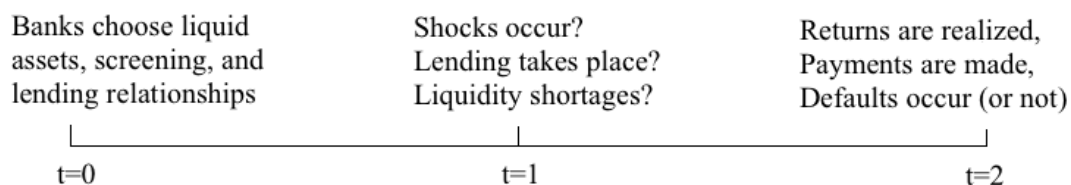
9. The normal rate of return is with respect to the lender's information set.

irrespective of whether they are distressed.<sup>10</sup> Second, I assume banks can only acquire liquidity through a direct lending relationship. As such, if bank  $i$  and bank  $j$  do not have a lending relationship at  $t = 0$ , bank  $i$  cannot borrow from bank  $j$  at  $t = 1$ .<sup>11</sup>

## 2.3 Timing

The timeline below summarizes the sequence of events in the model:

**Figure 2.1.** Timeline



At  $t = 0$ , each bank makes a portfolio allocation decision, a screening decision, and a decision over its lending relationships. At  $t = 1$ , banks are subject to liquidity and solvency shocks. A bank hit by a liquidity shock can use its lending relationships to help cover  $\phi$ , whereas unaffected banks can decide whether to lend. A liquidity-shocked bank that is unable to raise  $\phi$  defaults immediately and loses its charter value  $V$ . Finally, at  $t = 2$ , returns for the illiquid asset are realized, banks return money to their depositors, pay off any interbank debt they owe, and return remaining funds to equity. Any bank that is unable to meet its liabilities, defaults, and loses its charter value  $V$ .

---

10. In practice, a distressed borrower concerned about its ability to borrow from informed lenders may want to attempt to borrow from more relationships than a healthy borrower. However, such a situation will not be of concern in the equilibria on which I focus.

11. I will revisit this assumption in section 5.3.

Finally, for reference, below is a summary of some of the key notation used throughout.

Table 2.1: Key Parameter List

Variable	Parameter	Variable	Parameter
Liquid Asset Choice	$\alpha_i$	Solvency Shock Size	$\Delta$
Screening Choice	$e_i$	Bank Charter Value	$V$
Lending Relationship Choice	$B_{ij}$	Bank Private Benefits	$P$
Illiquid Asset Returns	$R_i$	Information Quality	$\theta$
Liquidity Shock Size	$\phi$	Pr. Liquidity (Solvency) Shock	$p_l$ ( $p_s$ )

I now move to the banks' optimization problem and a discussion of the baseline autarky economy.

## CHAPTER 3

### OPTIMIZATION PROBLEM AND BASELINE

Define the functions  $\Pi_i(\alpha_i, e_i, B_{ij}, l_i, s_i, \zeta_i, R_i)$ ,  $\mathcal{A}_i(\alpha_i, e_i, B_{ij}, l_i, s_i, \zeta_i, R_i)$ , and  $\mathcal{L}_i(\alpha_i, e_i, B_{ij}, l_i, s_i, \zeta_i, R_i)$  to be bank  $i$ 's  $t = 2$  profits, assets, and liabilities, respectively. Also define the function  $\zeta_i(\alpha_i, e_i, B_{ij}, l_i, s_i)$  to be the state-contingent fraction of bank  $i$ 's illiquid assets that remain following any liquidation at  $t = 1$ , and the function  $D_i(\alpha_i, e_i, B_{ij}, l_i, s_i, \zeta_i, R_i)$  to be the payment made by bank  $i$  to its depositors at  $t = 2$ . Finally, define the functions  $F_{ij}(\alpha_i, e_i, B_{ij}, l_i, s_i, \zeta_i, R_i)$  and  $r_{ij}(\alpha_i, e_i, B_{ij}, l_i, s_i, \zeta_i, R_i)$  to be the payment on interbank debt from bank  $i$  to bank  $j$ , and the associated interest rate, respectively. To save on notation, I will hereafter take the brackets to be implicit and shorten the respective functions to  $\Pi_i$ ,  $\mathcal{A}_i$ ,  $\mathcal{L}_i$ ,  $\zeta_i$ ,  $D_i$ ,  $F_{ij}$ , and  $r_{ij}$ . Note that as of  $t = 0$ , each of these functions is a random variable, and as of  $t = 1$ , all functions except  $\zeta_i$  remain random variables.

First, consider the function  $\zeta_i$ . If bank  $i$  receives a liquidity shock and cannot raise sufficient funds from its own liquid assets and/or the interbank market, it will have to liquidate its entire portfolio of illiquid assets. By contrast, if bank  $i$  can raise sufficient funds, no liquidation will be required. Thus, we have:

$$\zeta_i = \mathbf{1}_{\alpha_i + B_i \geq l_i \phi}$$

Given all decisions made prior to  $t = 2$ , and any realization of  $\{R_i\}$ , we can write bank  $i$ 's assets at  $t = 2$  as the sum of (i) liquid asset holdings, (ii) illiquid asset holdings, and (iii) interbank debt payments received. If bank  $i$  received a liquidity shock at  $t = 1$ , we must



also subtract payments made to the  $\phi$  depositors that withdrew early:

$$\mathcal{A}_i = \alpha_i + (1 - \alpha_i)\zeta_i R_i + \sum_j B_{ij} + \sum_j \bar{F}_{ji} - l_i \phi$$

Where  $\bar{F}_{ij}$  is bank  $i$ 's maximum repayment to bank  $j$  at  $t = 2$ . In a similar fashion, liabilities can be written as the sum of payments owed to depositors  $D - l_i \phi$  and other banks  $\sum_j \bar{F}_{ij}$ :

$$\mathcal{L}_i = D - l_i \phi + \sum_j \bar{F}_{ij}$$

Depositors are senior to interbank debt at  $t = 2$  and thus it follows that:

$$D_i = \min(\mathcal{A}_i, D - l_i \phi)$$

Bank  $i$ 's interbank debt is junior to its depositors, and I assume all banks  $j = \{1, 2, \dots, n\}$  from which bank  $i$  borrows are of equal seniority. If we define  $\bar{F}_i = \sum_j \bar{F}_{ij}$ ,<sup>1</sup> bank  $i$  pays its counter-parties a total of<sup>2</sup>

$$F_i = \max(\min(\mathcal{A}_i - D_i, \bar{F}_i), 0)$$

All lending relationships require lenders to make a normal rate of return. That is, lenders require repayment for both the principal they lend as well as the expected default costs incurred from lending. Define the function  $\hat{\pi}_{d,ij}(\alpha_i, e_i, B_{ij}, l_i, s_i, \zeta_i, R_i)$  to be

---

1. Note that the mapping between  $\bar{F}_i$  and the average interest rate  $r_i$  is  $\bar{F}_i = B_i(1 + r_i)$ , where  $B_i = \sum_j B_{ij}$ .

2. I make the simplifying assumption that when bank  $i$  is only able to partially repay the principal it borrows from its counter-parties, all repayments to counter parties are proportionate to each counterparty's share of the total principal borrowed. For a technical discussion of this assumption, see Appendix.

the probability of default for bank  $j$  resulting from lending to bank  $i$ ,<sup>3</sup> which I denote as  $\hat{\pi}_{d,ij}$  for short. Let  $\mathcal{I}_j$  be the information set of potential lender  $j$ . The lender's participation constraint at  $t = \{0, 1\}$  requires:

$$\mathbb{E}(\tilde{F}_{ij}|\mathcal{I}_j) \geq B_{ij} + \hat{\pi}_{d,ij}V$$

Borrowers also face a second constraint whereby they cannot borrow more liquid assets than the lender possesses in its portfolio:

$$B_{ij} \leq \alpha_j$$

We can write  $\Pi_i$  as the difference between bank  $i$ 's Assets,  $\mathcal{A}_i$ , and Liabilities,  $\mathcal{L}_i$ . We must also bound this difference at zero to reflect limited liability:

$$\begin{aligned} \Pi_i &= \max(\mathcal{A}_i - \mathcal{L}_i, 0) \\ &= \max(\alpha_i + (1 - \alpha_i)\zeta_i R_i + \sum_j B_{ij} + \sum_j F_{ji} - (D - l_i\phi) - \sum_j F_{ij}, 0) \end{aligned}$$

Moreover, let  $d_i$  be the indicator function that is equal to 1 if bank  $i$  defaults, and zero otherwise. Default occurs either when banks' fail to meet their liquidity shock at  $t = 1$  or when banks' liabilities are greater than their assets at  $t = 2$ , and in this case, a bank's profits are zero because of limited liability. With this notation, bank  $i$ 's optimization problem can

---

3. Bank  $j$  would have zero probability of default otherwise, so  $\hat{\pi}_{d,ij}$  is in fact the marginal default probability.

be written as:

$$\begin{aligned}
\max_{\alpha_i, e_i, B_{ij}} \Lambda_i(\alpha_i, \varepsilon_i, B_{ij}) &= \mathbb{E}(\Pi_i) + (1 - \mathbb{E}(d_i))V + (1 - \alpha_i)P_i^e \\
s.t. \quad \mathbb{E}(\tilde{F}_{ij} | \mathcal{I}_j) &\geq B_{ij} + \hat{\pi}_{d,ij}V \\
\alpha_j &\geq B_{ij}
\end{aligned}$$

Here, the expectations are taken over both the occurrence of liquidity shocks,  $l_i$ , and solvency shocks,  $s_i$  (at  $t = 1$ ), the realization of the illiquid asset returns,  $R_i$  (at  $t = 2$ ), and in the case of profits, lender behavior at  $t = 1$ . Then, because  $V$  is a constant, we can rewrite the expectation of the bank-default indicator function as a probability function for bank  $i$ 's default. Define the function  $\pi_{d,i}(\alpha_i, e_i, B_{ij}, l_i, s_i, \zeta_i, R_i)$  as bank  $i$ 's probability of default. Again, to save on notation, I will often drop the brackets and simply write  $\pi_{d,i}$ . Thus, we can rewrite the bank's objective function as:

$$\max_{\alpha_i, e_i, B_{ij}} \Lambda_i(\alpha_i, e_i, B_{ij}) = \mathbb{E}(\Pi_i) + (1 - \pi_{d,i})V + (1 - \alpha_i)P_i^e$$

Subject to the same set of constraints.

### 3.1 Autarky

Before continuing to the general economy, a useful baseline for the full model is an autarky economy in which lending between banks is prohibited. To this end, consider such an economy with  $N$  banks  $i = \{1, 2, \dots, N\}$  that are unable to lend to each other. We can write the profits of bank  $i$   $\Pi_i$  as the difference between its assets  $\mathcal{A}_i$  and liabilities  $\mathcal{L}_i$ :

$$\Pi_i = \max(\mathcal{A}_i - \mathcal{L}_i, 0) = \max(\alpha_i + (1 - \alpha_i)\zeta_i R_i - (D - l_i\phi), 0)$$

Note that no  $F_{ij}$  are here, because the interbank lending channel has been shut down. In similar fashion, we can proceed to write bank  $i$ 's objective function as:

$$\max_{\alpha_i, e_i} \Lambda_i(\alpha_i, e_i) = \mathbb{E}(\Pi_i) + (1 - \alpha_i)P_i^e + (1 - \pi_{d,i})V$$

Constraints are absent because banks are unable to form lending relationships. To ensure liquid and illiquid assets both play a role in the economy, I constrain the parameter space such that a mix of the two is optimal under autarky.

Moreover, to (later) highlight how lending relationships can align incentives in a multi-lateral setting, I also constrain the parameter space such that the social and private incentives of banks are misaligned under autarky. Because screening is socially optimal, any condition that provides incentives for banks to opt not to screen must work by giving banks the ability to push onto their depositors the losses from not screening. The following proposition provides a set of sufficient conditions:

**Proposition 1.** *If  $(p_l, p_s, \Delta, V, \phi, \bar{R}, P)$  satisfy the following three conditions, then all banks  $i$  choose  $\alpha_i = \phi$ ,  $e_i = 0$ , and obtain a realized value of their objective function  $\Lambda_0$ :*

$$\begin{aligned} (p_l - \phi) \frac{\bar{R} - \underline{R}}{2} - p_l p_s (1 - \phi) \Delta \left( 1 - \frac{\Delta/2}{\bar{R} - \underline{R}} \right) + p_l \left( 1 - p_s \frac{\Delta}{\bar{R} - \underline{R}} \right) V &\geq \phi P^e \\ \frac{\Delta p_l p_s}{\bar{R} - \underline{R}} \frac{V}{1 - \phi} - \frac{p_l p_s}{\bar{R} - \underline{R}} \frac{1}{2} (\bar{R} - \underline{R} - \Delta)^2 - (1 - p_l p_s) (\bar{R} - \underline{R}) \frac{1}{2} &\leq P^e \\ p_l (p_s^0 - p_s^1) \frac{\Delta}{\bar{R} - \underline{R}} \left( (\bar{R} - \underline{R} - \frac{1}{2} \Delta) + \frac{V}{1 - \phi} \right) &< P \end{aligned}$$

*Proof.* See Appendix. □

Proposition 1.1 and 1.2 ensure  $\alpha = \phi$ , and imply neither liquid assets ( $\alpha = 0$ ) nor illiquid assets ( $\alpha = 1$ ) are redundant in the autarky economy, whereas under Proposition 1.3, individual banks will opt not to screen their illiquid assets when lending relationships are prohibited.

Importantly, because all banks are identical, Proposition 1 applies to all banks, and thus implies aggregate liquidity in the  $N$  bank economy without liquidity sharing equals  $N\phi$ . Restricting banks such that they cannot lend to each other can be inefficient. Recall that at most one liquidity shock hits the economy, and thus the total liquidity needs of the economy are merely  $\phi$ . Therefore, if liquidity cannot be shared between any banks  $i$  and  $j$ , a minimum of  $(N - 1)\phi$  units of liquidity will remain unused at  $t = 1$ . However, if banks could share liquidity, each bank could potentially reduce its holdings of the liquid asset, shift investment to the more profitable illiquid asset, and still have sufficient funds to cover potential liquidity shocks.

## CHAPTER 4

### ECONOMY WITH PERFECT INFORMATION

To illustrate some of the key features of liquidity sharing in my model, starting with a simplified economy with perfect information ( $\theta = 1$ ) is useful. That is, if bank  $i$  and bank  $j$  formed a lending relationship at  $t = 0$ , the latter can perfectly observe at  $t = 1$  whether the former is distressed or healthy.

Recall bank  $i$ 's optimization problem is:<sup>1</sup>

$$\begin{aligned} \max_{\alpha_i, e_i, B_{ij}} \quad & \Lambda_i(\alpha_i, e_i, B_{ij}) = \mathbb{E}(\Pi_i) + (1 - \alpha_i)P^e + (1 - \pi_{d,i})V \\ \text{s.t.} \quad & \mathbb{E}(\tilde{F}_{ij} | \mathcal{I}_j) \geq B_{ij} + \hat{\pi}_{d,ij}V \\ & \alpha_j \geq B_{ij} \end{aligned}$$

I will first discuss in section 4.1 the properties of lending relationships with perfect information, before turning my focus in section 4.2 to the equilibrium and its properties.

#### 4.1 Lending Relationship Properties

Because banks that are not hit with a liquidity shock have little reason to borrow at  $t = 1$ , I focus on the borrowing behavior of the bank hit with the liquidity shock. Moreover, throughout section 4.1, I will assume banks hold  $\alpha < \phi$  liquid assets.<sup>2</sup>

---

1. Note the one difference from autarky is that in addition to choosing liquid assets  $\alpha_i$  and screening  $e_i$ , bank  $i$  must also choose which links to form with the other banks  $j$ ,  $B_{ij}$ . Moreover,  $B_{ji}$  does not enter bank  $i$ 's optimization problem, because any lending from bank  $i$  to bank  $j$  must provide bank  $i$  a normal rate of return.

2. If a bank held  $\alpha \geq \phi$  liquid assets, no reason would exist to secure a lending relationship.

The opportunity to engage in lending relationships serves two key roles in the economy: (i) reduces excess liquidity holdings, and (ii) potentially aligns bank-screening incentives with the social optimum.

Suppose liquid asset holdings from other banks are  $\alpha_j$ , and define  $\underline{\alpha}_i = \max(0, \phi - \sum_j \alpha_j)$ . The next proposition outlines joint properties of a bank's portfolio-allocation and lending-relationship decisions:

**Proposition 2.** *Suppose banks  $-i$  hold  $\alpha_{-i}$  liquid assets and Proposition 1 holds. If bank  $i$  chooses to form lending relationships with banks  $j \in \{1, 2, \dots, n\}$ , then the following are true:*

1. If bank  $i$  holds  $\alpha_i < \phi$  liquid assets, the total size of bank  $i$ 's lending relationships must satisfy  $\sum_j B_{ij} = B_i \geq \phi - \alpha_i$ .
2. If bank  $i$  opts to use lending relationships, instead of remain in autarky for some  $\alpha_i \in [\underline{\alpha}_i, \phi)$ , bank  $i$  will hold  $\alpha_i = \underline{\alpha}_i$  liquid assets.

Proposition 2.1 states that when holding  $\alpha_i$  liquid assets, bank  $i$  will not form lending relationships that leave it with too little liquidity to survive a liquidity shock. The intuition is straightforward. Any collection of liquid assets  $\alpha_i$  and lending relationships  $B_{ij}$  that fail to offer any insurance against liquidity shocks ( $\alpha_i + B_i < \phi$ ) must be strictly dominated by self-insurance ( $\alpha_i = \phi$ ). Thus, if banks use lending relationships, it must be the case that Proposition 2.1 is satisfied.

Proposition 2.2 states banks that opt to engage in lending relationships will prefer to hold as little liquidity possible. To understand this result, assume lenders only provide liquidity to healthy banks.<sup>3</sup> On the one hand, relative to autarky, bank  $i$  has higher default costs

---

3. The result is not contingent on this assumption.

when using lending relationships, because it now defaults whenever it becomes distressed. However, its probability of default is independent of  $\alpha_i$  over the interval  $\alpha_i \in [\underline{\alpha}_i, \phi)$ . On the other hand, a reduction in  $\alpha_i$  increases the profitability of bank  $i$ 's portfolio. Thus, when opting to engage in lending relationships, banks effectively pay a small fixed cost in the form of increased default costs. However, once paid, banks receive benefits that scale linearly with the amount of illiquid assets they choose to hold. Thus, if banks prefer using lending relationships to remaining in autarky, they will opt to use as many lending relationships as possible.

However, allowing banks to tilt their portfolios towards illiquid assets is not the only benefit of lending relationships. Lending relationships can also provide banks with incentives to screen. To understand how lending relationships induce screening, first note that because screening decisions are unobservable, any mechanism designed to induce screening cannot directly target screening. Rather, such a mechanism must act indirectly. Recall the decision to (not) screen trades off the increased probability of distress with larger private benefits. Thus, one possible approach to induce screening is to use lending relationships to punish distressed banks, which in turn increases the relative cost of forgoing screening.

To implement such an approach, lenders must be able to make liquidity provision at  $t = 1$  contingent on borrower health. Here, the lack of commitment inherent in lending relationships plays a key role.<sup>4</sup> Recall that a lack of commitment implies that if bank  $i$  has a lending relationship with bank  $j$ , bank  $j$  is under no obligation to provide liquidity to bank  $i$  when bank  $i$  is in distress. Moreover, even if bank  $j$  always provides liquidity to bank  $i$ , the lack of commitment in the lending relationship could allow bank  $j$  to make interest rates contingent on bank  $i$ 's health. However, bank  $j$ 's ability to charge higher interest rates (or engage in liquidity rationing) is constrained by bank  $j$ 's willingness to provide

---

4. Section 5.3 relaxes this assumption and discusses the trade-offs associated with commitment.



liquidity when offered a normal rate of return, or put differently, pure profit maximization. Thus, to confirm that lending relationships can indeed provide screening incentives, we must verify that higher interest rates (and/or liquidity rationing) are incentive compatible for the liquidity provider.

To this end, recall that under perfect information, all lenders know whether a borrower is healthy or distressed. Then, because a healthy borrower is always able to repay its lenders at  $t = 2$ ,<sup>5</sup> lenders incur no incremental default costs from lending to a healthy borrower, and require only  $\mathbb{E}(\tilde{F}_i) \geq B_i$  when  $s_i = 0$  for a normal rate of return. Consequently, healthy borrowers will request  $\bar{F}_i = B_i$ , which amounts to borrowing at a zero interest rate.

By contrast, distressed borrowers are not always able to repay their interbank debt.<sup>6</sup> An inability to repay, and the default costs it imposes on lenders, means lenders will only lend if the borrower promises a positive interest rate. If  $\Delta$  is relatively small, an interest rate exists which a distressed bank will be able to borrow from its potential lenders. However, for large values of  $\Delta$ , the distressed bank can offer no interest rate that provides lenders a normal rate of return, in which case, potential lenders will refuse to provide the distressed bank with liquidity, and the distressed bank will experience a liquidity shortage. Importantly, in each scenario ( $\Delta$  large or small), lenders effectively punish borrowers for their distress, and thus bank  $j$ 's decision to lend at  $t = 1$ , and the interest rate it charges, can be used as a tool to affect bank  $i$ 's screening incentives.

Suppose borrower  $i$  is distressed and needs to borrow  $B_i = \phi - \alpha_i$  liquidity to meet its own liquidity shock. Define  $\bar{\Delta}_i$  to be the maximum solvency shock such that borrower  $i$  is

---

5. If a borrower requires  $\phi - \alpha_i$  units of liquidity to cover a liquidity shock, and borrows  $B_i \geq \phi - \alpha_i$ , then following payments to depositors, it will have a minimum of  $B_i$  funds remaining at  $t = 2$  to repay its lenders.

6. In the worst state, a distressed bank has  $(\alpha_i + B_i) + (1 - \alpha_i)(\underline{R} - \Delta) - 1 = B_i - (1 - \alpha_i)\Delta$  funds from which to repay other banks, which is strictly less than  $B_i$ .

able to avoid a liquidity shortage. If  $\Delta > \bar{\Delta}_i$ , borrower  $i$  will not be able to raise the liquidity, and defaults immediately at  $t = 1$ , whereas if  $\Delta \leq \bar{\Delta}_i$ , borrower  $i$  will survive until  $t = 2$ .

The next proposition provides conditions for the alignment of banks' screening incentives through the use of lending relationships.

**Proposition 3.** *Suppose banks  $-i$  hold  $\alpha_{-i}$  liquid assets, Proposition 1 holds, and bank  $i$  chooses to form lending relationships with banks  $j \in \{1, 2, \dots, n\}$ . Bank  $i$ 's lending relationships will provide bank  $i$  with the incentive to screen when:*

1.  $P < p_l(p_s^0 - p_s^1) \left( \frac{\bar{R} - R}{2} + \frac{V}{1 - \alpha_i} \right)$  if  $\Delta > \bar{\Delta}$
2.  $P < p_l(p_s^0 - p_s^1) \left( \frac{\bar{R} - R}{2} + \frac{V}{1 - \alpha_i} - \frac{\psi_i}{\bar{R} - R} \left( \frac{1}{2} \psi_i + \frac{V}{1 - \alpha_i} \right) \right)$  if  $\Delta \leq \bar{\Delta}$ <sup>7</sup>

*Proof.* See Appendix. □

The two conditions in Proposition 3 represent the two channels through which lenders provide incentives. When solvency shocks are relatively small, incentives are provided through interest rates (prices), whereas when solvency shocks are relatively large, incentives come through liquidity rationing (quantities).<sup>8</sup> Although incentives can be aligned by lending relationships for both regions of  $\Delta$ , higher interest rates and liquidity rationing have vastly different ex-post consequences. I return to this distinction in section 5.

The channel through which incentive provision occurs when distress is large is also analogous to a setting where banks are disciplined by the threat of creditors refusing to roll

---

7. Here  $\psi_i$  is a function of the interest rate. Importantly, it is monotone decreasing in  $\Delta$ . See appendix for details.

8. This result is very similar to that of Diamond (1991) where borrowers with high profitability borrow at low interest rates, borrowers with intermediate profitability borrow at high interest rates and are monitored, and borrowers with low profitability are shut out of capital markets.

over bank short-term debt. For example, Calomiris and Kahn (1991) showed how demand-able (short-term) debt can align bank risk-taking incentives through forced liquidation. My work differs from Calomiris and Kahn by considering a multi-lateral environment, where each bank does not know ex-ante if it will be providing or receiving liquidity. As such, lending relationships play a dual role in incentive provision. That is, if an individual bank both borrows when hit by a liquidity shock, and engages in liquidity provision to other banks hit by liquidity shocks, then ex-ante it is both (potentially) experiencing the alignment of its own incentives by other banks, and simultaneously contributing to incentive provision within the financial sector.

## 4.2 Equilibrium and Discussion

I begin by defining the Bayes-Nash equilibrium concept used throughout the remainder of the paper:

**Definition.** An equilibrium at  $t = 0$  consists of vectors of investment and screening decisions by banks  $\vec{\alpha}, \vec{e}$ , a set of lending relationships between banks  $i$  and  $j$   $\{B_{ij}\} \forall i, j \in \{1, \dots, N\}$ , and a set of beliefs at  $t = 0$  for other banks' choices of screening at  $t = 0$  and lending behavior at  $t = 1$  such that

1. All banks maximize expected profits. No bank  $i$  can be strictly better off by changing  $(\alpha_i, e_i, B_{ij})$ .
2. Bank lending behaviors at  $t = 1$  are consistent with beliefs held at  $t = 0$ .
3. The borrower's incentive compatibility constraint and all lenders' participation constraints are satisfied.

I focus on symmetric equilibria where  $\alpha_i = \alpha_j \forall \alpha_i, \alpha_j$ . By abstracting away from asymmetric equilibria, I ensure tractability while still being able to generate several interesting insights. One nice property of the symmetric equilibrium is that  $\forall i, j \bar{\Delta}_i = \bar{\Delta}_j$ . Thus, a unique  $\bar{\Delta}$  will exist such that if  $\Delta \leq \bar{\Delta}$ , lending to distressed banks will take place under perfect information, whereas if  $\Delta > \bar{\Delta}$ , distressed banks will experience liquidity rationing.

Proposition 4 describes one symmetric equilibrium:<sup>9</sup>

**Proposition 4.** *Suppose the conditions in Propositions 1 and 3 are satisfied. If borrowing banks  $i$  hold beliefs in which lending banks  $j$  provide liquidity if and only if  $B_{ij} + \hat{\pi}_{d,ij}V \leq \mathbb{E}(\tilde{F}_{ij}|\mathcal{I}_j)$ , a symmetric equilibrium exists in which all banks  $i$  hold  $\alpha_i = \frac{\phi}{N}$  units of liquidity, opt to screen  $e_i = 1$ , and form lending relationships with all other banks  $j$  of size  $B_{ij} = \frac{\phi}{N}$ , providing  $\Lambda\left(\frac{\phi}{N}, 1, \{\frac{\phi}{N}\}\right) \geq \Lambda_0$ . Moreover, if  $\Delta \leq \bar{\Delta}$ , lenders provide incentives through interest rates, and if  $\Delta > \bar{\Delta}$ , lenders provide incentives through liquidity rationing.*

*Proof.* See Appendix. □

Under perfect information, Proposition 4 tells us that if banks form lending relationships, they will also hold as little liquidity as possible. Thus, in a symmetric equilibrium with perfect information, it must be that  $\forall i, j \alpha_i = \{\frac{\phi}{N}\}$  and  $B_{ij} = \{\frac{\phi}{N}\}$ . The decision to screen follows from Proposition 3, whereas the lenders' behavior matches what was discussed in the preceding section.

Note that unlike in autarky,  $\sum_i \alpha_i = \phi$ , and thus the efficient level of liquid assets are held in equilibrium. As a result, bank profits are substantially larger. However, although the efficient amount of liquidity is held in equilibrium,  $\phi$ , the distribution of liquidity is not

---

9. The equilibrium is not unique, but I will focus on it (and its extensions when  $\theta < 1$ ) throughout the rest of the paper.

always ex-post efficient. That is, a wedge exists between  $\bar{\Delta}$ , the threshold where lenders no longer provide liquidity, and  $\Delta^*$ , which is defined as the socially optimal threshold for liquidity provision.<sup>10</sup>

Finally, counterintuitively, contagious defaults<sup>11</sup> only occur when distress is small. To see why, note that a necessary condition for a contagious default is for liquidity to be provided to a distressed bank. With perfect information and large solvency shocks, lenders ration liquidity to distressed borrowers. Consequently, distressed banks are effectively isolated, and their distress is unable to spread to the rest of the financial system. Of course, the idea that greater distress leads to less contagion is somewhat unrealistic, and in section 5, with the introduction of imperfect information, I will show this intuition generally no longer holds. However, the intuition that lenders' responses to borrower distress can affect contagion is a key insight that will underscore much of the remaining analysis.

---

10. The wedge exists because borrowers are unable to offer  $V$  as repayment to lenders.

11. A contagious default is defined as a borrower default spilling over and causing a lender default.

## CHAPTER 5

### ECONOMY WITH IMPERFECT INFORMATION

I now turn my attention to the general economy with imperfect information,  $0 < \theta < 1$ . Recall that under imperfect information, a lending relationship formed at  $t = 0$  between bank  $i$  and bank  $j$ , only provides information about the health of bank  $i$  to bank  $j$  with probability  $\theta$ . Imperfect information introduces asymmetric information and heterogeneity. First, information asymmetry is present between the borrower and its lenders: the borrower knows whether it is healthy or distressed, but each individual lender only knows the borrower's health with probability  $\theta$ . Thus, banks that do not receive a signal are uninformed about the borrowing bank's health. Second, because each lender learns independently about the borrower's health, potential heterogeneity exists among lenders: those that are informed at  $t = 1$  about the health of the borrower and those that are uninformed. Both asymmetric information and heterogeneity will play a role in the outcomes of the model.

To preserve asymmetric information and heterogeneity, I make additional assumptions. First, all banks - including the borrower - are unable to share information. Second, I assume lenders cannot observe lending by other banks. Together, these assumptions prevent uninformed lenders from learning about the borrower's health from the informed lenders.

1

---

1. In practice, in all likelihood, no bank is completely uninformed. However, information can be both (a) relative in the sense that two potential lenders may each know something about the borrowers health, but one lender knows more precisely, and (b) revealed over time, such that one lenders learns of the borrowers distress before the other. Such considerations may provide fruitful avenues for future research.

Thus, recall bank  $i$ 's optimization problem is:

$$\begin{aligned} \max_{\alpha_i, e_i, B_{ij}} \Lambda_i(\alpha_i, e_i, B_{ij}) &= \mathbb{E}(\Pi_i) + (1 - \alpha_i)P^e + (1 - \pi_{d,i})V \\ \text{s.t. } \mathbb{E}(\tilde{F}_{ij}|\mathcal{I}_j) &\geq B_{ij} + \hat{\pi}_{d,ij}V \\ \alpha_j &\geq B_{ij} \end{aligned}$$

Because much of the following discussion focuses on what I call the distressed bank, I emphasize again that a distressed bank is a bank that has received a solvency shock.<sup>2</sup> Although many of the properties of lending relationships flow through from section 4, allowing for imperfect information adds new forces to the model. First, uninformed lenders - those who do not receive a signal - must infer distress from the borrower's behavior. Specifically, information might be conveyed through the borrower's requested interest rate. A distressed borrower has two options: reveal its distress to the lenders, or behave as if it were a healthy borrower. The former involves borrowing using what I refer to as the separating interest rate and similarly the latter, the pooling interest rate. Note that a distressed borrower can only borrow using the separating interest rate when it has sufficient future funds to pledge to its lenders. That is, the borrower can only reveal its distress when  $\Delta \leq \bar{\Delta}$ , where  $\bar{\Delta}$  is defined as it was under perfect information.

Thus, a separating interest rate implies the distressed borrower reveals its distress to all potential lenders, and requests liquidity at a rate that provides informed lenders with a normal rate of return. Provided  $p_s$  is small, the interest rate is greater than what an uninformed lender would accept. The distressed borrower receives liquidity with probability one, and may or may not default at  $t = 2$ . A pooling interest rate implies the distressed borrower attempts to conceal its distress by requesting liquidity at an interest rate that pro-

---

2. Note my definition of a distressed bank differs from other parts of the literature that considers distressed banks as solvent banks that merely lack liquidity.

vides uninformed lenders with a normal rate of return. Although pooling interest rates are lower, distressed borrowers run the risk of informed lenders refusing them liquidity. Consequently, if the number of informed lenders is sufficient, a distressed borrower that requests funds at the pooling interest rate will experience a liquidity shortage and default at  $t = 1$ .

Second, if banks request liquidity at pooling interest rates, they may consider securing promises of size  $B_i > \phi - \alpha_i$ . This behavior arises because banks recognize the pooling interest rate risks a liquidity shortage; thus, securing precautionary liquidity promises might be beneficial. In what follows, I will restrict focus to cases in which  $p_s$  is sufficiently small such that any competitive symmetric equilibrium with pooling offers has  $\alpha_i = \frac{\phi}{N}$  and  $B_{ij} = \frac{\phi}{N}$ . This restriction is not strong. Intuitively, the only scenario in which banks benefit from holding additional liquidity greater than  $\frac{\phi}{N}$  is when they are distressed. If the probability of a solvency shock is sufficiently small, the opportunity cost in the form of additional profits from illiquid assets will dominate the incentive to hold precautionary liquidity. Although banks may still prefer to obtain promises such that  $B_i > \phi - \alpha_i$ , all banks will have the incentive to hold as little liquidity as possible. Thus, in equilibrium, no bank can obtain liquidity promises greater than  $\phi - \alpha_i$ .

Third, the introduction of imperfect information has consequences for the amount of contagion in the financial system. Specifically, consider a distressed borrower that attempts to borrow from its lenders at the pooling interest rate, and suppose a subset of the lenders are informed. As noted above, informed lenders will reject the pooling offer because it does not provide them with a normal rate of return, whereas uninformed lenders will opt to provide liquidity. If sufficient lenders are informed, the distressed borrower will experience a liquidity shortage and default at  $t = 1$ . As a result, the uninformed banks that opted to lend will both lose their principal and experience a significant spike in their probability of a contagious default. Thus, whereas previously, decisions among lenders were synchronous,



imperfect information generates variation in lender behavior. The relationship between information and contagion is the focus of section 5.2.

## 5.1 Equilibrium and Discussion

Define  $r_i^p$  to be the pooling interest rate, and  $r_i^d$  to be the separating interest rate for the distressed bank. Recall  $s_i = \{0, 1\}$  is the indicator variable for whether bank  $i$  received a solvency shock, and let  $a_i \in [0, 1]$  be the lending strategy played by an uninformed potential lender. Define the function  $U(s_i, r_i, a_i)$  to be the bank  $i$ 's benefit from requesting liquidity with interest rate  $r_i$  at  $t = 1$ . Finally, define  $r_i^h$  to be the separating interest rate for the healthy bank.<sup>3</sup> The next proposition compares the symmetric separating and pooling equilibria with lending relationships.

**Proposition 5.** *Suppose the conditions in Propositions 1 and 3 are satisfied, and  $p_s$  is sufficiently small such that  $\alpha_i = \frac{\phi}{N}$  and  $B_{ij} = \frac{\phi}{N}$  is an equilibrium. If  $\Pr(\alpha_i + B_i \geq \phi | r_i^p, s_i = 1)U(1, r_i^p, 1) < U(1, r_i^d, 1)$ , the distressed bank separates in equilibrium, and the healthy bank picks  $r_i^h = \max(0, r_i^0)$ , where  $r_i^0$  is  $\arg \min(U(r_i^0) = U(r_i^d) | r_i^0 \leq r_i^p)$ . Otherwise, the distressed bank pools with the healthy bank in equilibrium.*

*Proof.* Sketch below. □

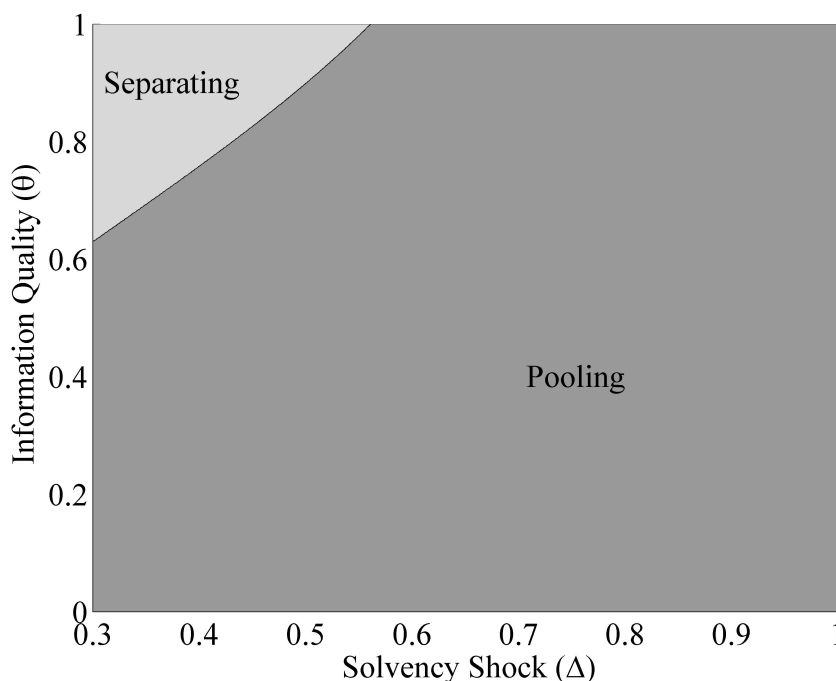
Proposition 5 describes when pooling and separating equilibria take place. Below, I plot the boundary between separating and pooling equilibria for the case when  $\bar{R} = 2.5$ ,  $\underline{R} = 1$ ,  $\phi = 0.3$ ,  $N = 5$ ,  $p_l = \frac{1}{N}$ ,  $V = 3$ , and  $P \approx 0.027$ . Above the line is the region where

---

3. Here,  $r_i^p$  satisfies the uninformed lender's participation constraint,  $r_i^d$  satisfies the informed lender's participation constraint, and  $r_i^h$  is the minimum interest rate a healthy bank can charge and have separation occur.

the equilibrium is of the separating variety, whereas below the line represents the pooling equilibrium. Note that when  $\Delta > 0.55 = \bar{\Delta}$ , the line is flat. The flat boundary arises because here, even under perfect information, a distressed bank has insufficient funds to separate. That is, when signaling its own distress, bank  $i$  will have insufficient funds in excess of its payments to depositors at  $t = 2$  to provide lenders a true normal rate of return.

**Figure 5.1.** Pooling and Separating Equilibria



From the plot, we see the incentive for a distressed bank to behave as a healthy bank is minimized when (potential) solvency shocks are small ( $\Delta$  small) and information is good ( $\theta$  large). Intuitively, when a distressed bank mimics a healthy bank, its requested interest rate from lenders does not provide informed lenders with a normal rate of return. Because all lenders are marginal, the distressed bank needs to receive liquidity from all lenders in order to survive until  $t = 2$ . When information is good, the probability of an early default is high, and when solvency shocks are small, the costs of an early default in the form of lost

future profits are relatively large. As such, for large  $\theta$  and small  $\Delta$ , distressed banks will tend to request liquidity at separating interest rates.

The implications of separating and pooling equilibria differ considerably. First, note that banks in aggregate are holding the efficient amount of liquidity,  $\phi$ . In the separating equilibrium, liquidity is always allocated efficiently. That is, the bank that needs liquidity, and has the means to pay lenders irrespective of distress, receives liquidity and survives until  $t = 2$ . Consequently, no inefficient liquidation takes place in the separating equilibrium.

However, under the pooling equilibrium, with probability  $1 - (1 - \theta)^n$ , the distressed bank does not receive sufficient liquidity to survive until  $t = 2$ . Moreover, a fraction of liquidity remains in the hands of banks for which it has no alternative use. If  $\Delta < \Delta^*$ , too little liquidity provision and too much inefficient liquidation takes place. This scenario corresponds to the bottom-left quadrant where bank distress is minimal and providing liquidity to distressed banks will be socially optimal. Conversely, when  $\Delta > \Delta^*$ , too much liquidity provision is possible, because uninformed banks will continue to naively provide liquidity to a bank that is almost certain to go under.<sup>4</sup>

Second, default probabilities for healthy borrowers are significantly higher when they are forced to pool due to the higher interest rates their debt requires. Higher interest rates are a result of the linear mapping between the interest rate and the probability of default for a healthy bank. By contrast, differences in the default probabilities for the distressed borrower when borrowing at separating versus pooling interest rates are less obvious. For example, if  $\Delta > \bar{\Delta}$ , separating interest rates induce higher default probabilities as distressed banks attempting to separate default with probability one. Conversely, if  $\Delta \leq \bar{\Delta}$ , liquidity requests at separating interest rates ensure the distressed bank survives until  $t = 2$ , whereas

---

4. Note  $\Delta > \Delta^*$  is not a sufficient condition for too much liquidity provision. The transfers to depositors, and the loss of  $V$  by the borrowing bank must be weighed against the losses of the lending banks.

pooling interest rate requests are likely to result in liquidity shortages and premature default.

Broadly speaking, one can view the two types of equilibria as representing two very different states of the economy. On the one hand, when information is good (high  $\theta$ ) and potential solvency shocks are small (small  $\Delta$ ), the economy can be viewed as being in a “normal” state. On the other hand, when either information is poor or potential solvency shocks are large (or both), one might think of the economy as being in times of high uncertainty, or perhaps a crisis state.

The preceding discussion appears to indicate the economy in “normal” times functions relatively well. Not only do lending relationships provide screening incentives; liquidity is always provided efficiently at a fair interest rate to those who need it. By contrast, when the economy experiences “bad” times, lending relationships do not function nearly as well. Liquidity can be both over- or under-provided to distressed banks, depending on the information quality and the size of distress, whereas healthy banks are charged inefficiently high interest rates that expose them to default risk significantly above what otherwise might be expected. Moreover, we will see in the next section that spillovers can be large if information is poor, even though banks have no previously existing interdependencies or correlated assets.

## 5.2 Contagion

This section looks at the impact of lending relationships on the distribution of lender defaults. Throughout this section, I rule out separating equilibria by considering large solvency shocks ( $\Delta > \bar{\Delta}$ ). I also focus on a symmetric equilibrium with  $\alpha_i = \frac{\phi}{N} \forall i$  and  $B_{ij} = \frac{\phi}{N} \forall i, j$ .

I define a contagious default to be a default by a bank that was not initially distressed. The only channel through which such a default can take place in my model is when distressed banks fail to repay healthy lenders, and thus contagious defaults only occur at  $t = 2$ . Contagion in this sense can only go one level deep in the system because following the realization of shocks at  $t = 1$ , healthy lenders do not have any other connections.

Suppose bank  $i$  receives both a liquidity shock and a solvency shock, and banks  $j = \{1, \dots, n\}$  represent bank  $i$ 's potential lenders. Given the previous discussion, define  $\hat{\pi}_{d,(0)}$  to be the conditional probability lender  $j$  defaults given that lender  $j$  opts to lend, but another lender opts to renege. Similarly, define  $\hat{\pi}_{d,(1)}$  to be the conditional probability lender  $j$  defaults given that lender  $j$  opts to lend, and all other potential lenders opt to lend. It can be shown that:

$$\begin{aligned}\hat{\pi}_{d,(0)} &= \frac{1}{\bar{R} - \underline{R}} \frac{\phi/N}{1 - \phi/N} \\ \hat{\pi}_{d,(1)} &= \frac{1}{(\bar{R} - \underline{R})^2} \frac{\phi/N}{1 - \phi/N} \left\{ \Delta - \frac{1}{2} \frac{(N-1)\phi}{1 - \phi} \right\}\end{aligned}$$

Note that  $\hat{\pi}_{d,(0)} > \hat{\pi}_{d,(1)}$ . Let  $C_k$  be the probability that  $k$  banks default from contagion when bank  $i$  is distressed, given all choices made at  $t = 0$ . For example,  $C_3$  is the conditional probability that three banks default from contagion when bank  $i$  is distressed. Define  $C$  to be the expected number of defaults from contagion in the economy when bank  $i$  is distressed.

Under these definitions, we have:

$$\begin{aligned}
C_k &= \sum_{m=1}^{n-k} \binom{n}{m} \theta^m (1-\theta)^{n-m} \binom{n-m}{k} \hat{\pi}_{d,(0)}^k (1 - \hat{\pi}_{d,(0)})^{n-m-k} \\
&\quad + (1-\theta)^n \binom{n}{k} \hat{\pi}_{d,(1)}^k (1 - \hat{\pi}_{d,(1)})^{n-k} \\
C &= \sum_{k=0}^n k C_k
\end{aligned}$$

In the Appendix, I show that:

$$C_k = \text{Bin}\left(k; n, \hat{\pi}_{d,(0)}(1-\theta)\right) + (1-\theta)^n \text{Bin}\left(k; n, \hat{\pi}_{d,(1)}\right) - (1-\theta)^n \text{Bin}\left(k; n, \hat{\pi}_{d,(0)}\right)$$

Where  $\text{Bin}(k; n, p)$  is the probability of  $k$  successes under a binomial distribution with parameters  $n$  and  $p$ . Moreover, using the above fact, it can be shown that:

$$C = n\hat{\pi}_{d,0}(1-\theta) + n(1-\theta)^n (\hat{\pi}_{d,1} - \hat{\pi}_{d,0})$$

The next two propositions look at the expected number of contagious defaults and the distribution of contagious defaults when a given bank is distressed.

**Proposition 6.** *Suppose bank  $i$  receives a solvency shock and has lending relationships with banks  $j = \{1, \dots, n\}$  and that  $\Delta > \bar{\Delta}$ . The following are true:*

1. The function  $C(\theta)$  is strictly concave.
2. If  $\frac{n-1}{n} \frac{\hat{\pi}_{d,(0)}}{\hat{\pi}_{d,(1)}} < 1$ ,  $C(\theta)$  obtains a unique maximum at  $\theta = 0$ . If  $\frac{n-1}{n} \frac{\hat{\pi}_{d,(0)}}{\hat{\pi}_{d,(1)}} > 1$ ,  $C(\theta)$  obtains a unique maximum at  $\bar{\theta}$ , where  $\bar{\theta}$  solves:  

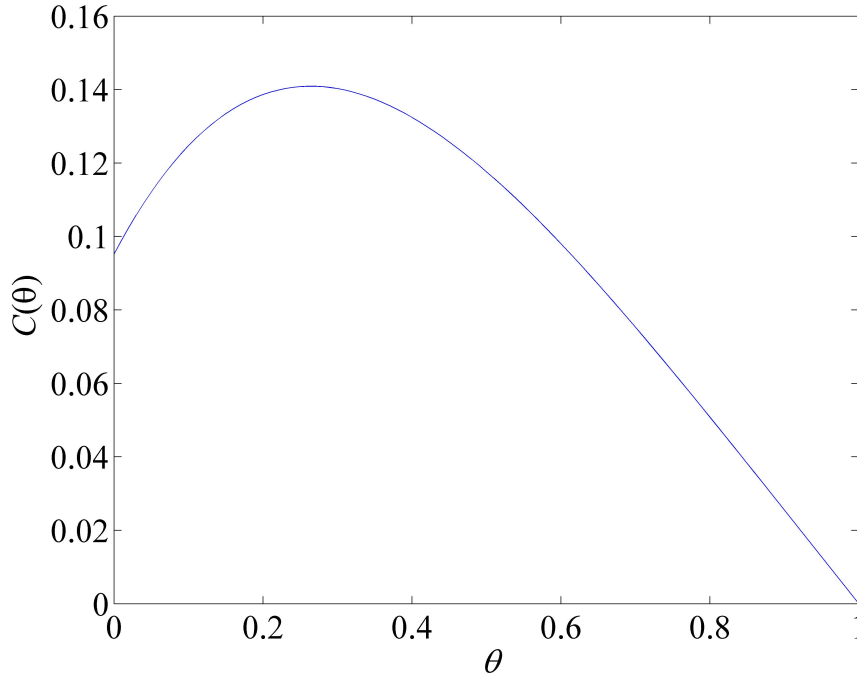
$$0 = \left( \left( n(1-\bar{\theta})^{n-1} - 1 \right) \hat{\pi}_{d,(0)} - n(1-\bar{\theta})^{n-1} \hat{\pi}_{d,(1)} \right).$$

*Proof.* See Appendix. □

Proposition 6 is best illustrated graphically, which I do below. The plot below depicts  $C(\theta)$  following a solvency shock in the  $N = 5$  bank economy, with parameters  $\bar{R} = 2$ ,  $\underline{R} = 1$ ,  $\Delta = 0.5$ ,  $\phi = 0.3$ ,  $p_l = \frac{1}{N}$ ,  $p_s^1 = 0.01$ ,  $p_s^0 = 0.05$ ,  $V = 6$ , and  $P \approx 0.027$ . Under this setup, each bank holds  $\alpha_i = \frac{\phi}{5}$  and has four potential lenders.

The plot highlights an interaction between the information quality and the probability of contagion in the economy. Define  $\underline{\theta}$  as that  $\theta$  that satisfies  $C(\underline{\theta}) = C(0)$ . For relatively poor information ( $0 < \theta < \underline{\theta}$ ), lending relationships lead to relatively high amounts of contagion following a solvency shock, with  $\bar{\theta} \approx 0.25$ , whereas for good information  $\theta$  ( $\underline{\theta} < \theta < 1$ ), contagious defaults are significantly more rare.

**Figure 5.2.** Expected Contagion



The non-monotonicity is a result of two opposing forces that act through the probability of the signal. The intuition for the non-monotonicity is as follows. Consider the perspective of bank  $j$ , which may or may not lend to the distressed bank  $i$ . First, because  $\Delta > \bar{\Delta}$ , all informed banks will refuse to provide liquidity. Second, with  $\alpha_i = \frac{\phi}{N}$ , banks that are hit by liquidity shocks require all banks to lend in order to survive until  $t = 1$ . Thus, whenever informed lenders exist, liquidity shortages ensue and the distressed bank defaults prematurely. As such, when the distressed bank defaults, the probability of default for all banks that continued to lend spikes from  $\hat{\pi}_{d,(1)}$  to  $\hat{\pi}_{d,(0)}$ . Thus, conditional on bank  $j$  lending to bank  $i$ , a higher  $\theta$  implies bank  $j$  is more likely to have a conditional probability of default equal to  $\hat{\pi}_{d,(0)}$ .

However, the probability of the signal also has implications for how many banks continue to lend, conditional on one bank opting to renege. For bank  $j$ , a higher  $\theta$  implies a greater chance of learning a borrower is distressed, and thus reduces the probability bank  $j$  lends to a distressed bank and experiences the state where its probability of default is  $\hat{\pi}_{d,(0)}$ . Moreover, this property is also true for all other potential lenders, and thus a higher  $\theta$  implies in expectation fewer banks lending when bank  $i$  is distressed. Thus, the signal probability  $\theta$  affects both the lender's conditional probability of default as well as the expected number of banks that continue to lend to the distressed bank and thus could default contagiously.

The aforementioned non-monotonicity is important for two reasons. First, on the one hand, when information is relatively imprecise following a solvency shock ( $\theta < \underline{\theta}$ ), the action of a small subset of lenders can cause significant harm to the rest of the financial sector. Banks that are able to get a head start on their competitors in terms of disassociating themselves from a distressed counter-party can potentially profit; however, such an action could come at the expense of the financial system as a whole. On the other hand,



when banks are relatively well informed ( $\theta > \underline{\theta}$ ), preventing bank distress from spreading through the system might be possible.<sup>5</sup>

Second, recall that lending relationships can potentially align banks' screening incentives with the social optimum and reduce the probability of individual bank distress. When information is sufficiently poor ( $\theta < \underline{\theta}$ ), incentive alignment is a double-edged sword. That is, although the use of lending relationships can reduce the probability of an individual bank becoming distressed, it can also drastically increase the probability of multiple concurrent defaults and systemic failure.

This result should not be confused with the finding that a more connected system can exacerbate the probability of a systemic failure.<sup>6</sup> Rather, this non-monotonicity emphasizes the role the types of connections in a financial system play in exacerbating or attenuating systemic risk. Another way to interpret this result is the following. To the extent that the information environment encapsulates the amount of liquidity provision, it also reflects the “tightness” of connections within the financial system. A rich information environment gives rise to connections that break apart frequently when borrowers become distressed, whereas conversely, a poor information environment is consistent with connections that are less likely to rupture. Taken together, this result implies that contagion is a function of how tight connections are, and how easily the financial system breaks apart.

---

5. One could extend the model to allow for endogenous information acquisition. Specifically, suppose we allow each lending bank  $j$  to pay  $c(\theta)$  at  $t = 0$  in order to obtain information with probability  $\theta$  about borrowing bank  $i$  at  $t = 1$ . The amount of information obtained within the financial system will depend on each bank's beliefs about other banks' actions and the nature of the cost function. Given the structure of the model, information acquisition would likely exhibit strong strategic complementarities. Moreover, if the cost function is very steep, the level of information acquired by banks in lending relationships likely corresponds to a moderate level of  $\theta$ , say  $\theta = 0.25$ .

6. Such a result is already well known. For example, see Blume et al. (2009) and Acemoglu et al. (2015a, 2015b) among others.

Looking at how information quality affects the distribution of contagious defaults conditional on a solvency shock is also interesting.

**Proposition 7.** *Suppose bank  $i$  receives a solvency shock and has lending relationships with banks  $j = \{1, \dots, n\}$  and that  $\Delta > \bar{\Delta}$ . The following are true:*

1. The functions  $C_0(\theta)$  and  $C_n(\theta)$  are globally convex. Also,  $C_n(\theta)$  is strictly decreasing, whereas  $C_0(\theta)$  is strictly increasing iff  $\frac{(1-\hat{\pi}_{d,(1)})^n}{(1-\hat{\pi}_{d,(0)})^{n-1}} > 1$ .
2. For  $0 < k < n$ , the functions  $C_k(\theta)$  are maximized for some  $\theta_k > 0$  if and only if  $\frac{n-k}{n} \left( \frac{\theta_a}{\theta_b} \right)^k \frac{(1-\hat{\pi}_{d,(0)})^{n-k-1}}{(1-\hat{\pi}_{d,(1)})^{n-k}} > 1$ .
3.  $C_0(1) = 1$ , and for  $k > 0$ ,  $C_k(1) = 0$ .

*Proof.* See Appendix. □

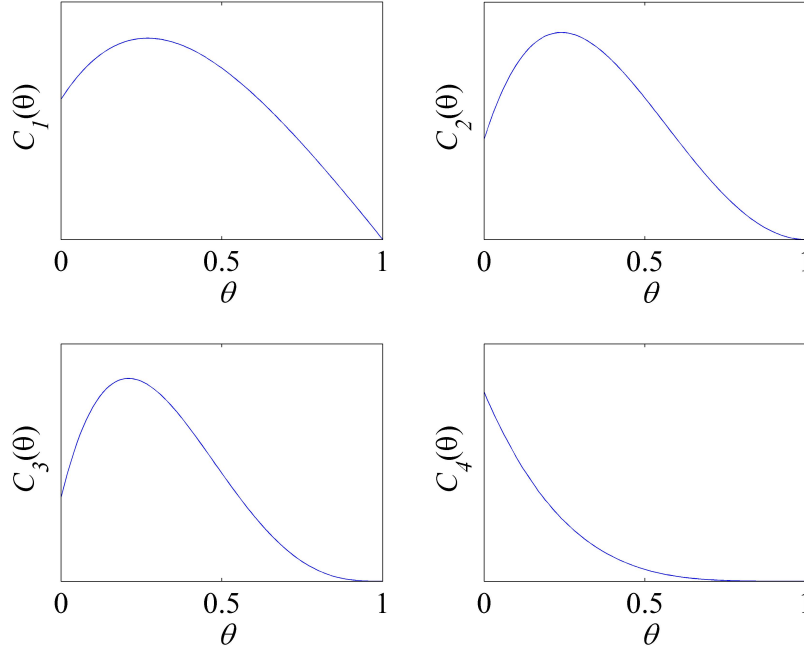
Proposition 7 provides conditions on the behavior of the functions  $C_k(\theta)$ . First, Proposition 7 states that the probability that  $n$  banks default contagiously is strictly decreasing in  $\theta$ . This result is reassuring because banks that do not lend cannot default, and the greater  $\theta$  is, the fewer the number of banks expected to lend.<sup>7</sup> Second, Proposition 6.1 provides a condition for which  $C_0(\theta)$  is strictly increasing. Equivalently, if  $C_0(\theta)$  is strictly increasing, it follows that the probability that at least one contagious default ( $\sum_{k>0} C_k(\theta)$ ) occurs must be strictly decreasing. However, unless  $\hat{\pi}_{d,(0)}$  and  $\hat{\pi}_{d,(1)}$  are approximately equal, or  $n$  is small, the probability that at least one contagious default occurs will not be maximized at  $\theta = 0$ . Proposition 6 also provides related conditions focusing on a specific number of defaults  $k$ .

---

7. In a network with pre-existing connections, this result may no longer hold.

Proposition 7 is depicted graphically below for the same parameters as the preceding plot.

**Figure 5.3.** Probability of  $k$  Contagious Defaults



Notice how the probability of  $k$  defaults for  $k < n$  are all maximized for  $\theta > 0$ . For  $\theta \approx 0.2$ , we see the economy is far more likely to experience one, two, or three contagious defaults following a solvency shock than if banks had opted instead for lending relationships that had some sort of inherent commitment. The exception is the case in which  $k = n = 4$ , but as explained above,  $C_4(\theta)$  must be decreasing in  $\theta$  as the probability that  $n$  banks remain connected (which is required for  $n$  defaults) is strictly decreasing in  $\theta$ .

If the model is extended to allow for pre-existing connections, so that banks may default contagiously even if they opt not to provide liquidity, overturning the  $k = 4$  case would be possible. I defer this extension for future research.

### 5.3 Commitment

One drawback of uncommitted lending relationships illustrated in section 5.2 is they increase the probability of multiple bank defaults within the financial system. In particular, when information is relatively poor, the expected number of contagious defaults is high. Note that contagious defaults occur only when a borrower fails to repay the principal to a lender. Moreover, contagious defaults are especially common when information is poor due to the failure of informed lenders to consider the impact of their decision on both the borrower and the uninformed lenders.

A natural question arises: What if lending relationships involved commitment? That is, what if lending relationships contractually required lenders to provide liquidity to a borrower with a liquidity shortage, regardless of whether the borrower was healthy or distressed?

Intuitively, commitment would prevent informed lenders from imposing costs *ex-post* on uninformed lenders, because all lenders would have the same obligation to lend. Moreover, relative to autarky, borrowers would seem to benefit because they would be able to hold more illiquid assets than if they remained in autarky, while still guaranteeing their own survival until  $t = 2$ .

To this end, suppose now that bank  $i$  and bank  $j$  can form lending relationships of size  $B_{ij}$  where bank  $j$  is committed to providing liquidity  $B_{ij}$  at  $t = 1$  should bank  $i$  request the liquidity. Because lending is still assumed to take place through standard debt contracts, I assume the interest rate is fixed at  $t = 0$ . Fixing the interest rate has the advantage of ensuring lenders make a normal rate of return *ex-ante*, as well as mitigating any potential hold-up problems. In practice, due to the associated credit risk, contracts similar to

committed lending relationships are uncommon in the financial sector, with probably the closest example being a loan guarantee.

Equivalently, one could instead assume such contracts are not possible, but allow banks to opt against forming any lending relationships at  $t = 0$  and instead go to a market consisting of all uninformed banks at  $t = 1$  should they require liquidity. Depending on the number of uncommitted lending relationships a bank possesses, access to this market may reveal information about its health. However, if banks do not hold any uncommitted lending relationships, information revelation should not be a problem. As such, the interest rates set on the contracts would be equivalent to the lending relationships with commitment. Henceforth, I consider lending relationships with commitment.

Thus, if bank  $i$  forms a committed lending relationship with bank  $j$  of size  $B_{ij}$  at  $t = 0$ , bank  $j$  will be required to lend bank  $i$   $B_{ij}$  if bank  $i$  is hit with a liquidity shock at  $t = 1$ . In return, bank  $i$  must make an expected payment to bank  $j$  at  $t = 2$  equal to the amount bank  $i$  borrowed plus bank  $j$ 's expected additional default costs from lending. That is, if bank  $i$  borrows from bank  $j$ , it sets a promised payment on its debt to bank  $j$  that satisfies:

$$\mathbb{E}(\tilde{F}_{ij} | \mathcal{I}_j) = B_{ij} + \hat{\pi}_{d,ij} V$$

Where the expectation is taken using bank  $j$ 's  $t = 0$  information set. The following proposition outlines the relevant properties of lending relationships with commitment.

**Proposition 8.** *Suppose banks  $-i$  hold  $\alpha_{-i}$  liquid assets and Proposition 1 holds. If bank  $i$  chooses to form committed lending relationships with banks  $j \in \{1, 2, \dots, n\}$ , then the following are true:*

1. If bank  $i$  holds  $\alpha_i$  liquid assets, the total size of bank  $i$ 's committed lending relationships will equal  $\sum_j B_{ij} = B_i = \phi - \alpha_i$ .

2. If bank  $i$  prefers committed lending relationships to autarky for some  $\alpha_i \in [\underline{\alpha}_i, \phi, )$ , bank  $i$  will hold  $\alpha_i = \underline{\alpha}_i$  liquid assets.
3. Bank  $i$  will not screen.
4. When banks use committed lending relationships, the expected number of contagious defaults conditional on a solvency shock equals  $C(0)$  for uncommitted lending relationships.

*Proof.* See Appendix. □

Propositions 8.1 and 8.2 are analogous to Proposition 2. Importantly, we see banks will use the lending relationships with commitment in a similar way. Specifically, banks will always secure sufficient lending relationships to cover their liquidity shocks, and if they prefer to use lending relationships with commitment to remaining in autarky, they will attempt to hold as few liquid assets as possible.

However, Proposition 8.3, which states that banks using lending relationships with commitment will not screen, is of particular interest. Recall that under autarky, banks opted against screening because they could profit by pushing some of the losses incurred onto their depositors whose payment was capped at a fixed amount  $D$ . Similarly, the use of lending relationships with commitment effectively increases bank  $i$ 's total future debt by a fixed amount in certain states of the world. Because screening is unobservable and unverifiable, committed lending relationships weaken banks' screening incentives.

Furthermore, bank  $i$ 's potential lenders recognize bank  $i$ 's weak screening incentives. As such, lenders will ex-ante request a higher interest rate than what would be required if bank  $i$  chose to screen. The higher interest rate required by bank  $i$ 's counter-parties both reduces bank  $i$ 's profits and increases bank  $i$ 's expected default costs following a liquidity

shock. In fact, bank  $i$ 's inability to commit to screen when using committed lending relationships harms bank  $i$ , because the costs associated with the higher interest rate typically outweigh the private benefits bank  $i$  receives from not screening. Moreover, although the interest rate is fixed at  $t = 0$ , the problem is unlikely to be resolved by a market at  $t = 1$ , because a set of uninformed lenders at  $t = 1$  will charge the same interest rate set by those with committed lending relationships at  $t = 0$ .

Proposition 8.4 states that when  $\theta = 0$ , the expected amount of contagion following a solvency shock is equivalent for committed and uncommitted lending relationships. Although this equivalence is perhaps unsurprising, what is crucial is that unlike uncommitted lending relationships, contagion under committed lending relationships is independent of  $\theta$ . Importantly, this difference between committed and uncommitted lending relationships suggests that the amount of contagion within the financial system is a function of the types of lending relationships within the economy. Lending relationships without commitment, akin to implicit promises between banks, allow lenders to renege at the first sign of danger. Although these lending relationships may be designed to prevent the spread of distress, they in fact may only serve to exacerbate problems within the financial system if the information quality is sufficiently poor. By contrast, committed lending relationships provide a type of insurance to the borrower, at the expense of leaving lenders with a lack of flexibility to respond to borrower distress. Relative to lending relationships without commitment, they may simply make today's problem tomorrow's disaster.

Finally, Proposition 8, combined with earlier results, suggests a trade-off between committed and uncommitted lending relationships. On the one hand, commitment ensures liquidity provision, but comes at the expense of good screening incentives. Moreover, although conditional on distress, commitment may reduce ex-post contagion when information is poor, the probability of an individual bank becoming distressed will be higher ex-ante. On the other hand, a lack of commitment aligns incentives by inducing potentially

undesirable ex-post outcomes such as liquidity shortages at distressed banks, early default, and significant contagion. Section 6 discusses some policy implications of this trade-off.

## 5.4 Social Planner

The symmetric equilibrium with uncommitted lending relationships represents a private solution to a dual investment-screening problem using a specific set of contracts. One question that arises asks whether a planner can improve on the private solution. This section focuses on whether a planner facing a set of contracting restrictions and liquid asset choices by each bank can design a liquidity-sharing contract for banks which increases total surplus. I also provide a brief discussion of two other options for the planner: liquidity requirements and alternate financial system configurations. Throughout this section, to remain consistent with the earlier analysis on financial contagion, I assume  $\Delta > \bar{\Delta}$  and  $\theta < 1$ .

Suppose each bank holds liquid assets of  $\alpha_i = \frac{\phi}{N}$  as per proposition 5, and the planner is tasked with designing a contract that increases total surplus subject to the following set of contracting restrictions.<sup>8</sup> First, no contract can be written between bank  $i$  and bank  $j$  that can provide bank  $j$  information about bank  $i$  with probability greater than  $\theta$ . Second, no bank is able to directly or indirectly (through the planner) share its information at  $t = 1$  with another bank. This assumption also means no bank can observe the actions of another banks at  $t = 1$  unless those actions are directed towards them. Third, contracts can be written on actions, but cannot be written on solvency shocks,  $s_i$ , or screening,  $e_i$ . This

---

8. Each contracting restriction is made to ensure the fairest comparison with the lending relationships discussed throughout the paper.



assumption implies the planner cannot tax banks in a state-contingent manner. Fourth, I assume that at  $t = 1$ , the following game takes place:<sup>9</sup>

1. Bank  $i$  receives a liquidity shock, and potentially a solvency shock.
2. Each bank  $j = \{1, 2, \dots, n\}$  that did not receive a shock but had a contract with bank  $i$  at  $t = 0$ , can potentially learn about bank  $i$ 's health with probability  $\theta$ .
3. Based on the contracts agreed upon at  $t = 0$ , bank  $i$  attempts to borrow liquidity by making simultaneous offers to each bank  $j$ .
4. Each bank  $j$  responds to bank  $i$ 's offer according to the pre-agreed-upon contract.

Finally, all contracts must satisfy individual rationality constraints in order for each bank to be willing to participate. Proposition 9 outlines when a planner can increase total surplus relative to the private benchmark through the use of a hybrid contract.<sup>10</sup> I define the hybrid contract as a contract between bank  $i$  and bank  $j$  in which bank  $i$  can request liquidity at any rate specified in the contract  $r_i = \{r_i^{(1)}, r_i^{(2)}, \dots\}$ , and bank  $j$  is required to provide liquidity at the requested rate or pay a tax  $\tau$  (that is redistributed to all other banks that provided liquidity). Privately, banks cannot implement the hybrid contract because they cannot enforce the payment of the tax when the borrowing bank defaults at  $t = 1$ .<sup>11</sup> By contrast, the planner implements the hybrid contract by imposing a fine  $f > \tau$  on any bank that does not pay a tax it owes.

---

9. This game nests what happened under committed and uncommitted lending relationships.

10. In the Appendix, I argue that given the assumptions, the hybrid contract represents the only method through which the planner can increase total surplus.

11. More specifically, lender  $j$  (which provided liquidity) is assumed to be unable to enforce the contract and require lender  $k$  (which did not provide liquidity) to pay the tax when borrower  $i$  is in default. Lender  $k$  recognizes this fact, and thus its effective tax when making its liquidity provision at  $t = 1$  is  $\tau = 0$ .

**Proposition 9.** Assume the above contracting restrictions,  $\theta < 1$ ,  $\Delta > \bar{\Delta}$ , and banks hold liquid assets  $\alpha_i = \frac{\phi}{N}$ . A planner can increase total surplus relative to the private benchmark through the use of the hybrid contract with interest rates  $r_i = \{r_i^{h'}, r_i^{d'}\}$  and tax  $\tau$  if and only if the following conditions are satisfied:

$$\begin{aligned}
(1 - p_s^1)U_j(0, A, r_i^{h'}, 1) + p_s^1 U_j(1, A, r_i^{d'}, 1) &\geq 0 \\
U_i(1, r_i^{d'}, 1) &\geq \mathbb{E}\left(U_i(1, r_i^{h'}, a_j)\right) \\
\Lambda_i(\alpha_i, 1, B_{ij}|r_i^{h'}, r_i^{d'}, \tau) &\geq \Lambda_i(\alpha_i, 0, B_{ij}|r_i^{h'}, r_i^{d'}, \tau) \\
U_j(1, A, r_i^{d'}, 1) &\geq -\tau
\end{aligned}$$

*Proof.* See Appendix. □

If the conditions in Proposition 9 are satisfied, the planner can implement the hybrid contract and increase total surplus relative to the private benchmark. The hybrid contract allows the planner to force a separating equilibrium that was not possible under uncommitted lending relationships when  $\Delta > \bar{\Delta}$ . The key lever at the planner's disposal is the ability to tax banks that refuse to provide liquidity. The tax is important because under the private benchmark, when a bank received both a liquidity shock and a solvency shock, the existence of an informed liquidity provider would cause the bank to suffer a liquidity shortage and a premature default. However, with the hybrid contract and its tax on liquidity providers that renege, the planner is able to shift the point of indifference between providing liquidity and reneging for potential liquidity providers, which in turn encourages a separating offers from the liquidity-shocked bank, and ensures that any bank that needs liquidity receives it. Under the separating equilibrium, defaults, depositor losses, and liq-

uidation are all reduced relative to the private benchmark, and thus total surplus is greater than the private benchmark.

However, should the conditions of Proposition 9 fail to be satisfied, the planner cannot use the hybrid contract to increase total surplus. In fact, in the Appendix, I argue the planner does not have any other levers through which it can increase total surplus, and thus it follows that when Proposition 9 is violated, uncommitted lending relationships are optimal.

Applying different contracts to the private benchmark is not the only way a planner might seek to increase total surplus. Two other alternatives worth a brief discussion are liquidity requirements and policies that lead to alternate structures of the financial system. I do not provide a complete analysis here, but a couple of important points are worth noting.

First, liquidity requirements can increase total surplus. In the equilibrium on which I focus, each bank picks  $\alpha_i = \frac{\phi}{N}$ , implying the economy has no precautionary liquidity. Here, each individual bank has chosen its liquid assets to maximize  $\Lambda_j$ . No precautionary liquidity is held, because it is valuable to each bank only when the bank itself receives a solvency shock, which occurs with probability  $\frac{p_s}{N}$ . However, each bank's choice fails to take into account of the value of its liquid assets to other banks that might receive a solvency shock. In particular, precautionary liquidity is potentially valuable from a social perspective whenever any bank receives a solvency shock, because it can reduce the amount of inefficient failures, and depending on the quality of information, expected financial contagion. That is, precautionary liquidity is potentially valuable with probability  $p_s$ . Thus, banks under-value liquid assets from a social perspective, and a planner that can institute a liquidity requirement can potentially increase total surplus.<sup>12</sup>

---

12. Of course the requirement must trade off the reduced liquidation / contagion with the lost profits from fewer illiquid assets and the potentially adverse impact on screening incentives.

Second, alternate network structures are another possibility the planner might consider. One structure a planner might want to induce in the financial system is a star. Here, one bank would exist at the center of the financial system, and it would distribute liquidity to a series of banks on the periphery. The advantage of this configuration is that by construction, no heterogeneity exists in information sets among liquidity providers because only one liquidity provider exists. As such, no externality is imposed on uninformed lenders as was the case in section 5.2. However, the star configuration has potential drawbacks. If information is poor (low  $\theta$ ), incentive provision by the bank at the center of the system will be weaker than the private benchmark discussed in section 5.1. Moreover, if the sole liquidity provider is able to make rents due to its monopoly power (which is outside the model, but potentially realistic), it will likely extort banks in need of liquidity for profits, and potentially reduce total surplus. A complete analysis of other networks is beyond the scope of this paper, and thus I defer associated questions to future research.

## CHAPTER 6

### POLICY IMPLICATIONS

I highlight two important policy implications. First, policy makers must understand commitment and liquidity provision in the interbank market go hand in hand with incentives. Policy aimed at increasing commitment ex-ante, will improve liquidity provision at the expense of incentives. For example, if the government restricted the use of material adverse change clauses, such an action would increase ex-ante commitment and improve liquidity provision in times of distress. However, with stronger liquidity promises ex-ante, borrower incentives will be adversely affected, and thus the policy will counterproductively increase the frequency of bank distress itself. Similarly, policies that relax commitment ex-ante align incentives through the threat of liquidity shortages.

Second, from a network perspective, a relatively recent but fast-growing body of research has focused on the nature of the systemic risk created within the financial system.<sup>1</sup> In particular, this literature seeks to understand the determinants of such risk and to design optimal policy responses. Two key determinants that have been identified are the size of the exposures between banks, and the density of the connections that make up the financial system.

In my model, to the extent that the information environment encapsulates the amount of liquidity provision, it also reflects the “tightness” of connections within the financial system. High-quality information is consistent with connections that break apart frequently when borrowers become distressed, whereas conversely, low-quality information corresponds to connections that are less likely to rupture. Taken together, the implication is

---

1. See Elliot, Golub, and Jackson (2014) and Acemoglu, Ozdaglar, and Tabhaz-Salehi (2015a, 2015b), amongst others.

contagion is a function of how tight connections are and how easily the financial system breaks apart. The model suggests the “tightness” of connections in the financial system should be an active consideration for policies targeting systemic risk-mitigation.<sup>2</sup>

Also note that the model predicts liquidity provision to distressed banks will be the most cost-effective when connections are of moderate tightness. Although “tightness” in my model is a function of information quality and commitment, other features of financial contracts such as covenants, collateral, and maturity are also likely to be important factors. In the case of LTCM, its connections indeed appear ex-post to have been of moderate tightness. Whereas LTCM’s liquidity-sharing agreements were often implicit, and thus loose, the hedge fund had many tight connections with counter-parties that stood to lose billions of dollars should it default. Again, the fact that the Federal Reserve was able to organize a “bailout” of LTCM that required zero public funds, is a testament to the fear of a catastrophic fallout stemming from the hedge fund’s failure. Thus, this relatively cheap<sup>3</sup> “bailout” is broadly consistent with the ex-post observation that LTCM’s connections were of moderate tightness.

Finally, the model provides a framework for future research on other important policy questions. For example, one extension of the model is to allow for a pre-existing network between banks. Presently, the only source of interdependencies in the model is through the lending relationships formed at  $t = 0$  and (potentially) activated at  $t = 1$ . A direct result of this setup is that banks that form lending relationships at  $t = 0$  but refuse to lend at  $t = 1$  face no consequences for their decisions. Alternative connections, such as long-term debt and cross-holdings of equity, introduce additional channels through which distress may spread.

---

2. Measuring tightness is difficult, but possible dimensions include maturity (shorter is less tight than longer), asset liquidity (liquid assets are less tight than illiquid ones), and contractual (loose covenants are less tight than strict covenants).

3. Zero public funds

Such connections are likely to dilute incentive provision, but exacerbate ex-post contagion when financial interconnections are severed. Moreover, asymmetries in such networks are particularly interesting. In particular, with asymmetric pre-existing connections, the potential for divergent bank incentives within the multi-lateral setting that is the financial system becomes increasingly likely, and as such, creates a new set of considerations for policy makers.

## **CHAPTER 7**

### **CONCLUSION**

At the core of the financial system, debt contracts are ubiquitous. A complete understanding of debt contracts within the financial system, and thus their importance to the economy, requires a careful examination of the characteristics of debt in a multi-lateral environment. To understand debt in a multi-lateral environment, this paper developed a model of liquidity sharing in which banks became interlinked through the use of committed and uncommitted lending relationships. The model illustrates that the choice of commitment in a multi-lateral setting trades off incentives with liquidity provision. I highlight how uncommitted lending relationships, such as the lending relationships between banks in the federal funds market, expose banks to potential liquidity shortages. However, liquidity shortages, which induce early default, can align banks' screening incentives. Moreover, if banks collectively use uncommitted lending relationships, incentive alignment can be multi-lateral. Liquidity shortages can also exacerbate contagion. I show that when banks use uncommitted lending relationships, the relationship between information quality and contagion is non-monotone. The non-monotonicity is the result of informed banks exerting externalities on uninformed banks.

The model produces two key policy implications. First, policy makers should consider the ex-ante impact of any intervention in interbank markets on bank incentives. This intuition does not rely on interventions in the form of a bailout. Second, the model underscores the "tightness" of connections in the financial system as an important additional consideration for any policy targeted at mitigating the systemic risk generated within the financial system. Note such policies must also take into account features not present in the model. Specifically, prices, existing connections such as cross-holdings of equity, and



network asymmetries are likely to also play fundamental roles in shaping incentives, liquidity provision, and contagion. Such considerations provide an interesting avenue for future research.

## APPENDIX A

### PROOFS OF PROPOSITIONS

This section outlines the proofs of the various propositions detailed throughout the paper.

#### A.1 Proof of Proposition 1

Let  $\Lambda_i(\alpha_i, e_i, B_i)$  be bank  $i$ 's objective function, which I will abbreviate to  $\Lambda_i$  where possible. The autarky objective function, derived in Appendix 2, is as follows:

$$\Lambda_i = \begin{cases} (1 - p_l)(1 - \alpha_i)\frac{\bar{R}-R}{2} + (1 - \alpha_i)P^e + (1 - p_l)V & \text{if } \alpha_i \in [0, \phi) \\ (1 - \alpha_i) \left\{ \frac{\bar{R}-R}{2} - p_l p_s \Delta \left( 1 - \frac{\Delta/2}{\bar{R}-R} \right) + P^e \right\} + \left( 1 - \frac{\Delta}{\bar{R}-R} p_l p_s \right) V & \text{if } \alpha_i \in [\phi, 1) \\ V & \text{if } \alpha_i = 1 \end{cases}$$

Within each region, the probability of default does not depend on  $\alpha_i$ , and  $\pi_{d,i}(\alpha_i \in [0, \phi)) > \pi_{d,i}(\alpha_i \in [\phi, 1)) > \pi_{d,i}(\alpha_i = 1)$ . By contrast,  $E(\Pi_i)$  and  $(1 - \alpha_i)P^e$  are strictly decreasing in  $\alpha_i$ . Together, these properties imply the objective function under autarky has three unique local maxima at  $\alpha_i = \{0, \phi, 1\}$ . Thus,  $\Lambda_i$  takes the following values at the local maxima:

$$\Lambda_i = \begin{cases} (1 - p_l)\frac{\bar{R}-R}{2} + P^e + (1 - p_l)V & \text{if } \alpha_i = 0 \\ (1 - \phi) \left\{ \frac{\bar{R}-R}{2} - p_l p_s \Delta \left( 1 - \frac{\Delta/2}{\bar{R}-R} \right) + P^e \right\} + \left( 1 - \frac{\Delta}{\bar{R}-R} p_l p_s \right) V & \text{if } \alpha_i = \phi \\ V & \text{if } \alpha_i = 1 \end{cases}$$

Thus,  $\alpha_i = \phi$  is optimal if and only if  $\Lambda_i(\phi, e_i, 0) \geq \Lambda_i(0, e_i, 0)$  and  $\Lambda_i(\phi, e_i, 0) \geq \Lambda_i(1, e_i, 0)$ , which is equivalent to:

$$\begin{aligned} (p_l - \phi) \frac{\bar{R} - \underline{R}}{2} - p_l p_s (1 - \phi) \Delta \left( 1 - \frac{\Delta/2}{\bar{R} - \underline{R}} \right) + p_l \left( 1 - p_s \frac{\Delta}{\bar{R} - \underline{R}} \right) V &\geq \phi P^e \\ (1 - \phi) \left( \frac{\bar{R} - \underline{R}}{2} - p_l p_s \Delta \right) + p_l p_s (1 - \phi) \Delta \frac{\Delta/2}{\bar{R} - \underline{R}} + (1 - \phi) P^e &\geq \frac{\Delta}{\bar{R} - \underline{R}} p_l p_s V \end{aligned}$$

The final condition of Proposition 1 implies that when bank  $i$  chooses  $\alpha_i^* = \phi$ , it will also pick  $e_i^* = 0$ . Again, we simply rearrange the objective function above. Note that if bank  $i$  chooses  $e_i = 0$ , it receives:

$$\Lambda_i(\phi, 0, 0) = (1 - \phi) \left\{ \frac{\bar{R} - \underline{R}}{2} - p_l p_s^0 \Delta \left( 1 - \frac{\Delta/2}{\bar{R} - \underline{R}} \right) + P \right\} + \left( 1 - \frac{\Delta}{\bar{R} - \underline{R}} p_l p_s^1 \right) V$$

And if it chooses  $e_i = 1$ , it receives:

$$\Lambda_i(\phi, 1, 0) = (1 - \phi) \left\{ \frac{\bar{R} - \underline{R}}{2} - p_l p_s^1 \Delta \left( 1 - \frac{\Delta/2}{\bar{R} - \underline{R}} \right) \right\} + \left( 1 - \frac{\Delta}{\bar{R} - \underline{R}} p_l p_s^1 \right) V$$

Thus, if  $\Lambda_i(\phi, 0, 0) \geq \Lambda_i(\phi, 1, 0)$ , bank  $i$  will choose  $e_i^* = 0$ , which occurs when:

$$p_l (p_s^0 - p_s^1) (1 - \phi) \Delta \left( 1 - \frac{\Delta/2}{\bar{R} - \underline{R}} \right) + \frac{\Delta}{\bar{R} - \underline{R}} p_l (p_s^0 - p_s^1) V < (1 - \phi) P$$

As required.

## A.2 Proof of Proposition 2

Assume bank  $i$  holds  $\alpha_i < \phi$  and needs to borrow the difference through uncommitted lending relationships. First, bank  $i$  would never form uncommitted lending relationships

of total size  $B_i < \phi - \alpha_i$  as no lender would ever lend at  $t = 1$  (and thus autarky strictly dominates this case). Thus,  $B_i \geq \phi - \alpha_i$ .

Second, with  $\theta = 1$ , if lenders play a strategy in which lending takes place only when solvency shocks do not occur, bank  $i$  always defaults following a solvency shock at  $t = 1$ . If a solvency shock does not occur, bank  $i$  has debt of  $1 - \phi + B_i$  and assets of  $(1 - \alpha_i)\tilde{R}_i + (B_i - (\phi - \alpha_i))$ . It follows that:

$$\min(\mathcal{A}_i | l_i = 1, s_i = 0) = (1 - \alpha_i)\underline{R} + B_i - (\phi - \alpha_i) = 1 - \phi + B_i$$

Thus, bank  $i$  is always able to pay its liabilities in the state where a solvency shock does not occur, and never defaults if it does not receive a solvency shock. As a result, the interest rate at which bank  $i$  borrows in the absence of a solvency shock is zero. Thus, given  $\alpha_i$ , bank  $i$  is indifferent for all  $B_i \geq \phi - \alpha_i$ .

For the second part of the statement, recall that under autarky, a choice of  $\alpha_i = \phi$  gave bank  $i$ :

$$\Lambda_i(\phi, e_i, 0) = (1 - \phi) \left\{ \frac{\bar{R} - \underline{R}}{2} - p_l p_s \Delta \left( 1 - \frac{\Delta/2}{\bar{R} - \underline{R}} \right) + P^e \right\} + \left( 1 - \frac{\Delta}{\bar{R} - \underline{R}} p_l p_s \right) V$$

Whereas uncommitted lending relationships with  $\theta = 1$  gives:

$$\Lambda_i(\phi - \delta, e_i, \delta) = (1 - \phi + \delta)(1 - p_l p_s) \frac{\bar{R} - \underline{R}}{2} + (1 - \alpha_i + \delta)P^e + (1 - p_l p_s)V$$

Taking the difference, we have:

$$\begin{aligned} \Lambda_i(\phi, e_i, 0) - \Lambda_i(\phi - \delta, e_i, \delta) &= -\delta(1 - p_l p_s) \frac{\bar{R} - \underline{R}}{2} + \frac{p_l p_s}{\bar{R} - \underline{R}} \frac{1 - \phi}{2} (\bar{R} - \underline{R} - \Delta)^2 \\ &\quad - \delta P^e + p_l p_s \left( 1 - \frac{\Delta}{\bar{R} - \underline{R}} \right) V \end{aligned}$$

If we then take the limit as  $\delta \rightarrow 0$ , we have:

$$\lim_{\delta \rightarrow 0} (\Lambda_i(\phi, e_i, 0) - \Lambda_i(\phi - \delta, e_i, \delta)) = \frac{p_l p_s}{\bar{R} - \underline{R}} (\bar{R} - \underline{R} - \Delta) \left( \frac{1 - \phi}{2} (\bar{R} - \underline{R} - \Delta) + V \right) > 0$$

Thus, a discontinuous drop occurs in the objective function immediately below  $\alpha_i = \phi$  when a bank engages in uncommitted lending relationships with perfectly informed lenders.

However, differentiating with respect to  $\delta$  gives:

$$\frac{\partial}{\partial \delta} (\Lambda_i(\phi, e_i, 0) - \Lambda_i(\phi - \delta, e_i, \delta)) = -(1 - p_l p_s) \frac{\bar{R} - \underline{R}}{2} - \delta P^e < 0$$

Thus, if engaging in uncommitted lending relationships leads to an increase in  $\Lambda_i$  for some feasible  $B_i$ , banks can maximize  $\Lambda_i$  by choosing the minimum possible  $\underline{\alpha}_i = \max(\sum_{j \neq i} \alpha_j, 0)$  and setting  $B_i = \phi - \underline{\alpha}_i$ .

As required.

### A.3 Proof of Proposition 3

If  $\Delta > \bar{\Delta}$ , lenders are unwilling to provide liquidity at any interest rate. Thus, under uncommitted lending relationships, bank  $i$ 's choices  $e_i = 0, e_i = 1$  give:

$$\begin{aligned} \Lambda_i(\alpha_i, 1, B_i) &= (1 - \alpha_i)(1 - p_l p_s^1) \frac{\bar{R} - \underline{R}}{2} + (1 - \alpha_i)P^1 + (1 - p_l p_s^1)V \\ \Lambda_i(\alpha_i, 0, B_i) &= (1 - \alpha_i)(1 - p_l p_s^0) \frac{\bar{R} - \underline{R}}{2} + (1 - \alpha_i)P^0 + (1 - p_l p_s^0)V \end{aligned}$$

Taking a difference, we have:

$$\Lambda_i(\alpha_i, 1, B_i) - \Lambda_i(\alpha_i, 0, B_i) = (1 - \alpha_i)p_l(p_s^0 - p_s^1)\frac{\bar{R} - \underline{R}}{2} - (1 - \alpha_i)P + p_l(p_s^0 - p_s^1)V$$

Thus, banks prefer  $e_i = 0$  if and only if:

$$p_l(p_s^0 - p_s^1)\left(\frac{\bar{R} - \underline{R}}{2} + \frac{V}{1 - \alpha_i}\right) < P$$

This is the first condition of Proposition 3.

If  $\Delta \leq \bar{\Delta}$ , lenders are willing to provide liquidity, but charge bank  $i$  a higher interest rate when it receives a solvency shock. Note that banks do not want to “over-borrow” in this instance, because doing so will only increase their total cost of borrowing (see Proposition 8 - this scenario is analogous to committed lending relationships). It can be shown that:

$$\begin{aligned} \Lambda_i(\alpha_i, 1, B_i) &= (1 - \alpha_i) \left\{ \frac{\bar{R} - \underline{R}}{2} - p_l p_s^1 \Delta \left( 1 + \frac{\Delta/2}{\bar{R} - \underline{R}} \right) \right\} \\ &\quad - \frac{p_l p_s^1}{\bar{R} - \underline{R}} \left( \Delta(\phi - \alpha_i) - \frac{1}{2} \frac{(\phi - \alpha_i)^2}{1 - \alpha_i} \right) Z_i \\ &\quad + \left( 1 - p_l p_s^1 + \frac{p_l p_s^1}{\bar{R} - \underline{R}} \sqrt{(\bar{R} - \underline{R} - \Delta)^2 - \left( 2 \frac{\Delta B_i}{1 - \alpha_i} - \frac{B_i^2}{(1 - \alpha_i)^2} \right) Z_i} \right) V \\ \Lambda_i(\alpha_i, 0, B_i) &= (1 - \alpha_i) \left\{ \frac{\bar{R} - \underline{R}}{2} - p_l p_s^0 \Delta \left( 1 + \frac{\Delta/2}{\bar{R} - \underline{R}} \right) \right\} \\ &\quad - \frac{p_l p_s^0}{\bar{R} - \underline{R}} \left( \Delta(\phi - \alpha_i) - \frac{1}{2} \frac{(\phi - \alpha_i)^2}{1 - \alpha_i} \right) Z_i + (1 - \alpha_i)P \\ &\quad + \left( 1 - p_l p_s^0 + \frac{p_l p_s^0}{\bar{R} - \underline{R}} \sqrt{(\bar{R} - \underline{R} - \Delta)^2 - \left( 2 \frac{\Delta B_i}{1 - \alpha_i} - \frac{B_i^2}{(1 - \alpha_i)^2} \right) Z_i} \right) V \end{aligned}$$

Where:

$$Z_i = 1 + \frac{1}{\bar{R} - \underline{R}} \sum_{-i} \frac{V}{1 - \alpha_{-i}}$$

Thus, taking differences and rearranging, we have that incentive alignment takes place when:

$$P < p_l \left( p_s^0 - p_s^1 \right) \left\{ \frac{\bar{R} - \underline{R}}{2} + \frac{V}{(1 - \alpha_i)} - \frac{\psi_i}{\bar{R} - \underline{R}} \left( \frac{\psi_i}{2} + \frac{V}{(1 - \alpha_i)} \right) \right\}$$

Where:

$$\psi_i = \sqrt{(\bar{R} - \underline{R} - \Delta)^2 + 2 \left( \frac{1}{2} \frac{(\phi - \alpha_i)^2}{(1 - \alpha_i)^2} - \frac{\phi - \alpha_i}{1 - \alpha_i} \Delta \right) Z_i}$$

As required.

## A.4 Proof of Proposition 4

If the conditions in Proposition 1 are satisfied, we know that  $\alpha_i = \phi$  and  $e_i = 0 \forall i$  is the unique equilibrium when no lending relationships exist. Now, suppose banks engage in lending relationships. Conjecture a set of beliefs whereby bank  $i$  believes it can borrow from bank  $j$  if and only if bank  $i$  is able to promise bank  $j$  with a normal rate of return. Proposition 2.1 implies banks with lending relationships must have  $\alpha_i < \phi$  and  $B_i \geq \phi - \alpha_i$ . Moreover, Proposition 2.2 implies banks that use lending relationships will hold the minimum liquid assets possible,  $\underline{\alpha}_i = \max(0, \phi - \sum_{j \neq i} \alpha_j)$ . Because every bank  $i$  that uses lending relationships wants to hold  $\underline{\alpha}_i$ , it immediately follows that an equilibrium involving lending relationships and the conjectured beliefs must satisfy both  $\sum_i \alpha_i = \phi$  and  $\alpha_i + B_i = \phi, \forall i$ . Focusing on a symmetric equilibrium, we have that  $\alpha_i = \frac{\phi}{N} \forall i$ , and  $B_{ij} = \frac{\phi}{N} \forall i, j$ . Because  $B_{ij} \leq \alpha_j, \forall i, j$ , we have that lenders' participation constraints are satisfied. Moreover, if the conditions in Proposition 3 are satisfied, it follows that  $e_i = 1$  in equilibrium. I appeal to Propositions 1-3 to argue that as long as  $\Lambda_i \left( \frac{\phi}{N}, 1, \left\{ \frac{\phi}{N} \right\} \right) \geq \Lambda_0$

(i.e., autarky is dominated), no bank  $i$  can unilaterally change its choice of  $\alpha_i, e_i, \{B_{ij}\}$  and increase  $\Lambda_i$ .

Next, note that when  $\Delta > \bar{\Delta}$ , lending does not take place following a solvency shock, because borrowers cannot provide lenders with a normal rate of return. Similarly, when  $\Delta < \bar{\Delta}$ , lending occurs following a solvency shock, because borrowers still have sufficient future funds they can promise lenders. Importantly, we see that beliefs over lending decisions are consistent with actions, and lenders' incentive compatibility constraints are satisfied. Finally, to prevent downward deviations in the choice of liquid assets by individual banks, I specify a set of off-equilibrium beliefs in which any bank that deviates to some  $\hat{\alpha}_i < \alpha_i^* = \frac{\phi}{N}$  will be unable to borrow in interbank markets.<sup>1</sup>

Thus, given the conjectured beliefs, a symmetric equilibrium exists with  $\alpha_i = \frac{\phi}{N} \forall i$ ,  $e_i = 1 \forall i$ , and  $B_{ij} = \frac{\phi}{N} \forall i, j$ , providing  $\Lambda_i\left(\frac{\phi}{N}, 1, \{\frac{\phi}{N}\}\right) \geq \Lambda_0$ .

## A.5 Proof of Proposition 5

Consider the signalling game between borrower  $i$  and a potentially uninformed lender  $j$ , which takes place at the intermediate date of the model. For exposition purposes, assume only one lender exists, and borrower  $i$  defaults at the intermediate date if the lender decides not to lend.<sup>2</sup>

Let  $U_i(s_i, r_i, a_j)$  be the utility of the borrower,  $U_j(s_i, t_j, r_i, a_j)$  be the utility of the lender,  $s_i = \{0, 1\}$  be the borrower's type, and  $t_j = \{A, B\}$  be the lender's type. A borrower of type

---

1. If, instead, bank  $i$  believed that a downwards deviation of size  $\varepsilon_i$  to some  $\hat{\alpha}_i$  would be met by a corresponding increase in liquidity of  $\varepsilon_j$  by bank  $j$  and continued access to interbank markets, the above set of choices would not constitute an equilibrium.

2. The proof has a natural extension to  $n$  lenders. In this situation, we assume lenders receive the same signal from borrowers but do not know each other's type or action.



$s_i = 0$  is a healthy borrower (did not receive solvency shock), whereas a borrower of type  $s_i = 1$  is a distressed borrower (received solvency shock). A lender of type  $t_j = A$  is an informed lender (knows  $s_i$ ), and a lender of type  $t_j = B$  is an uninformed lender (does not know  $s_i$ ).

Let  $r_i$  be the borrower's signal, which is a linear transformation of both  $\bar{F}_i$  and  $\gamma_i$ . I define a normal rate of return offered by a borrower as the  $r_i$  that sets  $\mathbb{E}(U_j(s_i, t_j, r_i, a_j) | \mathcal{I}_j) = 0$ . Recall that the lender's reservation utility is zero, and  $r_i = 0$  provides an informed lender with a normal rate of return if the borrower is type 0. I define  $r_i^d$  to be the interest rate the type 1 borrower offers that provides an informed lender a normal rate of return, and  $\bar{r}_i$  to be the interest rate offers that promises the entire future funds of the type 1 borrower to the lender. Similarly, let  $a_j \in [0, 1]$  be the lender's action, where  $a_j = 0$  is equivalent to providing liquidity with probability zero. I allow  $r_i$  to be conditional on the borrower's type, and  $a_j$  to be conditional on the borrower's signal and lender's type.

The structure of the game is as follows:

1. Borrower  $i$  learns its type  $s_i = \{0, 1\}$ , with  $Pr(s_i = 1) = p_s$ .
2. Lender  $j$  learns its type  $t_j = \{A, B\}$  with  $Pr(t_j = A) = \theta$ .
3. Borrower  $i$ , which knows its type, makes an offer to lender  $j$  to borrow the liquidity it requires to survive at interest rate  $r_i$ .
4. Conditional on its type  $t_j$ , lender  $j$  plays action  $a_j$ , accept with probability  $0 < a_j < 1$ .  
If lender  $j$  accepts, the borrower gets  $U_i(s_i, r_i, 1)$  and the lender gets  $U(s_i, t_j, r_i, 1)$ .  
If lender  $j$  rejects, both parties get 0.

Before continuing, I state without proof that for both types  $s_i = \{0, 1\}$ :

$$\begin{aligned}\frac{\partial U_i}{\partial r_i} &< 0 \quad r_i \in [0, r_i^d), \quad a_i > 0 \\ \frac{\partial U_i}{\partial a_i} &> 0 \quad r_i \in [0, r_i^d), \quad a_i > 0\end{aligned}$$

Note that regardless of the borrower's type, if  $r_i < 0$ , the lender's best response is  $a_j = 0$  because  $a_j > 0$  will give  $U_j < 0$ . Moreover, assuming  $r_i^d \leq \bar{r}_i$ , then regardless of the borrower's type, if  $r_i > r_i^d$ , the lender's best response is  $a_j = 1$  because  $a_j > 0$  will give  $U_j > 0$  and  $\frac{\partial U_j}{\partial a_j} > 0$ . Thus, because the borrower is profit maximizing, I restrict attention to signals  $r_i \in [0, r_i^d]$ . For exposition, I assume type  $A$  lenders take action  $a_j = 1$  if and only if the borrower promises them at least a normal rate of return.<sup>3</sup>

Assume for now that  $r_i \in [0, r_i^d)$ . I will first construct the best pooling and separating equilibria from the borrower's perspective, before later arguing why other equilibria do not survive the D1 criterion.

Consider the pooling equilibrium discussed in the paper. In this pooling equilibrium,  $r_i(s_i = 1) = r_i(s_i = 0)$  and  $a_j = 1$  for the type  $B$  lender.<sup>4</sup> Given the pooling strategy played by the borrower, the type  $B$  lender places prior probability  $p_s$  on the type 1 borrower. Moreover, the pooling interest rate  $r_i^p$  must be set to maximize borrower profits. I conjecture this interest rate is equivalent to the interest rate at which an uninformed lender makes exactly a normal rate of return:

$$p_s U_j(1, B, r_i^p, 1) + (1 - p_s) U_j(0, B, r_i^p, 1) \geq 0 \quad (\text{A.1})$$

---

3. One can generalize and allow for mixed strategies when the borrower offers exactly a normal rate of return, but doing so adds very little.

4. Note these interest rates do not mean the lender always provides liquidity; it simply means an uninformed lender always provides liquidity (those that do not receive the  $\theta$  signal).

Note that when this condition is satisfied, a type  $B$  lender will always (weakly) choose to lend. Thus,  $a_j = 1$  is a best response for the type  $B$  lender. Moreover, any pooling interest rate below  $r_i^P$  would result in the type  $B$  lender playing  $a_j = 0$ ; otherwise, it would receive  $U_j < 0$ . However, any pooling interest rate above  $r_i^P$  is not possible because such a rate would not be profit maximizing from the borrower's perspective. Given  $r_i^P$ , it is also immediate that the type  $A$  lender is playing its best response. A second condition is required for the pooling equilibrium to ensure both borrower types are playing a best response:

$$U_i(0, r_i^P, 1) \geq \theta U_i(0, 0, 1) \quad (\text{A.2})$$

Condition (2) guarantees a type 0 borrower does not want to deviate to an interest rate of  $r_i = 0$ . Note the type 1 borrower would never want to deviate downwards because doing so would result in  $U_j = 0$  (neither type of lender would be willing to lend to it). Thus, if we restrict  $r_i \in [0, r_i^d)$ , the conditions (1) and (2) are sufficient for the pooling equilibrium.

We now focus on separating equilibria. Conjecture a separating equilibrium where  $r_i(s_i = 1) \neq r_i(s_i = 0)$ . It follows that the type  $B$  lender's best response is  $a_i(r_i(s_i = 1)) = 0$  and  $a_i(r_i(s_i = 0)) = 1$  because lending to the type 1 borrower guarantees  $U_j < 0$  and lending to the type 0 borrower guarantees  $U_j > 0$  (with  $\frac{\partial U_j}{\partial a_j} > 0$ ). Note this strategy is identical to the one played by the type  $A$  lender (who is also informed). Because lending to the type 1 borrower never occurs, it always receives its reservation utility,  $U_i = 0$ . Thus, what is conjectured cannot constitute an equilibrium, because the type 1 borrower has the incentive to deviate and mimic the type 0 borrower for  $U_i > 0$ . Thus, no separating equilibrium exists when  $r_i(s_i = 1) < r_i^d$ .

However, a separating equilibrium might occur when  $r_i(s_i = 1) = r_i^d$ . To see why, note  $r_i^d$  gives the lender a normal rate of return, regardless of borrower type. Thus, if the type 1 borrower separates using  $r_i = r_i^d$ , both lender types would be willing to play  $a_j = 1$  in

response. Consequently, the type 1 borrower can achieve  $U_i > 0$  provided  $r_i^d < \bar{r}_i$ . I then conjecture that a type 0 borrower sets an interest rate  $0 \leq r_i^h \leq r_i^p$ . It follows that the type 1 borrower has no incentive to deviate and mimic the type 0 borrower so long as:

$$U_i(1, r_i^d, 1) > (1 - \theta)U_i(1, r_i^h, 1) \quad (\text{A.3})$$

Interestingly, because higher interest rates are more costly, one might think the type 0 borrower would signal through this more costly action. However, the possible presence of a type A lender is in fact what deters the type 1 borrower from mimicking the type 0 borrower when it chooses a lower interest rate in the separating equilibrium. That is, the threat of liquidity rationing is the deterrent, because it encourages the distressed borrower to accept the higher interest rate. As such, we are able to construct a separating equilibrium where  $r_i(s_i = 1) = r_i^d$ ,  $r_i(s_i = 0) = r_i^h$ , and  $a_j = 1$  for both lender types.

Note condition (3) is also a condition for the existence of the pooling equilibrium once we allow  $r_i \in [0, r_i^d]$ . If this condition is violated, the type 1 borrower would deviate to  $r_i = r_i^d$ , and the separating equilibrium would cease to exist. Moreover, I leave it to the reader to verify that if  $\bar{r}_i < r_i^d$ , only pooling equilibria exist.<sup>5</sup>

One can also generalize conditions (2) and (3) to a case with  $n$  lenders. For example, in the case in which each of the  $n$  lenders is marginal,<sup>6</sup> the conditions are:

$$\begin{aligned} U_i(0, r_i^p, 1) &\geq \theta^n U_i(0, 0, 1) \\ U_i(1, r_i^d, 1) &> (1 - \theta)^n U_i(1, r_i^h, 1) \end{aligned}$$

---

5. The argument relies on the fact that the type 0 borrower can send no (costly) signal that would encourage the type 1 borrower to pick a different interest rate.

6. Marginal here means each is required to lend in order for the borrower to survive until  $t = 2$ .

Finally, other equilibria are possible depending on the specified off-equilibrium beliefs. For example, we could have a pooling equilibrium with  $r_i = r_i^p + \delta$  for  $\delta$  small and positive. However, such an equilibrium does not survive the D1 criterion, because the type 0 borrower can increase its  $U_i$  by deviating downwards to  $r_i = r_i^p$  (and still have the lender provide liquidity as long as the off-equilibrium beliefs are not crazy). A similar argument can be applied to eliminate other pooling equilibria (supported by various off-equilibrium beliefs) where  $r_i(s_i = 0) > r_i^h$  and/or  $r_i(s_i = 1) > r_i^d$ .

However, note we cannot use D1 to eliminate the pooling equilibrium. Recall the separating equilibrium exists when (3) holds, because the type 1 borrower is unwilling to mimic the type 0 borrower due to the threat of liquidity rationing from the type A lender. Thus, in effect, the type 0 borrower's signal is costly to the type 1 borrower because it satisfies  $r_i^h < r_i^d$ .<sup>7</sup> Now, for any  $r_i^h < r_i^d$ , mimicking the type 0 borrower's choices of  $r_i$  is less costly for the type 1 borrower. Thus, because the type 0 borrower will never set  $r_i > r_i^p$ , if condition (3) is violated for  $r_i^h = r_i^p$ , a pooling equilibrium must ensue. Put differently, the most costly signal the type 0 borrower is willing to send is  $r_i = r_i^p$ , and if the type 1 borrower is willing to mimic it, then a pooling equilibrium is the only outcome.

## A.6 Proof of Proposition 6

Appendix 2 shows:

$$C(\theta) = \sum_{k=0}^n kC_k = n \left( \hat{\pi}_{d,(0)}(1 - \theta) + (1 - \theta)^n(\hat{\pi}_{d,(1)} - \hat{\pi}_{d,(0)}) \right)$$

---

7. In fact, conditional on being able to borrow, the signal is less costly in terms of interest rate.

Where:

$$\begin{aligned}\hat{\pi}_{d,(0)} &= \frac{1}{\bar{R} - \underline{R}} \frac{\phi/N}{1 - \phi/N} \\ \hat{\pi}_{d,(1)} &= \frac{1}{(\bar{R} - \underline{R})^2} \frac{\phi/N}{1 - \phi/N} \left\{ \Delta - \frac{1}{2} \frac{(N-1)\phi}{1 - \phi} \right\}\end{aligned}$$

The first statement follows immediately from the second derivative of the function  $C(\theta)$ :

$$\frac{\partial^2 C}{\partial \theta^2} = n^2(n-1)(1-\theta)^{n-2}(\hat{\pi}_{d,(1)} - \hat{\pi}_{d,(0)}) < 0$$

For the second statement, note the concavity of  $C(\theta)$  implies the maximum occurs at either  $\theta = 0$  (if  $C'(0) < 0$ ), or at  $\theta = 1$  (if  $C'(1) > 0$ ), or at an intermediate  $\bar{\theta}$  that solves  $C'(\bar{\theta}) = 0$ . Because by construction, (i)  $C(\theta) \geq 0$ , (ii)  $C(0) > 0$ , and (iii)  $C(1) = 0$ , we can rule out  $\theta = 1$  as a maximum. Thus, we simply need to check the derivative at  $\theta = 0$ , which can be shown to be:

$$\frac{\partial C}{\partial \theta} \Big|_{\theta=0} \propto (n-1)\hat{\pi}_{d,(0)} - n\hat{\pi}_{d,(1)}$$

Thus, the maximum occurs at  $\theta = 0$  if and only if:

$$\frac{\partial C}{\partial \theta} \Big|_{\theta=0} < 0 \iff \frac{(n-1)\hat{\pi}_{d,(0)}}{n\hat{\pi}_{d,(1)}} < 1$$

Likewise, if the inequality is reversed, it immediately follows that the maximum occurs at  $\theta = \bar{\theta}$  where  $\bar{\theta}$  solves:

$$n \left( -\hat{\pi}_{d,(0)} - n(1-\bar{\theta})^{n-1}(\hat{\pi}_{d,(1)} - \hat{\pi}_{d,(0)}) \right) = 0$$

As required.

## A.7 Proof of Proposition 7

For  $C_0(\theta)$ , note that:

$$C_0 = \left( \theta + (1 - \theta)(1 - \hat{\pi}_{d,(0)}) \right)^n + (1 - \theta)^n \left( (1 - \hat{\pi}_{d,(1)})^n - (1 - \hat{\pi}_{d,(0)})^n \right)$$

Then taking derivatives:

$$\begin{aligned} \frac{\partial C_0}{\partial \theta} &= n\hat{\pi}_{d,(0)} \left( \theta + (1 - \theta)(1 - \hat{\pi}_{d,(0)}) \right)^{n-1} \\ &\quad - n(1 - \theta)^{n-1} \left( (1 - \hat{\pi}_{d,(1)})^n - (1 - \hat{\pi}_{d,(0)})^n \right) \\ \frac{\partial^2 C_0}{\partial \theta^2} &= (n^2 - n)\hat{\pi}_{d,(0)}^2 \left( \theta + (1 - \theta)(1 - \hat{\pi}_{d,(0)}) \right)^{n-2} \\ &\quad + (n^2 - n)(1 - \theta)^{n-2} \left( (1 - \hat{\pi}_{d,(1)})^n - (1 - \hat{\pi}_{d,(0)})^n \right) \\ &> 0 \end{aligned}$$

Thus,  $C_0$  is globally convex. Now  $C_0$  will be strictly increasing if and only if  $\frac{\partial C_0}{\partial \theta}|_{\theta=0} >$

0. We can show:

$$\frac{\partial C_0}{\partial \theta}|_{\theta=0} = n(1 - \hat{\pi}_{d,(0)})^{n-1} - n(1 - \hat{\pi}_{d,(1)})^n$$

Which is positive if and only if:

$$\frac{(1 - \hat{\pi}_{d,(0)})^{n-1}}{(1 - \hat{\pi}_{d,(1)})^n} > 1$$

For  $C_n(\theta)$ , note that:

$$C_n = (1 - \theta)^n \hat{\pi}_{d,(1)}^n$$

Then taking derivatives:

$$\begin{aligned}\frac{\partial C_n}{\partial \theta} &= -n(1-\theta)^{n-1}\hat{\pi}_{d,(1)}^n < 0 \\ \frac{\partial^2 C_n}{\partial \theta^2} &= n(n-1)(1-\theta)^{n-2}\hat{\pi}_{d,(1)}^n > 0\end{aligned}$$

Which proves  $C_n$  is both globally convex and strictly decreasing in  $\theta$ .

Next, consider the second statement, which covers  $0 < k < n$ . Note that:

$$\begin{aligned}C_k &= \binom{n}{k} \left( \hat{\pi}_{d,(0)}(1-\theta) \right)^k \left( \theta + (1-\theta)(1-\hat{\pi}_{d,(0)}) \right)^{n-k} \\ &\quad + (1-\theta)^n \binom{n}{k} \left( \hat{\pi}_{d,(1)}^k (1-\hat{\pi}_{d,(1)})^{n-k} - \hat{\pi}_{d,(0)}^k (1-\hat{\pi}_{d,(0)})^{n-k} \right)\end{aligned}$$

It follows that:

$$\begin{aligned}\frac{\partial C_k}{\partial \theta} &= \binom{n}{k} \hat{\pi}_{d,(0)} \left( -k \left( \hat{\pi}_{d,(0)}(1-\theta) \right)^{k-1} \left( \theta + (1-\theta)(1-\hat{\pi}_{d,(0)}) \right)^{n-k} \right) \\ &\quad + \binom{n}{k} \hat{\pi}_{d,(0)}(n-k) \left( \hat{\pi}_{d,(0)}(1-\theta) \right)^k \left( \theta + (1-\theta)(1-\hat{\pi}_{d,(0)}) \right)^{n-k-1} \\ &\quad - \binom{n}{k} n(1-\theta)^{n-1} \left( \hat{\pi}_{d,(1)}^k (1-\hat{\pi}_{d,(1)})^{n-k} - \hat{\pi}_{d,(0)}^k (1-\hat{\pi}_{d,(0)})^{n-k} \right) \\ \frac{\partial C_k}{\partial \theta}|_{\theta=0} &= (n-k) \binom{n}{k} \hat{\pi}_{d,(0)}^k (1-\hat{\pi}_{d,(0)})^{n-k-1} - n \binom{n}{k} \hat{\pi}_{d,(1)}^k (1-\hat{\pi}_{d,(1)})^{n-k}\end{aligned}$$



Which means for fixed  $k$ , the maximizing  $\theta$  is greater than zero if and only if:

$$\frac{(n-k)}{n} \left( \frac{\hat{\pi}_{d,(0)}}{\hat{\pi}_{d,(1)}} \right)^k \frac{(1 - \hat{\pi}_{d,(0)})^{n-k-1}}{(1 - \hat{\pi}_{d,(1)})^{n-k}} > 1$$

Finally, to prove the third statement note that if  $\theta = 1$   $C_k(\theta)$  simplifies to

$$C_k = \binom{n}{k} (0)^k (1)^{n-k}$$

Which is equal to 1 if and only if  $k = 0$ , and zero otherwise.

## A.8 Proof of Proposition 8

For the first statement, note from bank  $i$ 's objective function that for a given  $\alpha_i$ , borrowing over and above  $B_i = \phi - \alpha_i$  affects  $\Lambda_i$  negatively through the compensation bank  $i$  owes its lenders  $\hat{\pi}_{d,i}V$  and through its impact on bank  $i$ 's own probability of default. The proof shows that both  $\hat{\pi}_{d,i}$  and  $\pi_{d,i}$  are increasing in  $B_i$  for  $B_i > \phi - \alpha_i$ .

Fix bank  $i$ 's choice of  $\alpha_i$ , and assume  $\alpha_i + B_i \geq \phi$  (note that if it were less, repayment would never be possible and the bank would be better off in autarky). Define  $\delta_i = B_i - (\phi - \alpha_i)$  to be the extra amount borrowed by bank  $i$  above what it needs to pay for a liquidity shock. Assume for now that  $\delta_i \leq \bar{\delta}_i = 1 - \phi - (1 - \alpha_i)(\underline{R} - \Delta)$ .

Step 1: I show the probability of default is increasing in  $B_i$  (equivalently  $\delta_i$ ) for  $\delta_i \in [0, \bar{\delta}_i]$ . Consider the following:

$$\hat{\pi}_{d,ij} = \frac{1}{(\bar{R} - \underline{R})^2} \frac{B_{ij}}{1 - \alpha_j} \left\{ \Delta - \frac{1}{2} \frac{B_i}{1 - \alpha_i} \right\}$$

Thus:

$$\begin{aligned}
\hat{\pi}_{d,i}V &= \sum_j \hat{\pi}_{d,ij}V \\
&= \sum_j \frac{1}{(\bar{R} - \underline{R})^2} \frac{B_{ij}}{1 - \alpha_j} \left\{ \Delta - \frac{1}{2} \frac{B_i}{1 - \alpha_i} \right\} V \\
\frac{\partial \hat{\pi}_{d,i}V}{\partial B_{ij}} &\propto (1 - \alpha_i)\Delta - \frac{1}{2} \sum_{-i \neq j} B_{i-i} - B_{ij} - \frac{1}{2} B_i \\
&= (1 - \alpha_i)\Delta - B_i \\
&\geq (1 - \alpha_i)\Delta - (\phi - \alpha_i + (1 - \phi - (1 - \alpha_i)(\underline{R} - \Delta)) \\
&= 0
\end{aligned}$$

Step 2: Consider the probability the principal is repaid in full:

$$\begin{aligned}
Pr(\delta_1 + (1 - \alpha_i)\tilde{R}_i \leq 1 - \phi + B_i) &= Pr\left(\tilde{R}_i \leq \frac{1 - \phi + B_i - \delta}{1 - \alpha_i}\right) \\
&= Pr\left(\tilde{R}_i \leq \frac{1 - \phi + B_i - (B_i - (\phi - \alpha_i))}{1 - \alpha_i}\right) \\
&= Pr(\tilde{R}_i \leq 1)
\end{aligned}$$

Thus, we see that conditional on  $\alpha_i + B_i \geq \phi$ , the probability the principal for interbank debt is repaid in full does not depend on  $\alpha_i$  or  $B_i$ . Thus, we can partition  $\tilde{R}_i$  into two disjoint and exhaustive subsets. Define  $\mathcal{E} = [\tilde{R}_i < 1]$  and  $\mathcal{E}^c = [\tilde{R}_i \geq 1]$ .

Step 3: For lender  $j$  to break even on its lending, we require:

$$\begin{aligned}
B_{ij} + \hat{\pi}_{d,ij}V &= \mathbb{E}(\tilde{F}_{ij}) \\
&= Pr(\mathcal{E})\mathbb{E}(\tilde{F}_{ij}|\mathcal{E}) + Pr(\mathcal{E}^c)\mathbb{E}(\tilde{F}_{ij}|\mathcal{E}^c) \\
&= Pr(\tilde{R}_i < 1)\mathbb{E}(\tilde{F}_{ij}|\mathcal{E}) + Pr(\tilde{R}_i \geq 1)\mathbb{E}(\tilde{F}_{ij}|\mathcal{E}^c) \\
\implies 0 &= Pr(\tilde{R}_i < 1)(B_{ij} - \mathbb{E}(\tilde{F}_{ij}|\mathcal{E})) + Pr(\tilde{R}_i \geq 1)(B_{ij} - \mathbb{E}(\tilde{F}_{ij}|\mathcal{E}^c)) + \hat{\pi}_{d,ij}V
\end{aligned}$$

Now, taking a derivative, and noting that  $Pr(\mathcal{E})$  and  $Pr(\mathcal{E}^c)$  are fixed, we have:

$$0 = Pr(\tilde{R}_i < 1) \frac{\partial(B_{ij} - \mathbb{E}(\tilde{F}_{ij}|\mathcal{E}))}{\partial B_{ij}} + Pr(\tilde{R}_i \geq 1) \frac{\partial(B_{ij} - \mathbb{E}(\tilde{F}_{ij}|\mathcal{E}^c))}{\partial B_{ij}} + \frac{\partial \hat{\pi}_{d,ij}V}{\partial B_{ij}}$$

Step 4: Solve for  $\frac{\partial(B_{ij} - \mathbb{E}(\tilde{F}_{ij}|\mathcal{E}))}{\partial B_{ij}}$ . Note that  $\mathcal{E} = \{\tilde{R}_i < 1\} = \{\tilde{F}_i < B_i\}$ . Also note that  $\{\mathcal{E}|s=0\} = \emptyset$ . We have:

$$\begin{aligned}
B_i - \mathbb{E}(\tilde{F}_i|\tilde{F}_i < B_i) &= B_i - \mathbb{E}(\tilde{F}_i|s=1, \tilde{F}_i < B_i) \\
&= Pr(\tilde{F}_i = 0|s=1, \tilde{F}_i < B_i)(B_i - \mathbb{E}(\tilde{F}_i|s=1, \tilde{F}_i = 0)) \\
&\quad + Pr(\tilde{F}_i > 0|s=1, \tilde{F}_i \leq B_i)(B_i - \mathbb{E}(\tilde{F}_i|s=1, 0 < \tilde{F}_i < B_i)) \\
&= G(R^{**}) \times 0 + (G(\underline{R}) - G(R^{**})) \left( B_i - \frac{B_i}{2} \right) \\
&= \frac{1}{\bar{R} - \underline{R}} (\underline{R} - R^{**}) \frac{B_i}{2} \\
&= \frac{1}{\bar{R} - \underline{R}} \left( \Delta - \frac{1}{2} \frac{B_i}{1 - \alpha_i} \right) \frac{B_i}{2}
\end{aligned}$$

Where  $R_i^{**}$  is defined as:

$$\forall R > R_i^{**} : F_{ij}(R_i^{**}) > 0$$

Then, taking the derivative with respect to  $B_i$  we have:

$$\begin{aligned}
\frac{\partial (B_i - \mathbb{E}(\tilde{F}_i | \tilde{F}_i < B_i))}{\partial B_i} &\propto (1 - \alpha_i)\Delta - B_i \\
&= (1 - \alpha_i)\Delta - (\phi - \alpha_i + \delta_i) \\
&\geq (1 - \alpha_i)\Delta - (\phi - \alpha_i + (1 - \phi - (1 - \alpha_i)(\underline{R} - \Delta))) \\
&= 0
\end{aligned}$$

Step 5: Thus, for  $B_i \leq 1 - \phi + (1 - \alpha_i)(\underline{R} - \Delta)$ ,  $\frac{\partial \hat{\pi}_{d,i}}{\partial B_i} V > 0$ , and  $\frac{\partial (B_i - \mathbb{E}(\tilde{F}_i | \mathcal{E}))}{\partial B_i} > 0$ , it follows that  $\frac{\partial (B_i - \mathbb{E}(\tilde{F}_i | \mathcal{E}^c))}{\partial B_i} < 0$ , which means  $\tilde{F}_i$  must be increasing more quickly than  $B_i$ , or in words, as we increase  $B_i$  over the range  $\delta_i \in [0, 1 - \phi - (1 - \alpha_i)(\underline{R} - \Delta)]$ , the promised payment must increase by even more.

Step 6: By extension, because  $F_i - B_i$  is increasing over the range  $\delta_i \in [0, 1 - \phi - (1 - \alpha_i)(\underline{R} - \Delta)]$ , we know the probability of default must be increasing:

$$\frac{\partial \pi_{d,i}}{\partial B_i} = \frac{1}{\bar{R} - \underline{R}} \frac{\partial}{\partial B_i} \left( \frac{\bar{F}_i - B_i}{1 - \alpha_i} + p_s \Delta \right) \propto \frac{\partial (\bar{F}_i - B_i)}{\partial B_i} > 0$$

Finally, to complete the proof, we must consider the case in which  $\delta_i > 1 - \phi - (1 - \alpha_i)(\underline{R} - \Delta)$ . When  $\delta_i = \bar{\delta}_i$ , notice the minimum amount of assets bank  $i$  has available at  $t = 2$  is:

$$\begin{aligned}
(1 - \alpha_i)(\underline{R} - \Delta) + B_i &= (1 - \alpha_i)(\underline{R} - \Delta) + 1 - \phi - (1 - \alpha_i)(\underline{R} - \Delta) \\
&= 1 - \phi \\
&= D_i
\end{aligned}$$

That is, if bank  $i$  borrows  $B_i = 1 - \phi - (1 - \alpha_i)(\underline{R} - \Delta)$ , it is always able to repay depositors at  $t = 2$ . Thus, any additional borrowing above this point does not alter either

the promised payment or the lender's probability of default. That is, the additional amount is borrowed at  $t = 1$ , and is available at  $t = 2$  for immediate repayment to whomever loaned it. Thus, for  $\forall \delta_i > \bar{\delta}_i$ ,  $F(\delta_i) = F(\bar{\delta}_i)$ ,  $\hat{\pi}_{d,i}(\delta_i) = \hat{\pi}_{d,i}(\bar{\delta}_i)$ , and therefore  $\pi_{d,i}(\delta_i) = \pi_{d,i}(\bar{\delta}_i)$ .

Putting the pieces together, we see  $\pi_{d,i}(\delta_i)$  is increasing for  $\delta_i \in [0, \bar{\delta}_i]$  and flat for  $\delta_i > \bar{\delta}_i$ . Thus, bank  $i$  would never want to borrow more than  $\phi - \alpha_i$  when using committed lending relationships, and will always obtain promises of  $B_i = \phi - \alpha_i$ .

For the second statement, the objective function can be written as:

$$\begin{aligned} \Lambda_i = & (1 - \alpha_i) \left( \frac{\bar{R} - \underline{R}}{2} - p_l p_s \Delta + p_l p_s \frac{1}{2} \frac{\Delta^2}{\bar{R} - \underline{R}} \right) \\ & - p_l \frac{p_s}{\bar{R} - \underline{R}} (\phi - \alpha_i) \left( \Delta - \frac{1}{2} \frac{\phi - \alpha_i}{1 - \alpha_i} \right) Z_i + (1 - \alpha_i) P^e \\ & + \left( 1 - p_l + \frac{p_l}{\bar{R} - \underline{R}} \sqrt{(\bar{R} - \underline{R} - p_s \Delta)^2 - 2 p_s \frac{\phi - \alpha_i}{1 - \alpha_i} \left( \Delta - \frac{1}{2} \frac{\phi - \alpha_i}{1 - \alpha_i} \right) Z_i} \right) V \end{aligned}$$

The key to the proof is to show the second derivative is positive (objective function is convex). If we have convexity, and some  $\alpha'_i \in [\underline{\alpha}_i, \phi)$  exists that gives a greater return (with its associated lending relationships) than  $\alpha_i = \phi$ , then convexity implies the objective function must be strictly decreasing over the range  $\alpha_i \in [\underline{\alpha}_i, \alpha'_i]$ , and therefore the optimal  $\alpha_i$  is  $\underline{\alpha}_i$ .

Thus, to show convexity, first note that due to the linearity of some terms, we have:

$$\begin{aligned} \frac{\partial^2 \Lambda_i}{\partial \alpha_i^2} & \propto \frac{\partial}{\partial \alpha_i^2} \left\{ -c_i + \frac{1}{\bar{R} - \underline{R}} \sqrt{(\bar{R} - \underline{R} - p_s \Delta)^2 - 2 \frac{1}{1 - \alpha_i} (\bar{R} - \underline{R}) c_i V} \right\} \\ & = -\frac{\partial c_i}{\partial \alpha_i^2} - \frac{V}{\bar{R} - \underline{R}} \frac{1}{X^{3/2}} \left( X \frac{\partial}{\partial \alpha_i^2} \left( \frac{(\bar{R} - \underline{R})}{1 - \alpha_i} c_i \right) + \left( \frac{\partial}{\partial \alpha_i} \frac{(\bar{R} - \underline{R})}{1 - \alpha_i} c_i \right)^2 \right) \end{aligned}$$

Where:

$$\begin{aligned} X &= (\bar{R} - \underline{R} - p_s \Delta)^2 - 2 \frac{1}{1 - \alpha_i} (\bar{R} - \underline{R}) c_i \\ c_i &= \frac{p_s Z_i}{\bar{R} - \underline{R}} (\phi - \alpha_i) \left( \Delta - \frac{1}{2} \frac{\phi - \alpha_i}{1 - \alpha_i} \right) \end{aligned}$$

It can then be shown:

$$\begin{aligned} \frac{\partial^2 c_i}{\partial \alpha_i^2} &= -\frac{p_s Z_i}{\bar{R} - \underline{R}} \frac{(1 - \phi)^2}{(1 - \alpha_i)^3} < 0 \\ \frac{\partial}{\partial \alpha_i} \left( \frac{(\bar{R} - \underline{R})}{1 - \alpha_i} c_i \right) &= -\frac{2(1 - \phi)}{(1 - \alpha_i)^3} (\Delta(1 - \alpha_i) - (\phi - \alpha_i)) p_s Z \\ \frac{\partial^2}{\partial \alpha_i^2} \left( \frac{(\bar{R} - \underline{R})}{1 - \alpha_i} c_i \right) &= -\frac{1}{(1 - \alpha_i)^4} ((1 - \phi) + 2(\Delta - \phi) + 2(1 - \Delta)\alpha_i) p_s Z < 0 \end{aligned}$$

To prove convexity, we just need to show the inner part of  $\frac{\partial \Lambda_i}{\partial \alpha_i}$  is negative. The first and second terms are strictly negative, as shown above. However, the third term is strictly positive. If we define:

$$Y = \left( X \frac{\partial}{\partial \alpha_i^2} \left( \frac{(\bar{R} - \underline{R})}{1 - \alpha_i} c_i \right) + \left( \frac{\partial}{\partial \alpha_i} \frac{(\bar{R} - \underline{R})}{1 - \alpha_i} c_i \right)^2 \right)$$

It can be shown that:

$$\begin{aligned}
Y &\propto -(1 - \alpha_i)^2 (\bar{R} - \underline{R} - p_s \Delta)^2 ((1 - \phi) + 2(\Delta - \phi) + 2(1 - \Delta)\alpha_i) \\
&\quad + (2(1 - \alpha_i)(\phi - \alpha_i)\Delta - (\phi - \alpha_i)) p_s Z ((1 - \phi) + 2(\Delta - \phi) + 2(1 - \Delta)\alpha_i) \\
&\quad + (1 - \phi) (\Delta(1 - \alpha_i) - (\phi - \alpha_i))^2 p_s Z \\
&\leq -(1 - \alpha_i)^2 (\bar{R} - \underline{R} - p_s \Delta)^2 ((1 - \phi) + 2(\phi - \phi) + 2(1 - \phi)\alpha_i) \\
&\quad + (2(1 - \alpha_i)(\phi - \alpha_i)\Delta - (\phi - \alpha_i)) p_s Z ((1 - \phi) + 2(\phi - \phi) + 2(1 - \phi)\alpha_i) \\
&\quad + (1 - \phi) (\Delta(1 - \alpha_i) - (\phi - \alpha_i))^2 p_s Z \\
&= \hat{Y}
\end{aligned}$$

$$\begin{aligned}
\hat{Y} &\propto - \left( (1 - \alpha_i)^2 (\bar{R} - \underline{R} - p_s \Delta)^2 - (2(1 - \alpha_i)(\phi - \alpha_i)\Delta - (\phi - \alpha_i)) p_s Z \right) (1 + 2\alpha_i) \\
&\quad + (\Delta(1 - \alpha_i) - (\phi - \alpha_i))^2 p_s Z \\
&\leq - (\bar{R} - \underline{R} - p_s \Delta)^2 \left( 1 - 3\alpha_i^2 + 2\alpha_i^3 \right) + (1 - \phi + \phi^2) p_s Z \\
&\propto - (\bar{R} - \underline{R} - p_s \Delta)^2 + \frac{(1 - \phi + \phi^2)}{(1 - 3\alpha_i^2 + 2\alpha_i^3)} p_s Z
\end{aligned}$$

Thus, a sufficient condition for convexity is

$$\frac{(1 - 3\alpha_i^2 + 2\alpha_i^3)}{(1 - \phi + \phi^2)} (\bar{R} - \underline{R} - p_s \Delta)^2 \geq p_s Z$$

Which is easily satisfied for any small  $p_s$ .

For the third part of the proposition, recall that under autarky risk-shifting was desired if and only if:

$$p_l \left( p_s^0 - p_s^1 \right) (1 - \phi) \Delta \left( 1 - \frac{\Delta/2}{\bar{R} - \underline{R}} \right) + \frac{\Delta}{\bar{R} - \underline{R}} p_l \left( p_s^0 - p_s^1 \right) V < (1 - \phi) P$$

Thus, assume for  $\delta_1 > 0$ , we can write:

$$P = p_l \left( p_s^0 - p_s^1 \right) \Delta \left( 1 - \frac{\Delta/2}{\bar{R} - \underline{R}} \right) + \frac{\Delta}{\bar{R} - \underline{R}} p_l \left( p_s^0 - p_s^1 \right) \frac{V}{1 - \phi} + \delta_1$$

Denote  $\eta_j^e$  as the premium paid to bank  $j$  to compensate bank  $j$  for its default risk, and  $\eta^e = \sum \eta_j^e$ . If lenders break even,  $\eta^e = c^e = B_i + \hat{\pi}_{d,i} V$ . However, note bank  $j$  cannot observe the effort choice of bank  $i$ , and because the lending relationship is committed, the promised payment is fixed regardless of effort choice. Thus, for a fixed promised payment  $\bar{F}_{ij}$ , it must be the case that  $\eta^1 > \eta^0$  as  $g(\tilde{R}|e_i = 1)$  first-order stochastically dominates  $g(\tilde{R}|e_i = 0)$ . Define  $\eta^1 - \eta^0 = \delta_2$

With a committed lending relationship, the objective function is:

$$\Lambda_i = (1 - \alpha_i) \left( \frac{\bar{R} - \underline{R}}{2} - p_l p_s^e \Delta + p_l p_s^e \frac{1}{2} \frac{\Delta^2}{\bar{R} - \underline{R}} \right) + p_l (B_i - \eta^e) + (1 - \alpha_i) P^e + (1 - \pi_{d,i}^e) V$$

Thus, banks choose low effort iff:

$$p_l (p_s^0 - p_s^1) \frac{1}{2} \frac{\Delta^2}{\bar{R} - \underline{R}} + P - p_l \Delta (p_s^0 - p_s^1) + p_l (1 - \alpha_i)^{-1} (\eta^0 - \eta^1) - \frac{\pi_{d,i}^0 - \pi_{d,i}^1}{1 - \alpha_i} V > 0$$

For a fixed promised payment  $\bar{F}_{ij}$ , we have that:

$$\pi_{d,i}^0 - \pi_{d,i}^1 = p_l \left( p_s^0 - p_s^1 \right) \frac{1}{\bar{R} - \underline{R}} \Delta$$

Substituting back in for each of  $P$ ,  $\eta^0 - \eta^1$ , and  $\pi_{d,i}^0 - \pi_{d,i}^1$ , and simplifying, we get:

$$p_l \left( p_s^0 - p_s^1 \right) \Delta \frac{1}{\bar{R} - \underline{R}} \frac{(\phi - \alpha_i) V}{(1 - \alpha_i)(1 - \phi)} + \delta_1 + \frac{p_l}{1 - \alpha_i} \delta_2 > 0$$



Notice the *RHS* is strictly greater than zero, providing  $\delta_1 > 0$ . Thus, we have shown that if banks want to risk shift without interbank borrowing, they will always desire to do so when using committed lending relationships.

Finally, for the last statement in the proposition, recall that the expected amount of contagion when  $\theta = 0$  is:

$$C(\theta) = \sum_{k=0}^n kC_k = n\hat{\pi}_{d,(1)}$$

However,  $\hat{\pi}_{d,(1)}$  is the level of contagion when no breakages occur. Committed lending relationships have exactly this property, and thus it follows that the  $\theta = 0$  case when uninformed banks always lend (despite having uncommitted lending relationships) must be equivalent to the committed lending relationships case.

## A.9 Proof of Proposition 9

First, let's consider the contract between bank  $i$  and bank  $j$ . For ease of exposition, I will assume throughout that bank  $i$  receives the liquidity shock (and potentially the solvency shock), and bank  $j$  is asked to provide liquidity. We know bank  $j$  cannot make negative profits, because individual rationality implies it would not engage in such a contract. Moreover, any positive profits bank  $j$  might make from its contract with bank  $i$  are merely a transfer from bank  $i$ . However, a contract giving bank  $j$  positive profits does not increase or decrease bank  $j$ 's probability of default, yet does increase bank  $i$ 's probability of default. Consequently, such a contract cannot be surplus maximizing, unless it affects bank  $i$ 's incentives.

Second, note that because bank  $i$  must go first and must make all its actions simultaneously, it cannot base its decisions at  $t = 1$  off the information sets of any bank  $j$ . Moreover, because bank  $i$  cannot write a contract on whether it receives a solvency shock, we can

rule out any contract that does not offer each bank  $j$  a normal rate of return when each bank  $j$  makes its liquidity-provision decision. Thus, bank  $i$  can offer bank  $j$  either (i) an unconditional contract in which bank  $j$  commits at  $t = 0$  to provide liquidity at a fixed rate were bank  $i$  to receive a liquidity shock at  $t = 1$ , or (ii) a conditional contract in which bank  $j$  has the option to refuse to provide liquidity based on bank  $i$ 's action when it receives a liquidity shock at  $t = 1$ .

The unconditional contract is equivalent to a committed lending relationship where lenders make zero profits. We know from Proposition 8 that this contract cannot align incentives and thus cannot be optimal. The conditional contract nests several possibilities, one of which is the uncommitted lending relationship. Consider the uncommitted lending relationships. If  $\Delta > \bar{\Delta}$ , we know bank  $i$  does not have sufficient funds to promise each bank  $j$  following a solvency shock at  $t = 1$ , and thus each informed bank  $j$  will refuse to provide liquidity. The only way to prevent liquidity rationing is through an unconditional penalty.<sup>8</sup>

Consider a penalty that takes the form of a tax  $\tau$  on informed liquidity providers that is paid to the uninformed liquidity providers.<sup>9</sup> Moreover, suppose for now that every bank plays the same strategy, which ensures each bank in expectation makes zero profits from the tax, and the tax does not affect the interest rate at which bank  $i$  requests liquidity. If the tax is sufficiently large,  $\tau > \bar{\tau}$  such that when implemented, informed bank  $j$  would be willing to provide liquidity at interest rate  $r_i^P$ , then bank  $i$  will no longer be disciplined if it opts against screening. However, we know  $r_i^P$  is lower than the interest rate paid on committed

---

8. The penalty is unconditional because contracts cannot be written on information sets or the occurrence of a solvency shock.

9. Paying the tax to bank  $i$  does not work, because bank  $i$  is going to default unless at  $t = 1$ . I rule out paying the tax to the planner that redistributes it optimally at  $t = 2$  in order to minimize defaults.

lending relationships.<sup>10</sup> Thus, if bank  $i$  were to deviate and opt against screening, each bank  $j$  would be unwilling to enter into the conditional contract with  $\tau > \bar{\tau}$ .

Suppose instead that  $\tau \leq \bar{\tau}$ . It can be shown that if banks  $j = \{1, \dots, k\}$  are informed (type A) and banks  $j = \{k+1, \dots, n\}$  are uninformed (type B), each bank's probability of default is:

$$\begin{aligned}\hat{\pi}_{d,A} &= \frac{1}{\bar{R} - \underline{R}} \frac{\tau}{1 - \alpha} \\ \hat{\pi}_{d,B} &= \max \left( \frac{1}{\bar{R} - \underline{R}} \frac{\alpha - \frac{k}{n-k} \tau}{1 - \alpha}, 0 \right)\end{aligned}$$

Summing these up, we have:

$$k\hat{\pi}_{d,A} + (n-k)\hat{\pi}_{d,B} = \frac{1}{\bar{R} - \underline{R}} \min \left( \frac{(n-k)\alpha}{1 - \alpha}, \frac{k\tau}{1 - \alpha} \right)$$

Thus, conditional on the interest rate bank  $i$  requests and a given realized signal to each bank  $j$ , the tax does not reduce the expected number of contagious defaults. Moreover, because depositors are senior claims and illiquid asset returned are uniformly distributed, it follows that the tax does not affect expected depositor losses. Thus, the only way such a contract can increase total surplus is if it affects bank  $i$ 's willingness to request liquidity at a separating interest rate. We find that if the following conditions are satisfied, the planner can improve on the private allocation by implementing conditional contracts that give bank  $i$  the option to borrow liquidity at interest rates  $r_i = \{r_i^{d'}, r_i^{h'}\}$ , and tax bank  $j$  at rate  $\tau$  if it

---

10. Otherwise, banks would strictly prefer committed lending relationships, and I focus throughout on cases in which this is not true.

refuses to provide liquidity to bank  $i$ .

$$\begin{aligned}
(1 - p_s^1)U_j(0, A, r_i^{h'}, 1) + p_s^1U_j(1, A, r_i^{d'}, 1) &\geq 0 \\
U_i(1, r_i^{d'}, 1) &\geq \mathbb{E}\left(U_i(1, r_i^{h'}, a_j)\right) \\
\Lambda_i(\alpha_i, 1, B_{ij}|r_i^{h'}, r_i^{d'}, \tau) &\geq \Lambda_i(\alpha_i, 0, B_{ij}|r_i^{h'}, r_i^{d'}, \tau) \\
U_j(1, A, r_i^{d'}, 1) &\geq -\tau
\end{aligned}$$

This contract works because (i) condition (1) guarantees bank  $j$  a normal rate of return, (ii) condition (2) ensures a distressed bank  $i$  will opt to borrow at a separating interest rate, (iii) condition (3) aligns bank  $i$ 's screening incentives, and (iv) condition (4) ensures bank  $j$  is always willing to provide liquidity. If any of these conditions are violated, a separating offer is not possible when  $\Delta > \bar{\Delta}$  under any potential tax instituted by the planner.

Finally, if separating offers are not possible, we are left with a set of conditional contracts in which liquidity is requested at an interest rate independent of bank  $i$ 's state, with each bank  $j$  granted the (potentially costly) option of refusal. In these scenarios, the expected number of defaults, expected losses to depositors, and the expected amount of liquidation are identical to what takes place under an uncommitted lending relationship.

## APPENDIX B

### FUNCTIONAL FORMS

This section provides the reader with a summary of a series of functional forms that are used throughout the paper.

#### B.1 Autarky

Under autarky, three distinct regions exist:  $\alpha_i \in [0, \phi)$ ,  $\alpha_i \in [\phi, 1)$ , and  $\alpha_i = 1$ .

Begin by deriving bank  $i$ 's probability of default. First, no default occurs in the absence of liquidity shocks. Second, if bank  $i$  sets  $\alpha_i \in [0, \phi)$ , it will have insufficient liquidity to pay for a liquidity shock, which implies  $\pi_{d,i} = p_l$ . Third, if bank  $i$  sets  $\alpha_i = 1$ , it cannot default, because it holds only liquid assets that will return exactly 1 in the future, and this amount is sufficient to pay all depositors. Finally, if bank  $i$  sets  $\alpha_i \in [\phi, 1)$ , it has sufficient liquidity to pay for the liquidity shock, and should no solvency shock occur, the bank will also have sufficient assets to pay its remaining depositors, because  $\min((\alpha_i - \phi) + (1 - \alpha_i)\tilde{R}_i) = 1 - \phi$ . Thus, bank  $i$  cannot default without receiving a solvency shock. If bank  $i$  does receive a solvency shock, it defaults with probability  $Pr((\alpha_i - \phi) + (1 - \alpha_i)\tilde{R}_i < 1 - \phi | s = 1) = Pr(\tilde{R}_i < 1 | s = 1) = \frac{\Delta}{\bar{R} - \underline{R}}$ . Thus, if bank  $i$  picks  $\alpha_i \in [\phi, 1)$ , its probability of default is  $p_l p_s \frac{\Delta}{\bar{R} - \underline{R}}$ . Thus, we have:

$$\pi_{d,i} = \begin{cases} p_l & \text{if } \alpha_i \in [0, \phi) \\ p_l p_s \frac{\Delta}{\bar{R} - \underline{R}} & \text{if } \alpha_i \in [\phi, 1) \\ 0 & \text{if } \alpha_i = 1 \end{cases}$$

Next, consider bank profits. If bank  $i$  picks  $\alpha_i \in [0, \phi)$ , and receives a liquidity shock (as it defaults immediately) at  $t = 1$ , it will be forced to liquidate its assets and receives nothing. Otherwise, it survives until  $t = 2$  and receives profits of  $\alpha_i + (1 - \alpha_i)\tilde{R}_i - D$ . Thus, we have:

$$\begin{aligned}\mathbb{E}(\Pi_i | \alpha_i \in [0, \phi)) &= (1 - p_l)(1 - \alpha_i)(E(\tilde{R}_i) - 1) \\ &= (1 - p_l)(1 - \alpha_i)\frac{\bar{R} - \underline{R}}{2}\end{aligned}$$

Second, if bank  $i$  picks  $\alpha_i \in [\phi, 1)$ , we know from before that bank  $i$  has sufficient liquid assets to survive until  $t = 2$ . Given this information, we also know that if bank  $i$  doesn't receive a solvency shock, it receives profits of  $\alpha_i + (1 - \alpha_i)\tilde{R}_i - D$  at  $t = 2$ . However, if the solvency shock hits bank  $i$ , with probability  $1 - \Delta/(\bar{R} - \underline{R})$ , it receives  $\alpha_i + (1 - \alpha_i)\tilde{R}_i - D$  (with  $\tilde{R}_i > 1$ ), and otherwise it receives nothing. Thus expected profits are:

$$\begin{aligned}\mathbb{E}(\Pi_i | \alpha_i \in [\phi, 1)) &= (1 - p_l p_s)(1 - \alpha_i)(\mathbb{E}(\tilde{R}_i) - 1) \\ &\quad + p_l p_s \left(1 - \frac{\Delta}{\bar{R} - \underline{R}}\right) (\alpha_i + (1 - \alpha_i)\mathbb{E}(\tilde{R}_i | s = 1, \tilde{R}_i > 1) - D) \\ &= (1 - \alpha_i) \left(\frac{\bar{R} - \underline{R}}{2} - p_l p_s \Delta\right) + \frac{p_l p_s}{\bar{R} - \underline{R}} (1 - \alpha_i) \frac{\Delta^2}{2}\end{aligned}$$

Finally, if bank  $i$  sets  $\alpha_i = 1$ , bank  $i$  has liquid assets that will always be worth 1, and always returns  $D = 1$  to depositors. Thus, we have  $\mathbb{E}(\Pi_i | \alpha_i = 1) = 0$ .

Given profits and the probability of default, we can write the autarky objective function in piecewise fashion as:

$$\Lambda_i = \begin{cases} (1 - p_l)(1 - \alpha_i)\frac{\bar{R}-R}{2} + (1 - \alpha_i)P^e + (1 - p_l)V & \text{if } \alpha_i \in [0, \phi) \\ (1 - \alpha_i) \left\{ \frac{\bar{R}-R}{2} - p_l p_s \Delta \left( 1 - \frac{\Delta/2}{\bar{R}-R} \right) + P^e \right\} + \left( 1 - \frac{\Delta}{\bar{R}-R} p_l p_s \right) V & \text{if } \alpha_i \in [\phi, 1) \\ V & \text{if } \alpha_i = 1 \end{cases}$$

## B.2 Lending Relationships

For the duration of this section, assume bank  $i$  holds  $\alpha_i < \phi$  liquid assets and has lending relationships equal to  $B_i \geq \phi - \alpha_i$ . Second, I assume throughout that lending banks provide liquidity if and only if borrowing banks promise a normal rate of return. I assume that beliefs are correct, and for simplicity, I abstract away from the bank's choice of screening. I begin with the probability of default for a bank that receives a liquidity shock and becomes a borrower.

First, if the borrower is only able to borrow  $B_i < \phi - \alpha_i$ , the probability of default is 1 because the borrower must liquidate all assets at  $t = 1$ . However, if the borrower is able to obtain  $B_i = \phi - \alpha_i$ , it survives until  $t = 2$ , and we have the probability of default as:

$$\begin{aligned} \pi_{d,i} &= Pr(\alpha_i + (1 - \alpha_i)\tilde{R}_i + B_i < 1 + \bar{F}_i) \\ &= Pr(\tilde{R}_i < R^*) \end{aligned}$$

Where:

$$R^* = \frac{1 + \bar{F}_i - B_i}{1 - \alpha_i}$$

Consider two specific cases. If  $\bar{F}_i \leq \bar{R} - \Delta$ , we have:

$$\begin{aligned}
\pi_{d,i} &= Pr(\tilde{R}_i < R^*) \\
&= (1 - p_s)Pr\left(\tilde{R}_i < \frac{1 - \phi + \bar{F}_i}{1 - \alpha_i} \middle| s = 0\right) + p_s Pr\left(\tilde{R}_i < \frac{1 - \phi + \bar{F}_i}{1 - \alpha_i} \middle| s = 1\right) \\
&= \frac{1}{\bar{R} - \underline{R}} \left( (1 - p_s) \left( \frac{1 - \phi + \bar{F}_i}{1 - \alpha_i} - \underline{R} \right) + p_s \left( \frac{1 - \phi + \bar{F}_i}{1 - \alpha_i} - (\underline{R} - \Delta) \right) \right) \\
&= \frac{1}{\bar{R} - \underline{R}} \left( \frac{\bar{F}_i - B_i}{1 - \alpha_i} + p_s \Delta \right)
\end{aligned}$$

If  $\bar{F}_i > \bar{R} - \Delta$ , it can be shown that:

$$\pi_{d,i} = \frac{1}{\bar{R} - \underline{R}} \left( p_s + (1 - p_s) \frac{\bar{F}_i - B_i}{1 - \alpha_i} \right)$$

If  $\hat{\theta}$  is the probability that the borrower (bank  $i$ ) defaults at  $t = 1$  because insufficient lenders followed through on their promises, we can write the borrowers probability of default as:

$$\begin{aligned}
\pi_{d,i} &= \hat{\theta} + (1 - \hat{\theta})Pr(\tilde{R}_i < R^*) \\
&= \begin{cases} \hat{\theta} + \frac{(1 - \hat{\theta})}{\bar{R} - \underline{R}} \left( \frac{\bar{F}_i - B_i}{1 - \alpha_i} + p_s \Delta \right) & \text{if } \bar{R} - \Delta \geq \bar{F}_i \\ \hat{\theta} + \frac{(1 - \hat{\theta})}{\bar{R} - \underline{R}} \left( p_s + (1 - p_s) \frac{\bar{F}_i - B_i}{1 - \alpha_i} \right) & \text{if } \bar{R} - \Delta < \bar{F}_i \end{cases}
\end{aligned}$$

Next, I derive the probability of default for a lender. By construction, a lender is some bank  $j$  that did not receive a liquidity shock, whereas a borrower is some bank  $i$  that both received a liquidity shock and has insufficient funds to pay for the liquidity shock on its own. Formally, suppose bank  $i$  holds  $\alpha_i < \phi$  liquid assets and has lending relationships of size  $B_{ij}$  with other banks  $j = \{1, 2, \dots, n\}$  such that  $\sum B_{ij} = B_i \geq \phi - \alpha_i$ . All lenders have some promised payment on their lending equal to  $\bar{F}_{ij}$ . As a technical note, I assume that when a borrower is unable to repay the entire principal it borrows, the payout to each lender



is proportionate to the size of the principal it lent. Thus, define  $\xi_j = \frac{B_{ij}}{B_i}$  as the fractional repayment lender  $j$  receives when bank  $i$  is unable to repay all borrowed principal.

Each lender's probability of default is conditional on both whether the borrower has sufficient liquidity to pay for its liquidity shock, and whether the borrower received a solvency shock. Without loss of generality, assume that if no solvency shock occurs, bank  $i$  is able to borrow liquidity such that  $\alpha_i + B_i \geq \phi$ . Thus, three potential states exist for the borrower; (i) no solvency shock; (ii) solvency shock and insufficient liquidity; and (iii) solvency shock and sufficient liquidity. An uninformed lender's probability of default will be an expectation across these three potential states.

Consider the first case in which the borrower did not receive a solvency shock. In this case, the minimum repayment made by the borrower is:

$$\begin{aligned} F_i(\underline{R}) &= \max(\min((1 - \alpha_i)\underline{R} - (1 - \phi), \bar{F}_i), 0) \\ &= B_i \end{aligned}$$

Which means the lender repays each borrower a minimum amount of  $B_{i,j}$ . When lenders receive the minimum repayment, their probability of default equals:

$$\begin{aligned} Pr(Def_j | F_i = B_{ij}) &= Pr((\alpha_j - B_{ij}) + (1 - \alpha_j)\tilde{R}_j + B_{ij} < D) \\ &= 0 \end{aligned}$$

Thus, if borrowers do not receive a solvency shock, lending is riskless.

Now consider the second case in which the borrower receives a solvency shock, and is unable to borrow sufficient liquidity from its lenders ( $\alpha_i + B_i < \phi$ ). In this case, if lender  $j$  were to provide liquidity of  $B_{ij}$  to borrower  $i$ , it would take a loss of  $B_{ij}$  because bank  $i$  is unable to cover its liquidity shock and is forced into early liquidation. Thus, the probability

of default for bank  $j$  conditional on lending amount  $B_{ij}$  is:

$$\begin{aligned} Pr(Def_j | F_{ij} = 0) &= Pr((\alpha_j - B_{ij}) + (1 - \alpha_j)\tilde{R}_j < D) \\ &= \frac{1}{\bar{R} - \underline{R}} \frac{B_{ij}}{1 - \alpha_j} \end{aligned}$$

Finally, consider the third case, in which the borrower receives a solvency shock and raises sufficient liquidity from its lenders ( $\alpha_i + B_i \geq \phi$ ). In what follows, I focus on the case in which  $\alpha_i + B_i = \phi$ . A similar method can be applied to the case in which  $\alpha_i + B_i > \phi$  but because this scenario is not of great importance in this paper, I leave the proof to the reader. Now, because the lender will not default if the borrower repays the principal in full, we only need to consider the region where the borrower repays  $F_i(R_i) \leq B_i$ . Moreover, note that  $F_i(\underline{R}) = B_i$ , and thus we can write:

$$F_{ij}(\tilde{R}_i | \tilde{R}_i \leq B_i) = \max \{0, \xi_j((1 - \alpha_i)R_i - (1 - \phi))\}$$

Define  $R_i^{**}$  as:

$$\forall R > R_i^{**} : F_{ij}(R_i^{**}) > 0$$

$$\implies R_i^{**} = \frac{1 - \phi}{1 - \alpha_i}$$

Finally, define the function  $\tilde{X}_j = \alpha_j - B_{ij} + (1 - \alpha_j)R_j + F_{ij}(\tilde{R}_i)$ , and the event  $\mathcal{E}_j = \{\alpha_j - B_{ij} + (1 - \alpha_j)\tilde{R}_j < D\}$ . Now, we can write the probability of default as:

$$\begin{aligned}
Pr(Def_j | s_i = 1, \alpha_i + B_i = \phi) &= Pr(\tilde{X}_j < D) \\
&= Pr(R_i^{**} < \tilde{R}_i < \underline{R})Pr(\tilde{X}_j < D | R_i^{**} < \tilde{R}_i < \underline{R}) \\
&\quad + Pr(\tilde{R}_i < R_i^{**})Pr(\tilde{X}_j < D | \tilde{R}_i < R_i^{**}) \\
&= Pr(\mathcal{E}_j)Pr(R_i^{**} < \tilde{R}_i < \underline{R})Pr(\tilde{X}_j < D | R_i^{**} < \tilde{R}_i < \underline{R}, \mathcal{E}_j) \\
&\quad + Pr(\mathcal{E}_j)Pr(\tilde{R}_i < R_i^{**})
\end{aligned}$$

We know  $Pr(\mathcal{E}_j) = G_0\left(1 + \frac{B_{ij}}{1 - \alpha_j}\right)$ ,  $Pr(R_i^{**} < \tilde{R}_i < \underline{R}) = G_1(\underline{R}) - G_1(R_i^{**})$ , and  $Pr(\tilde{R}_i < R_i^{**}) = G_1(R_i^{**})$ , which leaves  $Pr(\tilde{X}_j < D | R_i^{**} < \tilde{R}_i < \underline{R}, \mathcal{E}_j)$ . Notice that:

$$Pr(\tilde{X}_j < D | R_i^{**} < \tilde{R}_i < \underline{R}, \mathcal{E}_j) = Pr(c_1 R_i + c_2 R_j < c_3 | R_i^{**} < \tilde{R}_i < \underline{R}, \mathcal{E}_j)$$

Where I define the constants  $c_1 = \xi_j(1 - \alpha_i)$ ,  $c_2 = 1 - \alpha_j$ , and  $c_3 = (1 - \alpha_j + B_{ij}) + \xi_j(1 - \phi)$ . One can easily verify that:

$$c_1 \underline{R} + c_2 \underline{R} = c_1 R^{**} + c_2 \left(1 + \frac{B_{ij}}{1 - \alpha_j}\right) = c_3$$

Then, using convolution, it can be shown that:

$$\frac{1}{B_{ij}} \int_{\underline{z}}^{c_3} \frac{z - \underline{z}}{B_{ij}} dz = \frac{1}{2}$$

Where  $\underline{z} = c_1 R^{**} + c_2 \underline{R}$ . Finally, we can put the pieces together:

$$\begin{aligned}
Pr(Def_j | s_i = 1, \alpha_i + B_i = \phi) &= Pr(\mathcal{E}_j) Pr(R_i^{**} < \tilde{R}_i < \underline{R}) Pr(\tilde{X}_j < D | R_i^{**} < \tilde{R}_i < \underline{R}, \mathcal{E}_j) \\
&\quad + Pr(\mathcal{E}_j) Pr(\tilde{R}_i < R_i^{**}) \\
&= G_0 \left( 1 + \frac{B_{ij}}{1 - \alpha_j} \right) \left\{ \frac{1}{2} (G_1(\underline{R}) - G_1(R_i^{**})) + G_1(R_i^{**}) \right\} \\
&= \frac{1}{(\bar{R} - \underline{R})^2} \frac{B_{ij}}{1 - \alpha_j} \left\{ \Delta - \frac{1}{2} \frac{\phi - \alpha_i}{1 - \alpha_i} \right\}
\end{aligned}$$

Note that the probability of default when  $s_i = 1$  and  $\alpha_i + \sum B_{ij} = \phi$  has two main components. The term outside the brackets is an exposure-type term for the lending bank. The larger the amount lent, or the fewer the illiquid assets held by the lender (because illiquid assets are more profitable), the higher the probability of default. The part inside the brackets is how the risk of default for the borrower affects the lender.

In summary, we have:

$$\begin{aligned}
Pr(Def_j | s_i = 0, \alpha_i + B_i \geq \phi) &= 0 \\
Pr(Def_j | s_i = 1, \alpha_i + B_i < \phi) &= \frac{1}{\bar{R} - \underline{R}} \frac{B_{ij}}{1 - \alpha_j} \\
Pr(Def_j | s_i = 1, \alpha_i + B_i = \phi) &= \frac{1}{(\bar{R} - \underline{R})^2} \frac{B_{ij}}{1 - \alpha_j} \left\{ \Delta - \frac{1}{2} \frac{\phi - \alpha_i}{1 - \alpha_i} \right\}
\end{aligned}$$

Finally, to derive the expected probability of default for an uninformed lender, one simply takes an expectation of the conditional probabilities with respect to the uninformed lender's information set.

I now derive the interest rate on interbank debt. Let  $\bar{B}_i$  be the maximum amount bank  $i$  can borrow from all of its lenders. For simplicity, assume all lending relationships are of equal size. I define  $k^*$  to denote the maximum number of lenders that can refuse to provide liquidity before the borrower will be unable to pay for its liquidity shock. It can be shown

that:

$$\begin{aligned}
\mathbb{E}(\tilde{B}_i) + \hat{\pi}_{d,i}V &= \mathbb{E}\left(\max\left(\min\left(\tilde{X}_i, \frac{n-k}{n}\bar{F}_i\right), 0\right)\right) \\
&= (1-p_s)\left(\bar{F}_i - \frac{1}{2}\frac{1}{\bar{R}-\underline{R}}\left(\frac{\bar{F}_i-B_i}{1-\alpha_i}\right)\bar{F}_i + \frac{1}{2}\frac{1}{\bar{R}-\underline{R}}\left(\frac{\bar{F}_i-B_i}{1-\alpha_i}\right)\bar{B}_i\right) \\
&\quad + p_s \sum_{K=k}^n Pr(K=k) \left(1 + \frac{1}{\bar{R}-\underline{R}}\frac{1}{(1-\alpha_i)}\min(\underline{X}(k), 0)\right) \frac{n-k}{n}\bar{F}_i \\
&\quad - p_s \sum_{K=k}^n Pr(K=k) \frac{1}{2}\frac{1}{\bar{R}-\underline{R}}\frac{1}{(1-\alpha_i)} \left(\frac{n-k}{n}\bar{F}_i - \max(0, \underline{X}(k))\right)^2
\end{aligned}$$

We can then write the LHS as

$$\mathbb{E}(\tilde{B}_i) + \hat{\pi}_{d,i}V = (1-p_s)\bar{B}_i + p_s \sum_{k=0}^n \binom{n}{k} \theta^k (1-\theta)^{n-k} \frac{n-k}{n} \bar{B}_i + \pi_d V$$

Define  $\gamma_i$  as the “premium” that needs to be repaid (i.e., the amount of compensation a borrower must pay its lenders for (i) the lender’s default risk that results from providing liquidity and (ii) the losses lenders incur from a lender’s default). Note that:

$$\begin{aligned}
\gamma_i &= \bar{F}_i - B_i \\
r_i &= \frac{\gamma_i}{B_i}
\end{aligned}$$

It can be shown that:

$$\begin{aligned}
RHS = & -\frac{1}{\bar{R}-\underline{R}} \frac{1}{(1-\alpha_i)} \frac{\gamma_i^2}{2} \left( 1 - p_s + p_s \sum_{k \leq k^*} \left( \frac{n-k}{n} \right)^2 \frac{n!}{k!(n-k)!} \theta^k (1-\theta)^{n-k} \right) \\
& + \gamma_i \left( 1 - p_s + p_s \left( 1 - \frac{\Delta}{\bar{R}-\underline{R}} \right) \sum_{k \leq k^*} \left( \frac{n-k}{n} \right) \frac{n!}{k!(n-k)!} \theta^k (1-\theta)^{n-k} \right) \\
& + \frac{p_s}{\bar{R}-\underline{R}} \sum_{k \leq k^*} \binom{n}{k} \theta^k (1-\theta)^{n-k} \frac{1}{1-\alpha_i} X(k)
\end{aligned}$$

Where:

$$X(k) = \frac{1}{2} \bar{B}_i(k)^2 - (1-\alpha_i) \Delta \bar{B}_i(k) - \frac{1}{2} \min(0, (1-\alpha_i) \Delta - B_i(k))^2$$

Next, define  $p_1$  as the probability bank  $i$  defaults on its liquidity shock at  $t = 1$ :

$$p_1 = p_s \left( 1 - \sum_{k \leq k^*} \left( \frac{n-k}{n} \right) \frac{n!}{k!(n-k)!} \theta^k (1-\theta)^{n-k} \right)$$

Similarly, let:

$$\hat{p}_1 = p_s \left( 1 - \sum_{k \leq k^*} \left( \frac{n-k}{n} \right)^2 \frac{n!}{k!(n-k)!} \theta^k (1-\theta)^{n-k} \right)$$

Also define  $p_2$  to be the probability bank  $i$  receives a solvency shock at  $t = 1$  and survives until  $t = 2$ :

$$p_2 = p_s \sum_{k \leq k^*} \left( \frac{n-k}{n} \right) \frac{n!}{k!(n-k)!} \theta^k (1-\theta)^{n-k}$$

The final term is unique, in that it is the expected loss at  $t = 2$  to lenders when the borrower is unable to repay the principal:

$$X(k) = \begin{cases} \frac{1}{2}\bar{B}_i(k)^2 - (1 - \alpha_i)\Delta\bar{B}_i(k) & \text{if } B_i(k) - (1 - \alpha)\Delta \leq 0 \\ -\frac{1}{2}(1 - \alpha_i)^2\Delta^2 & \text{if } B_i(k) - (1 - \alpha)\Delta > 0 \end{cases}$$

Thus, we have:

$$\begin{aligned} c_i^{(2)} &= \frac{p_s}{\bar{R} - \underline{R}} \frac{1}{1 - \alpha_i} \sum_{k \leq k^*} \binom{n}{k} \theta^k (1 - \theta)^{n-k} X(k) \\ &= \frac{p_s}{\bar{R} - \underline{R}} \frac{1}{1 - \alpha_i} \sum_{k \leq k^*} \binom{n}{k} \theta^k (1 - \theta)^{n-k} \frac{(1 - \alpha_i)^2}{2} \Delta^2 \\ &\quad - \frac{p_s}{\bar{R} - \underline{R}} \frac{1}{1 - \alpha_i} \sum_{k \leq k^*} \binom{n}{k} \theta^k (1 - \theta)^{n-k} \max((1 - \alpha_i)\Delta - \bar{B}_i(k), 0)^2 \end{aligned}$$

Where I define this term as the expected losses to bank  $i$ 's lenders (a cost), conditional on bank  $i$ 's survival to  $t = 2$ , or  $c_i^{(2)}$ . We can also define the expected losses at  $t = 1$  in a similar way as:

$$c_i^{(1)} = \sum_{k > k^*}^n \binom{n}{k} \theta^k (1 - \theta)^{n-k} \frac{n-k}{n} \bar{B}_i$$

I will rewrite the equation and place  $c_i^{(2)}$  on the LHS. Thus, we have that the RHS is:

$$RHS = -\frac{1}{\bar{R} - \underline{R}} \frac{1}{(1 - \alpha_i)} \frac{\gamma_i^2}{2} (1 - \hat{p}_1) + \gamma_i \left( 1 - p_1 - p_2 \frac{\Delta}{\bar{R} - \underline{R}} \right)$$

Define  $c_i$  to be the expected cost of lending to bank  $i$  (for all banks in aggregate), which is equal to each bank's expected default costs plus its expected losses from lending to a risky

borrower aggregated across all lenders (conditional on the lender's providing liquidity). It follows that:

$$\begin{aligned}
LHS &= c_i \\
&= \frac{1}{2} \frac{p_s}{\bar{R} - \underline{R}} \frac{1}{(1 - \alpha_i)} \sum_{k \leq k^*} \binom{n}{k} \theta^k (1 - \theta)^{n-k} (1 - \alpha_i)^2 \Delta^2 \\
&\quad - \frac{1}{2} \frac{p_s}{\bar{R} - \underline{R}} \frac{1}{(1 - \alpha_i)} \sum_{k \leq k^*} \binom{n}{k} \theta^k (1 - \theta)^{n-k} \max((1 - \alpha_i)\Delta - \bar{B}_i(k), 0)^2 \\
&\quad + p_s \sum_{k > k^*}^n \binom{n}{k} \theta^k (1 - \theta)^{n-k} \frac{n-k}{n} \bar{B}_i + \hat{\pi}_d V \\
&= c_i^{(1)} + c_i^{(2)} + \hat{\pi}_{d,i} V
\end{aligned}$$

Combining LHS and RHS, we have the following quadratic:

$$0 = \frac{1}{\bar{R} - \underline{R}} \frac{1}{(1 - \alpha_i)} \frac{\gamma_i^2}{2} (1 - \hat{p}_1) - \gamma_i \left( 1 - p_1 - p_2 \frac{\Delta}{\bar{R} - \underline{R}} \right) + c_i$$

Which has solution:

$$\gamma_i = \frac{\left( 1 - p_1 - p_2 \frac{\Delta}{\bar{R} - \underline{R}} \right) - \sqrt{\left( 1 - p_1 - p_2 \frac{\Delta}{\bar{R} - \underline{R}} \right)^2 - 2(1 - \hat{p}_1)(\bar{R} - \underline{R})^{-1}(1 - \alpha_i)^{-1}c_i}}{(1 - \hat{p}_1)(\bar{R} - \underline{R})^{-1}(1 - \alpha_i)^{-1}}$$

Thus, we have derived the interest rate premium promised on lending. Finally, to map to promised payments and interest rates, we simply make the following transformation:

$$\begin{aligned}
r_i &= \frac{\gamma_i}{B_i} \\
\bar{F}_i &= \gamma_i + B_i
\end{aligned}$$



Now, consider profits, which can be written state contingently as:

$$\Pi_i = \alpha_i + (1 - \alpha_i)\tilde{R}_i - D_i + B_i - F_i$$

It can be shown that expected profits are:

$$\begin{aligned} \mathbb{E}(\Pi_i) = & (1 - \alpha_i) \left( \frac{\bar{R} - \underline{R}}{2} - p_l p_s \Delta + p_l p_s \frac{1}{2} \frac{\Delta^2}{\bar{R} - \underline{R}} \right) \\ & - p_l p_1 (1 - \alpha_i) \left( \frac{\bar{R} - \underline{R}}{2} - \Delta + \frac{1}{2} \frac{\Delta^2}{\bar{R} - \underline{R}} \right) - p_l c_i \end{aligned}$$

Where  $c_i$  is as before. Here, the first term is essentially baseline profits. That is, it represents what a bank would make if it could invest  $\alpha_i$  in the risk-less asset and borrow liquidity at an interest rate of  $r_i = 0$ . The second term represents the losses to the bank – from a profit standpoint – that occur when liquidity shortages take place. Finally, the third and final term is the cost of borrowing, which is the expected premium that must be paid to lenders on the interbank market.

### B.3 Contagion

Derive the functions  $C_k(\theta)$  and  $C(\theta)$ . Recall:

$$\begin{aligned} C_k(\theta) = & \sum_{m=1}^{n-k} \binom{n}{m} \theta^m (1 - \theta)^{n-m} \binom{n-m}{k} \pi_{d,(0)}^k (1 - \pi_{d,(0)})^{n-m-k} \\ & + (1 - \theta)^n \binom{n}{k} \pi_{d,(1)}^k (1 - \pi_{d,(1)})^{n-k} \end{aligned}$$

Here, I derive the case  $k = 0$  before specifying the general form for  $C_k(\theta)$ . I leave it to the reader to verify that all other cases for  $k$  are nested in the general form. Let  $\kappa = (\theta + (1 - \theta)(1 - \pi_{d,(0)}))^n$ :

$$\begin{aligned}
C_0(\theta) &= \sum_{m=1}^n \binom{n}{m} \theta^m (1 - \theta)^{n-m} (1 - \pi_{d,(0)})^{n-m} + (1 - \theta)^n \left( (1 - \pi_{d,(1)})^n \right) \\
&= \sum_{m=1}^n \binom{n}{m} \theta^m (1 - \theta)^{n-m} (1 - \pi_{d,(0)})^{n-m} + (1 - \theta)^n \left( (1 - \pi_{d,(1)})^n \right) \\
&\quad - (1 - \theta)^n (1 - \pi_{d,(0)})^n + (1 - \theta)^n (1 - \pi_{d,(0)})^n \\
&= \sum_{m=0}^n \binom{n}{m} \theta^m (1 - \theta)^{n-m} (1 - \pi_{d,(0)})^{n-m} \\
&\quad + (1 - \theta)^n \left( (1 - \pi_{d,(1)})^n - (1 - \pi_{d,(0)})^n \right) \\
&= \sum_{m=0}^n \binom{n}{m} \theta^m \left( (1 - \theta)(1 - \pi_{d,(0)}) \right)^{n-m} \\
&\quad + (1 - \theta)^n \left( (1 - \pi_{d,(1)})^n - (1 - \pi_{d,(0)})^n \right) \\
&= \kappa^n \sum_{m=0}^n \binom{n}{m} \frac{1}{\kappa^n} \theta^m \left( (1 - \theta)(1 - \pi_{d,(0)}) \right)^{n-m} \\
&\quad + (1 - \theta)^n \left( (1 - \pi_{d,(1)})^n - (1 - \pi_{d,(0)})^n \right) \\
&= \kappa^n \sum_{m=0}^n \binom{n}{m} \left( \frac{\theta}{\kappa} \right)^m \left( \frac{(1 - \theta)(1 - \pi_{d,(0)})}{\kappa} \right)^{n-m} \\
&\quad + (1 - \theta)^n \left( (1 - \pi_{d,(1)})^n - (1 - \pi_{d,(0)})^n \right) \\
&= \kappa^n + (1 - \theta)^n \left( (1 - \pi_{d,(1)})^n - (1 - \pi_{d,(0)})^n \right) \\
&= \left( \theta + (1 - \theta)(1 - \pi_{d,(0)}) \right)^n + (1 - \theta)^n \left( (1 - \pi_{d,(1)})^n - (1 - \pi_{d,(0)})^n \right)
\end{aligned}$$

The general form is:

$$C_k(\theta) = \binom{n}{k} \left( \pi_{d,(0)}(1-\theta) \right)^k \left( \theta + (1-\theta)(1-\pi_{d,(0)}) \right)^{n-k} \\ + (1-\theta)^n \binom{n}{k} \left( \pi_{d,(1)}^k (1-\pi_{d,(1)})^{n-k} - \pi_{d,(0)}^k (1-\pi_{d,(0)})^{n-k} \right)$$

Note that:

$$C_0(\theta) = \binom{n}{0} \left( \pi_{d,(0)}(1-\theta) \right)^0 \left( \theta + (1-\theta)(1-\pi_{d,(0)}) \right)^n \\ + (1-\theta)^n \binom{n}{0} \left( \pi_{d,(1)}^0 (1-\pi_{d,(1)})^n - \pi_{d,(0)}^0 (1-\pi_{d,(0)})^n \right) \\ = \left( \theta + (1-\theta)(1-\pi_{d,(0)}) \right)^n + (1-\theta)^n \left( (1-\pi_{d,(1)})^n - (1-\pi_{d,(0)})^n \right)$$

As above. Moreover:

$$1 - \theta - (1-\theta)(1-\pi_{d,0}) = (1-\theta)\pi_{d,(0)}$$

Which means we can rewrite the solution as a mixture of three binomial distributions:

$$C_k = \binom{n}{k} \left( \pi_{d,(0)}(1-\theta) \right)^k \left( \theta + (1-\theta)(1-\pi_{d,(0)}) \right)^{n-k} \\ + (1-\theta)^n \binom{n}{k} \left( \pi_{d,(1)}^k (1-\pi_{d,(1)})^{n-k} - \pi_{d,(0)}^k (1-\pi_{d,(0)})^{n-k} \right) \\ = \text{Bin} \left( n, \pi_{d,(0)}(1-\theta) \right) + (1-\theta)^n \text{Bin} \left( n, \pi_{d,(1)} \right) - (1-\theta)^n \text{Bin} \left( n, \pi_{d,(0)} \right)$$

Given the general form of  $C_k(\theta)$  we can now derive  $C(\theta)$ . Recall:

$$C(\theta) = \sum_{k=0}^n kC_k(\theta)$$

And thus:

$$C = \sum_{k=0}^n kC_k = n \left( \pi_{d,(0)}(1 - \theta) + (1 - \theta)^n(\pi_{d,(1)} - \pi_{d,(0)}) \right)$$

## REFERENCES

- [1] Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2015a. Systemic risk in endogenous financial networks. Working paper.
- [2] Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2015b. Networks, Shocks, and Systemic Risk. National Bureau of Economic Research.
- [3] Acharya, Viral, Heitor Almeida, Filippo Ippolito, and Ander Perez. 2014. Credit lines as monitored liquidity insurance: Theory and evidence. *Journal of Financial Economics* 112: 287-319.
- [4] Acharya, Viral, Douglas Gale, and Tanju Yorulmazer. 2011. Rollover Risk and Market Freezes. *Journal of Finance* 66 (4): 1177-1209.
- [5] Acharya, Viral, and Tanju Yorulmazer. 2008. Information Contagion and Bank Herding. *Journal of Money, Credit and Banking* 40 (1): 215-231.
- [6] Afonso, Gara, Anna Kovner, and Antoinette Schoar. 2011. Stressed not Frozen. *Journal of Finance* 66 (4): 1109-1139.
- [7] Allen, Franklin, Ana Babus, and Elena Carletti. 2012. Asset Commonality, Debt Maturity and Systemic Risk. *Journal of Financial Economics* 104: 519-534.
- [8] Allen, Franklin, and Douglas Gale. 2000. Financial contagion. *Journal of Political Economy* 108 (1): 1-33.
- [9] Babus, Ana. 2016. The formation of financial networks. *RAND Journal of Economics* Forthcoming.
- [10] Blume, Lawrence, David Easley, Jon Kleinberg, and Eva Tardos. 2009. Trading networks with price-setting agents. *Games and Economic Behavior* 67 (1): 36-50.
- [11] Bolton, Patrick, and David Scharfstein. 1996. Optimal Debt Structure and the Number of Creditors. *Journal of Political Economy* 104 (1): 1-25.
- [12] Caballero, Ricardo, and Alp Simsek. 2013. Fire sales in a model of complexity. *Journal of Finance* 68 (6): 2549-2587.
- [13] Calomiris, Charles, and Charles Kahn. 1991. The Role of Demandable Debt in Structuring Optimal Banking Arrangements. *American Economic Review* 81 (3): 497-513.
- [14] Cho, In-Koo, and David M. Kreps. 1987. Signalling games and stable equilibria. *Quarterly Journal of Economics* 102 (2): 179-221.

- [15] Cifuentes, Rodrigo, Gianluigi Ferrucci, and Hyun Song Shin. 2005. Liquidity Risk and Contagion. *Journal of the European Economic Association* 3 (2): 556-566.
- [16] Diamond, Douglas. 1984. Financial intermediation and delegated screening. *Review of Economic Studies* 51 (2): 393-414.
- [17] Diamond, Douglas. 1991. Monitoring and Reputation: The choice between Bank Loans and Directly Placed Debt. *Journal of Political Economy* 99 (4): 689-721.
- [18] Diamond, Douglas, and Philip Dybvig. 1983. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 91 (3): 401-419.
- [19] Diamond, Douglas, and Raghuram Rajan. 2005. Liquidity Shortages and Banking Crises. *Journal of Finance* 60 (2): 615-647.
- [20] Elliott, Matthew, Benjamin Golub, and Matthew Jackson. 2014. Financial networks and contagion. *American Economic Review* 104 (10): 3115-3153.
- [21] Farboodi, Maryam. 2014. Intermediation and Voluntary Exposure to Counterparty Risk. Working paper.
- [22] Flannery, Mark. 1994. Debt Maturity Structure and the Deadweight Cost of Leverage: Optimally Financing Banking Firms. *American Economic Review* 84 (1): 320-331.
- [23] Freixas, Xavier, Bruno Parigi, and Jean Charles Rochet. 2000. Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank. *Journal of Money, Credit and Banking* 3 (3), Part II: 611-638.
- [24] Garleanu, Nicolae, Stavros Panageas, and Jiafeng Yu. 2015. Financial Entanglement: A theory of Incomplete Integration, Leverage, Crashes, and Contagion. *American Economic Review* 105 (7): 1979-2010.
- [25] Gorton, Gary, and Lixin Huang. 2003. Banking Panics and the Origin of Central Banking. *Evolution and Procedures in Central Banking*.
- [26] Gorton, Gary, and Andrew Metrick. 2012. Securitized banking and the run on the repo. *Journal of Financial Economics* 104: 425-451.
- [27] Hart, Oliver, and John Moore. 1994. A Theory of Debt Based on the Inalienability of Human Capital. *Quarterly Journal of Economics* 109 (4): 841-879.
- [28] Heider, Florian, Marie Hoerova, and Cornelia Holthausen. 2009. Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk. *ECB Working Paper Series* 1126.

- [29] Holmstrom, Bengt, and Jean Tirole. 1998. Public and Private Supply of Liquidity. *Journal of Political Economy* 106 (1): 1-40.
- [30] Kiyotaki, Nobuhiro, and John Moore. 1997. Credit Cycles. *Journal of Political Economy*, 105 (2), 1997. *Quarterly Journal of Economics* 112 (3): 663-691.
- [31] Lowenstein, Roger. 2000. *When Genius Fails: The Rise and Fall of Long-Term Capital Management*. Random House.
- [32] Rajan, Raghuram, and Andrew Winton. 1995. Covenants and Collateral as Incentives to Monitor. *Journal of Finance* 50 (4): 1113-1146.
- [33] Rochet, Jean Charles, and Jean Tirole. 1996. Interbank Lending and Systemic Risk. *Journal of Money, Credit and Banking* 28 (4): 733-762.
- [34] Sufi, Amir. 2009. Bank Lines of Credit in Corporate Finance: An Empirical Analysis. *Review of Financial Studies* 22 (3): 1057-1088.
- [35] Wagner, Wolf. 2010. Diversification at financial institutions and systemic crises. *Journal of Financial Intermediation* 19: 373-386.
- [36] Zawadowski, Adam. 2013. Entangled financial systems. *Review of Financial Studies* 26 (5): 1291-1323.